Quark and lepton mass matrices in modular symmetric vacuum

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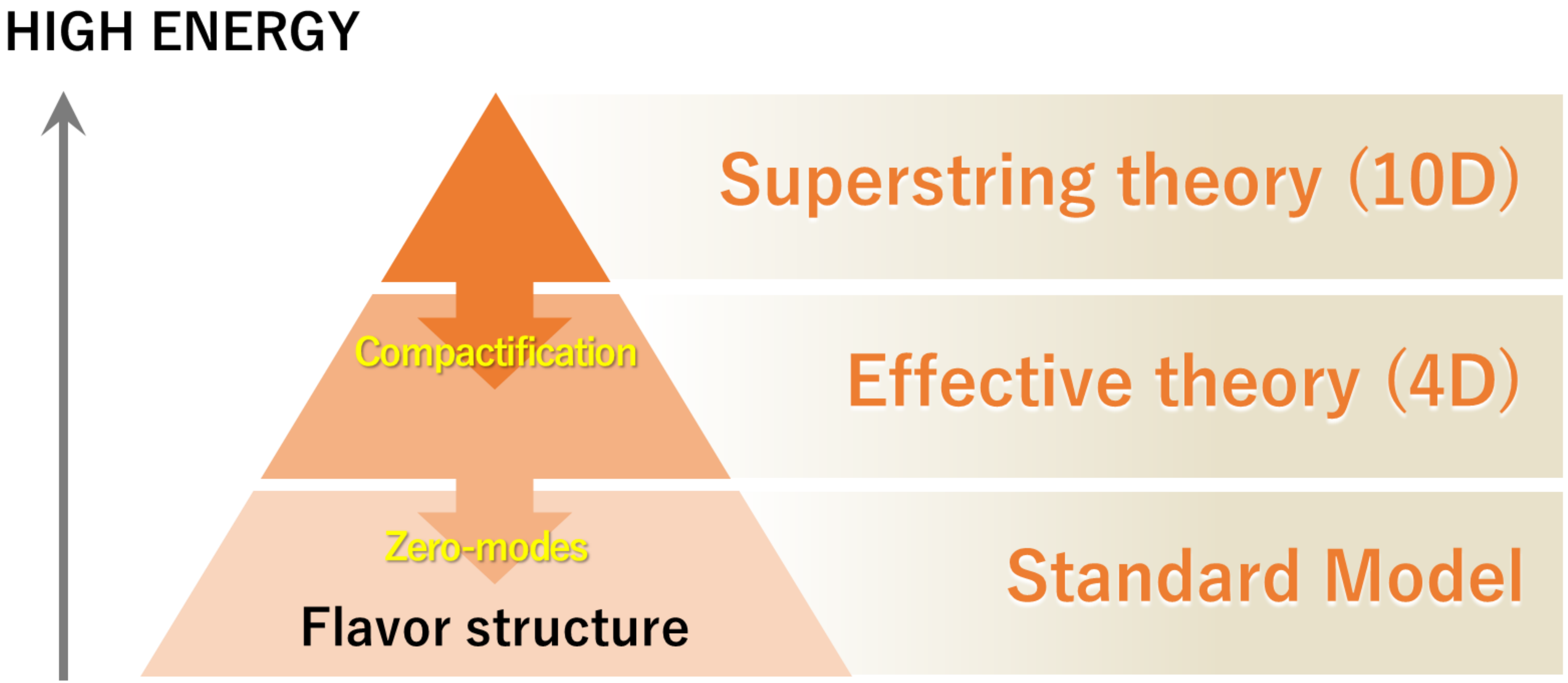
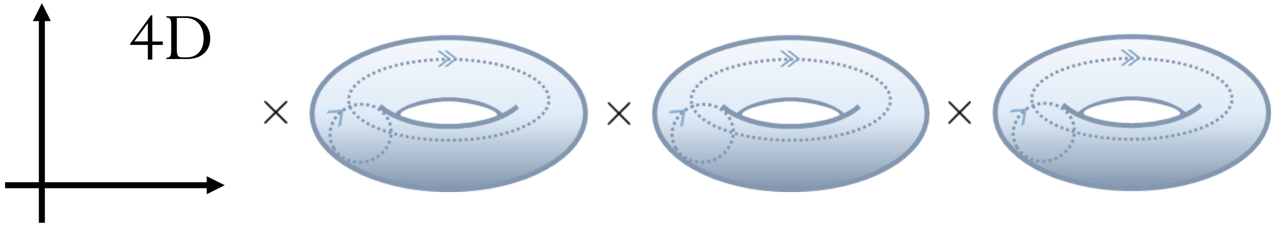
1 . Introduction

