A new measurement of the cosmic birefringence

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New Extraction of the Cosmic Birefringence from the Planck 2018 Polarization Data

Yuto Minami and Eiichiro Komatsu

Physics See synopsis: Hints of Cosmic Birefringence?

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ABSTRACT

We search for evidence of parity-violating physics in the Planck 2018 polarization data and report on a new measurement of the cosmic birefringence angle β. The previous measurements are limited by the systematic uncertainty in the absolute polarization angles of the Planck detectors. We mitigate this systematic uncertainty completely by simultaneously determining β and the angle miscalibration using the observed cross-correlation of the E- and B-mode polarization of the cosmic microwave background and the Galactic foreground emission. We show that the systematic errors are effectively mitigated and achieve a factor-of-2 smaller uncertainty than the previous measurement, finding $\beta = 0.35 \pm 0.14$ deg (68% C.L.), which excludes $\beta = 0$ at 99.2% C.L. This corresponds to the statistical significance of 2.4σ.

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Cosmic Birefringence

The Universe filled with a "birefringent material"

➢ If the Universe is filled with a pseudo-scalar field, ϕ, (e.g., an axion field) coupled to the electromagnetic tensor via a Chern-Simons coupling:

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

Chern-Simons term

$$\mathcal{L} \supset -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\boxed{g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}} \quad \cdots (1)$$

Turner & Widrow (1988)

In electromagnetic fields:

Parity Even
$$F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$

Parity Odd
$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B}\cdot\mathbf{E}$$

*The axion field, ϕ, is a "pseudo scalar", which is parity odd; thus, the last term in Eq (1) is parity even as a whole.

Cosmic Birefringence

Carroll, Field & Jackiw (1990);
Harari & Sikivie (1992); Carroll (1998)

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Chern-Simons term

$$\mathcal{L} \supset -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\boxed{g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}} \quad \cdots(1)$$

Turner & Widrow (1988)

$$\beta = \frac{g_{\phi\gamma}}{2}\int_{emission}^{observer} dt\,\dot{\phi}$$

$$= \frac{g_{\phi\gamma}}{2}\left(\phi_{observer} - \phi_{emission}\right) \quad \cdots(2)$$

Difference of the field values rotates the linear polarization!

Motivation

ordinary matter
5%

dark matter
26%
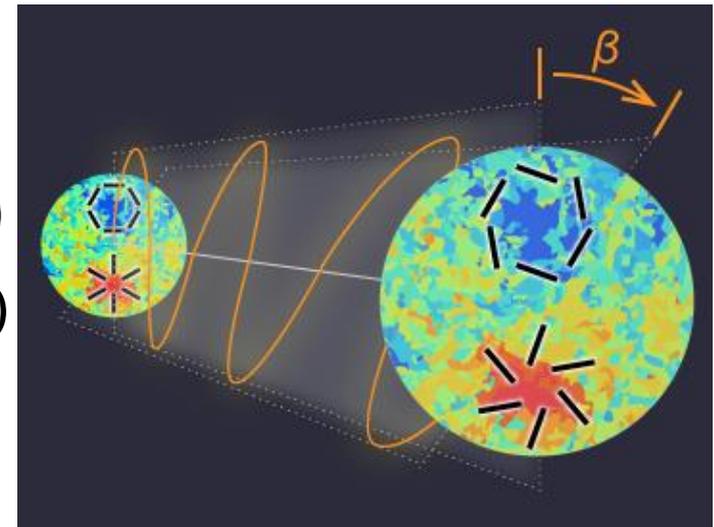
dark energy
69%

Credit: ESA

➢ The Universe's energy budget is dominated by two dark components:
  ➢ Dark Energy
  ➢ Dark Matter
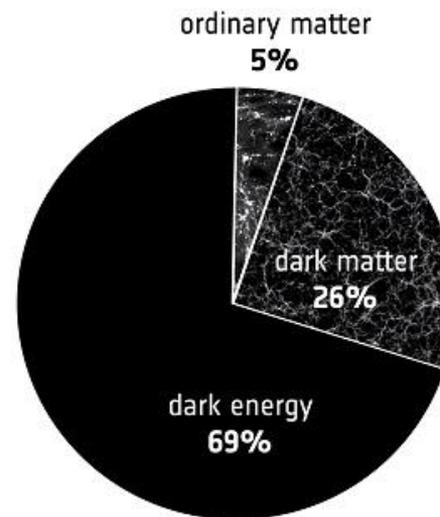
➢ We know that the weak interaction violates parity (Lee & Yang 1956; Wu et al. 1957)

Why should the laws of physics governing the Universe conserve parity?

Test with cosmic microwave background (CMB)

Origin of the CMB

Hot plasma state

Cosmic microwave background 380,000 years

Clouds of neutral hydrogen

First stars form

First galaxies form

Travel of CMB photons

380 thousands years
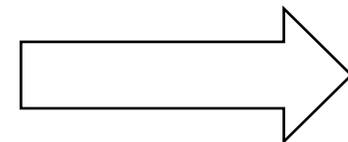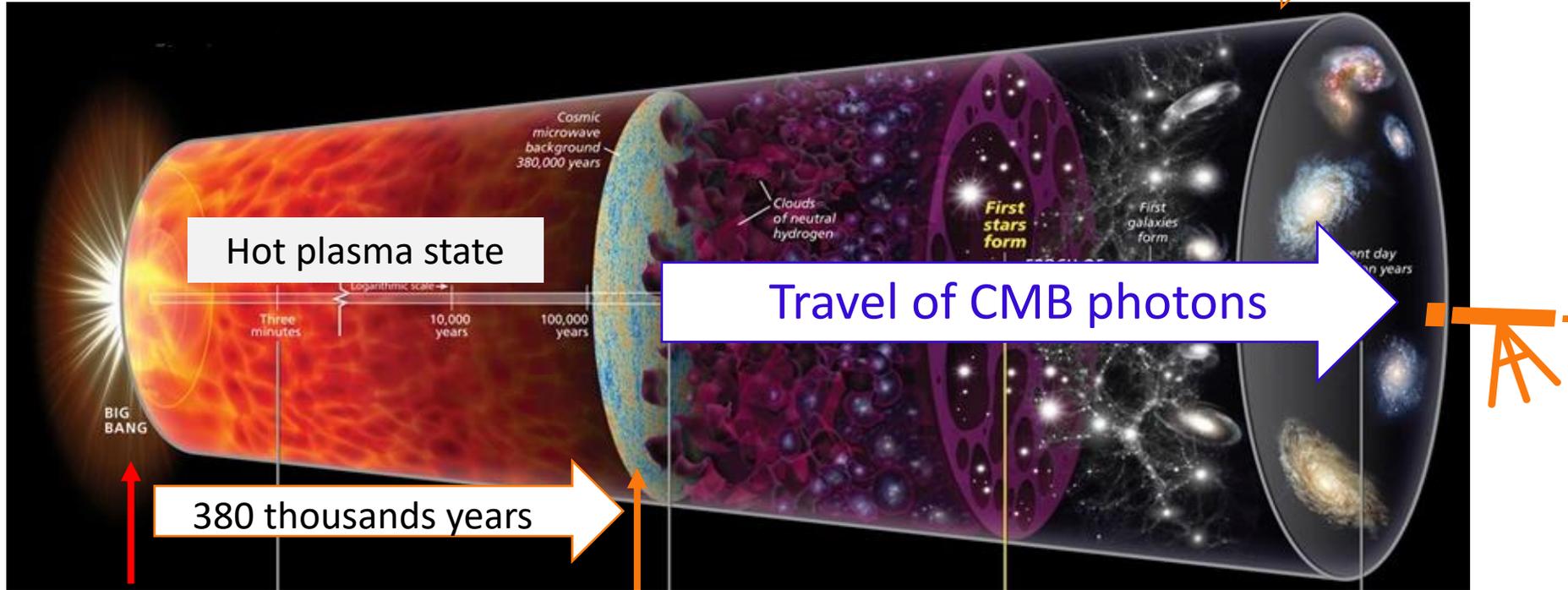
Big Bang?   Recombination   Credit: Roen Kelly, Discovermagazine

Travel of CMB photons

➢ Before the recombination : photons cannot travel long distance because of hot plasma

➢ After the recombination : photons travel the Universe after the last scattering by electrons

