Improved indirect limits on muon EDM

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Based on 2108.05398 and 2207.01679 with T. Gao and M. Pospelov

Electric dipole moment

- Electric dipole moment of a particle is proportional to spin:

$$\mathcal{H} = -\overrightarrow{B}\cdot\overrightarrow{\mu} - \overrightarrow{E}\cdot\overrightarrow{d} = -2\vec{s}\cdot\left(\mu\overrightarrow{B} + d\overrightarrow{E}\right).$$

* μ : magnetic dipole moment, d :electric dipole moment.

➡ EDM violates P and T (or CP).

	$\overrightarrow{B}$	$\overrightarrow{E}$	$\vec{s}$
P	$+$	$-$	$+$
T	$-$	$+$	$-$

- Flavor diagonal: standard model contribution extremely suppressed.

e.g. $d_e^{(\text{equiv})}(\delta_{\text{CKM}}) \simeq 10^{-35}\, e\,\text{cm}\,.$ [YE, Gao, Pospelov 22]

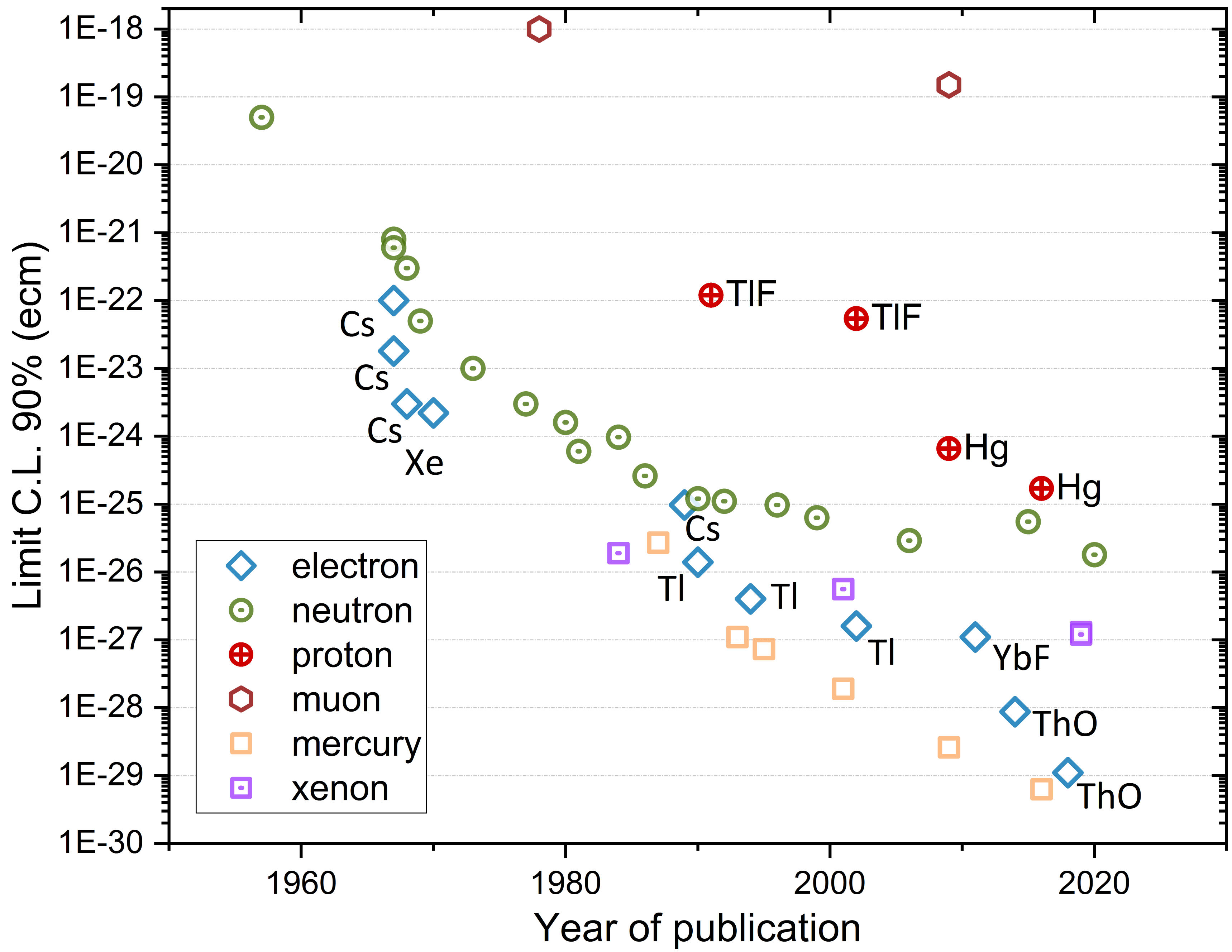
➡ Background free probe of CP-odd new physics.

- CP violation motivated by baryogenesis, BSM such as 2HDM, SUSY, ...

EDM experiments

Many efforts on detecting EDM in different systems

[Taken from Kirch & Schmidt-Wellenburg 20]



Today's goal: understand muon EDM contributions to atomic experiments.

Muon EDM

- Recently FNAL confirms BNL muon g-2 result:

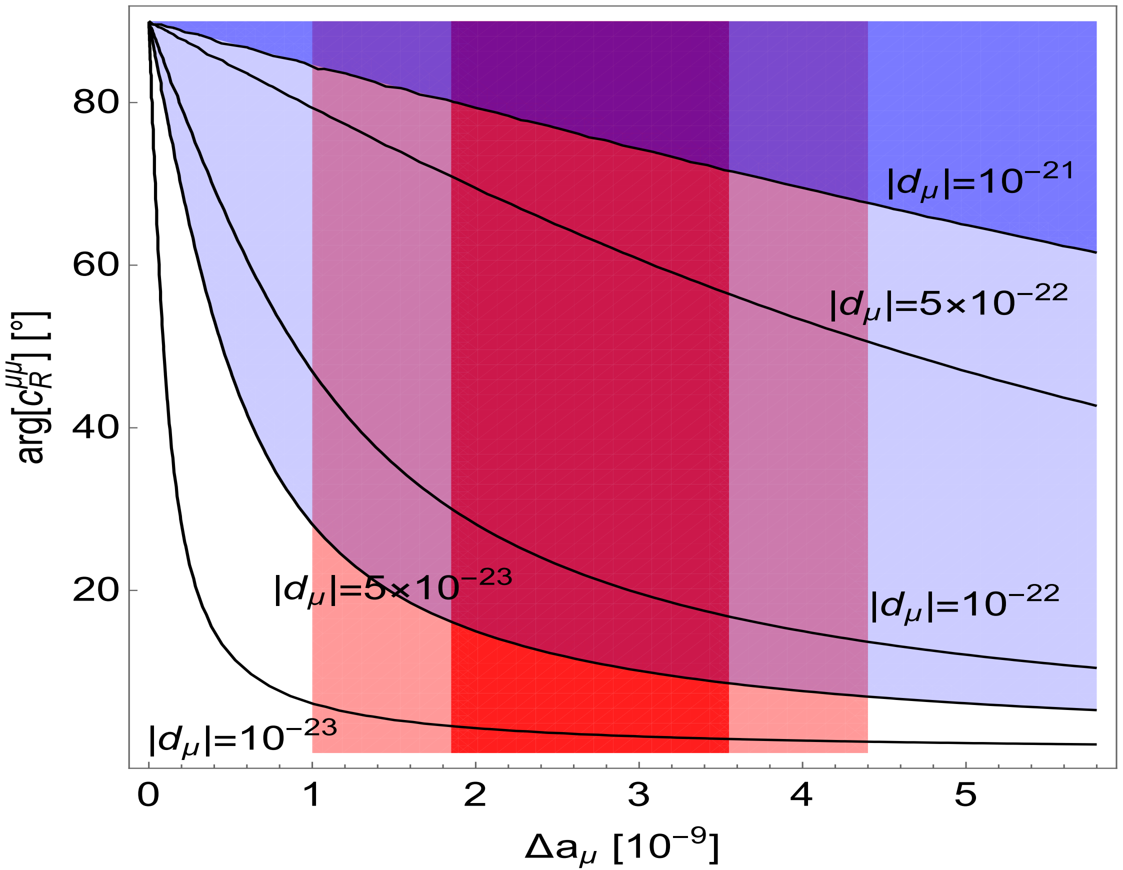
$$a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = \left(251 \pm 59\right) \times 10^{-11} \quad (4.2\sigma)$$ [FNAL muon g-2 21]