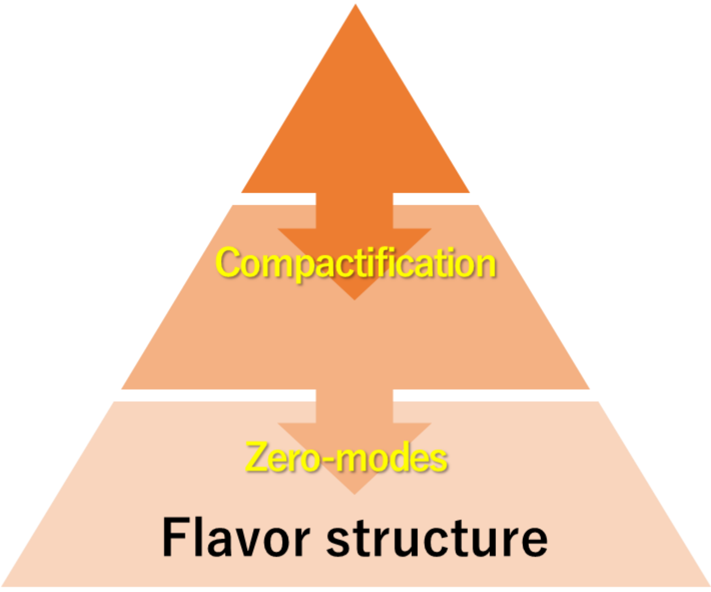


Realize **flavor structure** from superstring theory.

1. Torus compactification

10D =





2. Magnetic flux

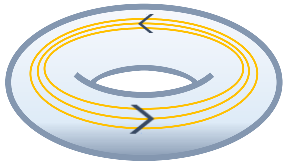
Free fermion:

$$\Gamma^{\alpha}\partial_{\alpha}\psi(z,\bar{z}) = 0$$

+ background magnetic flux A_{α} :

$$\Gamma^{\alpha}(\partial_{\alpha} - iA_{\alpha})\psi(z,\bar{z}) = 0$$

Magnetic flux A_{α} on torus



1 . Introduction

Flux compactification model (torus + flux)

▶ **Solutions to EOM:** $\Gamma^{\alpha}(\partial_{\alpha} - iA_{\alpha})\psi(z,\bar{z}) = 0$ $\left(\int_{T^2} dA = M \ (\in \text{integer})\right)$

$$\psi = \begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}, \quad \psi_+ = \sum_{j=0}^{|M|-1} \psi^{j,|M|}(z,\tau), \quad \psi_- = \psi_+(\bar{z},\bar{\tau})$$

$$\psi^{j,|M|}(z,\tau) = \mathcal{N}_j e^{i\pi|M|z\frac{\text{Im}\, z}{\text{Im}\, \tau}} \theta \begin{bmatrix} \frac{j}{|M|} \\ 0 \end{bmatrix} (|M|z, |M|\tau)$$

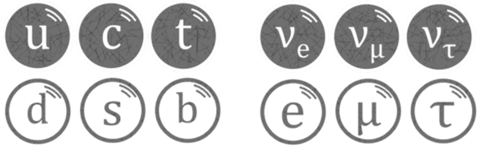
Theta function

Property of theta function

	$M>0$	$M<0$
ψ_+	$\lvert M \rvert$ gen.	✕
ψ_-	✕	$\lvert M \rvert$ gen.

