NATIONAL SCIENCE CENTRE
POLAND

UNIVERSITY OF WARSAW

On the phenomenology of sphaleron-induced processes at the LHC and beyond

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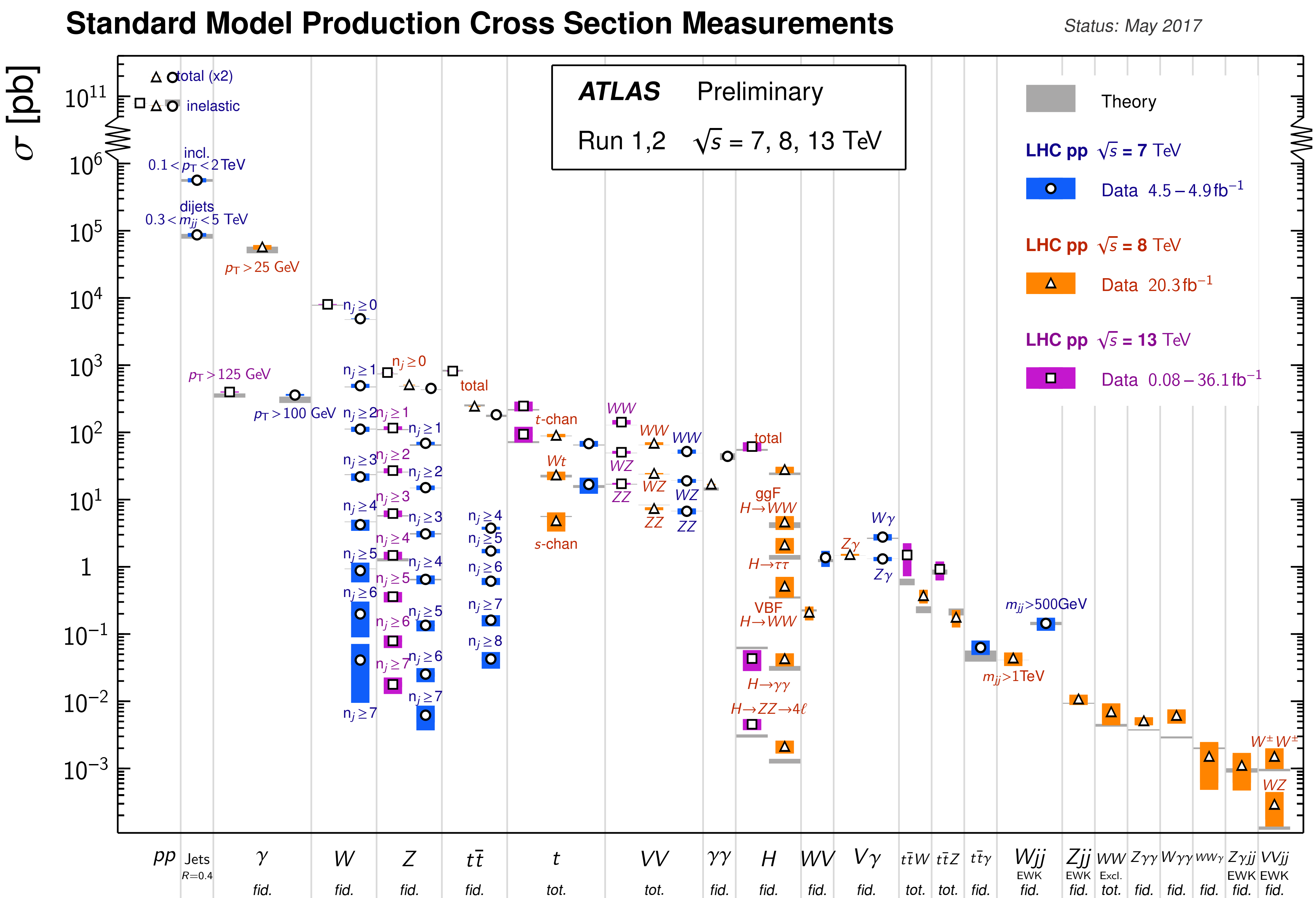
Perturbative sector of EW theory is very well tested!

- Remarkable agreement between experimental results and perturbative calculations.



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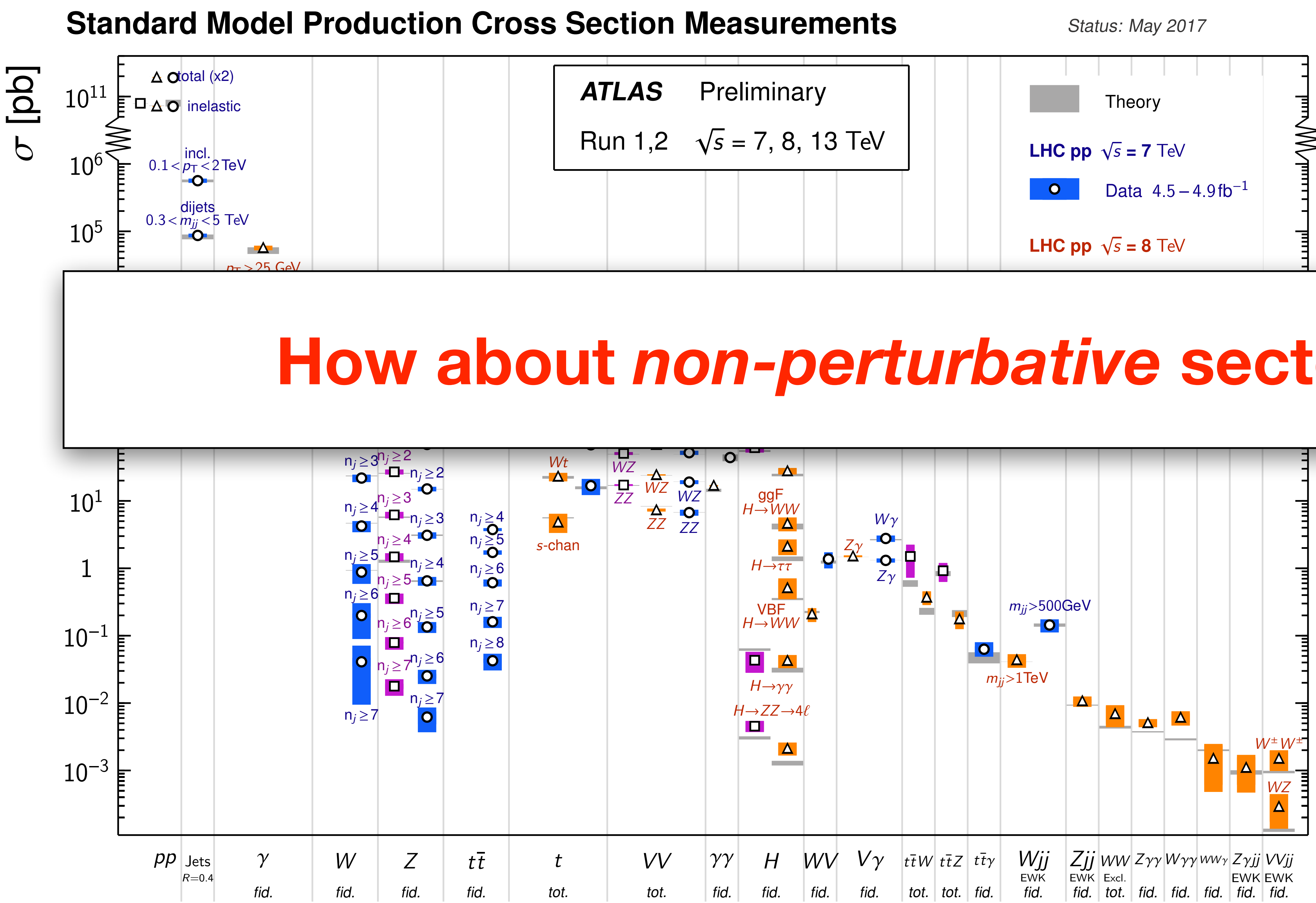

How about *non-perturbative* sector?

Vacua of EW theory

action: $S_{\mathrm{EW}} = -\dfrac{1}{2g^2}\int d^4x\, \mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$ $\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$

gauge trans.: $A_\mu \to U^\dagger[A_\mu + i\partial_\mu]U$

a vacuum: $A_\mu = 0 \quad \leftrightarrow \quad A_\mu = U^\dagger \partial_\mu U$

- There are as many vacua as $U_{ij}(\mathbf{x})$

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gauge trans.: $A_\mu \to U^\dagger[A_\mu + i\partial_\mu]U$

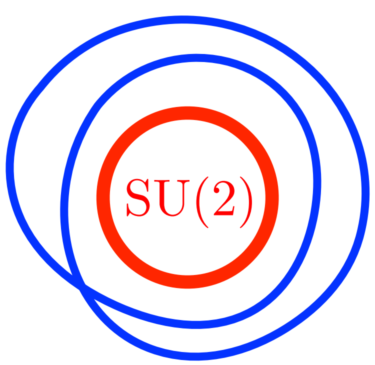
a vacuum: $A_\mu = 0 \quad \leftrightarrow \quad A_\mu = U^\dagger \partial_\mu U$ $SU(2) \ni U = a + i(\mathbf{b}\cdot\sigma)$

$a^2 + \mathbf{b}^2 = 1$

- There are as many vacua as $U_{ij}(\mathbf{x})$

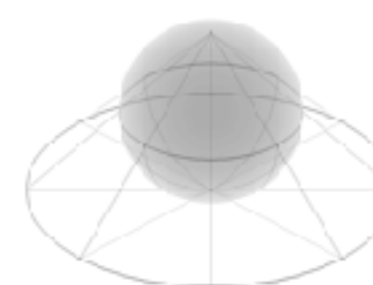
topological gauge

$U(\infty) \to \mathbf{1}$

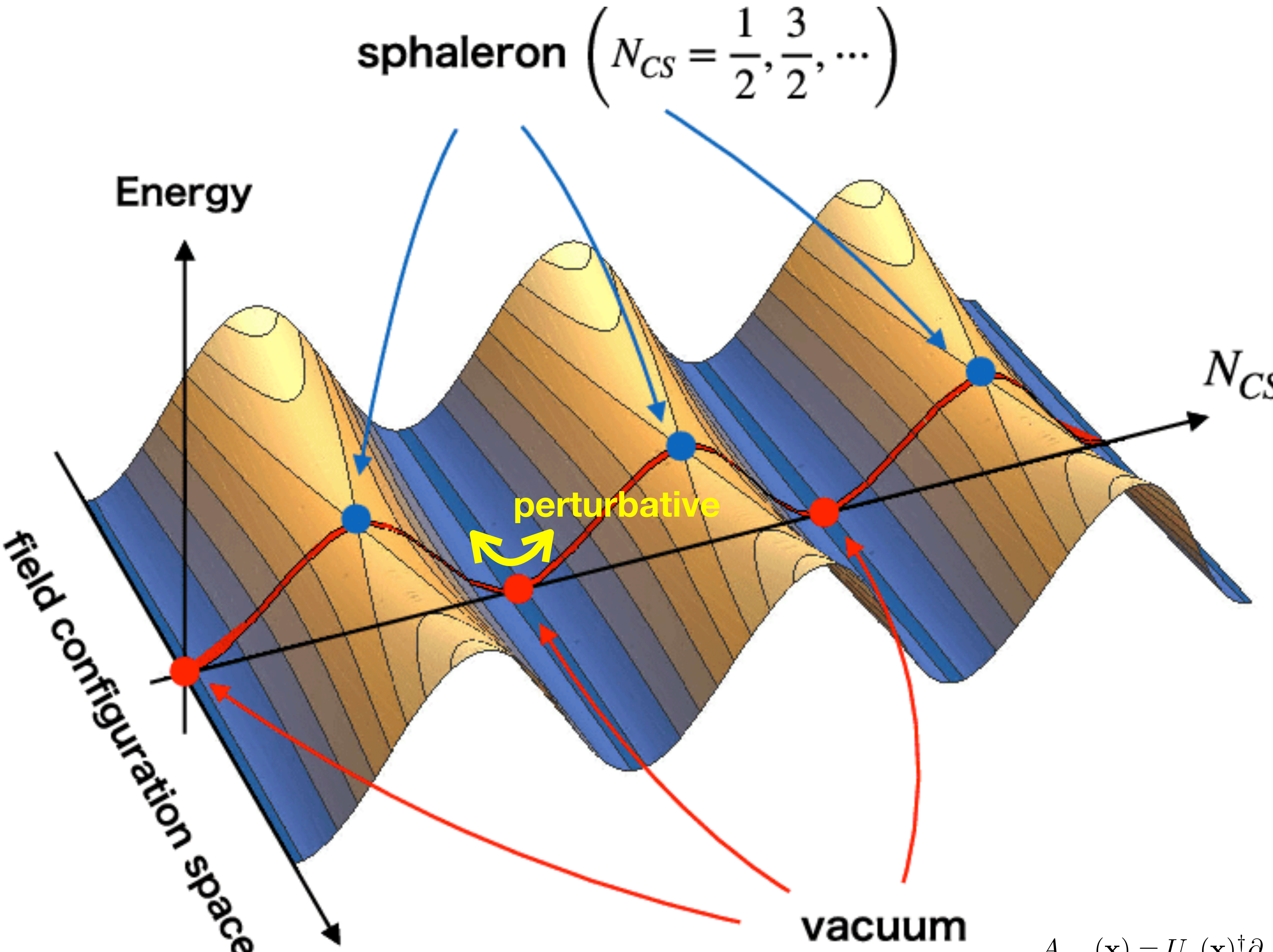


$\pi_3(S^3) = \mathbb{Z}$

$\mathrm{SU}(2) \cong S^3 \xleftarrow{\text{map}} S^3 \cong \mathbf{R}^3 \cup \{\infty\}$

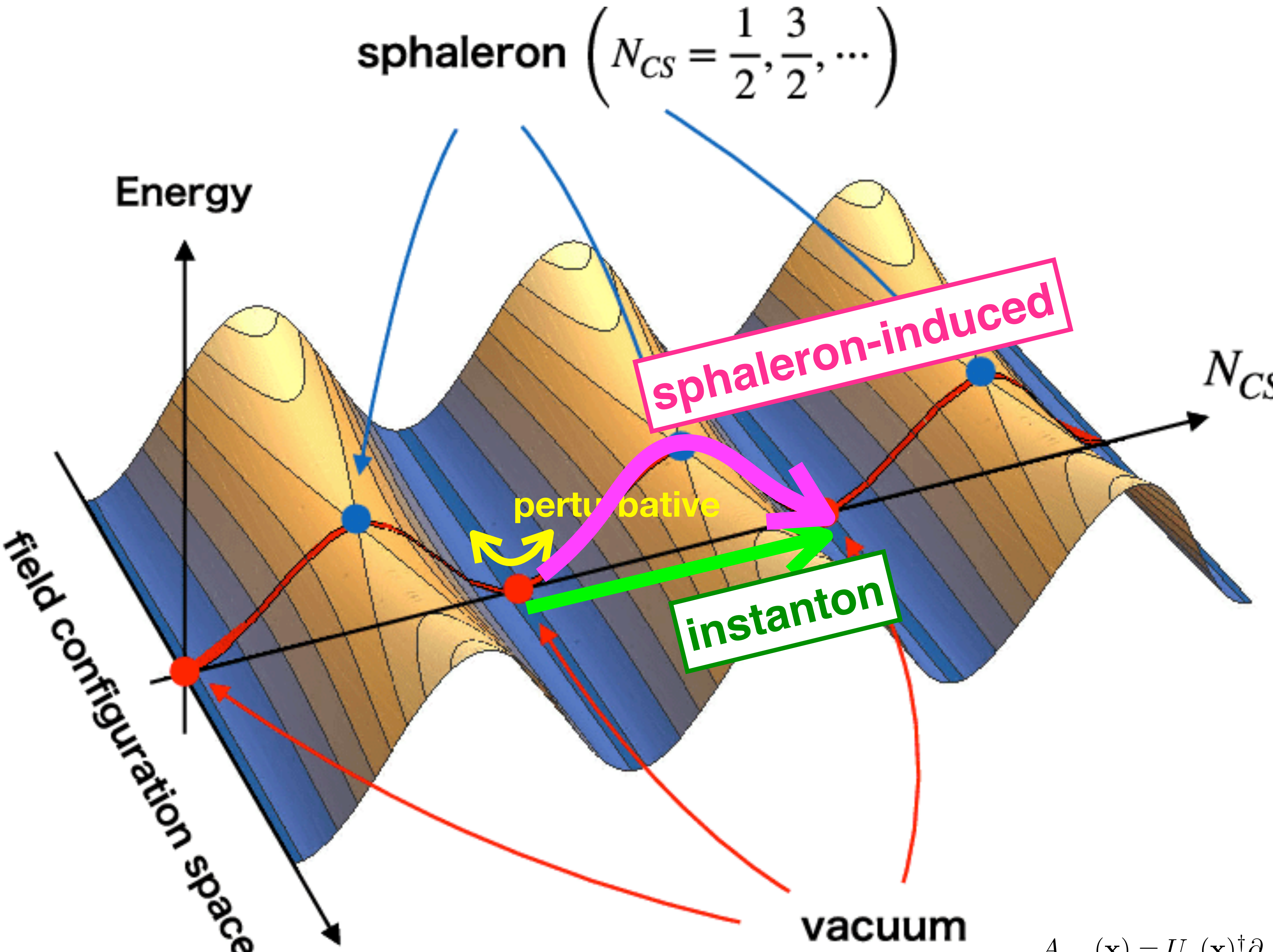


The map has distinctive sectors classified by the winding number!



$$A_{n,\mu}(\mathbf{x}) = U_n(\mathbf{x})^\dagger \partial_\mu U_n(\mathbf{x})$$

$$U_n(\mathbf{x}) = \exp\left(in\pi \frac{\mathbf{x}\cdot\sigma}{\sqrt{\mathbf{x}^2 - \rho^2}}\right)$$



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How does it look like?

- A "current" carrying the winding number:

$$K_\mu \;=\; \frac{1}{8\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}A^\nu(\partial^\rho A^\sigma + \frac{2}{3}A^\rho A^\sigma)$$

- One can show

$$\int K_0(A_n(\mathbf{x}))d^3x = n\,, \qquad \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \;=\; \partial^\mu K_\mu$$

- This implies

$$\int \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}d^4x \;=\; \int \partial^\mu K_\mu d^3x dt \;\;=\;\; \Big[\int K_0(t,\mathbf{x})d^3x\Big]_{t=-\infty}^{t=\infty}$$

$$=\; n(t=\infty) - n(t=-\infty) \;\;=\; \Delta n$$

How does it look like?

$$\Delta n \; = \; \frac{g^2}{16\pi^2} \int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x$$

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$$\Delta n = \frac{g^2}{16\pi^2} \int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x \underbrace{=}_{\text{anomaly}} \Delta N_F$$

SU(2) charged fermion

How does it look like?

$$\Delta n \;=\; \frac{g^2}{16\pi^2}\int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x \underset{\text{anomaly}}{=} \left\{ \begin{array}{l} \color{red}{\Delta N_{q_1^r}} \\ \color{green}{\Delta N_{q_1^g}} \\ \color{blue}{\Delta N_{q_1^b}} \\ \Delta N_{\ell_1} \end{array} \right. \quad \times\, 3\ \text{flavour}$$

How does it look like?

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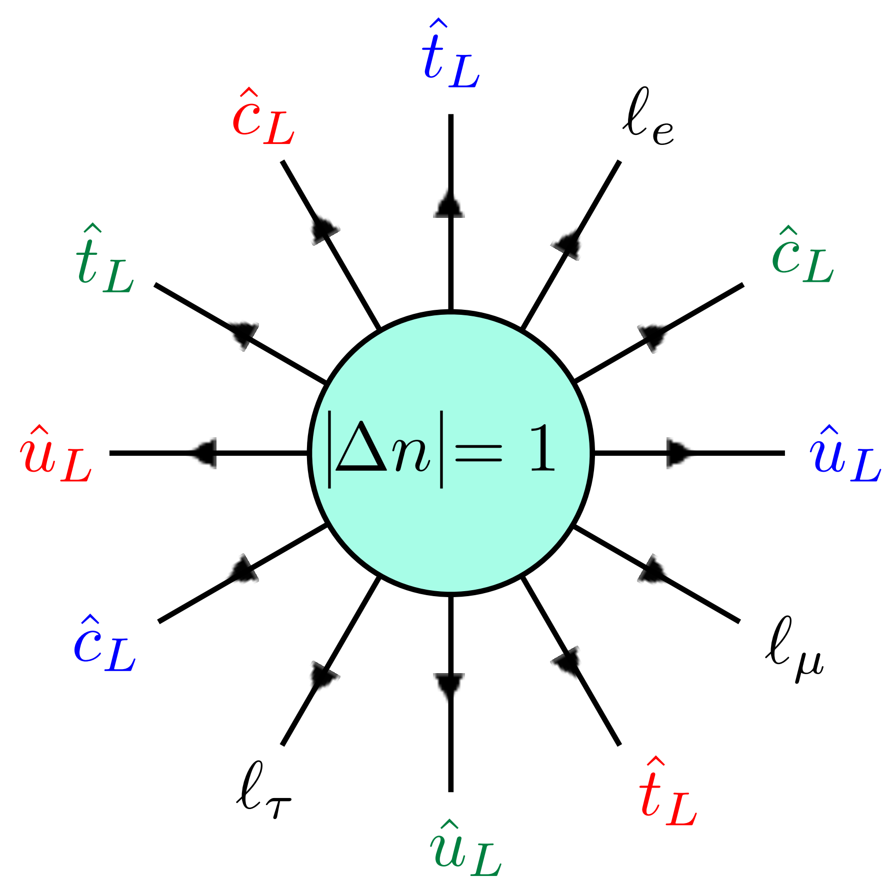
- ΔN_{CS} is related to the change of SU(2) charged fermion numbers.

$$\Delta B = \Delta L = 3\Delta N_{CS}$$

$$\Delta(B+L) \neq 0$$

$$\Delta(B-L) = 0$$



|Δn| = 1 transition creates 12 fermions altogether!

Party at collider!

The tunnelling rate can be estimated using the WKB approximation as

$$\langle n | n + \Delta n \rangle \sim e^{-\hat{S}_E}$$

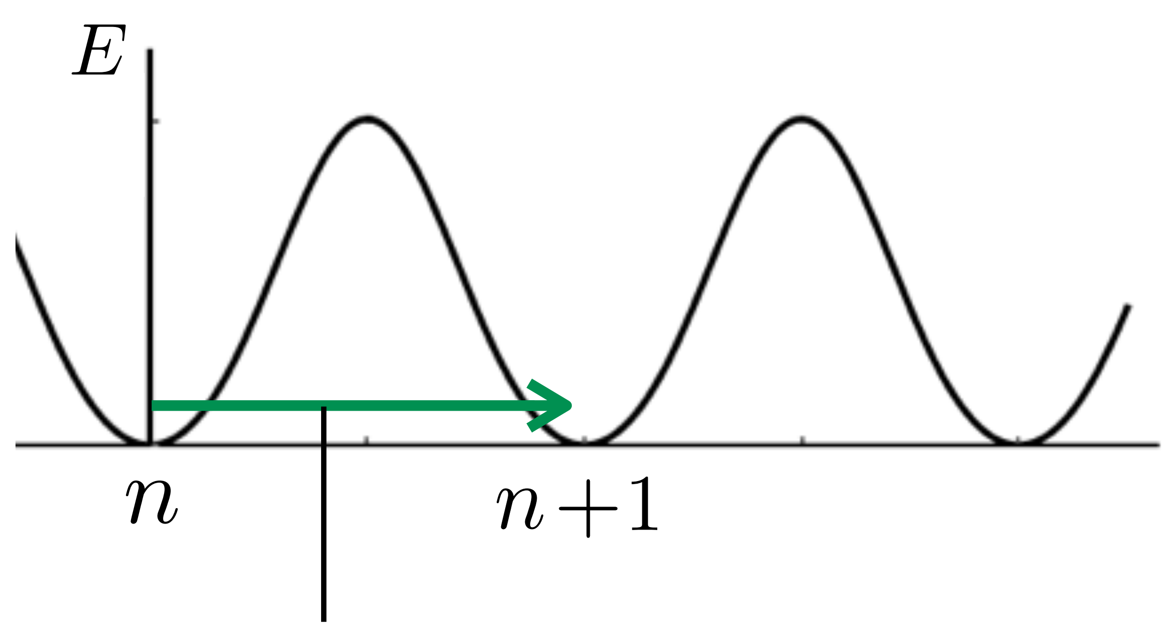
S_E is the Euclidean action at the stationary point, which is given by

$$\begin{aligned} \hat{S}_E &= \frac{1}{2g^2} \int F F d^4x \\ &= \frac{1}{2g^2} \left| \int F \widetilde{F} d^4x \right| \\ &= \frac{8\pi^2}{g^2} |\Delta n| \end{aligned}$$

Note that:

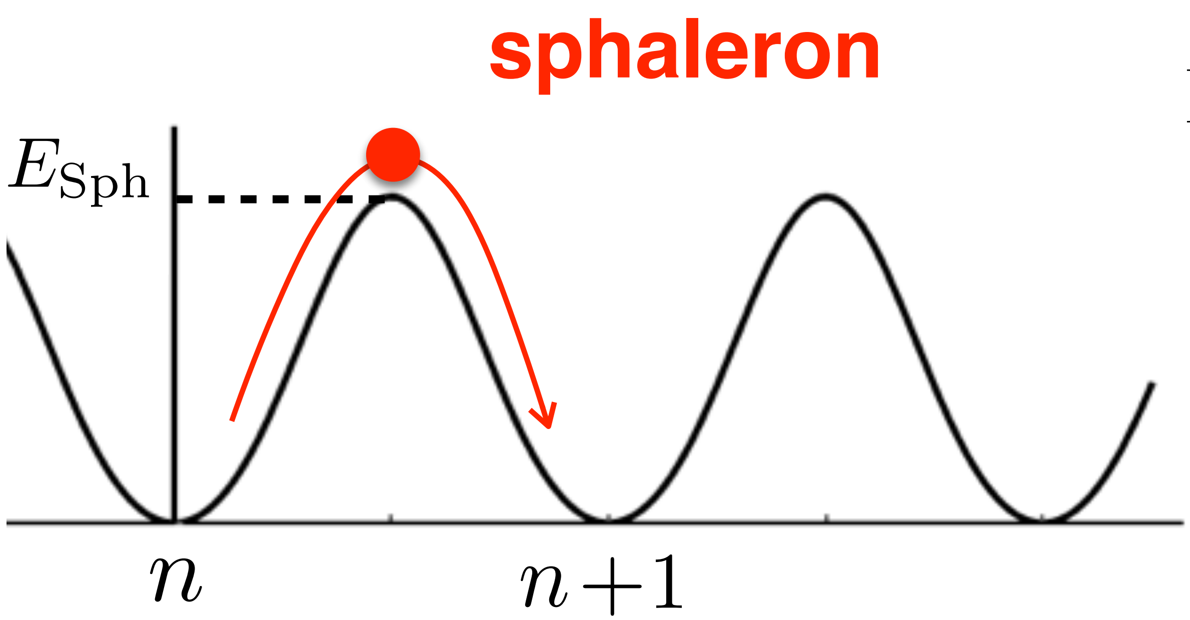
$$\int (F \pm \tilde{F})^2 d^4x \geq 0$$

$$\Longrightarrow \int F F d^4x \geq \left| \int F \tilde{F} d^4x \right|$$



$$e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-170}$$

The tunnelling rate is unobservably small



The barrier hight was calculated by F.R.Klinkhamer and N.S.Manton (1984)

$$E_{\rm Sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right)$$

$$\simeq 9\,\mathrm{TeV} \quad (\text{for } m_H = 125\mathrm{GeV})$$

- At high temperature, the sphaleron rate may be unsuppressed.

$$\Gamma \propto \exp\left(-\frac{E_{\rm Sph}(T)}{T}\right)$$

It plays an important role in baryo(lepto)genesis.

What happens for the high energy (zero temperature) case?

Cross-section estimate

LSZ formula:

$$\langle f\,|\,S\,|\,i\rangle \;=\; \left[i\int d^4x_1 e^{-ip_1x_1}(\Box_1+m_1^2)\right]\cdots\left[i\int d^4x_n e^{-ip_nx_n}(\Box_n+m_n^2)\right]\cdot\langle\Omega\,|\,T\{\phi_1(x_1)\cdots\phi_n(x_n)\}\,|\,\Omega\rangle$$

Path-integral:

$$\langle\Omega\,|\,T\{\phi_1(x_1)\cdots\phi_n(x_n)\}\,|\,\Omega\rangle \;=\; \frac{\int\mathscr{D}\phi\;\phi_1(x_1)\cdots\phi_n(x_n)\,e^{iS[\phi]}}{\int\mathscr{D}\phi\;e^{iS[\phi]}}$$

- Matrix elements for Δn = 1 processes may be obtained by

$$i\mathscr{M}(\Delta n=1) = \left.\frac{\int\mathscr{D}\phi\,|_{\Delta n=1}\;\phi_1(x_1)\cdots\phi_1(x_n)\,e^{iS}}{\int\mathscr{D}\phi\;e^{iS}}\right|_{\rm LSZ}$$

$$\sim\left.\frac{\int\mathscr{D}\delta\phi\;\tilde\phi_1(x_1)\cdots\tilde\phi_1(x_n)\,e^{iS(\tilde\phi+\delta\phi)}}{\int\mathscr{D}\phi\;e^{iS}}\right|_{\rm LSZ}$$

stationary configuration with Δn = 1

$\downarrow$

$$\phi_i \;=\; \tilde\phi_i \;+\; \delta\phi_i$$

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h})\exp(-S_E)\Big|_{\mathrm{LSZ}}$$

[Ringwald '90, Espinosa '90]

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- Evaluate it at the instanton configuration:

[Ringwald '90, Espinosa '90]

$$W^{\mu a}_{\mathrm{inst}} \simeq \frac{2\rho^2}{g}\underbrace{U_{ab}}_{\text{orientation}}\frac{\bar{\eta}_{b\mu\nu}(x-x_0)_\nu}{(x-x_0)^2[(x-\underbrace{x_0}_{\text{position}})^2+\underbrace{\rho^2}_{\text{size}}]} \qquad \phi_{\mathrm{inst}}(x) \simeq v\left[\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right]^{1/2}$$

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orientation position size

- Integration over orientation, position, size and phase-space:

$$\sigma(n_W, n_h) \sim \int |\mathcal{M}|^2 \cdot d\Phi_{\rm PS}$$

- The LO Matrix Element in the *instanton* **background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12}) W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h})\exp(-S_E)\Big|_{\rm LSZ}$$

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- Result [Ringwald '90, Espinosa '90]

$$\sigma_{\rm LO}(n_W, n_h) \sim \mathscr{G}^2 2^n v^{-2n}\left[\frac{\Gamma(n+103/12)}{\Gamma(103/12)}\right]^2 \frac{1}{n_B!\,n_H!} \qquad \mathscr{G} = 1.6\times 10^{-101}\ \mathrm{GeV}^{-14}$$

$$\times \int \prod_{i=1}^{10} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} E_i \prod_{j=1}^{n_B} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j} \frac{2(4E_j^2 - m_W^2)}{m_W^2} \prod_{k=1}^{n_H} \frac{\mathrm{d}^3 p_k}{(2\pi)^3 2E_k} (2\pi)^4 \delta^{(4)}\left(P_{\rm in} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n_B} p_j - \sum_{k=1}^{n_H} p_k\right)$$

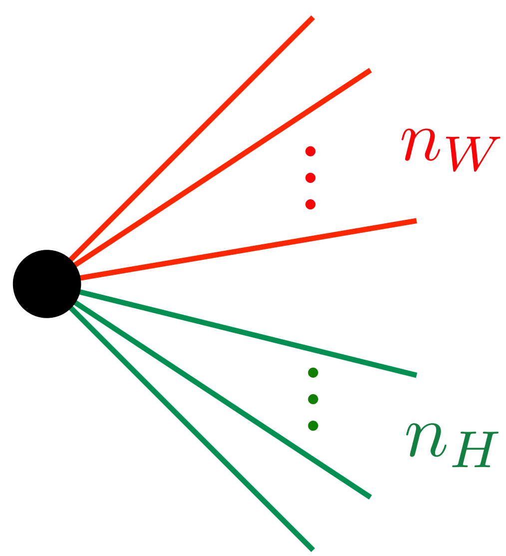
The cross-section grows with energy and the number of final state bosons.

