Stringy origin of Minimal Flavor Violation

Hajime Otsuka (KEK)

References :

M. Honda, T. Kobayashi and H.O., arXiv: 1812.03357
T. Kobayashi and H.O., arXiv: 2108.02700

“素粒子物理学の進展2021”, September 8, 2021 (Online)

Introduction (1/3)

- Origin of flavor and CP symmetries : important issue in the SM

- Standard Model Effective Field Theory (SMEFT)
  - powerful bottom-up approach to find signatures of new physics
  - flavor/CP violating processes are sensitive to high-dim. operators

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_n \frac{c_n}{\Lambda_{\rm NP}^{d-4}} O_n^d$$

constructed by assuming
i) matter contents
ii) Lorentz, global/gauge symmetries

e.g., 2499 dim-6 op. in the SMEFT in the absence of any flavor sym.
Grzadkowski-Iskrzynski-Misiak-Rosiek, 1008.4884

Flavor structure of higher-dimensional operators ?

Introduction (2/3)

- Flavor symmetry in the SM ($\bar{\psi} i D \psi$)

  – $G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$

  broken by Yukawa couplings of quarks/leptons: $\bar{Q}_L Y_U^{ij} u_R^j \bar{H}$, …

- Minimal Flavor Violation (MFV) hypothesis :

D' Ambrosio-Giudice-Isidori-Strumia, hep-ph/0207036

i) Yukawa couplings = spurion fields under G_F flavor symmetry

E.g., $Q_L: (3, 1,1,1,1)$ $u_R: (1, 3,1,1,1)$ $Y_U: (3, \bar{3}, 1,1,1)$

ii) All higher-dim. operators are constructed in a G_F invariant way

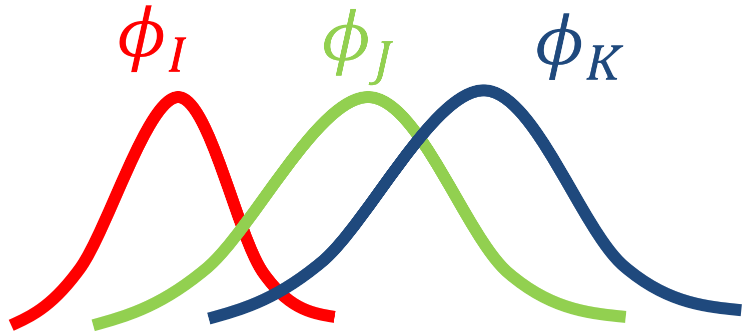
E.g., $\bar{Q}_L Y_U Y_U^\dagger Q_L$ $Y_U Y_U^\dagger: (8,1,1,1,1)$

The MFV hypothesis from UV point of view ?

Introduction (3/3)

- 4D Yukawa couplings of matter zero-modes (in the higher-dim. theory)
  = Overlap integrals of zero-mode wavefunctions on the internal space

$$Y_{IJK} = \int$$



Extra-dimensional space

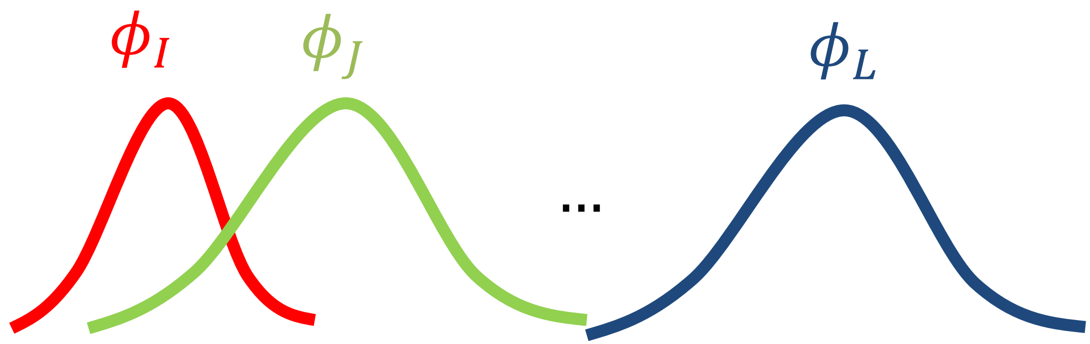
- Geometrical symmetries (and stringy selection rule)
  → Flavor structure of quarks/leptons

- Yukawa couplings $Y(\tau)$ depend on moduli τ (geometric parameters)
  — $< \tau >$ determine the flavor structure and CP violation
  — $Y(\tau)$ transform non-trivially under geometrical symmetry

Short Summary

- 4D n-point couplings of matter zero-modes (in the higher-dim. theory)
  = Overlap integrals of zero-mode wavefunctions on the internal space

$$Y_{IJ\ldots L} = \int$$



Extra-dimensional space

- In various string EFTs,

$$n\text{-point couplings} = (\text{Yukawa coupling})^{n-2}$$

e.g., 4-point = $(\text{Yukawa})^2$

1. Yukawa couplings transform non-trivially under geometrical symmetries (spurions)
2. Flavor/CP-violating processes are controlled by Yukawa couplings

These observation strongly supports the MFV hypothesis !

