TON DUC THANG UNIVERSITY

Electroweak Baryogenesis

Eibun Senaha (Ton Duc Thang U, AIMaS)

September 10, 2021@PPP2021 Online

Outline

- part 1: RG invariance of resummed effective potential

K. Funakubo and E.S., in progress

- part 2: Band structure effect on sphaleron rate

K. Funakubo, K. Fuyuto, E.S., arXiv:1612.05431

Baryon Asymmetry of the Universe (BAU)

Our Universe is baryon-asymmetric.

$$\eta^{\rm BBN} = \frac{n_B}{n_\gamma} = (5.8 - 6.5) \times 10^{-10},$$
$$\eta^{\rm CMB} = \frac{n_B}{n_\gamma} = (6.105 - 0.055) \times 10^{-10}.$$

PDG2020

Sakharov's conditions [Sakharov, JETP Lett. 5 (1967) 24]

(1) Baryon number violation
(2) C and CP violation
(3) Out of equilibrium

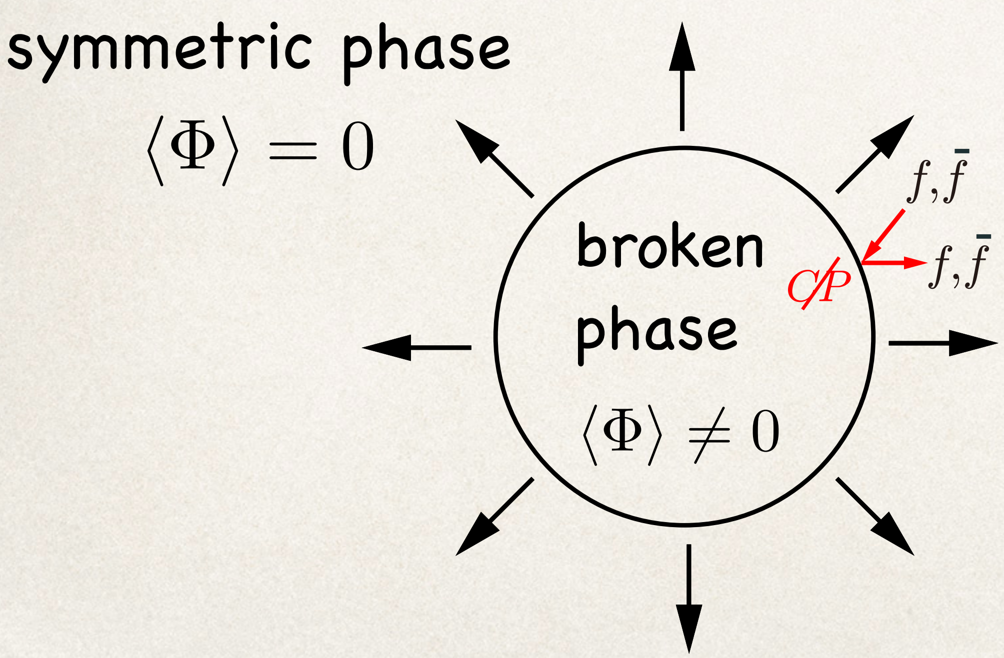
- ❒ after inflation (scale is model dependent)
- ❒ before Big-Bang Nucleosynthesis ($T \simeq O(1)$ MeV)

EW baryogenesis (EWBG)

Sakharov's conditions

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]

- B violation: anomalous (sphaleron) process
- C violation: chiral gauge interaction
- CP violation: KM phase and/or other sources in beyond the SM
- Out of equilibrium: 1st-order EW phase transition (EWPT) with expanding bubble walls



$n_B = 0 \to n_B \neq 0$ (sphaleron proecess)

baryogengesis occurs outside bubbles!

sphaleron decoupling condition

$$\Gamma_B^{(\mathrm{br})}(T) < H(T) \longrightarrow \frac{v_C}{T_C} \gtrsim 1$$

Precise calculation of this condition is indispensable for experimental verification of EWBG.

Our concern

- Effective potential is commonly used to analyze EWPT.
- Scale dependences is one of the theoretical uncertainties and can be diminished by renormalization group (RG) improvement.
- Ordinary RG equation at T=0 has been used.

Question

Is it really correct to use the ordinary RGE to improve resummed effective potential?

Today's topic

- We devise RG equations in the resummed perturbation theory.
- As a 1st step, φ^4 theory up to 2-loop level is considered.

Effective potential - MS-bar scheme -

$$V_{\rm eff}(\varphi;T) = V_0(\varphi) + V_1(\varphi;T)$$

$$V_0(\varphi) = \Omega - \frac{\nu^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4, \qquad V_1(\varphi;T) = \frac{m^4}{4(16\pi^2)}\left(\ln\frac{m^2}{\bar{\mu}^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} I_B(a^2),$$

$$m^2 = \frac{\partial^2 V_0}{\partial\varphi^2} = -\nu^2 + \frac{\lambda}{2}\varphi^2, \qquad I_B(a^2) = \int_0^\infty dx\, x^2 \ln\left(1 - e^{-\sqrt{x^2+a^2}}\right), \quad a^2 = \frac{m^2}{T^2}$$

RG invariance

$$0 = \frac{dV_{\rm eff}}{dt} = \left[\frac{\partial}{\partial t} + \beta_X \frac{\partial}{\partial X} - \gamma_\varphi \varphi \frac{\partial}{\partial\varphi}\right] V_{\rm eff} \equiv \mathcal{D} V_{\rm eff}, \quad X = \Omega, \nu^2, \lambda, \quad t = \ln\bar{\mu},$$

@1-loop

$$\mathcal{D}V_0 = \beta_\Omega^{(1)} - \frac{1}{2}\beta_{\nu^2}^{(1)}\varphi^2 + \frac{1}{4!}\beta_\lambda^{(1)}\varphi^4 = \frac{1}{16\pi^2}\left[\frac{\nu^4}{2} - \frac{1}{2}\lambda\nu^2\varphi^2 + \frac{\lambda^2}{8}\varphi^4\right] = \frac{m^4}{32\pi^2},$$

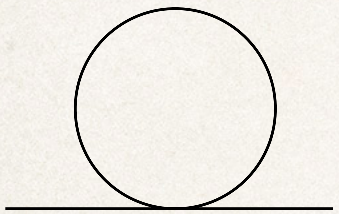
$$\mathcal{D}V_1 = \frac{\partial}{\partial t} V_1 = \frac{m^4}{64\pi^2}(-2) = -\frac{m^4}{32\pi^2}. \qquad \therefore \quad \mathcal{D}(V_0 + V_1) = 0$$

Effective potential at this order is RG invariant.

Need of resummation

Perturbative expansion breaks down at high T.

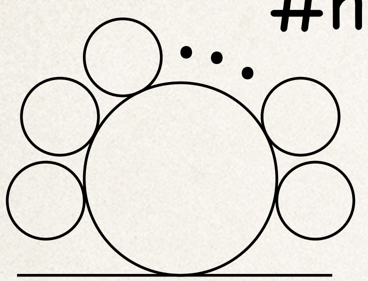
$$M^2 = m^2 + \Sigma(\varphi;T), \quad \Sigma(\varphi;T) = \frac{\lambda}{2} I(m^2)$$



$$= \frac{\lambda}{2} I(m^2) \underset{T>0}{\rightarrow} \frac{\lambda}{2}\frac{I'_B(a^2)}{\pi^2} = \frac{\lambda}{2}\left[\frac{T^2}{12} + \cdots\right]$$

grow with O(λT²)

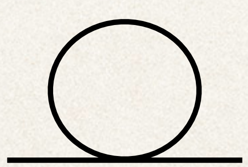
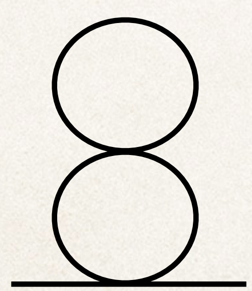
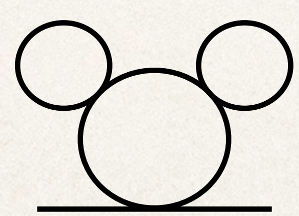
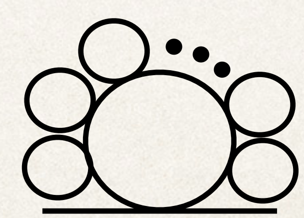
#n sub-bubbles



$$\sim \frac{\lambda^2 T^3}{m}\left(\frac{\lambda T^2}{m^2}\right)^{n-1}$$

sizable at $T \simeq \frac{m}{\sqrt{\lambda}}$

Daisy resummation

$-i\Sigma(\varphi;T) =$  +  +  + ⋯ +  +

$$= \left(\frac{-i\lambda}{2}\right) I\left(m^2 + \frac{\lambda T^2}{24}\right)$$

mass is shifted by $\frac{\lambda T^2}{24}$

Resummed effective potential

[Parwani (92) etc.]

Dominant T-dep. terms are added and subtracted in the Lagrangian.

$$\mathcal{L}_B = \mathcal{L}_R + \mathcal{L}_{\rm CT}$$

$$= \left[\mathcal{L}_R - \frac{1}{2}\Sigma(T)\phi^2\right] + \left[\mathcal{L}_{\rm CT} + \frac{1}{2}\Sigma(T)\phi^2\right] \qquad \Sigma(T) = \frac{\lambda T^2}{24}$$

new 0th-order part — new counterterm (CT)

Resummed 1-loop effective potential

$$V_0(\varphi;T) = \frac{1}{2}\left(-\nu^2 + \Sigma(T)\right)\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

$$V_1(\varphi;T) = \frac{M^4}{4(16\pi^2)}\left(\ln\frac{M^2}{\bar{\mu}^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2}I_B(M^2/T^2) - \frac{1}{2}\Sigma(T)\varphi^2$$

thermal CT

$$\underset{\rm HTE}{\to} -\frac{T}{12\pi}(M^2)^{3/2} + \frac{M^4}{64\pi^2}\left(\ln\frac{T^2}{\bar{\mu}^2} + 2c_B\right) + \cdots$$

where $M^2 = m^2 + \Sigma(T) = -\nu^2 + \frac{\lambda}{2}\varphi^2 + \Sigma(T)$ $\qquad$ c_B: const.

This resummed $V_{\rm eff}$ is often used to study EWPT in BSM.

