Monte Carlo study of SU(2) gauge theories coupled to multiple Higgs fields

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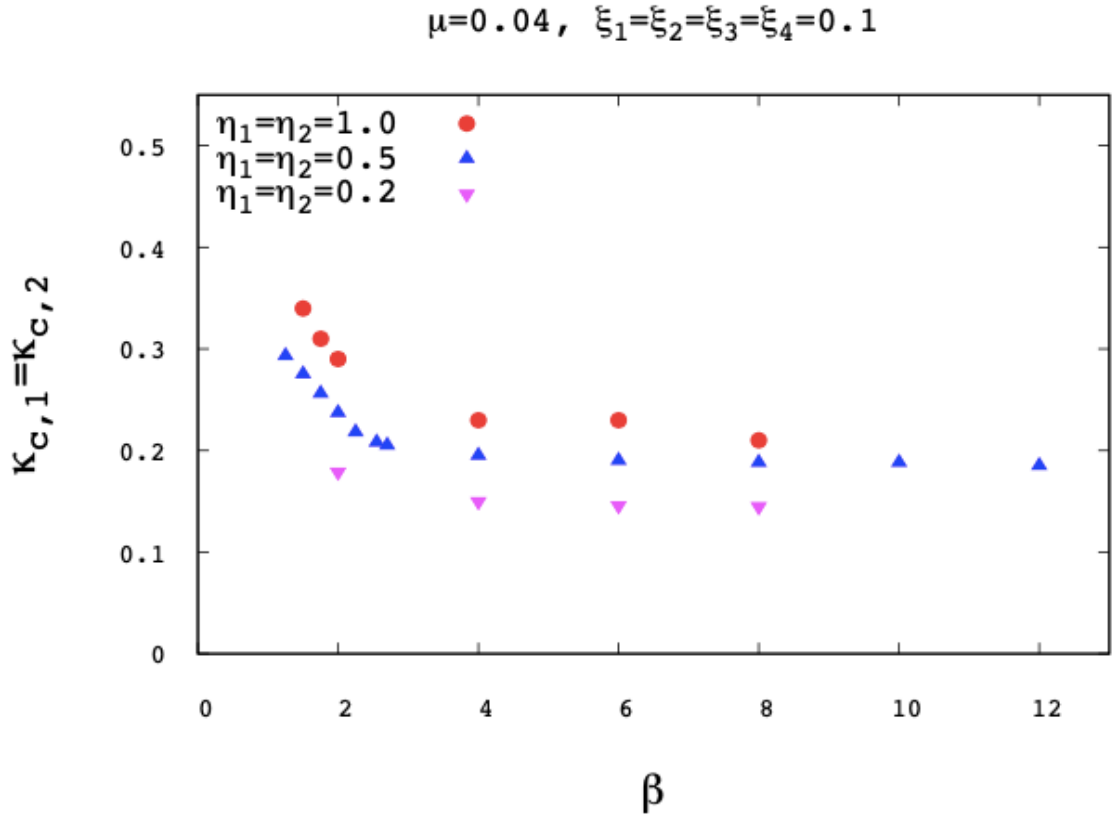
Motivation