Outline

1. Introduction/Short summary

2. ***n*-point couplings on T^2**

3. *n*-point couplings on *Calabi-Yau manifolds*
   (vacuum solutions in string theory)

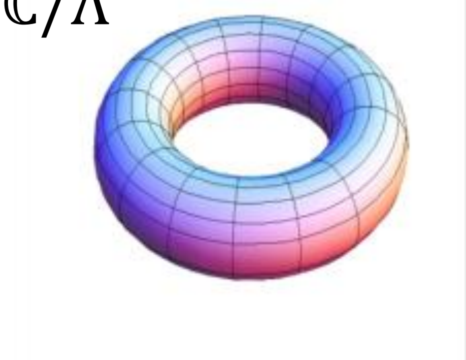
4. Phenomenological implications

5. Conclusion

T^2 torus

- $SL(2,\mathbb{Z})$ geometrical (modular) symmetry

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$



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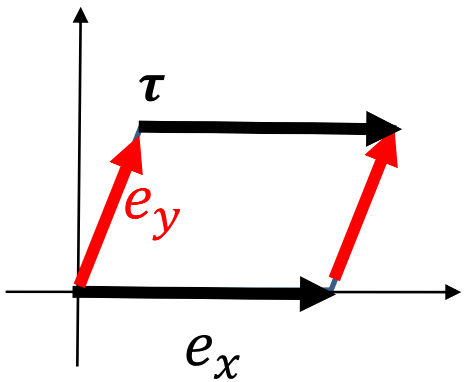


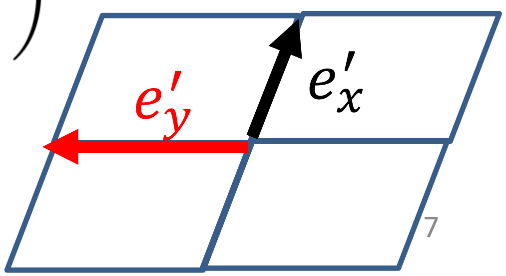
- Lattice vectors are related under $SL(2,\mathbb{Z})$ modular transformation:

$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$p, q, s, t \in \mathbb{Z}$ satisfying $pt - qs = 1$

Two generators : S and T

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



T^2 torus

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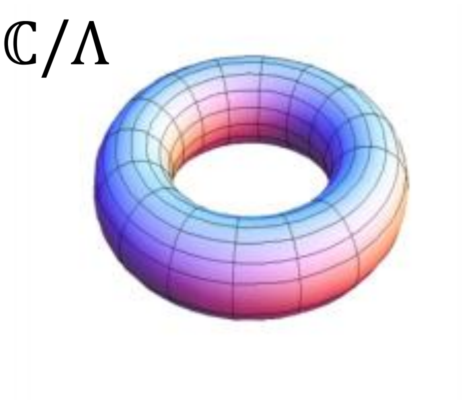

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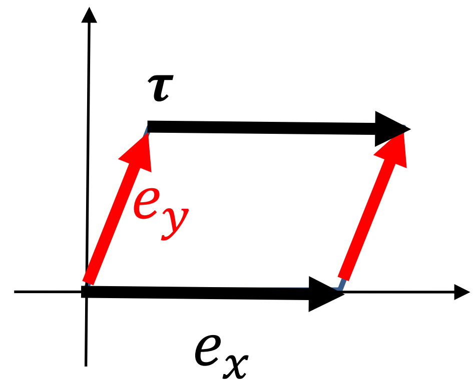

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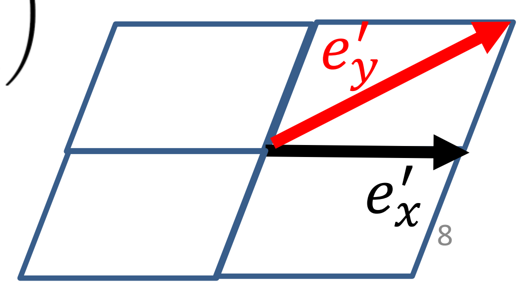
$p, q, s, t \in \mathbb{Z}$ satisfying $pt - qs = 1$

Two generators : S and T

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



Finite subgroups of modular group

$SL(2,\mathbb{Z})$ $\boxed{\{S,T \mid S^2 = -\mathbb{I}, S^4 = (ST)^3 = \mathbb{I}\}}$

Finite subgroups : $\boxed{\Gamma_N = \{S,T \mid S^2 = (ST)^3 = T^N = \mathbb{I}\}}$

$\Gamma_2 \simeq S_3,\ \Gamma_3 \simeq A_4, \Gamma_4 \simeq S_4, \Gamma_5 \simeq A_5, \dots.$

identified with the flavor symmetries of quarks/leptons

F. Feruglio, 1706.08749

- Matter fields and Yukawa couplings : non-trivial representations of Γ_N

$$y_{ij}(\tau)Q_i H U_j$$

$$\tau \to \tau' = \frac{p\tau + q}{s\tau + t}$$

$$y_{ij} \to y'_{ij}$$

Example : D=6 U(N) SYM on $R^{1,3} \times T^2$

$$L = -\frac{1}{4g^2}\ \mathrm{Tr}\,(F^{MN}F_{MN}) + \frac{i}{2g^2}\ \mathrm{Tr}\left(\bar{\lambda}\Gamma^M D_M \lambda\right)$$