RG non-invariance of resummed V_{eff}

- common argument in the literature -

Using HTE, resummed 1-loop V_{eff} takes the form

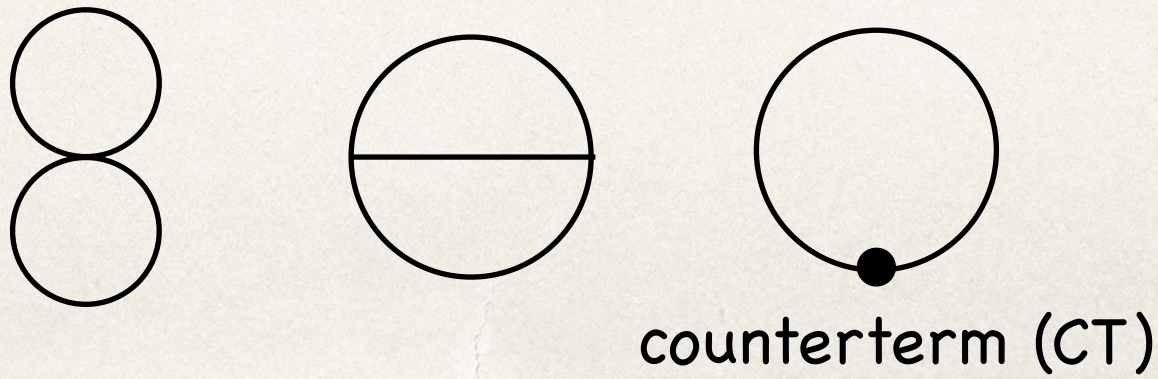
$$
\begin{aligned}
V_{\rm eff}^{\rm HTE}(\varphi;T) &= V_0(\varphi;T) + V_1^{\rm HTE}(\varphi;T) \\
&\simeq \frac{1}{2}\left[-\nu^2\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right) + \frac{\lambda(-\nu^2+\Sigma)}{16\pi^2}c_B + \boxed{\frac{\lambda T^2}{24}\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)}\right]\varphi^2 \\
&\quad - \frac{T}{12\pi}(M^2)^{3/2} + \frac{1}{4!}\left[\lambda\left(1+\frac{3\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right) + \frac{3\lambda^2 c_B}{16\pi^2}\right]\varphi^4 + \cdots
\end{aligned}
$$

Solutions of 1-loop beta functions (MS-bar)

$$
\nu^2(T) \simeq \nu^2\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right), \quad \lambda(T) \simeq \lambda\left(1+\frac{3\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)
$$

After thermal resummation, RG invariance is broken.

way-out   Include 2-loop corrections to restore RG invariance!



RG non-invariance of resummed $V_{\rm eff}$

- common argument in the literature -

HTE of resummed 2-loop $V_{\rm eff}$ [Arnold and Espinosa, PRD47 (1993) 3546]

$$V_2(\varphi;T) \underset{\rm HTE}{\to} \frac{\lambda^2 T^2}{48(16\pi^2)}\left(2\ln\frac{M^2}{T^2} + \ln\frac{T^2}{\bar{\mu}^2} + 1 + c_H\right)\varphi^2 + V_2(\varphi, T=0)\Big|_{m^2\to M^2}$$

Adding the mu-dependent term into $V_{\rm eff}$,

$$\begin{aligned}
V_{\rm eff}^{\rm HTE}(\varphi;T) &= V_0(\varphi;T) + V_1^{\rm HTE}(\varphi;T) + V_2^{\rm HTE}(\varphi;T) \\
&\simeq \frac{1}{2}\left[-\nu^2\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right) + \frac{\lambda(-\nu^2+\Sigma)}{16\pi^2}c_B + \frac{\lambda T^2}{24}\left(1+\frac{(1+2)\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)\right]\varphi^2 \\
&\quad - \frac{T}{12\pi}(M^2)^{3/2} + \frac{1}{4!}\left[\lambda\left(1+\frac{3\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right) + \frac{3\lambda^2 c_B}{16\pi^2}\right]\varphi^4 + \cdots
\end{aligned}$$

RG invariance is restored to $O(\lambda^2)$ "in the high-T limit".

Questions

- Why should we use the ordinary β function for ν^2 even in the resummed perturbation theory?
- What happens if HTE is not used? (*HTE of V_2 is not valid in EWBG-favored regions)

β-functions in the resummed theory

After resummation, T-dependent divergences at 1-loop level arise from

$$\mu^\epsilon V_1(\varphi;T)\Big|_{\rm div} = -\frac{1}{\epsilon}\frac{1}{32\pi^2}\left(-\nu^2+\Sigma(T)+\frac{\lambda\mu^\epsilon}{2}\varphi^2\right)^2$$

(1) MS-bar scheme: $\delta^{(1)}\nu^2 = \frac{\lambda\nu^2}{16\pi^2}\frac{1}{\epsilon}$ * T-dep. div. are cancelled by higher-order terms

(2) Our scheme: $\delta^{(1)}\nu^2 = \frac{\lambda}{16\pi^2}\frac{1}{\epsilon}(\nu^2-\Sigma)$

[β—functions (in our scheme)]

We expand bare ν² as $\nu_B^2 = \nu^2\left(1+\sum_{n=1}^{\infty}\frac{b_n(\lambda)}{\epsilon^n}\right)+\Sigma(T)\sum_{n=1}^{\infty}\frac{\tilde{b}_n(\lambda)}{\epsilon^n},\quad \Sigma(T)=\frac{\lambda T^2}{24}$

* all-order sum = 0

From scale-independence of bare ν²

$$\beta_{\nu^2} = \frac{d\nu^2}{d\ln\mu} = \lambda\nu^2\frac{db_1(\lambda)}{d\lambda}+\Sigma(T)\left(\cancel{\tilde{b}_1(\lambda)}+\lambda\frac{d\tilde{b}_1(\lambda)}{d\lambda}\right)$$

required by RG-invariance of $V_{\rm eff}$ (at least 2-loop order)

At 1-loop level $\beta_{\nu^2}^{(1)} = \frac{\lambda}{16\pi^2}(\nu^2-\Sigma)$ * self-coupling $\beta_\lambda^{(1)} = \frac{3\lambda^2}{16\pi^2}$

RG invariance of resummed V_{eff}

Using HTE, resummed 1-loop V_{eff} takes the form

$$\begin{aligned} V_{\rm eff}^{\rm HTE}(\varphi;T) &= V_0(\varphi;T) + V_1^{\rm HTE}(\varphi;T) \\ &\simeq \frac{1}{2}\left[(-\nu^2+\Sigma)\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)+\frac{\lambda(-\nu^2+\Sigma)}{16\pi^2}c_B\right]\varphi^2 \\ &\quad -\frac{T}{12\pi}(M^2)^{3/2}+\frac{1}{4!}\left[\lambda\left(1+\frac{3\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)+\frac{3\lambda^2 c_B}{16\pi^2}\right]\varphi^4+\cdots \end{aligned}$$

Solutions of 1-loop beta functions

$$-\nu^2(T)+\Sigma = (-\nu^2+\Sigma)\left(1+\frac{\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right), \quad \lambda(T)=\lambda\left(1+\frac{3\lambda}{32\pi^2}\ln\frac{T^2}{\bar{\mu}^2}\right)$$

V_{eff} is RG invariant even at this order in our scheme.

@2-loop

$$\beta_{\nu^2}=\beta_{\nu^2}^{(1)}+\beta_{\nu^2}^{(2)}=\frac{\lambda}{16\pi^2}(\nu^2\boxed{-\Sigma})+\frac{\lambda^2}{(16\pi^2)^2}\left(-\frac{5\nu^2}{6}+\Sigma\right)+\boxed{\frac{\lambda\Sigma}{16\pi^2}}=\frac{\lambda\nu^2}{16\pi^2}+\frac{\lambda^2}{(16\pi^2)^2}\left(-\frac{5\nu^2}{6}+\Sigma\right)$$

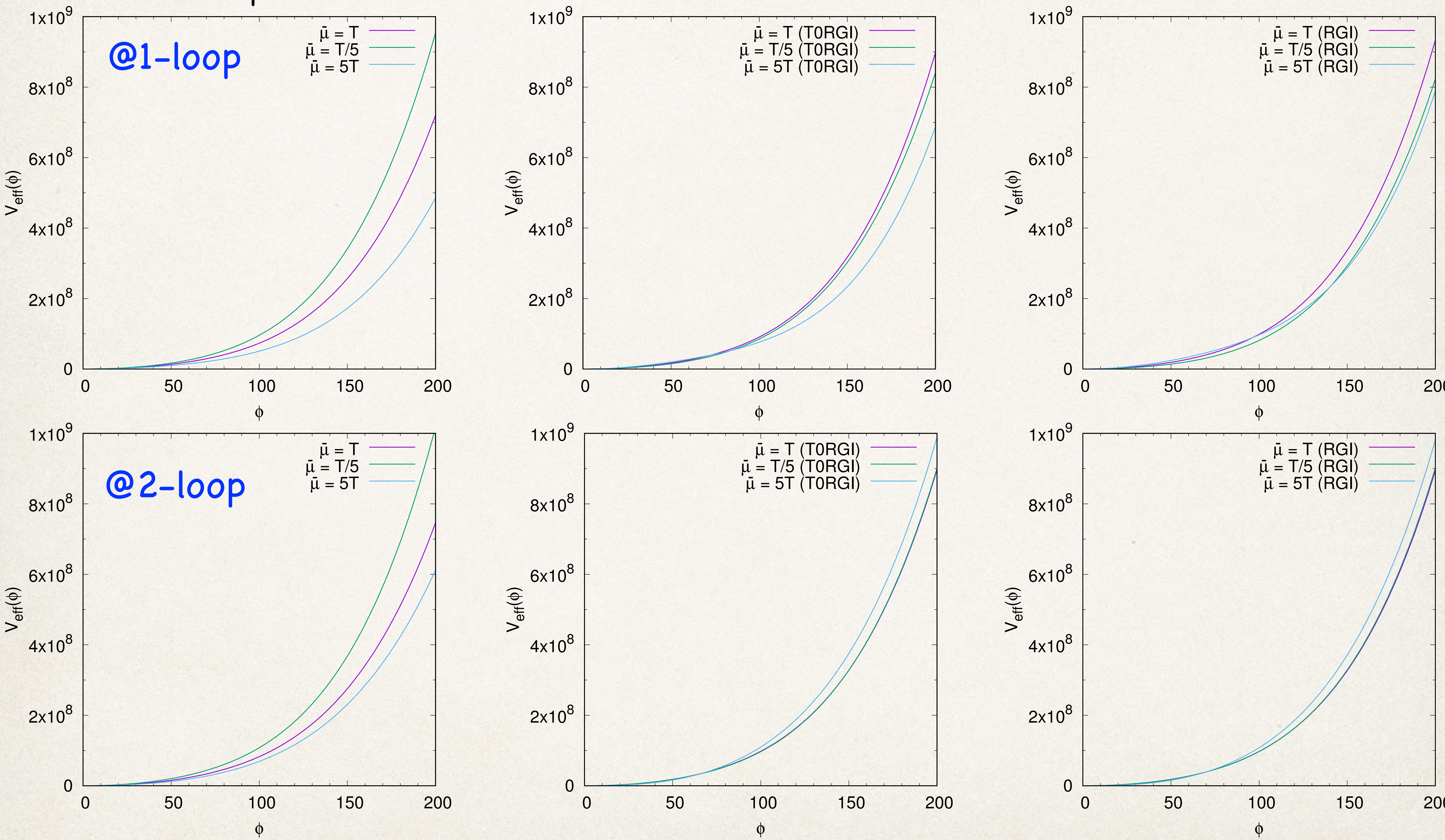
cancelled!

- 2 schemes give the same potential "in the high-T limit".
- However, total V_{eff} satisfies RGE in our scheme (not in the MS-bar scheme).
- The difference could get bigger for $\nu^2 \ll \Sigma(=\lambda T^2/24)$.

Numerical analysis

Inputs: $\bar{\mu}_0 = 90,\ v(\bar{\mu}_0) = 50,\ m_\phi(\bar{\mu}_0) = 90,\ T = 250$ [any mass units] $[\lambda(\bar{\mu}_0) \simeq 10]$

no RG improvement | MS-bar RGI | our RGI



- Our scheme gives less scale dependences at 1-loop level.
- Application to other models (1st-order PT case) is the next step.

