

# Scalar Dark Matter with a $\mu\tau$ Flavored Mediator

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Based on

**KA**, C. Miyao, S. Okawa, K. Tsumura, PRD **106** (2022) 035017, arXiv : [2205.08998](https://arxiv.org/abs/2205.08998) [hep-ph]

# Introduction

# Introduction

● Introduction

● Model

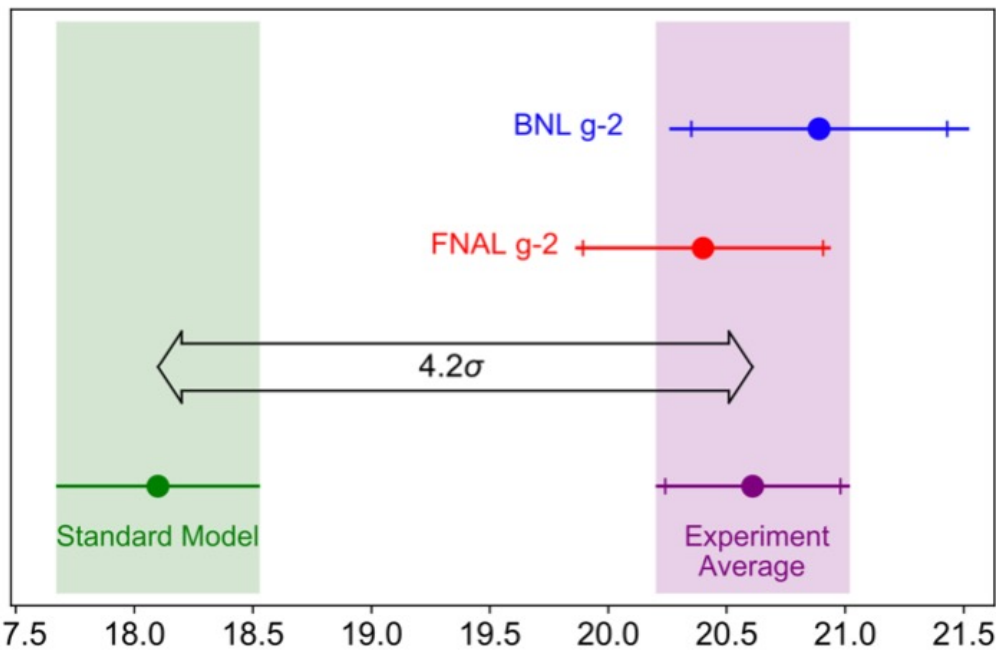
● DM Physics

● Result

● Appendix

## Muon anomalous magnetic moment

There is a discrepancy between experimental value and theoretical prediction of the muon  $g-2$



$$\begin{aligned}\Delta a_\mu &= a_\mu^{\text{BNL+FNAL}} - a_\mu^{\text{SM}} \\ &= (25.1 \pm 5.9) \times 10^{-10} \quad (4.2\sigma)\end{aligned}$$



$$\frac{m_\mu^2}{16\pi^2} \frac{g_{\text{NP}}^2}{m_{\text{NP}}^2}$$

New Physics ?

$a_\mu \times 10^9 - 1165900$  Muon  $g-2$  collaboration,  
Phys. Rev. Lett. **126**, 141801 (2021)

## Muon anomalous magnetic moment

### New Physics for muon $g-2$

- $U(1)_{L_\mu - L_\tau}$  gauge boson

S. Baek, N.G. Deshpande, X.G. He, P. Ko, PRD **64**, 055006 (2001); ...

- Supersymmetry

J. L. Lopez, D. V. Nanopoulos, X. Wang, PRD **49** (1994) 366-372;

U. Chattopadhyay, P. Nath, PRD **53** (1996) 1648-1657;

T. Moroi, PRD **53** (1996) 6565-6575; ...

- Leptoquark

D. Chakraverty, D. Choudhury, A. Datta, PLB **506** (2001) 103-108;

K.-M. Cheung, PRD **64** (2001) 033001; ...

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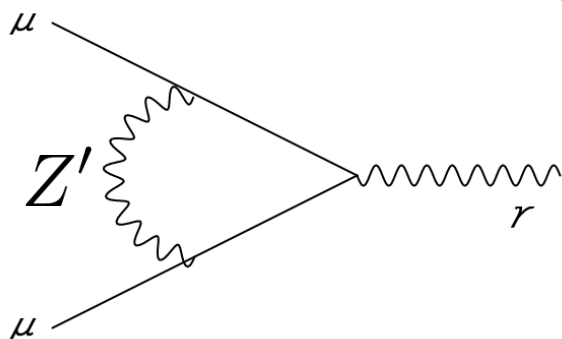
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## Muon anomalous magnetic moment

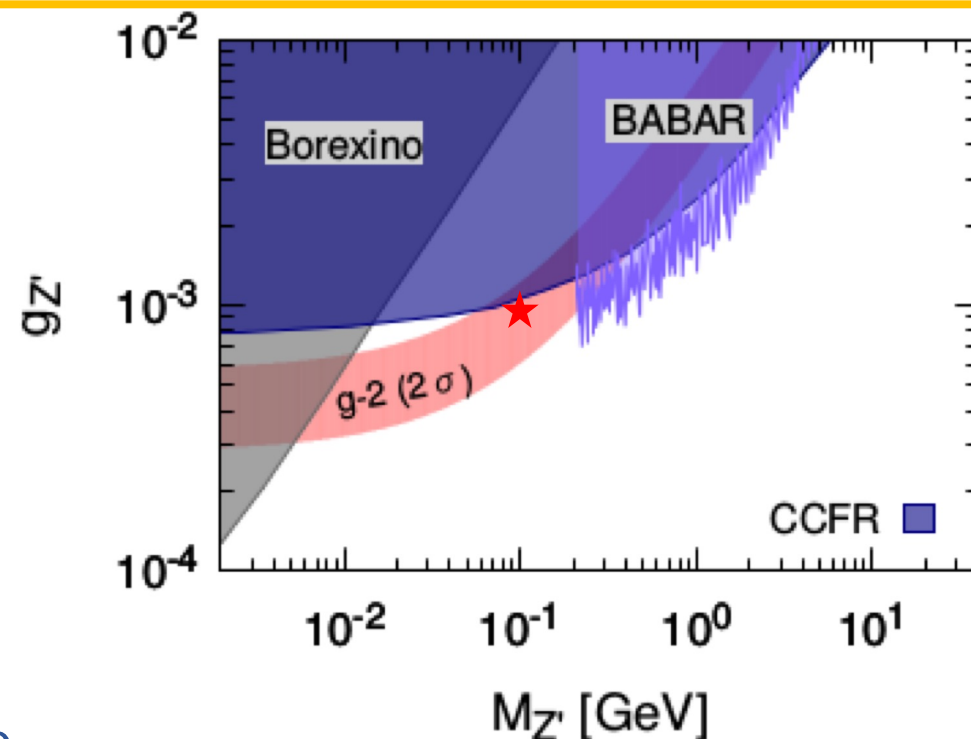
### New Physics for muon g-2

- $U(1)_{L_\mu - L_\tau}$  gauge boson

$$\Delta a_\mu = \mathcal{O}(10) \times 10^{-10} \left( \frac{g_{Z'}}{10^{-3}} \right)^2 \left( \frac{100 \text{ MeV}}{m_{Z'}} \right)^2$$



No tree level coupling to electron & quarks



Araki, Hoshino, Ota, Sato, Shimomura,  
PRD **95**, 5, 055006 (2017)

## Muon anomalous magnetic moment

### New Physics for muon g-2

- Supersymmetry

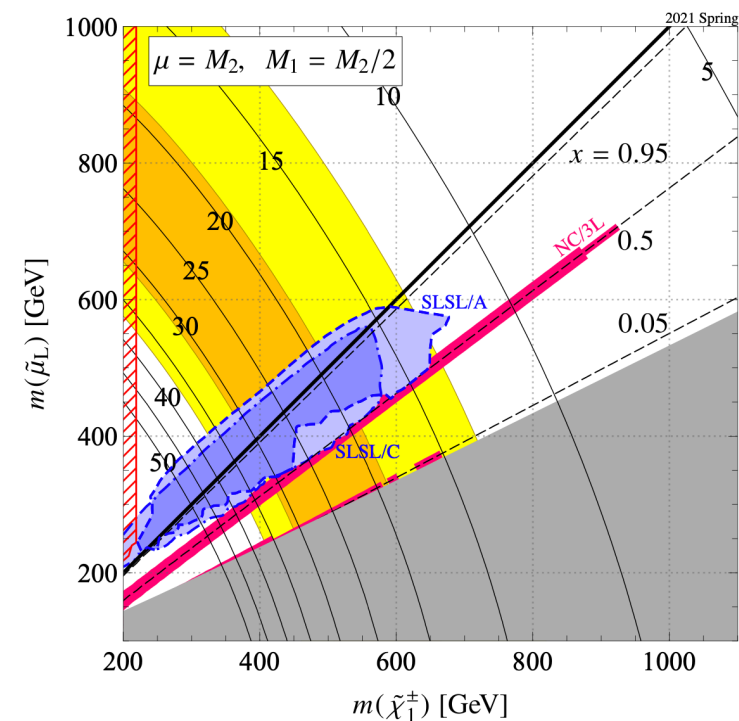
M. Endo, K. Hamaguchi, S. Iwamoto, T. Kitahara, JHEP **07**, 075 (2021)

$$\mathcal{O}(1) \times 10^{-10} \left( \frac{\alpha_2}{0.03} \right) \left( \frac{400 \text{ GeV}}{m_{\text{SUSY}}} \right)^2$$



$\tan \beta$  enhancement

$$\Delta a_{\mu}^{\text{SUSY}} = \mathcal{O}(10) \times 10^{-10} \left( \frac{\tan \beta}{10} \right) \left( \frac{\alpha_2}{0.03} \right) \left( \frac{400 \text{ GeV}}{m_{\text{SUSY}}} \right)^2$$



## Muon anomalous magnetic moment

### New Physics for muon g-2

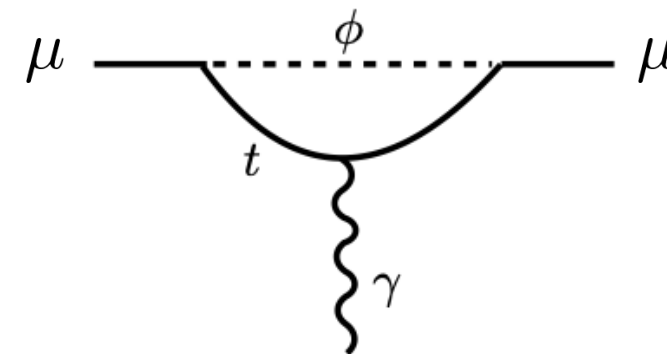
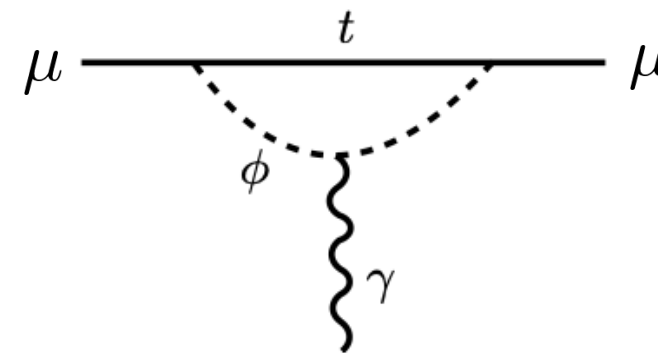
- Leptoquark

$$\mathcal{O}(10^{-2}) \times 10^{-10} \left( \frac{\lambda_{t\mu}}{0.1} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\text{LQ}}} \right)^2$$