$M, N = 0, 1, 2, 3, 4, 5$

Constant U(1) magnetic flux F on T^2

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a\times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b\times N_b} \end{pmatrix}.$$

$N = N_a + N_b$

$M_a, M_b \in \boldsymbol{Z}$

- Flux compactifications lead to
  (i) Gauge symmetry breaking $U(N) \to U(N_a) \times U(N_b)$
  (ii) Chiral fermions

Bi-fundamental field

$$\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$$

KK 展開

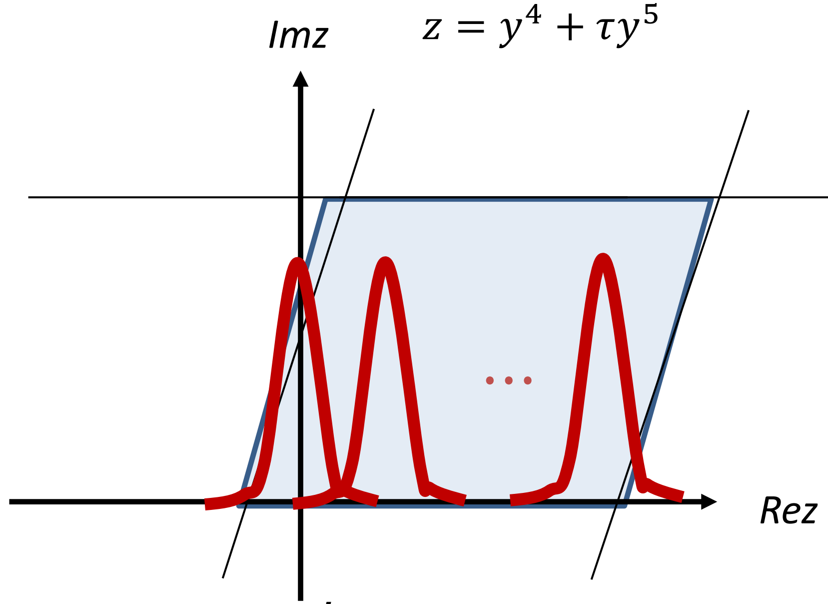
$$\sum_{n=0} \chi_n(x)\psi_n(y)$$

- Zero-mode solution is given by Jacobi-theta function :

$$i\gamma^m D_m \psi_0 = 0 \to \quad \psi^{j,M} \propto \vartheta\begin{bmatrix} \frac{j}{M} \\ 0 \end{bmatrix}(Mz, M\tau)$$

Cremades-Ibanez-Marchesano ('04)

— ***M* degenerate** zero-modes : $\psi^{j,M}$ $(j = 1, \ldots, M)$



- Same functional form for massless scalars $\phi^{j,M}$

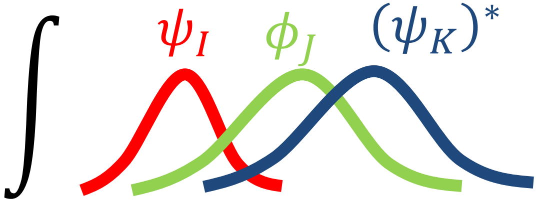
EOM for scalar: $g^{z\bar{z}} D_{\bar{z}} D_z \phi = 0 \to D_z \phi = 0$

- Modular transformations under *S*-transformation ($M =$ even):

$$\psi^{j,M} \to \sum_k e^{2\pi i jk/M} \psi^{k,M}$$

Kobayashi-Nagamoto-Takada-Tamba-Tatsuishi, 1804.06644

- 3-point couplings (fermion-fermion-scalar) :



$$p_{IJK} = \int d^2z\, \psi^{I,M_1} \phi^{J,M_2} (\psi^{K,M_3})^*$$

ゲージ不変性： $M_1 + M_2 - M_3 = 0$

$$i\gamma^m D_m^{(M_1+M_2)}(\psi^{I,M_1} \cdot \phi^{J,M_2}) = 0 \rightarrow \psi^{I,M_1} \cdot \phi^{J,M_2} = \sum_k p_{IJK}\, \psi^{K,M_1+M_2}$$

完全正規直交系

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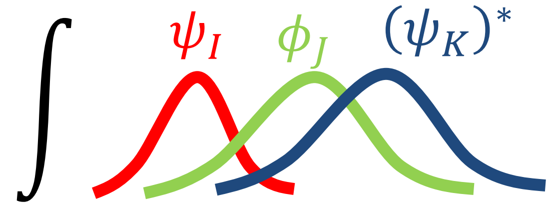