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})\, W(y_1)\cdots W(y_{n_W})\, \phi(z_1)\cdots \phi(z_{n_h}) \exp(-S_E)\Big|_{\mathrm{LSZ}}$$

$$A^{\mathrm{inst}\,a}_{\ \mu}(x_i) \xrightarrow{\mathbf{FT}} \frac{4i\pi^2\rho^2}{g}\,\frac{\bar\eta^a_{\mu\nu}p_i^\nu}{p_i^2(p_i^2+m_W^2)}\,e^{ip_i x_0} \xrightarrow{\mathbf{LSZ}} \boxed{\frac{4i\pi^2\rho^2}{g}\,\frac{\bar\eta^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_i x_0}}$$

$$H^{\mathrm{inst}}(x_j) \to -\frac{2\pi^2\rho^2 v}{(p_j^2+m_H^2)}\,e^{ip_j x_0} \to \boxed{-2\pi^2\rho^2 v\, e^{ip_j x_0}}$$

- **Multi-particle interaction under the instanton BG is (almost) a point-like vertex**



$$i\mathcal{M} \sim \left[\frac{4i\pi^2\rho^2}{g}\,\frac{\bar\eta^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_i x_0}\right]^{n_W}\left[-2\pi^2\rho^2 v\, e^{ip_j x_0}\right]^{n_H}$$

$$\Phi_n(Q) \sim (Q^2)^{n-2}$$

↑
n-body phase-space

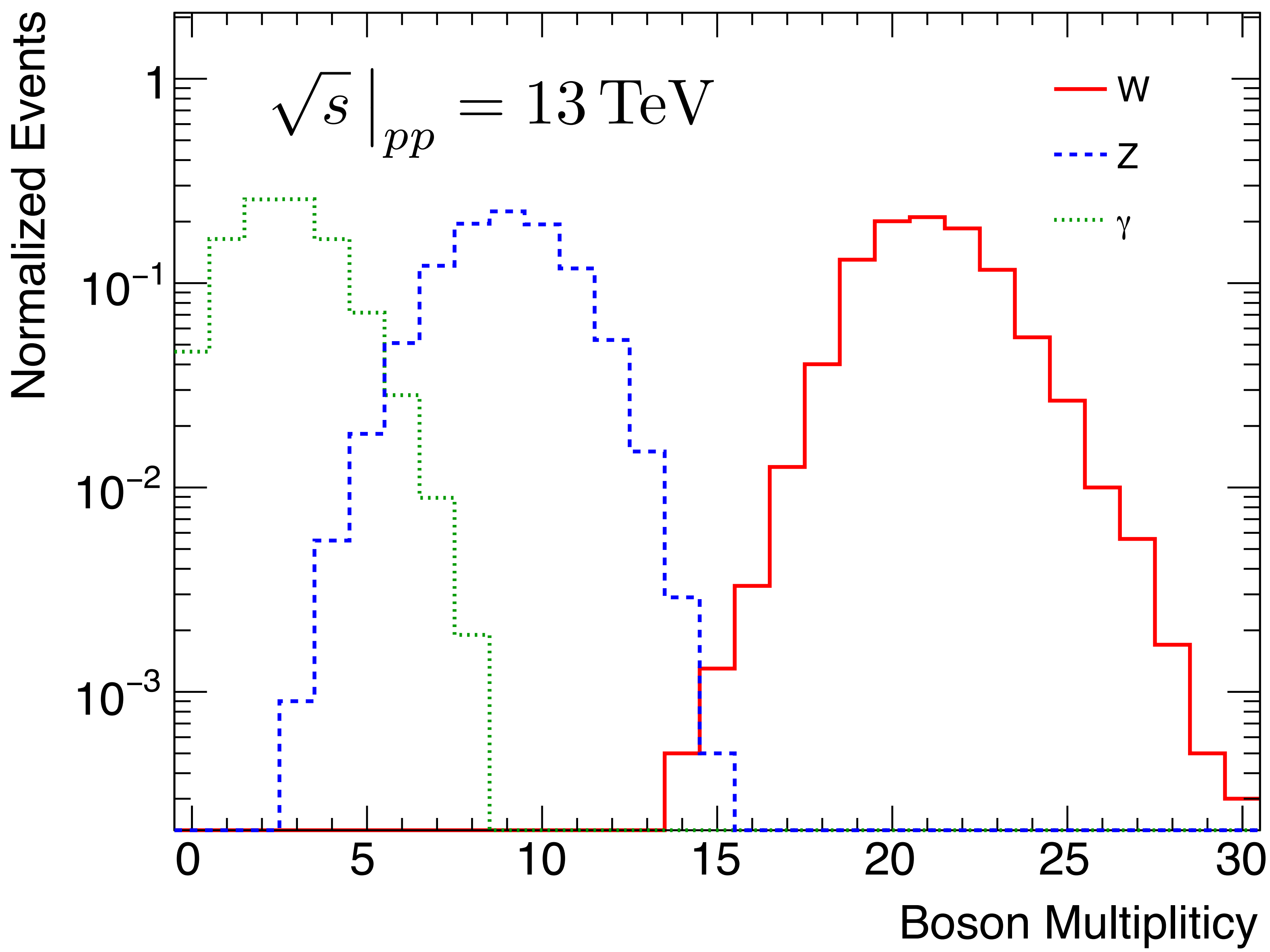
Such a vertex is highly unrenormalisable and high energy behaviour is not regulated.

Enhancement at large *nW* and *nH*.

• The MC Event Generator (**HERBVI**) using the LO ME formula:

[Gibbs, Webber '95]

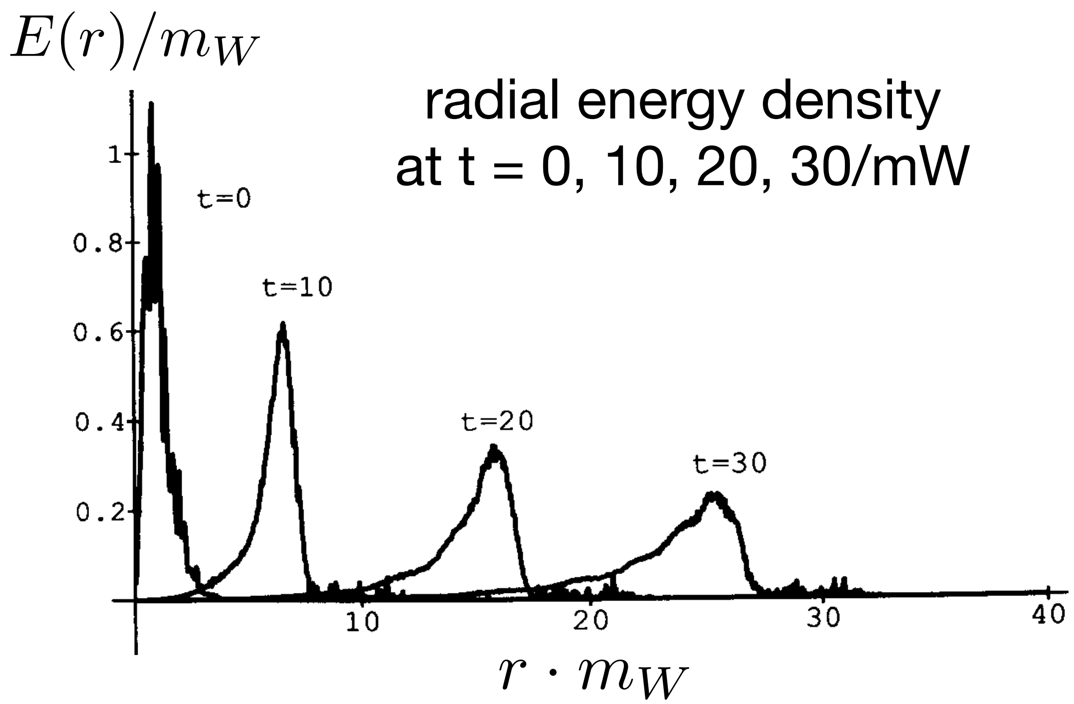
[Ringwald, KS, Webber '18]



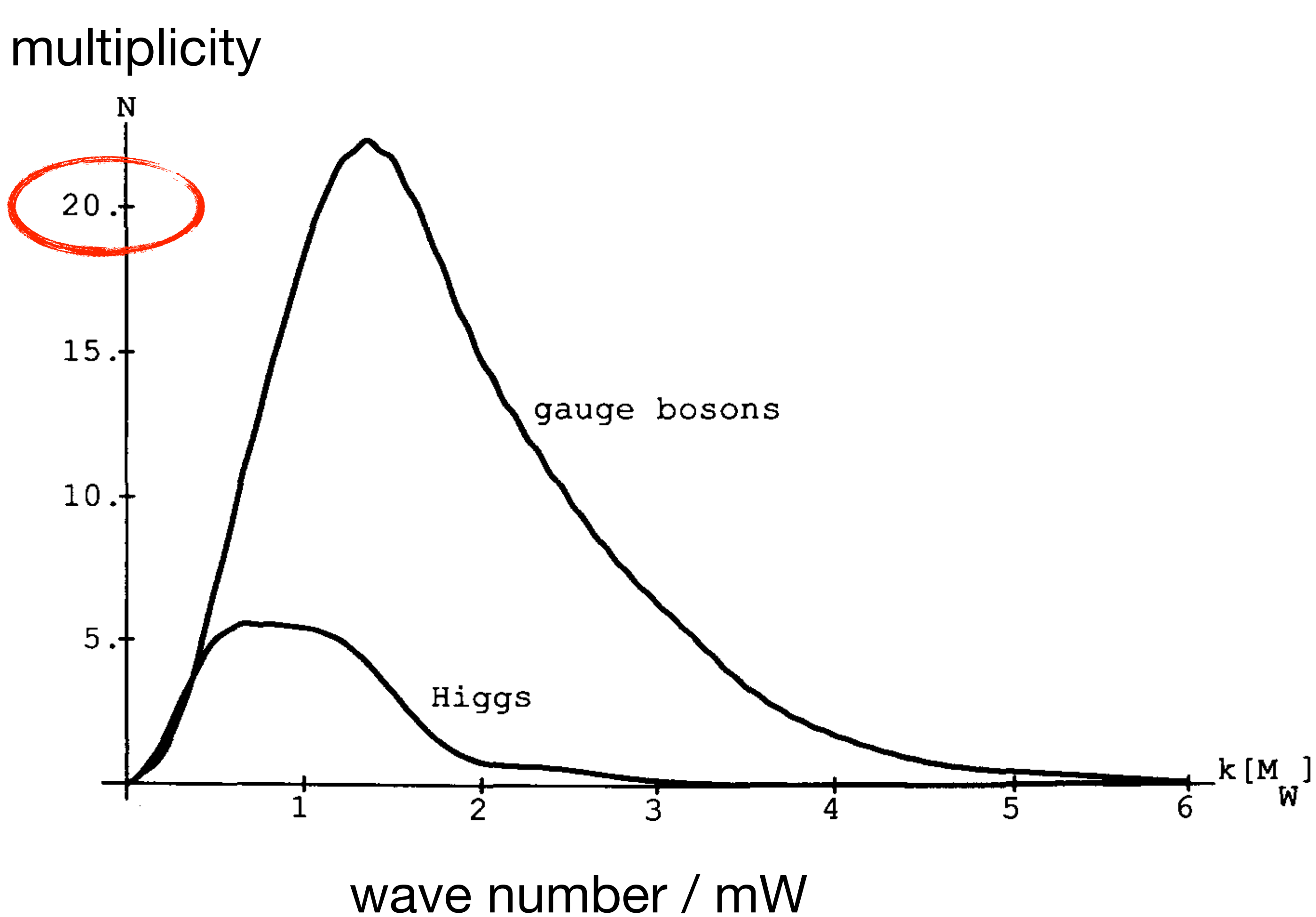
O(30) EW gauge bosons are produced!

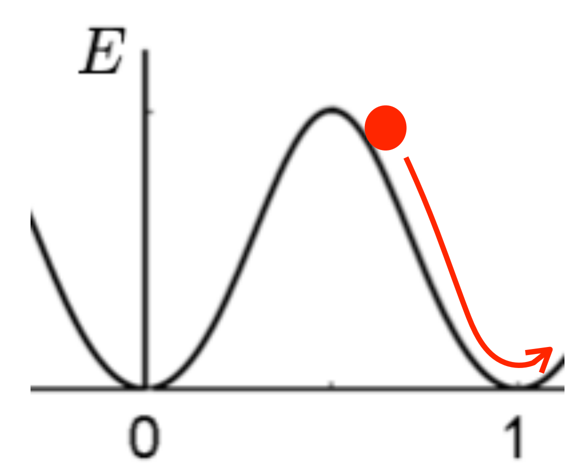
Festival at collider!

Real time evolution [Heilmund, Kripfganz '91]



- Prepare an *almost* sphaleron configuration, deviated slightly to the unstable direction.
- Evolve it with EoM and observe the field rump dissipates.
- Fourier expand (expansion in terms of fee particle modes) the final state and count the number of W and H bosons.



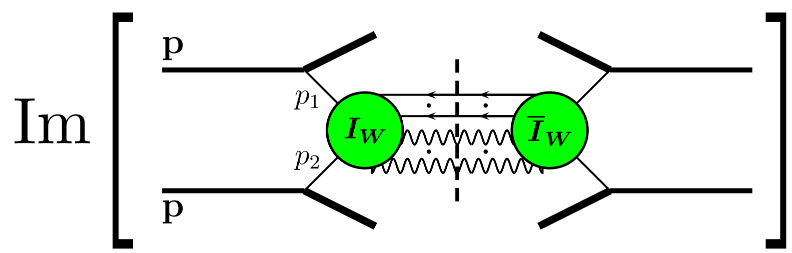


Cross-section Estimate

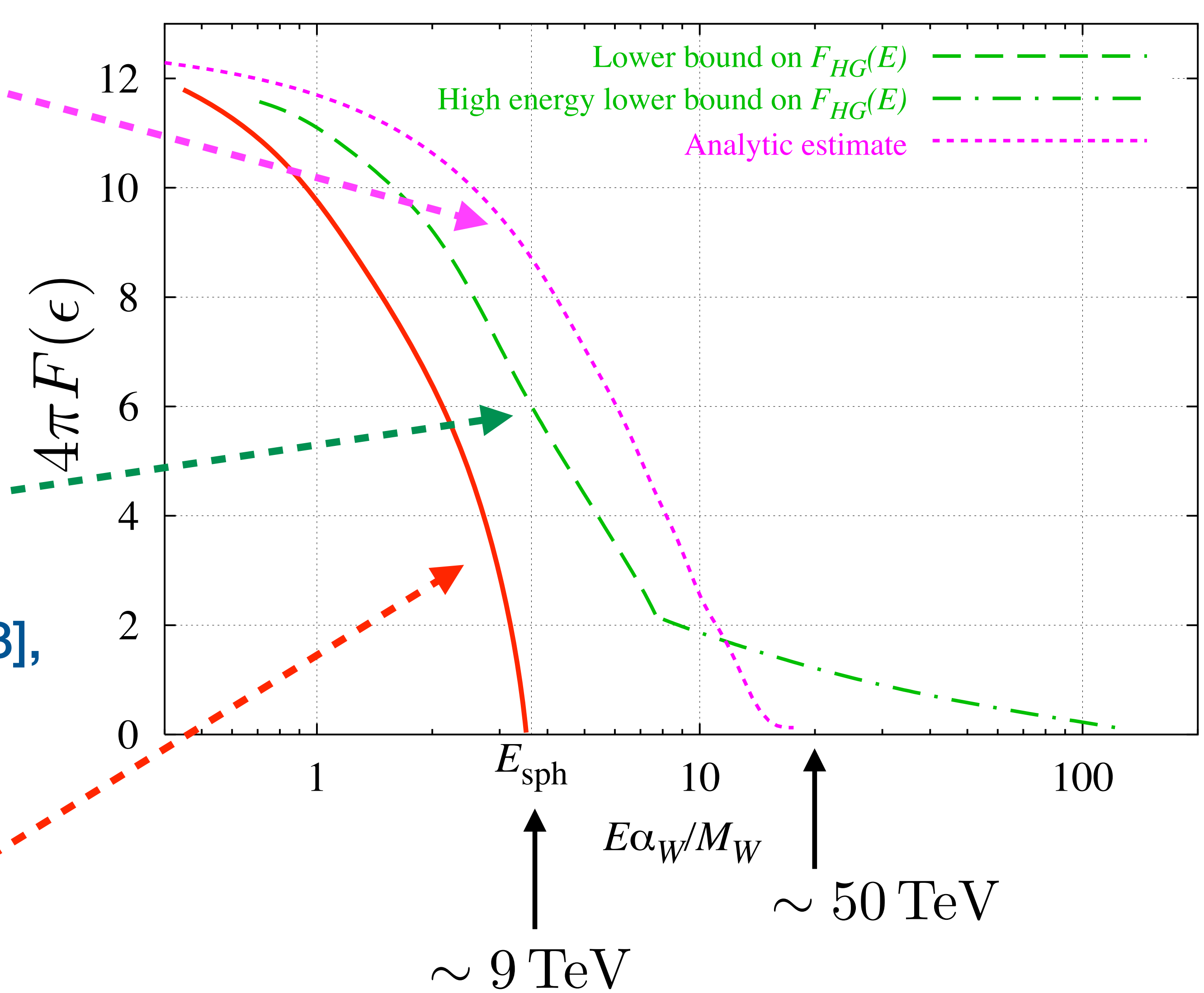
$$\boxed{\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\rm BV}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]} \qquad \epsilon \equiv E \cdot \frac{\alpha_W}{4\pi m_W}$$

- **Optical theorem**

  [Khoze, Ringwald '91], …



[Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03]



- **Semi-Classical method**

  [Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03],

  [Rubakov, Tinyakov '92], …

- **Treating N_{CS} as a dynamical variable**

  [Tye, Wong '15 '16]

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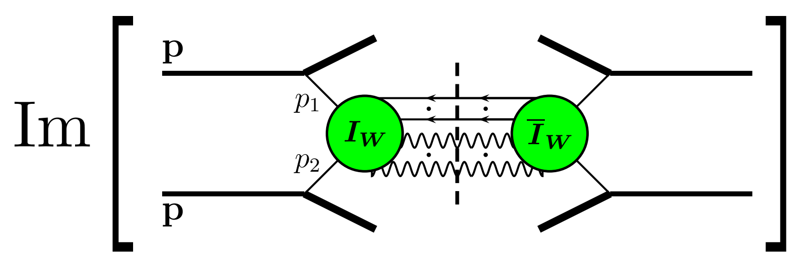

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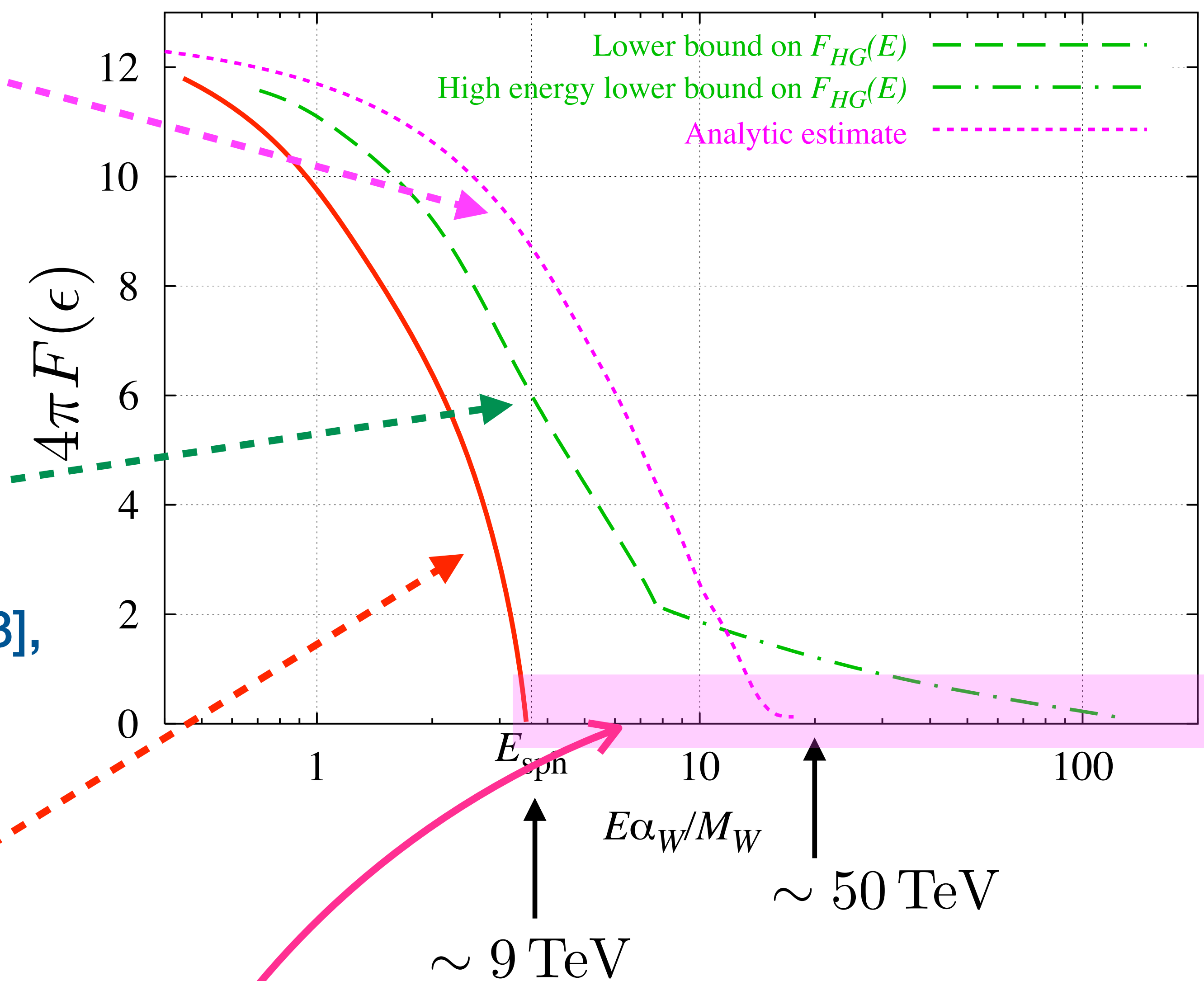


huge theo. unc. on the energy at which σ terns on

Phenomenological parametrization

partonic:

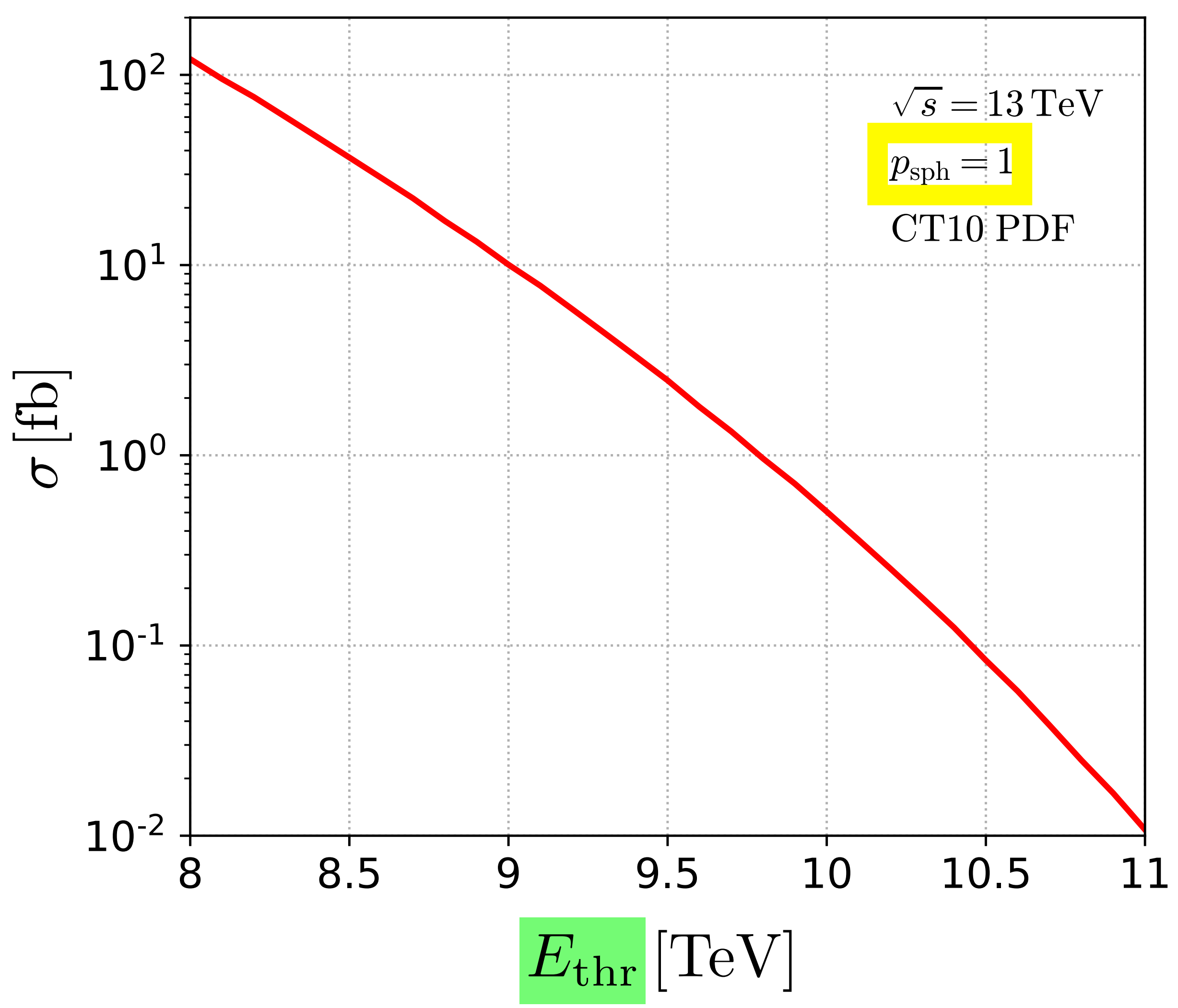
$$\boxed{\hat{\sigma}_0(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm thr})}$$

$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W, n_h} \hat{\sigma}^{\rm BV}_{n_W, n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]$$

hadronic:

$$\sigma_{pp}(\sqrt{s}) \sim \sum_{ab} \left(\frac{1}{2}\right)^2 \int dx_1 dx_2 f_a(x_1) f_b(x_2) \hat{\sigma}_0(\sqrt{s x_1 x_2})$$

[Ellis, KS, 1601.03654]



CMS-EXO-17-023

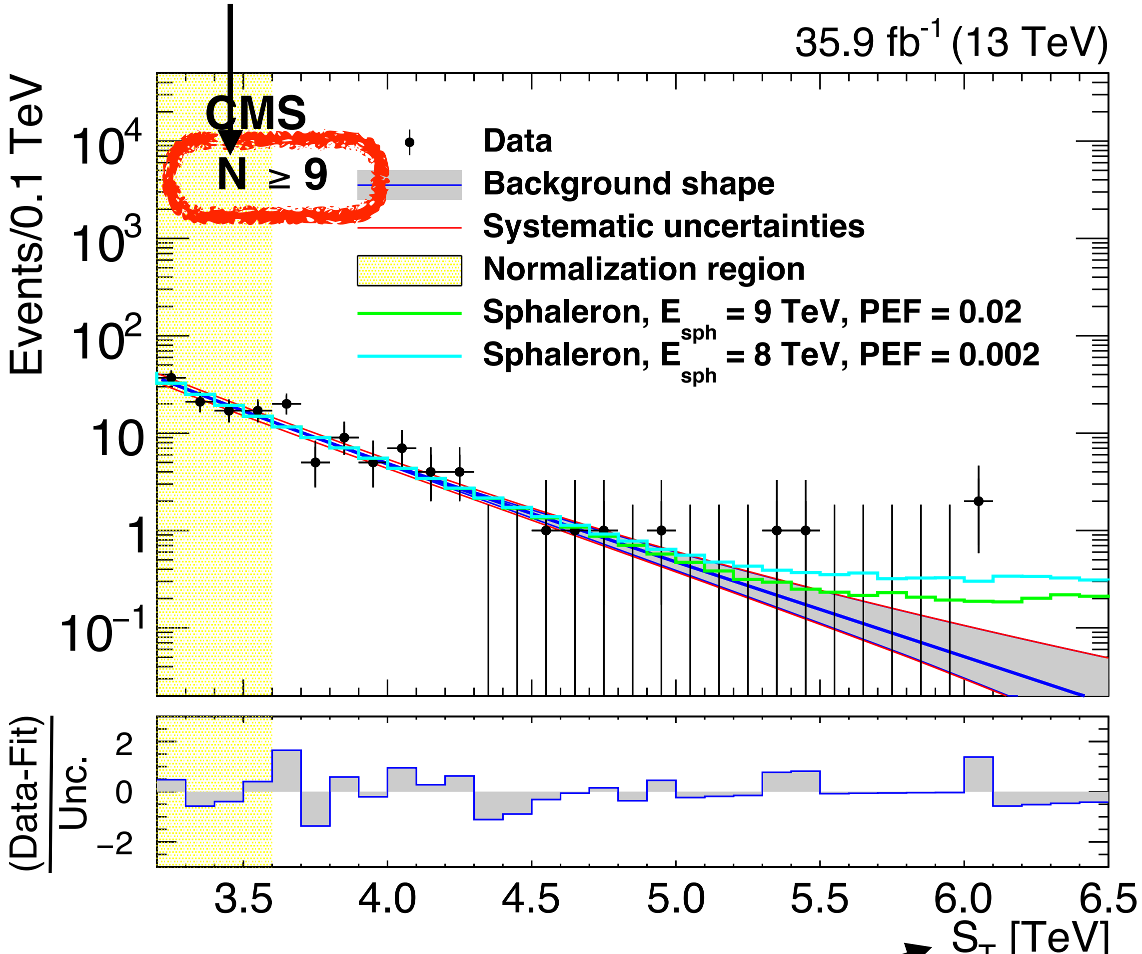
CERN-EP-2018-093
2018/11/16

[1805.06013]