Lattice field theory ➡ Mass spectrum in two Higgs doublet model

Confinement-Higgs phase line in (β, κ) for various parameters



Contents

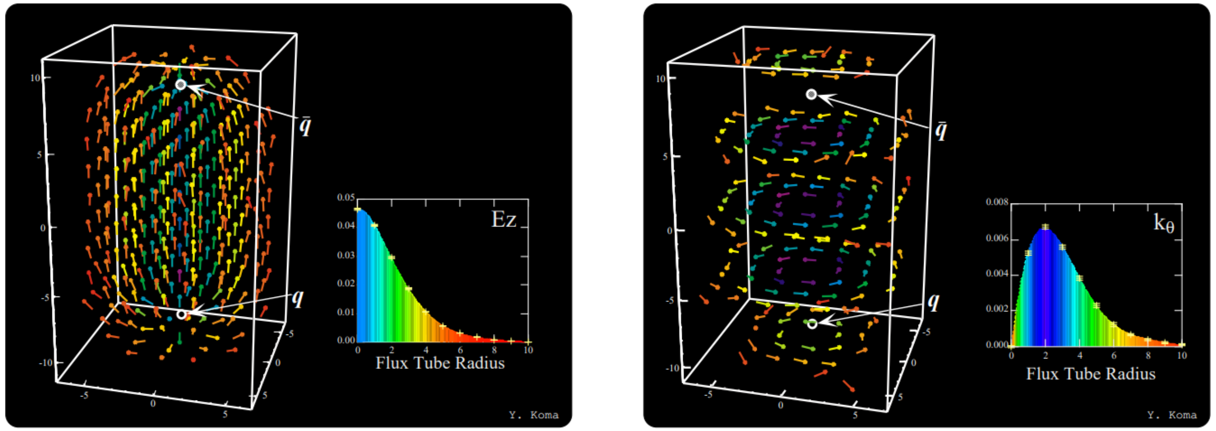
平口敦基

New Phys. Rev. D 106 (2022), 014515

- 2018.04 ~2021.03 高知大学 博士課程

  2019 金沢大

The dual Meissner effect



Y. Koma et al., Phys. Rev. D68, 094018 (2003).

- 2021.06 ~ 國立陽明交通大学 PD

SU(2) Higgs model

平口敦基

・2018.04 ~ 2021.03 高知大学 博士課程

2019 金沢大

FW: 【注意喚起】クマ目撃情報について／[WARNING] About Bear Sighting 受信トレイ ×

・2021.06 ~ 2023.07 國立陽明交通大学 PD

カツオのたたき

https://www.maff.go.jp/j/keikaku/syokubunka/k_ryouri/search_menu/menu/katuonotataki_kochi.html