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完全正規直交系

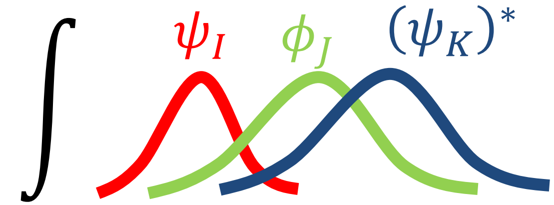
Honda-Kobayashi-Otsuka, 1812.03357

$$\gamma^m D_m^{(M_1+M_2)} (\psi^{I,} \cdot \phi^J)$$
$$= \gamma^m \left[\left(\partial_m + \frac{1}{4} w_{m\alpha\beta} \gamma^{\alpha\beta} \right) \psi^I - i \left[A_m^{(M_1)}, \psi^I \right] \right] \cdot \phi^J + \gamma^m \psi^I \left[\partial_m \phi^J - i \left[A_m^{(M_2)}, \phi^J \right] \right]$$
$$= \gamma^m D_m^{(M_1)} \psi^I \cdot \phi^J + \gamma^m \psi^I D_m^{(M_2)} \phi^J$$
$$= 0$$

$$A_m^{(M_1+M_2)} = A_m^{(M_1)} + A_m^{(M_2)}$$

EOM for scalar: $D_z \phi = 0$

- 3-point couplings (fermion-fermion-scalar) :



$$p_{IJK} = \int d^2z\, \psi^{I,M_1} \phi^{J,M_2} (\psi^{K,M_3})^*$$

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$$i\gamma^m D_m(\psi^{I,M_1} \cdot \phi^{J,M_2}) = 0 \rightarrow \psi^{I,M_1} \cdot \phi^{J,M_2} = \sum_K p_{IJK}\, \psi^{K,M_1+M_2}$$

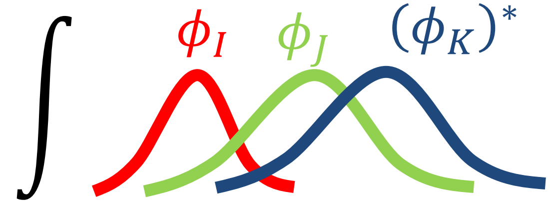
完全正規直交系

$$p_{IJK} = \int d^2z\, p_{IJK} \psi^{M_1+M_2} (\psi^{K,M_3})^* = p_{IJK}$$

$$\boxed{\psi^{I,M_1} \cdot \phi^{J,M_2} = \sum_K p_{IJK}\, \psi^{K,M_1+M_2}}$$

OPE in CFT

- 3-point couplings (scalar-scalar-scalar) :



$$q_{IJK} = \int d^2 z\, \phi^{I,M_1} \phi^{J,M_2} (\phi^{K,M_3})^*$$

ゲージ不変性: $M_1 + M_2 - M_3 = 0$

$$D_z(\phi^{I,M_1} \cdot \phi^{J,M_2}) = 0 \rightarrow \phi^{I,M_1} \cdot \phi^{J,M_2} = \sum_K q_{IJK}\, \phi^{K,M_1+M_2}$$

完全正規直交系

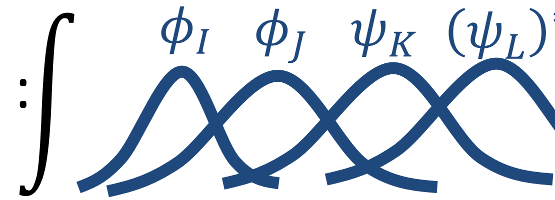
EOM : $g^{z\bar{z}} D_{\bar{z}} D_z \phi = 0$

$$q_{IJK} = \int d^2 z\, q_{IJK} \phi^{M_1+M_2} (\phi^{K,M_3})^* = q_{IJK}$$

$$\boxed{\phi^{I,M_1} \cdot \phi^{J,M_2} = \sum_K q_{IJK}\, \phi^{K,M_1+M_2}}$$

OPE in CFT

- 4-point couplings (scalar-scalar-fermion-fermion) :



(i) $$y_{IJKL} = \int d^2y\, \phi^{I,M_1}\phi^{J,M_2}\psi^{K,M_3}(\psi^{L,M_4})^*$$

$$= \int d^2y \int d^2z\, \phi^{I,M_1}(y)\phi^{J,M_2}(y)\delta^2(y-z)\psi^{K,M_3}(z)(\psi^{L,M_4}(z))^*$$

$$\| $$

$$\sum_{n=0,S=1} \left(\phi_n^{S,M_1+M_2}(y)\right)^* \phi_n^{S,M_1+M_2}(z)$$ *完全系*

$$= \sum_{n,S} \int d^2y\, q_{IJK}\phi^{K,M_1+M_2}(y)\left(\phi_n^{S,M_1+M_2}(y)\right)^* \int d^2z\, \phi_n^{S,M_1+M_2}(z)\psi^{K,M_3}(z)(\psi^{L,M_4}(z))^*$$