Searching for cosmic birefringence with the CMB

Credit: ESA

ESA's Planck
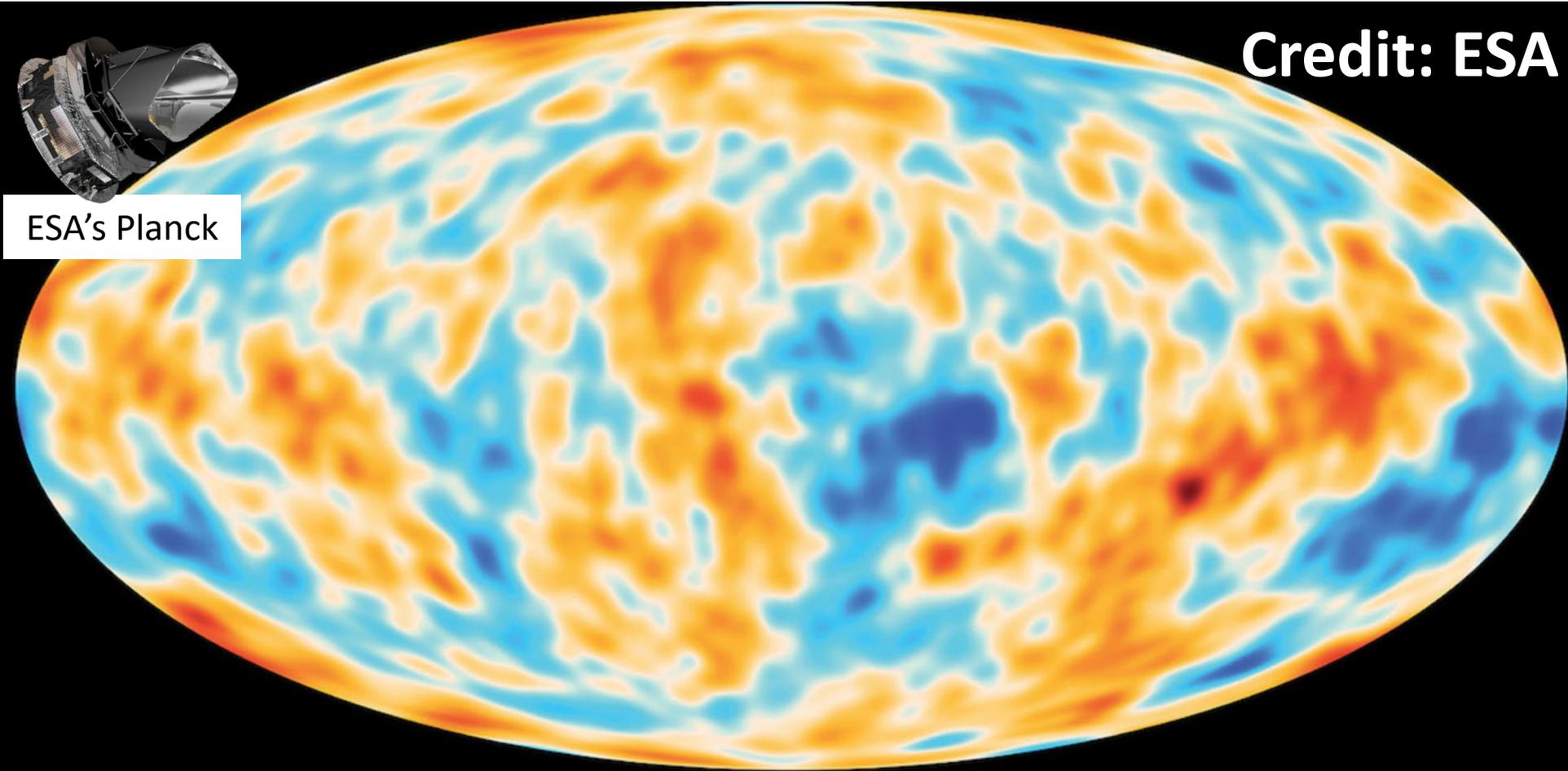
Temperature (smoothed)

Emitted 13.8 billion years ago
at the last scattering surface (LSS)

Temperature anisotropy + polarisaion

ESA's Planck

Credit: ESA
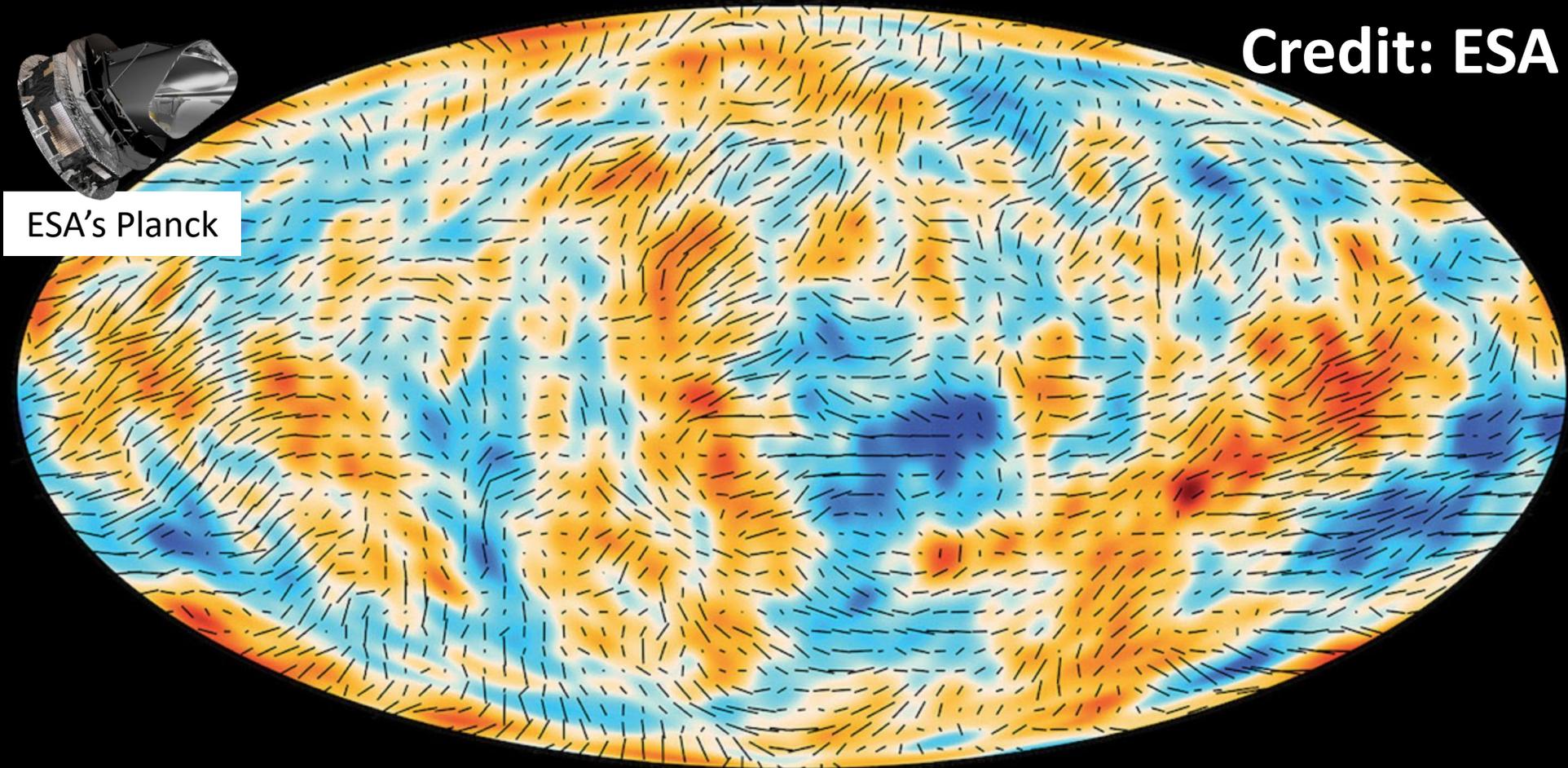
Temperature (smoothed) + Polarisation

We know the initial $\beta = 0$

In the case of axion like particles (ALPs)

➤ Which is possible when we search with cosmic microwave background (CMB):

Dark Energy ?

Or

Dark Matter ?

$$\beta = \frac{g_{\phi\gamma}}{2}\underbrace{(\phi_{observer} - \phi_{LSS})}_{=\Delta\bar{\phi}}$$
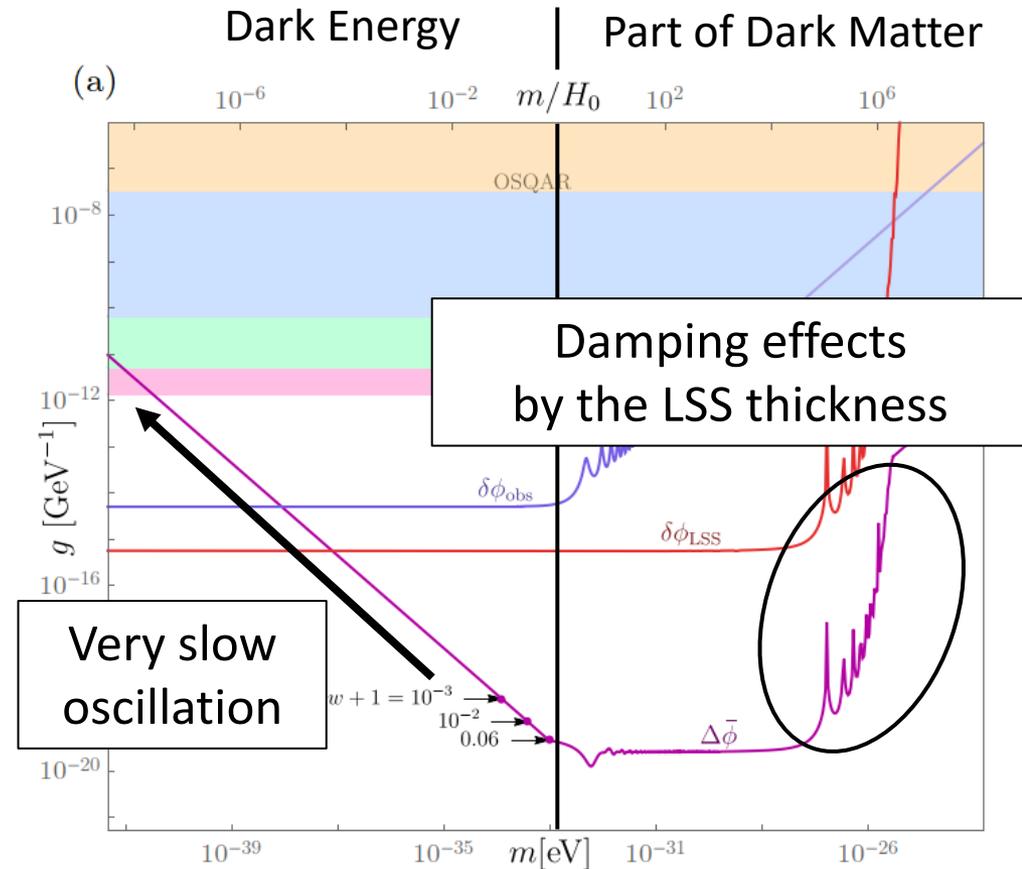
➤ See Marsh (2016) and Ferreira (2020) for reviews of ALPs

