Evolution of lepton number for neutrinos with wave packet-like effects

基研研究会　素粒子物理学の進展２０２２

Nicholas J. Benoit†, A. S. Adam‡, Y. Kawamura, Y. Matsuo, T. Morozumi†, Y. Shimizu†, and N. Toyota
†Graduate School of Science, Hiroshima University; ‡Graduate School of Advanced Science and Engineering, Hiroshima University;
‡Research Center for Quantum Physics, National Research and Innovation Agency (BRIN)

Introduction

We present a novel formulation, based on quantum field theory, for neutrino oscillation phenomenology that can be applied to nonrelativistic and relativistic energies for neutrinos. The formulation is constructed as the time evolution of a lepton family number density operator. We also introduce a Gaussian momentum distribution for the initial state that incorporates wave packet-like decoherence effects. At time t=0, we assign either a Dirac or Majorana mass to the neutrino. Then, the time evolution of the lepton family number density operator becomes dependent on the mass and new features appear. We show in the nonrelativistic regime, by taking the expectation value of the density operator, the type of neutrino mass is distinguishable even under the presence of decoherence effects.

How does the density of the lepton family number evolve when neutrinos have a Dirac or Majorana mass?

Lepton Number

» Three active neutrinos sometimes called flavors or families exsist as part of the Standard Model [LEP (2006)]
» Lepton family number is a global symmetry of the Standard Model

Electron family number $L_e = n_e - n_{\bar{e}}$

Muon family number $L_\mu = n_\mu - n_{\bar{\mu}}$

Tauon family number $L_\tau = n_\tau - n_{\bar{\tau}}$

Broken symmetry due to massive neutrinos [S. M. Bilenky and C. Giunti (2001)]

» Total lepton number is a sum over the families and it also conserved in the Standard Model

Total lepton number $\sum_\alpha L_\alpha = L_e + L_\mu + L_\tau$

Broken if neutrinos have Majorana mass $\left(\frac{m_{\alpha\beta}}{2}\overline{(\nu_{L\alpha})^C}\nu_{L\beta} + \text{h.c}\right)$

Conserved for Dirac mass $(\overline{\nu_{R\alpha}} m_{\alpha\beta}\nu_{L\beta} + \text{h.c.})$

» We define a lepton family number from a *SU(2)$_L$* doublet based on the charged weak interaction

Dirac mass case: $L^D_\alpha(t) = \int d^3x\, [l^L_\alpha(t,\mathbf{x}) + l^R_\alpha(t,\mathbf{x})] = \int d^3x\, [:\overline{\nu_{L\alpha}}(t,\mathbf{x})\gamma^0\nu_{L\alpha}(t,\mathbf{x}): + :\overline{\nu_{R\alpha}}(t,\mathbf{x})\gamma^0\nu_{R\alpha}(t,\mathbf{x}):]$

Majorana mass case: $L^M_\alpha(t) = \int d^3x\, l^M_\alpha(t,\mathbf{x}) = \int d^3x\; :\overline{\nu_{L\alpha}}(t,\mathbf{x})\gamma^0\nu_{L\alpha}(t,\mathbf{x}):$

Expectation Value

» For an observable quanity we take the expectation values of the densities

Initial Gaussian distribution in 1 dimension

$$|\psi^{(L)}_\sigma(q^0;\sigma_q)\rangle = \frac{1}{\sqrt{\sigma_q}(2\pi)^{3/4}\delta(0)}\int' \frac{dq}{2\pi\sqrt{2|q|}} e^{-\frac{(q-q^0)^2}{4\sigma_q^2}} a^\dagger_{(L)\sigma}(0,q,0)|0\rangle;$$

» The result is observable evolutions for the densities with wave packet-like decoherence

Dirac expectation value: $\lambda^D_{\sigma\to\alpha}(t,x_2) = \iint dx_1 dx_3 \langle\psi^L_\sigma(q^0;\sigma_q)|l^L_\alpha(t,\mathbf{x}) + l^R_\alpha(t,\mathbf{x})|\psi^L_\sigma(q^0;\sigma_q)\rangle$

Majorana expectation value: $\lambda^M_{\sigma\to\alpha}(t,x_2) = \iint dx_1 dx_3 \langle\psi_\sigma(q^0;\sigma_q)|l^M_\alpha(t,\mathbf{x})|\psi_\sigma(q^0;\sigma_q)\rangle$

Wave packet-like effects

» The decoherence of the oscilliations occur due to differences in the velocities of the mass eigenstate Gaussian distributions