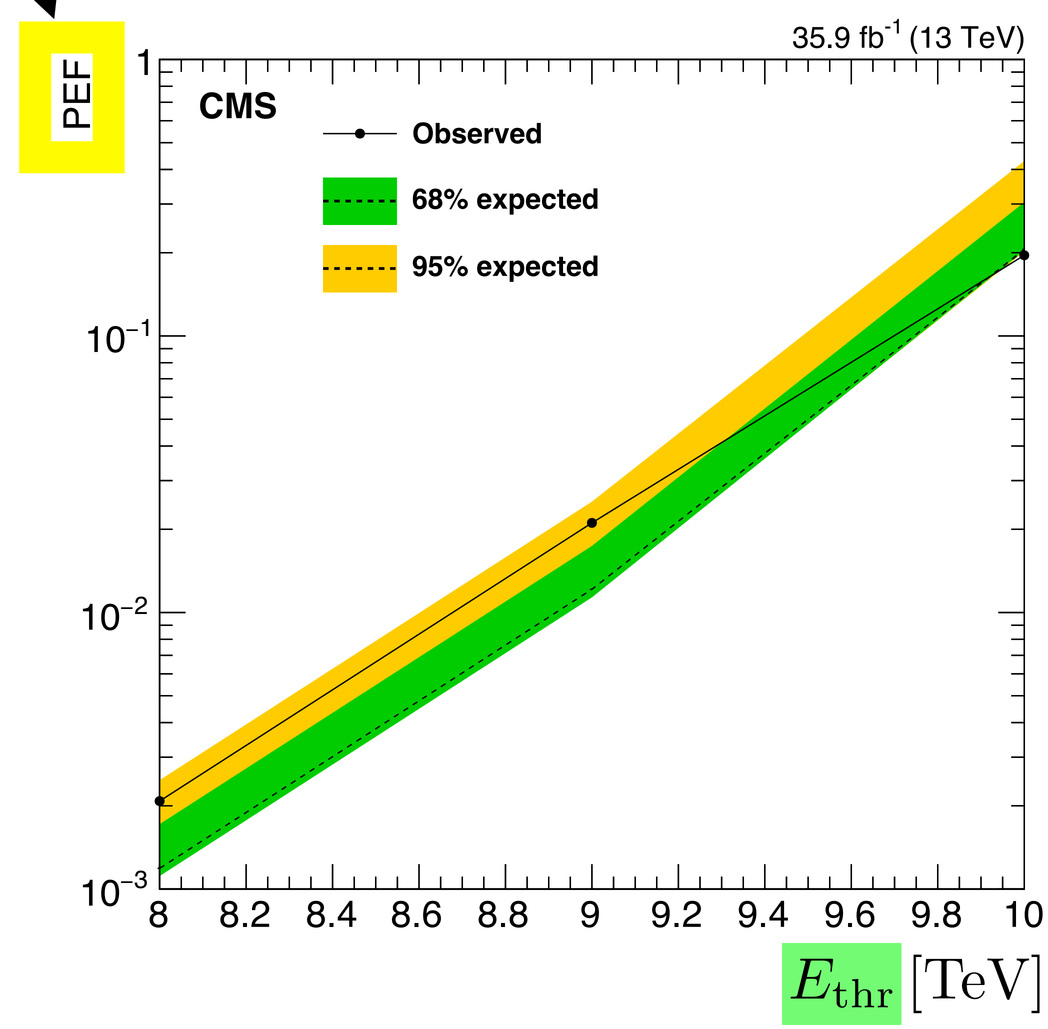
Search for black holes and sphalerons in high-multiplicity final states in proton-proton collisions at $\sqrt{s} = 13$ TeV†

$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\mathrm{sph}}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\mathrm{thr}})$$

# of jets + leptons + photons



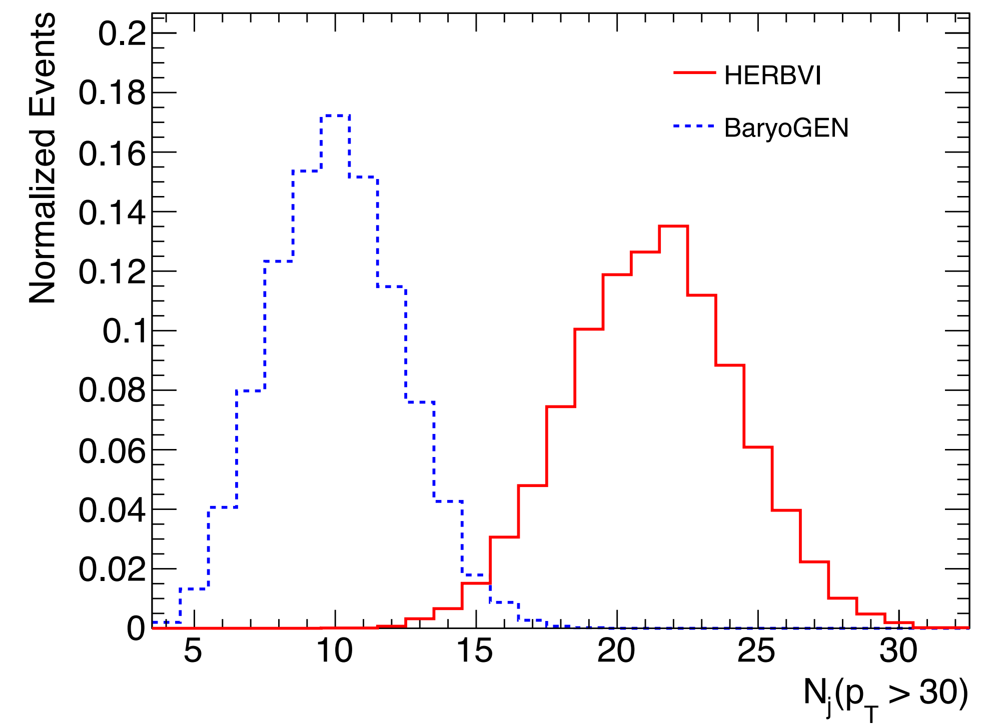
Sum of all pT in the final state



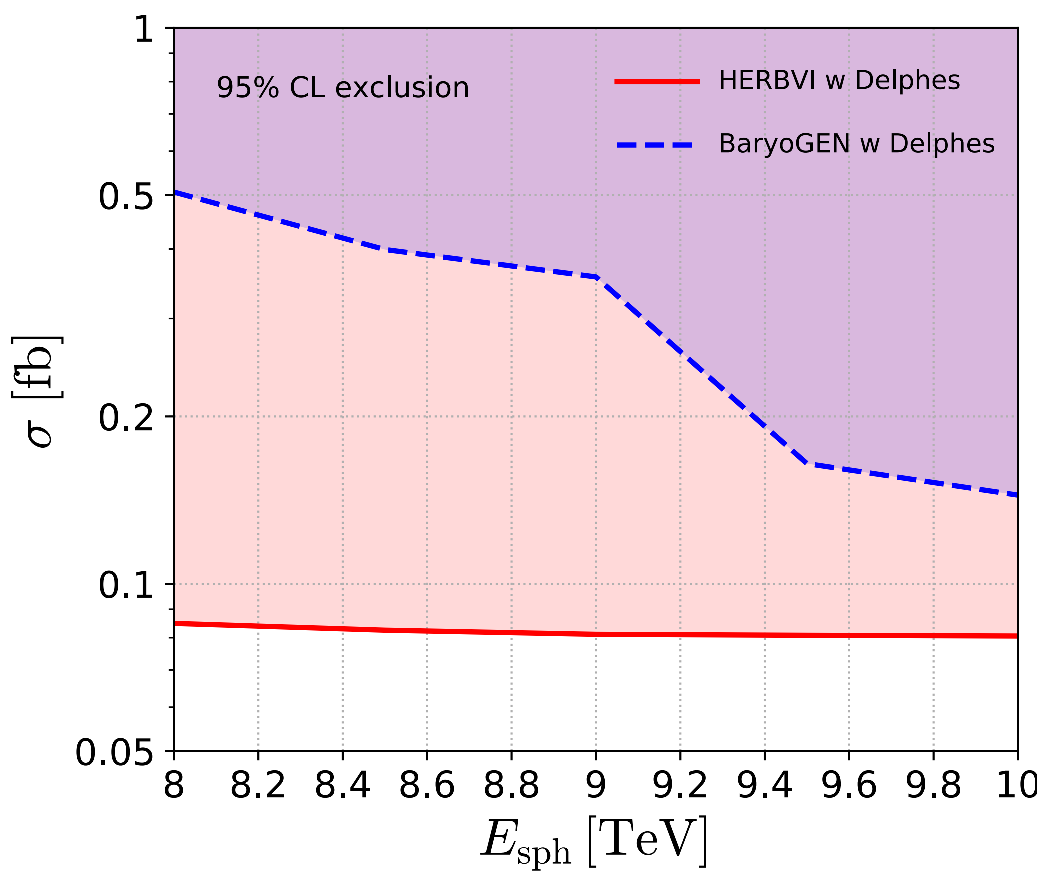
- CMS analysis assumes sphaleron final states ***DO NOT*** involve any EW bosons.

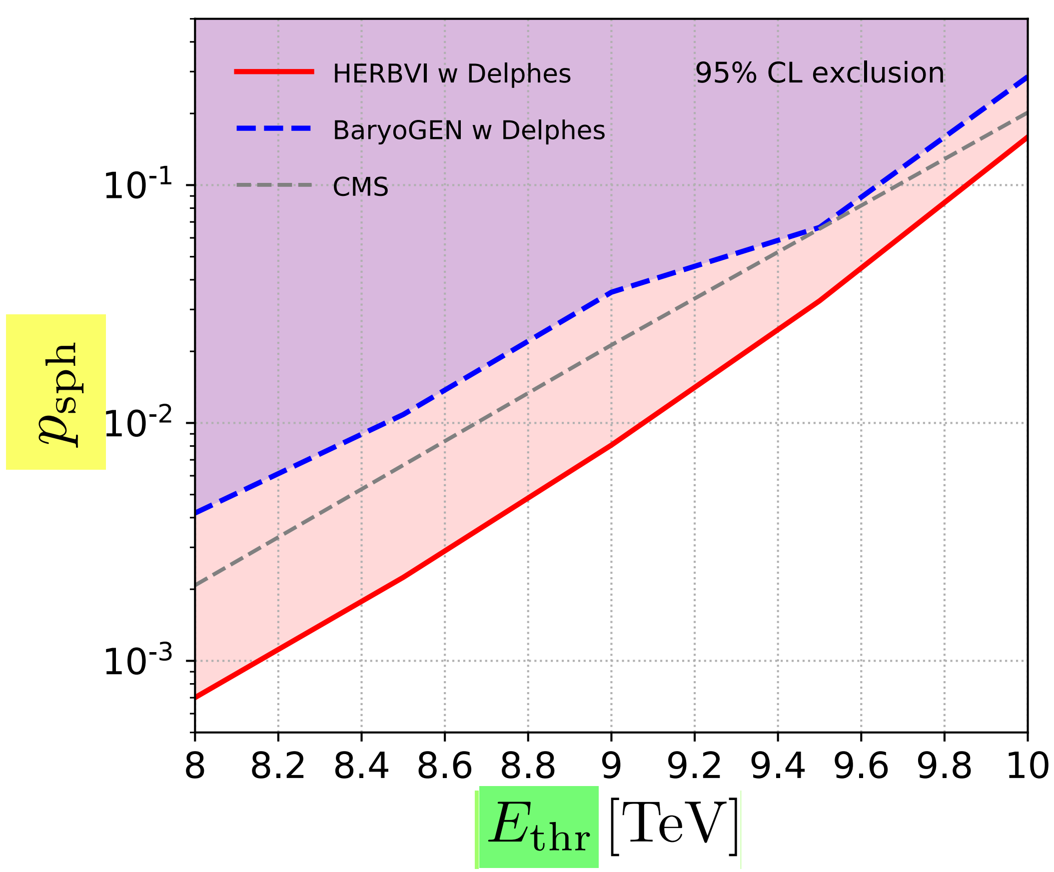
$$qq \to \begin{cases} n_q q + 3\ell & \text{[BaryoGEN]} \\ 7q + 3\ell + \sum n_B B & \text{[HERBVI]} \end{cases}$$

jet multiplicity



[Ringwald, KS, Webber 1809.10833]

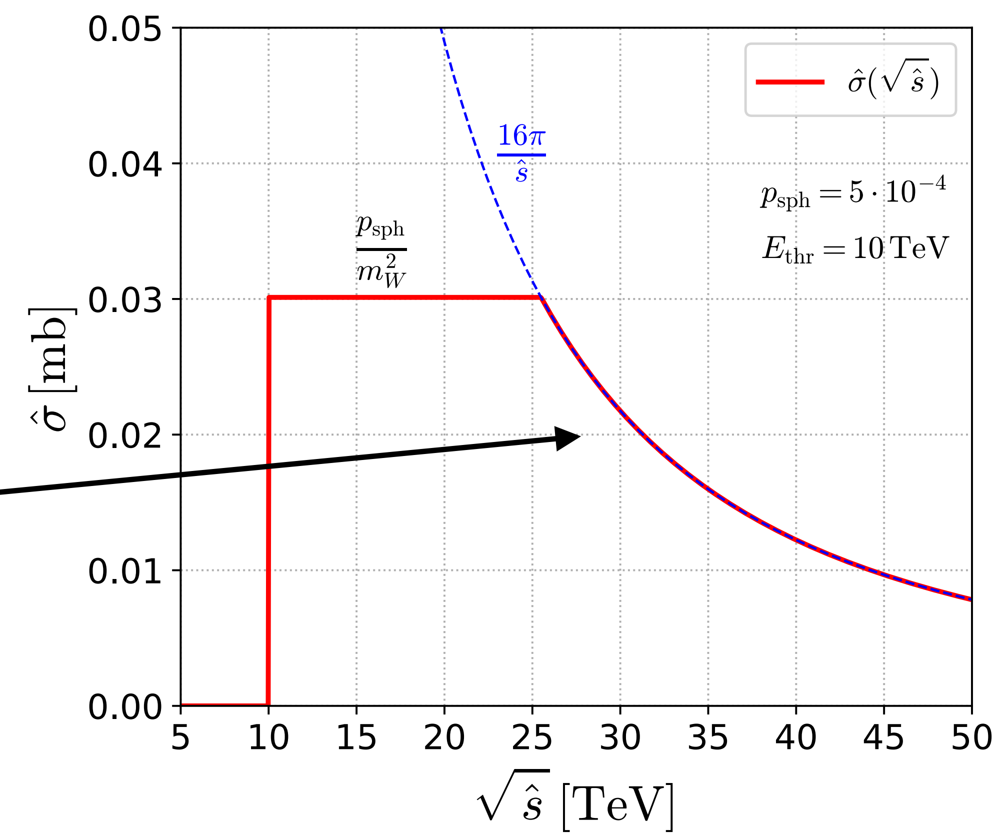




MC Event Generators

	written in	Multi-Boson	Unitarity
HERBVI	Fortran	**LO**	No (default)
BaryoGEN	**C++**	No	No
HERWIG7	**C++**	**LO + E_{freeze}**	**Yes**

[A.Papaefstathiou, S.Plätzer, KS 1910.04761]

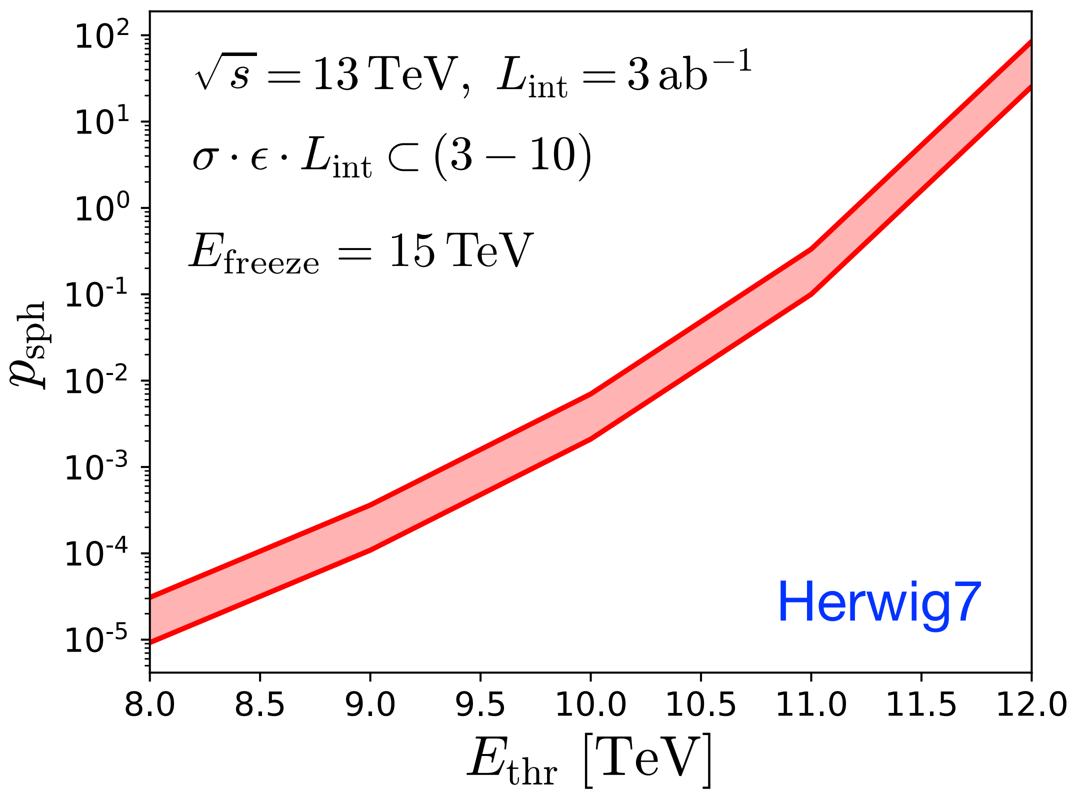


LO prediction cannot be trusted for $\sqrt{s}$ >~ E_{sph}.

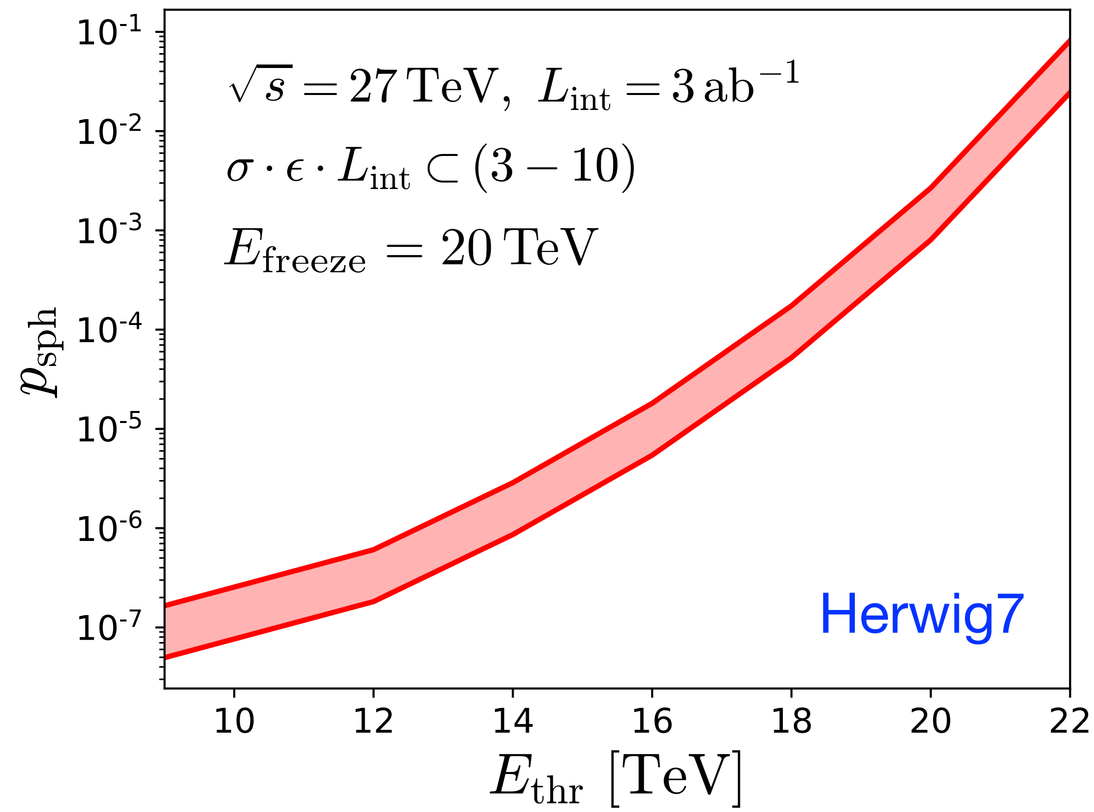
For $\sqrt{s} > E_{freeze}$, the LO Boson multiplicity distribution is calculated with $\sqrt{s} = E_{freeze}$.

Future Colliders

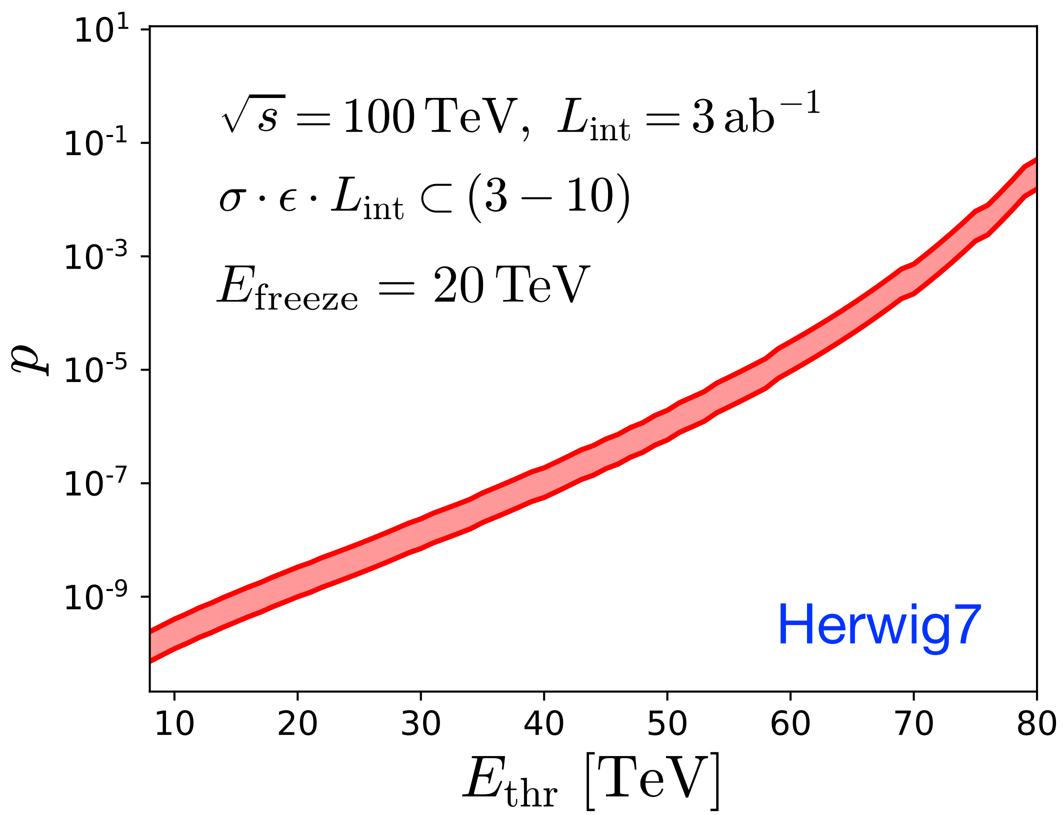
HL-LHC



HE-LHC



FCC100



- Event selection

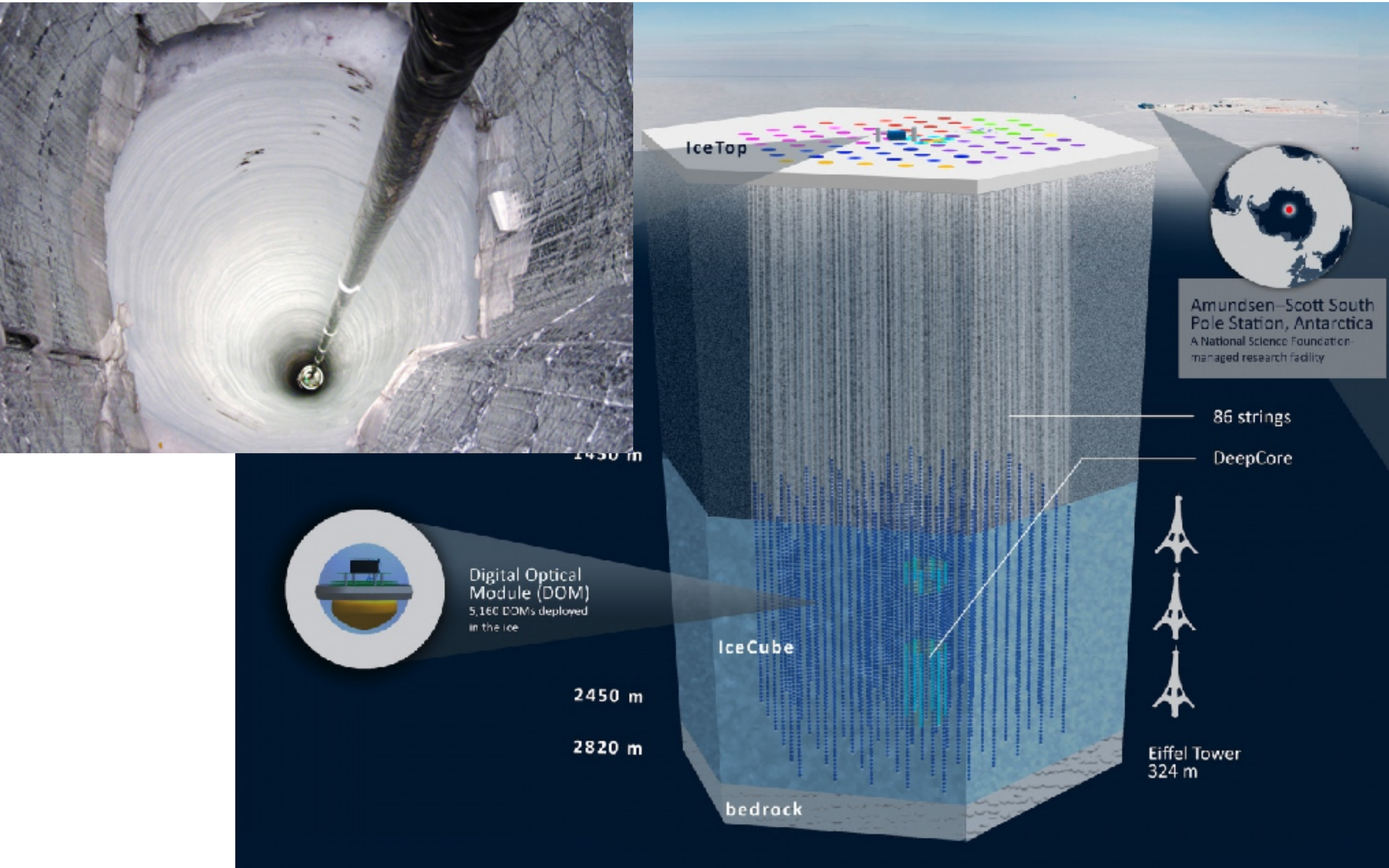
$$\begin{cases} N(p_T > 100) \geq 11,\ S_T^{100} > 4\,\mathrm{TeV} & \text{... HL-LHC} \\ N(p_T > 100) \geq 15,\ S_T^{100} > 7\,\mathrm{TeV} & \text{... HE-LHC, FCC100} \end{cases}$$

$N(p_T > 100)$: # of visible objects with p_T >100GeV

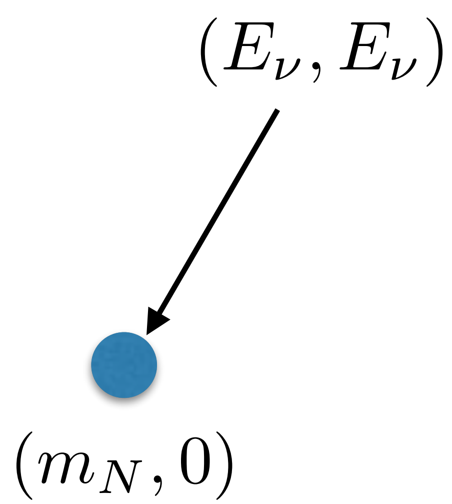
S_T^{100} : scalar p_T sum of visible objects with p_T > 100GeV

[A.Papaefstathiou, S.Plätzer, KS 1910.04761]

Sphalerons @ IceCube



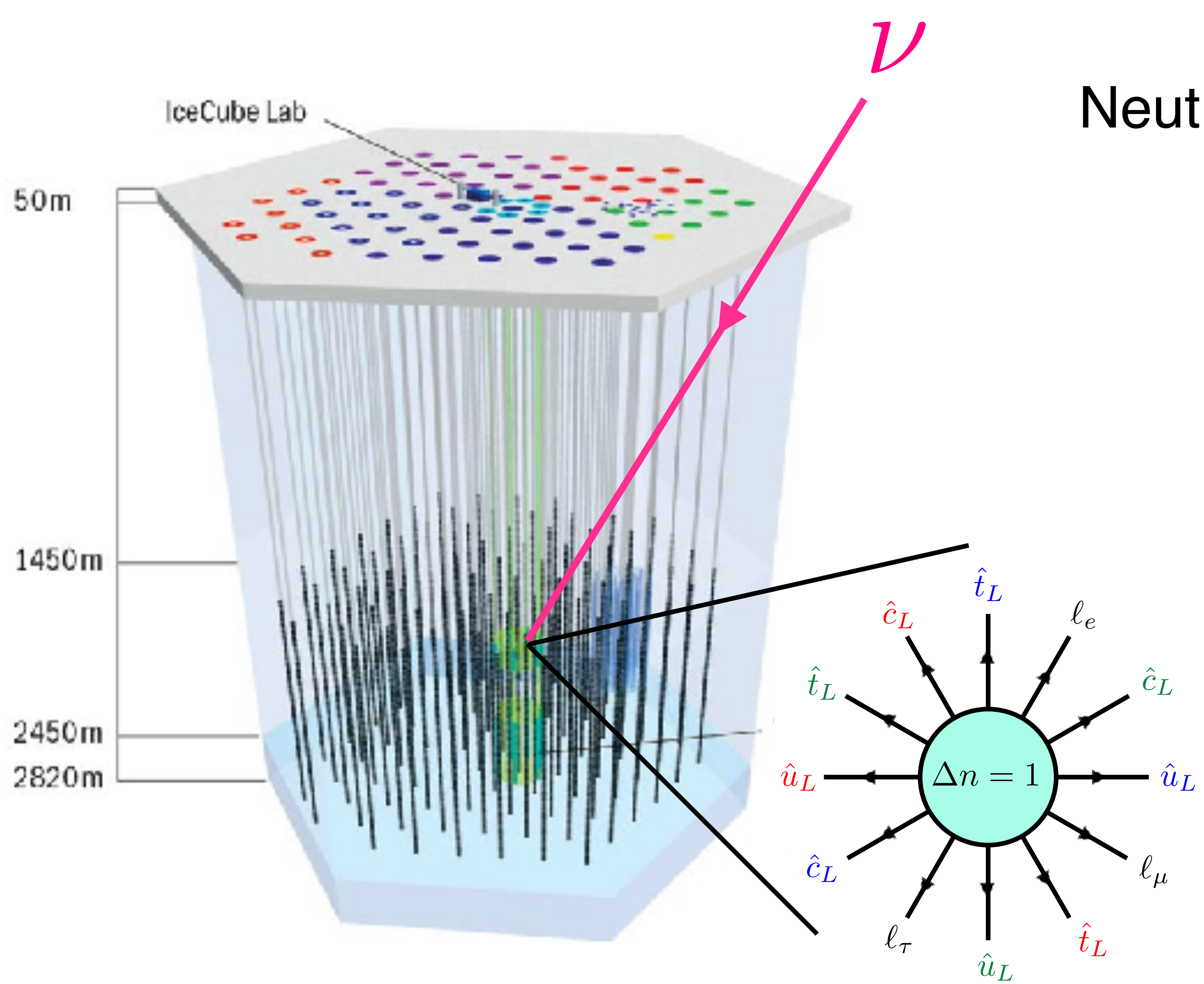
What neutrino energy is required to create a sphaleron?