Chiral fermion + $|M|$ generation

Gen. = Flux $|M|$

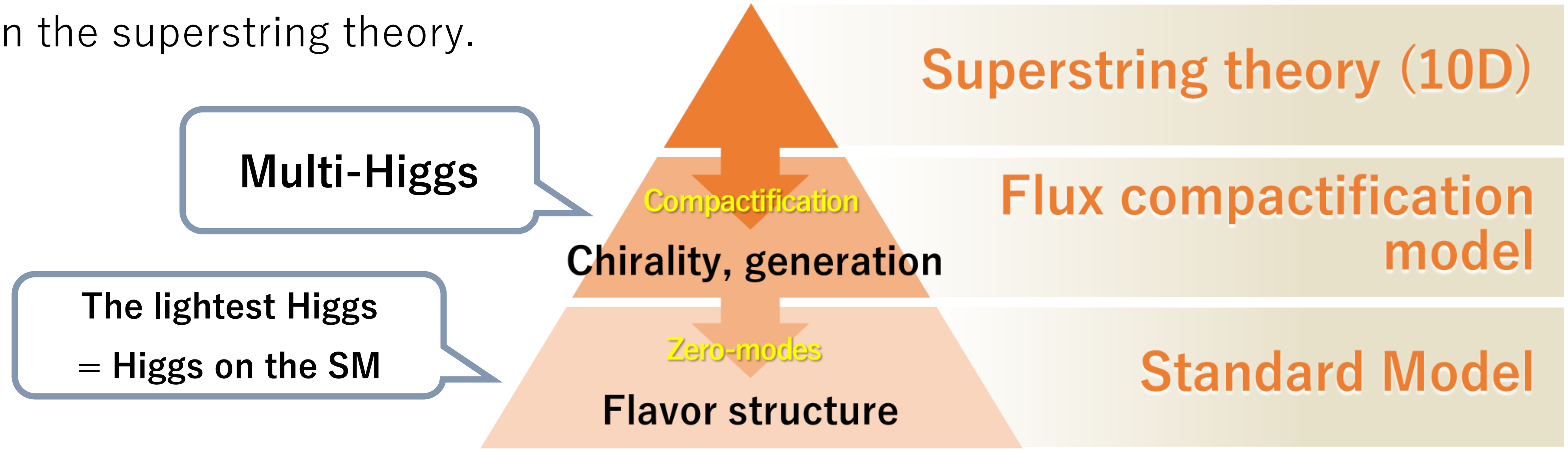


1 . Introduction

Flux compactification model (torus + flux)

Similarly, multi-Higgs appears*.

*In general, multi-Higgs appears in the superstring theory.



1 . Introduction

Flux compactification model

$$\psi^{j,|M|}(z,\tau) = \mathcal{N}_j e^{i\pi|M|z\frac{\mathrm{Im}\, z}{\mathrm{Im}\,\tau}} \theta \begin{bmatrix} \frac{j}{|M|} \\ 0 \end{bmatrix} (|M|z, |M|\tau)$$

The directions of the lightest Higgs

Yukawa coupling

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \int d^2z\ \psi_L^{i,M_L}\psi_R^{j,M_R}\left(\psi_H^{k,M_H}\right)^* \cdot \langle H_k\rangle$$

Higgs VEVs

Flavor structures

Flavor structures strongly depend on Higgs VEVs.

1. Introduction

How find Higgs VEVs (the lightest Higgs direction)?

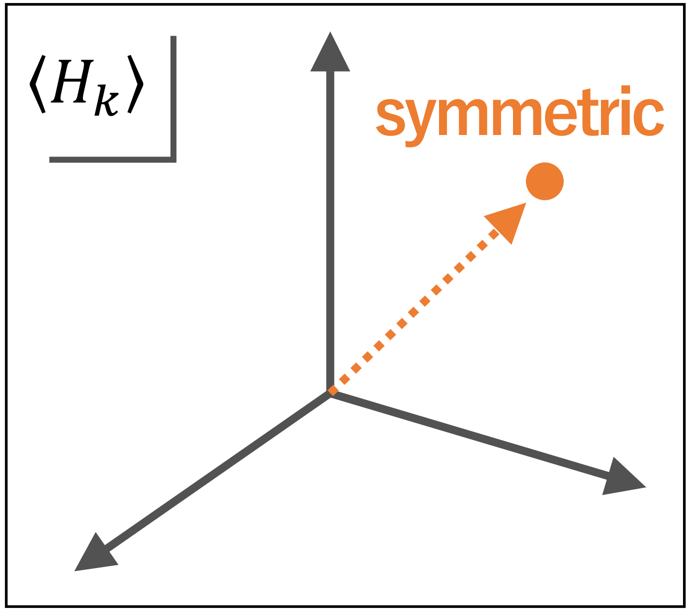
1. Higgs mass term (μ-term)

   → Find the lightest Higgs

2. Symmetry

2 Symmetry

If vacuum has a symmetry, VEVs are aligned in a symmetric direction.

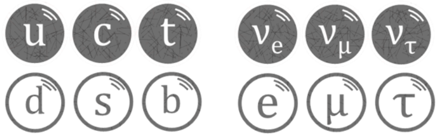


Mass term

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \int d^2z\ \psi_L^{i,M_L}\psi_R^{j,M_R}\left(\psi_H^{k,M_H}\right)^* \cdot \langle H_k\rangle$$

Can lead realistic flavor model?