Band structure effect on sphaleron process

based on the collaborators with

Koichi Funakubo (Saga U), Kaori Fuyuto (Los Alamos Natl Lab)

Ref. arXiv:1612.05431

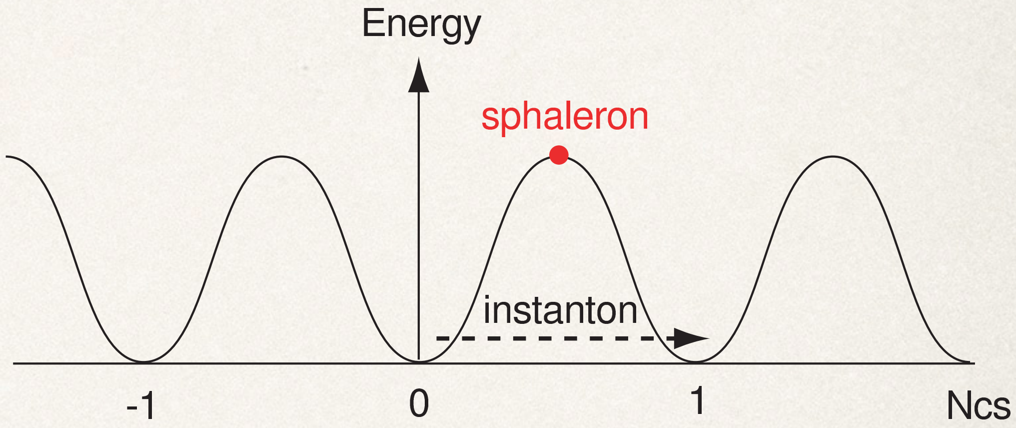
B+L violation

- (B+L) is violated by a chiral anomaly in EW theories.

Vacuum transition (instanton)

['t Hooft, PRL37,8 (1976), PRD14,3432 (1976)]

$$\sigma_{\rm instanton} \simeq e^{-2S_{\rm instanton}} = e^{-4\pi/\alpha_W} \simeq 10^{-162}$$



Transition rate at finite-E

[Ringwald, NPB330,(1990)1, Espinosa, NPB343 (1990)310]

instanton-based

holy-grail function

$$\sigma(E) \sim \exp\left(\frac{4\pi}{\alpha_W} F(E)\right) \qquad F(0) = -1 \qquad |F(E_{\rm sph})| \ll 1?$$

- But, instanton-based calculation is not valid at $E \gtrsim E_{sph}$

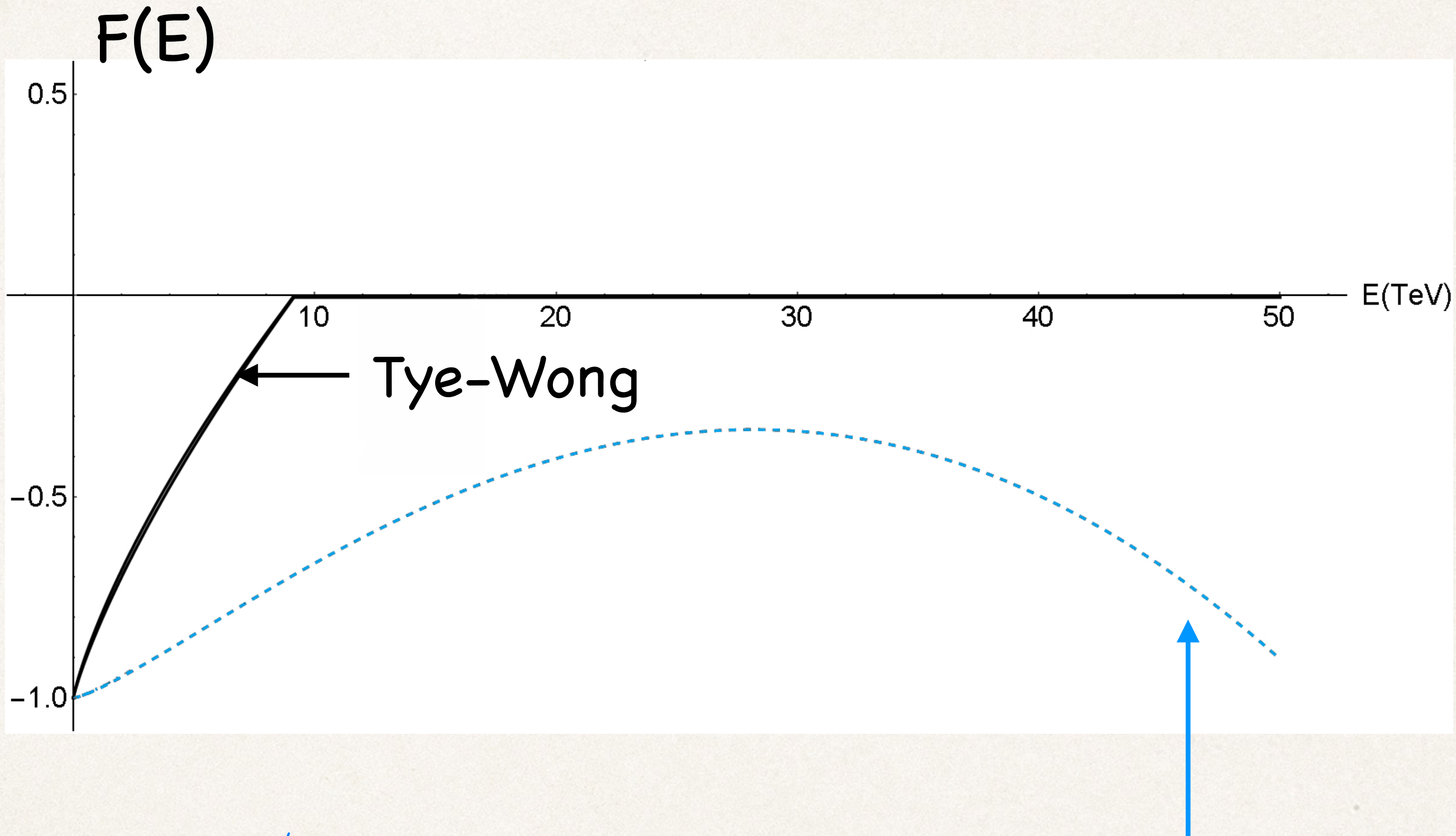
Bounce is more appropriate (transition between the finite-E states)

-> Reduced model

[Aoyama, Goldberg, Ryzak, PRL60, 1902 ('88)]
[Funakubo, Otsuki, Takenaga, Toyoda, PTP87,663('92), PTP89,881('93)]
[H. Tye, S. Wong, PRD92,045005 ('15)]

Tye-Wong's work

[H. Tye, S. Wong, PRD92,045005 (2015)]



$$F(E) = -1 + \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3} - \frac{9}{16}\left(\frac{E}{E_0}\right)^2 + \cdots$$ (instanton calculus) $E_0 \simeq 15$ TeV

F(E) = 0 for $E > E_{sph}$ (Tye-Wong) ∵ a band structure

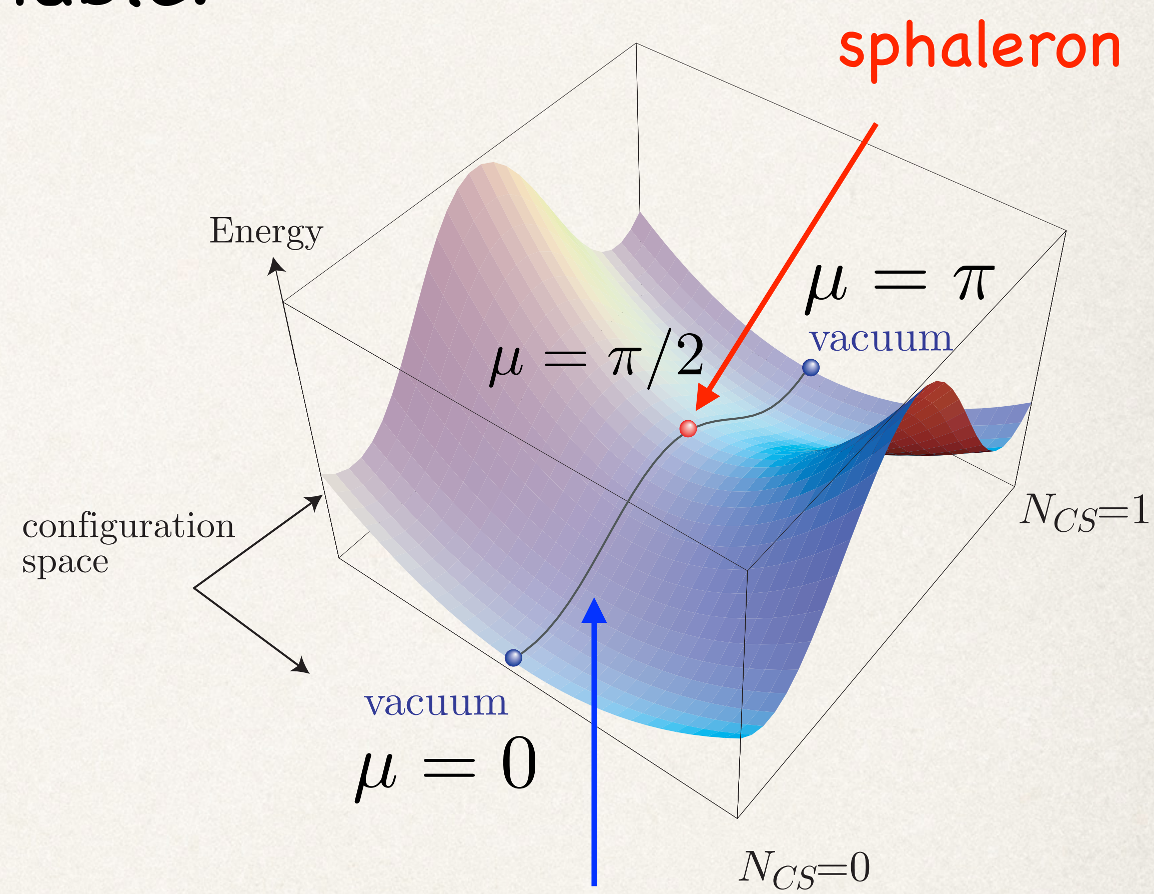
Reduced model

[Aoyama, Goldberg, Ryzak, PRL60, 1902 (1988)]
[Funakubo, Otsuki, Takenaga, Toyoda, PTP87, 663 (1992), PTP89, 881 (1993)]
[H. Tye, S. Wong, PRD92,045005 (2015)]

Let us promote μ to a dynamical variable:

$$\mu \Rightarrow \mu(t)$$

$\mu(-\infty)=0$, $\mu(+\infty)=\pi$: vacuum,
$\mu(t_{sph})=\pi/2$: sphaleron



Non-contractible loop
(least energy path)

- We construct a reduced model by adopting a Manton's ansatz.