Chirality enhancement

$$\Delta a_{\mu}^{\text{LQ}} = \mathcal{O}(10) \times 10^{-10} \left( \frac{m_t/m_{\mu}}{10^3} \right) \left( \frac{\lambda_{t\mu}}{0.1} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\text{LQ}}} \right)^2$$



## Muon anomalous magnetic moment

### Advantage

- $U(1)_{L_\mu - L_\tau}$  gauge boson
- Supersymmetry
- Leptoquark

No tree level coupling to  $e^-$  &  $q$

Enhancement factor

Enhancement factor

### Another possibility

$\mu\tau$ -philic doublet scalar

Y. Abe, T. Toma, K. Tsumura, JHEP **06**, 142 (2019)

No tree level coupling to  $e^-$  &  $q$

Enhancement factor



## $\mu\tau$ -philic doublet scalar

Y. Abe, T. Toma, K. Tsumura, JHEP **06**, 142 (2019)

### Symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

&

$Z_4$  flavor symmetry

### Fields

SM fields

+  $\Phi$  :  $\mu\tau$ -philic doublet scalar

particles	$(L_e, L_\mu, L_\tau)$	$(e_R, \mu_R, \tau_R)$	$H$	$\Phi$
SM	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{2})_{1/2}$
$Z_4$	$(1, i, -i)$	$(1, i, -i)$	1	-1

# Introduction

## $\mu\tau$ -philic doublet scalar

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● DM Physics

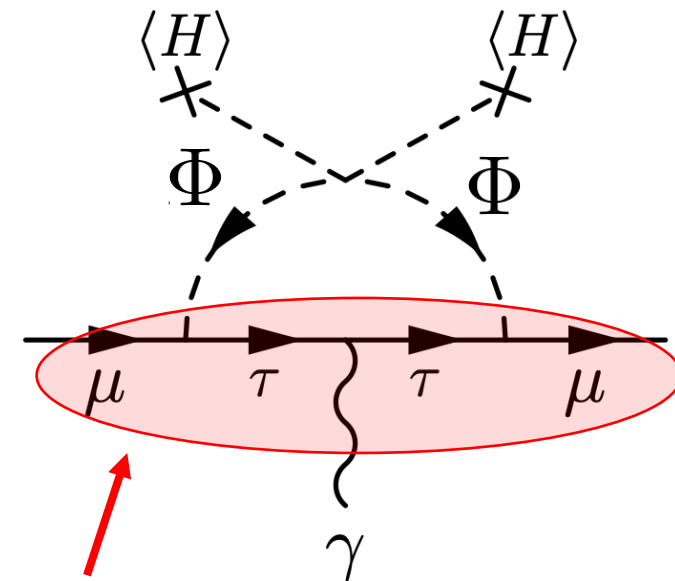
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### Muon $g-2$

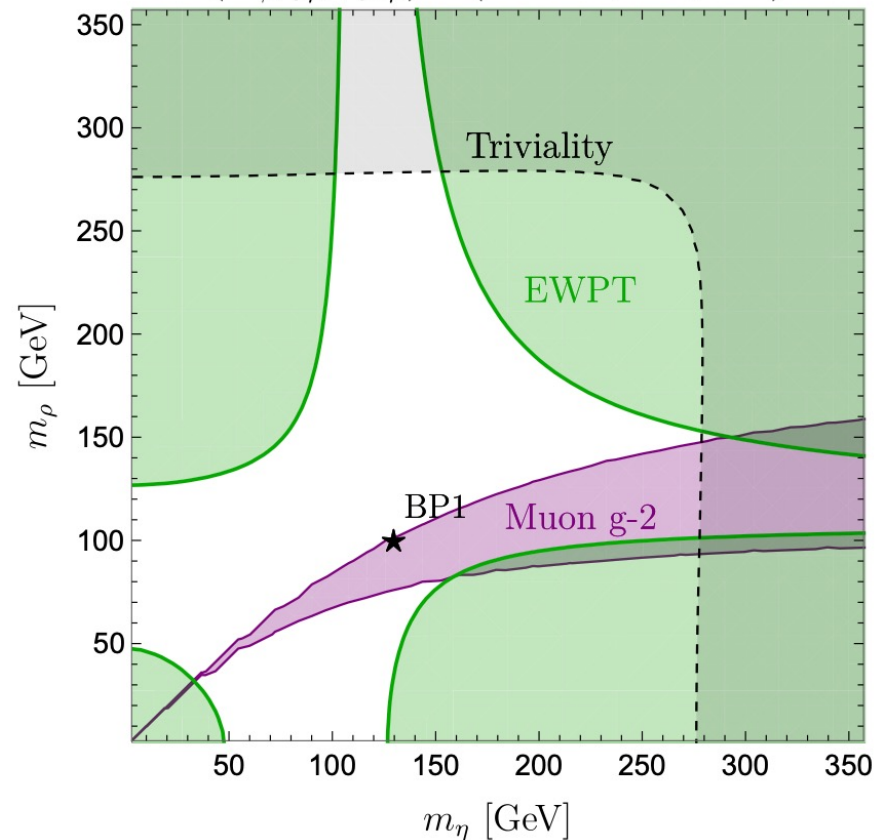
KA, C. Miyao, S. Okawa, K. Tsumura, PRD **106** (2022) 035017

$\rho$  : CP-even  
 $\eta$  : CP-odd heavy scalar  
 $\phi$  : charged

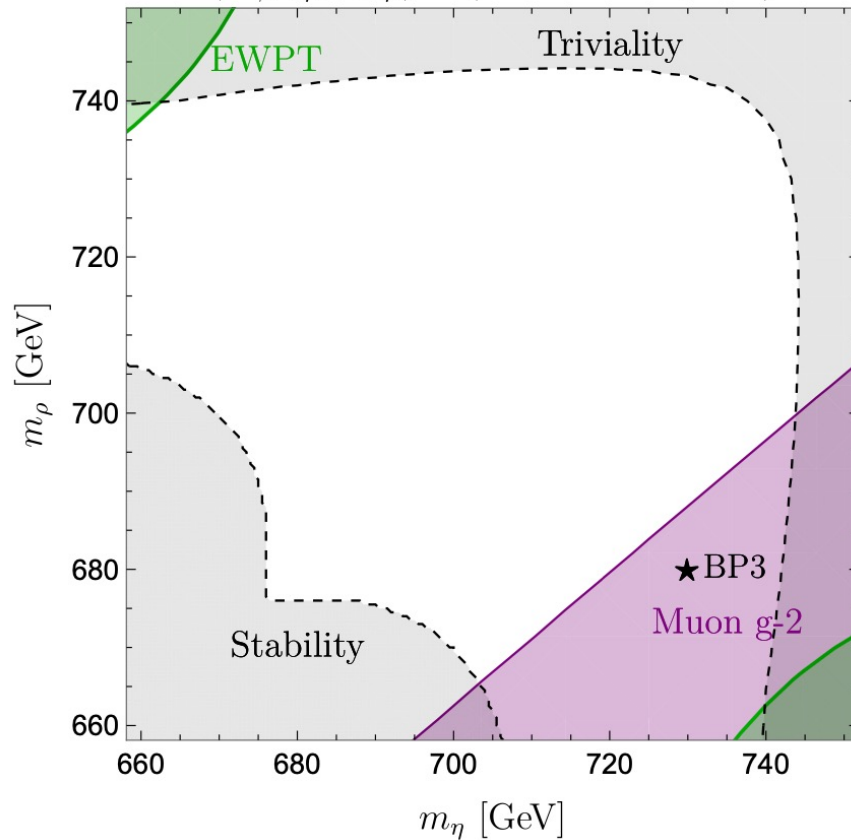


Chirality enhancement  $\times m_\tau / m_\mu$

$(m_\phi, y_{\mu\tau}, y_{\tau\mu}) = (100\text{GeV}, 0.07, 0.07)$



$(m_\phi, y_{\mu\tau}, y_{\tau\mu}) = (700\text{GeV}, 0.70, 0.70)$



# Introduction

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● DM Physics

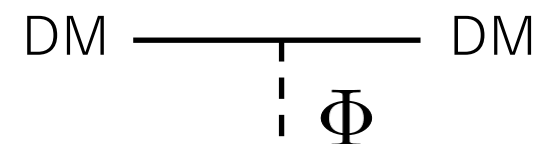
● Result

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## $\mu\tau$ -philic doublet scalar & dark matter

$\mu\tau$ -philic doublet scalar is very attractive for DM model

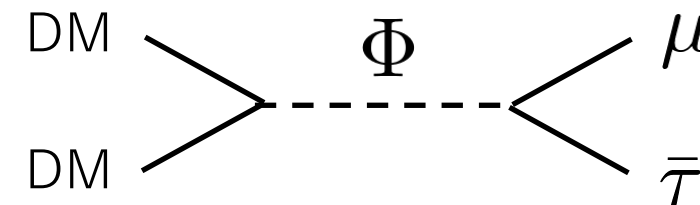
- $\mu\tau$ -philic doublet scalar does not couple to electron & nucleon at tree level



➔ Avoid severe constraints from direct detection



- $\mu\tau$ -philic doublet scalar couples to  $\mu$  &  $\tau$



➔ DM can annihilate into  $\mu$  &  $\tau$

- $Z_4$  flavor symmetry guarantees DM stability ?

## $\mu\tau$ -philic doublet scalar & dark matter

$\mu\tau$ -philic doublet scalar is very attractive for DM model

- No coupling to electron and nucleon  
➔ Avoiding severe direct detection constraints
- $Z_4$  flavor symmetry  
➔ DM stability

### Questions

- Muon  $g - 2$  can be explained simultaneously ?
- DM can be detected by direct & indirect detection exps.?

# Model

## DM model with $\mu\tau$ -philic mediator

### Symmetry

$$\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$$

&

$$\text{Z}_4 \text{ flavor symmetry}$$

### Fields

SM fields

+  $\Phi$  :  $\mu\tau$ -philic doublet scalar

+  $\Sigma$  : complex scalar DM

particles	$(L_e, L_\mu, L_\tau)$	$(e_R, \mu_R, \tau_R)$	$H$	$\Phi$	$\Sigma$
SM	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{1})_0$
$\text{Z}_4$	$(1, i, -i)$	$(1, i, -i)$	1	-1	$i$

## DM model with $\mu\tau$ -philic mediator

### Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + |D_\mu \Phi|^2 + |D_\mu \Sigma|^2 - (y_{\mu\tau} L_\mu^\dagger \Phi \tau_R + y_{\tau\mu} L_\tau^\dagger \Phi \mu_R + \text{H.c.}) - V(H, \Phi, \Sigma)$$

$$V(H, \Phi, \Sigma) = \mu_\Phi^2 |\Phi|^2 + \lambda_2 |\Phi|^4 + \lambda_3 |H|^2 |\Phi|^2 + \lambda_4 |H^\dagger \Phi|^2 + \frac{\lambda_5}{2} [(H^\dagger \Phi)^2 + \text{H.c.}]$$

$$+ \mu_\Sigma^2 |\Sigma|^2 + \lambda_\Sigma |\Sigma|^4 + [\lambda'_\Sigma \Sigma^4 + \text{H.c.}] + \cancel{\lambda_{H\Sigma} |H|^2 |\Sigma|^2} + \lambda_{\Phi\Sigma} |\Phi|^2 |\Sigma|^2$$

$$+ \kappa [(H^\dagger \Phi) \Sigma^2 + \text{H.c.}] ,$$

$H$	$\Phi$	$\Sigma$
$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{1})_0$
1	-1	$i$

$\Phi$  and  $\Sigma$  have no VEV

Subgroup of  $Z_4$  flavor symmetry



Accidental  $Z_2$  symmetry guarantees DM stability ( $\Sigma \rightarrow -\Sigma$ )