Non-zero for $n=0$ *(直交系)*

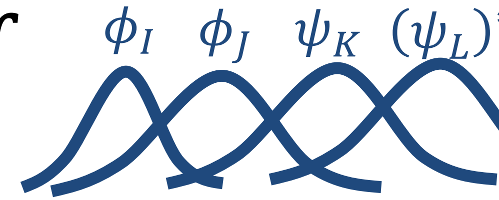
$$\phi^{I,M_1}\cdot\phi^{J,M_2} = \sum_K q_{IJK}\,\phi^{K,M_1+M_2}$$

$$= \sum_S q_{IJS}p_{SKL}$$

q_{IJK}: (scalar-scalar-scalar)
p_{IJK}: (fermion-fermion-scalar)

Abe-Choi-Kobayashi-Ohki, 0903.3800
Honda-Kobayashi-Otsuka, 1812.03357

- 4-point couplings (scalar-scalar-fermion-fermion) : $\int$



(i)

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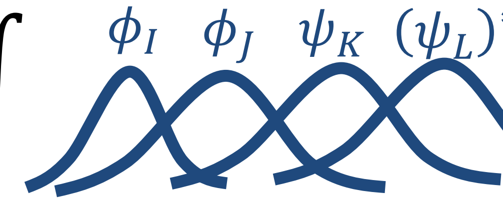
$$\parallel$$

$$\sum_{n=0,S=1} \left(\phi_n^{S,M_1+M_2}(y)\right)^* \phi_n^{S,M_1+M_2}(z) \quad 完全系$$

$$= \sum_S q_{IJS}p_{SKL}$$

$$\phi^{I,M_1}\cdot\phi^{J,M_2} = \sum_K q_{IJK}\,\phi^{K,M_1+M_2}$$

- 4-point couplings (scalar-scalar-fermion-fermion) : $\int$



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$$\parallel$$

$$\sum_{n=0,s=1} \left(\phi_n^{S,M_1+M_2}(y)\right)^* \phi_n^{S,M_1+M_2}(z) \quad 完全系$$

$$= \sum_S q_{IJS}p_{SKL}$$

$$\phi^{I,M_1}\cdot\phi^{J,M_2} = \sum_K q_{IJK}\,\phi^{K,M_1+M_2}$$

(ii) $$y_{IJKL} = \int d^2y \int d^2z \phi^{I,M_1}(z)\phi^{J,M_2}(y)\delta^2(y-z)\psi^{K,M_3}(y)(\psi^{L,M_4}(z))^*$$

$$\parallel$$

$$\sum_{n=0,T=1} \left(\psi_n^{T,M_2+M_3}(y)\right)^* \psi_n^{T,M_2+M_3}(z) \quad 完全系$$

$$= \sum_T p_{ITL}p_{JTK}$$

$$\psi^{I,M_1}\cdot\phi^{J,M_2} = \sum_K p_{IJK}\,\psi^{K,M_1+M_2}$$

- 4-point couplings (scalar-scalar-fermion-fermion) :

q_{IJK}: (scalar-scalar-scalar)
p_{IJK}: (fermion-fermion-scalar)

$$y_{HIJK} = \sum_S q_{HIS} p_{SJK} = \sum_T p_{ITL} p_{JTK}$$

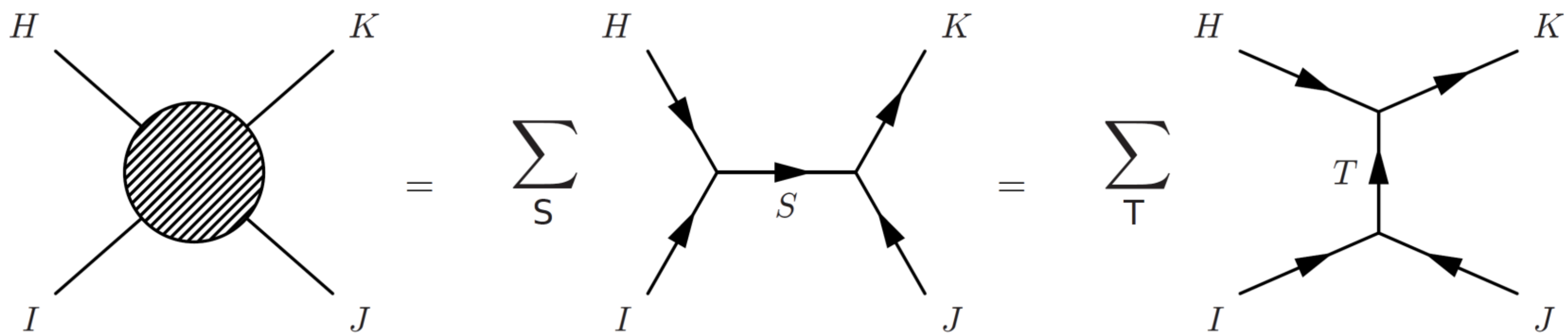

Figure 1: Four-point couplings and crossing symmetry

from 1812.03357

- 4-point = $(\mathrm{Yukawa})^2$ relation hold for $(\mathrm{scalar})^4, (\mathrm{fermion})^4$

- Similarly, *n*-point = $(\mathrm{Yukawa})^{n-2}$

- Matter fields and Yukawa couplings transform non-trivially under (subgroups of) $SL(2,\mathbb{Z})$ (i.e., Yukawa = spurion)