ズグロミゾゴイ

https://zukan.com/jbirds/internal15482

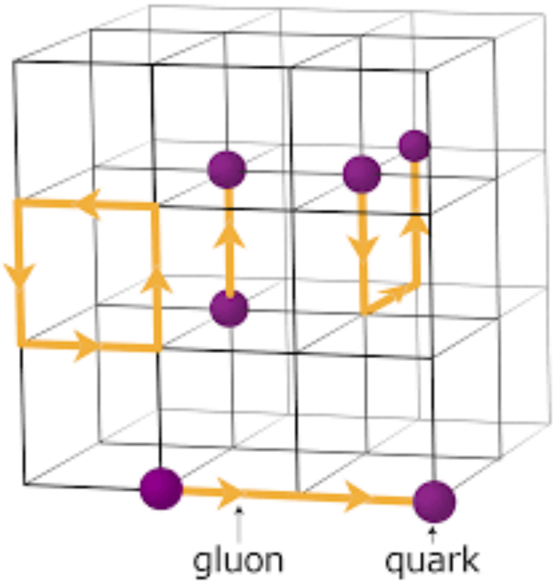
ゴイのピンバッジ

Introduction

- **Lattice field theory**

Link variables

$U_\mu(x)$

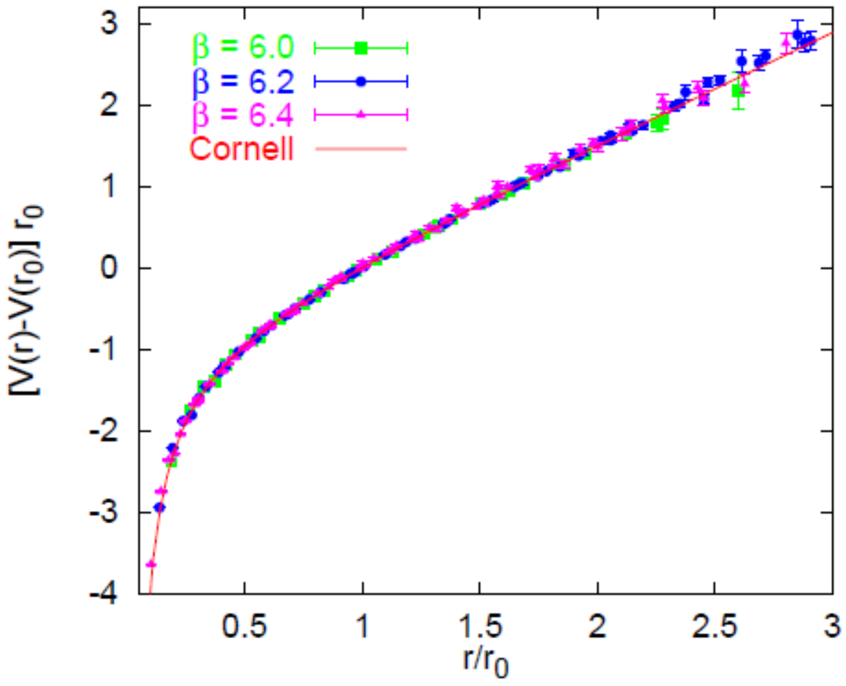


http://vietnam.in2p3.fr/2018/windows/transparencies/05_friday/01_morning/HEP/03_Patella.pdf

Non–perturbative properties:

Color confinement in pure gauge theory

Gunnar S.Bail Phys.Rept.343:1-136,2001

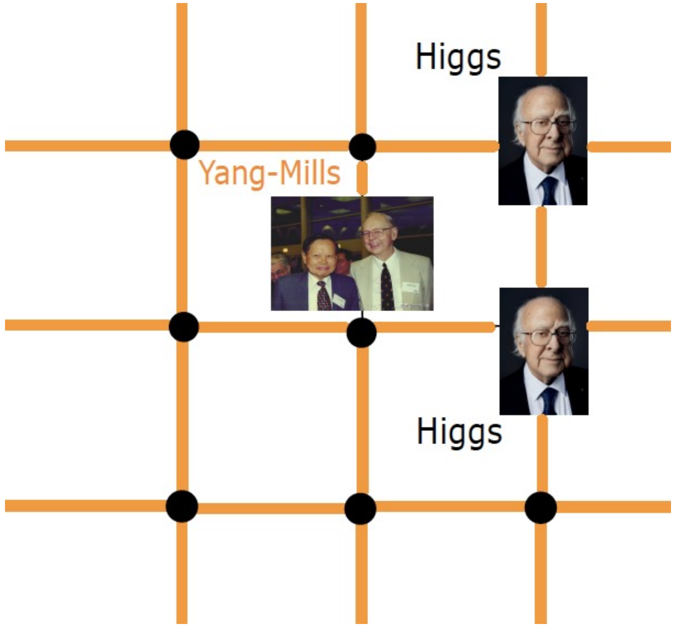


Linear potential

$$V(r) = \sigma r - c/r + \mu$$

Introduction

- SU(2) Higgs model



$$S = \beta \sum_{\square} \left[1 - \frac{1}{2} \mathrm{Tr}\, U_{\square}\right]$$
$$+ \sum_{x} \left[\lambda[\frac{1}{2} \mathrm{Tr}(\phi_x^\dagger \phi_x) - 1]^2 + \frac{1}{2} \mathrm{Tr}(\phi_x^\dagger \phi_x)) - \kappa \sum_{\mu} \mathrm{Tr}(\phi_x^\dagger U_{x,\mu} \phi_{x+\hat{\mu}})\right]$$