Classical action

$$S[\mu] = g_2 v \int dt \left[\frac{M(\mu)}{2} \left(\frac{d}{dt} \frac{\mu(t)}{g_2 v} \right)^2 - V(\mu) \right],$$

where $M(\mu) = \frac{4\pi}{g_2^2} \left(\alpha_0 + \alpha_1 \cos^2 \mu + \alpha_2 \cos^4 \mu \right), \quad V(\mu) = \frac{4\pi}{g_2^2} \sin^2 \mu \left(\beta_1 + \beta_2 \sin^2 \mu \right).$

$$\alpha_0 = 19.42, \quad \alpha_1 = -1.937, \quad \alpha_2 = -2.656,$$
$$\beta_1 = 1.313, \quad \beta_2 = 0.603,$$

$$M_{\rm sph} = g_2 v M\left(\frac{\pi}{2}\right) \simeq 92.01 \text{ TeV}, \quad E_{\rm sph} = g_2 v V\left(\frac{\pi}{2}\right) \simeq 9.08 \text{ TeV}.$$

- We quantize this system and evaluate energy spectrum.

-> band structure

Band structure

E_{sph}=9.08 TeV E_{sph}=9.11 TeV

this work	Units:	TeV	Tye-Wong
Band Centre E	Band Width	Band Centre E	Band Width
14.054	0.0744	?	?
13.980	0.0741	?	?
E_{sph} ⋮	⋮	⋮	⋮
9.072	0.0104	9.113	0.0156
9.044	4.85×10^{-3}	9.081	7.19×10^{-3}
9.012	1.61×10^{-3}	9.047	2.62×10^{-3}
⋮	⋮	⋮	⋮
0.1015	1.88×10^{-199}	0.1027	$\sim10^{-177}$
0.03383	1.31×10^{-202}	0.03421	$\sim10^{-180}$

# of band $<E_{sph}$ = 158 # of band $<E_{sph}$ = 148

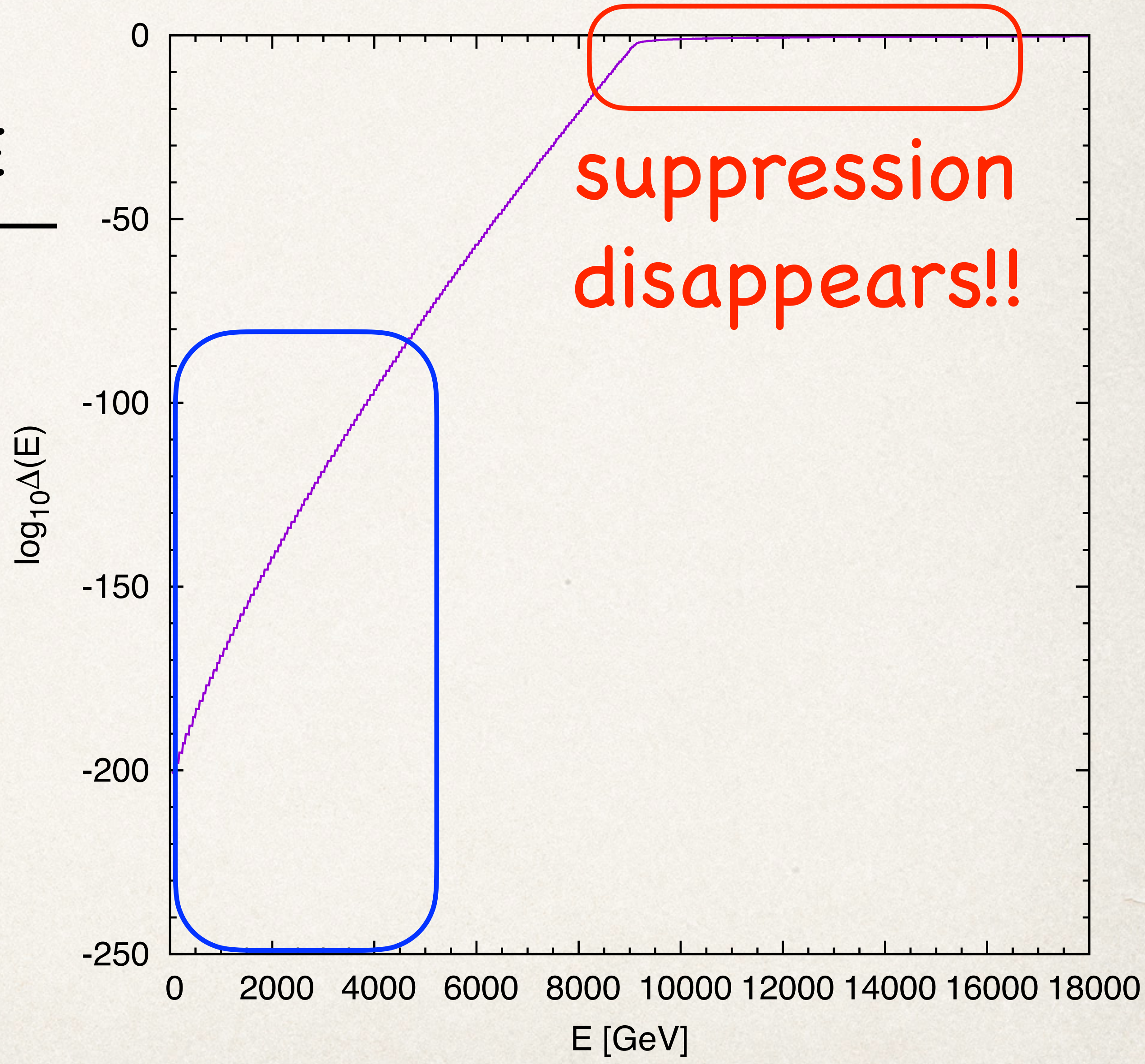
Band gaps still exist $E>E_{sph}$ due to nonzero reflection rate.

Transition factor

tunneling factor

$$\sigma_{\Delta(B+L)=\pm 1} \propto \begin{cases} 1 \times \exp\left(\frac{4\pi}{\alpha_W} F(E)\right) & \text{instanton calculus} \\ \Delta(E) \times 1 & \text{band picture} \end{cases}$$

$$\Delta(E) \simeq \frac{\text{sum of band widths up to E}}{\text{energy (E)}}$$



Band picture:

- density of state is restricted.

- Exponential suppression at $E \ll E_{sph}$ is due to the tiny band width.

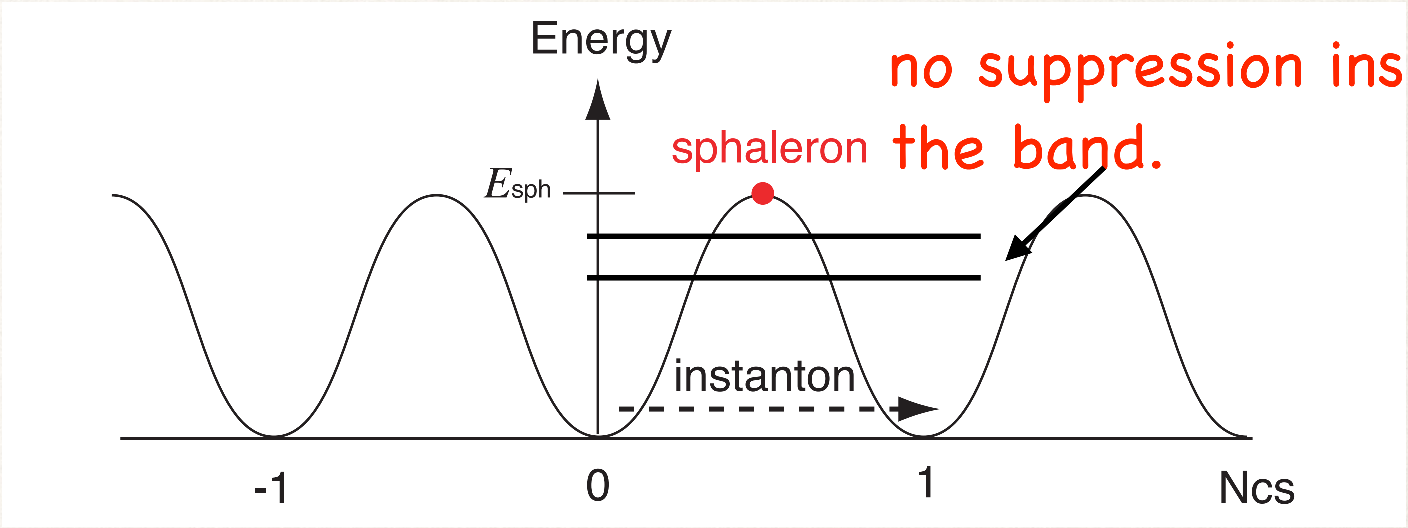
Question: Can this modification affect EWBG?

Impact on EWBG

K. Funakubo, K. Fuyuto, E.S., 1612.05431

If Γ_B is not suppressed,

B would be washed out!!

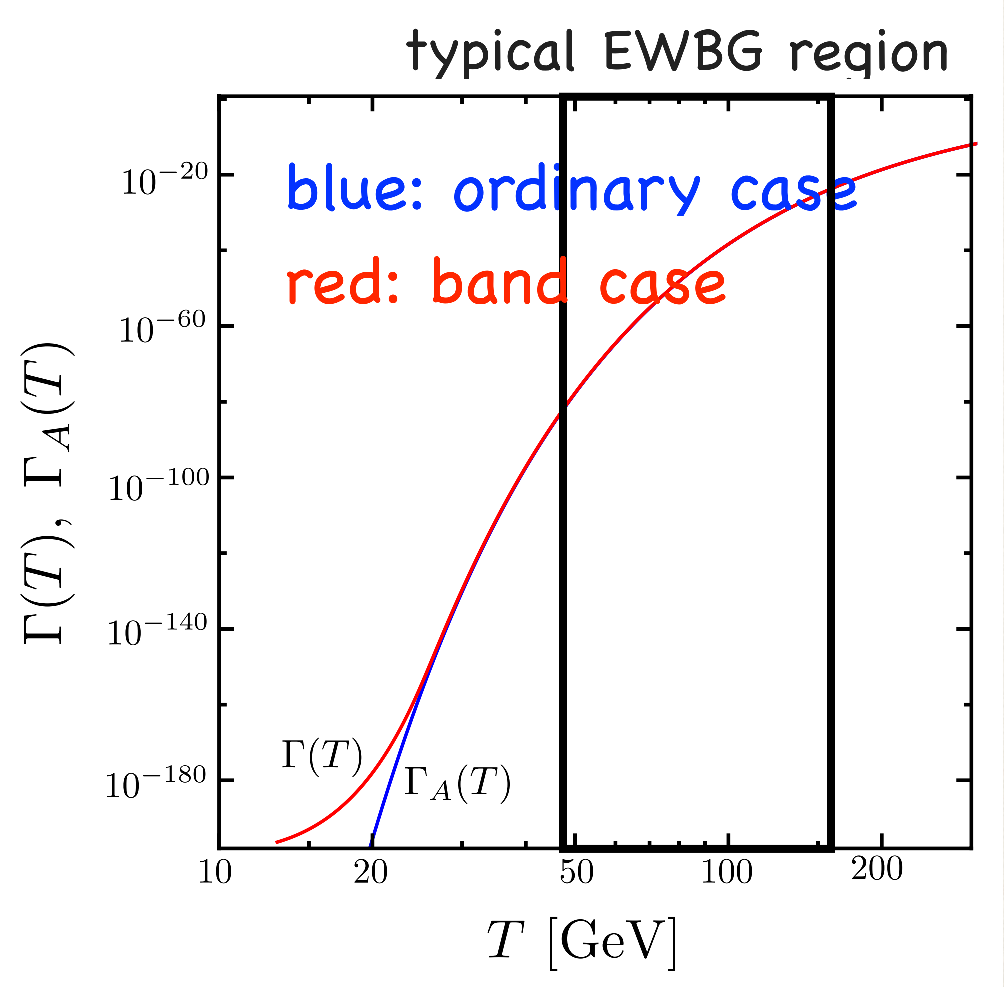


For T=100 GeV, Γ / Γ_A = 1.06.