$$s_{N\nu} = E^2 - p^2 = (m_N + E_N)^2 - E_N^2 \simeq 2m_N E_\nu$$

$$s_{q\nu} \simeq 2x m_N E_\nu \quad (x = E_q/E_N)$$

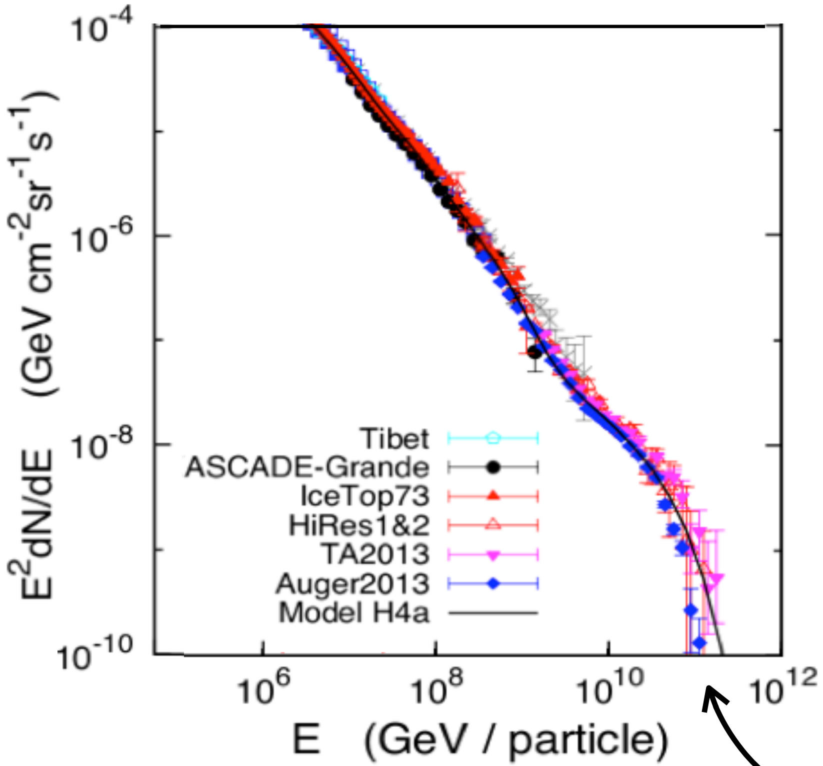
$$E_\nu \geq \frac{E_{\text{Sph}}^2}{2x m_N} \simeq \frac{(9\,\text{TeV})^2}{2x(0.94\,\text{GeV})} \simeq \frac{4\cdot 10^7}{x}\,\text{GeV}$$



Neutrino energy needed to create sphalerons:

$$E_\nu \gtrsim 10^{8-10}\,\text{GeV}$$

$$(\text{for } 10^{-3} \lesssim x \lesssim 10^{-1})$$

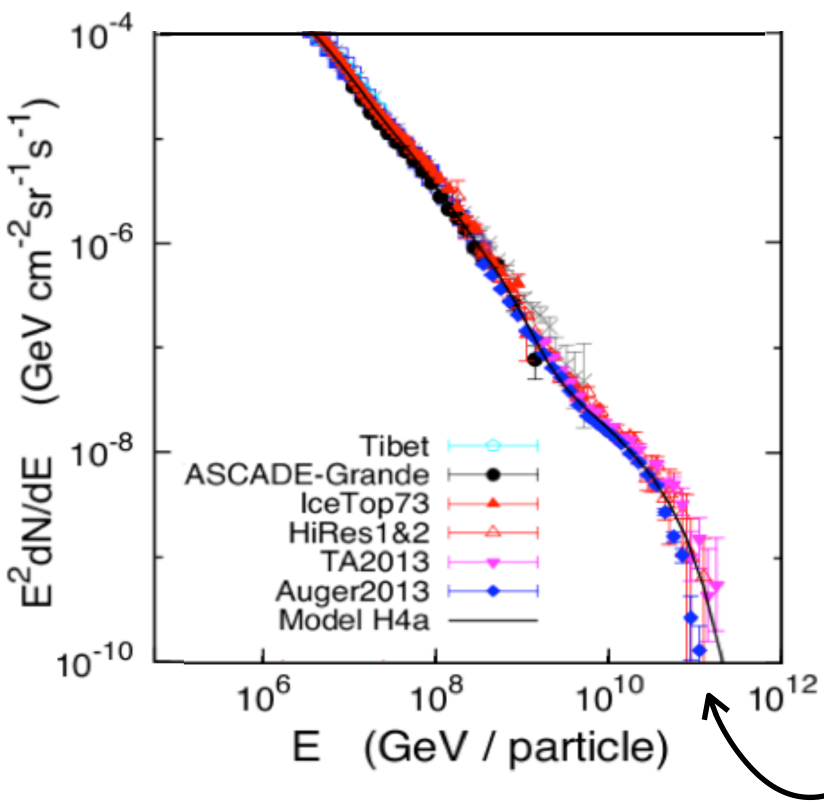


Greisen–Zatsepin–Kuzmin (GZK) processes:

$$p + \gamma_{\rm CMB} \rightarrow p + \pi^0$$

$$p + \gamma_{\rm CMB} \rightarrow n + \pi^+$$

GZK cutoff energy: $E_p \sim 3 \cdot 10^{11}\,\mathrm{GeV}$



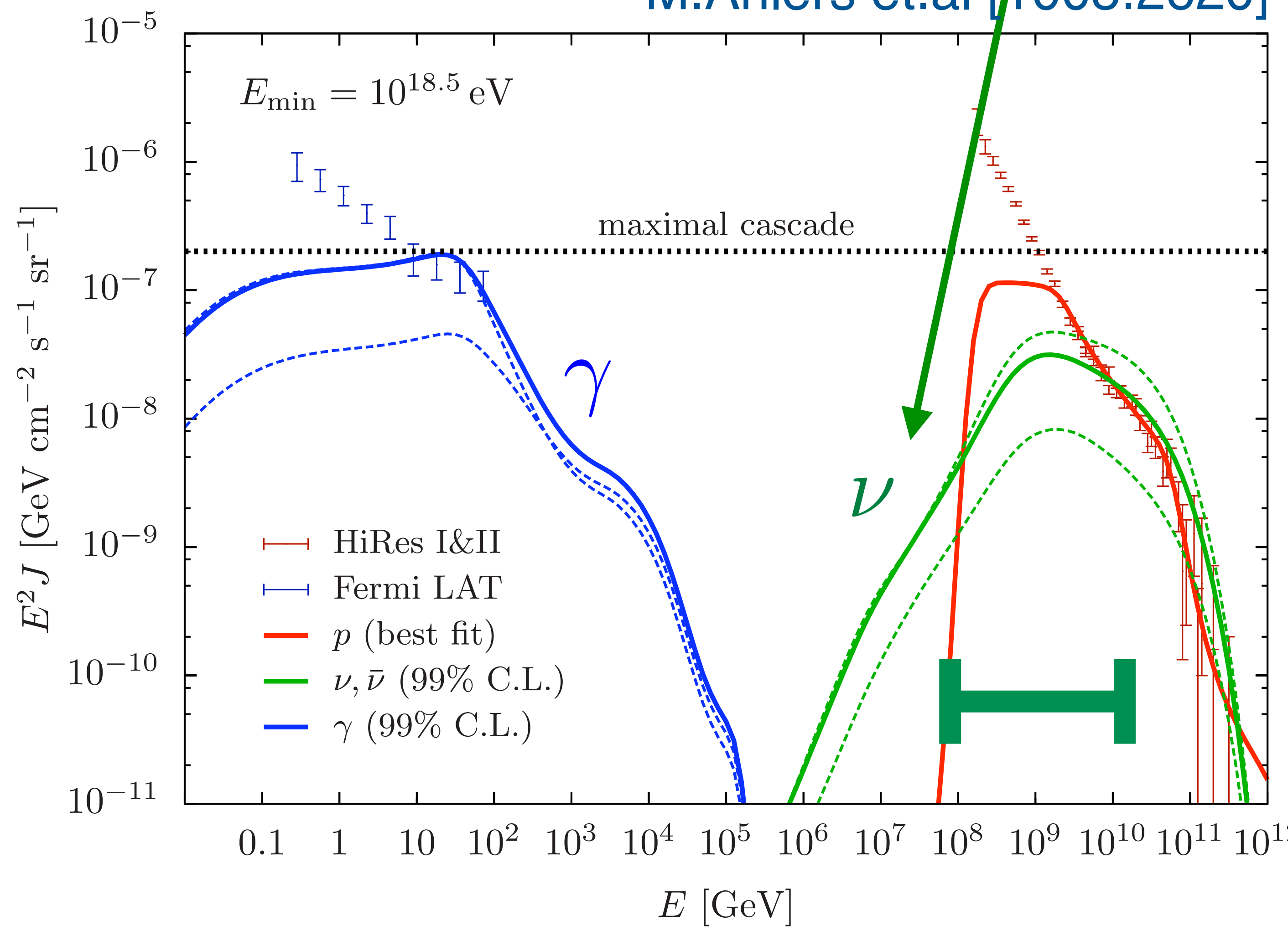
Greisen–Zatsepin–Kuzmin (GZK) processes:

$$p + \gamma_{\rm CMB} \to p + \pi^0 \quad : \pi^0 \to \gamma\gamma$$

$$p + \gamma_{\rm CMB} \to n + \pi^+ : \pi^+ \to \mu^+ \nu_\mu \to e^+ \boxed{\nu_e \bar{\nu}_\mu \nu_\mu}$$

GZK cutoff energy: $E_p \sim 3 \cdot 10^{11}\,\mathrm{GeV}$

M.Ahlers et.al [1005.2620]

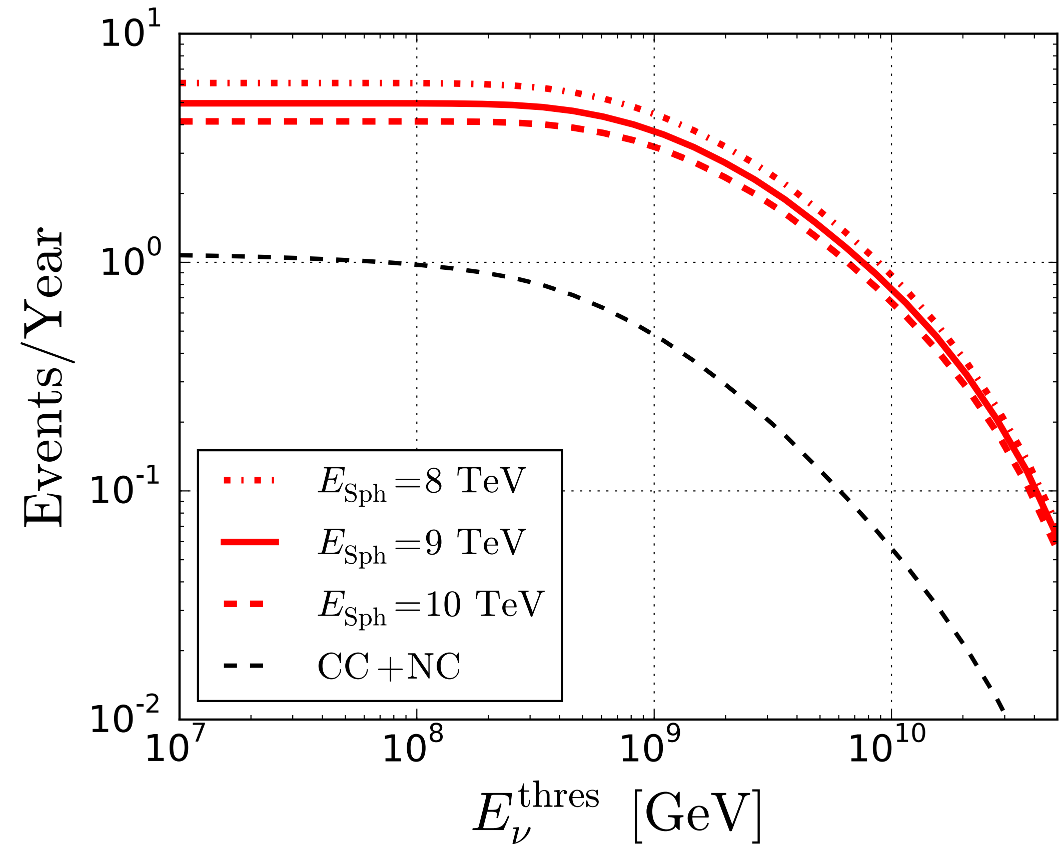


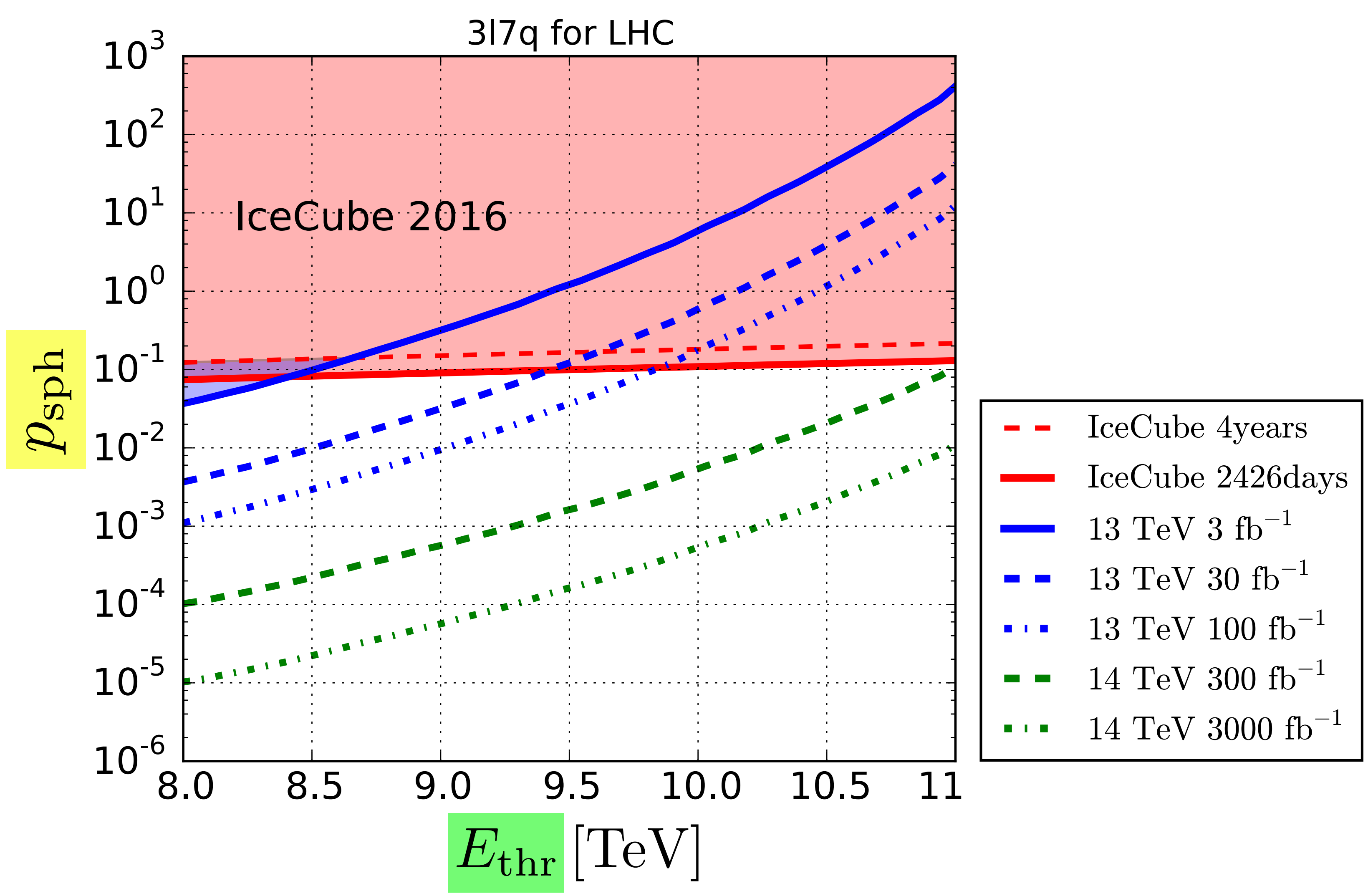
$$E_\nu^{\rm Sph} \gtrsim 10^{8-10}\,\mathrm{GeV}$$

[Ellis, KS, Spannowsky 1603.06573]

$$\frac{dN_{\rm Sph}}{dt} = \int_{E_{\rm three}} dE_\nu \frac{\sigma^{\rm Sph}_{\nu N}(E_\nu)}{\sigma^{CC/NC}_{\nu N}(E_\nu)} A_{\rm eff}(E_\nu) \frac{d^2\Phi}{dE_\nu dt}$$

$$\hat\sigma(\sqrt{\hat s}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat s} - E_{\rm thr})$$





EW instantons/sphalerons at cosmic ray air showers:

[Brooijmans, Schichtel, Spannowsky 1602.00647]

[Y.Jho, S.Chan.Park 1806.03063]

Conclusions

- The rate of zero-temperature high-energy instanton-induced processes is still an open question.
- Some studies suggest that the rate of such processes might be observably large at (future) colliders but the current estimate suffers from very large uncertainties.
- It is important to tackle this issue from the experimental side and to set limits on the EW instanton processes.
- The LHC can probe the region E_{thr} ~ 9TeV, while 100TeV collider can probe the realistic region up to E_{thr} ~ 80TeV.
- More theoretical understanding on the cross-section and final state multiplicity is necessary to fully exploit the power of future high-energy colliders.
- Experiments exploiting ultra high-energy cosmic rays may also be useful to probe the EW instanton processes.

Backup

Effect of BSM fermions

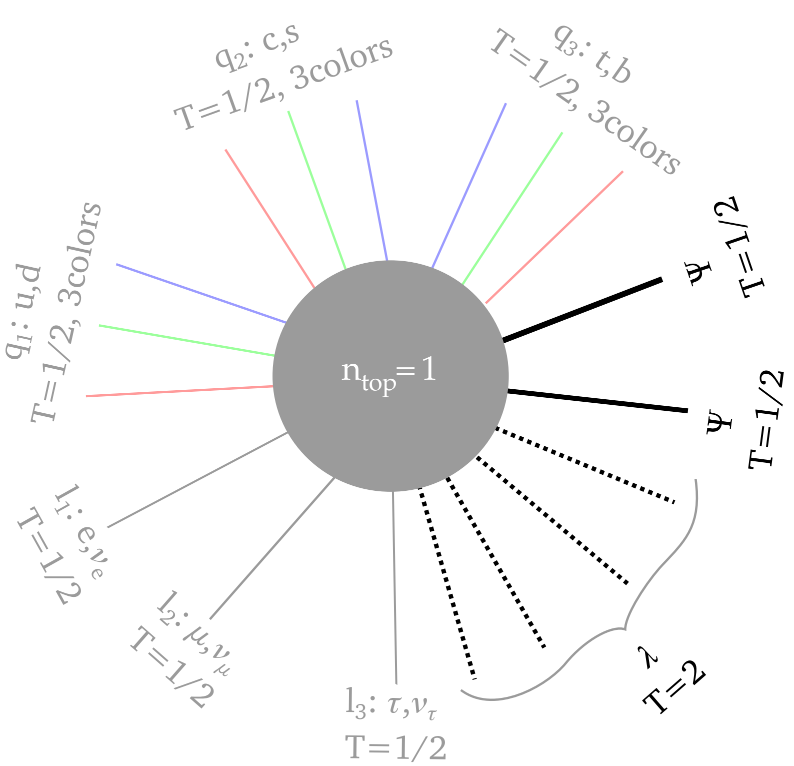
[Cerden, Reimitzb, KS, Tamaritd 1908.00065]

$$M_F \bar{\Psi}^i \Psi_i + M_A \lambda^a \lambda^a$$

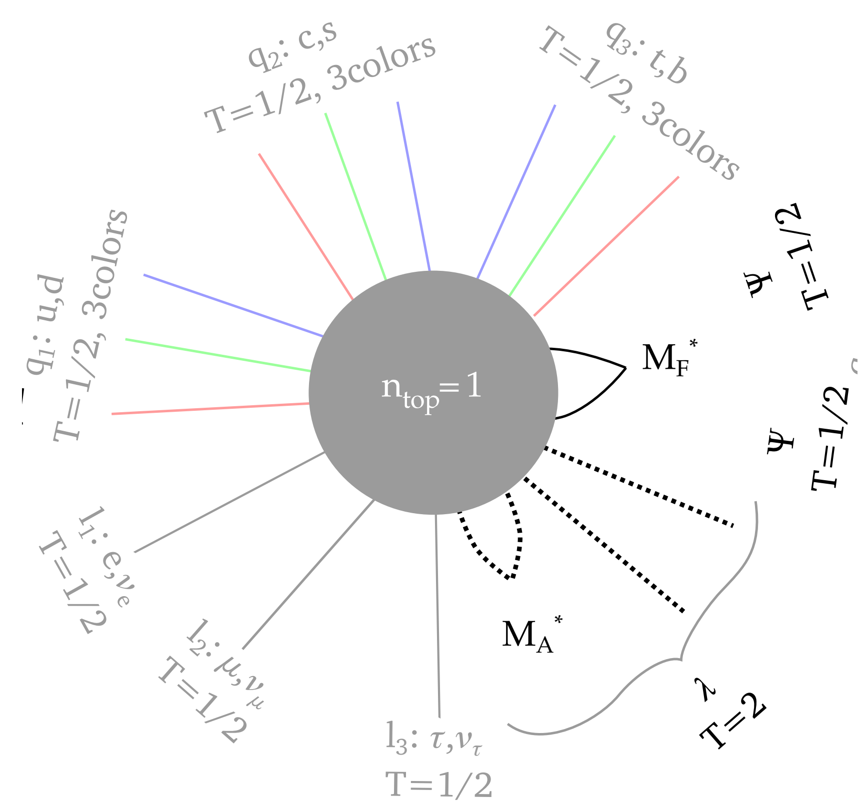
fundamental rep. (Higgsino)

adjoint rep. (Wino)

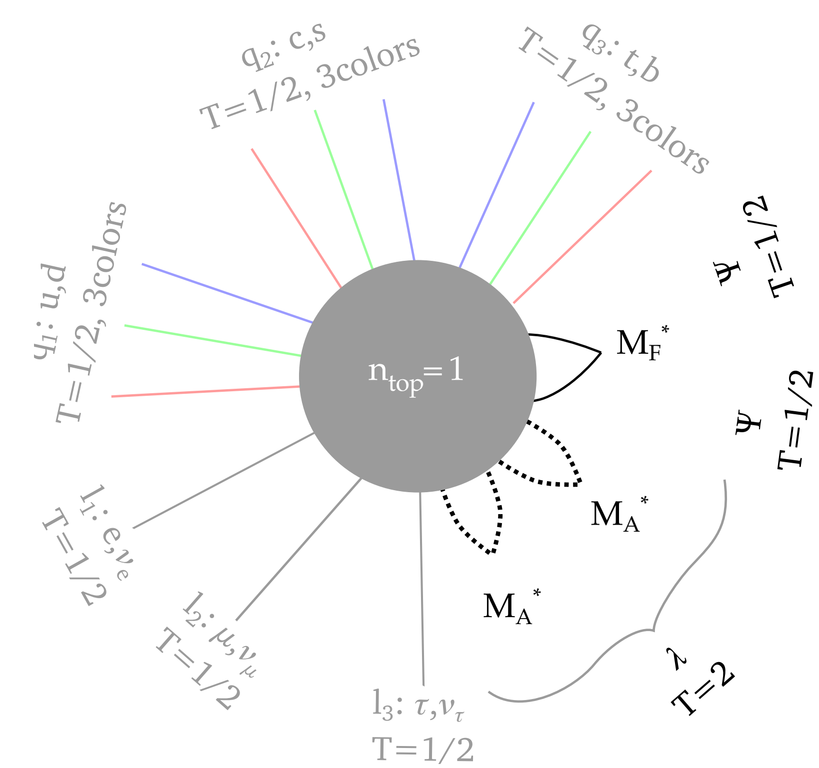
12 SM fermions
+
6 BSM fermions



12 SM fermions
+
2 BSM fermions



12 SM fermions
+
0 BSM fermions

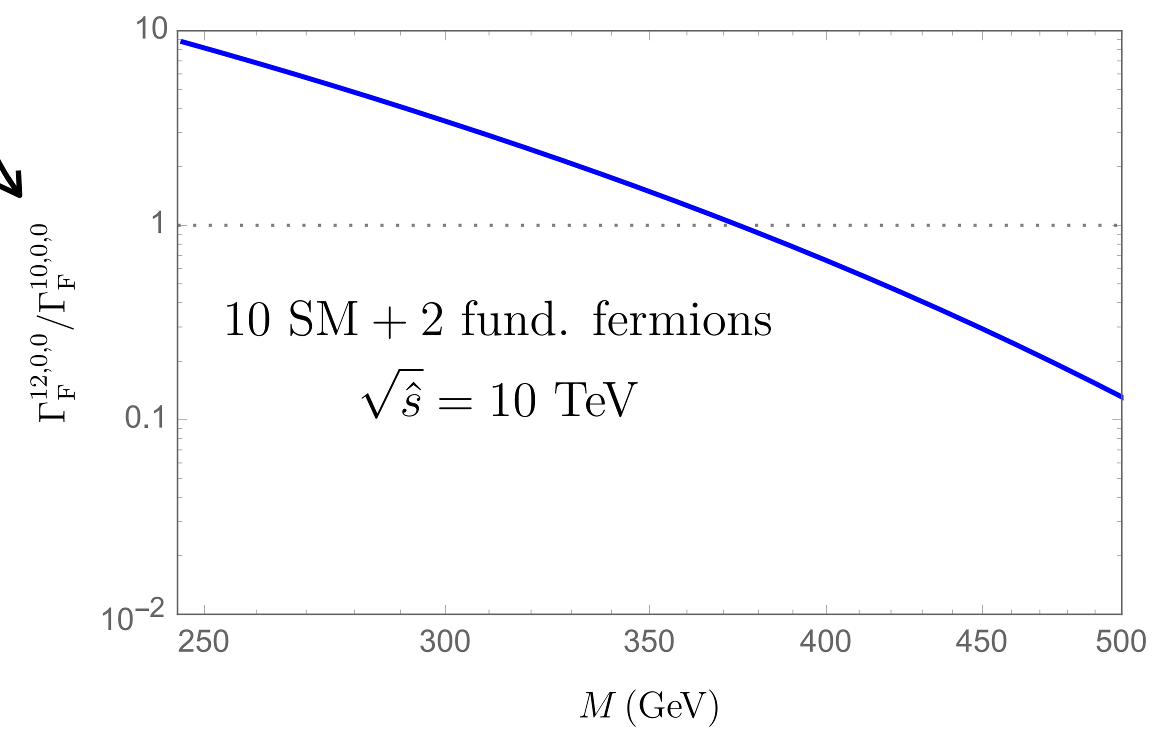


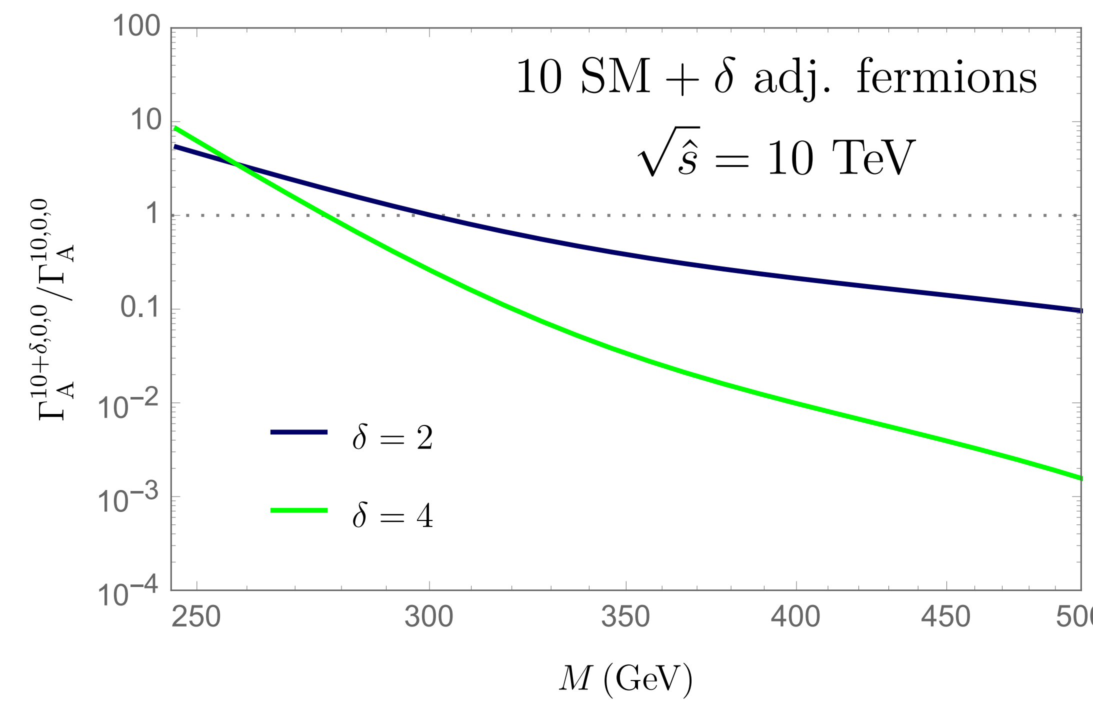
Effect of BSM fermions

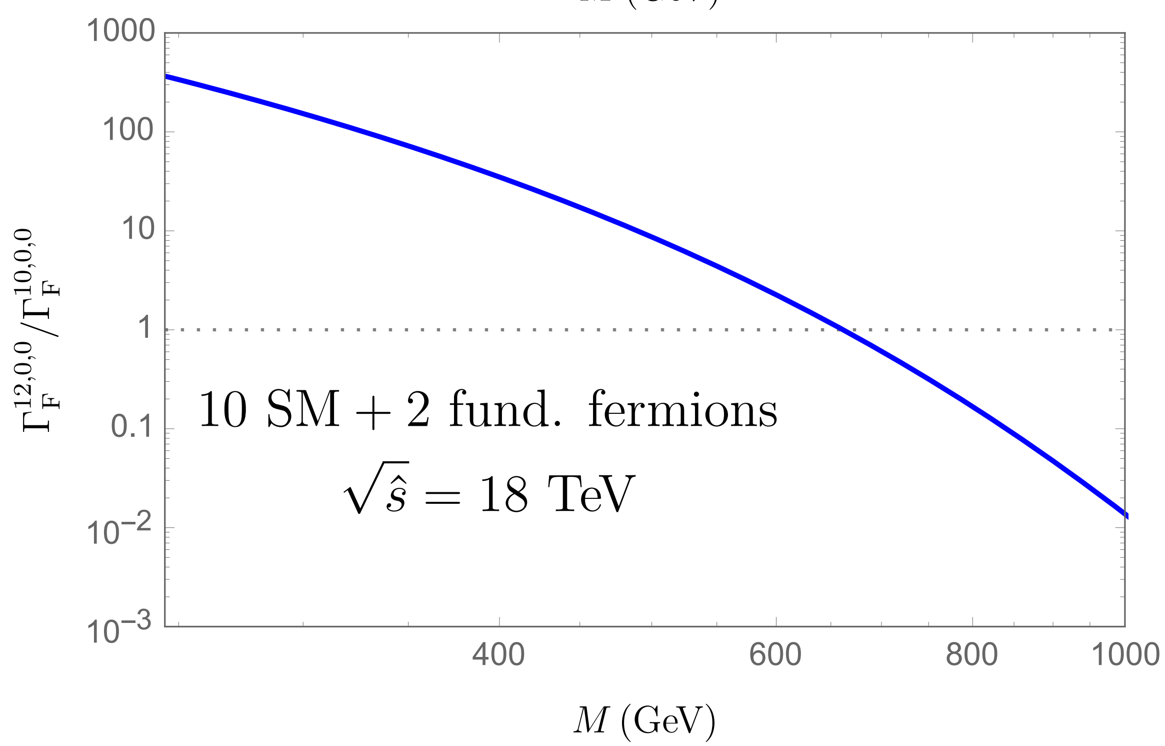
[Cerden, Reimitzb, KS, Tamaritd 1908.00065]