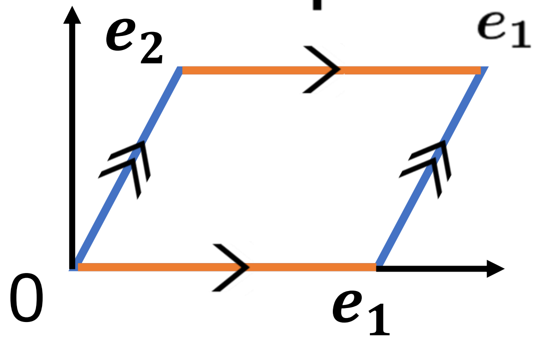
Mass ratios, CKM matrix,⋯

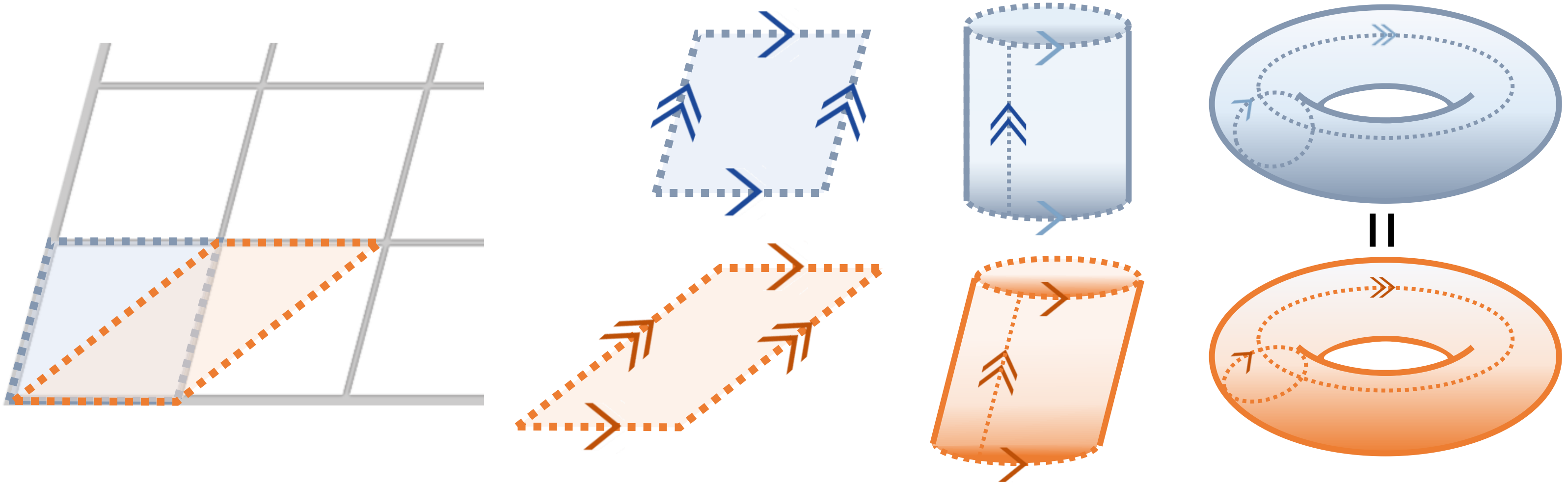


2 . Modular symmetry

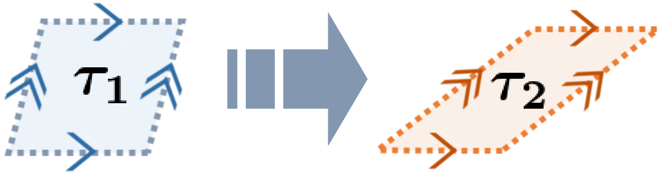
Same size, different shape on lattice → **same torus**

Shape $= \dfrac{e_2}{e_1} \equiv \tau$





Modular transformation



$$\tau \to \frac{a\tau + b}{c\tau + d},\quad z \to \frac{z}{c\tau + d},\quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

generated by

- S trans. $\tau \to \tau' = -\dfrac{1}{\tau}$
- T trans. $\tau \to \tau' = \tau + 1$