Measurement of the polarisation

We measure linear polarisation with two orthogonal parameters
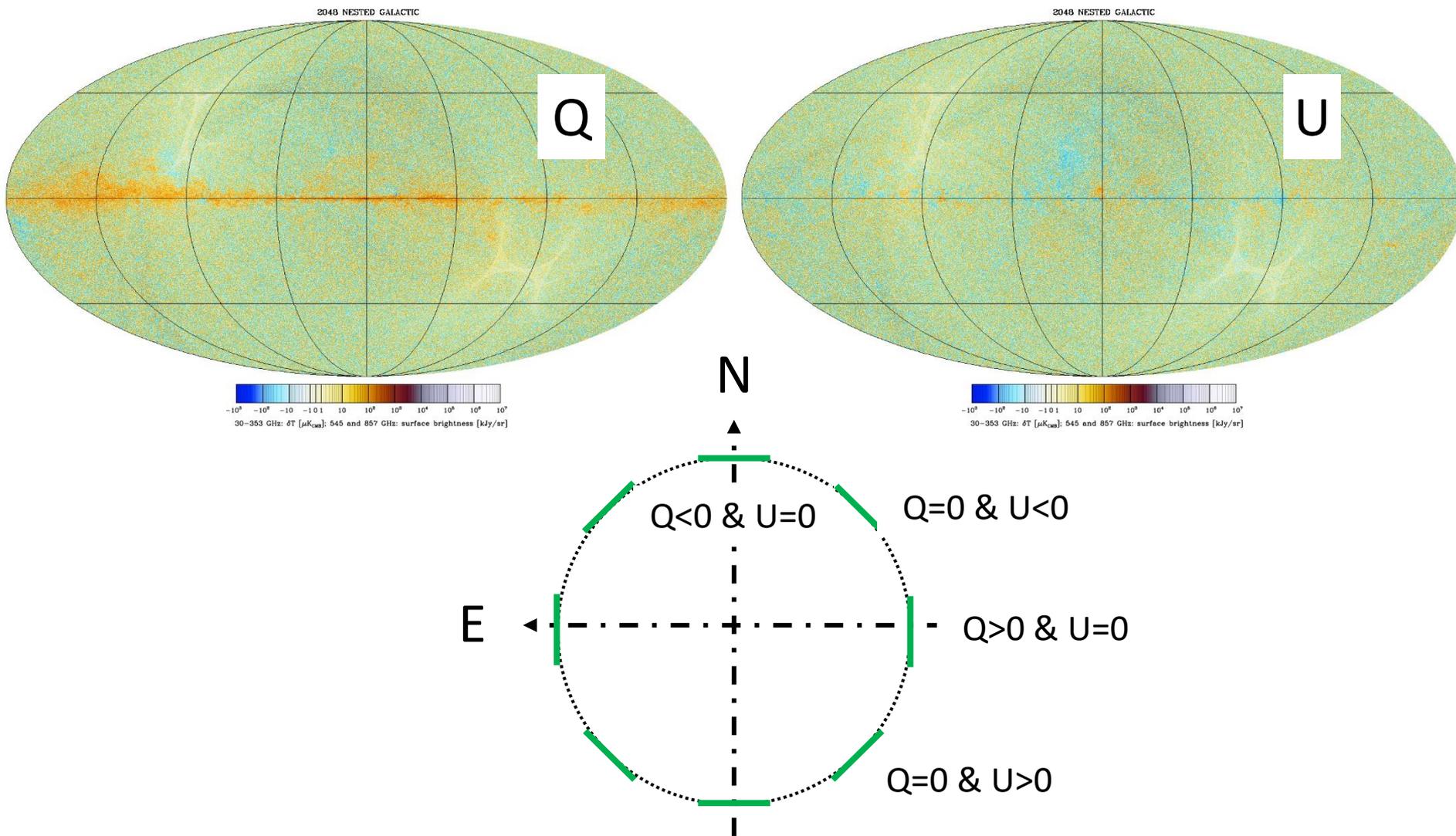
E- and B-mode: decomposition of linear polarisation

$$E(\ell) \pm iB(\ell) = e^{\mp 2i\phi_\ell} \int d\hat{\boldsymbol{n}} \left[Q(\hat{\boldsymbol{n}}) + iU(\hat{\boldsymbol{n}}) \right] e^{-i\ell \cdot \hat{\boldsymbol{n}}}$$

E mode

B mode

Direction of the Fourier wavenumber vector
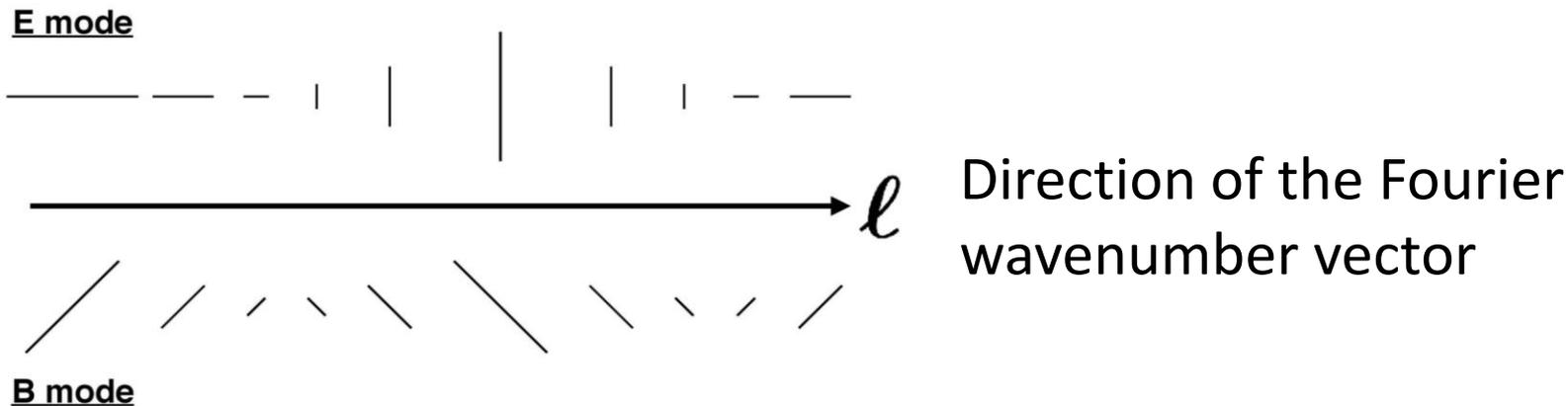
- ➢ E-mode: Polarisation directions are **parallel or perpendicular** to the wavenumber direction

- ➢ B-mode: Polarisation directions are **45 degrees tilted** w.r.t the wavenumber direction

IMPORTANT": These "E- and B-modes" are jargons in the CMB community, and completely unrelated to the electric and magnetic fields of the electromagnetism!!

Parity transformation

Parity flip

E mode

B mode

ℓ ℓ
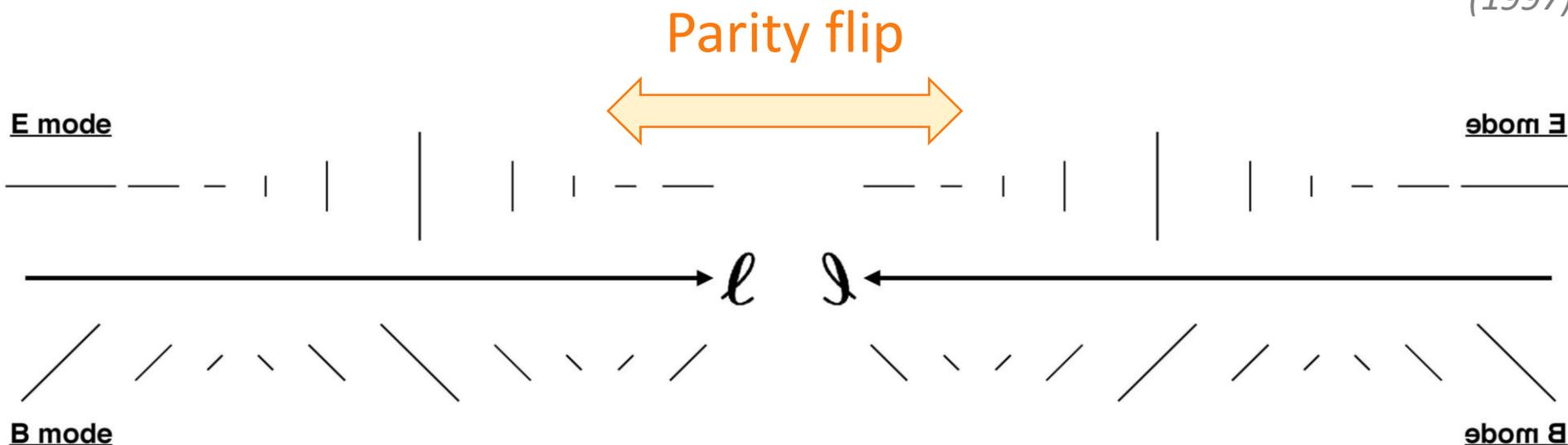
> Two-point correlation functions invariant under the parity flip:

auto correlation
$$\langle E_\ell E_{\ell'}^* \rangle = (2\pi)^2 \delta^{(2)}(\ell - \ell') C_\ell^{EE}$$
$$\langle B_\ell B_{\ell'}^* \rangle = (2\pi)^2 \delta^{(2)}(\ell - \ell') C_\ell^{BB}$$

even × even
$$\langle T_\ell E_{\ell'}^* \rangle = \langle E_\ell T_{\ell'}^* \rangle = (2\pi)^2 \delta^{(2)}(\ell - \ell') C_\ell^{TE}$$

$\cdots (3)$

> The others, e.g., $\langle T_\ell B_{\ell'}^* \rangle$ and $\langle E_\ell B_{\ell'}^* \rangle$, are not invariant
> > We can use these to probe parity-violating physics!