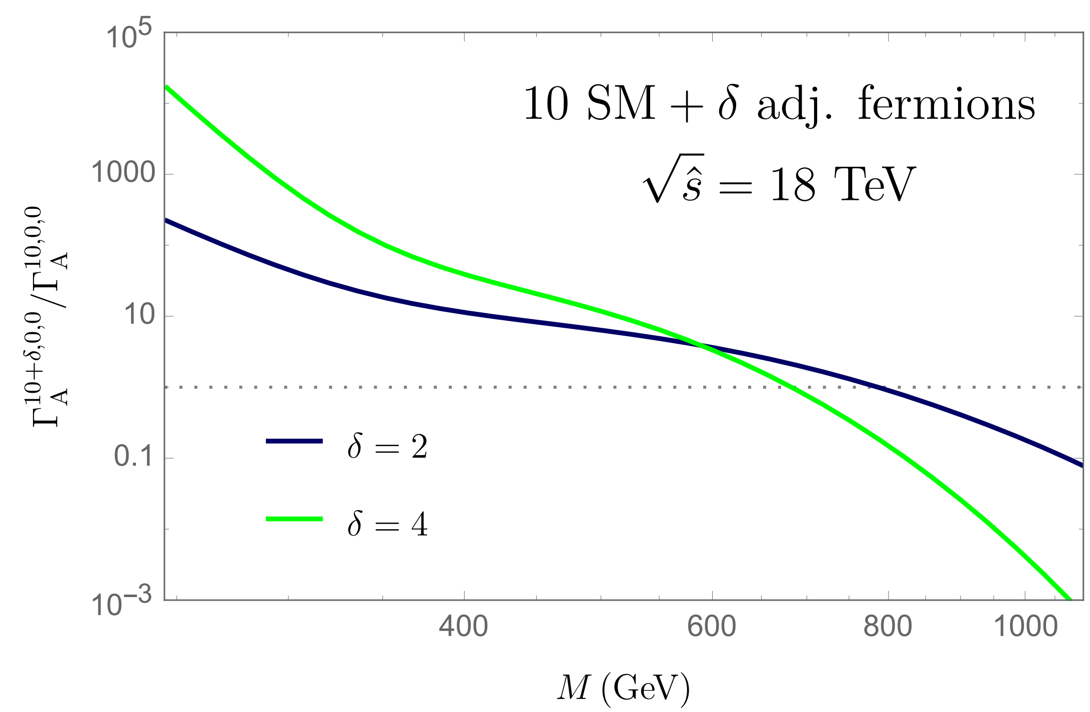
LO Matrix Element: qq -> N fermions

enhancement over SM



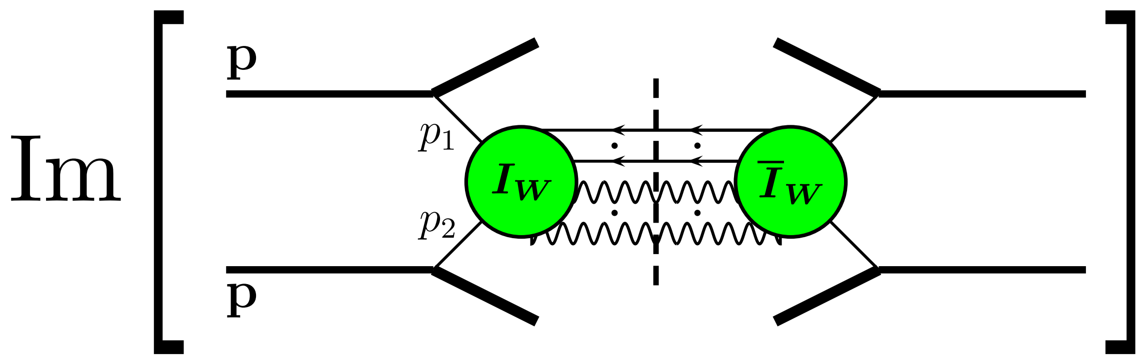






- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



$$\hat{\sigma}_{\rm incl}^{\rm BV} \equiv \sum_{n_W, n_h} \hat{\sigma}_{n_W, n_h}^{\rm BV} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]$$

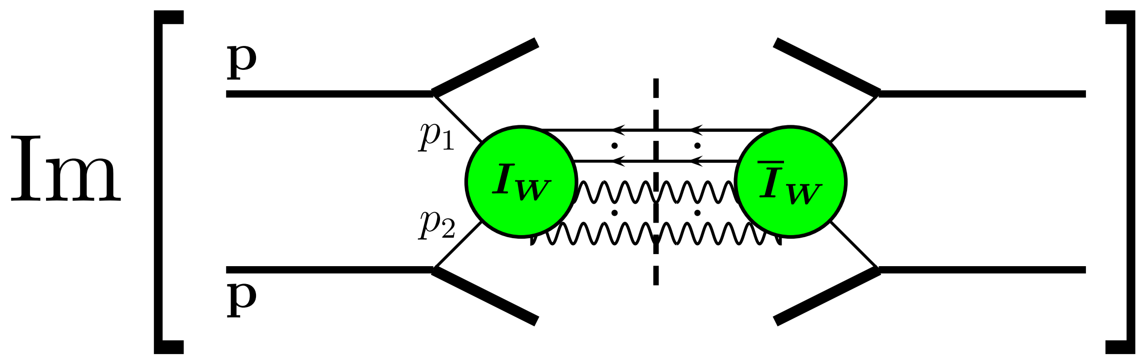
$$F(\epsilon) = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot \text{const})$$

$$P(\epsilon) = \frac{\pi^{15/2}}{1024}\left(\frac{3}{2}\right)^{\frac{2}{3}} d^2 \left(\frac{4\pi}{\alpha_W}\right)^{7/2} \epsilon^{\frac{74}{9}}\left[1 + \mathcal{O}\left(\epsilon^{2/3}\right)\right] \qquad d \simeq 0.15$$

$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3 \text{ TeV}$$

- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



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'Holy grail' function

$$\boxed{F(\epsilon)} = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot {\rm const})$$

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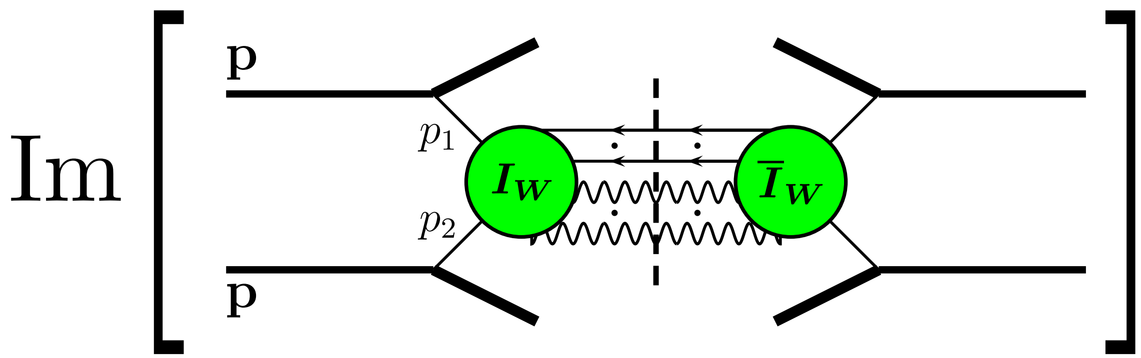

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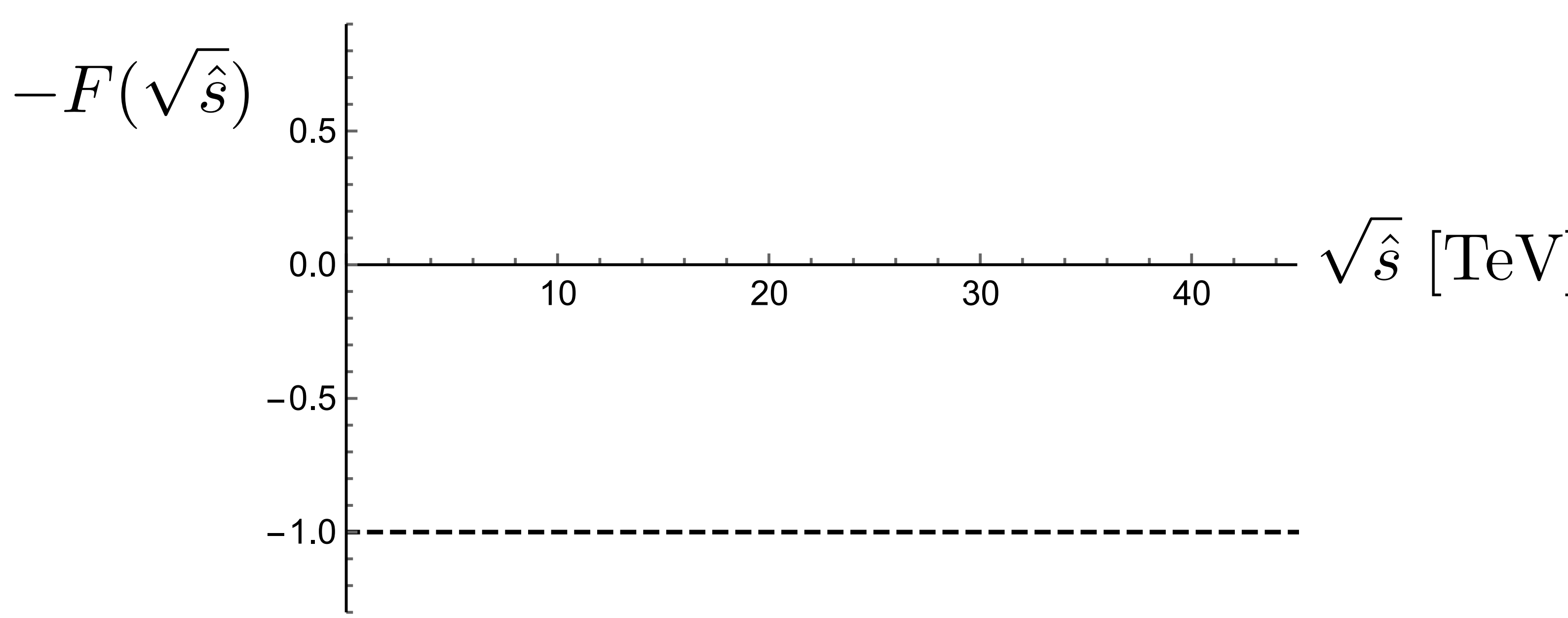

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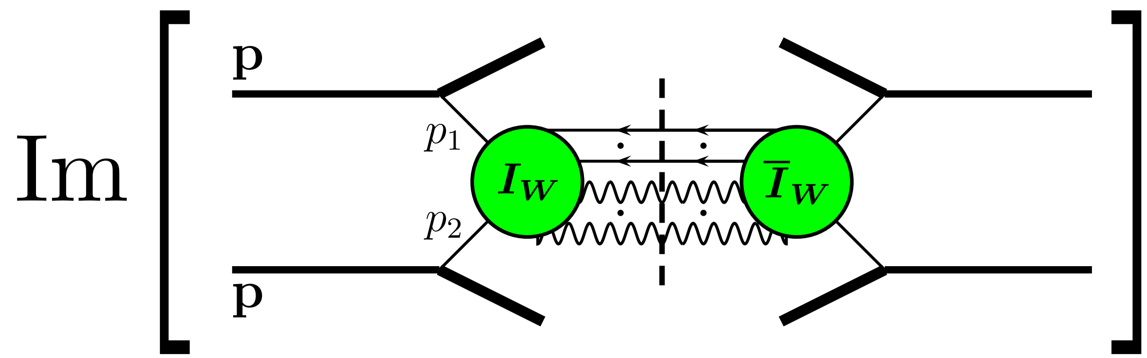

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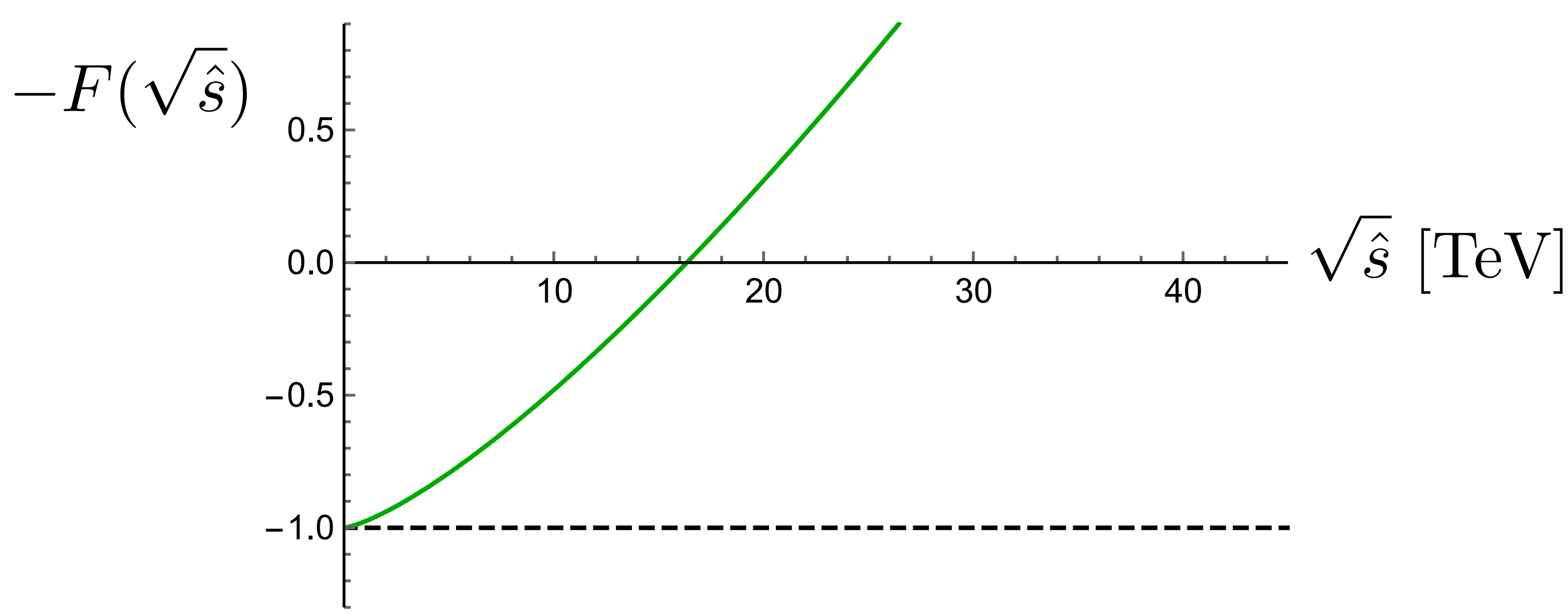

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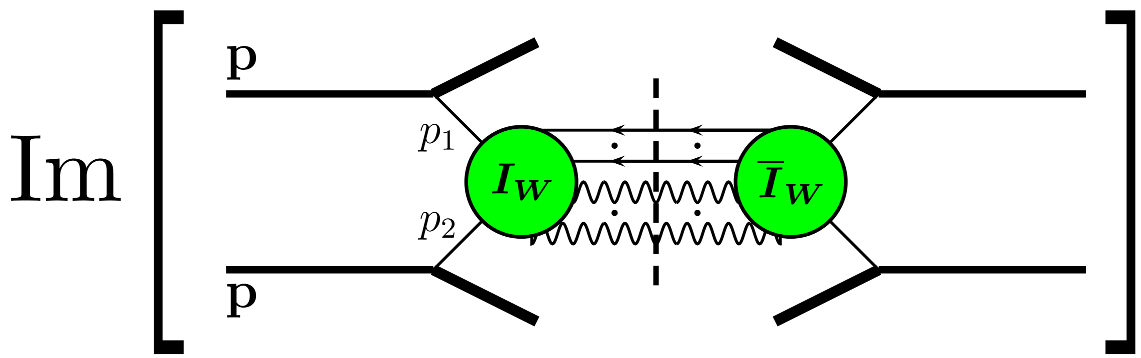

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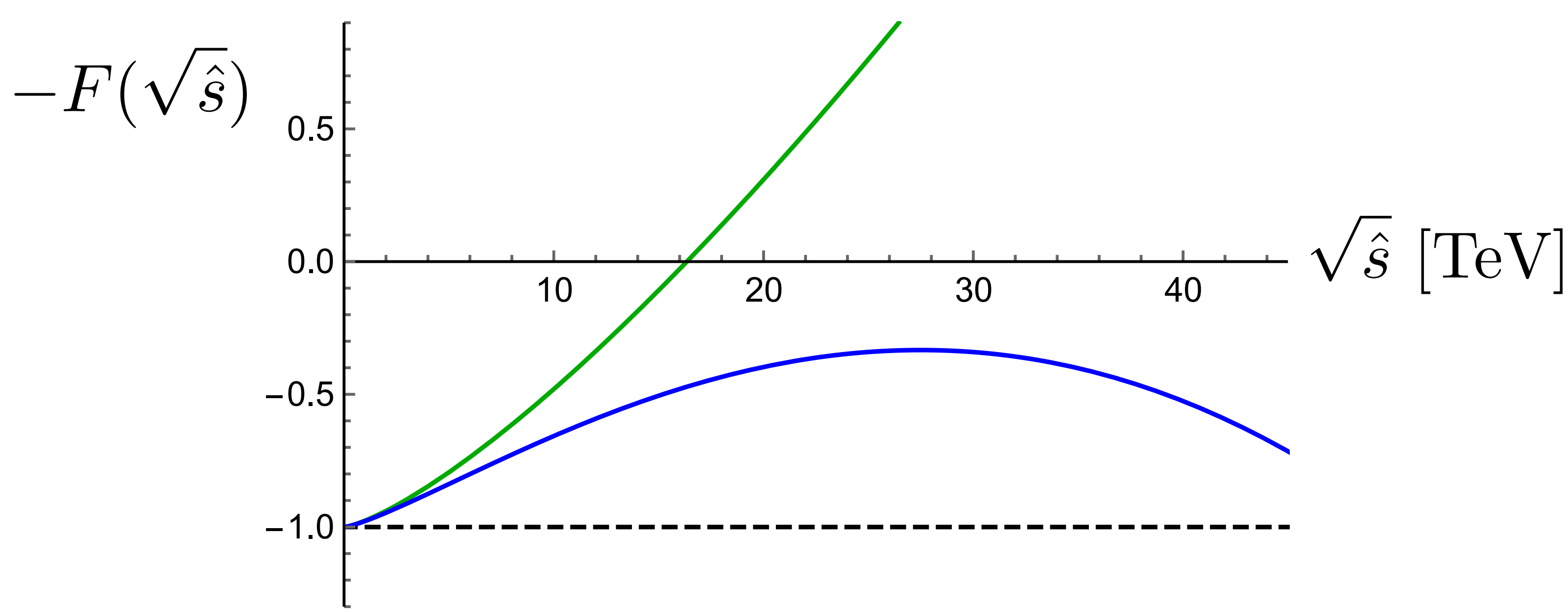

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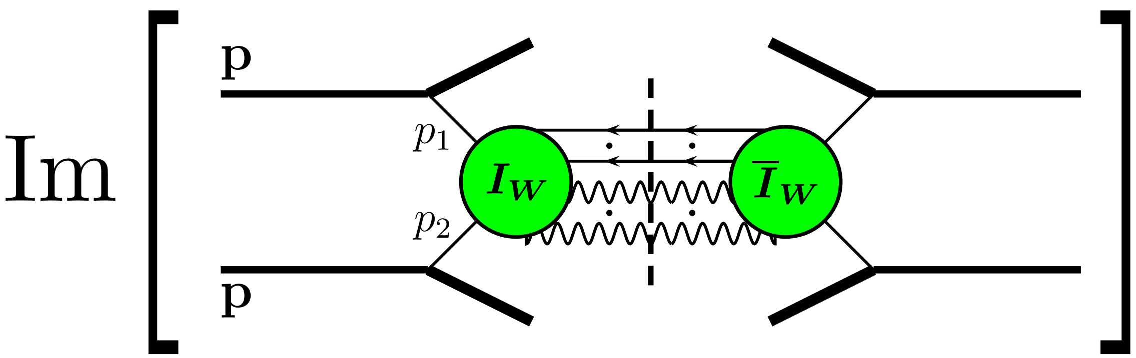

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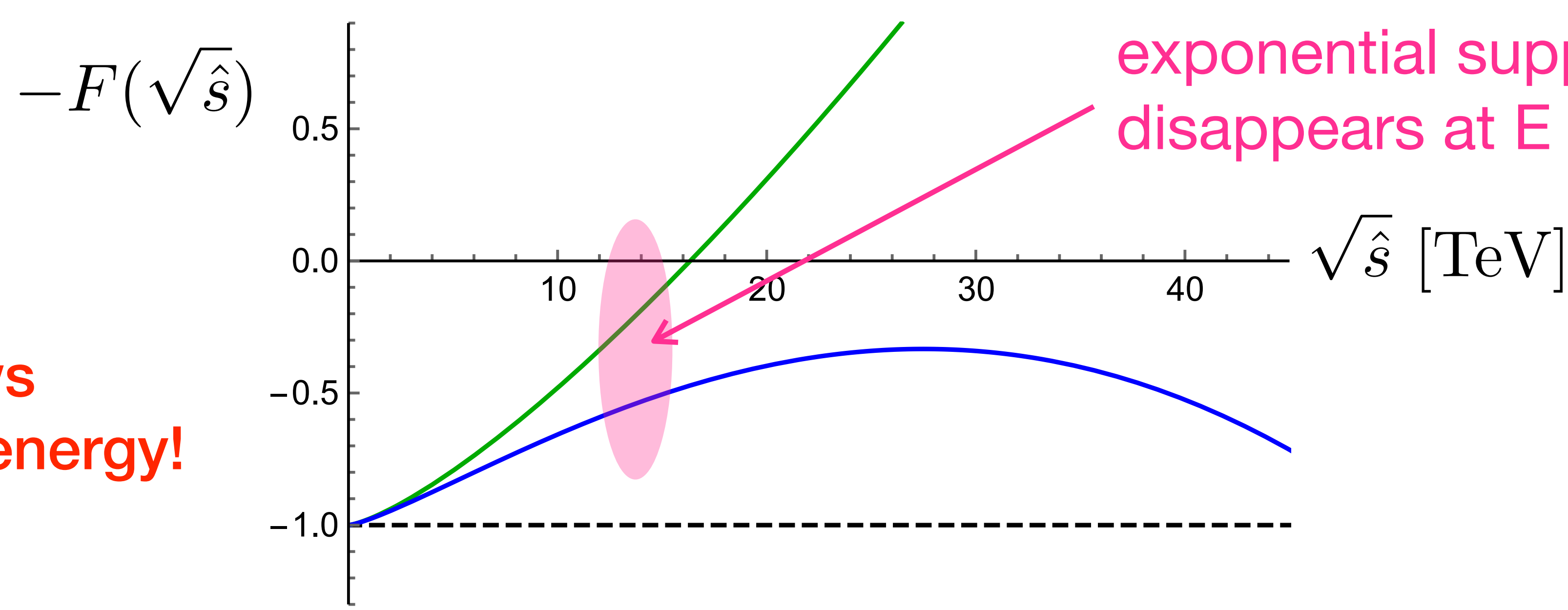


exponential suppression disappears at E ~ E_{sph}?

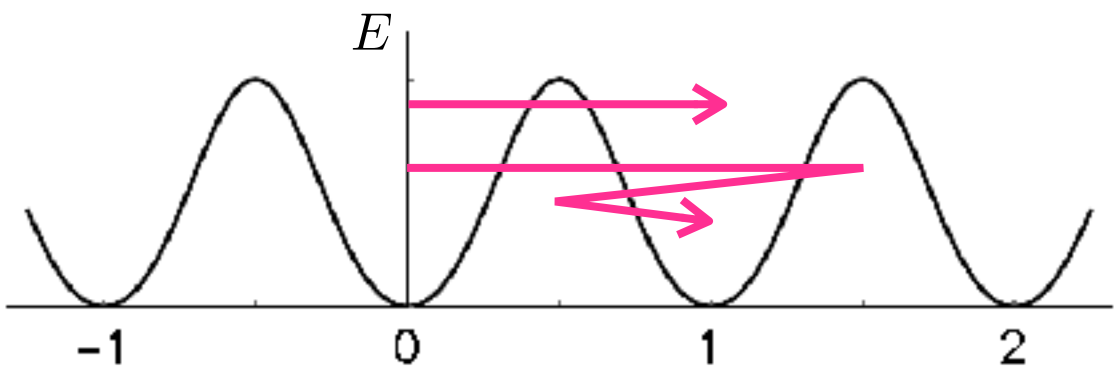
Cross-section grows exponentially with energy!

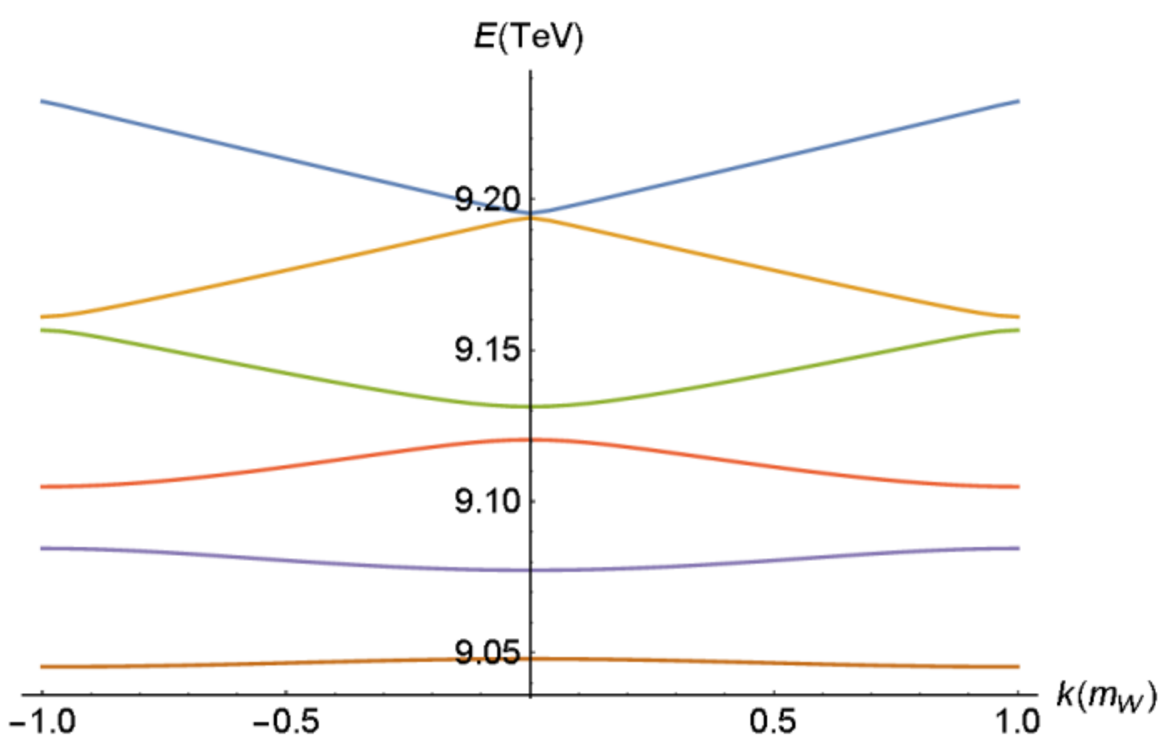
- More recently (2015), it has been pointed out that at zero temperature instanton rate may be able to overcome the exponential suppression for $E > E_{sph} \sim 9$ TeV, if the periodicity of the EW potential is taken into account, due to *resonant tunnelling*.

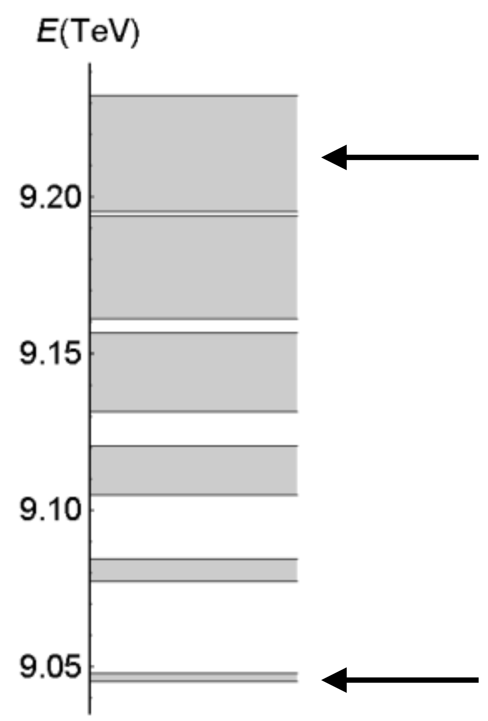
Tye, Wong [1505.03690, 1710.07223]
Qiu, Tye [1812.07181]

Resonant tunneling:

Different paths coherently interfere at particular energies, forming a conducing band structure



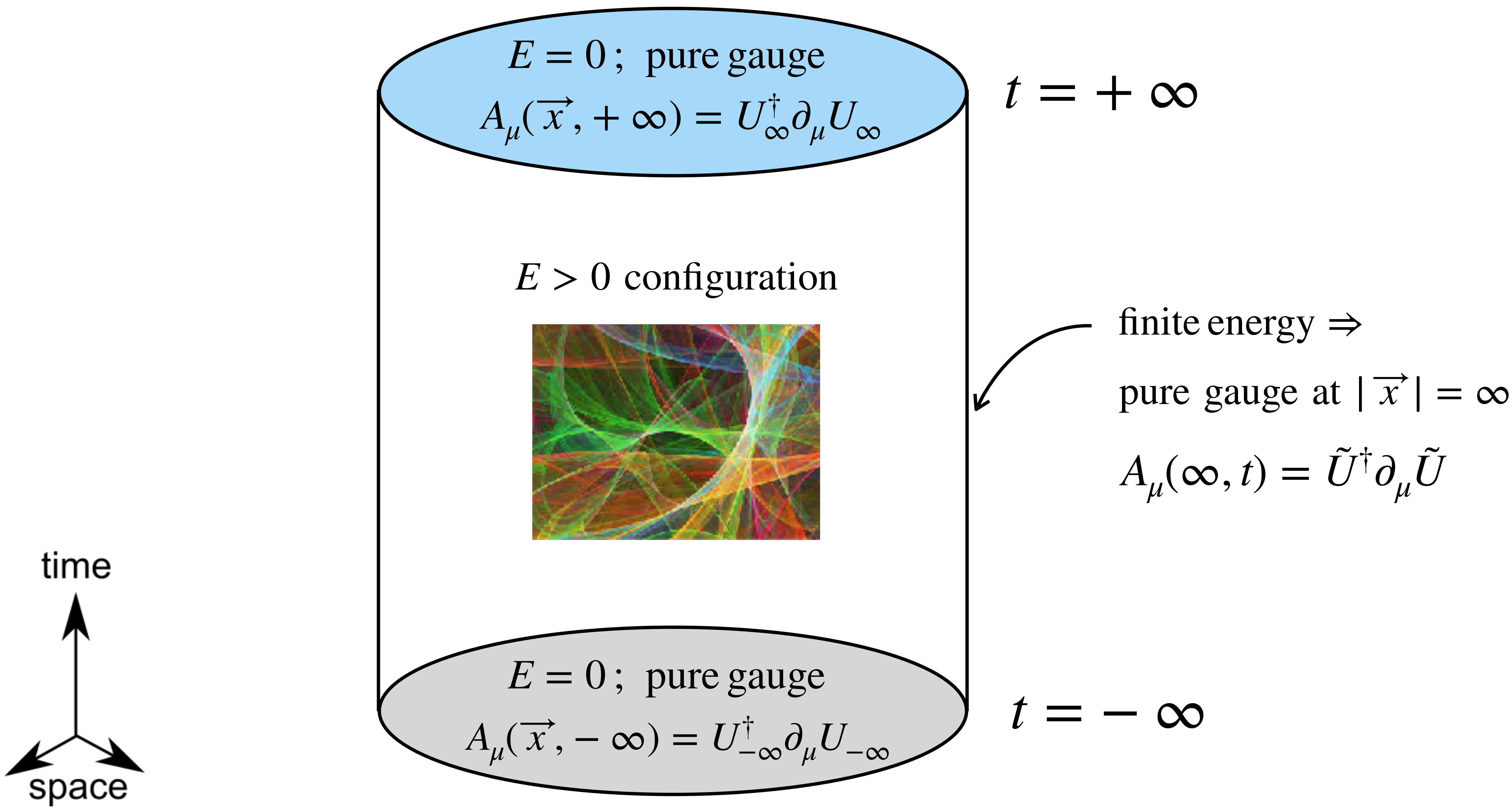




the band widths are unsuppressed for E > 9.1 TeV.