2. Modular symmetry

Modular transformation for wavefunctions

$$S: \psi^{j,M}(z,\tau) \to \psi^{j,M}\left(-\frac{z}{\tau}, -\frac{1}{\tau}\right) = (-\tau)^{\frac{1}{2}} e^{\frac{i\pi}{4}} \frac{1}{\sqrt{M}} e^{2\pi i \frac{jk}{M}} \psi^{k,M}(z,\tau) \equiv \rho(S)^{jk} \psi^{k,M}(z,\tau)$$

$$T: \psi^{j,M}(z,\tau) \to \psi^{j,M}(z,\tau+1) = e^{i\pi \frac{j^2}{M}} \psi^{j,M}(z,\tau) \equiv \rho(T)^{jk} \psi^{k,M}(z,\tau)$$

Unitary

Yukawa coupling

$$Y^{ijk} \to \rho_L^{ii'} \rho_R^{jj'} \left(\rho_H^{kk'}\right)^* Y^{i'j'k'}$$

3 . Texture structures

Modular symmetry restrict the forms of Yukawa matrices.

1. S-symmetry at $\tau = i$
2. ST-symmetry at $\tau = e^{\pm 2\pi i/3}$
3. T-symmetry at $\mathrm{Im}\,\tau = \infty$

3 . Texture structures

1 S-symmetry at $\tau = i$

At $\tau = i$, Yukawa matrices are invariant as $S: \tau = -\frac{1}{\tau}$.

$$Y^{ijk} = \rho_L^{ii'}(S)\rho_R^{jj'}(S)\left(\rho_H^{kk'}(S)\right)^* Y^{i'j'k'} \quad at \;\; \tau = i$$

where $\rho(S)$ is given by

Unitary

$$S: \psi^{j,M}(z,\tau) \to \psi^{j,M}\left(-\frac{z}{\tau}, -\frac{1}{\tau}\right) = (-\tau)^{\frac{1}{2}} e^{\frac{i\pi}{4}} \frac{1}{\sqrt{M}} e^{2\pi i \frac{jk}{M}} \psi^{k,M}(z,\tau) \equiv \rho(S)^{jk} \psi^{k,M}(z,\tau)$$

3 . Texture structures

1 S-symmetry at $\tau = i$

On $T^2/\mathbb{Z}_2$ orbifold, Yukawa matrices restricted to two types:

$$\begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}, \quad \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix},$$

and these matrices correspond to different S-eigenstates. Then,

$$M^{ij} = Y^{ijk}\langle H_k \rangle = \sum_m \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_m^{ij} \langle H_m \rangle + \sum_n \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}_n^{ij} \langle H_n \rangle$$

3. Texture structures

② ST-symmetry at $\tau = e^{\pm 2\pi i/3}$

On $T^2/\mathbb{Z}_2$ orbifold, Yukawa matrices restricted to three types:

$$\begin{pmatrix} * & & \\ & & * \\ & * & \end{pmatrix}, \quad \begin{pmatrix} & & * \\ & * & \\ * & & \end{pmatrix}, \quad \begin{pmatrix} & * & \\ * & & \\ & & * \end{pmatrix},$$

and these matrices correspond to different ST-eigenstates. Then,

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \sum_{\ell} \begin{pmatrix} * & & \\ & & * \\ & * & \end{pmatrix}_{\ell}^{ij} \langle H_n\rangle + \sum_{m} \begin{pmatrix} & & * \\ & * & \\ * & & \end{pmatrix}_{m}^{ij} \langle H_m\rangle + \sum_{n} \begin{pmatrix} & * & \\ * & & \\ & & * \end{pmatrix}_{n}^{ij} \langle H_n\rangle$$

3 . Texture structures

3 T-symmetry at $\mathrm{Im}\,\tau = \infty$

Non-realistic

On $T^2/\mathbb{Z}_2$ orbifold, almost elements of Yukawa matrices become zero.

For example, $\begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}, \quad \begin{pmatrix} & & \\ & * & \\ & & \end{pmatrix}, \quad \begin{pmatrix} & & \\ & & \\ & & * \end{pmatrix},$

and these matrices correspond to different T-eigenstates. Then,

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \sum_{\ell} \begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}_{\ell}^{ij} \langle H_n\rangle + \sum_{m} \begin{pmatrix} & & \\ & * & \\ & & \end{pmatrix}_{m}^{ij} \langle H_m\rangle + \sum_{n} \begin{pmatrix} & & \\ & & \\ & & * \end{pmatrix}_{n}^{ij} \langle H_n\rangle$$

4 . Rank one mass matrix

We have seen Yukawa matrices are restricted by modular symmetry.
Hereafter, we focus on textures by *S*-symmetry.