Power spectra

➢ This is the typical figure that you find in many talks on the CMB

➢ The temperature anisotropy and E- and B-mode polarisation power spectra have been measured well

➢ Focus is the EB cross spectrum, which is not shown here

Temperature anisotropy (sound waves)

E-**mode (sound waves)**

B-**mode (lensing)**

r=0.05

Planck ●
BICEP2/Keck □
SPTPol ✳
POLARBEAR △

B-**mode (primordial gravitational wave, if any)**

$l(l+1)C_l^{TT,EE,BB}/2\pi$ [μK^2]
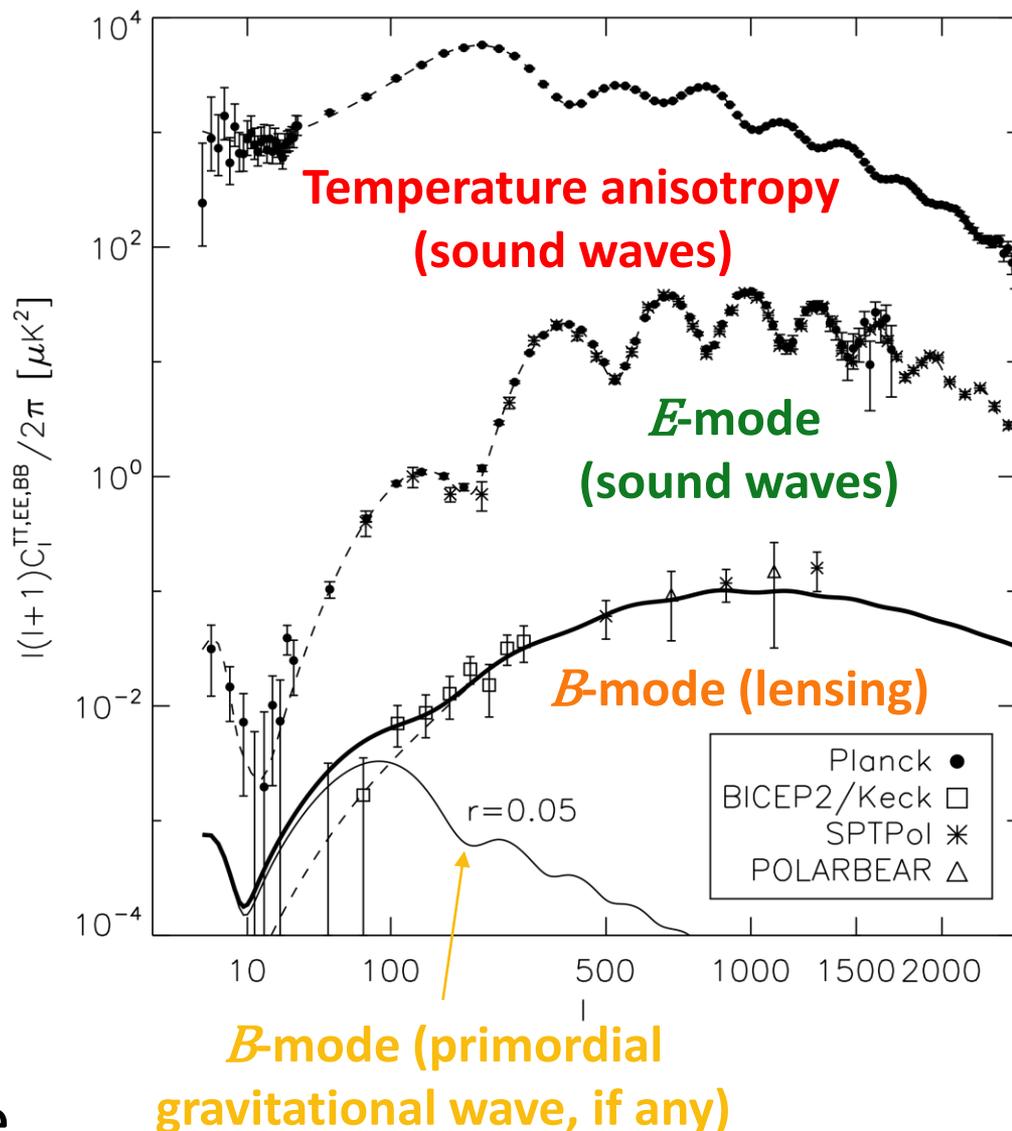
EB correlation from the cosmic birefringence

Lue, Wang & Kamionkowski (1999); Feng et al. (2005, 2006); Liu, Lee & Ng (2006)
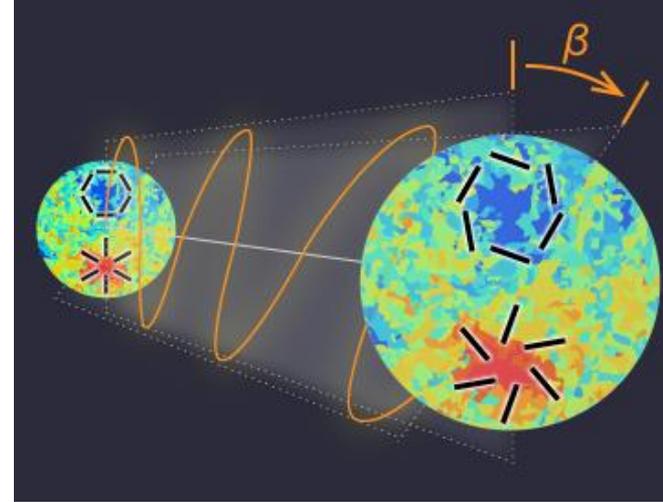
➤ Cosmic birefringence convert $E <-> B$ as

$$\begin{pmatrix} E_{\ell m} \\ B_{\ell m} \end{pmatrix}^{obs} = \begin{pmatrix} \cos(2\boldsymbol{\beta}) & -\sin(2\boldsymbol{\beta}) \\ \sin(2\boldsymbol{\beta}) & \cos(2\boldsymbol{\beta}) \end{pmatrix} \begin{pmatrix} E_{\ell m} \\ B_{\ell m} \end{pmatrix} \cdots (4)$$

➤ In power spectra:

$$\langle C_\ell^{EB,obs} \rangle = \frac{1}{2} \boxed{\left(\langle C_\ell^{EE} \rangle - \langle C_\ell^{BB} \rangle \right)} \sin(4\boldsymbol{\beta}) + \langle C_\ell^{EB} \rangle \cos(4\boldsymbol{\beta})$$

0

Need to assume a model!

Vanish at the LSS $\cdots (5)$

➤ Traditionally, one would find $\boldsymbol{\beta}$ by fitting $C_\ell^{EE,CMB} - C_\ell^{BB,CMB}$ to the observed $C_\ell^{EB,obs}$ using the best-fitting CMB model
  ➤ Assuming the intrinsic $\langle C_\ell^{EB} \rangle = 0$, at the last scattering surface (LSS) (justified in the standard cosmology)

Only with observed data

Zhao et al. 2015;Minami et al. 2019
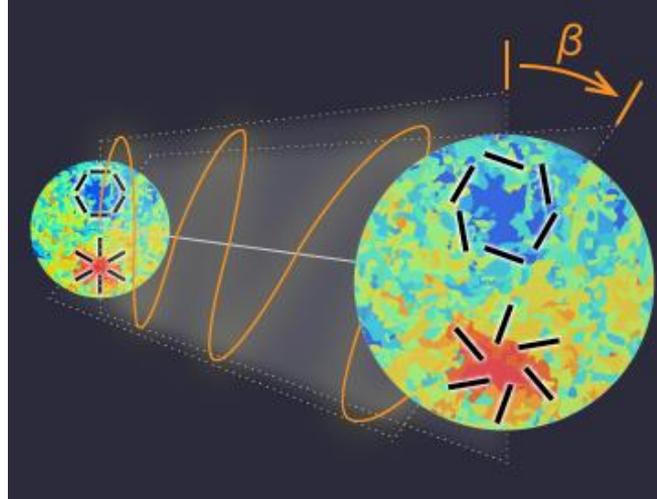
➢ Cosmic birefringence convert $E <-> B$ as

$$\begin{pmatrix} E_{\ell m} \\ B_{\ell m} \end{pmatrix}^{obs} = \begin{pmatrix} \cos(2\boldsymbol{\beta}) & -\sin(2\boldsymbol{\beta}) \\ \sin(2\boldsymbol{\beta}) & \cos(2\boldsymbol{\beta}) \end{pmatrix} \begin{pmatrix} E_{\ell m} \\ B_{\ell m} \end{pmatrix} \cdots (4)$$