Real exponentials damp the oscillations

$$e^{-\sigma_q^2[(x_2\pm v_{i0}t)^2+(x_2\pm v_{j0}t)^2]}$$

$$e^{-\sigma_q^2[(x_2\pm v_{i0}t)^2+(x_2\mp v_{j0}t)^2]}$$

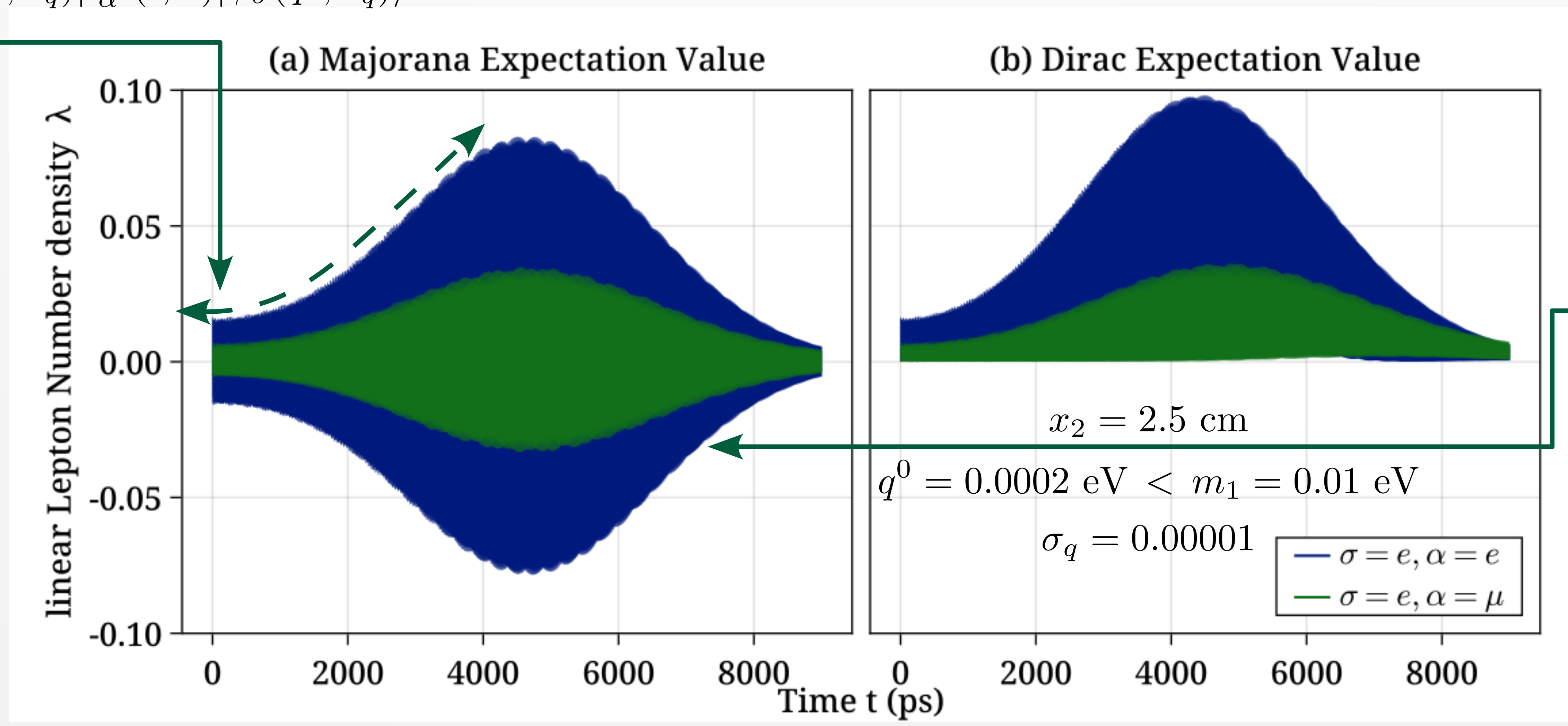
» Behaivor is the similar as wave packet decoherence in quantum mechanics
[C. Giunti, C. W. Kim and U. W. Lee (2001)]

Dirac Vs. Majorana

» The expectation values have different evolutions depending on the mass type
» Easy to see for the total lepton number

Dirac mass case:
$$\sum_\alpha \lambda^D_{\sigma\to\alpha}(t,x_2) \simeq \frac{\sigma_q}{\sqrt{2\pi}}\sum_i |V_{\alpha i}|^2\left[(1+v_{i0})e^{-2\sigma_q^2(x_2-v_{i0}t)^2} + (1-v_{i0})e^{-2\sigma_q^2(x_2+v_{i0}t)^2}\right]$$

Majorana mass case:
$$\sum_\alpha \lambda^M_{\sigma\to\alpha}(t,x_2) \simeq \frac{\sigma_q}{\sqrt{2\pi}}\sum_i |V_{\sigma i}|^2\left[v_{i0}(1+v_{i0})e^{-2\sigma_q^2(x_2-v_{i0}t)^2} - v_{i0}(1-v_{i0})e^{-2\sigma_q^2(x_2+v_{i0}t)^2} + 2(1-v_{i0}^2)e^{-2\sigma_q^2(v_{i0}^2t^2+x_2^2)}\cos 2E_i(q^0)t\right]$$



Majorana mass case breaks total lepton number
» The breaking of lepton number depends on time
» This appears in the time evolution of the density as negative values

The Dirac mass case conserves lepton number, so it is always postive definite

Always true even with decoherence effects

Contact:
Nicholas J. Benoit ⁜ d19506@hiroshima-u.ac.jp

References:
※ A. S. Adam, etc.; *PTEP*, ptab025, arXiv:2101.07751 [hep-ph]
https://doi.org/10.1093/ptep/ptab025
※ A. S. Adam, etc.; arXiv:2106.02783v2 [hep-ph]