## Constraints on $\mu\tau$ -philic mediator

### Theoretical

- Electroweak precision test

$$S = 0.00 \pm 0.07, \quad T = 0.05 \pm 0.06, \quad U = 0$$

- Triviality bound

$$|\lambda| \leq 4\pi, \quad |y| \leq \sqrt{4\pi}$$

- Potential stability

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad 2\sqrt{\lambda_1\lambda_2} + \lambda_3 > 0, \quad 2\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0$$



# Model

## Constraints on $\mu\tau$ -philic

### Collider experiment

- LEP

$$m_\phi \gtrsim 93.5 \text{ GeV}$$

- Slepton search

$$\phi^+ \rightarrow \tau^+ \nu_\tau \quad \Rightarrow \quad m_\phi \gtrsim 350 \text{ GeV}$$

$$\phi^+ \rightarrow \mu^+ \nu_\mu \quad \Rightarrow \quad m_\phi \gtrsim 550 \text{ GeV}$$

- Neutral scalar search

$$pp \rightarrow W^{\pm*} \rightarrow \phi^\pm \rho, \phi^\pm \eta$$

$$pp \rightarrow Z^* \rightarrow \rho\eta$$

L. Wang, Y. Zhang, PRD 100 (2019) 095005;

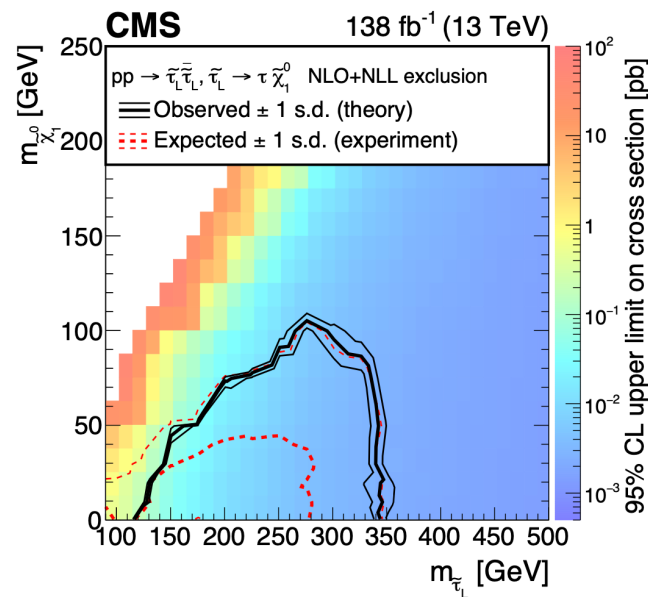
S. Iguro, Y. Omura, M. Takeuchi, JHEP 11 (2019) 130

Exclude

$$200 \text{ GeV} \lesssim m_\eta \lesssim 500 \text{ GeV}$$

$$\left( \begin{array}{l} m_\rho < m_\eta \simeq m_\phi \\ \text{or} \\ m_\eta < m_\rho \simeq m_\phi \end{array} \right)$$

$100 \text{ GeV} \lesssim m_\phi \lesssim 115 \text{ GeV}$  is allowed



CMS collaboration,  
2207.02254

# DM Physics

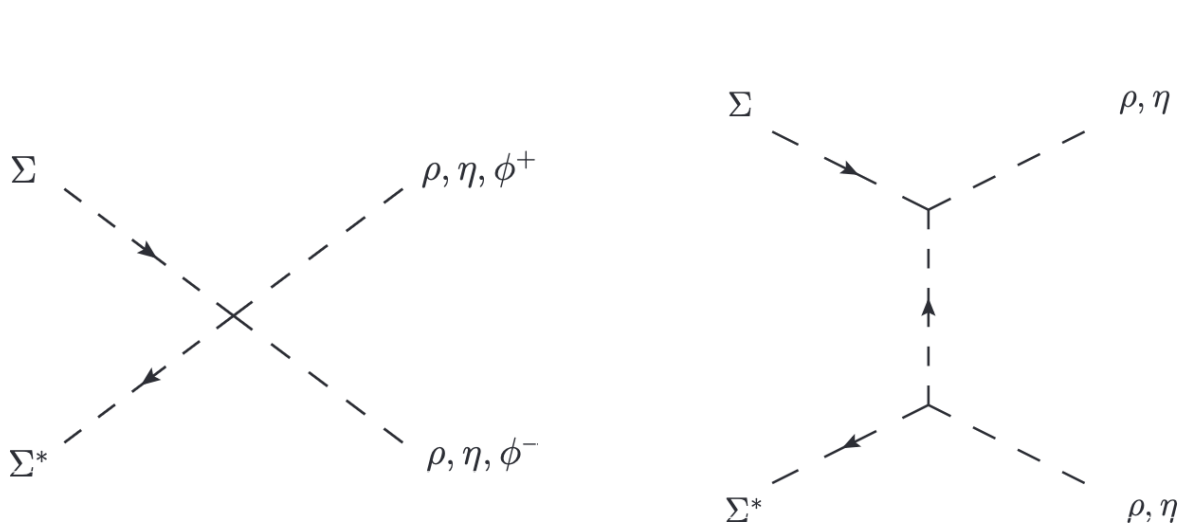
# DM Physics

## Relic density

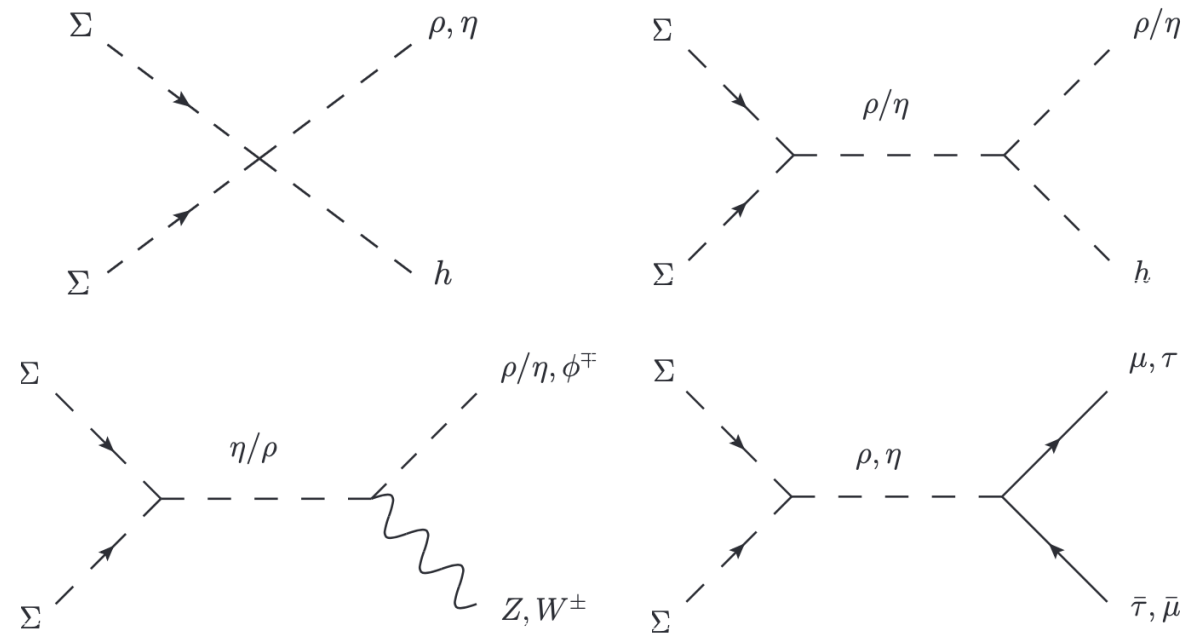
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Observed DM relic density is realized by thermal freeze-out mechanism

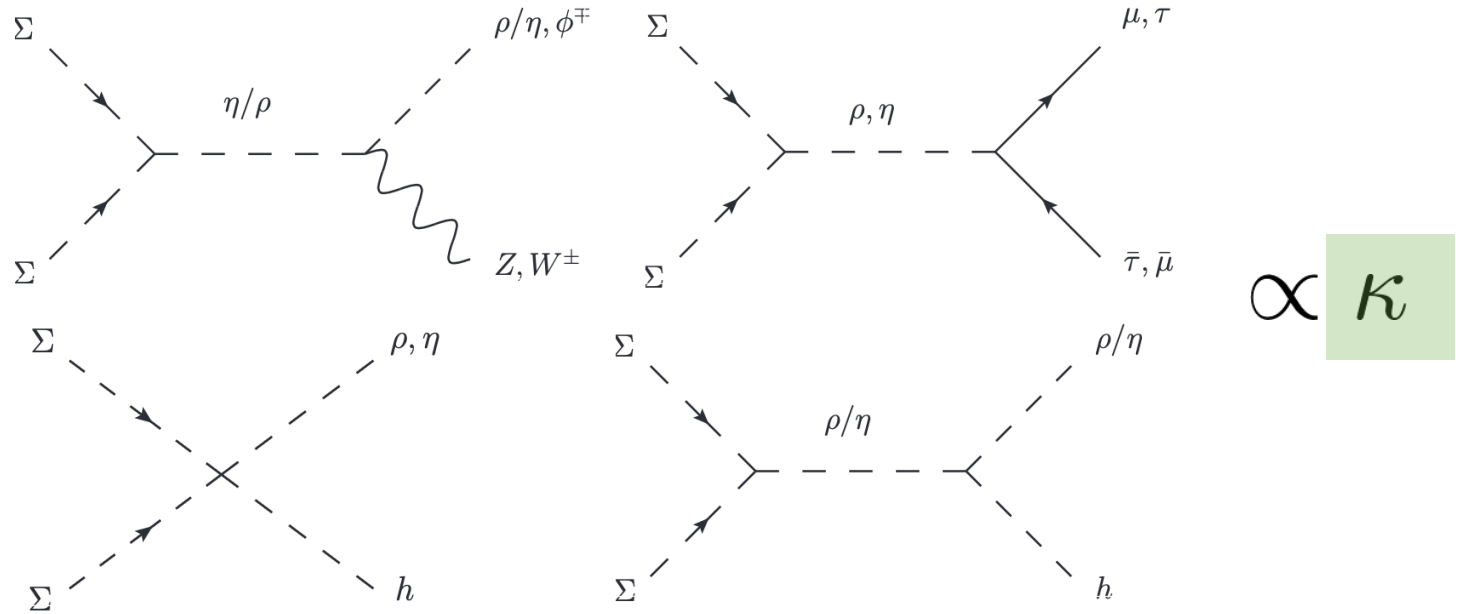
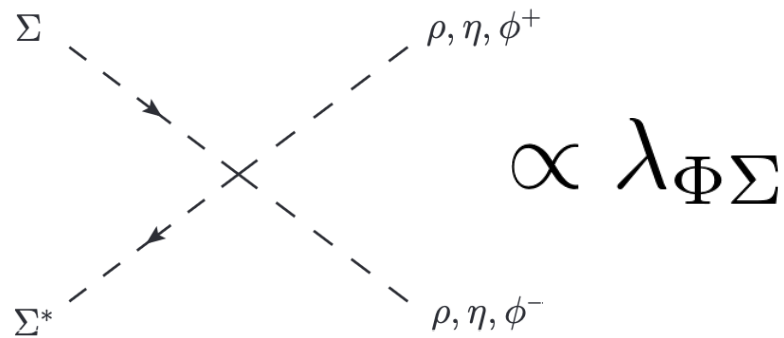
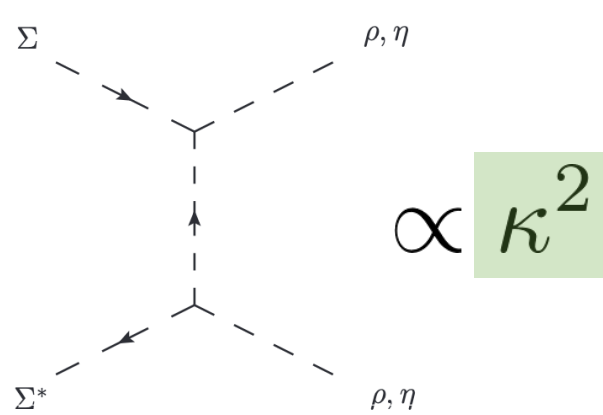
### 1, $\Sigma \Sigma^*$ annihilation