-> Band effect has little effect.

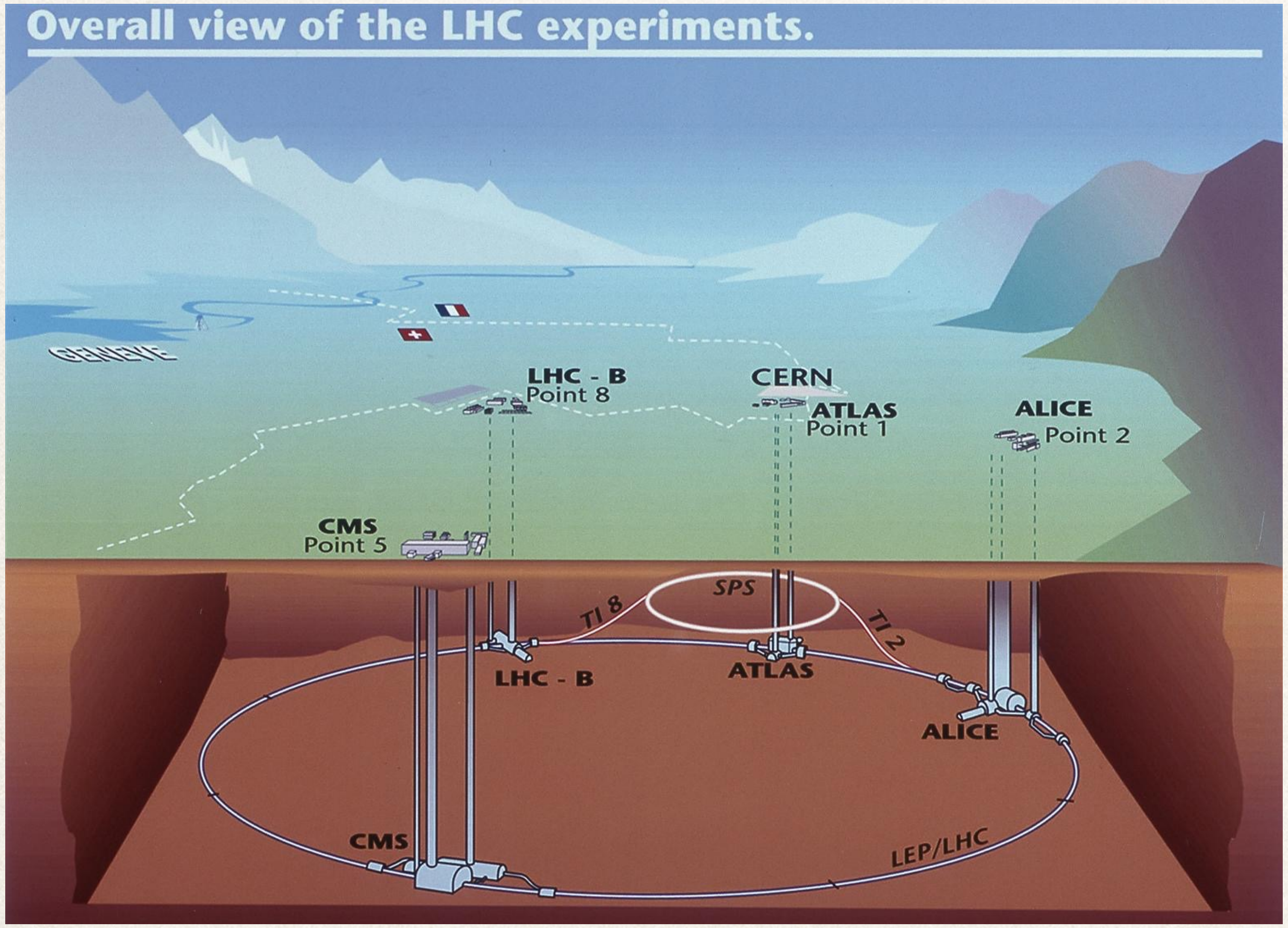
- Structure below the top of the barrier is essentially irrelevant.

- EWBG remains intact even if the band exists.

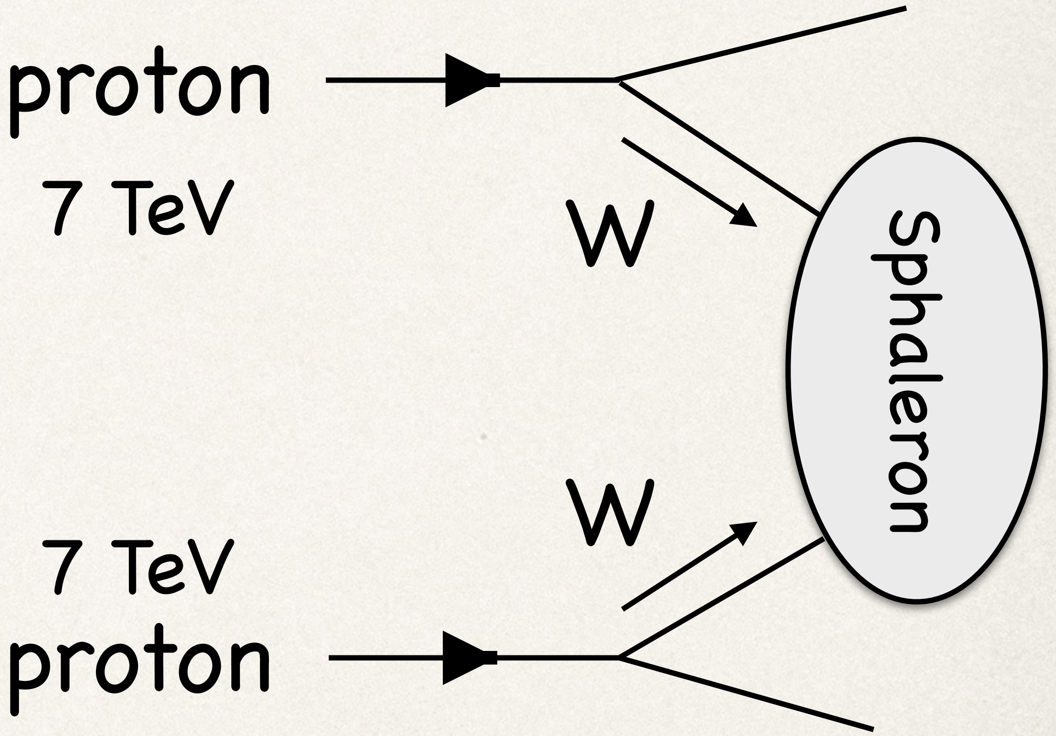


Tye-Wong:
We may observe (B+L)-changing processes at colliders.

13 TeV proton-proton collision machine



courtesy CERN



possible?

Δ(B+L)≠0 process

[Funakubo, Otsuki, Takenaga, Toyoda, PTP87, 663 (1992), PTP89, 881 (1993)]

Transition amplitude

final state — tunneling — initial state

$$S_{fi} = \langle f|\hat{S}|i\rangle \sim \int\int \langle f|\phi(y),\pi(y)\rangle\langle\phi(y),\pi(y)|\hat{S}|\phi(x),\pi(x)\rangle\langle\phi(x),\pi(x)|i\rangle$$

Path integral using coherent state $|\phi,\pi\rangle$

Tunneling (evaluated by WKB approximation)

- Tunneling factor: $e^{-S_{\text{classical}}} \xrightarrow{\text{band picture}} \Delta(E)$

Initial/final states

- overlap issue: suppressions from $\langle f|\phi,\pi\rangle$ and $\langle\phi,\pi|i\rangle$ (few-to-many suppression).

Overlap factor

Inner product between n particle state and coherent state:

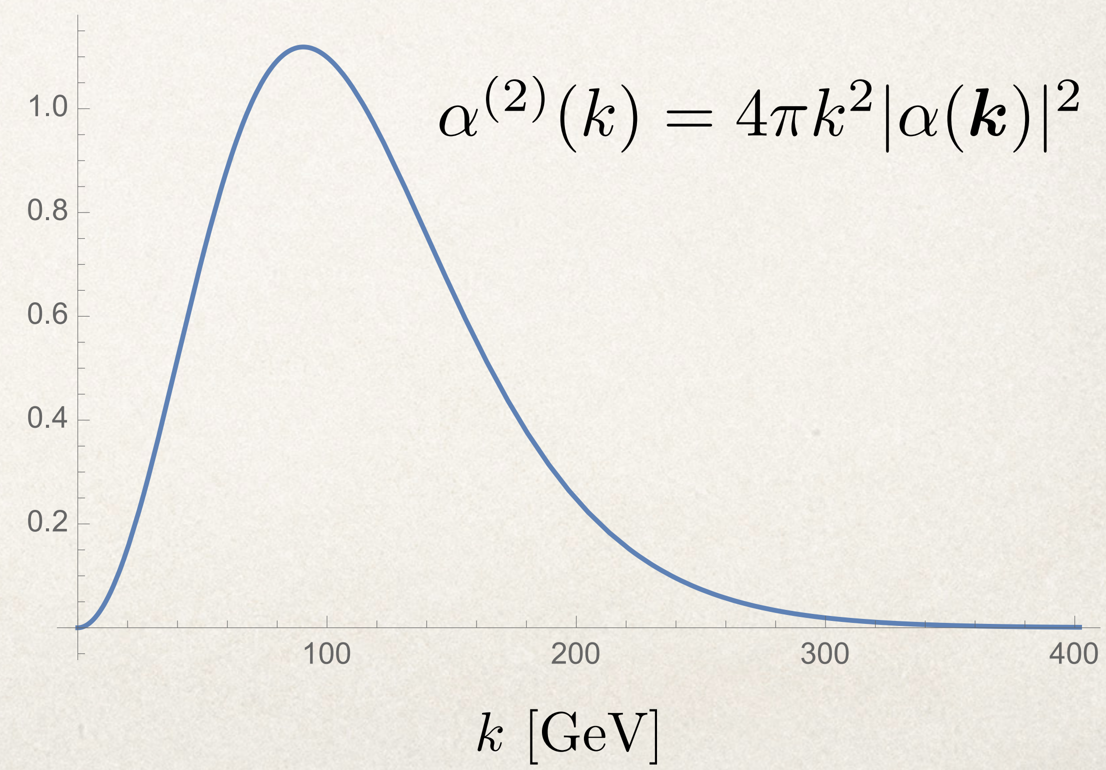
$$\langle 0|\hat{a}(\boldsymbol{k}_1)\hat{a}(\boldsymbol{k}_2)\cdots\hat{a}(\boldsymbol{k}_n)|\phi(x),\pi(x)\rangle = \exp\left[-\frac{1}{2}\int d\boldsymbol{k}|\alpha(\boldsymbol{k})|^2\right]\alpha(\boldsymbol{k}_1)\alpha(\boldsymbol{k}_2)\cdots\alpha(\boldsymbol{k}_n)$$

$$\alpha(k) = \int \frac{d^{d-1}\boldsymbol{x}}{(2\pi)^{d-1}}\frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}}\Big[\omega_{\boldsymbol{k}}\phi(x) + i\pi(x)\Big]e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

classical configuration (bounce, sphaleron)