***S*-symmetry**

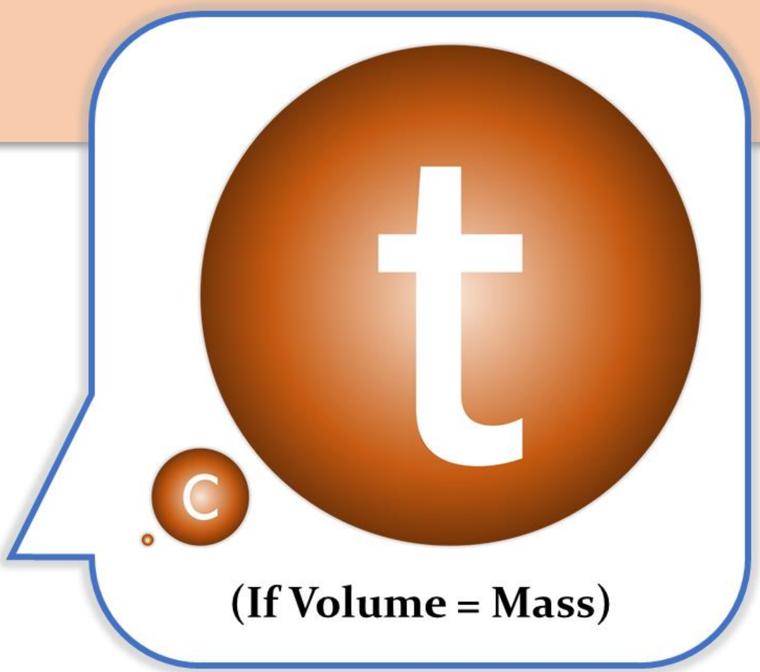
$$M^{ij} = Y^{ijk}\langle H_k\rangle = \sum_m \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_m^{ij} \langle H_m\rangle + \sum_n \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}_n^{ij} \langle H_n\rangle \qquad (※\, \tau = i)$$

From this mass matrix, we will see what vacuum $\langle H_k\rangle$ is favored for realistic quark mass matrix.

4 . Rank one mass matrix

Quark has large hierarchy.

Quarks mass ratios	Experimental Values
$(m_u, m_c, m_t)/m_t$	$(0.0000126, 0.00738, 1)$
$(m_d, m_s, m_b)/m_b$	$(0.00112, 0.0222, 1)$



Quark mass matrix is approximately rank one matrix.

$$M^{ij} = Y^{ijk}\langle H_k\rangle \propto U_L \begin{pmatrix} 0.0000126 & & \\ & 0.00738 & \\ & & 1 \end{pmatrix} U_R^\dagger \sim \underbrace{U_L \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} U_R^\dagger}_{\text{Rank one}}$$

Rank one

4 . Rank one mass matrix

Can rank one mass matrix be realized by textures?

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \sum_m \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_m^{ij} \langle H_m\rangle + \sum_n \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}_n^{ij} \langle H_n\rangle = \text{Rank one} \qquad (※\,\tau = i)$$

Rank one is realized if mass matrix includes,

1. Three or more of $\begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}$. Higgs VEVs leading to rank one exist in S-eigenstates.

2. Besides 1, includes one or more of $\begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}$. Rank one exist in not S-eigenstates, too.

3. Three or more of non-symmetric $\begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}$. Rank one exist in S-eigenstates.

4 . Rank one mass matrix

Three-generation fermion models on $T^2/\mathbb{Z}_2$ orbifold	The directions of Higgs VEVs leading to rank one
5 pair Higgs (M_H=8, even)	**S-invariant**, not S-eigenstate
5 pair Higgs (M_H=9, even)	**S-invariant**, not S-eigenstate
6 pair Higgs (M_H=10, even)	**S-invariant**, not S-eigenstate
5 pair Higgs (M_H=11, odd)	i eigenstate, not S-eigenstate
5 pair Higgs (M_H=12, odd)	i eigenstate, not S-eigenstate
6 pair Higgs (M_H=13, odd)	$\pm i$ eigenstate, not S-eigenstate
8 pair Higgs (M_H=14, even)	-1 eigenstate, not S-eigenstate
8 pair Higgs (M_H=15, even)	**S-invariant**, -1 eigenstate, not S-eigenstate
9 pair Higgs (M_H=16, even)	-1 eigenstate, not $S-$eigenstate