- Muon g-2 and EDM can be closely related:

$$\mathscr{L} = -\frac{c}{2}\bar{\psi}_R \sigma \cdot F \psi_L + \mathrm{h.c.} \quad \Rightarrow \quad \mathrm{Re}[c] = \frac{e a_\mu}{2m}, \quad \mathrm{Im}[c] = d_\mu .$$

- $\mathcal{O}(1)$ phase directly probed in near future.

➡ understand indirect limits from atomic/molecular EDM experiments.



[Figure taken from Crivellin et.al.18]

Outline

1. Introduction

2. Review on atomic EDM

3. Indirect limits on muon EDM

4. Summary

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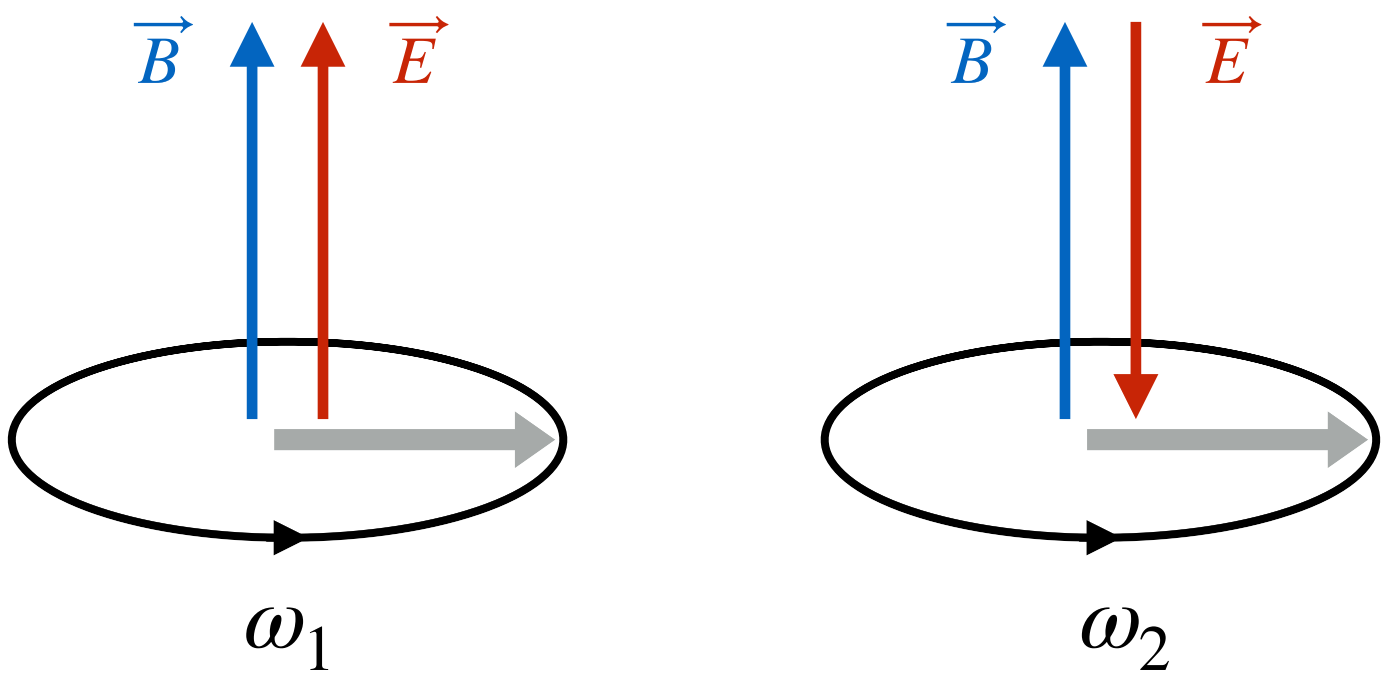
Spin precession

- Atomic EDM observable: spin precession (as for other cases)

$$\frac{d\vec{s}}{dt} = \vec{\omega} \times \vec{s}, \quad \vec{\omega} = 2\mu \vec{B} + 2d\vec{E}\,.$$

- Extract EDM by flipping $\vec{E}$:



$$\Delta\omega = \omega_1 - \omega_2 = 4dE\,.$$

- Atomic EDM actually has complications (screening → next slides).

Shielding theorem

- (Non-relativistic) atomic Hamiltonian with external $\vec{E}$ and EDM:

$$\mathscr{H}_A = \mathscr{H}_N + \mathscr{H}_e + \Phi - \sum_k \left(e_k \vec{r}_k \cdot \vec{E}_{\rm ext} + \vec{d}_k \cdot \vec{E}(\vec{r}_k) \right),$$

where Φ : coulomb potential btw particles and $\vec{E} = \vec{E}_{\rm int} + \vec{E}_{\rm ext}$.

* $\vec{E}_{\rm int}(\vec{r}_k) = -\frac{\vec{\nabla}_k}{e_k}\Phi = -\frac{i}{e_k}\left[\vec{p}_k, \mathscr{H}_0\right]$ where $\mathscr{H}_0 = \mathscr{H}_N + \mathscr{H}_e + \Phi$.