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   (vacuum solutions in string theory)

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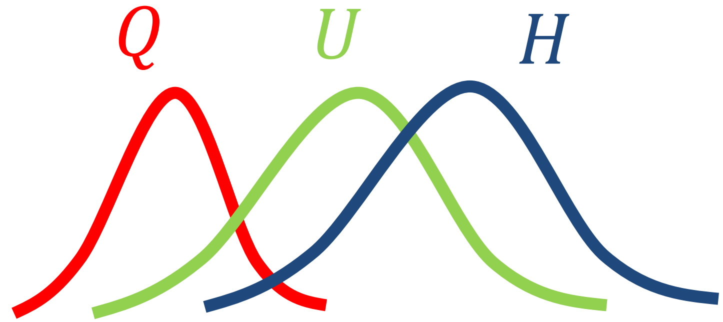
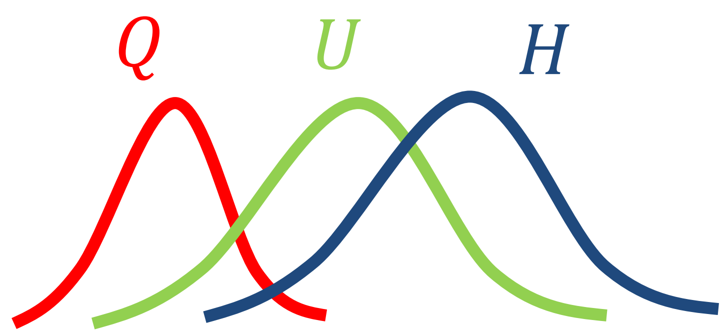
$$Y = \int_{\text{Torus}}$$



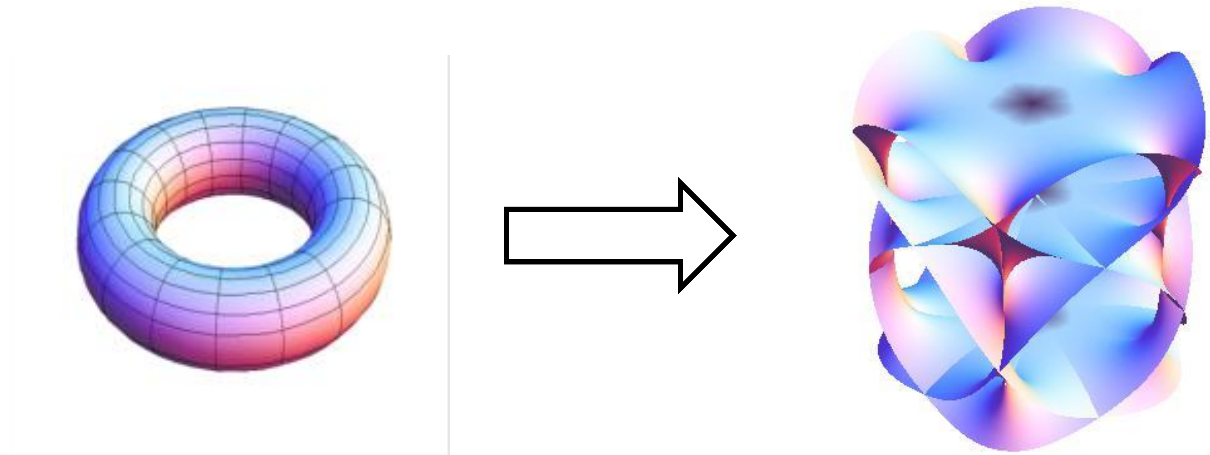
<u>Geometrical symmetry on 6D Calabi-Yau threefolds</u> ?

(Vacuum solutions in string theory)

$$Y = \int_{\text{Calabi-Yau}}$$



6D CY : $Sp(2h+2,\mathbb{Z})$ symplectic modular symmetry



h : # of moduli fields

$Sp(2,\mathbb{Z}) \simeq SL(2,\mathbb{Z})$

A. Strominger ('90),
P. Candelas, X. de la Ossa ('91)

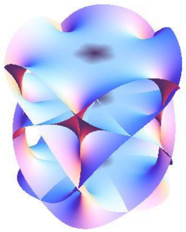
- (dual) basis $\{\alpha^I, \beta_I\}$ of three cycles is related under $Sp(2h+2,\mathbb{Z})$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \to \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \qquad I = 0,1,\ldots,h$$

$\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ preserves the symplectic matrix: $\Sigma = \begin{pmatrix}0 & \mathbf{1}\\ -\mathbf{1} & 0\end{pmatrix}$

$E_8 \times E_8$ Heterotic string on 6D CY (standard embedding)

Candelas-Horowitz-Strominger-Witten ('85)



- 4D Gauge symmetry :

$$E_6 \times E_8$$

- Moduli $\approx$ Matters (E_6 : 27 or $\overline{27}$)