Consistent with S-invariant vacuum

5 . Numerical example: 5 pair Higgs (M_H=8, even)

5 pair = 3 (S-even)
+ 2 (S-odd)

Three-generation fermion models on $T^2/\mathbb{Z}_2$ orbifold	The directions of Higgs VEVs leading to rank one
5 pair Higgs (M_H=8, even)	***S*-invariant**, not S-eigenstate

If vacuum is *S*-invariant, quark mass matrix can be rank one.

$$M^{ij} = Y^{ijk}\langle H_k\rangle = \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}^{ij}\langle H_0\rangle + \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}^{ij}\langle H_1\rangle + \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}^{ij}\langle H_2\rangle$$

S-even (invariant)

$$+\begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}^{ij}\langle H_3\rangle + \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}^{ij}\langle H_4\rangle$$

S-odd

5. Numerical example: 5 pair Higgs (M_H=8, even)

5 pair = 3 (S-even)
+ 2 (S-odd)

Three-generation fermion models on $T^2/\mathbb{Z}_2$ orbifold	The directions of Higgs VEVs leading to rank one
5 pair Higgs (M_H=8, even)	***S*-invariant**, not S-eigenstate

If vacuum is *S*-invariant, quark mass matrix can be rank one.

$\langle H_k \rangle \to U^{kk'} \langle H_{k'} \rangle = \langle H'_k \rangle$

Rank one

$$M^{ij} = Y^{ijk}\langle H_k \rangle = \begin{pmatrix} * & * & \\ * & * & \\ & & 0 \end{pmatrix}^{ij} \langle H'_0 \rangle + \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}^{ij} \langle H'_1 \rangle + \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}^{ij} \langle H'_2 \rangle$$

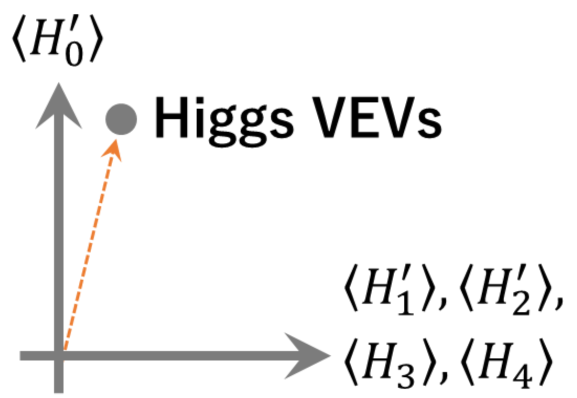
***S*-even (invariant)**

$$+ \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}^{ij} \langle H_3 \rangle + \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}^{ij} \langle H_4 \rangle$$

***S*-odd**

5 . Numerical example: 5 gen. Higgs (M_H=8, even)

<Best fit>



Up Higgs

$\langle H_0' \rangle = 0.99995$

$|\langle H_1' \rangle, \langle H_2' \rangle, \langle H_3 \rangle, \langle H_4 \rangle| = 0.0097539$

Down Higgs

$\langle H_0' \rangle = 0.99879$

$|\langle H_1' \rangle, \langle H_2' \rangle, \langle H_3 \rangle, \langle H_4 \rangle| = 0.049086$

	Theoretical value	Experimental value
$(m_u, m_c, m_t)/m_t$	$(6.84 \times 10^{-6}, 7.86 \times 10^{-3}, 1)$	$(5.58 \times 10^{-6}, 2.69 \times 10^{-3}, 1)$
$(m_d, m_s, m_b)/m_b$	$(1.84 \times 10^{-3}, 4.08 \times 10^{-2}, 1)$	$(6.86 \times 10^{-4}, 1.37 \times 10^{-2}, 1)$
$\lvert V_{\mathrm{CKM}} \rvert$	$\begin{pmatrix} 0.975 & 0.223 & 0.00211 \\ 0.223 & 0.975 & 0.0230 \\ 0.0719 & 0.0220 & 1.00 \end{pmatrix}$	$\begin{pmatrix} 0.974 & 0.227 & 0.00361 \\ 0.226 & 0.973 & 0.0405 \\ 0.00854 & 0.0398 & 0.999 \end{pmatrix}$

6 . Conclusion

Conclusion

・We could evaluate three-generation models by finding the directions of Higgs VEVs leading to rank one mass matrix.

・S-invariant vacuum is preferred for several models.