- EDM without $\vec{E}_{\rm ext}$ induces mixing of (unperturbed) states as

$$|\Psi\rangle \simeq |0\rangle - \sum_{n\neq 0} \frac{\langle n | \sum_k \vec{d}_k \cdot \vec{E}_{\rm int}(\vec{r}_k) | 0 \rangle}{E_0 - E_n} |n\rangle = \left(1 + \sum_k \frac{i}{e_k} \vec{d}_k \cdot \vec{p}_k \right) |0\rangle .$$

- This cancels the direct contribution to the atomic EDM:

$$\vec{d}_A = \langle \Psi | \sum_k \left(\vec{d}_k + e_k \vec{r}_k \right) | \Psi \rangle \simeq \sum_k \langle 0 | \left(\vec{d}_k - \sum_l \frac{i e_l}{e_k} \left[\vec{d}_k \cdot \vec{p}_k, \vec{r}_l \right] \right) | 0 \rangle = 0,$$

"Schiff shielding theorem"

[Purcell, Ramsey 50; Garwin, Lederman 59; Schiff 63]

Shielding theorem

- Two contributions to atomic EDM:

  (1) direct contribution from the constituent particle's EDM

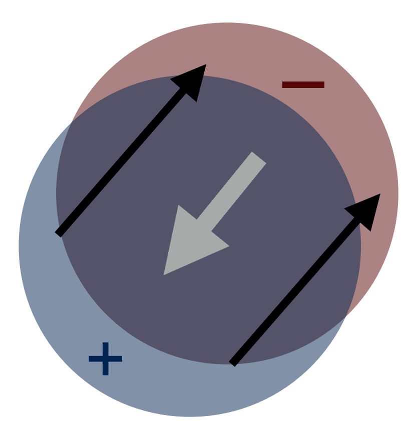
  (2) mixing of opposite parity wave functions through P,CP-odd interaction

  ↖ doesn't have to be EDM

$$\vec{d}_A \ni 2 \sum_{n \neq 0} \frac{\langle 0 | \sum_k e\vec{r}_k | n \rangle \langle n | \mathscr{H}_{\rm int} | 0 \rangle}{E_n - E_0} .$$

➡ these two cancel for non-relativistic point particle's EDM.

- This is a rearrangement due to the constitutions.



- Two ways out:

  (a) relativistic correction → paramagnetic atom (an unpaired electron)

  (b) finite size correction → diamagnetic atom (all electrons paired)

Paramagnetic atom/molecule

- Electron actually relativistic $v \sim Z\alpha \ \to d_e$ can induce d_A:

$$\vec{d}_A = d_e \sum_{i=1}^{Z} \left[\langle 0_e | \underline{\left(\gamma_0^{(i)} - 1\right)} \vec{\Sigma}^{(i)} | 0_e \rangle + 2 \sum_{n \neq 0} \frac{\langle 0_e | e\vec{r}_i | n_e \rangle}{E_0 - E_n} \langle n_e | \underline{\left(\gamma_0^{(i)} - 1\right)} \vec{\Sigma}^{(i)} \cdot \vec{E}_{\rm int} | 0_e \rangle \right]$$

relativistic correction to shielding

where $\vec{\Sigma} = \gamma_5 \gamma^0 \vec{\gamma}$.

$(\gamma_0 - 1)\vec{\Sigma} = -2 \begin{pmatrix} 0 & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$ in Dirac rep. → need an unpaired electron = paramagnetic atom.

- The latter (mixing of states) dominant,

  [Sandars 65; ...]

  and this is actually an enhancement: $d_A / d_e \sim Z^3 \alpha^2 \sim \mathcal{O}(10^2)$.

- CP-odd operator $C_S (G_F/\sqrt{2})\, \bar{e} i \gamma_5 e \times \bar{N} N$ also induces mixing of states.

➡ d_e and C_S degenerate.

$$d_e^{\rm equiv} = d_e + C_S \times 1.5 \times 10^{-20}\, e{\rm cm} \ \ \text{for} \ \ {\rm ThO}.$$

- Experimental constraint: $|d_e^{\rm equiv}| < 1.1 \times 10^{-29}\, e{\rm cm}$ [ACME 18].