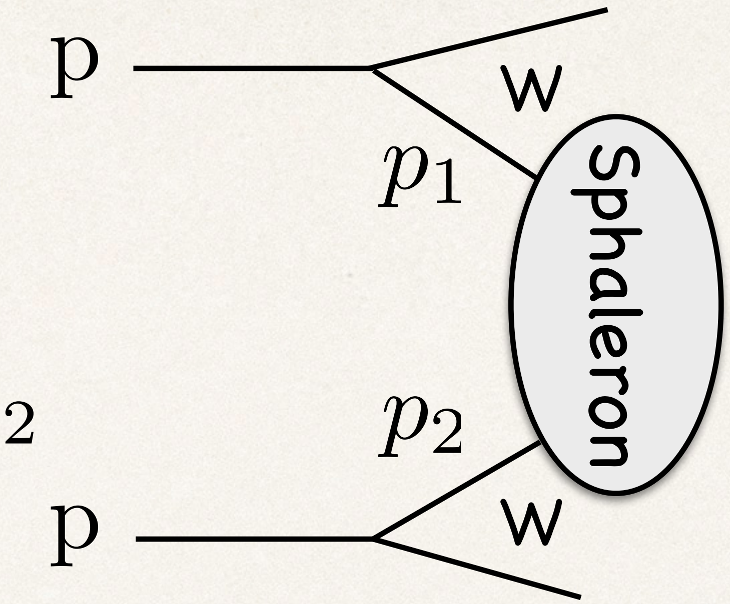
- cross section ∝ $|\alpha_1|^2 \ldots |\alpha_n|^2$

- $|\alpha|^2$ has a peak at $k=m_W$.



Few-to-many suppression

Case1: 2 -> sphaleron



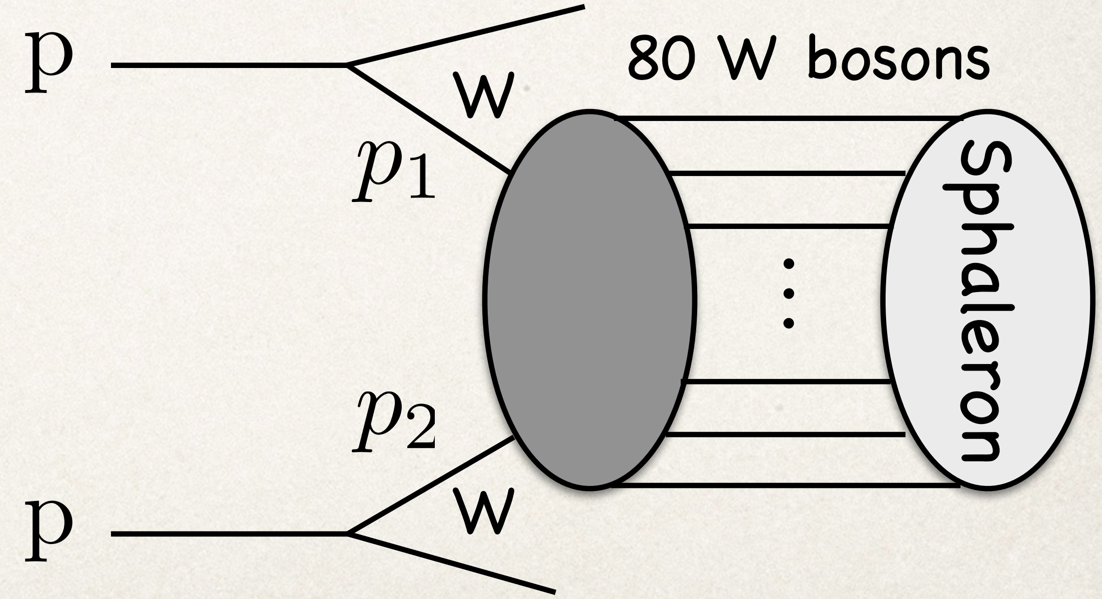
For $|p_1|=|p_2|\simeq E_{sph}/2$

$$|\langle\phi(x),\pi(x)|\boldsymbol{p}_1\boldsymbol{p}_2\rangle|^2 \ni |\alpha(\boldsymbol{p}_1)|^2|\alpha(\boldsymbol{p}_2)|^2$$

$$\sim e^{-\pi E_{\rm sph}/m_W} \sim \boxed{10^{-155}}$$

Creation of sphaleron from the 2 energetic particles is difficult.

Case2: 2 -> n W -> sphaleron



$n\simeq80$ since $E_{sph}/\sqrt{2}m_W$

Producing additional W boson costs "g".

$$\alpha_W^{80} = e^{-80|\ln\alpha_W|} \sim \boxed{10^{-118}}$$

$$\alpha_W = \frac{g^2}{4\pi} \sim \frac{1}{30}$$

difficult to produce about 80 W bosons.

Summary

part 1

- We have discussed the renormalization scale dependence of the resummed effective potential and proposed a scheme in which RG invariance is manifest order by order in resummed perturbation theory.

part 2

- We also discussed the band effect (claimed by Tye and Wong) on the sphaleron processes at zero and non-zero temperatures.

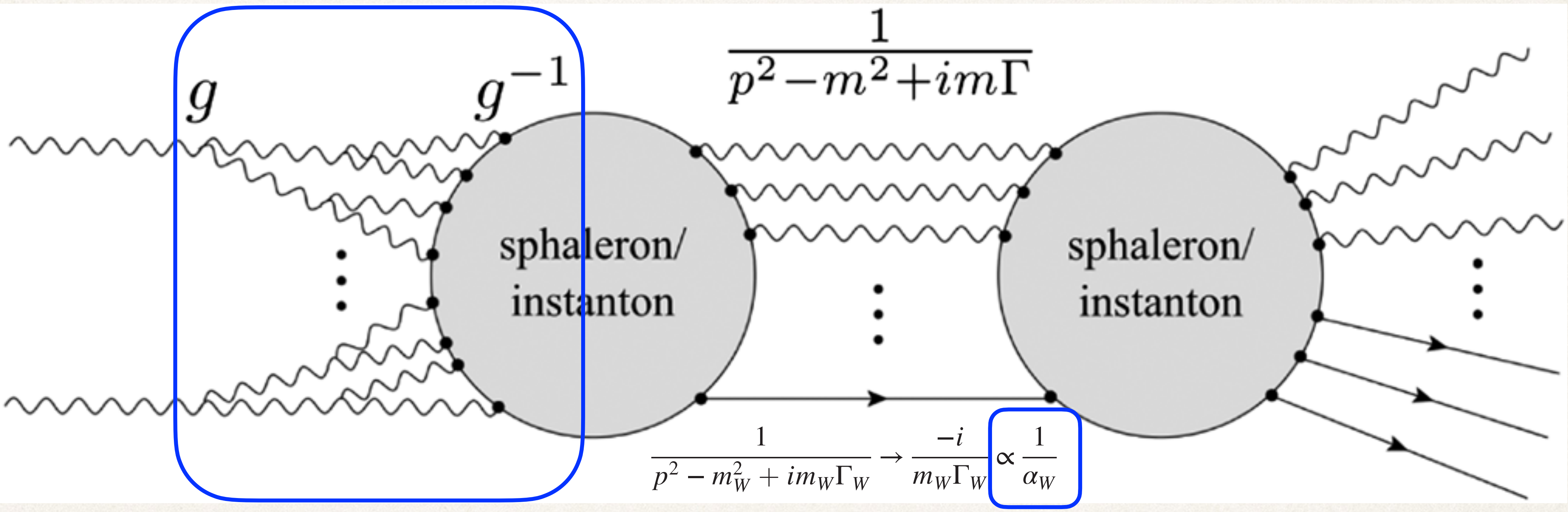
- Band effect has virtually no impacts on EWBG and is unlikely to affect (B+L)-violating processes at colliders unless the few-to-many suppression is overcome somehow.

Backup

Resonant tunneling

Few-to-many suppression can be compensated by "resonant tunneling".

[H. Tye, S Wong, PRD96 093004 ('17), Y.-C. Qiu, H. Tye, PRD100, 033006 ('19)]



Naive power counting given in the papers

- Producing additional W boson in the initial state costs g. However, such g is cancelled once it is attached to sphaleron because its coupling to sphaleron is g^{-1}.

- If intermediate W's are on-shell, it gives a factor $1/g^2$(propagator) x $1/g^2$(couplings) = $1/\alpha^2_W \sim 10^3$ for each W boson.

Sphaleron in SU(2) gauge-Higgs system

$$\mathcal{L}_{\text{gauge+Higgs}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi),$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_2\epsilon^{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\Phi = \left(\partial_\mu + ig_2 A^a_\mu \frac{\tau^a}{2}\right)\Phi, \quad V(\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2.$$

We consider the static classical solution.

$$E = \int d^3\boldsymbol{x}\left[\frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\Phi)^\dagger D_i\Phi + \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2\right].$$

Since $\lim_{|\boldsymbol{x}|\to\infty} E =$ finite, A and Φ must have the form of the vacuum configurations at $|\boldsymbol{x}| = \infty$:

$$A_i^\infty(\boldsymbol{x}) = \frac{i}{g_2}\partial_i U(\theta,\phi)U^{-1}(\theta,\phi),$$

$$\Phi^\infty(\boldsymbol{x}) = U(\theta,\phi)\begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix},$$

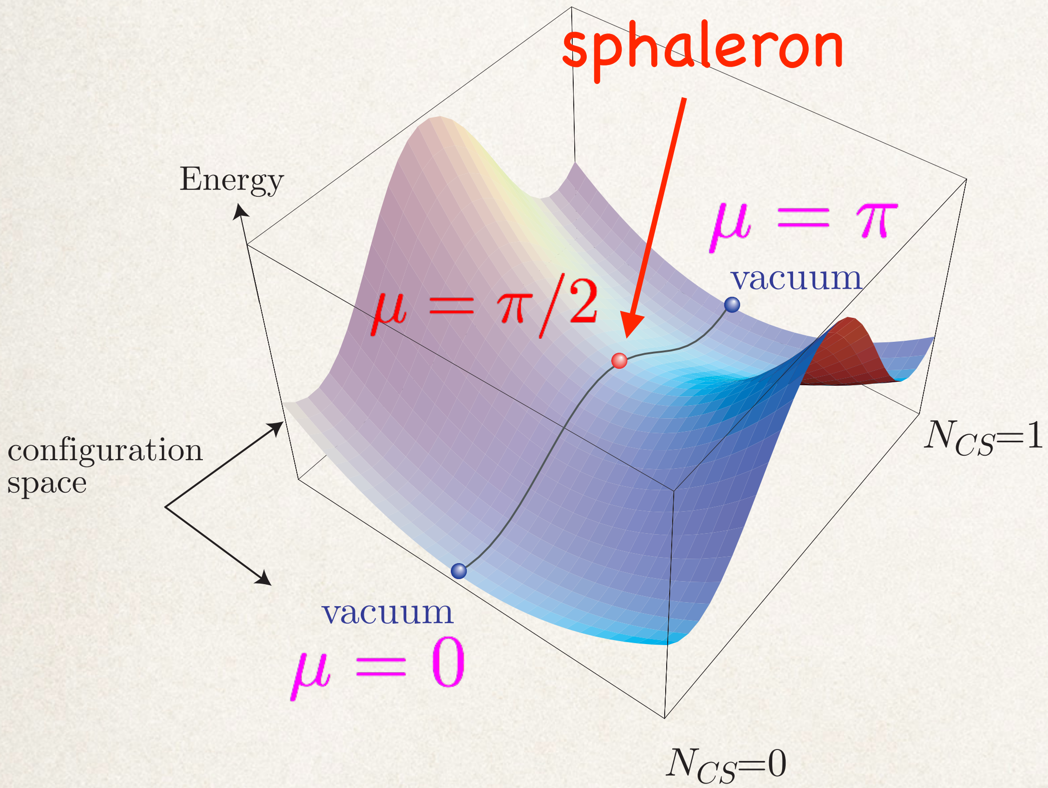
where $U(\theta,\phi)$ such that $S^2 \to SU(2) \simeq S^3$. $U(\theta=0,\phi) = 1$ should be taken.