### 2, $\Sigma \Sigma$ annihilation



### Annihilation process



$\kappa$  is important for DM annihilation

(Assume  $\lambda_{\Phi\Sigma} = 0$  for simplicity)

## Relic density

### Boltzmann equation

$$\frac{dn_{\text{DM}}}{dt} + 3Hn_{\text{DM}} = -\frac{1}{2}(\sigma v_{\text{rel}})_{\text{eff}} [n_{\text{DM}}^2 - (n_{\text{DM}}^{\text{eq}})^2]$$

with

$$n_{\text{DM}}^{(\text{eq})} = n_{\Sigma}^{(\text{eq})} + n_{\Sigma^*}^{(\text{eq})}$$

$$(\sigma v_{\text{rel}})_{\text{eff}} = \sum_{ij} \left[ (\sigma v_{\text{rel}})_{\Sigma\Sigma^* \rightarrow ij} + \frac{1}{2}(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow ij} + \frac{1}{2}(\sigma v_{\text{rel}})_{\Sigma^*\Sigma^* \rightarrow ij} \right]$$

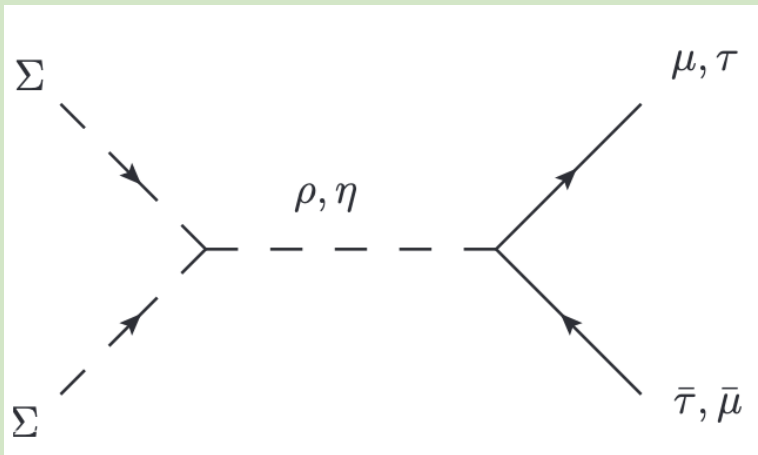
$$\simeq 6 \times 10^{-26} \text{ cm}^3/\text{s} \quad \rightarrow \quad \Omega_{\text{DM}} h^2 \simeq 0.12$$

# DM Physics

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## Indirect detection

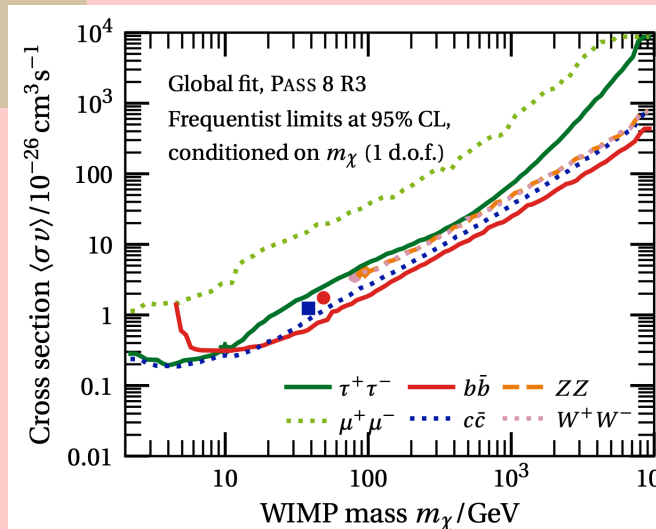
DM annihilates into  $\mu^+\tau^-$  and  $\tau^+\mu^-$



DM DM  $\rightarrow \phi^+\phi^-, \rho\rho, \rho Z, \phi^-W^+ \dots$   
 $\rightarrow$  SM particles

future work

Fermi-LAT gives bound for DM DM  $\rightarrow \mu^+\mu^-, \tau^+\tau^-$  process



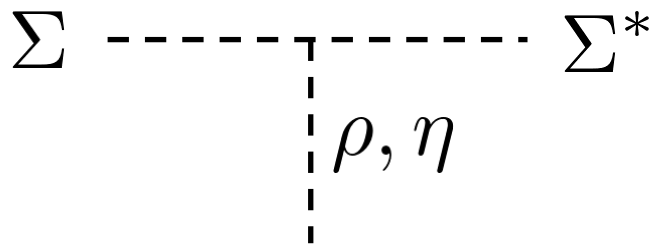
S. Hoof, A. Geringer-Sameth, R. Trotta, JCAP **02**, 012 (2020)

One  $\mu$  &  $\tau$  are produced by  $\Sigma\Sigma$  and  $\Sigma^*\Sigma^*$  annihilation

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \mu\bar{\mu}, \tau\bar{\tau}} \equiv \frac{(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \mu\bar{\tau}} + (\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \tau\bar{\mu}}}{2}$$

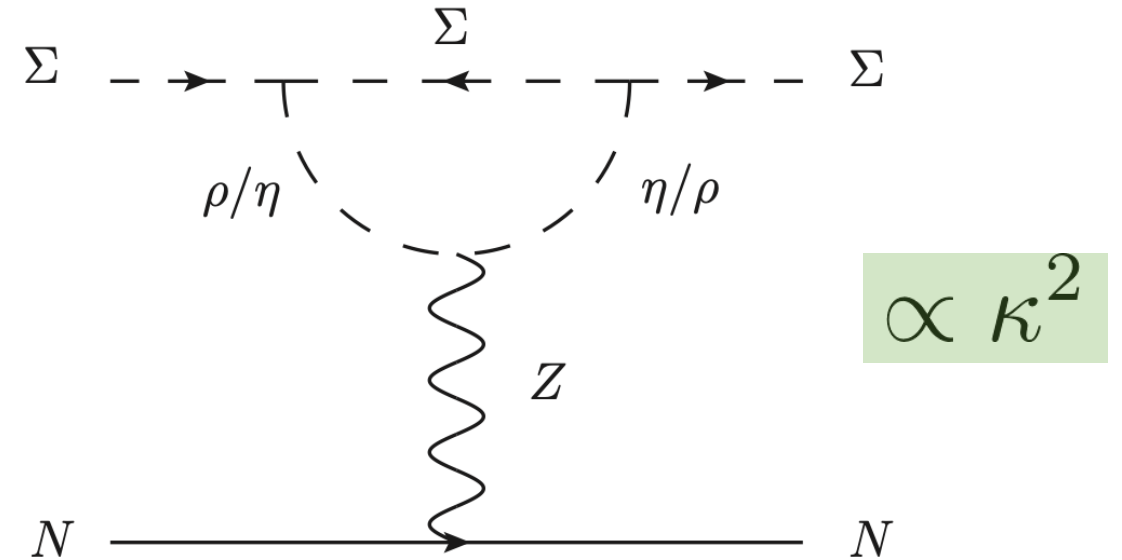
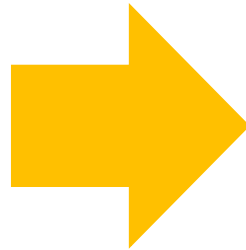
## Direct detection

### Elastic scattering



$N, e$  -----  $N, e$

 Tree-level scattering



 One-loop penguin diagram

$\kappa$  is important for not only relic density but also direct detection

## Direct detection

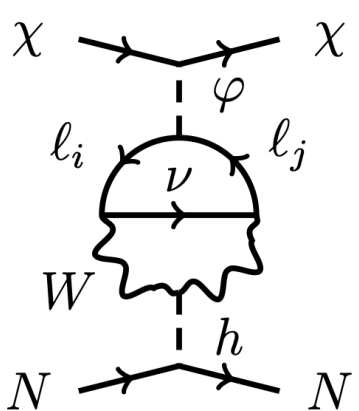
### Previous work

I. Galon, A. Kwa, P. Tanedo, JHEP **03**, 064 (2017)

Lepton flavor off-diagonal mediator in EFT framework

$$\mathcal{L}_\varphi\chi = \frac{1}{2}y_{S\eta}\varphi\bar{\chi}\chi + \frac{i}{2}y_{P\eta}\varphi\bar{\chi}\gamma^5\chi$$

$$\mathcal{L}_{\varphi\text{SM}} = g_{ij}\varphi\bar{l}_i P_L l_j + g_{ji}^*\varphi^*\bar{l}_j P_R l_i$$

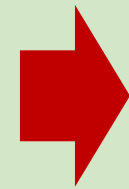
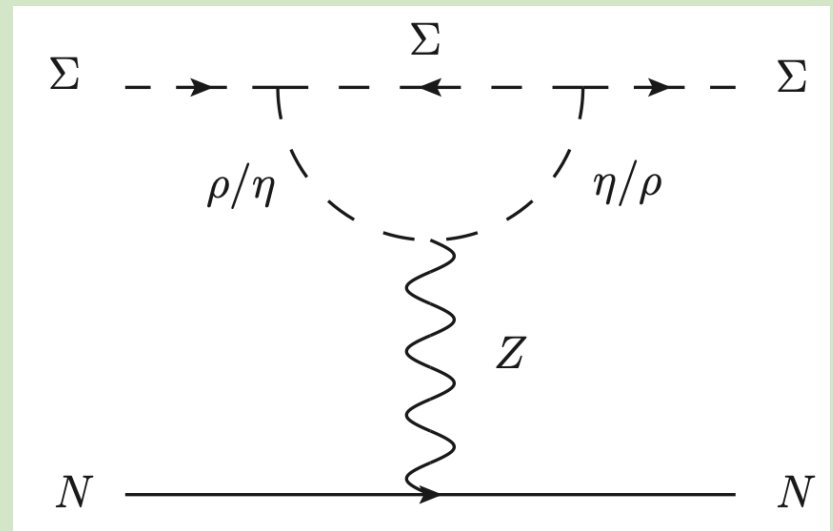


Origin of lepton flavor off-diagonal interaction ?

Impossible direct detection ?