➢ We find additional relations

$$\langle C_\ell^{EB,obs} \rangle = \frac{1}{2} (\langle C_\ell^{EE} \rangle - \langle C_\ell^{BB} \rangle) \sin(4\beta) + \langle C_\ell^{EB} \rangle \cos(4\beta)$$

$$\langle C_\ell^{EE,obs} \rangle - \langle C_\ell^{BB,obs} \rangle = (\langle C_\ell^{EE} \rangle - \langle C_\ell^{BB} \rangle) \cos(4\beta) - 2\langle C_\ell^{EB} \rangle \sin(4\beta)$$

- $\langle C_\ell^{EE,obs} \rangle = \langle C_\ell^{EE} \rangle \cos^2(2\beta) + \langle C_\ell^{BB} \rangle \sin^2(2\beta) - \langle C_\ell^{EB} \rangle \sin(4\beta)$

- $\langle C_\ell^{BB,obs} \rangle = \langle C_\ell^{EE} \rangle \sin^2(2\beta) + \langle C_\ell^{BB} \rangle \cos^2(2\beta) + \langle C_\ell^{EB} \rangle \sin(4\beta)$

$$\langle C_\ell^{EB,o} \rangle = \frac{1}{2} \boxed{(\langle C_\ell^{EE,o} \rangle - \langle C_\ell^{BB,o} \rangle)} \tan(4\beta) + \frac{\overset{0}{\langle C_\ell^{EB} \rangle}}{\cos(4\beta)} \qquad \cdots (6)$$

No need to assume a model   Vanish at the LSS

The Biggest Problem: Miscalibration of detectors

Miscalibration of detectors

Cosmic or Instrumental?
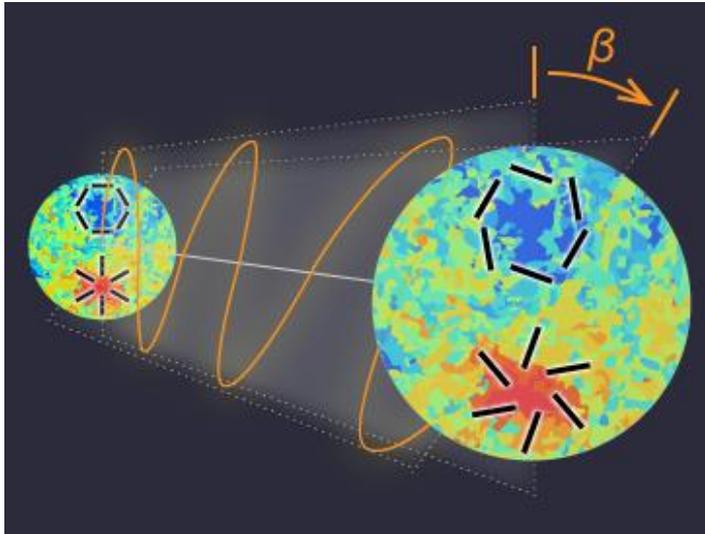
Polarisation-sensitive detectors on the focal plane

OR
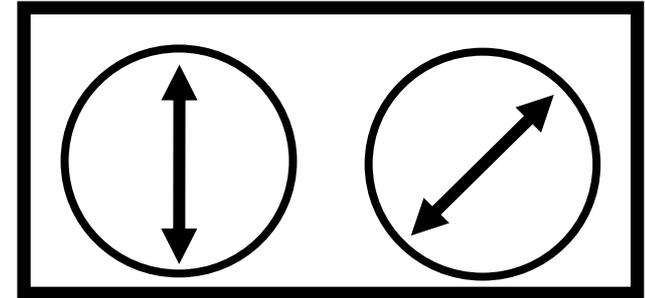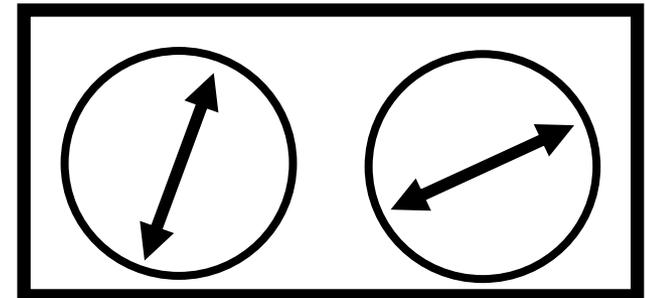
Miscalibration

rotated by an angle "α" (but we do not know it)

➤ Is the polarization plane rotated by the genuine cosmic birefringence, $\boldsymbol{\beta}$?

➤ Are the polarisation-sensitive detectors rotated by miscalibration, α, on the sky coordinate (and we did not know)?

We can only measure the sum, $\alpha + \beta$

The past measurements

The quoted uncertainties are all statistical only (68% C.L.)

Measurement	$\alpha + \beta$ +stats. (deg.)
Feng et al. 2006	-6.0 ± 4.0
WMAP Collaboration, Komatsu et al. 2009; 2011	-1.1 ± 1.4
QUaD Collaboration, Wu et al. 2009	-0.55 ± 0.82
...	...
Planck Collaboration 2016	0.31 ± 0.05
POLARBEAR Collaboration 2020	-0.61 ± 0.22
SPT Collaboration, Bianchini et al. 2020	0.63 ± 0.04
ACT Collaboration, Namikawa et al. 2020	0.12 ± 0.06
ACT Collaboration, Choi et al. 2020	0.09 ± 0.09

First measurement

Why not yet discovered?

The past measurements

Now including the estimated systematic errors on α

Measurement	β + stat. + sys. (deg.)	
Feng et al. 2006	$-6.0 \pm 4.0 \pm$**??**	First measurement
WMAP Collaboration, Komatsu et al. 2009; 2011	$-1.1 \pm 1.4 \pm \mathbf{1.5}$	
QUaD Collaboration, Wu et al. 2009	$-0.55 \pm 0.82 \pm \mathbf{0.5}$	
…	…	
Planck Collaboration 2016	$0.31 \pm 0.05 \pm \mathbf{0.28}$	Uncertainty in the calibration of α has been the major limitation
POLARBEAR Collaboration 2020	$-0.61 \pm 0.22 +$**??**	
SPT Collaboration, Bianchini et al. 2020	$0.63 \pm 0.04 +$**??**	
ACT Collaboration, Namikawa et al. 2020	$0.12 \pm 0.06 +$**??**	
ACT Collaboration, Choi et al. 2020*	$0.09 \pm 0.09 +$**??**	

*used optical model , "as-designed" angles

➢ Other way to calibrate?

Crab nebula, Tau A (Celestial source)	0.27 deg. (Aumont et al.(2018))
Wire grid	1.00 deg. ? (Planck pre launch)

The Key Idea: The polarized Galactic foreground emission as a calibrator

ESA's Planck

Polarised dust emission within our Milky Way!

Emitted "right there" - it would not be affected by the cosmic birefringence.

Directions of the magnetic field inferred from polarisation of the thermal dust emission in the Milky Way

Searching for birefringence

<u>Idea</u>: Miscalibration of the polarization angle α rotates both the FG and CMB, but β affects only the CMB

$$E_{\ell,m}^{\mathrm{o}} = E_{\ell,m}^{\mathrm{fg}} \cos(2\alpha) - B_{\ell,m}^{\mathrm{fg}} \sin(2\alpha) + E_{\ell,m}^{\mathrm{CMB}} \cos(2\alpha + 2\beta) - B_{\ell,m}^{\mathrm{CMB}} \sin(2\alpha + 2\beta) + E_{\ell,m}^{\mathrm{N}}$$

$$B_{\ell,m}^{\mathrm{o}} = E_{\ell,m}^{\mathrm{fg}} \sin(2\alpha) + B_{\ell,m}^{\mathrm{fg}} \cos(2\alpha) + E_{\ell,m}^{\mathrm{CMB}} \sin(2\alpha + 2\beta) + B_{\ell,m}^{\mathrm{CMB}} \cos(2\alpha + 2\beta) + B_{\ell,m}^{\mathrm{N}}$$

noise

$\cdots$ (7)

From them, we derived

$$\langle C_\ell^{EB,\mathrm{o}} \rangle = \frac{\tan(4\alpha)}{2} \left(\underbrace{\langle C_\ell^{EE,\mathrm{o}} \rangle - \langle C_\ell^{BB,\mathrm{o}} \rangle}_{\text{Measured}} \right) + \frac{\sin(4\beta)}{2\cos(4\alpha)} \left(\underbrace{\langle C_\ell^{EE,\mathrm{CMB}} \rangle - \langle C_\ell^{BB,\mathrm{CMB}} \rangle}_{\text{Known accurately}} \right) \quad \cdots (8)$$

$$+ \frac{1}{\cos(4\alpha)} \langle C_\ell^{EB,\mathrm{fg}} \rangle + \frac{\cos(4\beta)}{\cos(4\alpha)} \langle C_\ell^{EB,\mathrm{CMB}} \rangle.$$

➢ For the baseline result, we ignore the intrinsic EB correlations of the FG and the CMB
  ➢ The latter is justified but the former is not
  ➢ We will revisit this important issue at the end

Likelihood for determination of α and β

Single frequency case, full sky data

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}}\right)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)} \quad \cdots (9)$$

➢ We determine α and β simultaneously using this likelihood

Likelihood for determination of α and β

Single frequency case, full sky data

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\text{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\text{o}} - C_\ell^{BB,\text{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\text{CMB}} - C_\ell^{BB,\text{CMB}}\right)\right]^2}{\text{Var}\left(C_\ell^{EB,\text{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\text{o}} - C_\ell^{BB,\text{o}}\right)\right)} \quad \cdots (9)$$

➢ We determine α and β simultaneously using this likelihood

➢ For analysing the Planck data, we use the multi-frequency likelihood developed
in Minami and Komatsu (2020a)

➢ First, validate the algorithm using simulated data

How does it work?