Future work

・Realization of lepton flavors
→ Neutrino mass can be induced by D-brane instanton effects.
Then, what vacuum is favored?

・Other orbifold models

7 . Appendix: How to restrict Yukawa matrices

1 S-symmetry at $\tau = i$

Ex)

$$Y^{ijk} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}^{ii'} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}^{jj'} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}^{kk'} Y^{i'j'k'}$$

Eigenvalue 1 Higgs ($k = 0$)

$$\begin{pmatrix} & & \\ & Y^{ij0} & \\ & & \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} & & \\ & Y^{ij0} & \\ & & \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$$Y^{ij0} = \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}$$

Eigenvalue -1 Higgs ($k = 1$)

$$\begin{pmatrix} & & \\ & Y^{ij1} & \\ & & \end{pmatrix} = - \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} & & \\ & Y^{ij1} & \\ & & \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$$Y^{ij1} = \begin{pmatrix} & & * \\ & & * \\ * & * & \end{pmatrix}$$

7 . Appendix: Rank one condition at $\tau = i$

Rank one is realized if mass matrix includes,

1. Three or more of $\begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}$. Higgs VEVs leading to rank one exist in S-eigenstates.

Proof: "Higgs VEVs leading to rank one exist in S-eigenstates."

"Unitary transformation for Higgs leading to rank one matrix exists."

We should find the transformation $\langle H_k \rangle \to U^{kk'} \langle H_{k'} \rangle = \langle H'_k \rangle$ such that

$$Y^{ij0} \to U^{0k'} Y^{ijk'} = U^{00} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_0^{ij} + U^{01} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_1^{ij} + U^{02} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2^{ij} = \begin{pmatrix} A & B & \\ C & D & \\ & & 0 \end{pmatrix} \text{ (rank one).}$$

7 . Appendix: Rank one condition at $\tau = i$

1

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_0 \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_1 \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} a & b & \\ c & d & \\ & & 0 \end{pmatrix}_{0'} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_{1'} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2 \end{pmatrix}$$

2

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \begin{pmatrix} a & b & \\ c & d & \\ & & 0 \end{pmatrix}_{0'} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_{1'} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} a & b & \\ c & d & \\ & & 0 \end{pmatrix}_{0'} \\ \begin{pmatrix} e & f & \\ g & h & \\ & & 0 \end{pmatrix}_{1''} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_{2'} \end{pmatrix}$$

3

$$\begin{pmatrix} \cos\omega & e^{i\beta}\sin\omega \\ -\sin\omega & e^{i\beta}\cos\omega \end{pmatrix} \begin{pmatrix} \begin{pmatrix} a & b & \\ c & d & \\ & & 0 \end{pmatrix}_{0'} \\ \begin{pmatrix} e & f & \\ g & h & \\ & & 0 \end{pmatrix}_{1''} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} A & B & \\ C & D & \\ & & 0 \end{pmatrix}_{0''} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & 0 \end{pmatrix}_{1'''} \end{pmatrix}$$

such that $AD - BC = 0$.

$x \in \mathbb{C}$ $\equiv$ $e^{i\beta}\tan\omega$

$$\det\left(\cos\omega\begin{pmatrix} a & b \\ c & d \end{pmatrix} + e^{i\beta}\sin\omega\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right) = \cos^2\omega\det\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + e^{i\beta}\tan\omega\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right) = 0$$

This is a quadratic equation for x. Thus, the transformation such that $AD - BC = 0$ exists.

7 . Appendix: Rank one condition at $\tau = i$

Now, we obtain the unitary transformation,

$$U_3 U_2 U_1 \begin{pmatrix} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_0 \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_1 \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} A & B & \\ C & D & \\ & & 0 \end{pmatrix}_{0''} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & 0 \end{pmatrix}_{1'''} \\ \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_{2'} \end{pmatrix}$$ such that $AD - BC = 0$.

When $AD - BC = 0$, $\begin{pmatrix} A & B & \\ C & D & \\ & & 0 \end{pmatrix}$ is rank one. Therefore, we could find $U \equiv U_3 U_2 U_1$ such that

$$Y^{ij0} \to U^{0k'} Y^{ijk'} = U^{00} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_0^{ij} + U^{01} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_1^{ij} + U^{02} \begin{pmatrix} * & * & \\ * & * & \\ & & * \end{pmatrix}_2^{ij} = \begin{pmatrix} A & B & \\ C & D & \\ & & 0 \end{pmatrix}$$ (rank one).

QED.