### Our work

UV complete model



Can be detectable

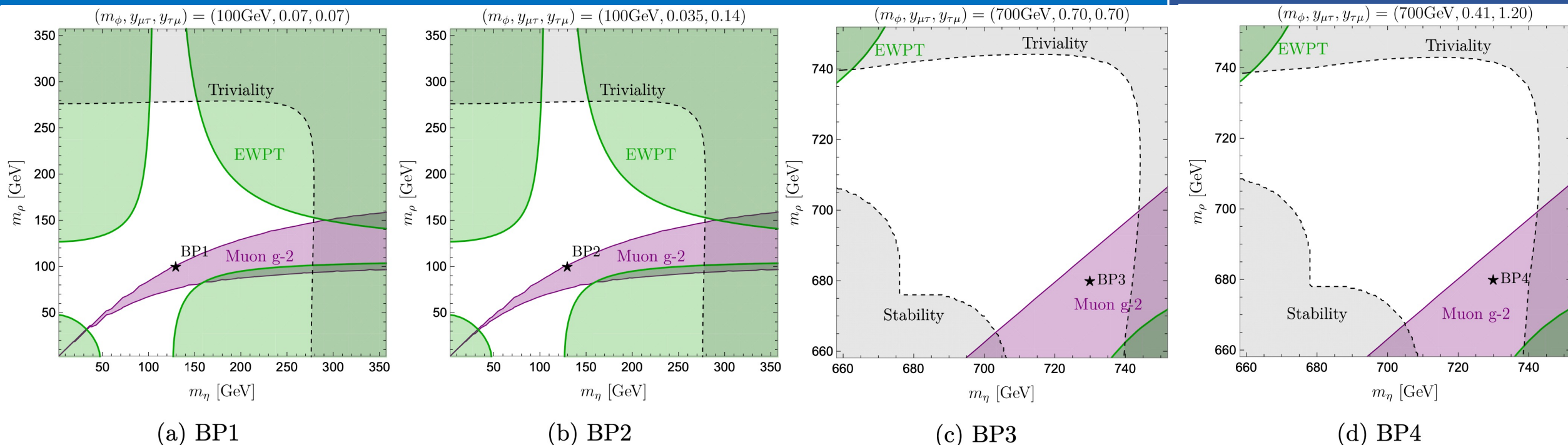


# Result

# Result

## Benchmark point

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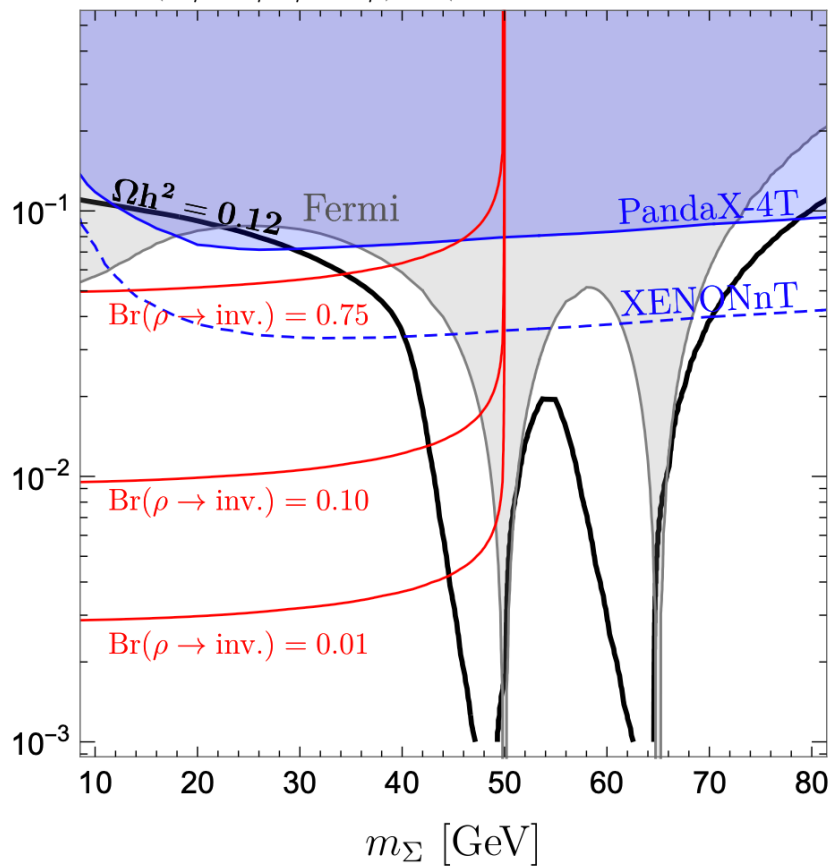
- BP1 :  $(m_{\phi^\pm}, m_\rho, m_\eta) = (100\text{ GeV}, 100\text{ GeV}, 130\text{ GeV}), (y_{\mu\tau}, y_{\tau\mu}) = (0.07, 0.07),$
- BP2 :  $(m_{\phi^\pm}, m_\rho, m_\eta) = (100\text{ GeV}, 100\text{ GeV}, 130\text{ GeV}), (y_{\mu\tau}, y_{\tau\mu}) = (0.035, 0.14),$
- BP3 :  $(m_{\phi^\pm}, m_\rho, m_\eta) = (700\text{ GeV}, 680\text{ GeV}, 730\text{ GeV}), (y_{\mu\tau}, y_{\tau\mu}) = (0.7, 0.7),$
- BP4 :  $(m_{\phi^\pm}, m_\rho, m_\eta) = (700\text{ GeV}, 680\text{ GeV}, 730\text{ GeV}), (y_{\mu\tau}, y_{\tau\mu}) = (0.41, 1.2).$

# Result

## Limit on $\kappa$ (Light DM case)

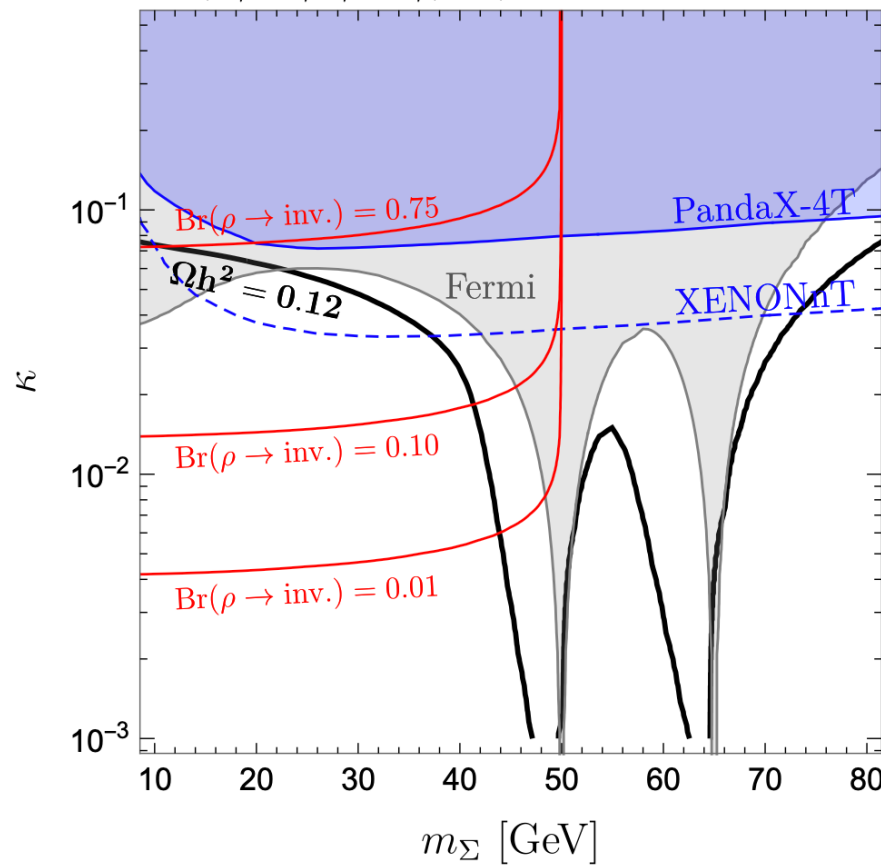
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BP1:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (100\text{GeV}, 130\text{GeV}, 0.07, 0.07)$



(a) BP1

BP2:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (100\text{GeV}, 130\text{GeV}, 0.035, 0.1)$



(b) BP2

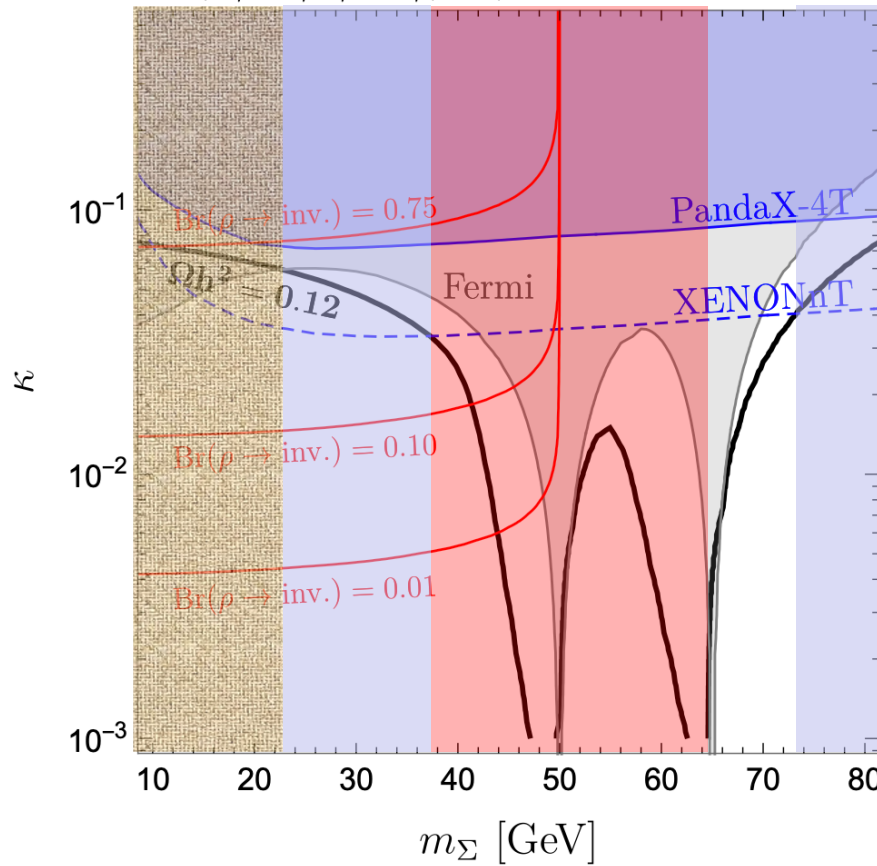
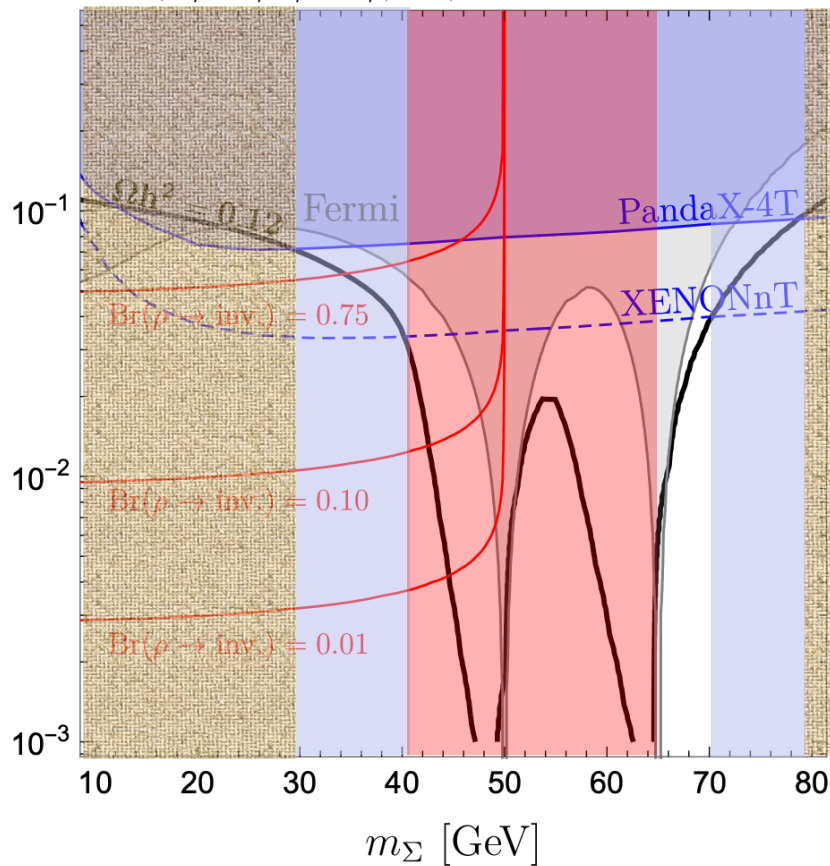
# Result

## Limit on $\kappa$ (Light DM case)

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BP1:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (100\text{GeV}, 130\text{GeV}, 0.07, 0.07)$

BP2:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (100\text{GeV}, 130\text{GeV}, 0.035, 0.1)$



- already excluded
- can explore by XENONnT
- can explore  $\rho$  and  $\eta$  by collider ?