How does it work?

Simulation with future CMB data (LiteBIRD)
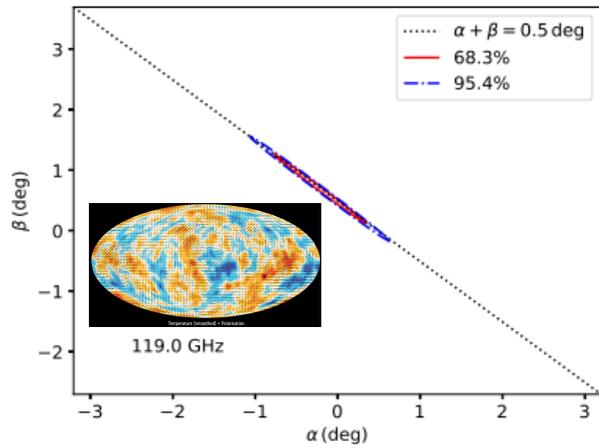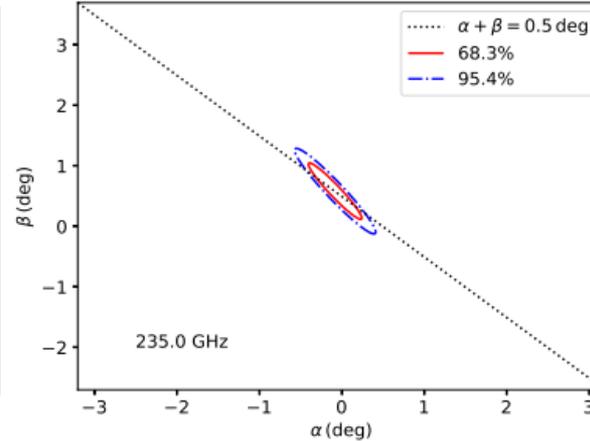
CMB channel (119 GHz)   **Mid freq. (235 GHz)**   **Dust FG channel (337 GHz)**
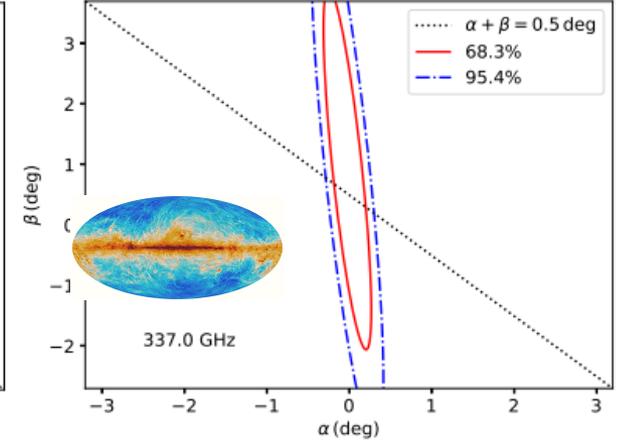
➢ The CMB signal determines the sum of two angles, $\alpha + \beta$
   ➢ Diagonal line

➢ The FG determines only α

➢ Mid freq. : breaking the degeneracy with FG signal!
   ➢ $\sigma(\beta) \sim \sigma(\alpha)$, since $\sigma(\alpha + \beta) \ll \sigma(\alpha)$

Application to the Planck Data (PR3, released in 2018)

Application to the Planck Data (PR3, released in 2018)

$\ell_{\min} = 51, \ell_{\max} = 1500$ (the same values used by Planck team)

➢ We used Planck High Frequency Instrument (HFI) data
   ➢ 4 channels: 100, 143, 217, and 353 GHz

Information for experts

➢ Power spectra calculated from "Half Missions" (HM1 and HM2 maps)
➢ Mask (using NaMaster [Alonso et al.]), apodization by "Smooth" with 0.5 deg
   ➢ Bright CO regions. Bright point sources. Bad pixels.
➢ $I \rightarrow P$ leakage due to the beam is corrected using QuickPol [Hivon et al.]
   ➢ It does not change the result even if we ignore this correction: good news!

Validation with FFP10

FFP10 = Planck team's "Full Focal Plane Simulation"

➢ There are 4 α_ν's and one β

➢ 10 simulations, without foreground samples because no beam systematics is applied to them

    ➢ We can check only β ($\alpha_\nu = 0$) and only α_ν ($\beta = 0$)

Angles	$\alpha_\nu = 0$ (deg.)	$\beta = 0$ (deg.)
β	0.010 ± 0.030	-
α_{100}	-	-0.008 ± 0.047
α_{143}	-	0.013 ± 0.033
α_{217}	-	0.017 ± 0.065
α_{353}	-	0.14 ± 0.41

➢ No bias found. The test passed.

Main Results

Main results: $\beta > 0$ at 99.2% (2.4σ)

Angles	$\alpha_\nu = 0$	Results (deg.)
β	0.289 ± 0.048	0.35 ± 0.14
α_{100}	-	-0.28 ± 0.13
α_{143}		0.07 ± 0.12
α_{217}		-0.07 ± 0.11
α_{353}	-	-0.09 ± 0.11

Planck (Int. XLIX):
0.29 ± 0.05 (stat.)
± 0.28 (syst.)
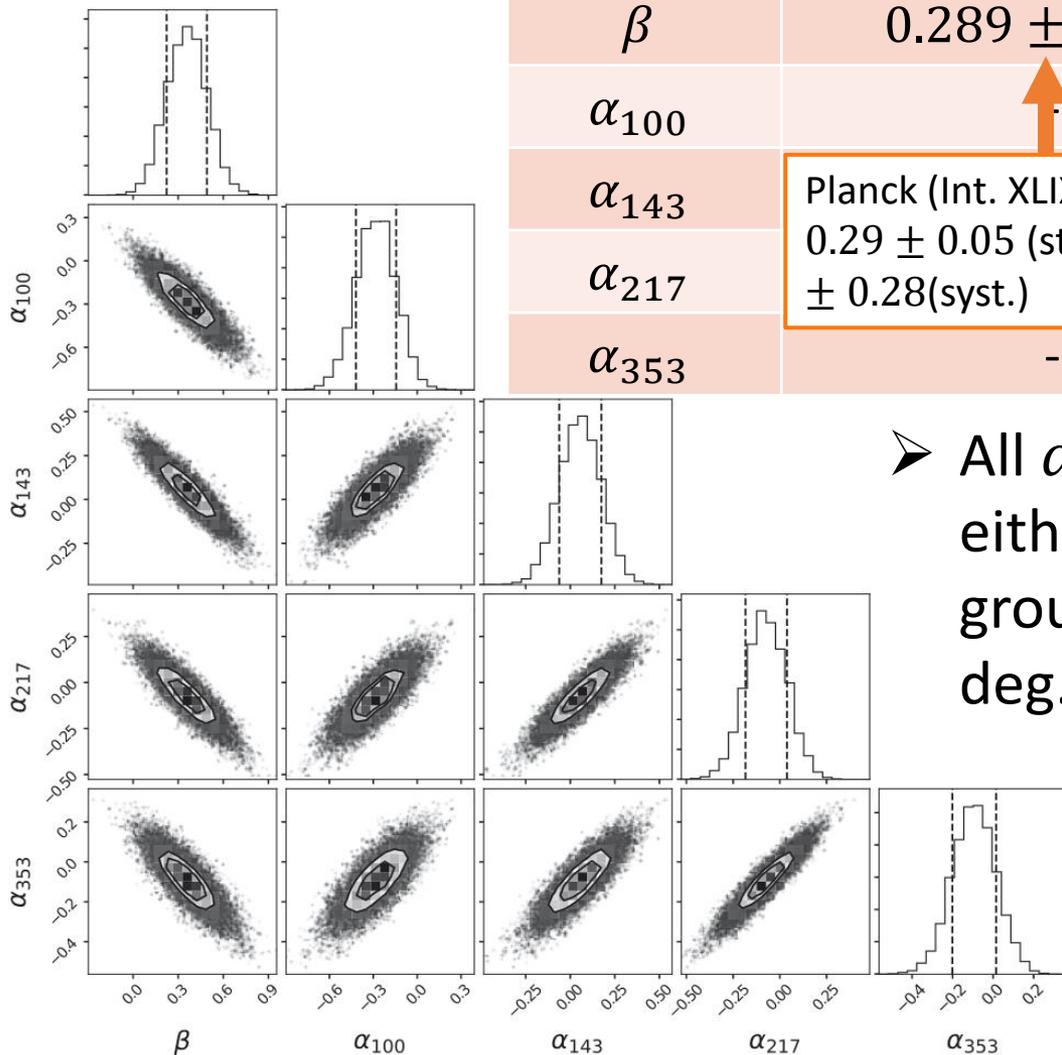
➤ All α_ν are consistent with zero either statistically, or within the ground calibration error of 0.28 deg.