27^i : Kaehler moduli
$\overline{27}^a$: Complex structure moduli

- Yukawa couplings (27^3)

$$W = F_{ijk} 27^i 27^j 27^k$$

$F_{ijk} = \partial_i \partial_j \partial_k F$
F : prepotential

Symplectic modular symmetry in CY moduli space
$\sim$ Flavor symmetry

- S_4 flavor symmetry

$$W = 27^1 27^2\, 27^3$$

$F_{123} = 1$, otherwise 0

— Invariant under two generators

$$\mathrm{P} : 1 \to 3,\, 2 \to 1,\, 3 \to 2$$
$$\mathrm{Q} : 1 \to -1,\, 2 \to -3,\, 3 \to 2$$

— S_4 triplet : $\{27^1, 27^2, 27^3\}$

— $S_4 \subset Sp(2 \times 3 + 2, \mathbb{Z}) = \boxed{Sp(8, \mathbb{Z})}$ $\quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \to \boxed{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

- realized in some classes of 6D Calabi-Yaus and toroidal orbifolds

Ishiguro-Kobayashi-Otsuka, 2107.00487

- Symplectic transf. of matters (27^i) and Yukawa couplings (F_{ijk})

$$27^i \to \widetilde{27}^i = \left(\tilde{X}^0\right)^{1/3} \frac{\partial \tilde{X}^i}{\partial X^j} 27^j$$

$$F_{ijk} \to \tilde{F}_{ijk} = \frac{\partial X^l}{\partial \tilde{X}^i} \frac{\partial X^m}{\partial \tilde{X}^j} \frac{\partial X^n}{\partial \tilde{X}^k} F_{lmn}$$

(X^0, X^i): projective coordinates
with the gauge $X^0 = 1$
($i = 1,2, \ldots, h$)

— Non-trivial representations of $Sp(2h+2, \mathbb{Z})$

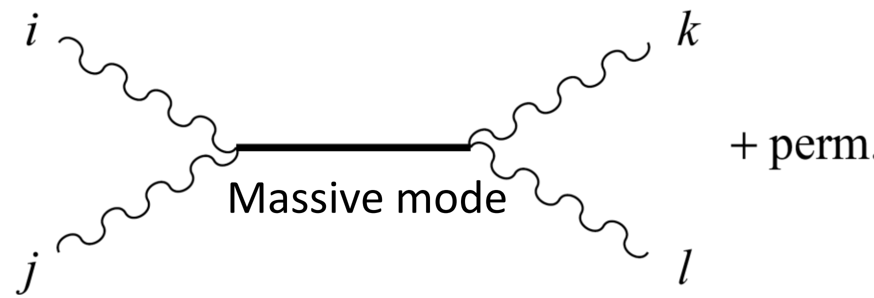
— Flavor symmetry : $\mathrm{G}_{\mathrm{flavor}} \subset Sp(2h+2, \mathbb{Z})$

Higher-order couplings in SUSY E_6 GUT

Bershadsky-Cecotti-Ooguri-Vafa ('93)

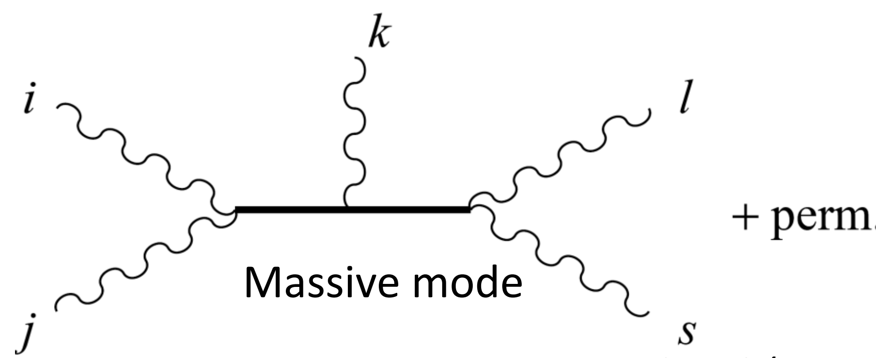
- Dimension-5

$$\frac{F_{ijkl}}{\Lambda} 27_i 27_j 27_k 27_l$$



- Dimension-6

$$\frac{F_{ijkls}}{\Lambda^2} 27_i 27_j 27_k 27_l 27_s$$



From hep-th/9309140

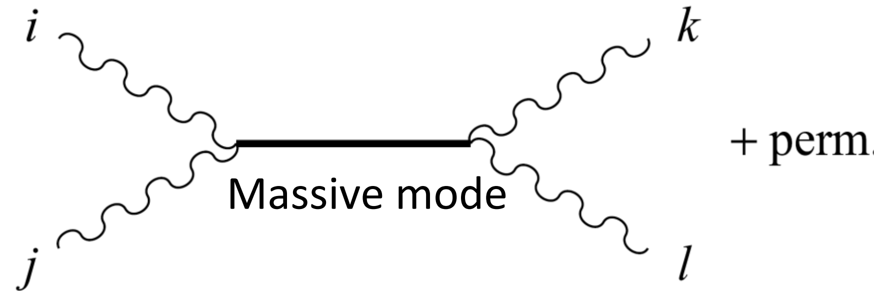
- n-point couplings : $F_{ij\ldots n} = \partial_i \partial_j \ldots \partial_n F$ F : prepotential
  — Non-trivial representations under symplectic modular symmetry of CY moduli space