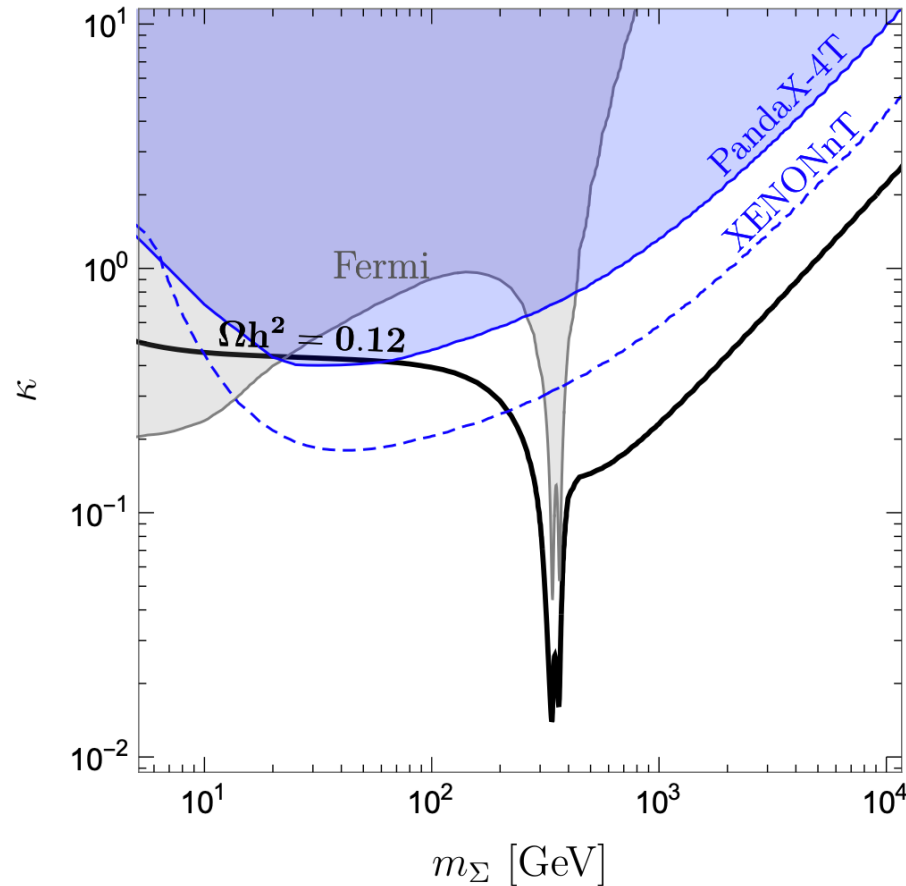
Future direct detection exp. can explore DM except for resonant region

# Result

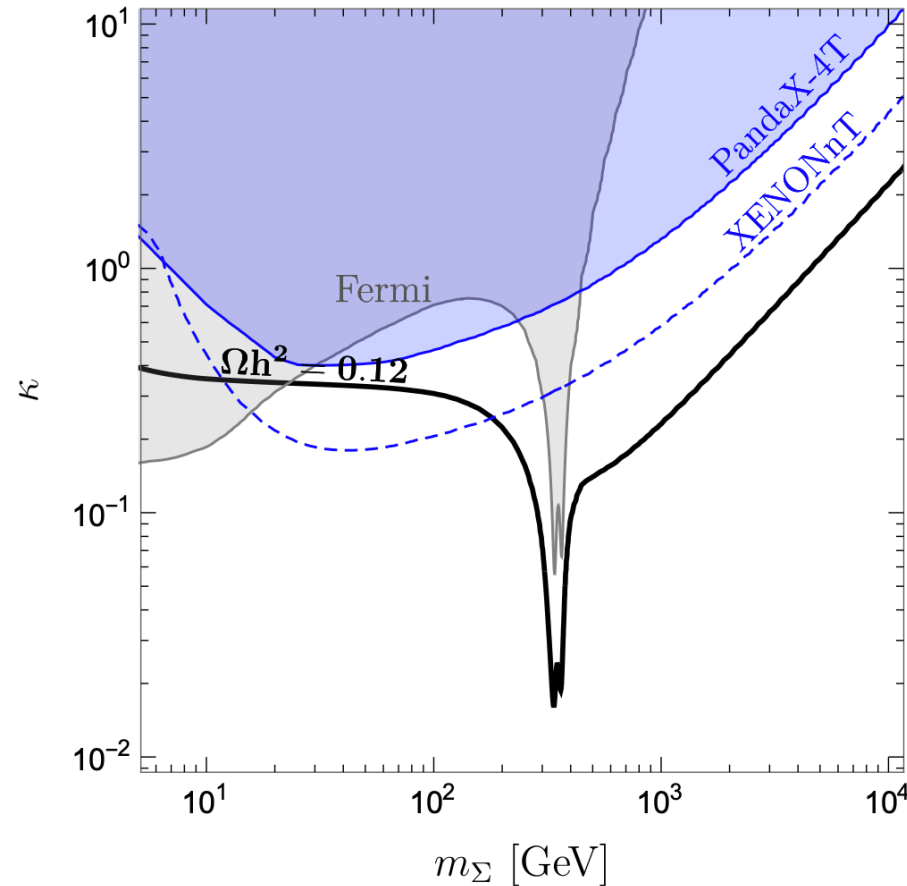
## Limit on $\kappa$ (Heavy DM case)

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BP3:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (680\text{GeV}, 730\text{GeV}, 0.7, 0.7)$



BP4:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (680\text{GeV}, 730\text{GeV}, 0.41, 1.2)$



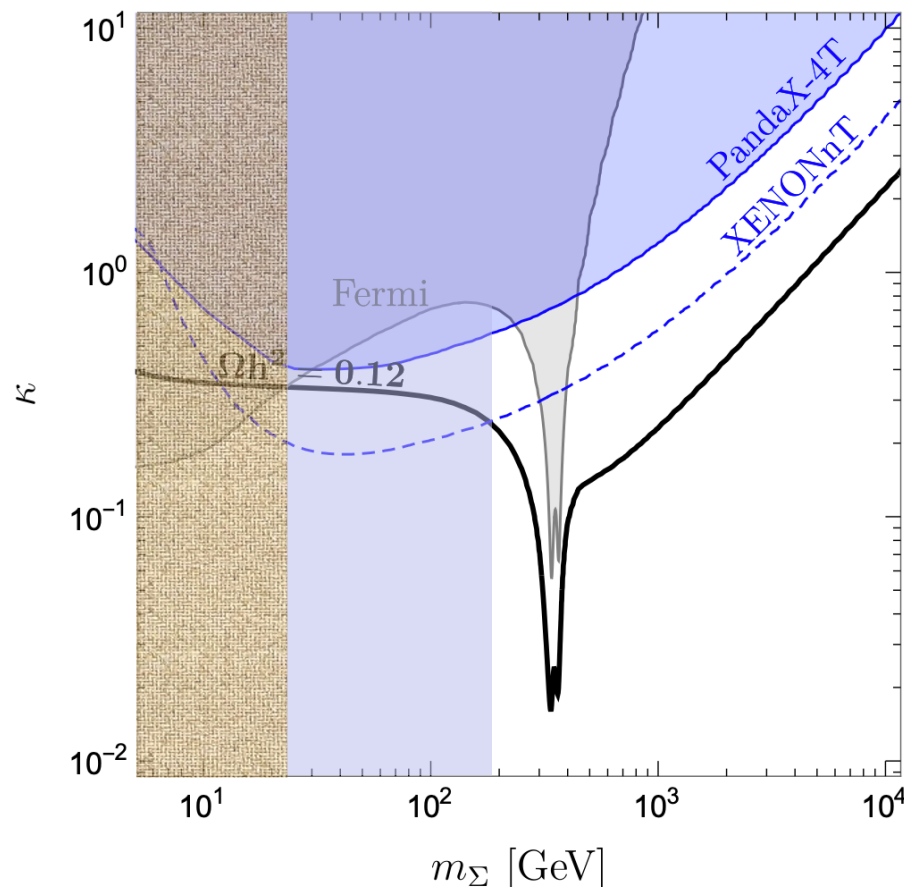
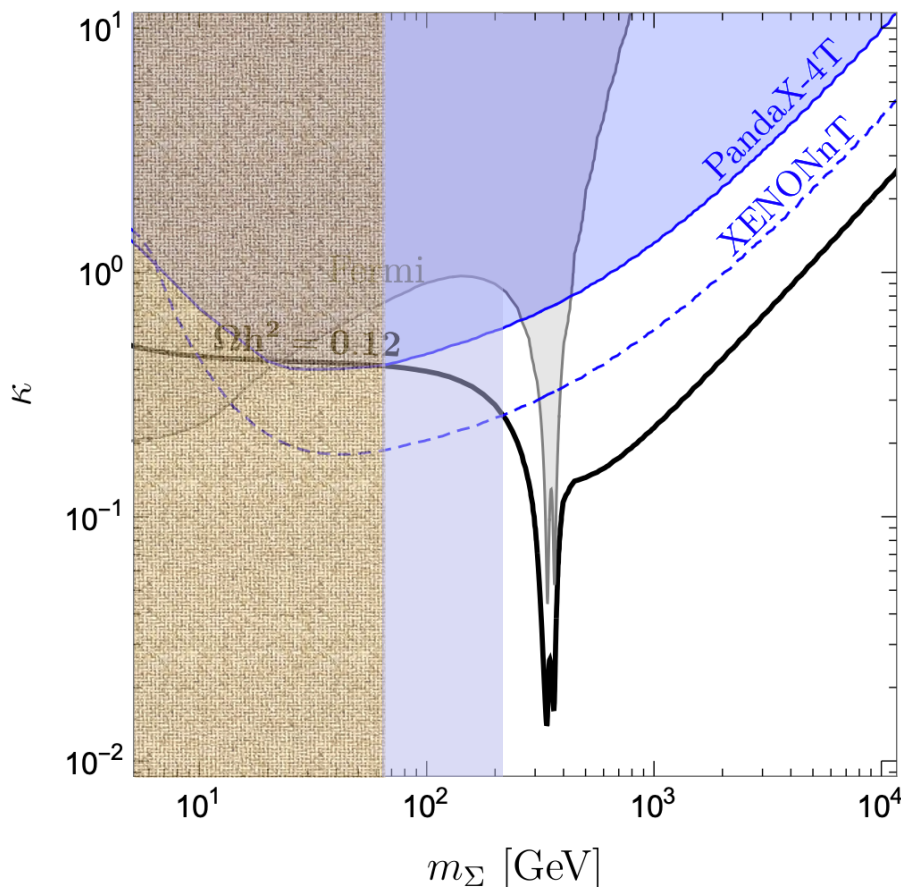
# Result

## Limit on $\kappa$ (Heavy DM case)

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BP3:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (680\text{GeV}, 730\text{GeV}, 0.7, 0.7)$

BP4:  $(m_\rho, m_\eta, y_{\mu\tau}, y_{\tau\mu}) = (680\text{GeV}, 730\text{GeV}, 0.41, 1.2)$



already excluded  
 can explore by XENONnT

DM with  $m_\Sigma \lesssim 200 \text{ GeV}$  can be searched by XENONnT

# Summary

- We've considered **dark matter model** with  $\mu\tau$ -philic scalar mediator.
- $\mu\tau$ -philic mediator has no coupling to electron and quarks at one-loop level
  - ➔  $\mu\tau$ -philic mediator can explain DM relic abundance and muon  $g-2$  simultaneously.
- We've explored the possibility of the DM direct detection through the one-loop process.