➤ Removing α_{100} did not change β

➤ $\beta = 0.35$ is consistent with the Planck's result

$EE - BB$ power spectra

143 GHz-HM1 x 143 GHz-HM2 — CMB theory EE - BB, observed EE- BB
217 GHz-HM1 x 143 GHz-HM2
143 GHz-HM1 x 217 GHz-HM2
217 GHz-HM1 x 217 GHz-HM2
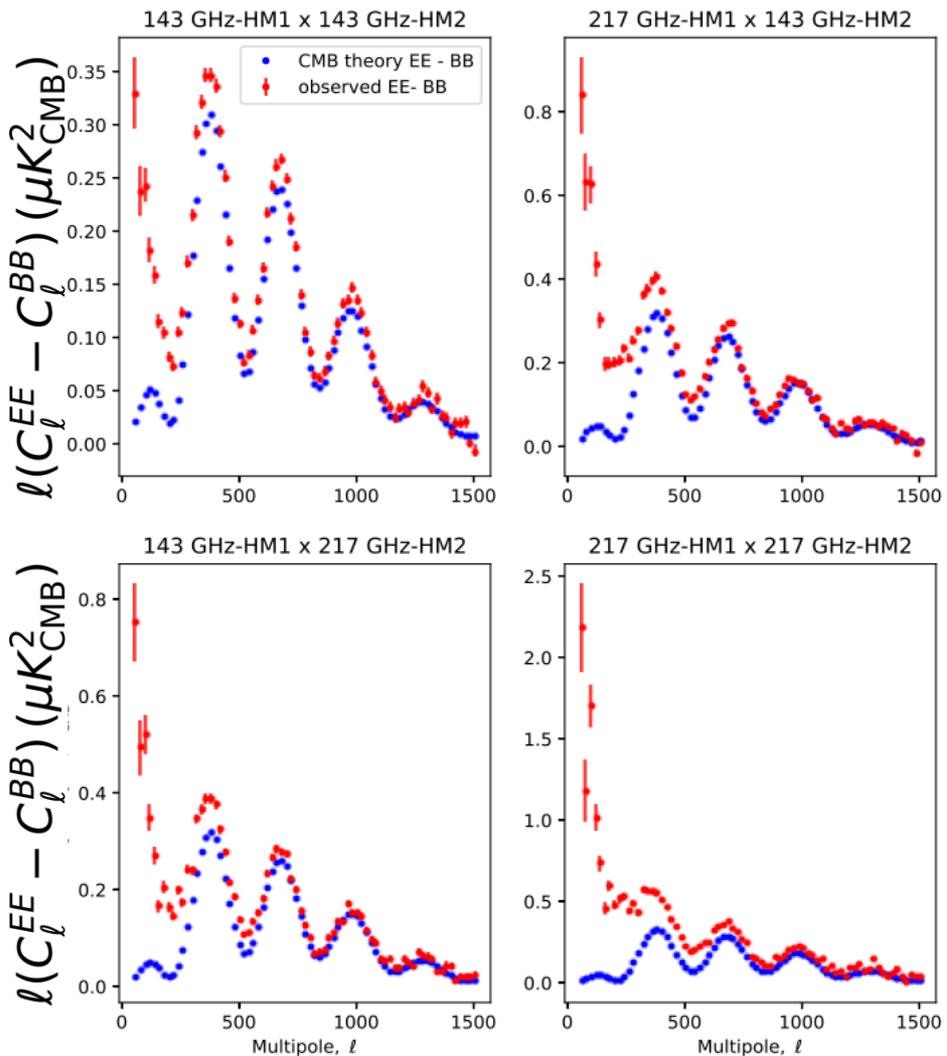
➢ Can we see $\beta = 0.35 \pm 0.14$ by eyes?

➢ First, take a look at the observed $EE - BB$ spectra

➢ Red: Observed total

➢ Blue: The best-fitting CMB model

*The difference is due to the FG (and potentially systematics)

EB power spectra (Black dots)
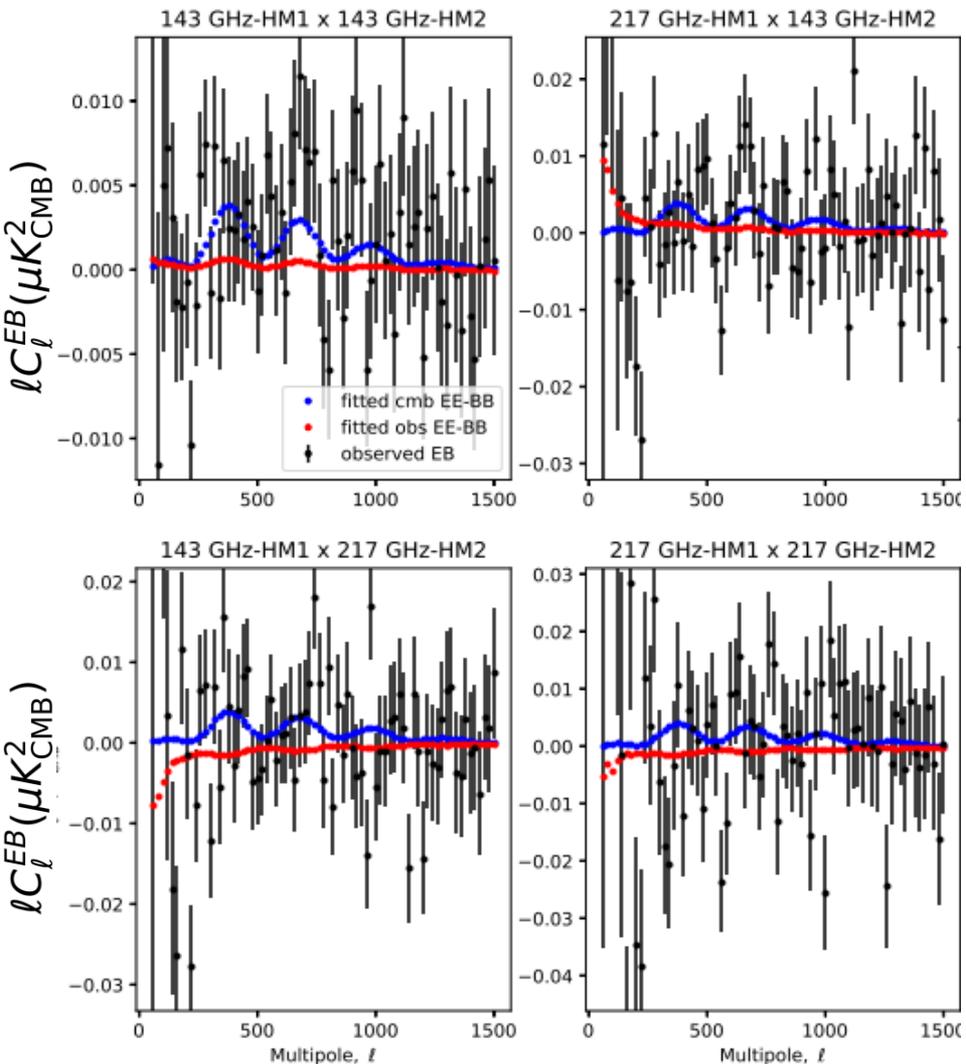
➤ Can we see $\beta = 0.35 \pm 0.14$ by eyes?

➤ Red: The observed signal attributed to the miscalibration angle, α_ν

➤ Blue: The CMB signal attributed to the cosmic birefringence, β

➤ Red + Blue is the best-fitting model for explaining the data points (black dots)

How about the foreground EB?

Minami et al. (2019); Minami (2020);Minami & Komatsu (2020b)

If the intrinsic foreground (FG) EB exists, our method interprets it as a miscalibration angle α

➢ Thus, $\alpha \rightarrow \alpha + \gamma$, where γ is the parameter of the intrinsic EB
  ➢ The sign of γ is the same as the sign of the foreground EB

➢ We thus can determine:

$$\text{FG: } \alpha + \gamma$$
$$\text{CMB: } \alpha + \beta$$

$$\boldsymbol{\beta - \gamma = 0.35 \pm 0.14 \text{ deg.}}$$

➢ There is evidence for the dust-induced $TE_{dust} > 0 \, \& \, TB_{dust} > 0$; then, we'd expect $EB_{dust} > 0$ [Huffenberger et al.], i.e., $\gamma > 0$. If so, β increased further...
  ➢ We can give a lower bound on β

Implications

What does it mean for your models of dark matter and energy?

➢ When a Lagrangian density includes a Chern-Simons coupling between a pseudo-scalar field and the electromagnetics tensor as:

$$\mathcal{L} \supset \frac{1}{4} g_{\phi\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \cdots (10)$$

➢ The birefringence angle is

$$\beta = \frac{g_{\phi\gamma}}{2} \left(\bar{\phi}_{obs} - \bar{\phi}_{LSS} + \delta\phi_{obs} \right) \cdots (11)$$

Carroll, Field & Jackiw (1990); Harari & Sikivie (1992); Carroll (1998); Fujita, Minami, et al. (2020)

$\bar{\phi}_{LSS}$

$\bar{\phi}_{obs} + \delta\phi_{obs}$

➢ This measurement yields

$$g_{\phi\gamma} \left(\bar{\phi}_{obs} - \bar{\phi}_{LSS} + \delta\phi_{obs} \right) = (1.2 \pm 0.5) \times 10^{-2} \text{ rad.} \quad \cdots (12)$$
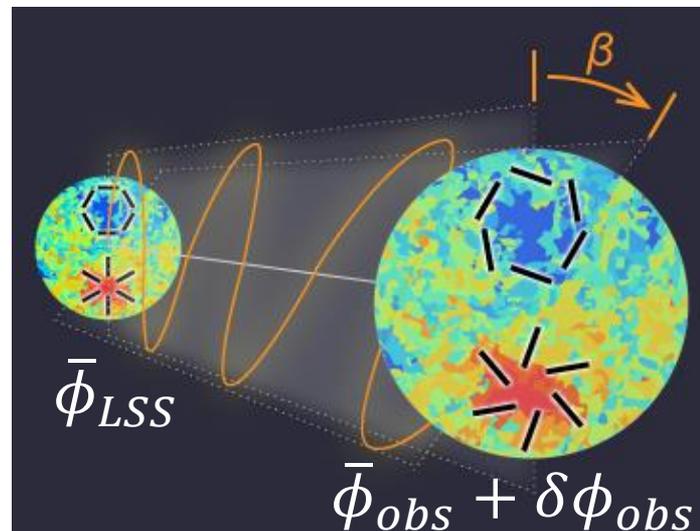
Implications (examples)