Higgs fields

$$\phi_x = \rho_x \alpha_x, \quad \alpha_x \in SU(2)$$

β : gauge coupling
λ : Higgs coupling
κ : Hopping parameter

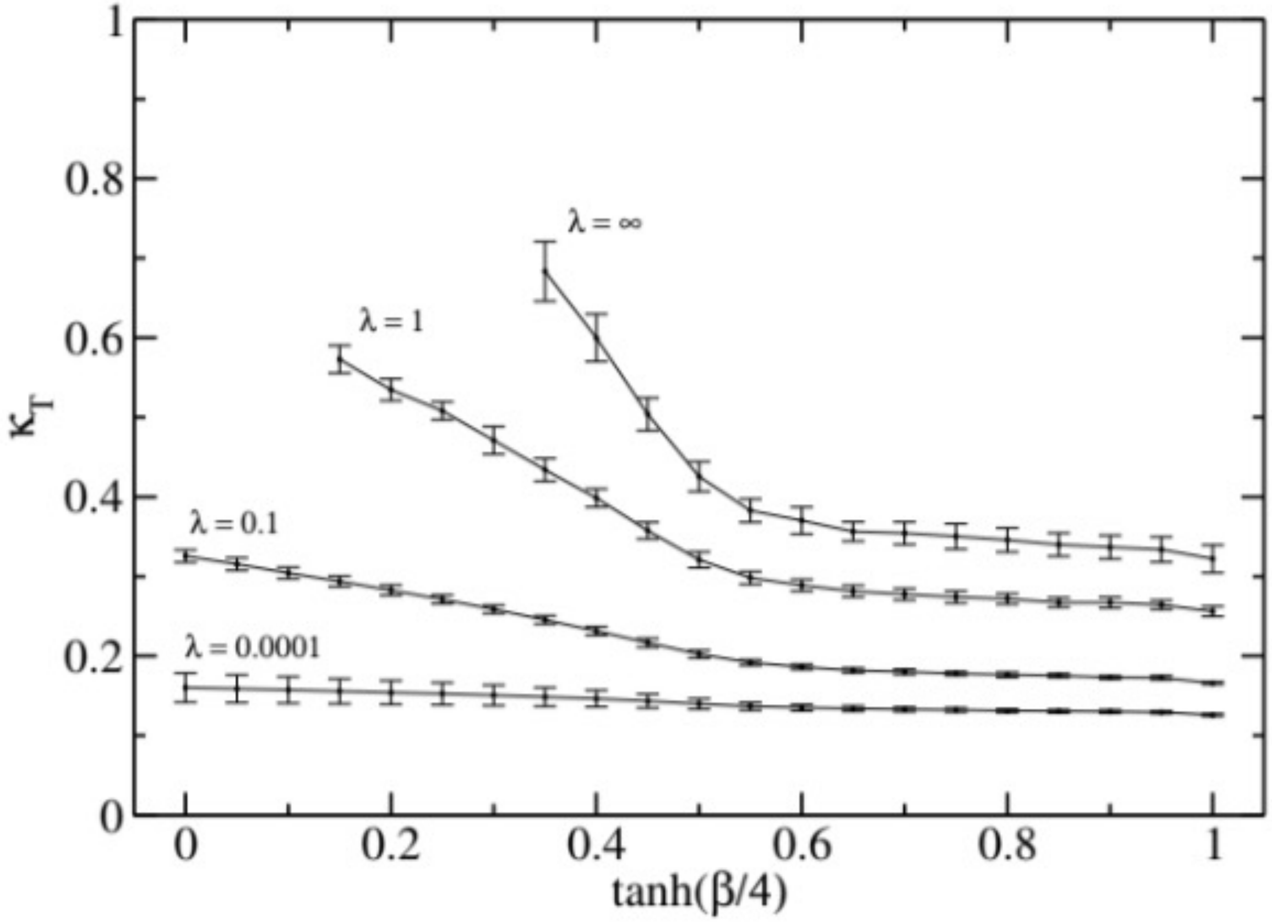
$$a\phi(x) = \sqrt{\kappa}\phi_x,$$
$$\lambda_0 = \frac{\lambda}{\kappa^2},$$
$$a^2 m_0^2 = \frac{1 - 2\lambda - 8\kappa}{\kappa}$$