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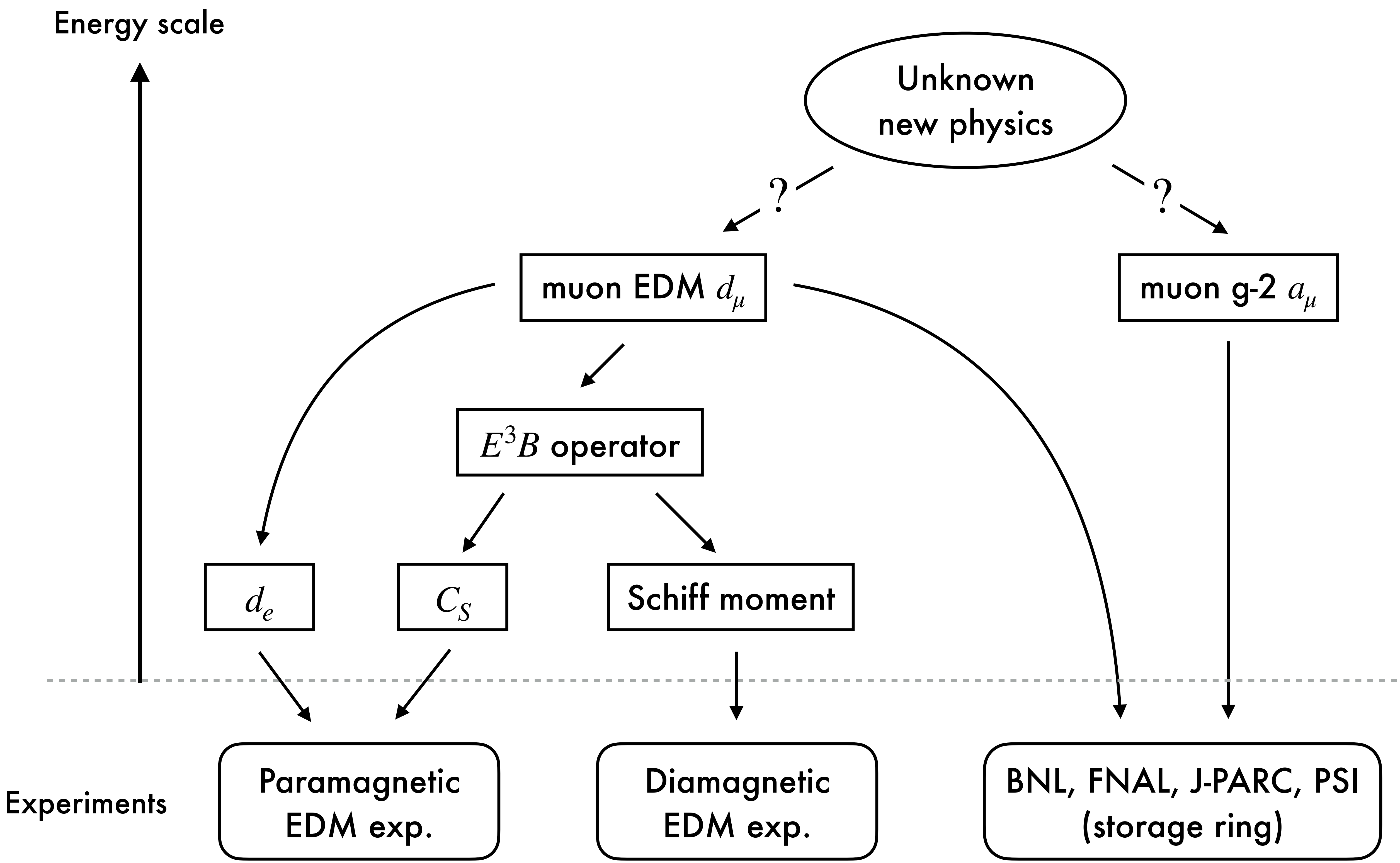
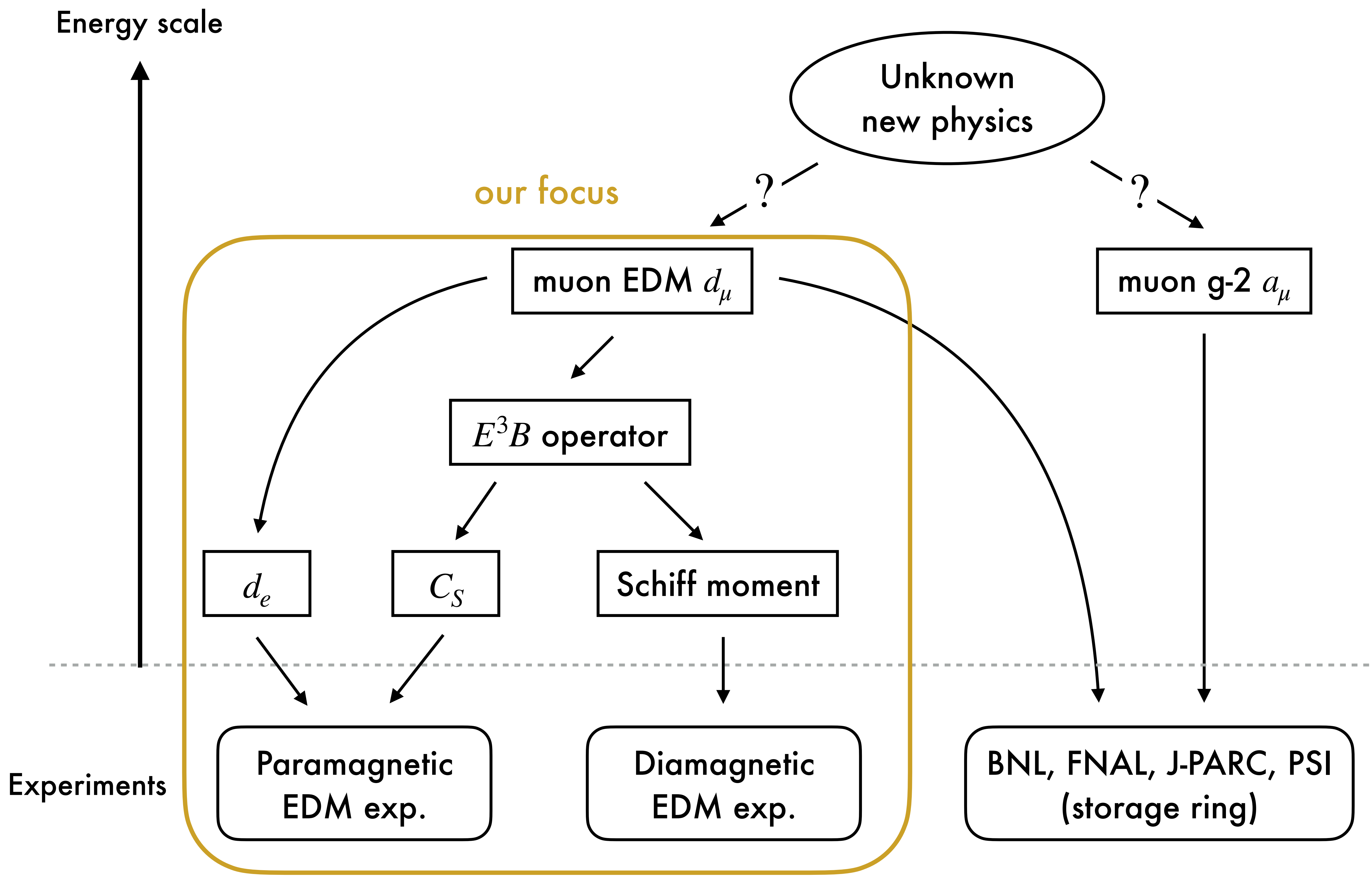
Toward observables

Many observables and many ways to achieve them



Toward observables

Many observables and many ways to achieve them



Toward observables

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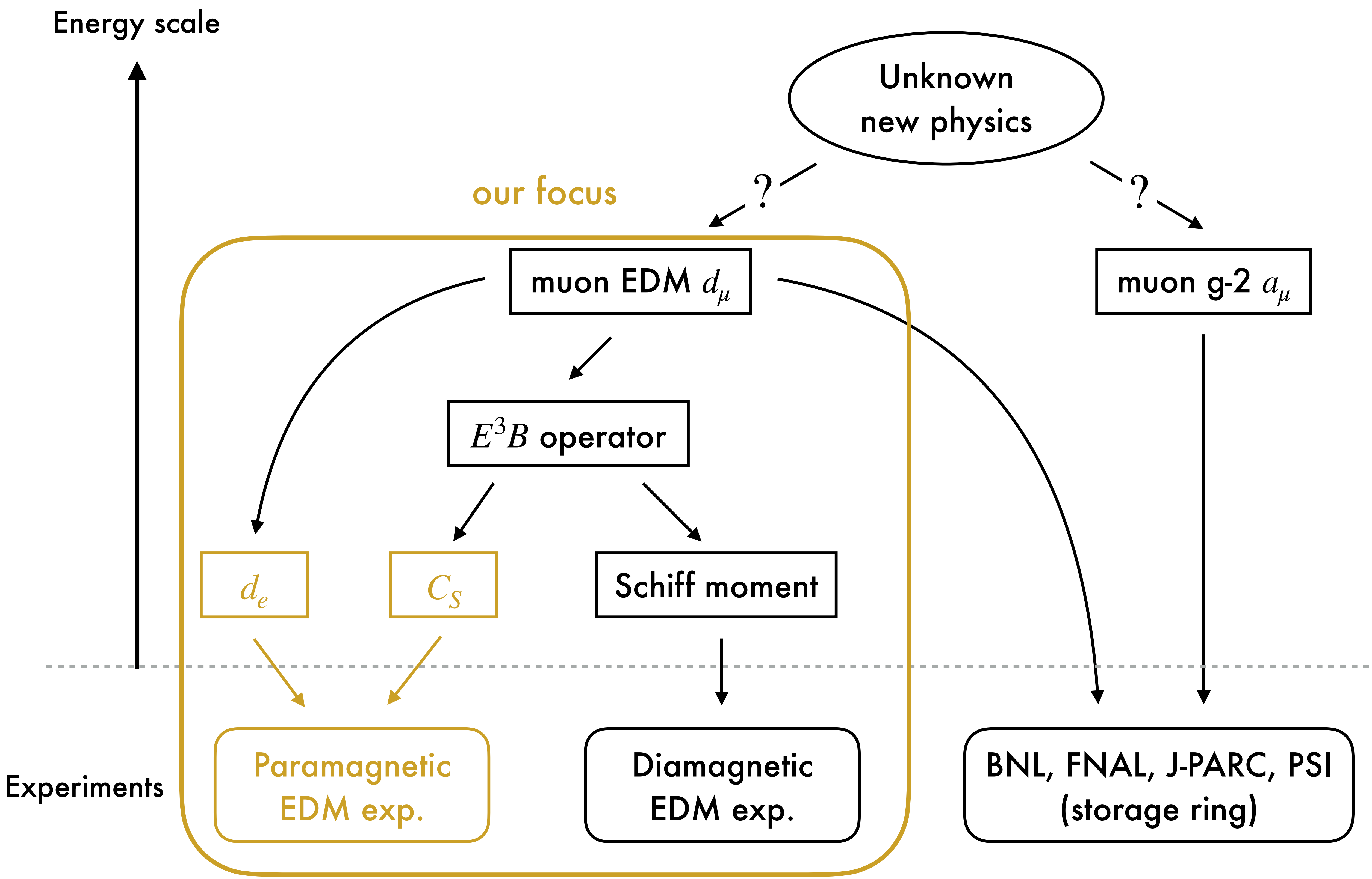
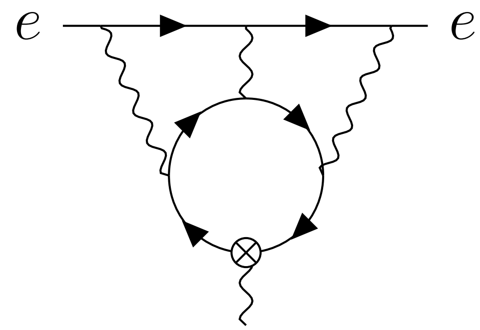