Noncontractible loop

1 parameter family of $U(\mu,\theta,\phi)$ $(\mu \in [0,\pi])$ with a finite E
$[S^1 \times S^2 \to S^3]$

where
$$U(\mu,\theta,\phi) = U(\mu,\theta+\pi,\phi) = U(\mu,\theta,\phi+2\pi), \quad \text{for } {}^\forall\mu,$$
$$U(0,\theta,\phi) = U(\pi,\theta,\phi) = 1, \quad U(\mu,0,\phi) = 1, \quad \text{vacuum}$$

$(\mu,\theta,\phi) \in S^3$, $U(\mu,\theta,\phi)$ is noncontractible since $\pi_3(SU(2)) \simeq \mathbb{Z}$.



- saddle point = maximum energy along the least energy path -> μ =π/2

- vacuum -> μ =0,π.

$$U(\mu,\theta,\phi) = \begin{pmatrix} e^{i\mu}(\cos\mu - i\sin\mu\cos\theta) & e^{i\phi}\sin\mu\sin\theta \\ -e^{-i\phi}\sin\mu\sin\theta & e^{-i\mu}(\cos\mu + i\sin\mu\cos\theta) \end{pmatrix}.$$

Ansatz

Let us consider the configuration space spanned by the following:

$$A_i^\infty(\mu, \boldsymbol{x}) = \frac{i}{g_2}\partial_i U(\mu,\theta,\phi)U^{-1}(\mu,\theta,\phi),$$

$$\Phi^\infty(\mu, \boldsymbol{x}) = U(\mu,\theta,\phi)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

in the limit of $r = |\boldsymbol{x}| = \infty$. The most highest symmetry configurations in this space are

$$A_i(\mu, r, \theta, \phi) = \frac{i}{g_2} f(r)\partial_i U(\mu,\theta,\phi)U^{-1}(\mu,\theta,\phi),$$

$$\Phi(\mu, r, \theta, \phi) = \frac{v}{\sqrt{2}}\left[(1-h(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h(r)U(\mu,\theta,\phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right].$$

$\mu = \pi/2 \quad \Rightarrow \quad$ saddle point configuration (sphaleron)

$\mu = 0, \pi \quad \Rightarrow \quad$ vacuum configuration

Changing the variable $r = \sqrt{\boldsymbol{x}^2}$ to ξ, one gets

Energy functional $\left(\mu = \frac{\pi}{2}\right)$

$$
\begin{aligned}
E_{\rm sph} &= \frac{4\pi v}{g_2}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f - f^2)^2 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 + h^2(1-f)^2 + \frac{\lambda}{4g_2^2}\xi^2(h^2-1)^2\right] \\
&= \frac{4\pi v}{g_2}\mathcal{E}_{\rm sph}, \quad \text{where} \quad \xi = g_2 v r.
\end{aligned}
$$

input: $\frac{\lambda}{g_2^2} \simeq 0.3 \text{ (SM)}$

Equations of motion for the sphaleron

$$
\frac{d^2}{d\xi^2}f(\xi) = \frac{2}{\xi^2}f(\xi)(1-f(\xi))(1-2f(\xi)) - \frac{1}{4}h^2(\xi)(1-f(\xi)),
$$

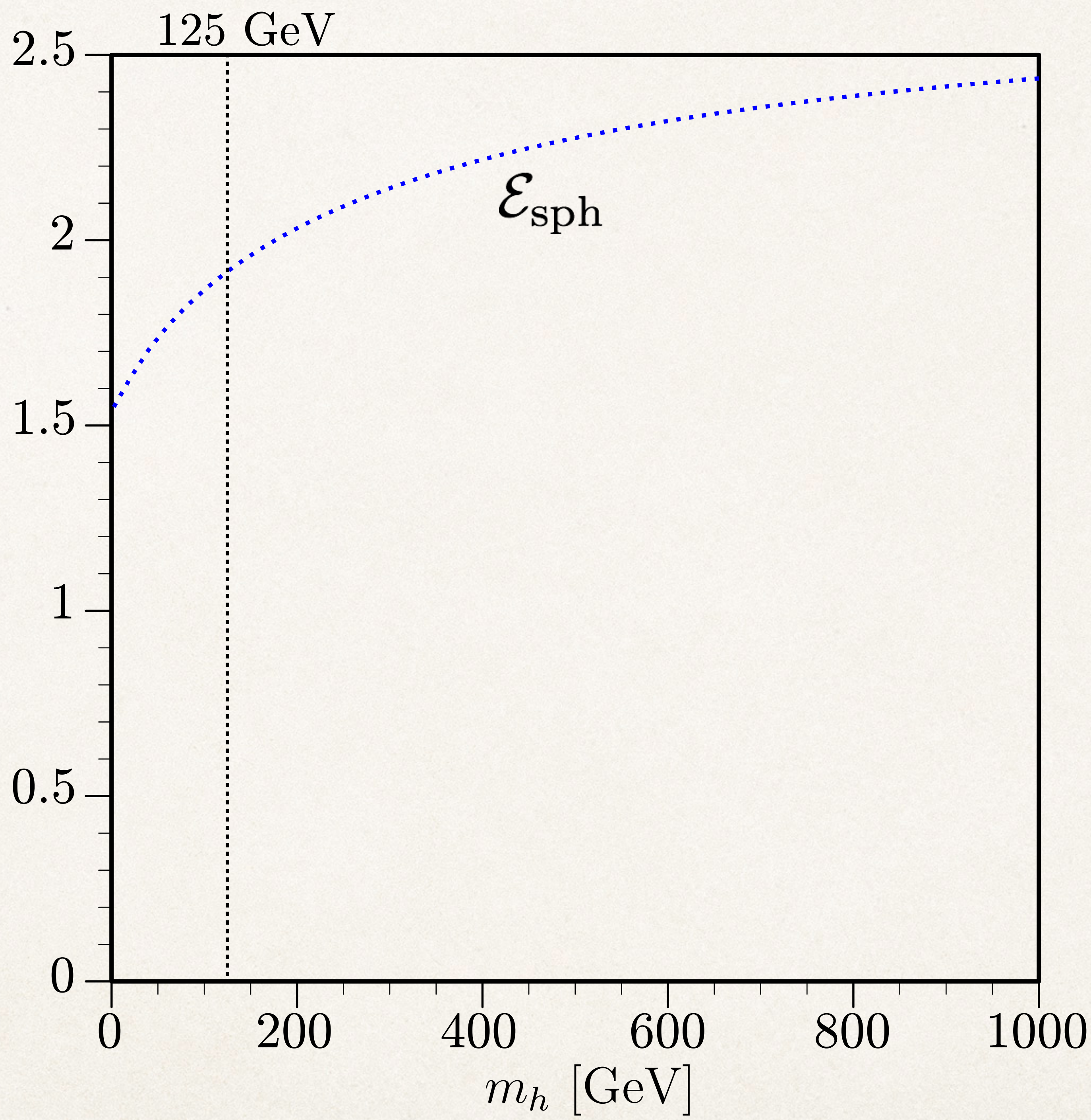
$$
\frac{d}{d\xi}\left(\xi^2\frac{dh(\xi)}{d\xi}\right) = 2h(\xi)(1-f(\xi))^2 + \frac{\lambda}{g_2^2}(h^2(\xi)-1)h(\xi)
$$

with the boundary conditions

$$
\lim_{\xi\to 0} f(\xi) = 0, \quad \lim_{\xi\to 0} h(\xi) = 0,
$$

$$
\lim_{\xi\to\infty} f(\xi) = 1, \quad \lim_{\xi\to\infty} h(\xi) = 1.
$$

Sphaleron energy



For $m_h = 125$ GeV, $\mathcal{E}_{\rm sph} = 1.92$ ($E_{\rm sph} = 9.08$ TeV)

Comparison with Tye-Wong's work

Some differences between our work and Tye-Wong's (TW's).

	A_0	Sphaleron mass	Method for band structure
this work	$A_0 \neq 0$	μ-dependent	WKB w/ 3 connection formulas
Tye-Wong	$A_0 = 0$	μ-independent	Schroedinger eq. numerically

We use the Manton's ansatz with $A_0 = \frac{i}{g_2} f(r) \partial_0 U U^{-1}$.

N.B.

If $A_0=0$ is naively adopted with the Manton's ansatz, an unwanted divergence would appear in DΦ at the region r=∞. -> some prescription is needed!!

[Aoyama, Goldberg, Ryzak, PRL60, 1902 (1988)]

Classical action

$$S[\mu] = g_2 v \int dt \left[\frac{M(\mu)}{2} \left(\frac{d}{dt} \frac{\mu(t)}{g_2 v} \right)^2 - V(\mu) \right],$$

$$\begin{aligned} M(\mu) = \frac{4\pi}{g_2^2} \int_0^\infty d\xi \; \xi^2 \Bigg[4 \left\{ f'^2 \frac{4 + 2c_\mu^2}{3} + \frac{4}{\xi^2}(f - f^2)^2 \frac{8 + 2c_\mu^2}{3} s_\mu^2 \right\} \\ + (1-h)^2 + 2h(1-h)(1-f) + 2(1-h)^2 f c_\mu^2 \\ + \frac{4 + 2c_\mu^2}{3} \left\{ h^2(1-f)^2 + \Big((1-h)^2(f^2 - 2f) - 2h(1-h)f(1-f) \Big) c_\mu^2 \right\} \Bigg] \end{aligned}$$

$$\begin{aligned} V(\mu) = \frac{4\pi}{g_2^2} \int_0^\infty d\xi \; \xi^2 \Bigg[\frac{4}{\xi^2} \left\{ f'^2 + \frac{2}{\xi^2}(f - f^2)^2 s_\mu^2 \right\} s_\mu^2 \\ + \frac{s_\mu^2}{2} \left\{ h'^2 + \frac{2}{\xi^2} \Big(h^2(1-f)^2 - 2h(1-h)f(1-f)c_\mu^2 + f^2(1-h)^2 c_\mu^2 \Big) \right\} \\ + \frac{\lambda}{4g_2^2}(1-h^2)^2 s_\mu^4 \Bigg]. \end{aligned}$$

f, h are determined by the EOM for the sphaleron.