Introduction

・Phase structure

1. Higgs phase

2. Confinement phase



M. Wurtz, R. Lewis, T.G. Steele,
Phys.Rev.D 79 (2009) 074501

Introduction

E. Fradkin and S. Shenker, Phys. Rev. D 19, 3682 (1979).

- **Previous Monte Carlo researches**

1980s, Abelian Higgs model in 4d
non-Abelian Higgs model in 4d

I. Montvay, Phys. Lett. B 150, 441 (1985).

J. Jersák, C. B. Lang, T. Neuhaus and G. Vones, Phys. Rev. D 32, 2761 (1985).

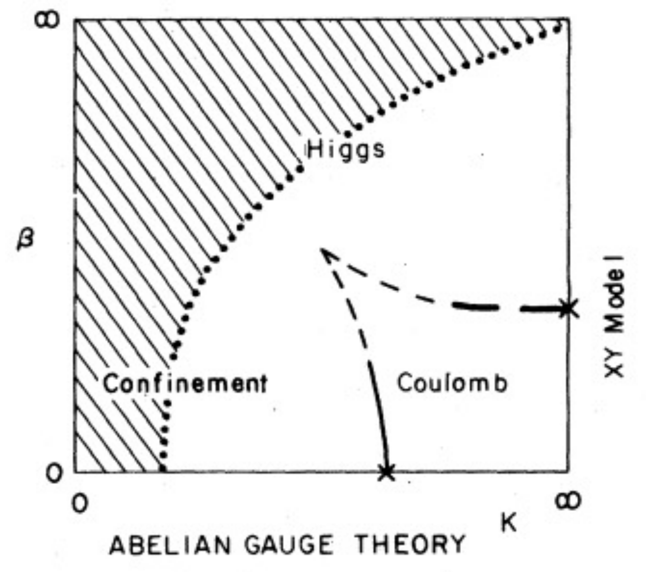


FIG. 2. Phase diagram for the Abelian model with Higgs fields in the fundamental representation ($d=4$). The broken line emerging from the XY transition ($K=\infty$) is a line of first-order transitions. The full line that emerges from the pure gauge transition ($\beta=0$) is a line of transitions of the *same* order as the pure gauge critical point. Notice the curvature of the lines. The phases are described in the text.

1990s

Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay, Nucl. Phys. B 439 (1995) 147.

Kajantie, K., Laine, M., Rummukainen, K., and Shaposhnikov, M., Phys. Rev. Lett. 77, 2887 (1996).

$m_H \sim 80\ \mathrm{GeV}$ **First order**

125 GeV ➡ **Cross over**

M. D'Onofrio and K. Rummukainen, Phys. Rev. D 93, 025003 (2016)

Introduction

- **Two Higgs doublet models**

Wei-Shu Hou and Mariko Kikuchi, EPL, 1231(2018)111001

$$V(\Phi, \Phi') = \mu_{11}^2|\Phi|^2 + \mu_{22}^2|\Phi'|^2 - \left(\mu_{12}^2\Phi^\dagger\Phi' + \text{h.c.}\right)$$
$$+ \frac{1}{2}\eta_1|\Phi|^4 + \frac{1}{2}\eta_2|\Phi'|^4 + \eta_3|\Phi|^2|\Phi'|^2 + \eta_4|\Phi^\dagger\Phi'|^2$$
$$+ \left[\frac{1}{2}\eta_5(\Phi^\dagger\Phi')^2 + \left(\eta_6|\Phi|^2 + \eta_7|\Phi'|^2\right)\Phi^\dagger\Phi' + \text{h.c.}\right]$$

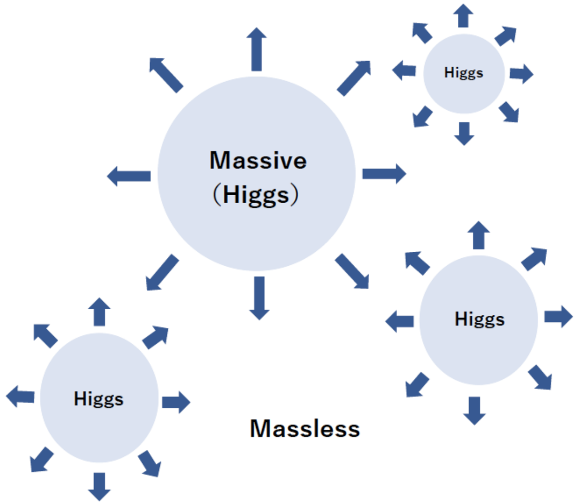
☆Electro weak phase transition

☆Mass spectrum

➡ Quartic self-coupling could be strong.

➡ We need a non-perturbative treatment.