Electron EDM

- Muon EDM induces electron EDM at three-loop:



+ permutations

* cross-dot: EDM operator insertion

- There are two types of contributions:

* [Grozin, Khriplovich, Rudenko 08] computed only $S^{(1)}$.

$$i\mathscr{M} = i\tilde{F}^{\mu\nu}\,\bar{e}(p)\left[S^{(1)} m_e \sigma_{\mu\nu} + S^{(2)}\left\{\sigma_{\mu\nu}, \not{p}\right\}\right] e(p)\,.$$

- Combining two, the result is $\sim 40\,\%$ larger:

[YE, Gao, Pospelov 22]

$$d_e = 2.75 \times d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 2 \times 10^{-10} \times d_\mu\,.$$

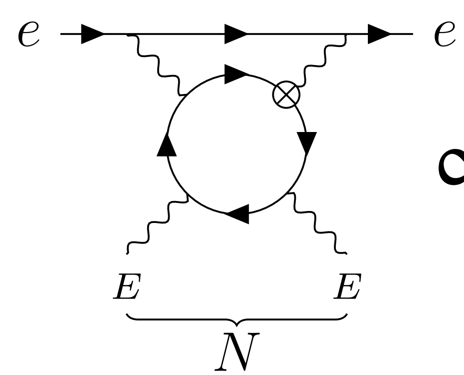
- But paramagnetic atom sensitive only to linear combination of d_e and C_S:

$$d_e^{\rm equiv} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,{\rm cm} \ \ \text{for} \ \ {\rm ThO}.$$

➡ Need to evaluate semi-leptonic CP-odd operator C_S.

Semi-leptonic CP-odd operator

- Paramagnetic atom EDM depends on C_S : $\mathscr{L} = C_S \frac{G_F}{\sqrt{2}} \bar{e} i\gamma_5 e \, \bar{N}N \,.$
- Muon EDM induces



$\propto \; d_\mu \times \bar{e} i\gamma_5 e \times E_N^2 \,.$

* We evaluated this with leading-log accuracy.

- Nuclear electric field E_N^2 localized around nucleus.

$\bar{e} i\gamma_5 e \times E_N^2 \;\sim\; \bar{e} i\gamma_5 e \times \bar{N}N$: equivalent to C_S.

* Fudge factor included in our actual computation.

- ACME experiment: $|d_e^{\text{equiv}}| < 1.1 \times 10^{-29} \, e\,\text{cm}$ [ACME 18]

$|d_\mu(\text{ThO})| < 1.7 \times 10^{-20} \, e\,\text{cm}$ [YE, Gao, Pospelov 21, 22]

* C_S dominates over d_e by a factor 4, better than direct bound by BNL $|d_\mu| < 1.8 \times 10^{-19} \, e\,\text{cm}$.

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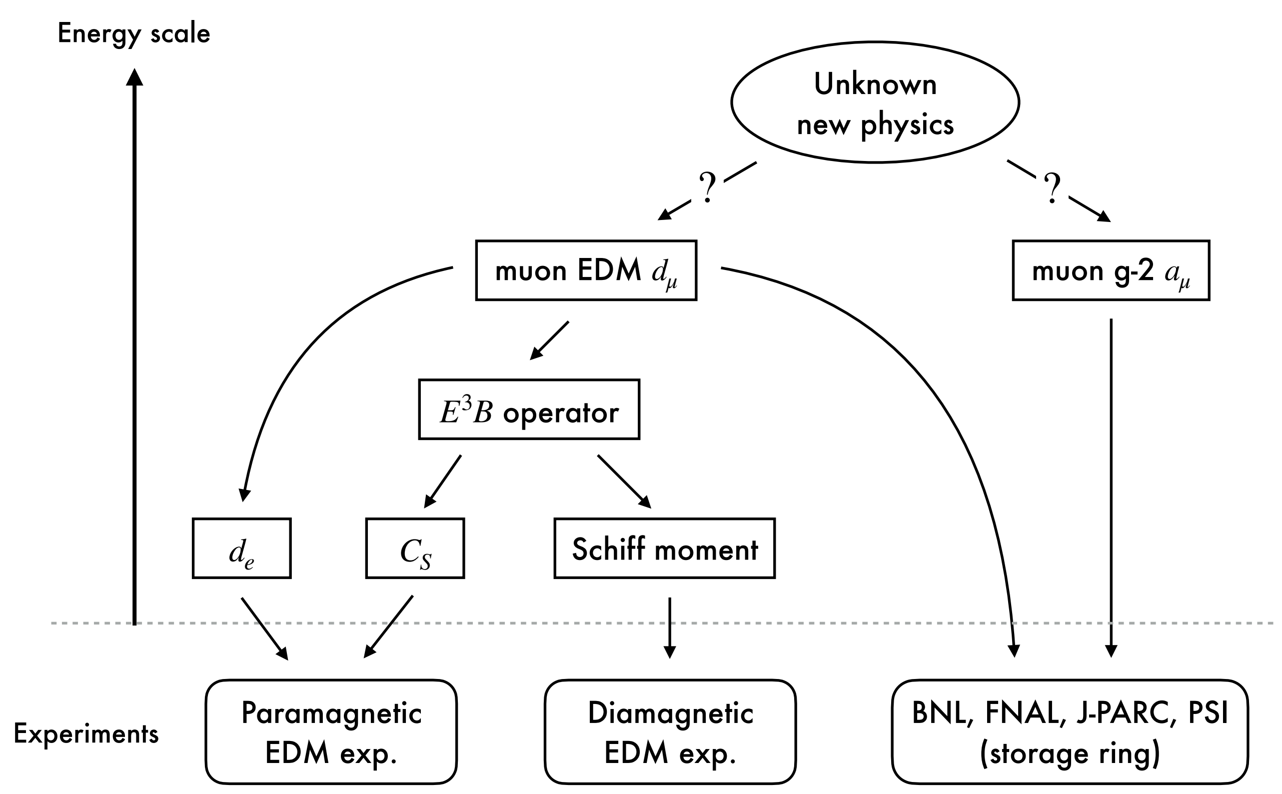
Summary