Exponentially small widths for E < 9.1 TeV corresponds to the exponential suppression in instantons

Vacuum-to-vacuum transitions



Vacuum-to-vacuum transitions

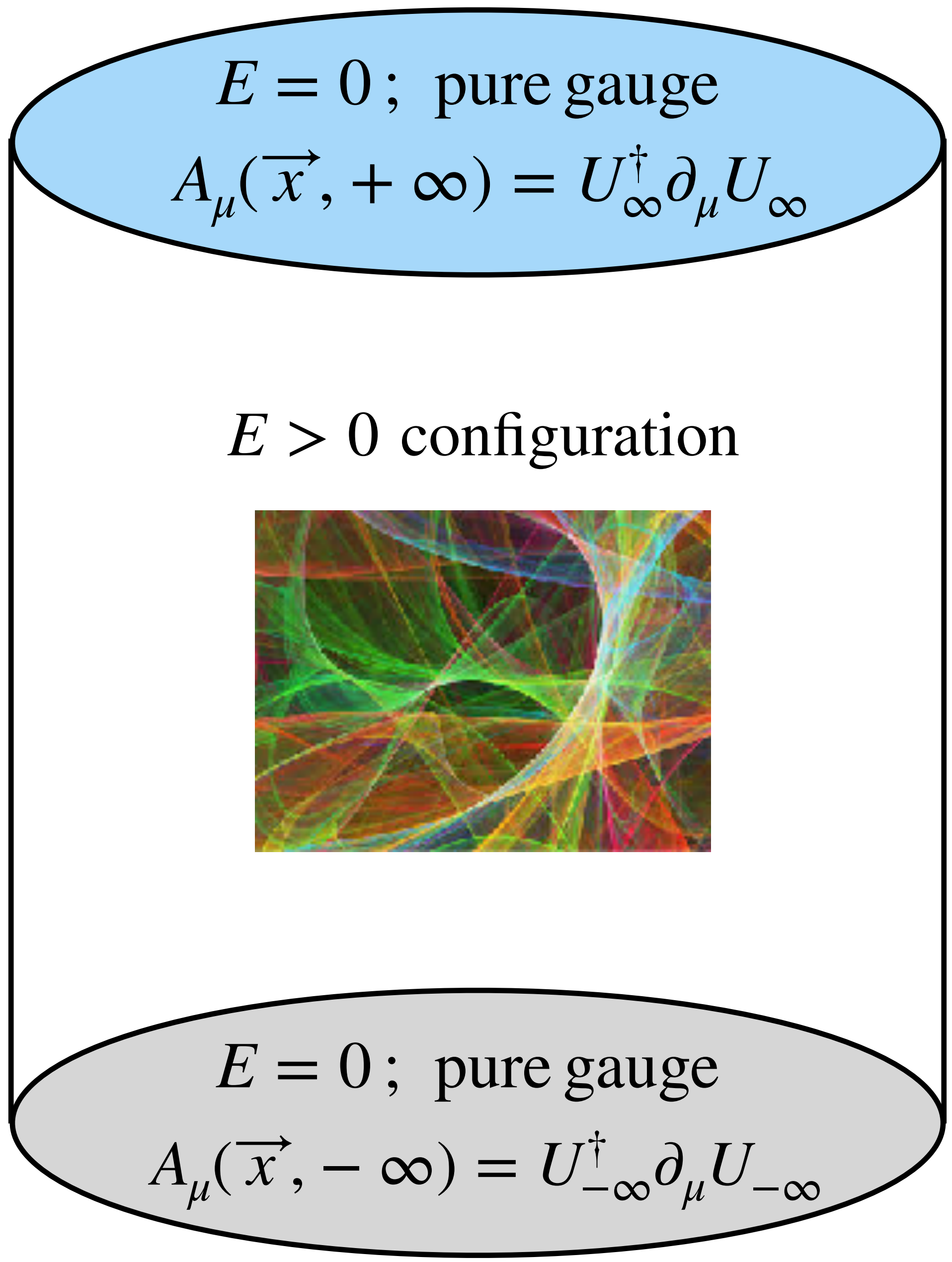
Temporal gauge : $A_0 = 0$

$\Rightarrow \; U^\dagger \partial_0 U = 0$

$\Rightarrow \; U(\vec{x}) = \; t$ independent

Fix the rest such that :

$A_\mu(\vec{x}, -\infty) = 0$



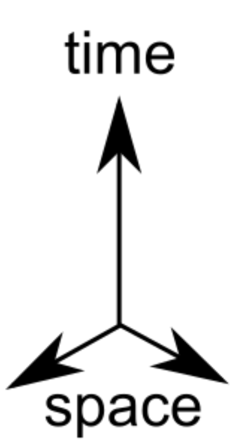
$t = +\infty$

$t = -\infty$

finite energy $\Rightarrow$

pure gauge at $|\vec{x}| = \infty$

$A_\mu(\infty, t) = \tilde{U}^\dagger \partial_\mu \tilde{U}$



Vacuum-to-vacuum transitions

Temporal gauge : $A_0 = 0$

$\Rightarrow \; U^\dagger \partial_0 U = 0$

$\Rightarrow \; U(\overrightarrow{x}) = \; t$ independent

Fix the rest such that :

$A_\mu(\overrightarrow{x}, -\infty) = 0$

in particular $U_{-\infty} = \mathbf{1}$

$E = 0\,;$ pure gauge

$A_\mu(\overrightarrow{x}, +\infty) = U^\dagger_\infty \partial_\mu U_\infty$

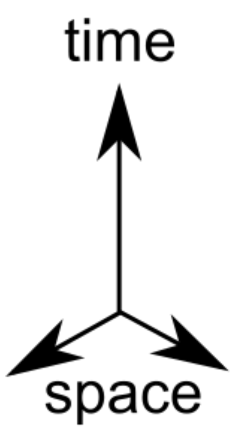
$t = +\infty$

$E > 0$ configuration

finite energy $\Rightarrow$

pure gauge at $|\overrightarrow{x}| = \infty$

$A_\mu(\infty, t) = \tilde{U}^\dagger \partial_\mu \tilde{U}$



$E = 0\,;$ pure gauge

$A_\mu(\overrightarrow{x}, -\infty) = U^\dagger_{-\infty} \partial_\mu U_{-\infty}$

$t = -\infty$

$U = \mathbf{1}$

Vacuum-to-vacuum transitions

$U_\infty(\vec{x}) \xrightarrow{|\vec{x}| \to \infty} \mathbf{1}$

$\Rightarrow \mathbf{R}^3 \cup \{\infty\} \cong S^3$

Temporal gauge : $A_0 = 0$

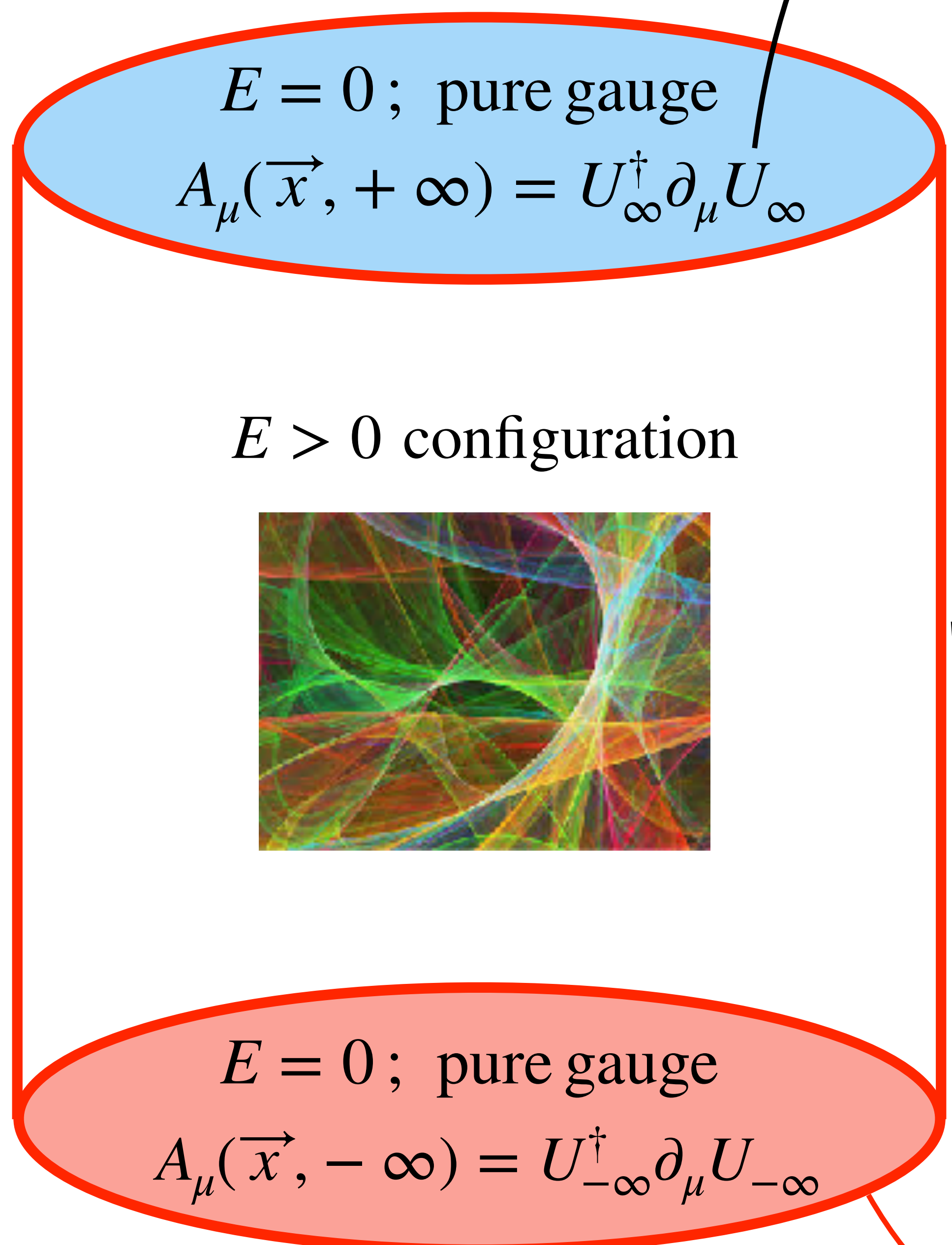
$\Rightarrow U^\dagger \partial_0 U = 0$

$\Rightarrow U(\vec{x}) = t$ independent

Fix the rest such that :

$A_\mu(\vec{x}, -\infty) = 0$

in particular $U_{-\infty} = \mathbf{1}$



$t = +\infty$

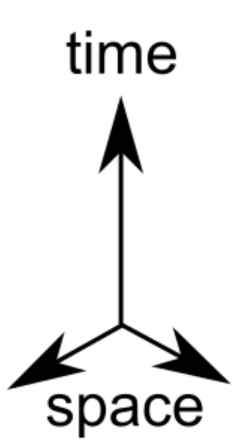
finite energy $\Rightarrow$

pure gauge at $|\vec{x}| = \infty$

$A_\mu(\infty, t) = \tilde{U}^\dagger \partial_\mu \tilde{U}$

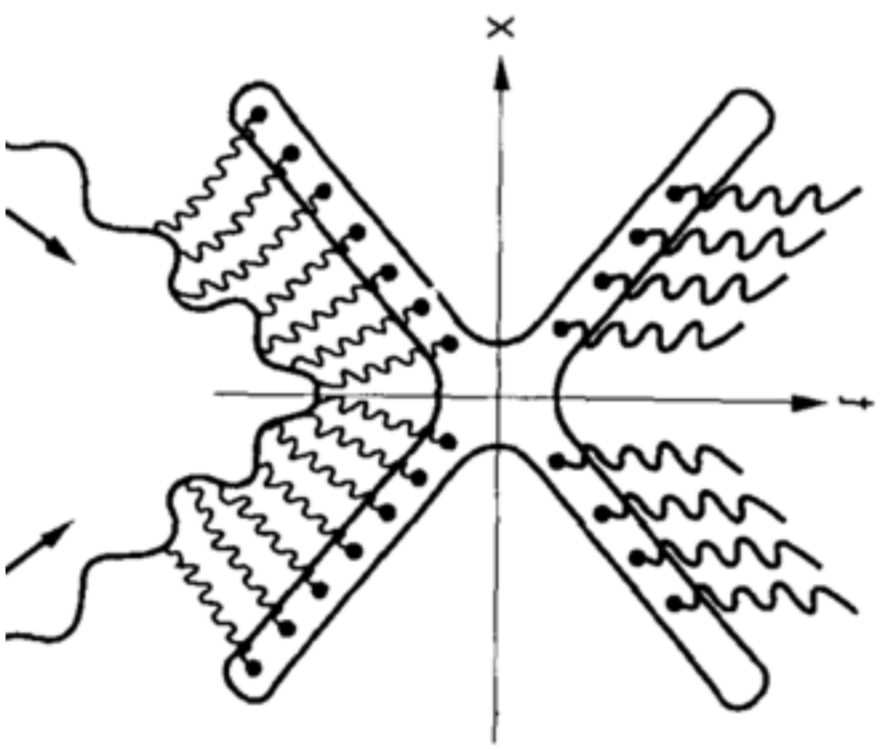
$t = -\infty$

$U = \mathbf{1}$

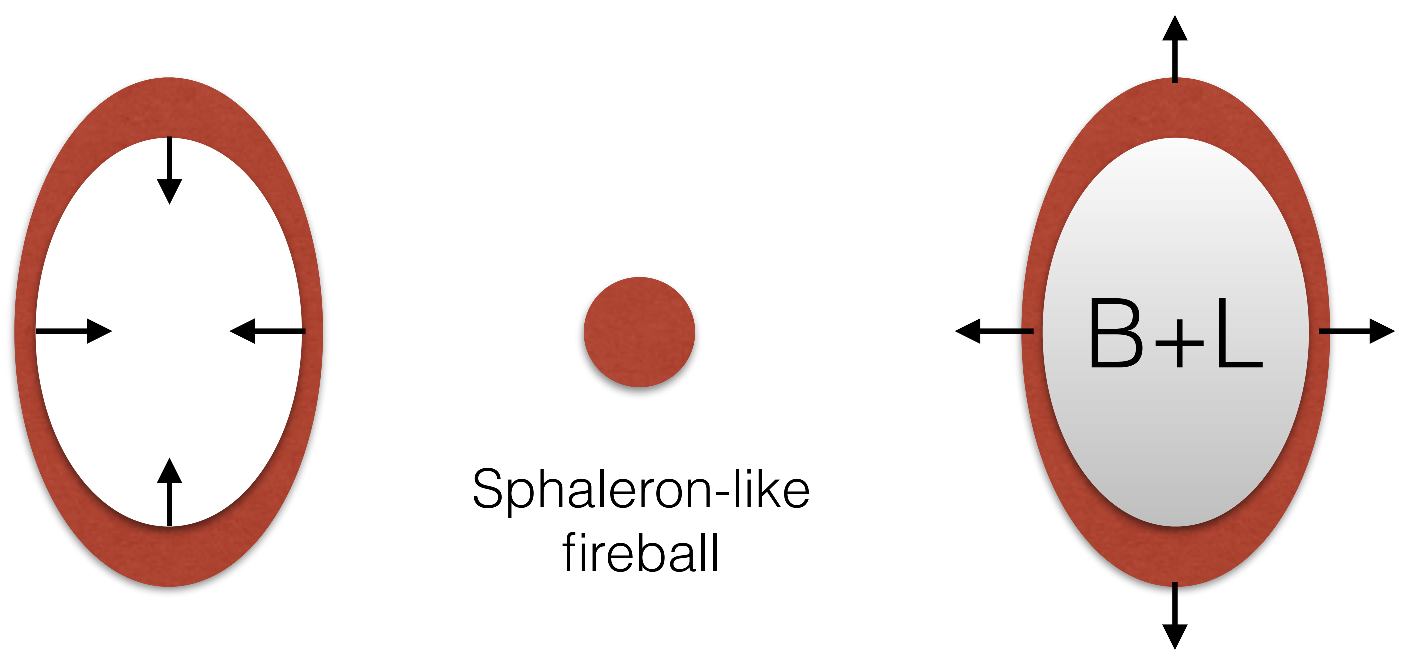


Optimistic view:

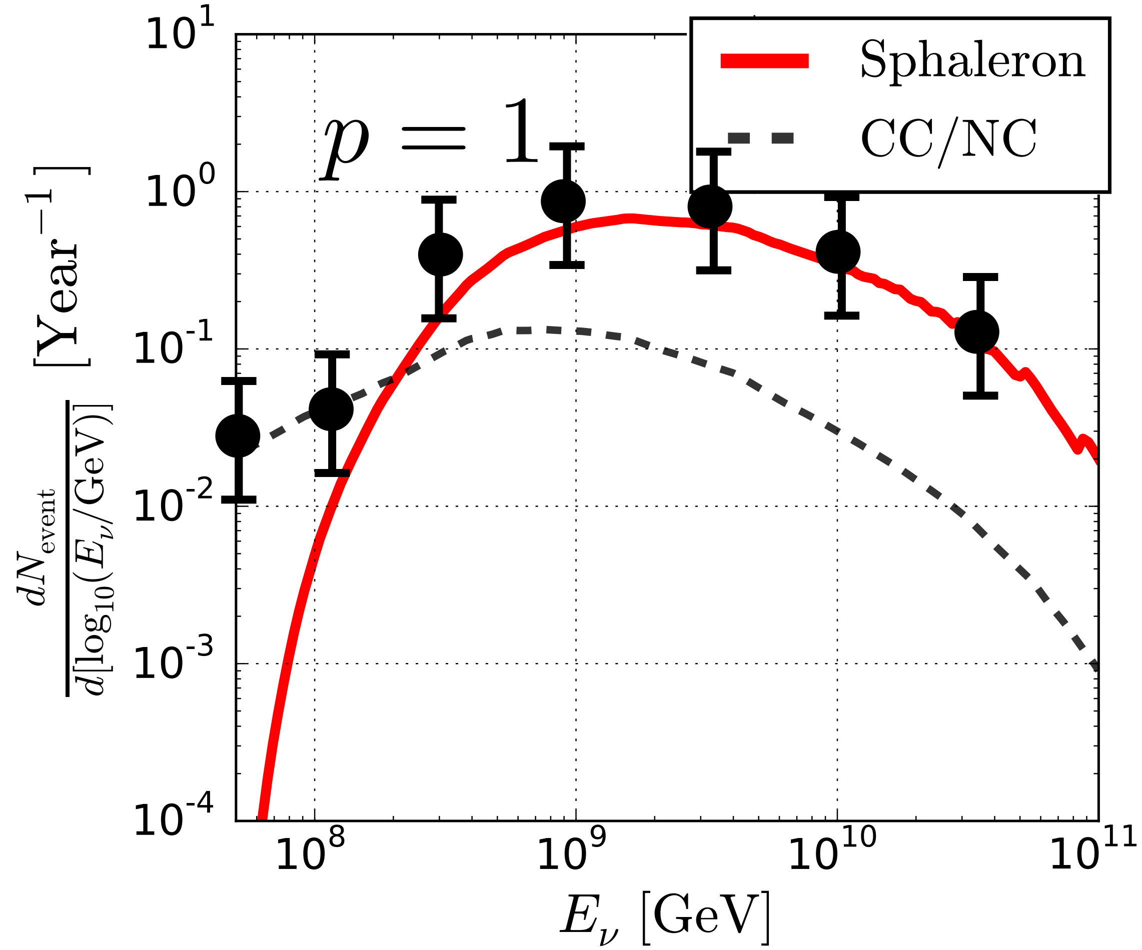
1. It is not the sphaleron which is directly created in the initial collision
2. Instantons in Minkwoski space are not point-like configurations; they are localized near the light-cone:



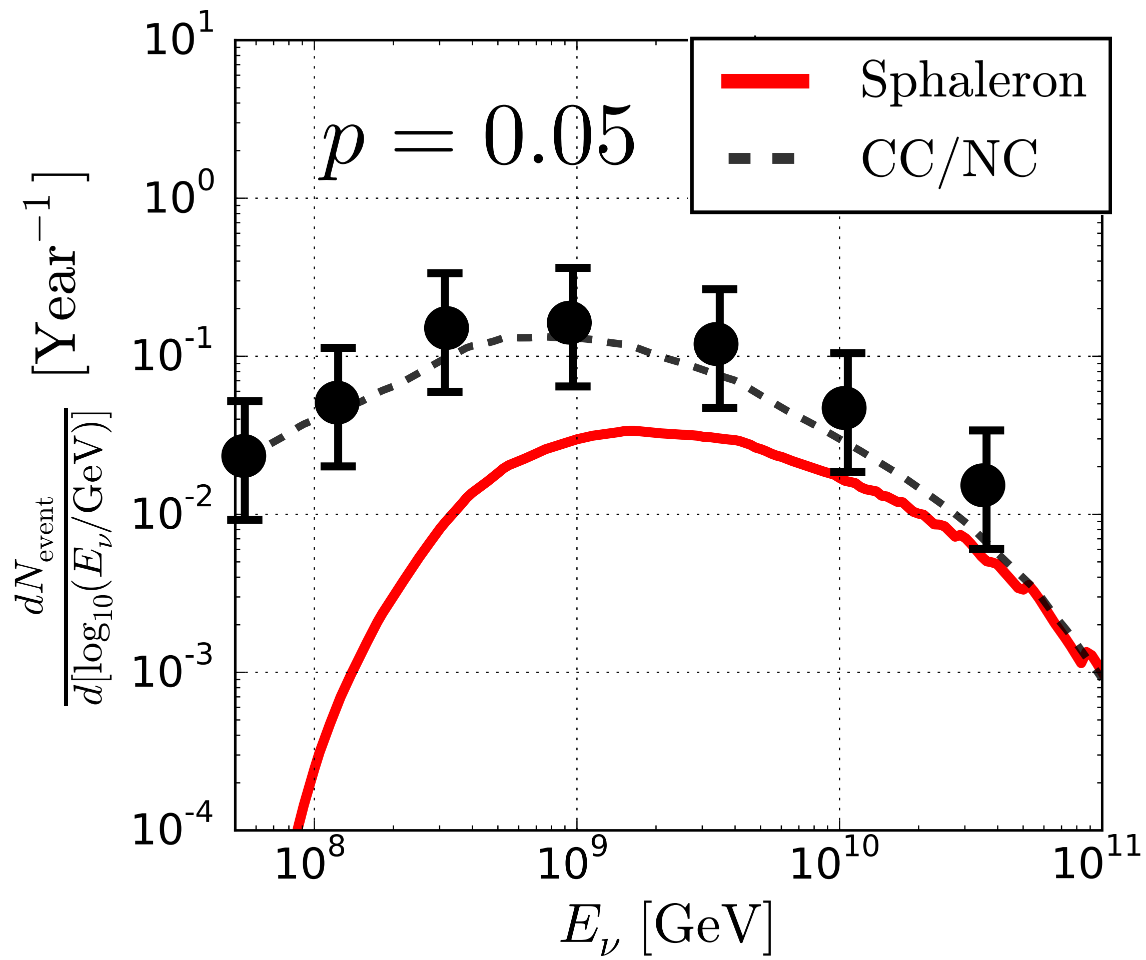
Cartoon of snapshots in time:



Taken from Valya K



- If unknown pre-factor p is small, the sphaleron events may be hidden in the GZK neutrino events via the ordinary EW interaction.
- In this case, discrimination using the event shape is important.

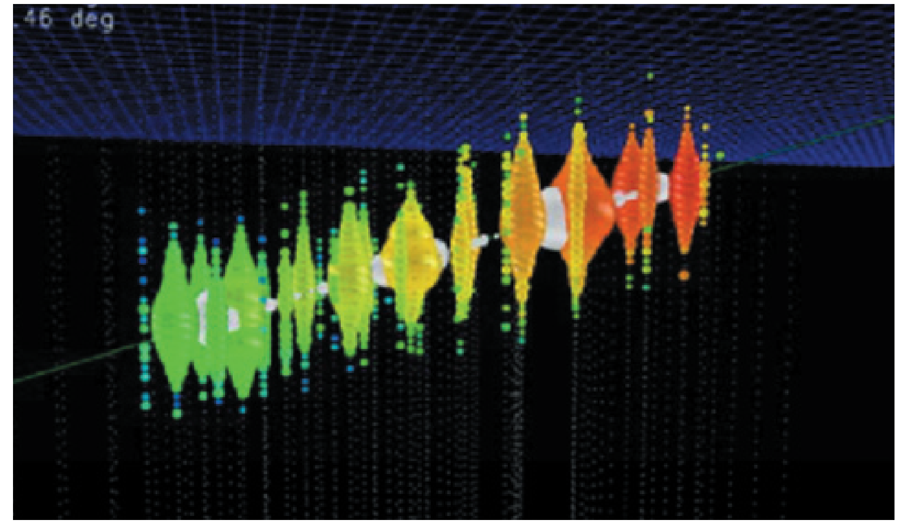


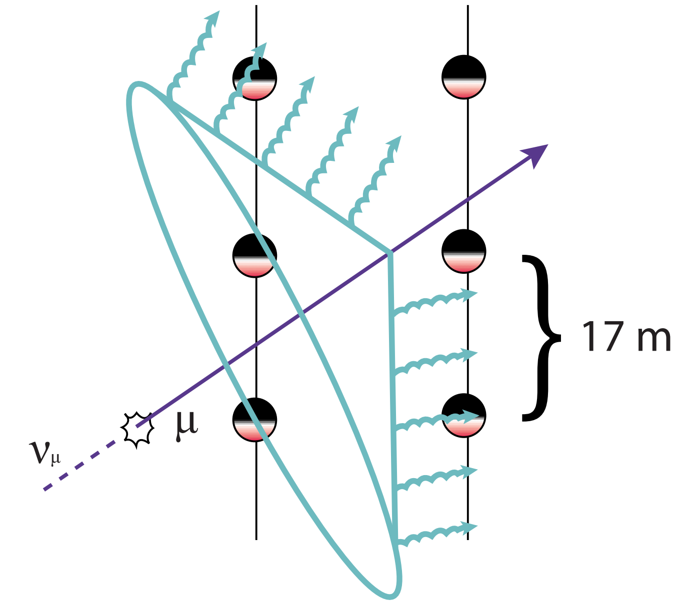
How do sphaleron events look different from the ordinary neutrino events at IceCube?

IceCube Events:

"muon bundle"

$$\nu_\mu N \to \mu X$$



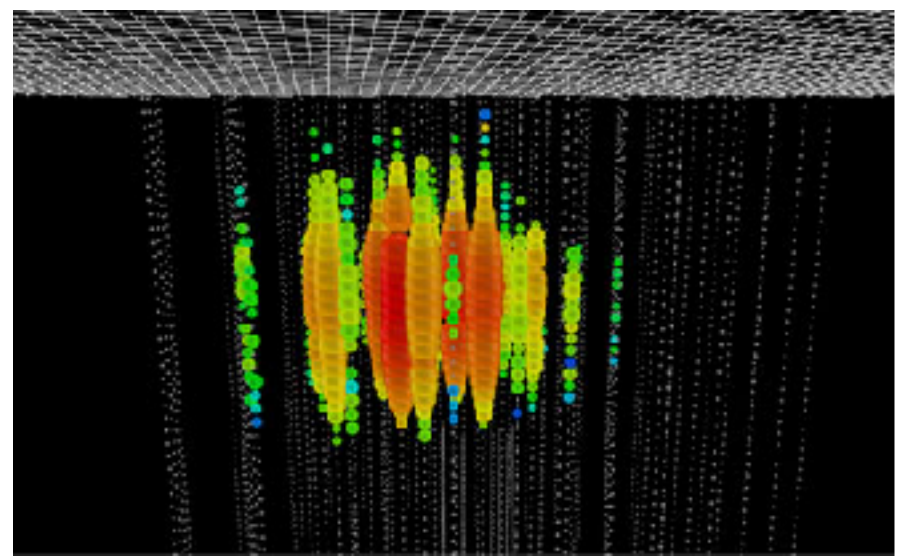


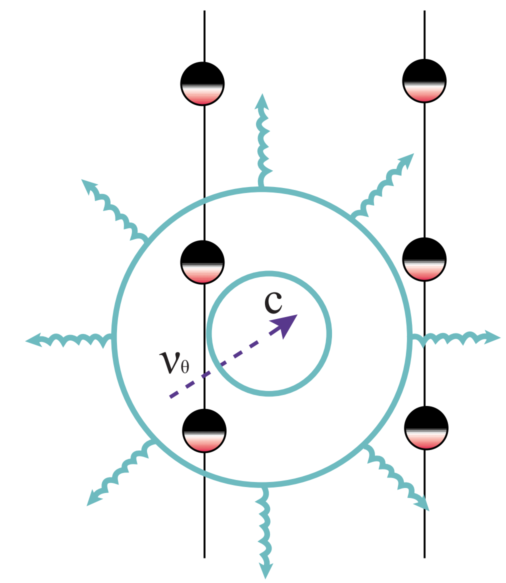
"shower"

$$\nu_e N \to e X$$

$$\nu_\tau N \to \tau X$$

$$\nu_i N \to \nu_i X$$

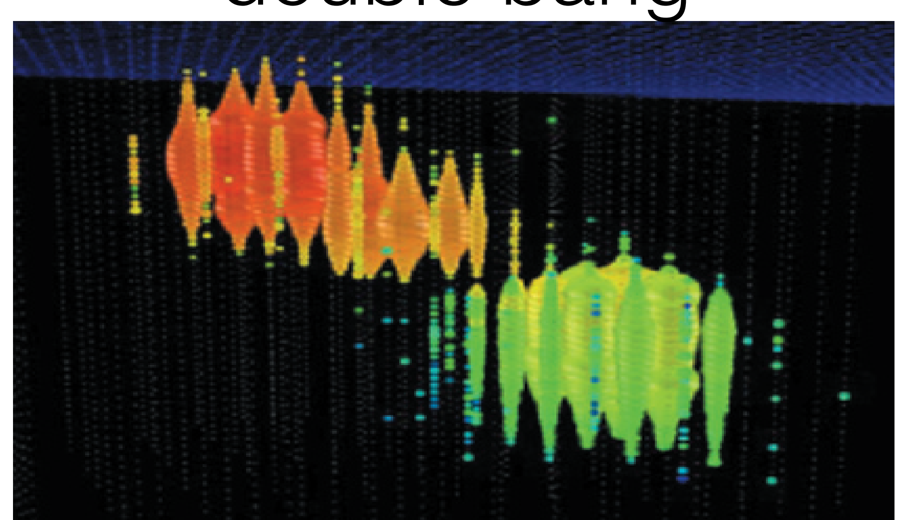




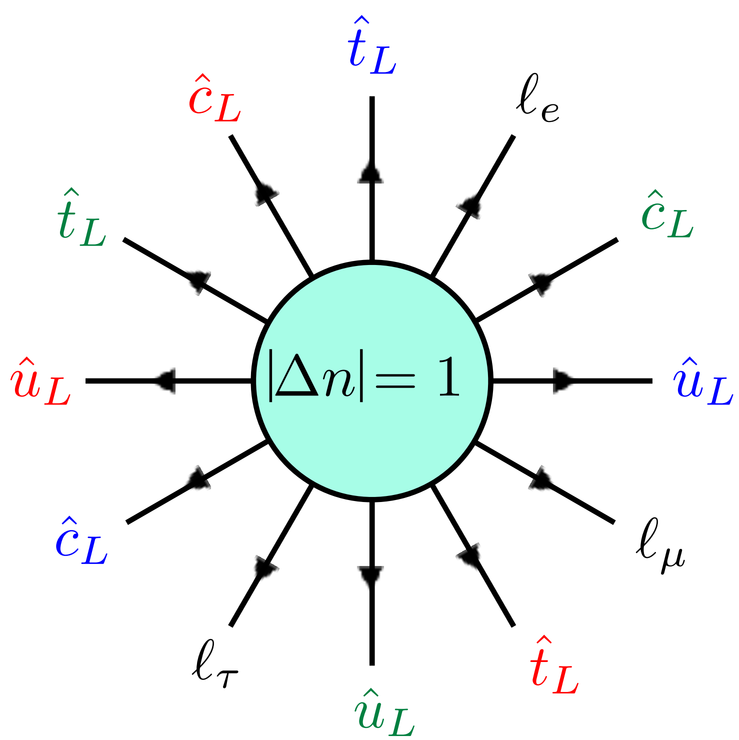
"double bang"