# Appendix



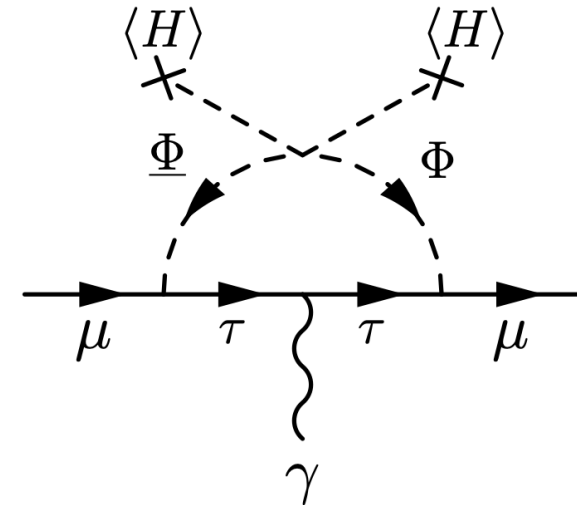
### Contribution from $\mu\tau$ -philic scalar

Y. Abe, T. Toma, K. Tsumura, JHEP **06**, 142 (2019)

$$\Delta a_{\mu}^{\text{new}} = \frac{\text{Re}(y_{\mu\tau}y_{\tau\mu})}{(4\pi)^2} \left[ \frac{m_{\mu}m_{\tau}}{m_{\rho}^2} I_1(m_{\mu}^2/m_{\rho}^2, m_{\tau}^2/m_{\rho}^2) - \frac{m_{\mu}m_{\tau}}{m_{\eta}^2} I_1(m_{\mu}^2/m_{\eta}^2, m_{\tau}^2/m_{\eta}^2) \right] \\ + \frac{|y_{\mu\tau}|^2 + |y_{\tau\mu}|^2}{2(4\pi)^2} \left[ \frac{m_{\mu}^2}{m_{\rho}^2} I_2(m_{\mu}^2/m_{\rho}^2, m_{\tau}^2/m_{\rho}^2) + \frac{m_{\mu}^2}{m_{\eta}^2} I_2(m_{\mu}^2/m_{\eta}^2, m_{\tau}^2/m_{\eta}^2) \right]$$

with

$$I_1(\alpha, \beta) \equiv \int_0^1 dx \frac{(1-x)^2}{x - x(1-x)\alpha + (1-x)\beta}, \\ I_2(\alpha, \beta) \equiv \frac{1}{2} \int_0^1 dx \frac{x(1-x)^2}{x - x(1-x)\alpha + (1-x)\beta}$$




## Scalar masses

### Scalar potential

$$V(H, \Phi, \Sigma) = \mu_{\Phi}^2 |\Phi|^2 + \lambda_2 |\Phi|^4 + \lambda_3 |H|^2 |\Phi|^2 + \lambda_4 |H^\dagger \Phi|^2 + \frac{\lambda_5}{2} [(H^\dagger \Phi)^2 + \text{H.c.}] \\ + \mu_{\Sigma}^2 |\Sigma|^2 + \lambda_{\Sigma} |\Sigma|^4 + [\lambda'_{\Sigma} \Sigma^4 + \text{H.c.}] + \cancel{\lambda_{H\Sigma} |H|^2 |\Sigma|^2} + \lambda_{\Phi\Sigma} |\Phi|^2 |\Sigma|^2 \\ + \kappa [(H^\dagger \Phi) \Sigma^2 + \text{H.c.}] ,$$

$\Phi$  and  $\Sigma$  have no VEV

### Scalar mass


$$m_{\phi^\pm}^2 = \mu_{\Phi}^2 + \frac{1}{2} \lambda_3 v^2 , \quad m_{\eta}^2 = \mu_{\Phi}^2 + \frac{1}{2} (\lambda_3 + \lambda_4 - \lambda_5) v^2 , \\ m_{\rho}^2 = \mu_{\Phi}^2 + \frac{1}{2} (\lambda_3 + \lambda_4 + \lambda_5) v^2 , \quad m_{\Sigma}^2 = \mu_{\Sigma}^2 + \frac{1}{2} \lambda_{H\Sigma} v^2 .$$

## Annihilation cross section

### 1, $\Sigma \Sigma^*$ annihilation

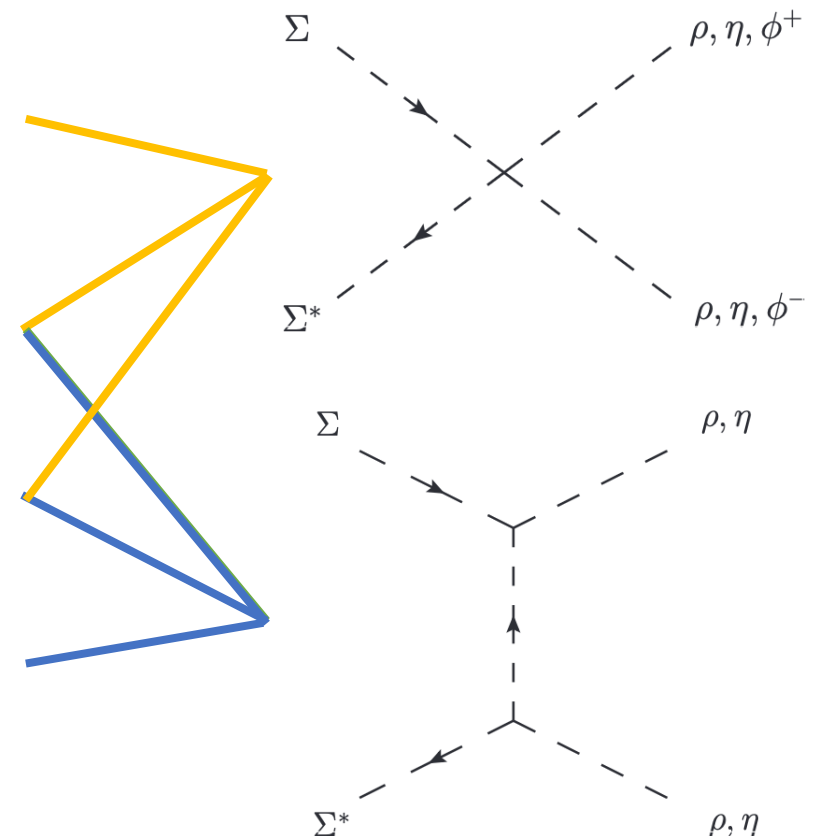
$$(\sigma v_{\text{rel}})_{\Sigma \Sigma^* \rightarrow \phi^+ \phi^-} = \frac{\lambda_{\Phi \Sigma}^2}{32\pi m_{\Sigma}^2} \sqrt{1 - \frac{m_{\phi}^2}{m_{\Sigma}^2}},$$

$$(\sigma v_{\text{rel}})_{\Sigma \Sigma^* \rightarrow \rho \rho} = \frac{1}{64\pi m_{\Sigma}^2} \left( \frac{2\kappa^2 v^2}{2m_{\Sigma}^2 - m_{\rho}^2} - \lambda_{\Phi \Sigma} \right)^2 \sqrt{1 - \frac{m_{\rho}^2}{m_{\Sigma}^2}},$$

$$(\sigma v_{\text{rel}})_{\Sigma \Sigma^* \rightarrow \eta \eta} = (\sigma v_{\text{rel}})_{\Sigma \Sigma^* \rightarrow \rho \rho} \Big|_{m_{\rho} \rightarrow m_{\eta}},$$

$$(\sigma v_{\text{rel}})_{\Sigma \Sigma^* \rightarrow \rho \eta} = \frac{2v_{\text{rel}}^2}{3\pi} \frac{(\kappa v)^4 m_{\Sigma}^2}{(4m_{\Sigma}^2 - m_{\rho}^2 - m_{\eta}^2)^4} \beta_{\rho \eta}^3$$

with  $\beta_{ij} = \sqrt{1 - \frac{m_i^2 + m_j^2}{2m_{\Sigma}^2} + \frac{(m_i^2 - m_j^2)^2}{(4m_{\Sigma}^2)^2}}$



# Appendix

## Annihilation cross section

### 2, $\Sigma\Sigma$ annihilation

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \rho h} = \frac{\kappa^2}{32\pi m_\Sigma^2} \left( 1 + \frac{v^2(\lambda_3 + \lambda_4 + \lambda_5)}{4m_\Sigma^2 - m_\rho^2} \right)^2 \beta_{\rho h},$$

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \rho Z} = \frac{\kappa^2}{32\pi m_\Sigma^2} \left( \frac{4m_\Sigma^2}{4m_\Sigma^2 - m_\eta^2} \right)^2 \beta_{\rho Z},$$

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \eta h, \eta Z} = (\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \rho h, \rho Z} \Bigg|_{\substack{m_\rho \leftrightarrow m_\eta \\ \lambda_5 \rightarrow -\lambda_5}},$$

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \phi^- W^+} = \frac{\kappa^2}{32\pi m_\Sigma^2} \left( \frac{4m_\Sigma^2}{4m_\Sigma^2 - m_\rho^2} + \frac{4m_\Sigma^2}{4m_\Sigma^2 - m_\eta^2} \right)^2 \beta_{\phi W^+},$$

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \phi^+ W^-} = \frac{\kappa^2}{32\pi m_\Sigma^2} \left( \frac{4m_\Sigma^2}{4m_\Sigma^2 - m_\rho^2} - \frac{4m_\Sigma^2}{4m_\Sigma^2 - m_\eta^2} \right)^2 \beta_{\phi W^-},$$