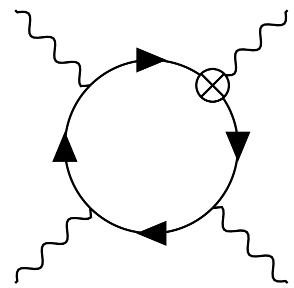
- We derived indirect limits on muon EDM, motivated by muon g-2.
- $|d_\mu(\mathrm{ThO})| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}$ from ThO.
- $|d_\mu(\mathrm{Hg})| < 6.4 \times 10^{-20}\, e\,\mathrm{cm}$ from $^{199}\mathrm{Hg}$.
- Two different observables, so cancellation by chance less likely.
- Can be applied to tau EDM: $|d_\tau| < 1.1 \times 10^{-18}\, e\,\mathrm{cm}$ (dominantly from d_e).



Back up

CP-odd photon operator

- Muon EDM induces CP-odd photon operator at one-loop:



$$= -\frac{e^3 d_\mu}{96\pi^2 m_\mu^3}\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$$

where cross-dot: muon EDM $d_\mu \bar{\mu}\sigma \cdot \tilde{F}\mu$ insertion.

- Atomic EDM exp. has large Z → strong nuclear electric field.

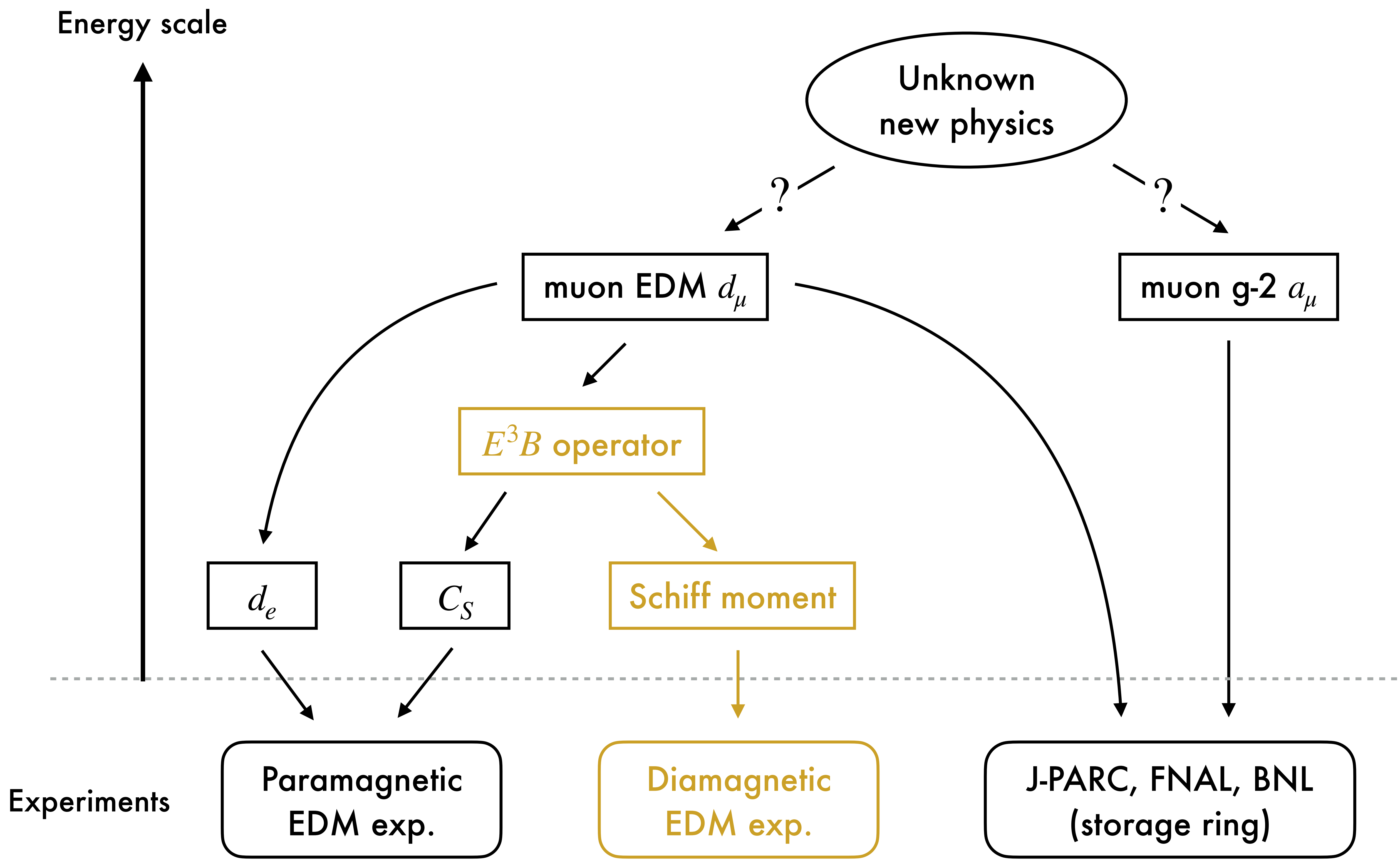
➡ $\tilde{F}_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} \ni E^3 B$ can induce sizable CP-odd effects.

- In particular this operator induces

$\{$ semi-leptonic CP-odd operator → paramagnetic EDM (ThO)
Schiff moment → diamagnetic EDM (Hg)

Toward observables

Many observables and many ways to achieve them



Diamagnetic atom

- Nucleus actually not point-like → d_N can induce d_A through mixing of states.

$$\vec{d}_A = \sum_{n\neq 0} \frac{1}{E_0 - E_n} \left[\langle 0_e | e \sum_{i=1}^{Z} \vec{r}_i | n_e \rangle \langle n_e | \mathcal{H}_{\rm int} | 0_e \rangle + {\rm h.c.} \right]$$

where $\mathcal{H}_{\rm int} = \int d^3 r \underbrace{\left(\frac{\vec{d}_N(\vec{r})}{e} - \rho_q(\vec{r}) \frac{\langle \vec{d}_N \rangle}{e} \right)}_{\text{(partial) shielding}} \cdot \vec{\nabla}_e \frac{\alpha}{|\vec{r} - \vec{r}_e|}$: difference btw charge and EDM distribution.

➡ avoid complete suppression: $d_A / d_N \sim 10 Z^2 (R_N/R_A)^2 \sim \mathcal{O}(10^{-3})$.