The first order phase transition



Methods

Quaternion representation

$\hat{\Phi}_n(x) = \frac{1}{\sqrt{2}} \sum_{\alpha=0}^{3} \theta_\alpha \hat{\phi}_\alpha^{(n)}(x)$

$\theta_0 = \mathbf{1}_{2\times 2}, \theta_i = \sigma_i$

- **Lattice two Higgs doublet models**

$$S_{\mathsf{2HDM}} = S_{\mathsf{YM}} + \sum_x \sum_{n=1}^{2} \Big\{ \sum_\mu -2k_n \operatorname{Tr}\left(\hat{\Phi}_n^\dagger(x) U_\mu(x) \hat{\Phi}_n(x+\hat{\mu})\right)$$
$$+ \operatorname{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) + \eta_n \left[\operatorname{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) - 1\right]^2 \Big\} + 2\mu^2 \operatorname{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right)$$
$$+ \xi_2 \operatorname{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right)^2 + \xi_1 \operatorname{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) \operatorname{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right)$$
$$+ 2 \operatorname{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right) \left[\xi_3 \operatorname{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) + \xi_4 \operatorname{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right)\right],$$

- **HMC**
- **Metropolis**

$\Rightarrow$ $\kappa_1 = \kappa_2,\ \eta_1 = \eta_2,\ \mu = 0.04,\ \xi_1 = \xi_2 = \xi_3 = \xi_4 = 0.1$

Methods

- **Gauge invariant observables**

Plaquette

$$P = \frac{1}{6V} \sum_{\text{Plaquette}} \frac{1}{2} \mathrm{Tr}(U_{\text{Plaquette}})$$

Higgs length

$$\rho^2 = \frac{1}{V} \sum_x \frac{1}{2} \mathrm{Tr}(\Phi^\dagger(x)\Phi(x))$$

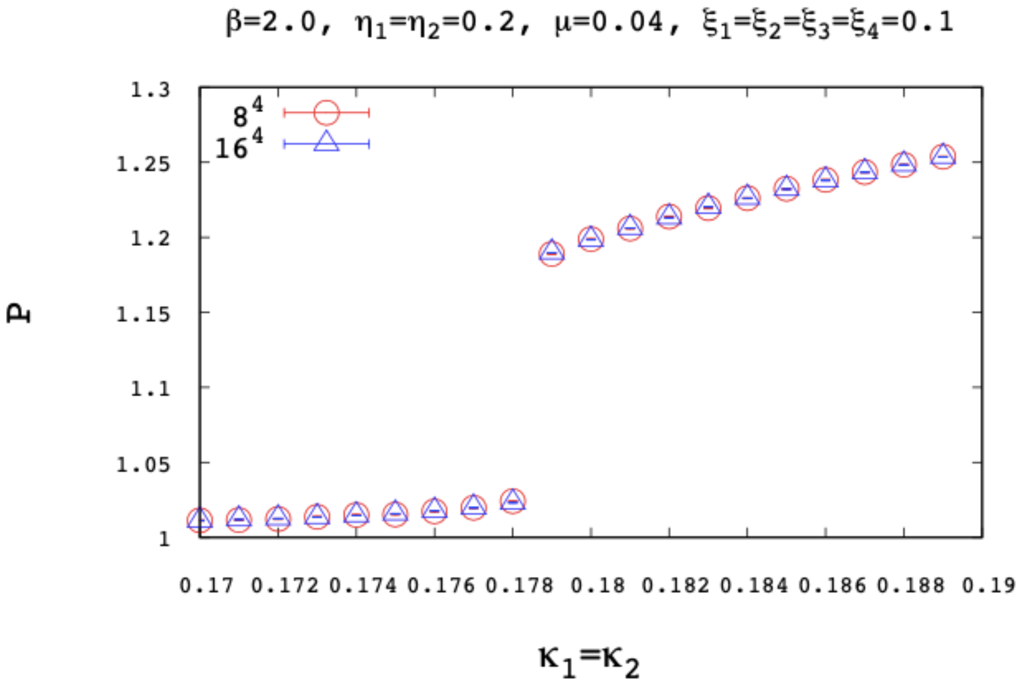
$$L_\Phi = \frac{1}{4V} \sum_x \frac{1}{2} \mathrm{Tr}(\Phi^\dagger(x) U_\mu(x) \Phi(x+\hat{\mu}))$$