Higher-order couplings in SUSY E_6 GUT

Bershadsky-Cecotti-Ooguri-Vafa ('93)

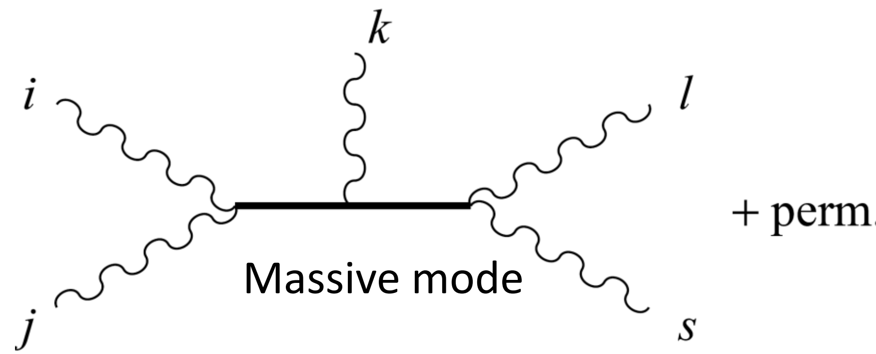
- <u>Dimension-5</u>

$$\frac{F_{ijkl}}{\Lambda} 27_i 27_j 27_k 27_l$$



- <u>Dimension-6</u>

$$\frac{F_{ijkls}}{\Lambda^2} 27_i 27_j 27_k 27_l 27_s$$



From hep-th/9309140

- Prepotential : $F = F_{\text{cubic polynomial}} + F_{\text{instanton}}$

  E.g., $F_{ijkl} = \partial_i \partial_j \partial_k \partial_l F_{\text{instanton}}$ are exponentially suppressed

  -> no dangerous flavor/CP-violating processes

Outline

1. Introduction/Short summary

2. *n*-point couplings on T^2

3. *n*-point couplings on *Calabi-Yau manifolds*
   (vacuum solutions in string theory)

4. **Phenomenological implications**

5. Conclusion

($*$) n-point couplings $\propto$ (Yukawa coupling)$^{n-2}$

- $\Delta F = 2$ quark operators :

$$\frac{c_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i\gamma_\mu P_{L,R}Q_j)(\bar{Q}_k\gamma^\mu P_{L,R}Q_l)$$

- $\Delta F = 1$ semileptonic operators :

$$\frac{d_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i\gamma_\mu P_{L,R}Q_j)(\bar{L}_k\gamma^\mu P_{L,R}L_l)$$

—These couplings are decomposed into moduli-dependent Yukawa :

$$\sum_m y_{ijm}y_{mkl}$$

ϕ_m: heavy superpartners or stringy modes

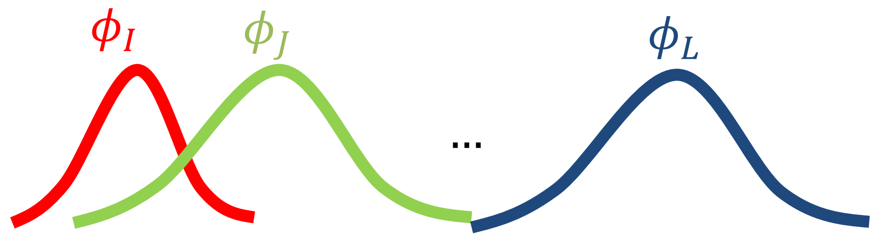
- Yukawa couplings in the string EFTs : Rank 1 (at the leading order) (corresponding to U(2) flavor symmetry)

— interesting to discuss recent flavor anomalies

Conclusion

- <u>String EFTs satisfy the criterion of MFV hypothesis</u>

$$Y_{IJ\ldots L} = \int$$



Extra-dimensional space

— n-point couplings $\propto$ (Yukawa coupling)$^{n-2}$
inherited from OPE in the 2D CFT

> 1. Flavor/CP-violating processes are controlled by Yukawa couplings
> 2. Yukawa couplings transform non-trivially under geometrical symmetries (spurions)

- The MFV scenario works in the low-energy EFT below the compactification scale

($*$) *n*-point couplings $\propto$ $(\text{Yukawa coupling})^{n-2}$

- Magnitudes of CP violation in Yukawa and higher-order couplings have a common origin in string axions

- String EFTs satisfy the MFV hypothesis at the compactification scale Λ
- Various physical stages may occur between Λ and Λ_{EW}

(i) Some modes become massive
-> New operators appear after integrating out Φ_{massive} keeping (*)

(ii) Some modes develop their VEVs
— E.g., $y_{ijkl}\langle\phi_i\rangle\phi_j\phi_k\phi_l$
New couplings $y_{ijkl}\langle\phi_i\rangle$: spurion as a function of $\langle\phi_i\rangle$ and moduli