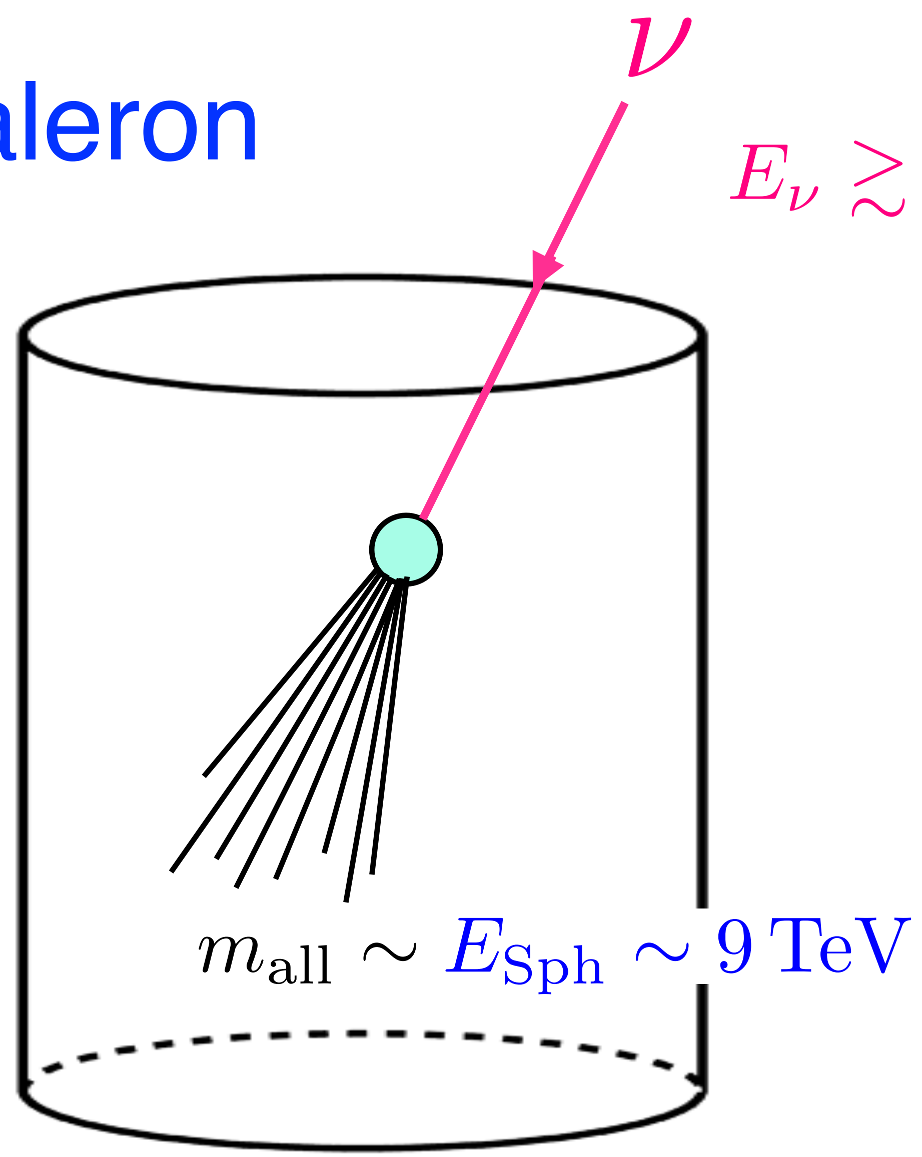
$$\nu_\tau N \to \tau X_1 \to X_1 \nu_\tau X_2$$

$$E_\tau \in [10^6, 10^7]\,\mathrm{GeV}$$

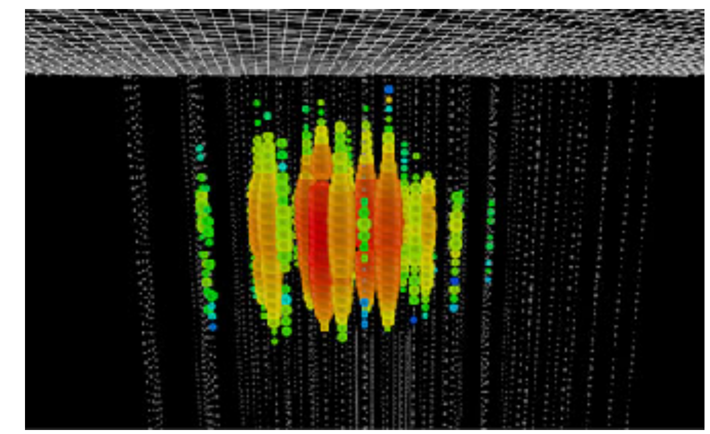


What does the sphaleron event look like?

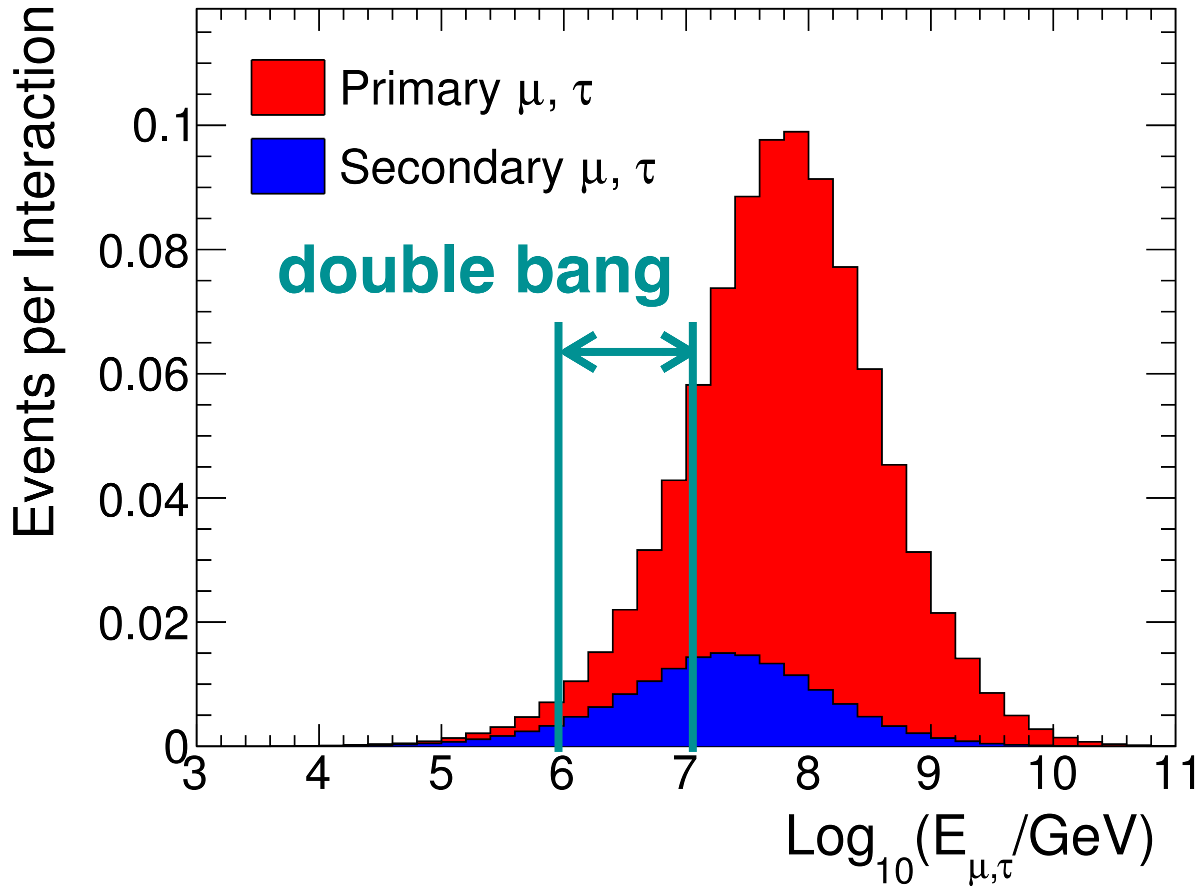




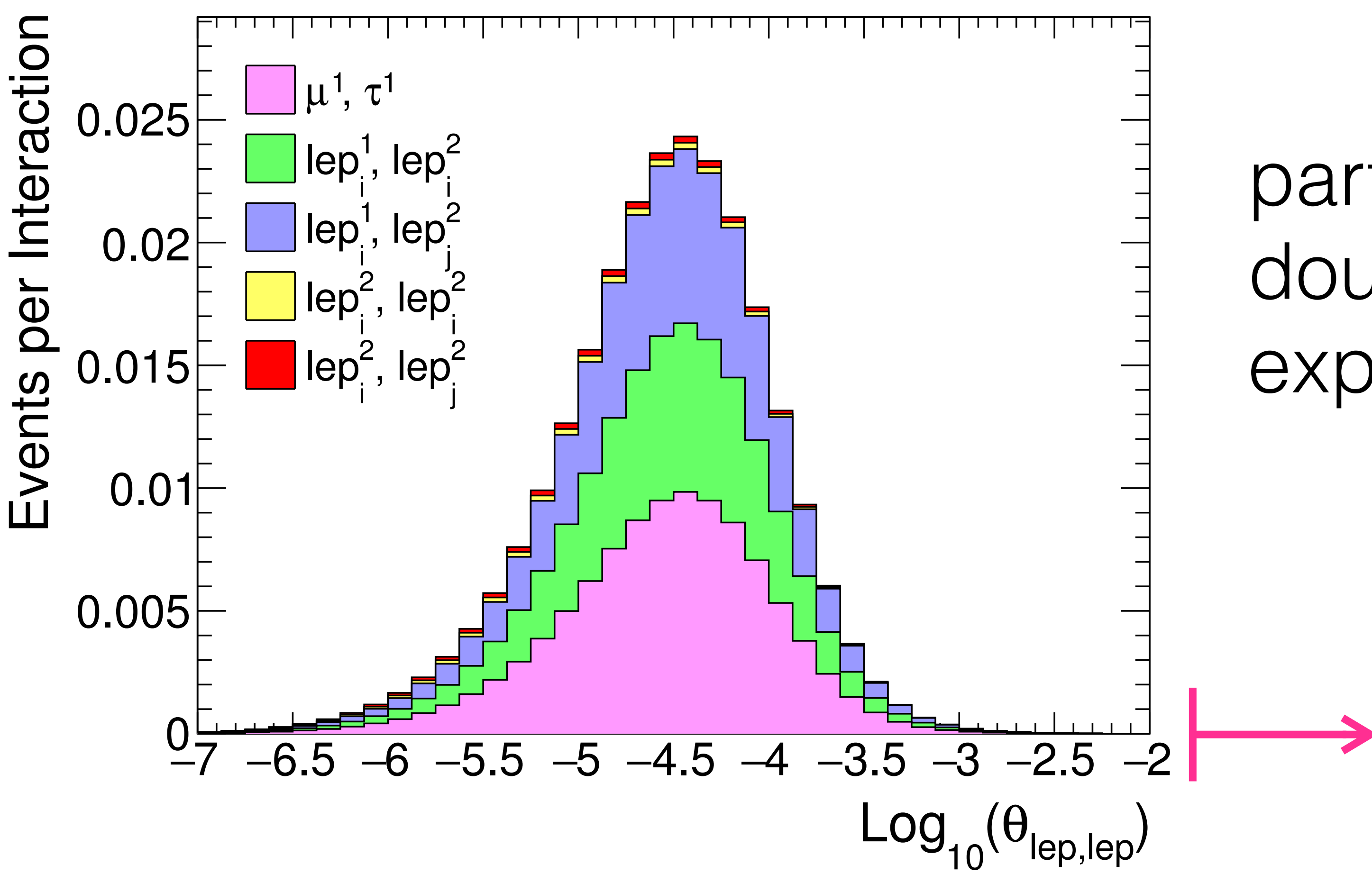
"shower"



- quarks and leptons are stopped in the ice (except for μ). ⇒ "shower"
- If μ is produced. ⇒ "bundle"
- If τ is produced with $E_\tau \in [10^6, 10^7]$ GeV. ⇒ "double bang"
- If primary μ and a μ from a top-quark decay has an opening angle with $\theta > 10^{-2}$ rad ⇒ "double bundle"??



Only 5% of the sphaleron-induced events have double bang taus.



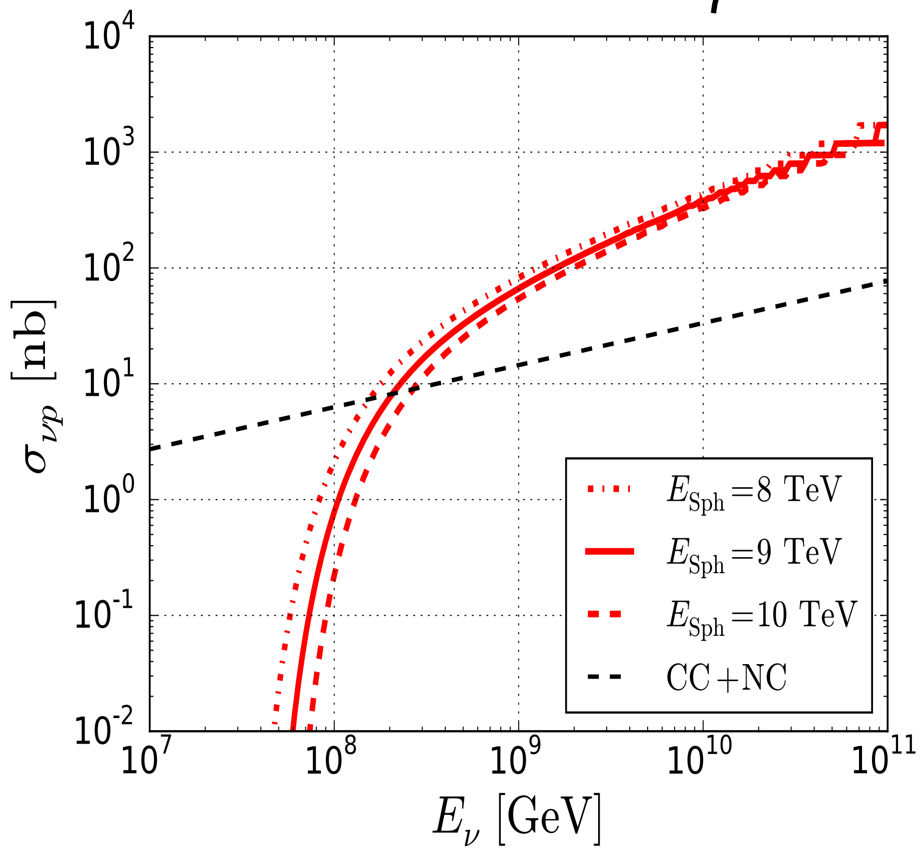
particles are highly collimated and double bundles cannot be expected.

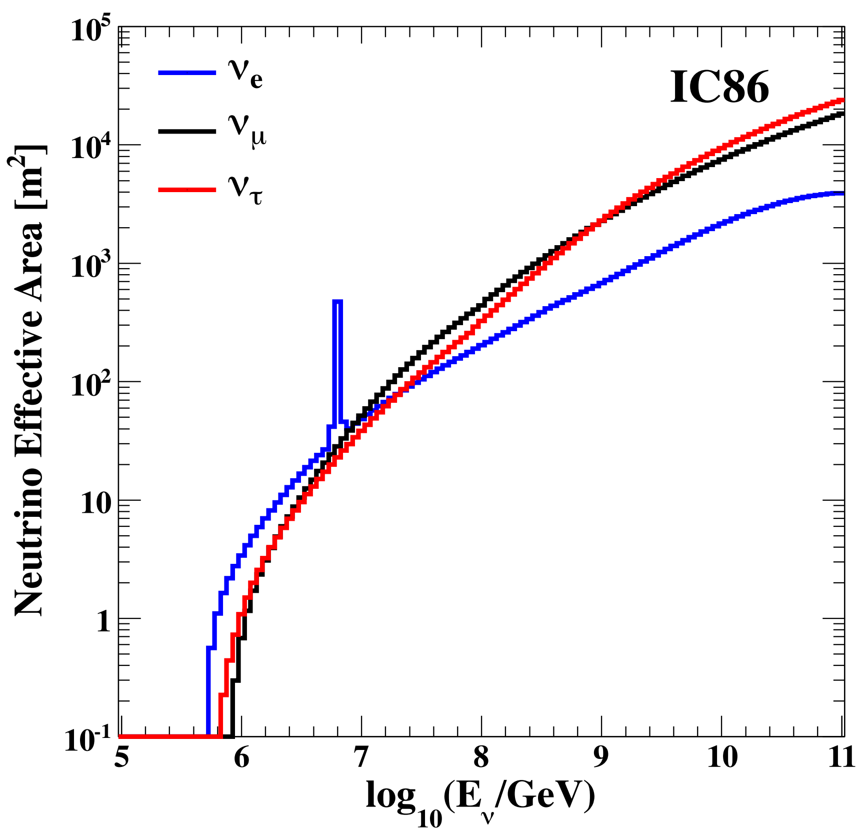
double bundle

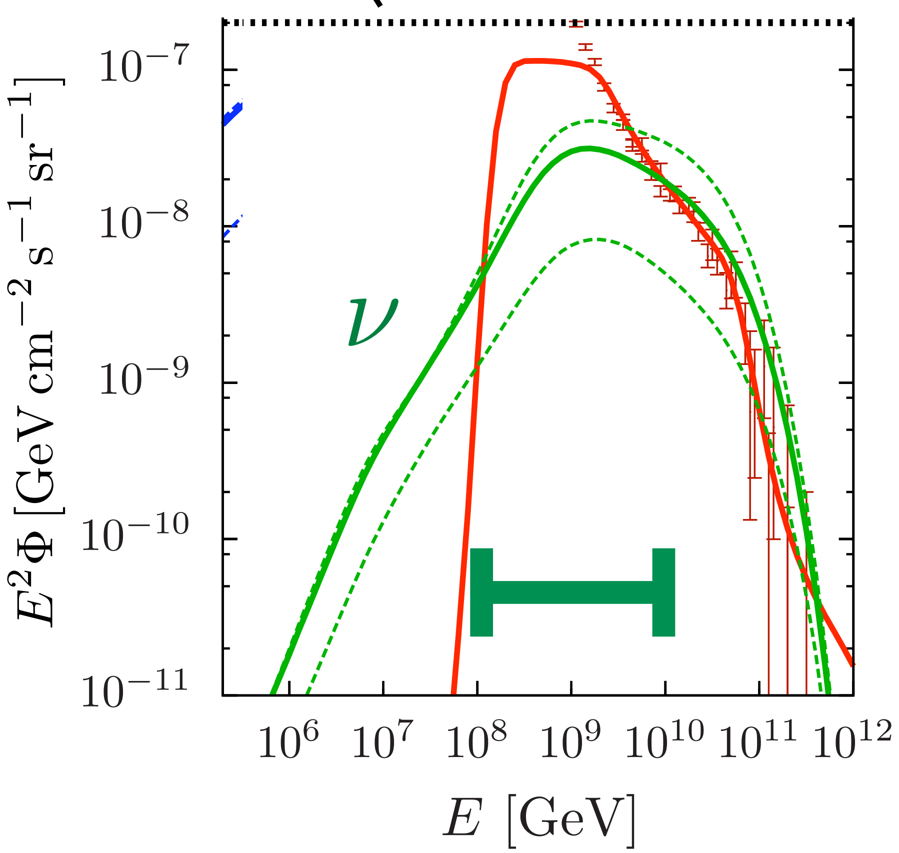
Event rate can be calculated using the energy dependent effective neutrino detection area.

$$\frac{dN_{CC/NC}}{dt} = \int_{E_{\rm three}} dE_\nu A_{\rm eff}(E_\nu) \frac{d^2\Phi}{dE_\nu dt}$$

$$\frac{dN_{\rm Sph}}{dt} = \int_{E_{\rm three}} dE_\nu \frac{\sigma_{\nu N}^{\rm Sph}(E_\nu)}{\sigma_{\nu N}^{CC/NC}(E_\nu)} A_{\rm eff}(E_\nu) \frac{d^2\Phi}{dE_\nu dt}$$







J.Ellis, KS, M.Spannowsky [1603.06573]

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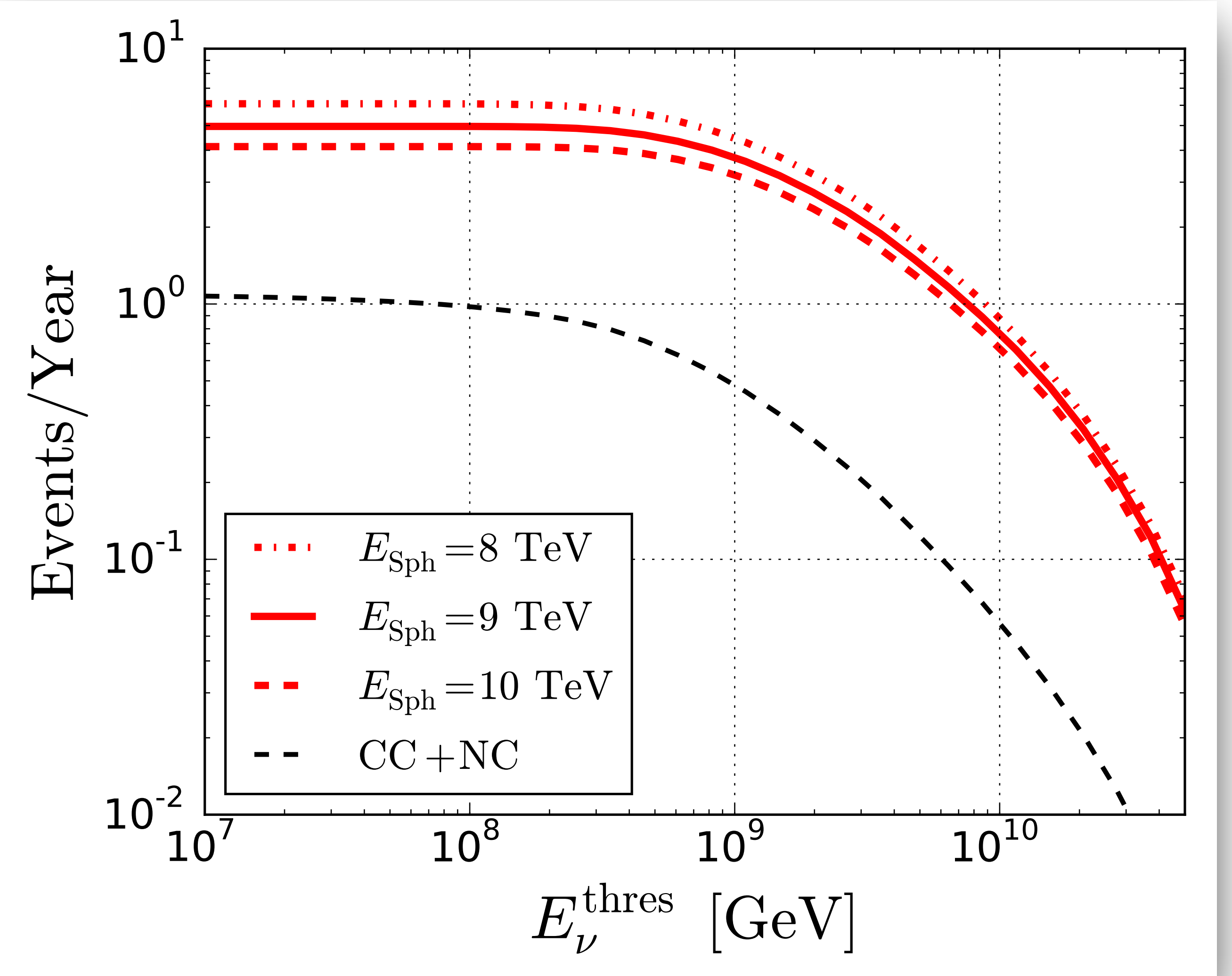
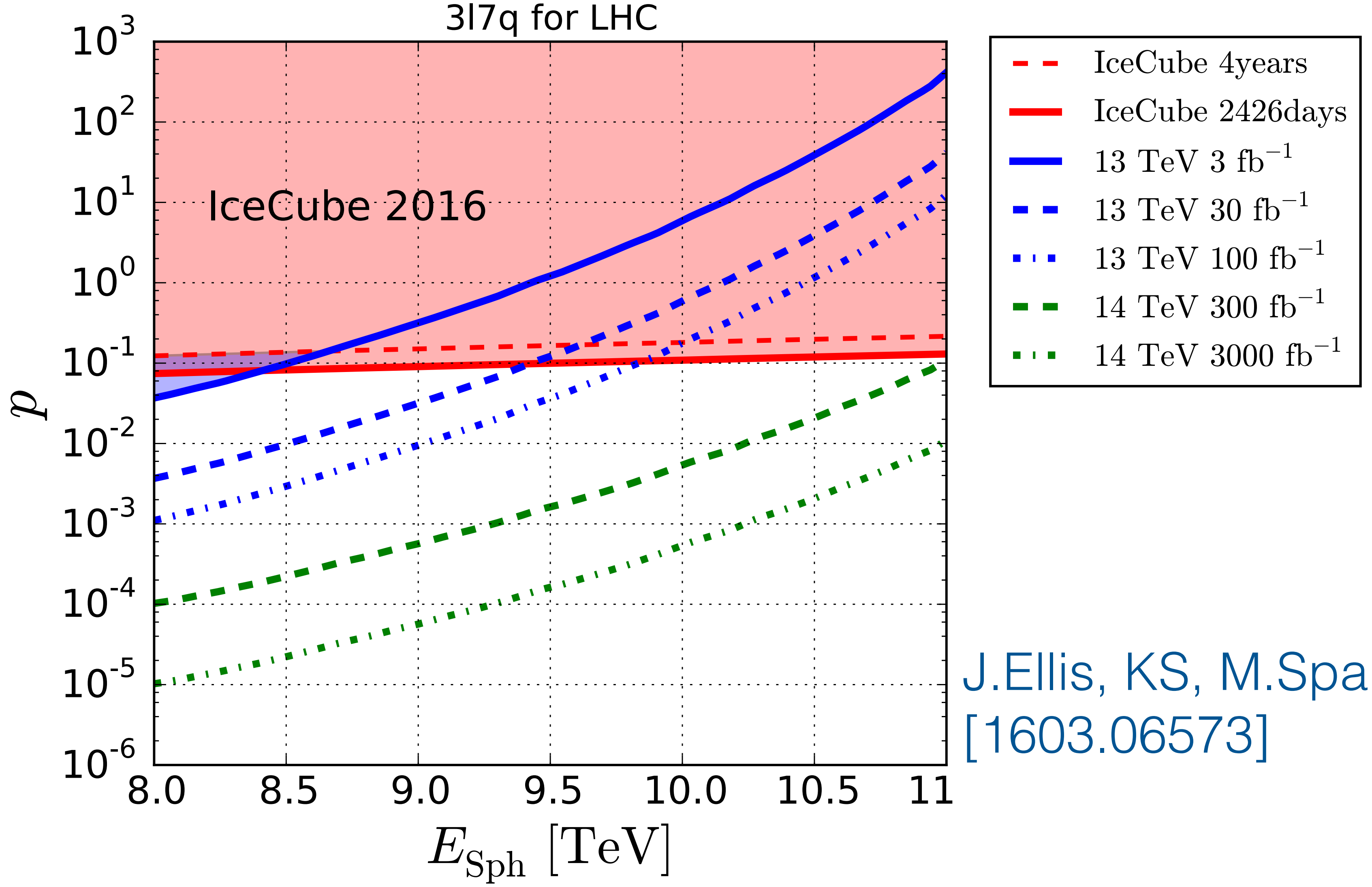
J.Ellis, KS, M.Spannowsky
[1603.06573]



Sensitivity



J.Ellis, KS, M.Spannowsky [1603.06573]

- For E_{Sph} ~ 9TeV, IceCube and LHC sensitivities are comparable.
- Good IceCube sensitivity persists for $E > E_{\mathrm{Sph}}$.