- Expanding this w.r.t. $r \sim r_N \ll r_e$:

$$\mathcal{H}_{\rm int} = \int d^3 r \left(\frac{\vec{d}_N}{e} - \rho_q \frac{\langle \vec{d}_N \rangle}{e} \right) \cdot \vec{\nabla}_e \frac{\alpha}{|\vec{r} - \vec{r}_e|} = -4\pi\alpha \frac{\vec{S}}{e} \cdot \vec{\nabla}_e \delta^{(3)}(\vec{r}_e) + \cdots$$ [Schiff 63]

where $\vec{S}$: Schiff moment.

- Diamagnetic atom EDM exp. puts constraints on this S.

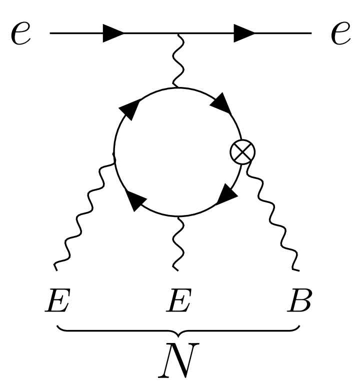
e.g. ^{199}Hg constraint: $|S_{^{199}\rm Hg}| < 3.1 \times 10^{-13}\, e{\rm fm}^3$. [Graner et.a. 16]

Schiff moment

- Schiff moment: $\mathcal{H}_{\rm int} = -4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)\,.$

$$\vec{d}_A = \sum_{i=1}^{Z}\langle\Psi|e\vec{r}_i|\Psi\rangle = -\sum_{n\neq 0}\frac{2}{E_0-E_n}\sum_{i=1}^{Z}\langle 0_e|e\vec{r}_i|n_e\rangle\langle n_e|4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_i\delta^{(3)}(\vec{r}_i)|0_e\rangle\,.$$

- E^3B with two E_N and one B_N induces effective EDM distribution:



$$= \int d^3r\left(\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|}\right)\cdot\frac{\vec{d}_N(\vec{r})}{e}\,,\qquad \vec{d}_N \propto d_\mu\left(2\vec{E}_N(\vec{E}_N\cdot\vec{B}_N)+\vec{B}_N E_N^2\right).$$

- Difference between EDM and charge distribution gives Schiff moment:

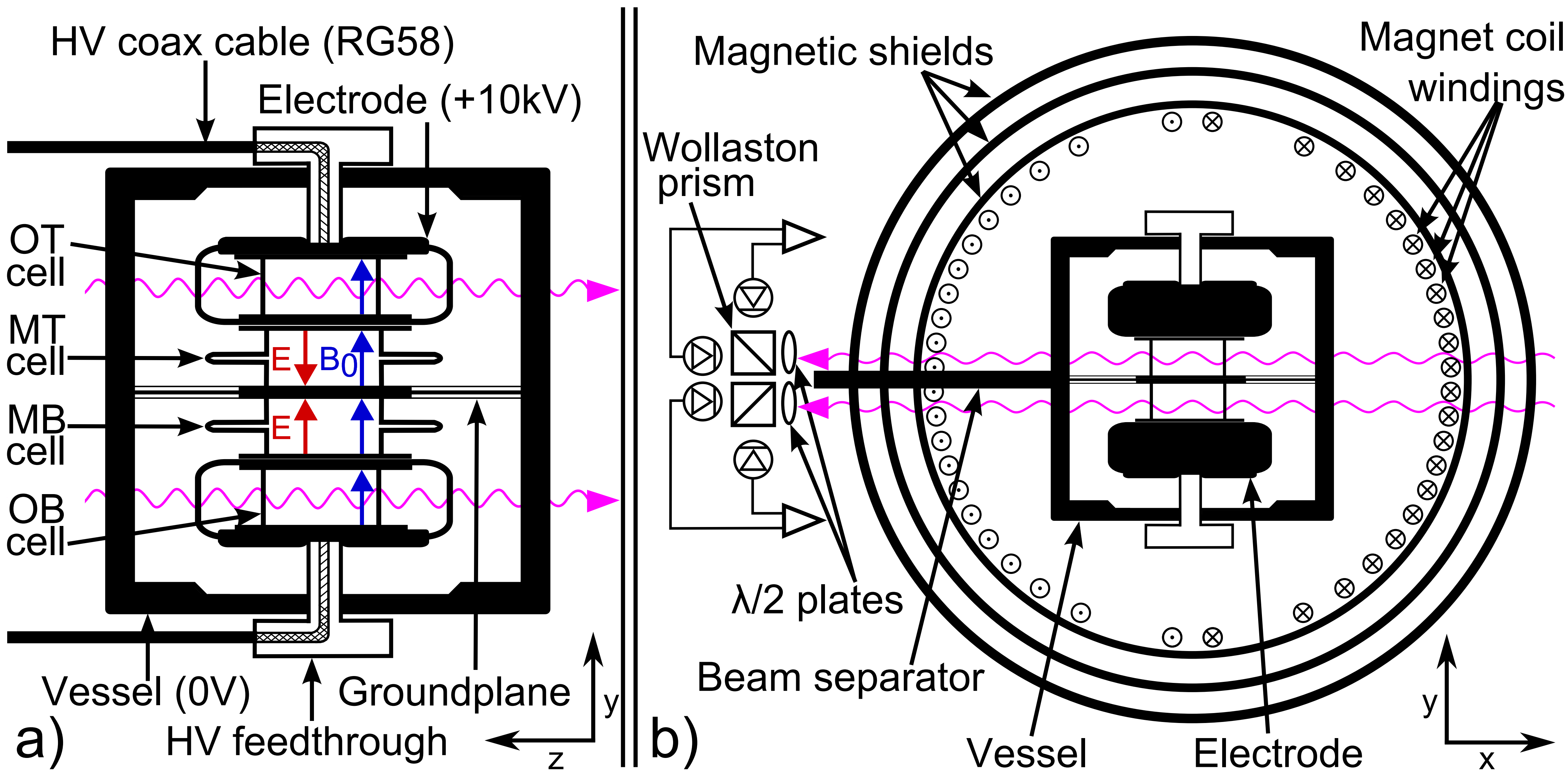
$$\mathcal{H}_{\rm eff} = \int d^3r\left(\frac{\vec{d}_N}{e}-\rho_q\frac{\langle\vec{d}_N\rangle}{e}\right)\cdot\vec{\nabla}_e\frac{\alpha}{|\vec{r}-\vec{r}_e|} = -4\pi\alpha\frac{\vec{S}}{e}\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)+\cdots\,.$$