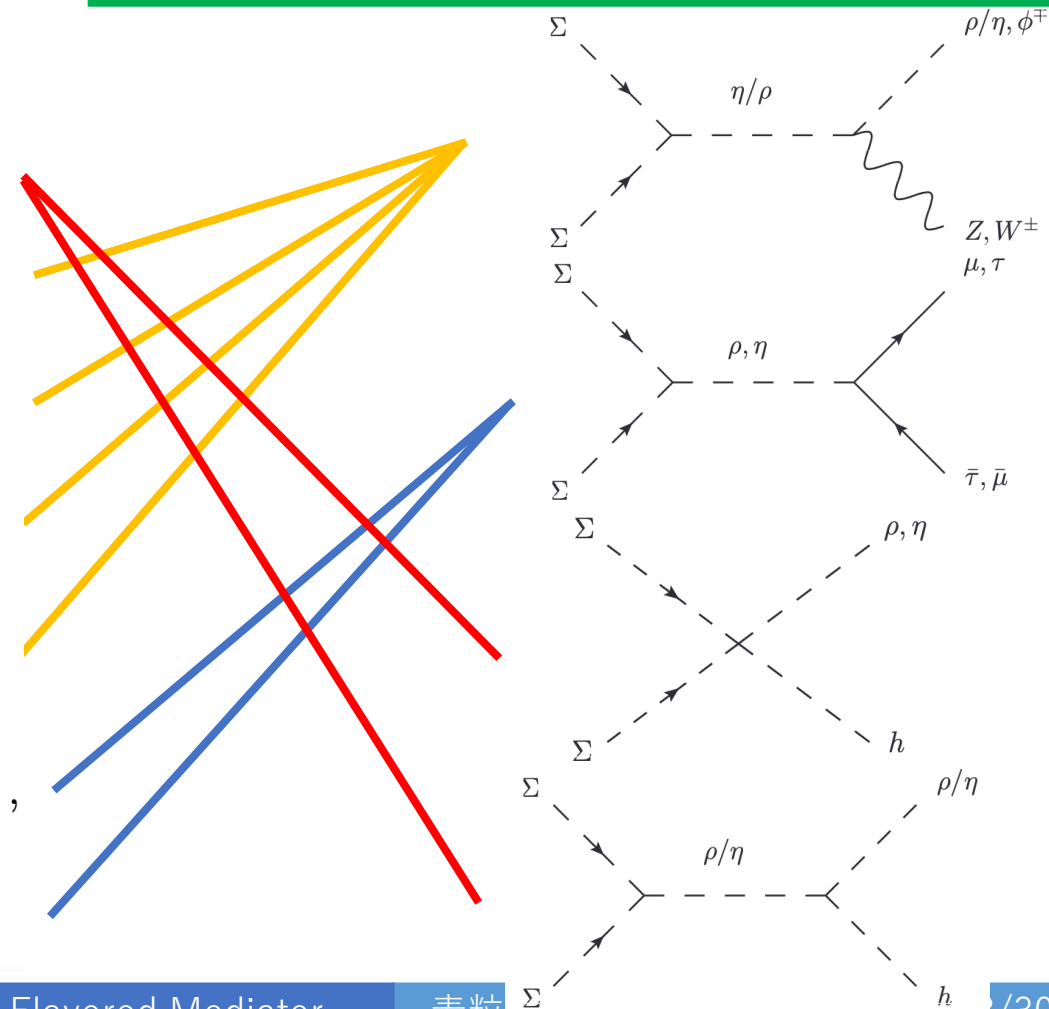
$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \mu\bar{\tau}} = \frac{\kappa^2 v^2}{64\pi m_\Sigma^2} [a |y_{\mu\tau}|^2 + b |y_{\tau\mu}|^2 + c \text{Re}(y_{\mu\tau} y_{\tau\mu})] \beta_{\mu\tau},$$

$$(\sigma v_{\text{rel}})_{\Sigma\Sigma \rightarrow \tau\bar{\mu}} = \frac{\kappa^2 v^2}{64\pi m_\Sigma^2} [b |y_{\mu\tau}|^2 + a |y_{\tau\mu}|^2 + c \text{Re}(y_{\mu\tau} y_{\tau\mu})] \beta_{\mu\tau}$$

$$a = (4m_\Sigma^2 - m_\mu^2 - m_\tau^2) \left( \frac{1}{4m_\Sigma^2 - m_\rho^2} - \frac{1}{4m_\Sigma^2 - m_\eta^2} \right)^2,$$

$$b = (4m_\Sigma^2 - m_\mu^2 - m_\tau^2) \left( \frac{1}{4m_\Sigma^2 - m_\rho^2} + \frac{1}{4m_\Sigma^2 - m_\eta^2} \right)^2,$$

$$c = -4m_\mu m_\tau \left[ \left( \frac{1}{4m_\Sigma^2 - m_\rho^2} \right)^2 - \left( \frac{1}{4m_\Sigma^2 - m_\eta^2} \right)^2 \right].$$



### Relevant interaction Lagrangian

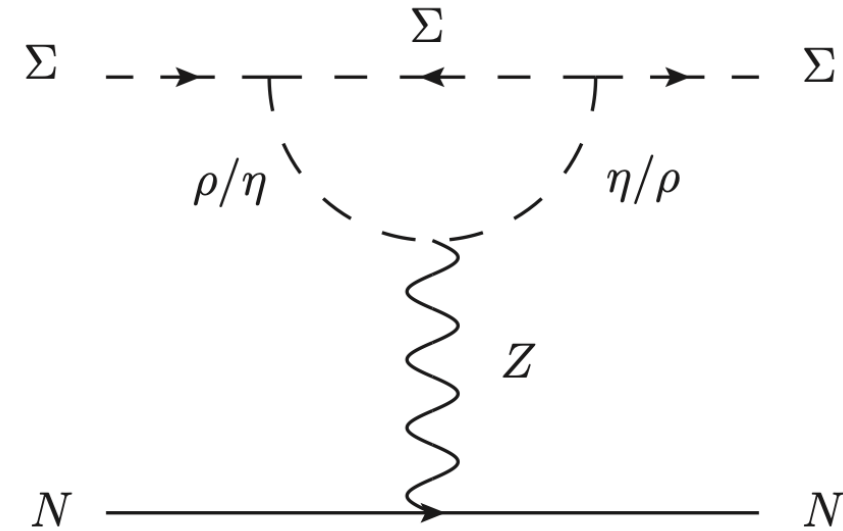
$$\mathcal{L}_{\Sigma q}^{\text{eff}} = \sum_{q=u,d,s} C_{V,q} \{ \Sigma^* i \partial_\mu \Sigma - (i \partial_\mu \Sigma^*) \Sigma \} (\bar{q} \gamma^\mu q)$$

with

$$C_{V,q} = a_Z \frac{1}{m_Z^2} \frac{g}{2 \cos \theta_W} (T_3 - 2Q_q \sin^2 \theta_W),$$

$$a_Z = \frac{(\kappa v)^2}{(4\pi)^2} \frac{g}{2 \cos \theta_W} \times \frac{1}{m_\rho^2 - m_\eta^2} [f(m_\rho/m_\Sigma) - f(m_\eta/m_\Sigma)]$$

$$f(x) = x^2 + x^2(2 - x^2) \log x + x^3 \sqrt{x^2 - 4} \log \left( \frac{x + \sqrt{x^2 - 4}}{2} \right)$$



## Direct detection

### Spin-independent elastic scattering

$$\sigma_{\text{SI}} = \frac{\mu_N^2 [ZC_{V,p} + (A - Z)C_{V,n}]^2}{\pi}$$

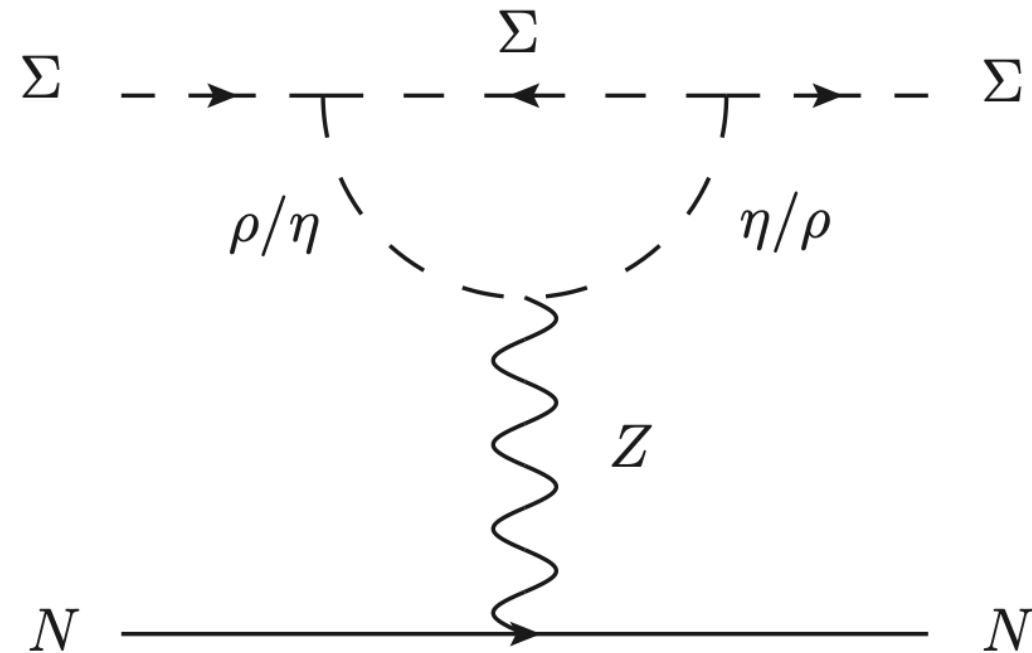
with

$$\mu_N = m_\Sigma m_N / (m_\Sigma + m_N)$$

$$C_{V,p} = 2C_{V,u} + C_{V,d}, \quad C_{V,n} = C_{V,u} + 2C_{V,d}$$

$Z(A)$  : atomic (mass) number

$m_N$  : target nucleus mass



## Neutrino mass & mixing

3 right-handed neutrinos

➔ Neutrino mass & its lightness through seesaw mechanism

But ...

Exact  $Z_4$  flavor symmetry

➔ Observed neutrino mixing cannot be obtained

$$\mathcal{L}_N = -\frac{1}{2} \begin{pmatrix} \overline{N_e^c} & \overline{N_\mu^c} & \overline{N_\tau^c} \end{pmatrix} \begin{pmatrix} M_{ee} & & \\ & M_{\mu\tau} & \\ & & M_{\mu\tau} \end{pmatrix} \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} - \begin{pmatrix} \overline{L_e} & \overline{L_\mu} & \overline{L_\tau} \end{pmatrix} \begin{pmatrix} y_{ee}\tilde{H} & & \\ & y_{\mu\mu}\tilde{H} & y_{\mu\tau}\tilde{\Phi} \\ & y_{\tau\mu}\tilde{\Phi} & y_{\tau\tau}\tilde{H} \end{pmatrix} \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} + \text{H.c.}$$

## $Z_4 \times Z_2$ symmetric model

### Fields

- SM fields +  $\Phi$  :  $\mu\tau$ -philic doublet scalar
- +  $\Sigma$  : complex scalar DM
- +  $N_\alpha$  : right-handed neutrino
- +  $S$  :  $Z_4$ -breaking singlet scalar

particles	$(L_e, L_\mu, L_\tau)$	$(e_R, \mu_R, \tau_R)$	$H$	$\Phi$	$\Sigma$	$(N_e, N_\mu, N_\tau)$	$S$
SM	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{1}, \mathbf{1})_0$
$Z_4$	$(1, i, -i)$	$(1, i, -i)$	1	-1	$i$	$(1, i, -i)$	$i$
$Z_2$	+	+	+	+	-	+	+



# Appendix

## $Z_4 \times Z_2$ symmetric model


### Lagrangian

$$\mathcal{L}_N = -\frac{1}{2} \begin{pmatrix} \overline{N_e^c} & \overline{N_\mu^c} & \overline{N_\tau^c} \end{pmatrix} \begin{pmatrix} M_{ee} & \lambda_{e\mu} S^* & \lambda_{e\tau} S \\ \lambda_{e\mu} S^* & & M_{\mu\tau} \\ \lambda_{e\tau} S & M_{\mu\tau} & \end{pmatrix} \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} - \begin{pmatrix} \overline{L_e} & \overline{L_\mu} & \overline{L_\tau} \end{pmatrix} \begin{pmatrix} y_{ee} \tilde{H} \\ y_{\mu\mu} \tilde{H} & y_{\mu\tau} \tilde{\Phi} \\ y_{\tau\mu} \tilde{\Phi} & y_{\tau\tau} \tilde{H} \end{pmatrix} \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} + \text{H.c.}$$

  Neutrino mass & mixing

Neutrino mass matrix has  
two-zero minor structure

$$\mathcal{M}_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

 Detail in KA, K. Hamaguchi, N. Nagata, S.-Y. Tseng, K. Tsumura, PRD 99, 5, 055029 (2019)

particles	$(L_e, L_\mu, L_\tau)$	$(e_R, \mu_R, \tau_R)$	$H$	$\Phi$	$\Sigma$	$(N_e, N_\mu, N_\tau)$	$S$
SM	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{-1}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{1}, \mathbf{1})_0$	$(\mathbf{1}, \mathbf{1})_0$
$Z_4$	$(1, i, -i)$	$(1, i, -i)$	1	-1	$i$	$(1, i, -i)$	$i$
$Z_2$	+	+	+	+	-	+	+