(because the fall of PDF is faster than that of GZK neutrino spectrum)

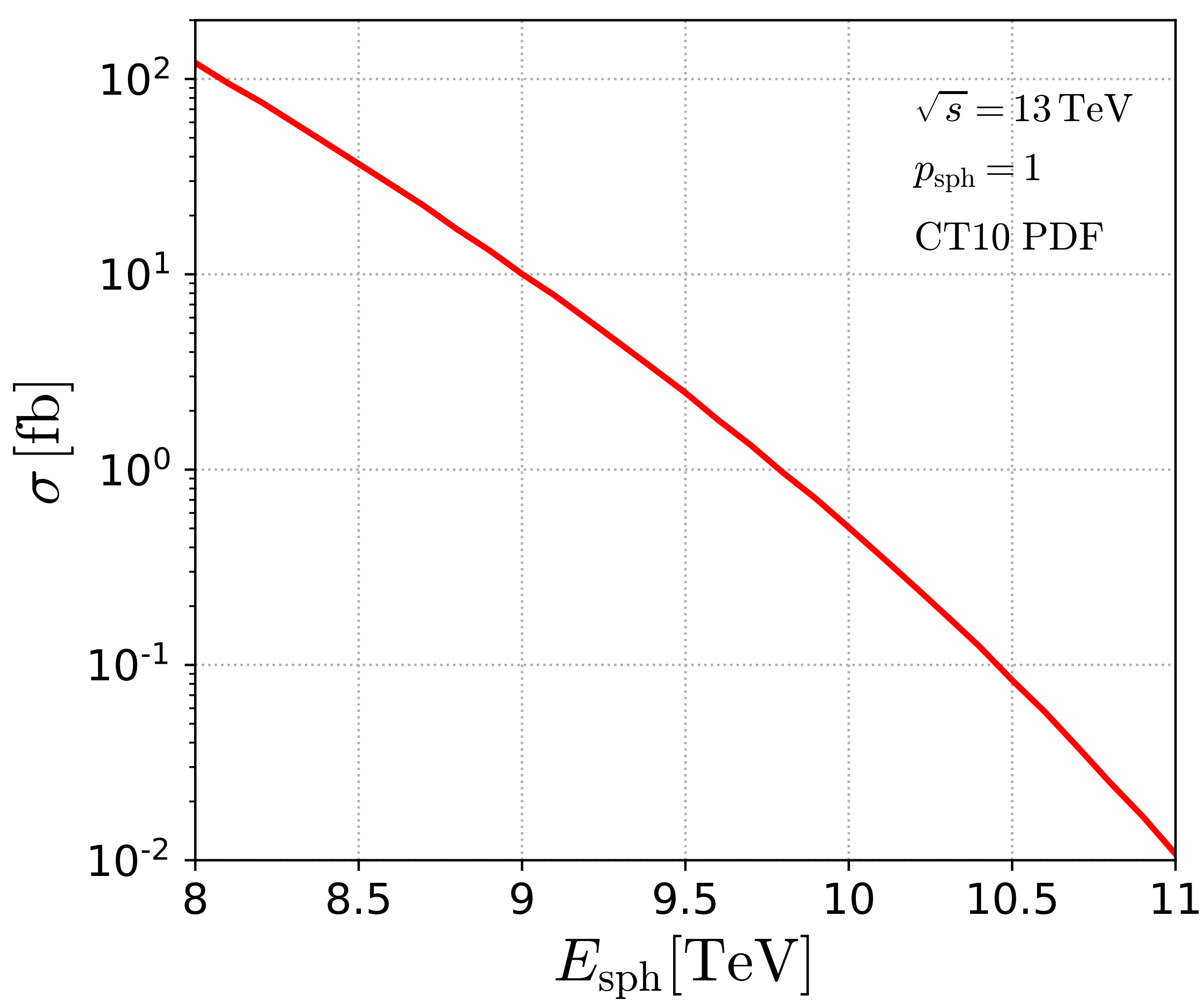
Phenomenological parametrisation for cross-section:

partonic: $$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm sph})$$

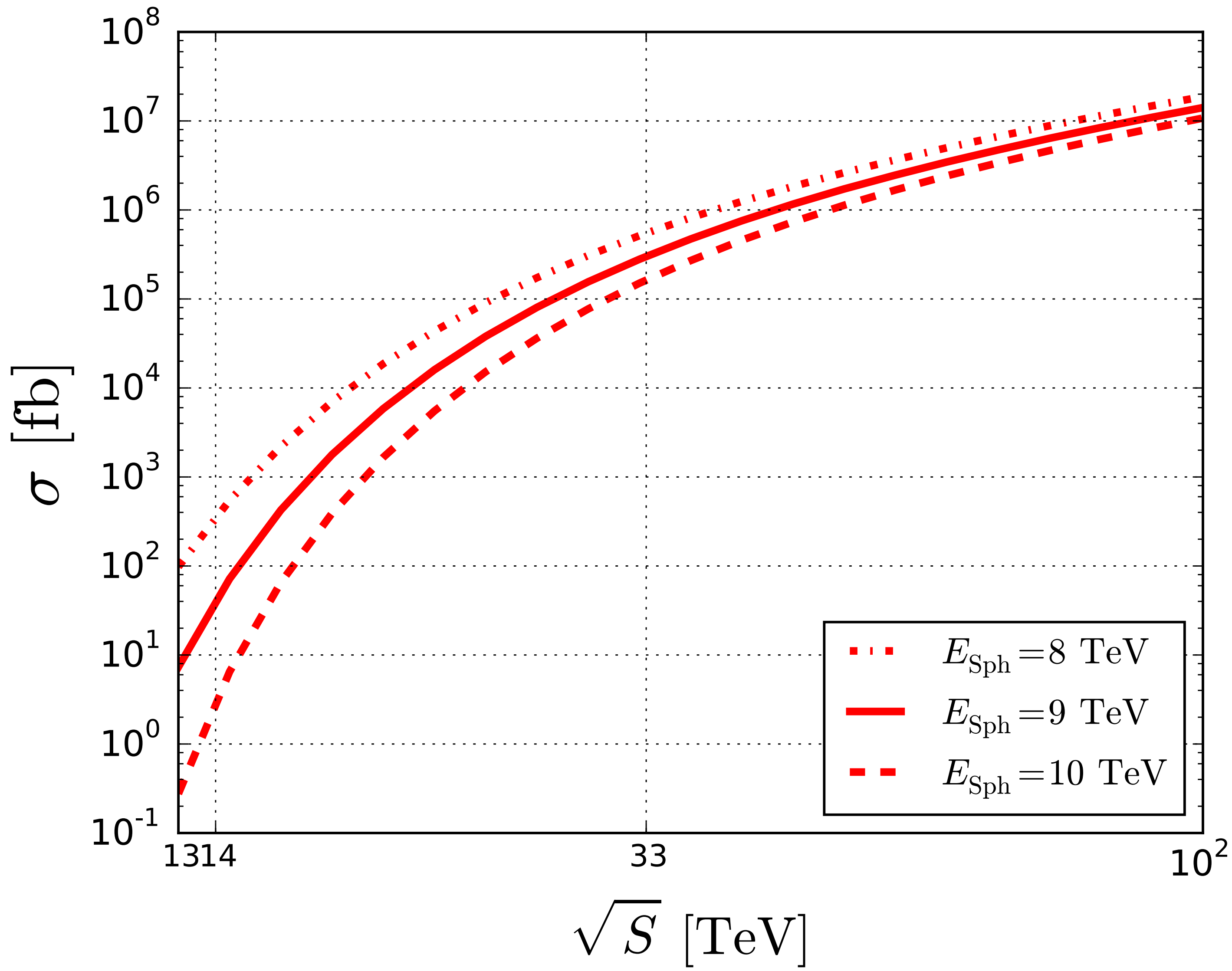
$$c_{ab} = \begin{cases} 2/3 & a, b \text{ same generation} \\ 1 & \text{otherwise} \end{cases}$$

hadronic: $$\sigma_{pp}(\sqrt{s}) = \sum_{ab}\left[c_{ab}\left(\frac{1}{2}\right)^2\right]\int dx_1 x_2 f_a(x_1) f_b(x_2) \hat{\sigma}(\sqrt{s x_1 x_2})$$

anti-symmetric colour construction **L-handed only**



Cross Section

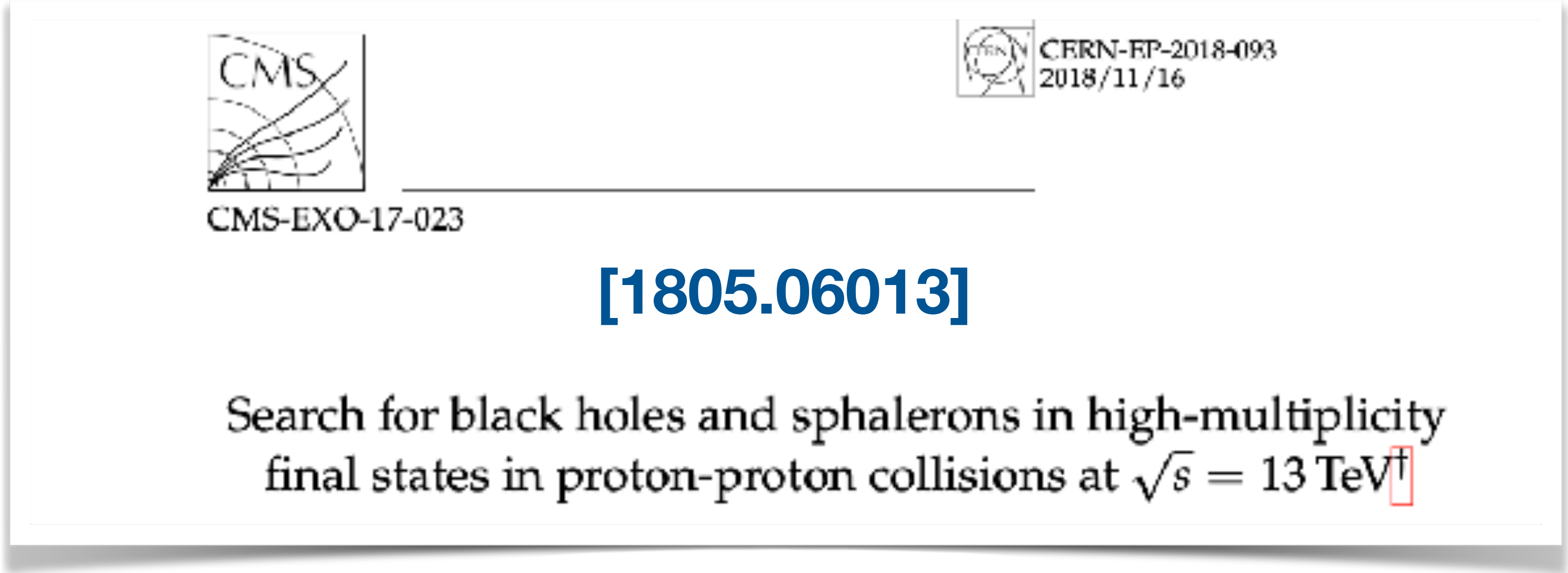


$$p = 1$$

$$E_{\rm Sph} = 9\,\text{TeV}$$

	Sphaleron	gg→H
13 TeV	7.3 fb	44 x 10^3 fb
14 TeV	41 fb	50 x 10^3 fb
33 TeV	0.3 x 10^6 fb	0.2 x 10^6 fb
100 TeV	141 x 10^6 fb	0.7 x 10^6 fb

J. Ellis, KS
[1601.03654]

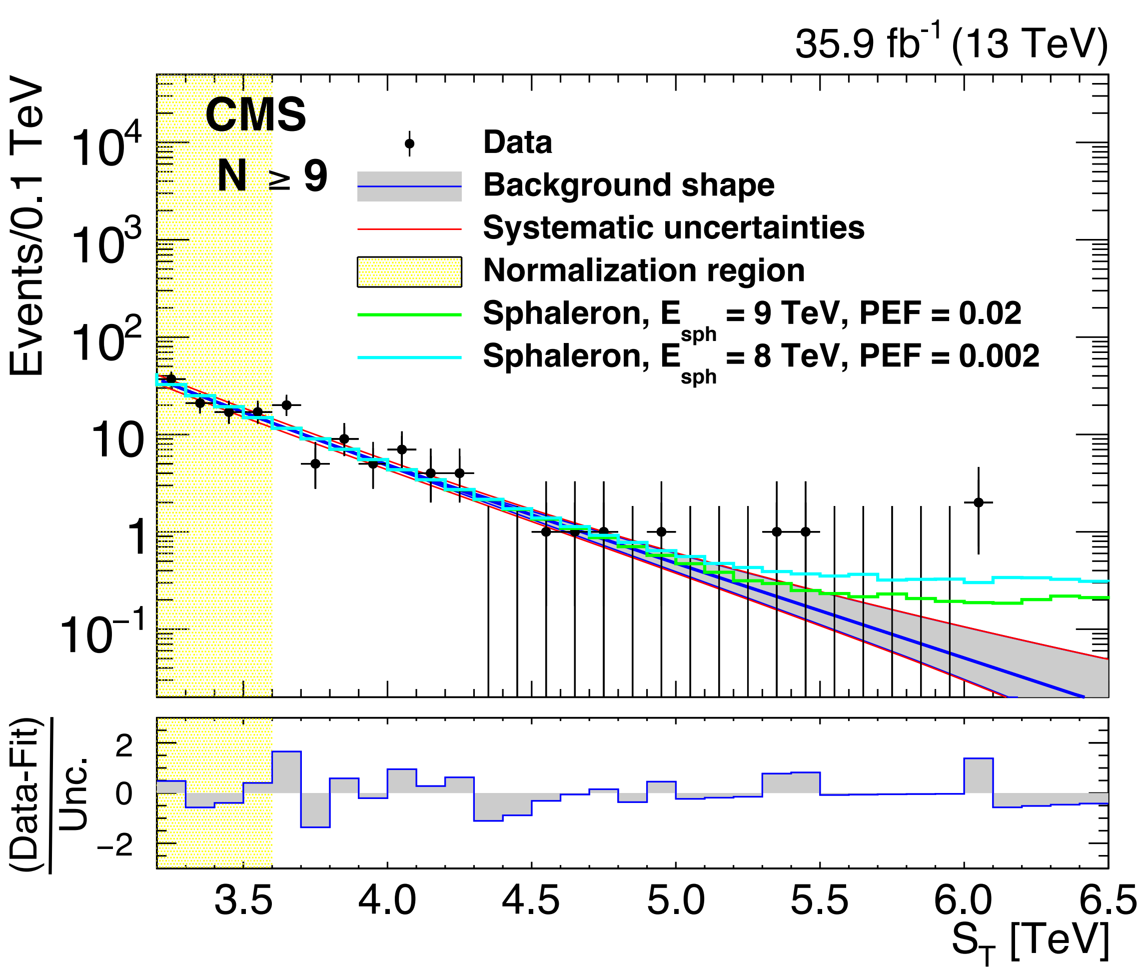

CMS
CERN-EP-2018-093
2018/11/16
CMS-EXO-17-023

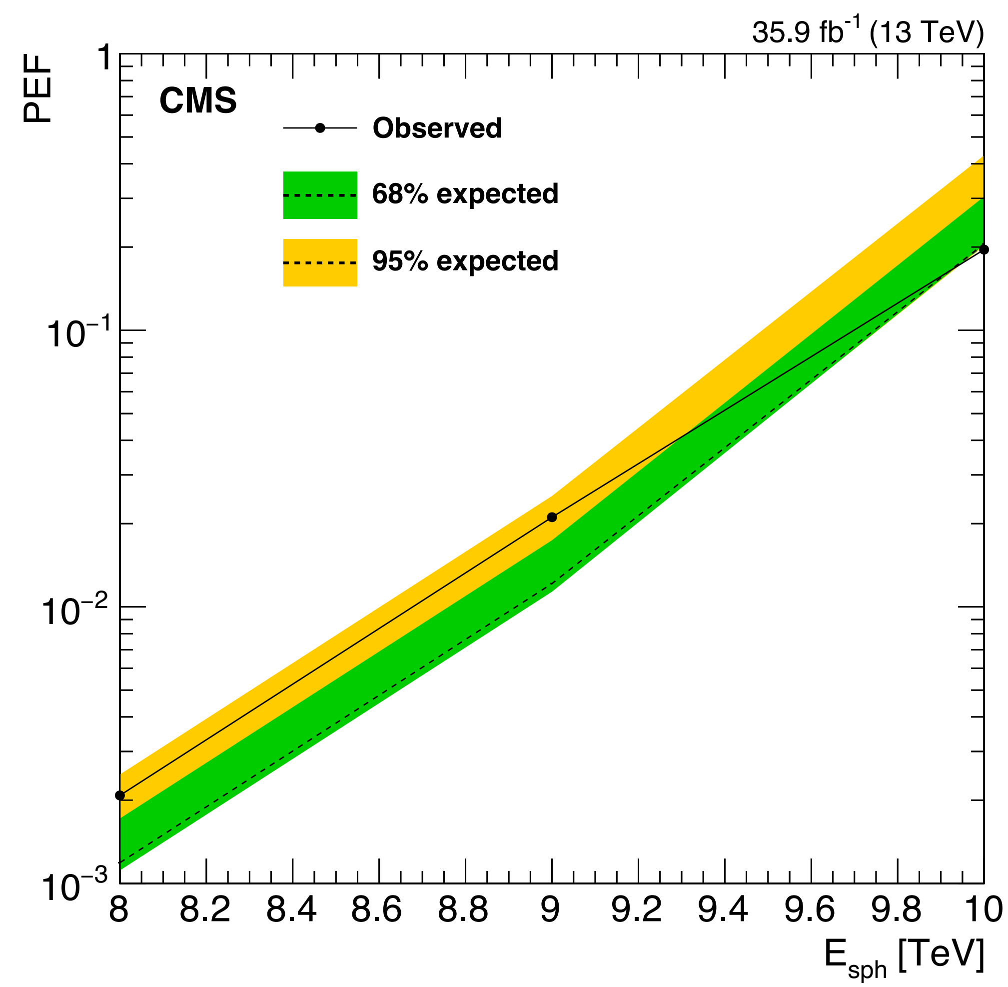
[1805.06013]

Search for black holes and sphalerons in high-multiplicity final states in proton-proton collisions at $\sqrt{s} = 13$ TeV[†]

$$S_T \equiv E_T^{\text{miss}} + \sum_i^{p_T > 70\text{GeV}} p_T^{(i)} \;\; > S_T^{\min} \qquad 3.8 < S_T^{\min}/\text{TeV} < 8$$

$$N(p_T > 70\,\text{GeV}) \;\; \geq N_{\min} \qquad N_{\min} = 3, \ldots, 11$$





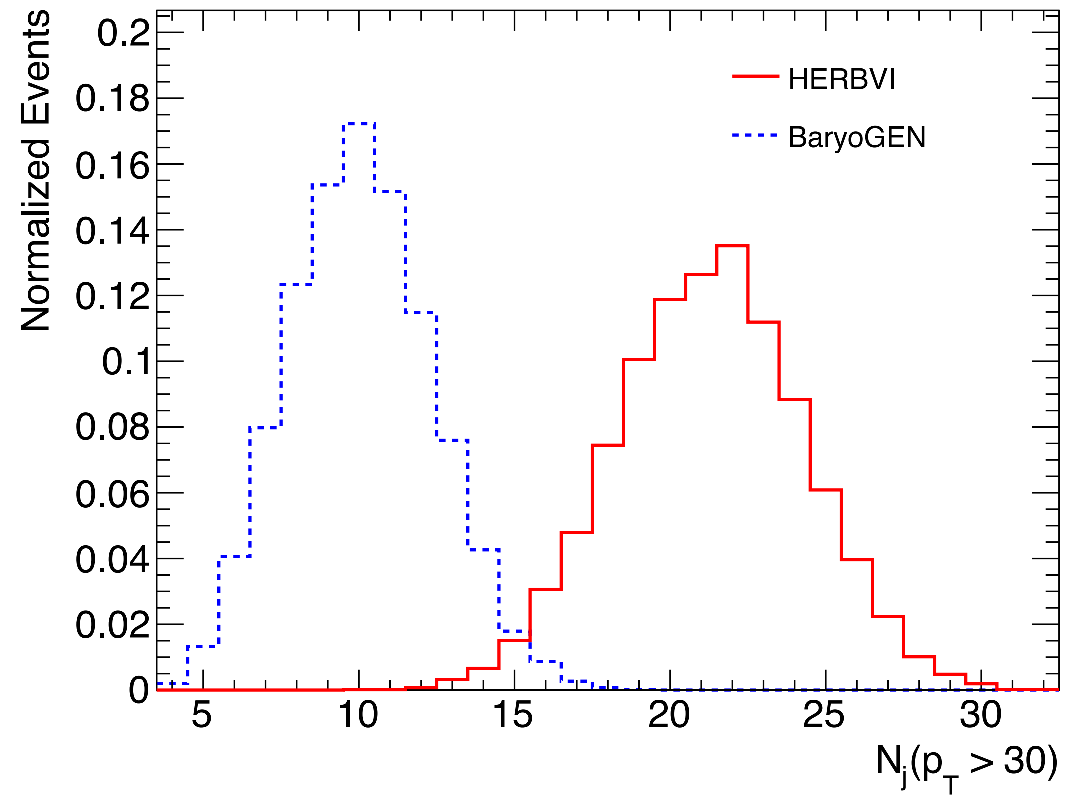
[Ringwald, KS, Webber 1809.10833]

- CMS analysis assumes sphaleron final states ***DO NOT*** involve any EW bosons.

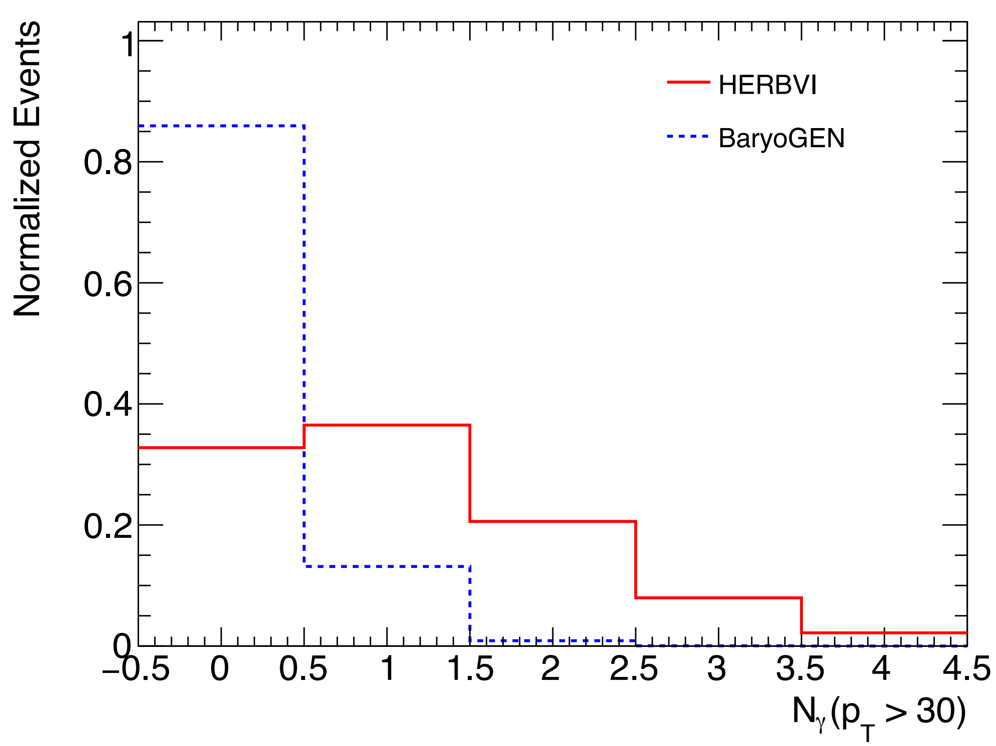
$$
qq \to \begin{cases} n_q q + 3\ell & \text{[BaryoGEN]} \\ 7q + 3\ell + \sum n_B B & \text{[HERBVI]} \end{cases}
$$

- **Delphes** is used for detector simulation

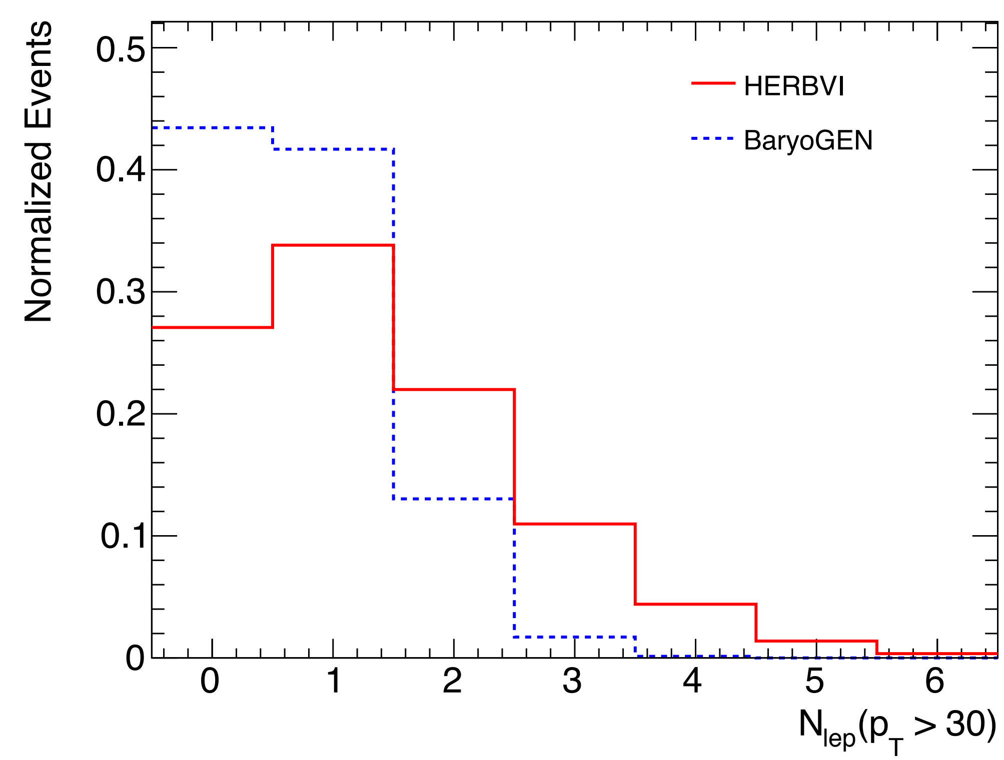
jet multiplicity



lepton multiplicity



photon multiplicity



Comparison in signal efficiencies

$$S_T \equiv E_T^{\rm miss} + \sum_i^{p_T > 70{\rm GeV}} p_T^{(i)} \; > S_T^{\rm min} \qquad 3.8 < S_T^{\rm min}/{\rm TeV} < 8$$

$$N(p_T > 70\,{\rm GeV}) \; \geq N_{\rm min} \qquad N_{\rm min} = 3, ..., 11$$

[Ringwald, KS, Webber 1809.10833]

$E_{\rm sph}$ [TeV]		8	8.5	9	9.5	10
multi-boson	$(N_{\rm min}, S_T^{\rm min}\,[{\rm TeV}])^*$	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)	(11, 4.2)
	$\epsilon^{(a^*)}$ [%]	94.8	97.5	99.2	99.6	99.9
	$N_{\rm obs}^{{\rm max}(a^*)}$	3.0	3.0	3.0	3.0	3.0
Zero Boson	$(N_{\rm min}, S_T^{\rm min}\,[{\rm TeV}])^*$	(9, 5.4)	(9, 5.6)	(9, 5.6)	(8, 6.2)	(8, 6.2)
	$\epsilon^{(a^*)}$ [%]	37.7	40.5	45.3	50.5	57.5
	$N_{\rm obs}^{{\rm max}(a^*)}$	6.9	5.8	5.8	3.0	3.0

most sensitive SR

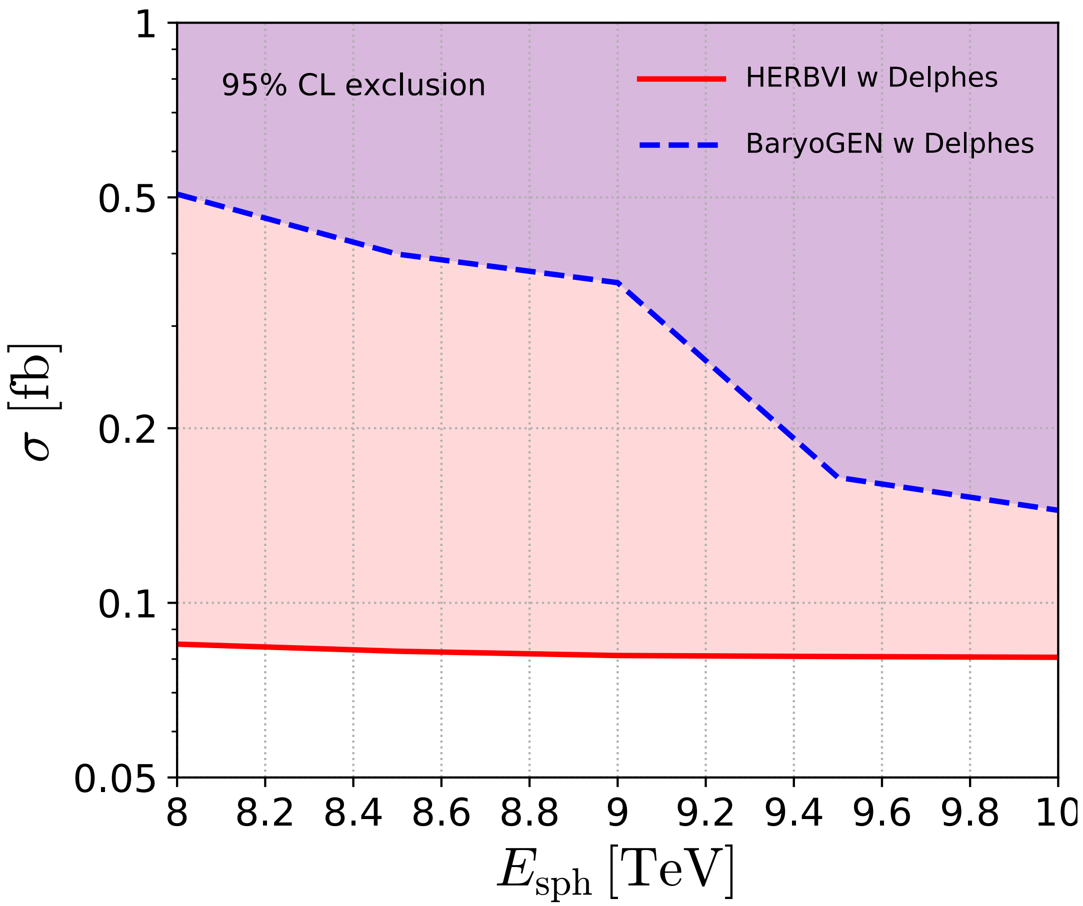
signal efficiency

limit on signal events

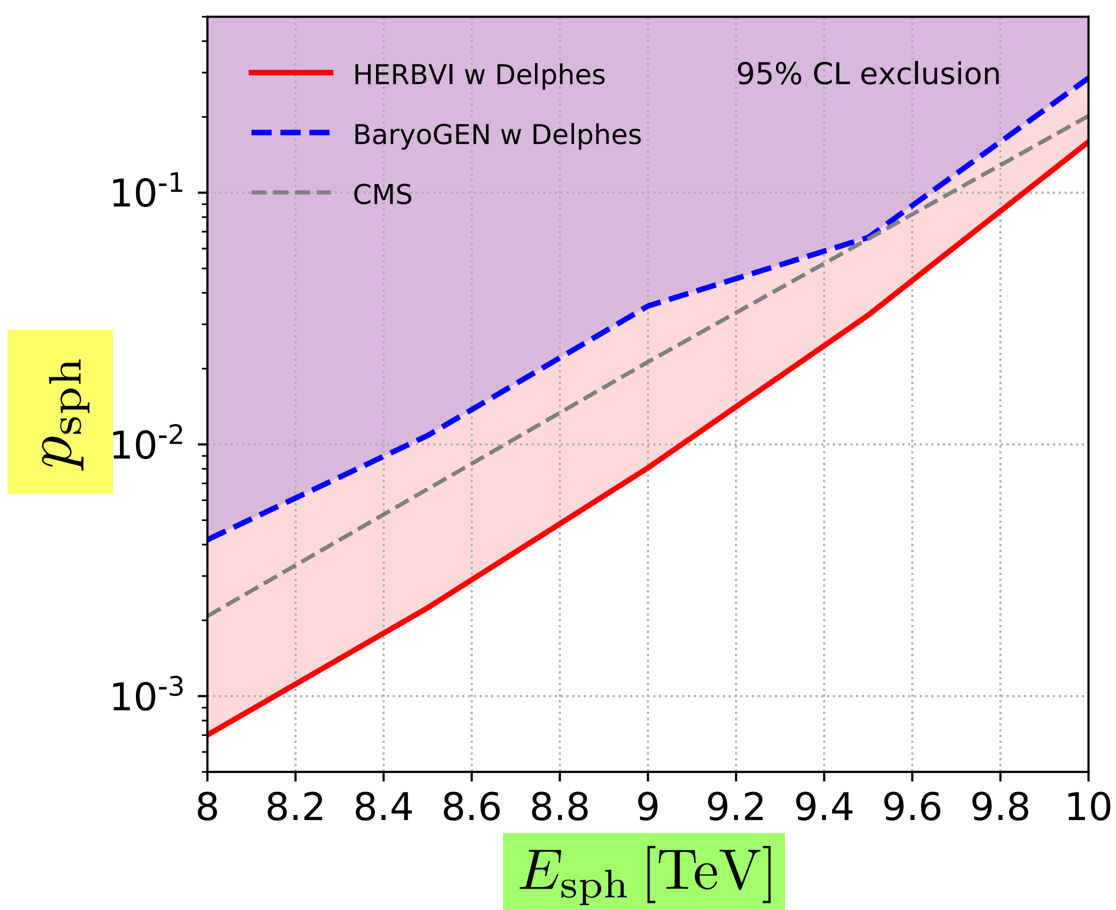
signal efficiencies are much larger in the multi-boson case

Exclusion limit

[Ringwald, KS, Webber 1809.10833]



$$\hat{\sigma}(\sqrt{\hat{s}}) = \frac{p_{\rm sph}}{m_W^2}\Theta(\sqrt{\hat{s}} - E_{\rm sph})$$



$$qq \to \begin{cases} n_q q + 3\ell & (\texttt{BaryoGEN}) \\ 7q + 3\ell + \sum n_B B & (\texttt{HERBVI}) \end{cases}$$

- The limit on the multi-boson cross-section: $\sigma_{\rm sph} < 0.8\,{\rm fb}$