Models with ALPs

➢ Explanation of β and H_0 tension simultaneously (Fujita, Murai, Nakatsuka, & Tsujikawa 2020)

➢ "Kilobyte Cosmic Birefringence from ALP Domain Walls" (Takahashi & Yin 2020)

➢ Hidden-monopole-DM gives ALP mass to oscillate during matter dominant era   (Nakagawa, Takahashi, & Yamada 2021), next talk

➢ Make VEV of the Higgs vacuum lighter to explain ^{7}Li puzzle (Fung et al. 2021)

➢ etc…

Non−zero foreground EB

➢ Magnetically misaligned filamentary dust structures introduce nonzero EB (Clark, Kim, Hill, & Hensley 2021)

Conclusion

➤ We find a hint of the parity violating-physics in the CMB polarization:

$$\beta = 0.35 \pm 0.14 \text{ deg. (68\% C.L.)}$$
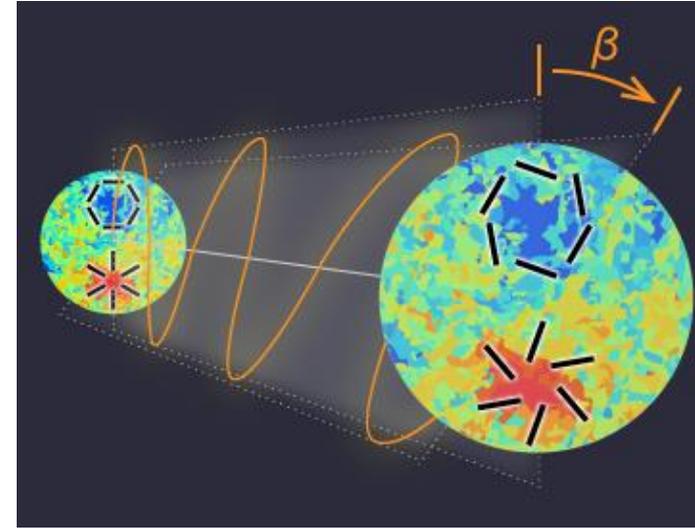
*Higher statistical significance is needed to confirm this signal

➤ New method finally makes impossible to possible:
  ➤ Use foreground signal to calibrate detector rotations
  ➤ Our method can be applied to any of the **existing** and **future** CMB experiments
➤ We should be possible to test the signal is true or only a coincidence immediately
  ➤ If confirmed, it would have important implications for the dark matter/energy.

Backups

➤ Birefringence from background and fluctuation
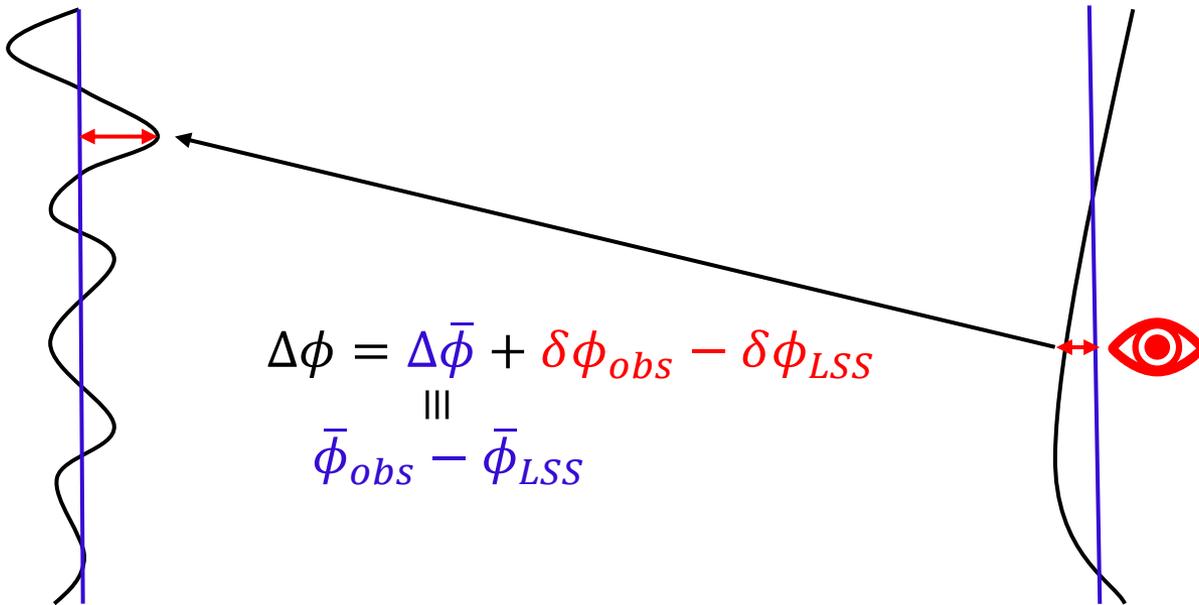
$$\Delta\phi = \Delta\bar{\phi} + \delta\phi_{obs} - \delta\phi_{LSS}$$
$$\overset{\text{III}}{\bar{\phi}_{obs} - \bar{\phi}_{LSS}}$$

$$\phi_{LSS}(t = t_{LSS}, d_{LSS}\hat{n}) = \bar{\phi}_{LSS} + \delta\phi_{LSS} \qquad \phi_{obs}(t = t_0, \mathbf{0}) = \bar{\phi}_{obs} + \delta\phi_{obs}$$

$$\beta(\hat{\boldsymbol{n}}) = \frac{g_{\phi\gamma}}{2}\left(\underbrace{\Delta\bar{\phi} + \delta\phi_{obs}}_{\beta} - \underbrace{\delta\phi_{LSS}(\hat{\boldsymbol{n}})}_{A_\alpha}\right)$$

When we detect only β (no A_α)

- If β is from $\delta\phi_{obs}$, non-zero A_α is expected via r as

$$\beta(\hat{\boldsymbol{n}}) = \frac{g_{\phi\gamma}}{2}\Big(\underbrace{\Delta\bar{\phi} + \delta\phi_{obs}}_{\beta} \underbrace{- \delta\phi_{LSS}(\hat{\boldsymbol{n}})}_{A_\alpha}\Big)$$

$$|\beta| \simeq 0.05°\left(\frac{A_\alpha}{4\times 10^{-3}\,\deg^2}\right)^{1/2}.$$

with $1\sigma(\delta\phi_{obs})$ level in $m < H_0$

- If this A_α is not detected

⟹ $\Delta\bar{\phi}$ should be the origin

- Then we can have

$$10^{-8}\left(\frac{|\beta|}{0.3°}\right) < \frac{m}{H_0} \lesssim 10^8\left(\frac{|\beta|}{0.3°}\right)^{-1},$$
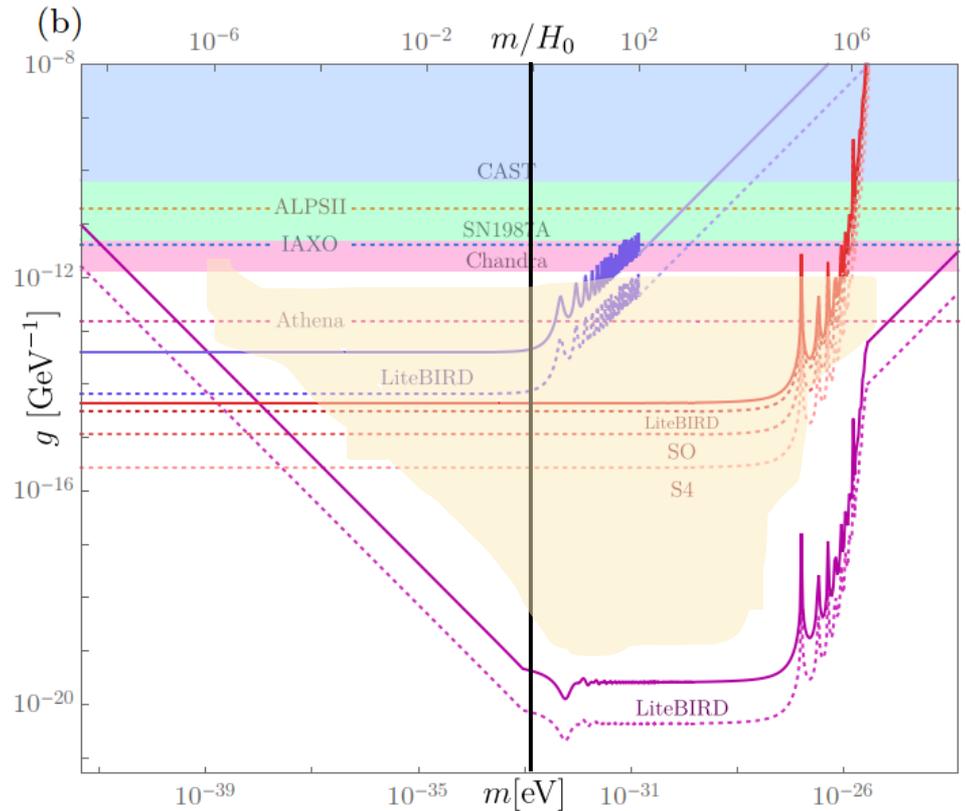
We can constrain the mass range