Eigenvalue problem

Hamiltonian:

$$\hat{H}(\mu,p) = g_2 v\left[\hat{p}\frac{1}{2M(\hat{\mu})}\hat{p} + V(\hat{\mu})\right], \qquad [\hat{\mu},\hat{p}] = i$$

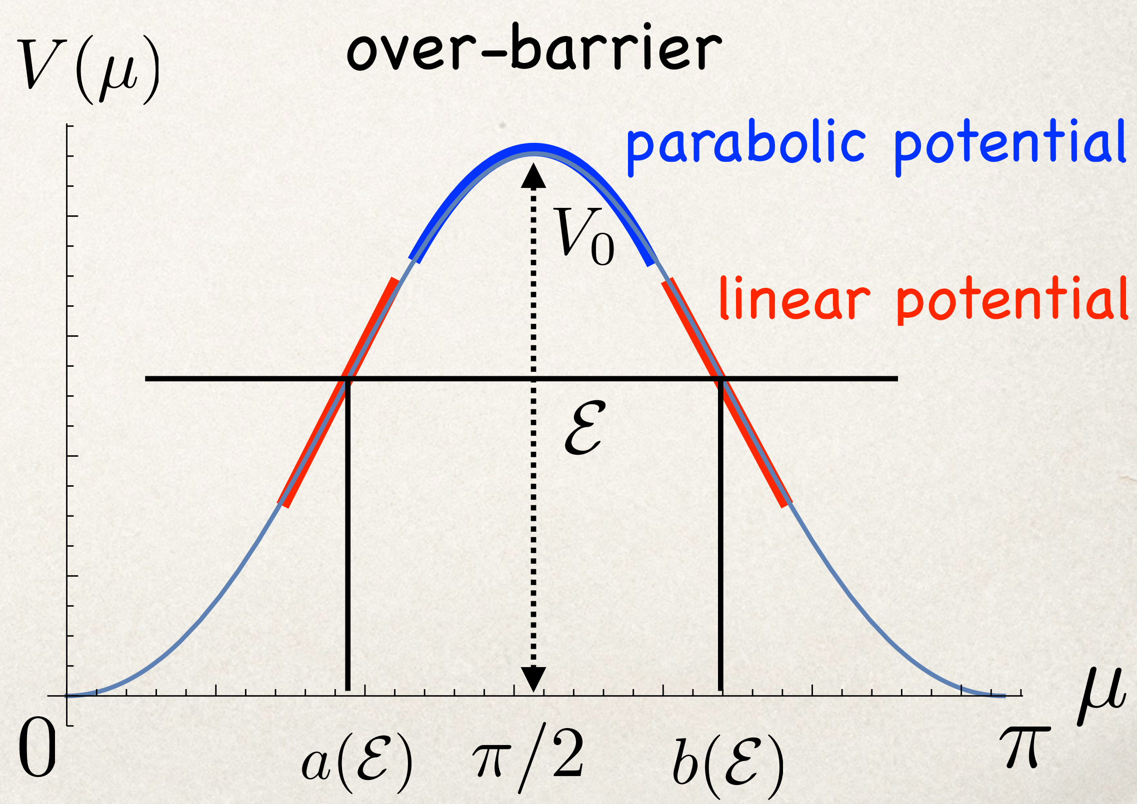
Band energy is determined by solving [N.L.Balazs, Ann.Phys.53,421 (1969)]

$$\cos(\Phi(\mathcal{E})) = \pm\sqrt{T(\mathcal{E})}$$

$$\Phi(\mathcal{E}) = \begin{cases} \frac{1}{\hbar}\int_{b(\mathcal{E})}^{a(\mathcal{E})} d\mu\, p(\mu) & \text{for } \mathcal{E} < V_0, \\ \frac{1}{\hbar}\int_{-\pi/2}^{\pi/2} d\mu\, p(\mu) & \text{for } \mathcal{E} \geq V_0, \end{cases}$$

$$p(\mu) = \sqrt{M(\mu)(\mathcal{E} - V(\mu))}$$

with 3 connection formulas depending on energy.



Vacuum decay rate at finite-T

Ordinary case: [Affleck, PRL46,388 (1981)]

$$\Gamma_A(T) = \frac{1}{Z_0(T)} \int_0^\infty dE\ J(E) e^{-E/T}$$
$$\simeq \frac{1}{Z_0} \frac{\omega_-}{4\pi \sin\left(\frac{\omega_-}{2T}\right)} e^{-E_{\rm sph}/T} \quad \text{for } T > \frac{\omega_-}{2\pi},$$

≃14 GeV

$$J(E) = \frac{T(E)}{2\pi}, \quad Z_0(T) = \left[2\sinh\left(\frac{\omega_0}{2T}\right)\right]^{-1}, \quad \frac{\omega_0}{g_2 v} = \sqrt{\frac{V''(0)}{M(0)}}, \quad \frac{\omega_-}{g_2 v} = \sqrt{\frac{V''(\pi/2)}{M(\pi/2)}}$$

≃0.42 ≃0.51

Band case: $J(E) \to \eta(E)/2\pi$

$$\Gamma(T) = \frac{1}{Z_0(T)} \int_0^\infty dE\ \frac{\eta(E)}{2\pi} e^{-E/T}$$

$\eta(E) = 1$ for the conducting band, $\eta(E) = 0$ for the band gap

Quantum mechanics analogy

1-dim. $[\hat{x},\hat{p}]=i,\quad [\hat{x},\hat{x}]=[\hat{p},\hat{p}]=0$

$$\hat{a}=\frac{1}{\sqrt{2}}(\hat{x}+i\hat{p}),\quad \hat{a}^\dagger=\frac{1}{\sqrt{2}}(\hat{x}-i\hat{p})$$

coherent state: $\hat{a}|z\rangle=z|z\rangle,\quad |z\rangle=e^{-\frac{1}{2}|z|^2+z\hat{a}^\dagger}|0\rangle$

$$\langle z|z'\rangle=e^{-\frac{1}{2}(|z|^2+|z'|^2)+z^*z'},\quad \int\frac{d^2z}{\pi}|z\rangle\langle z|=1,\quad d^2z=d(\mathrm{Re}z)d(\mathrm{Im}z)$$

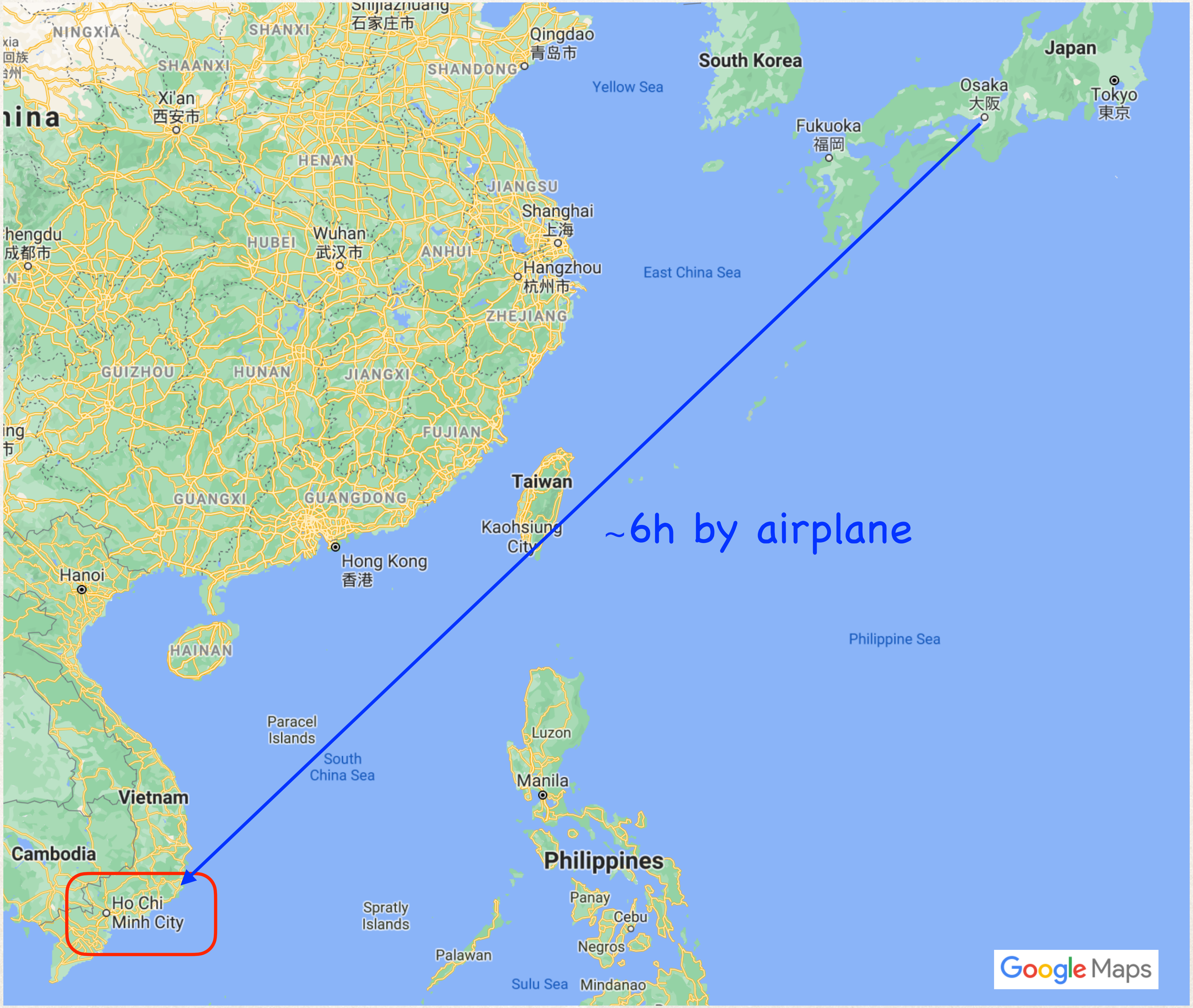
n-dim. $[\hat{x}_i,\hat{p}_j]=i\delta_{ij},\quad [\hat{x}_i,\hat{x}_j]=[\hat{p}_i,\hat{p}_j]=0,\quad i=1,2,\cdots,n$

overlap with the coherent state

a state w/ definite particle numbers: $\hat{a}^\dagger_{i1}\hat{a}^\dagger_{i2}\cdots\hat{a}^\dagger_{iN}|0\rangle$

$$\langle 0|\hat{a}_{i1}\hat{a}_{i2}\cdots\hat{a}_{iN}|\boldsymbol{z}\rangle=z_{i1}z_{i2}\cdots z_{iN}e^{-\frac{1}{2}|\boldsymbol{z}|^2}$$

Where is Ton Duc Thang Univ.?



Where is Ton Duc Thang Univ.?

I belong to Theoretical Particle Physics and Cosmology Research Group (TPPC) in Advanced Institute of Materials Science (AIMaS).