- ^{199}Hg constraint: $|S_{^{199}\rm Hg}| < 3.1\times 10^{-13}\,e{\rm fm}^3\,.$ [Graner et.a. 16]

$$|d_\mu({\rm Hg})| < 6.4\times 10^{-20}\,e{\rm cm}$$

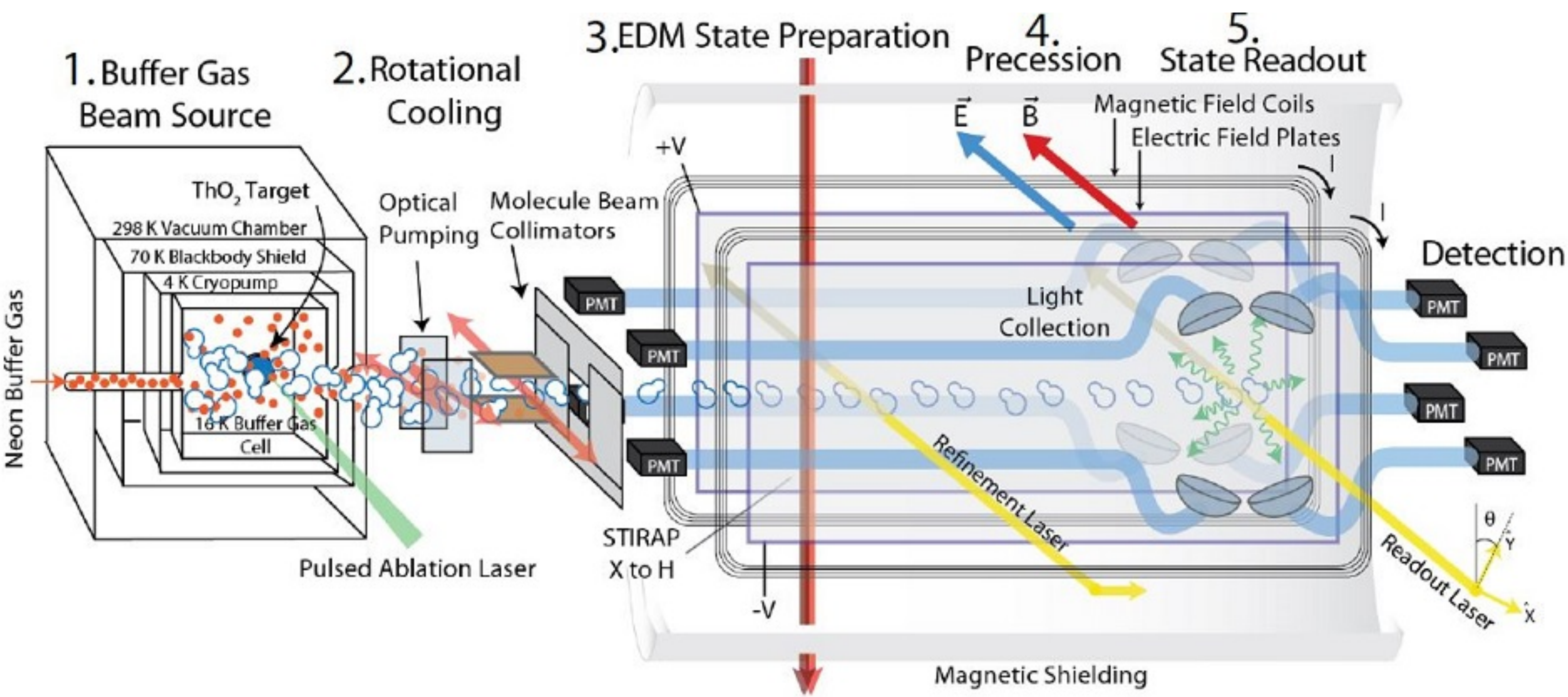
[YE, Gao, Pospelov 21]

^{199}Hg experiment



[Graner et.a. 16]

ACME ThO experiment



[ACME 18]

Nuclear electric field

- Nuclear electric field given by the charge distribution inside nuclei:

$$e\vec{E}_N(\vec{r}) = \frac{Ze^2}{4\pi}\int d^3r_N \rho_q(\vec{r}_N)\,\vec{\nabla}\frac{1}{|\vec{r}-\vec{r_N}|}\,.$$

- We simply take the charge distribution as

$$\rho_q(r_N) = \frac{3}{4\pi R_N^3}\Theta\left(R_N - r_N\right), \quad R_N = \sqrt{\frac{5}{3}}r_c, \quad r_c = 5.45\,\mathrm{fm}\ \text{ for } {}^{199}\text{Hg}.$$

- The Woods-Saxon shape different only within $10\,\%$ in the final result.

Nuclear magnetic field

- ^{199}Hg has an unpaired outermost neutron with $2p_{1/2}$ $(n=2, l=1, j=1/2)$.

  ➡ $\overrightarrow{B}_N$ dominantly provided by this neutron.

- As a result $\overrightarrow{B}_N$ is given by

$$e\overrightarrow{B}_N(\vec{r}) = \frac{2e\mu_n}{3}\psi_n^\dagger(\vec{r})\overrightarrow{\sigma}\psi_n(\vec{r}) + \frac{e\mu_n}{4\pi}\left[\overrightarrow{\nabla}\left(\overrightarrow{\nabla}\cdot\right) - \frac{\overrightarrow{\nabla}^2}{3}\right]\int d^3r_n \frac{\psi_n^\dagger(\vec{r}_n)\overrightarrow{\sigma}\psi_n(\vec{r}_n)}{|\vec{r}_n - \vec{r}|}$$

$$= e\mu_n \frac{\left|R(r)\right|^2}{4\pi}\chi^\dagger\left[\left(\overrightarrow{n}\cdot\overrightarrow{\sigma}\right)\overrightarrow{n} - \overrightarrow{\sigma}\right]\chi + \frac{e\mu_n}{4\pi}\int_0^\infty dr_n\, r_n^2 \left|R(r_n)\right|^2 \chi^\dagger\overrightarrow{g}(\vec{r}, r_n)\chi,$$

where $\mu_n \simeq -1.91\dfrac{e}{2m_p}$: neutron magnetic moment, ψ_n : neutron wave function,

R_n: neutron radial wave function, χ neutron spinor, $\overrightarrow{n} = \vec{r}/r$.

- We use the nuclear shell model to obtain ψ_n.

Fudge factor from E_N^2 to $\bar{N}N$

- E_N^2 and $\bar{N}N$ are both localized around nuclei, but not exactly the same.

- Relevant electron transition btw $s_{1/2}$ and $p_{1/2}$ states.

  matrix element:

$$\begin{cases} \int d^3 r \rho_N \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right) & \text{for} \quad \bar{N}N\bar{e}i\gamma_5 e, \\ \int d^3 r \,|\overrightarrow{E}_N|^2 \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right) & \text{for} \quad E_N^2 \bar{e} i \gamma_5 e\,. \end{cases}$$

- We compute the fudge factor κ by solving the Dirac equation and get

$$\kappa = \frac{\int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right)}{\int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right)} \simeq 0.66.$$

Magnetic quadrupole moment

- Magnetic quadrupole moment (MQM) also violates P and CP:

$$\mathscr{H}_{\rm eff} = -\frac{M}{6}\nabla_j B_i I_{ij}, \quad I_{ij} \equiv \frac{3}{2I(I-1)}\left(I_i I_j + I_j I_i - \frac{2}{3}\delta_{ij} I(I+1)\right).$$

- The E^3B operator converts EQM Q to MQM as

$$\frac{M}{e} \simeq -\frac{Z^2\alpha^3 d_\mu/e}{5\pi m_\mu^2 R_N^3}\frac{Q}{e} \simeq 1.1\times 10^{-4}\,\mathrm{fm}\left(\frac{Q/e}{300\,\mathrm{fm}^2}\right)\times d_\mu/e\,.$$

- Q can be large in nuclei with $I \geq 1$ and large deformation.

➡ can be an interesting observable in future.