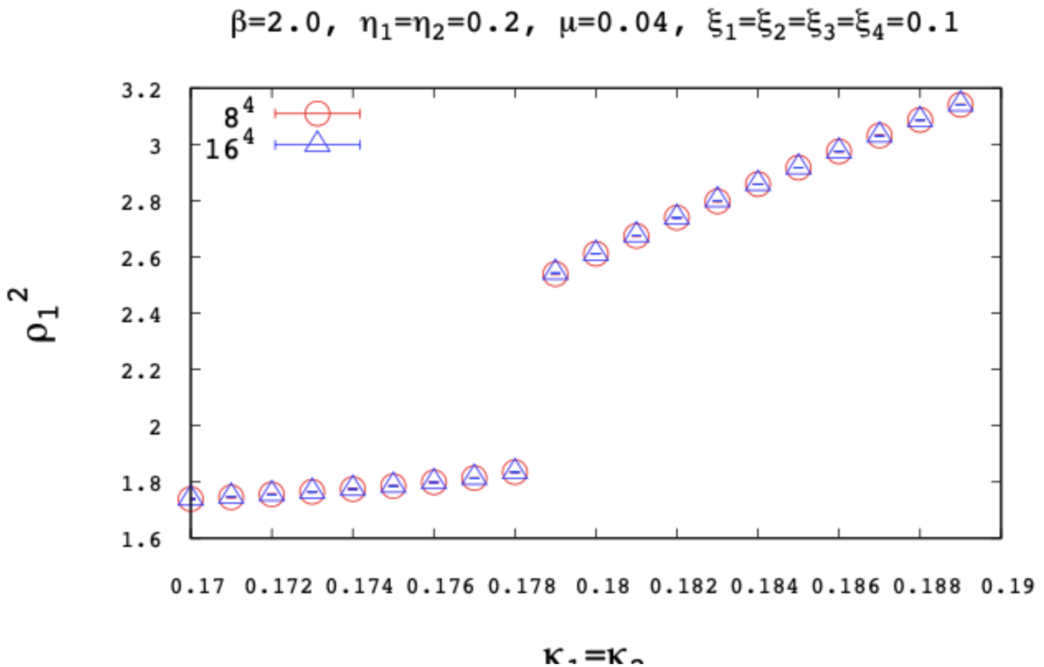
Gauge-invariant link

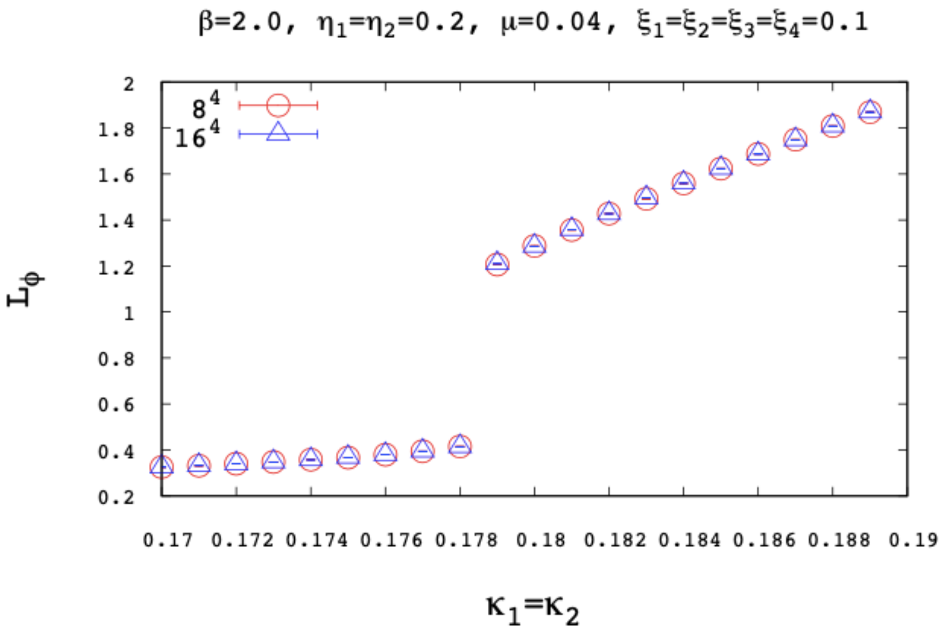
$$L_\alpha = \frac{1}{4V} \sum_x \frac{1}{2} \mathrm{Tr}(\alpha^\dagger(x) U_\mu(x) \alpha(x+\hat{\mu}))$$

Preliminary results

β=2.0, η1=η2, μ=0.04, ξi=0.1

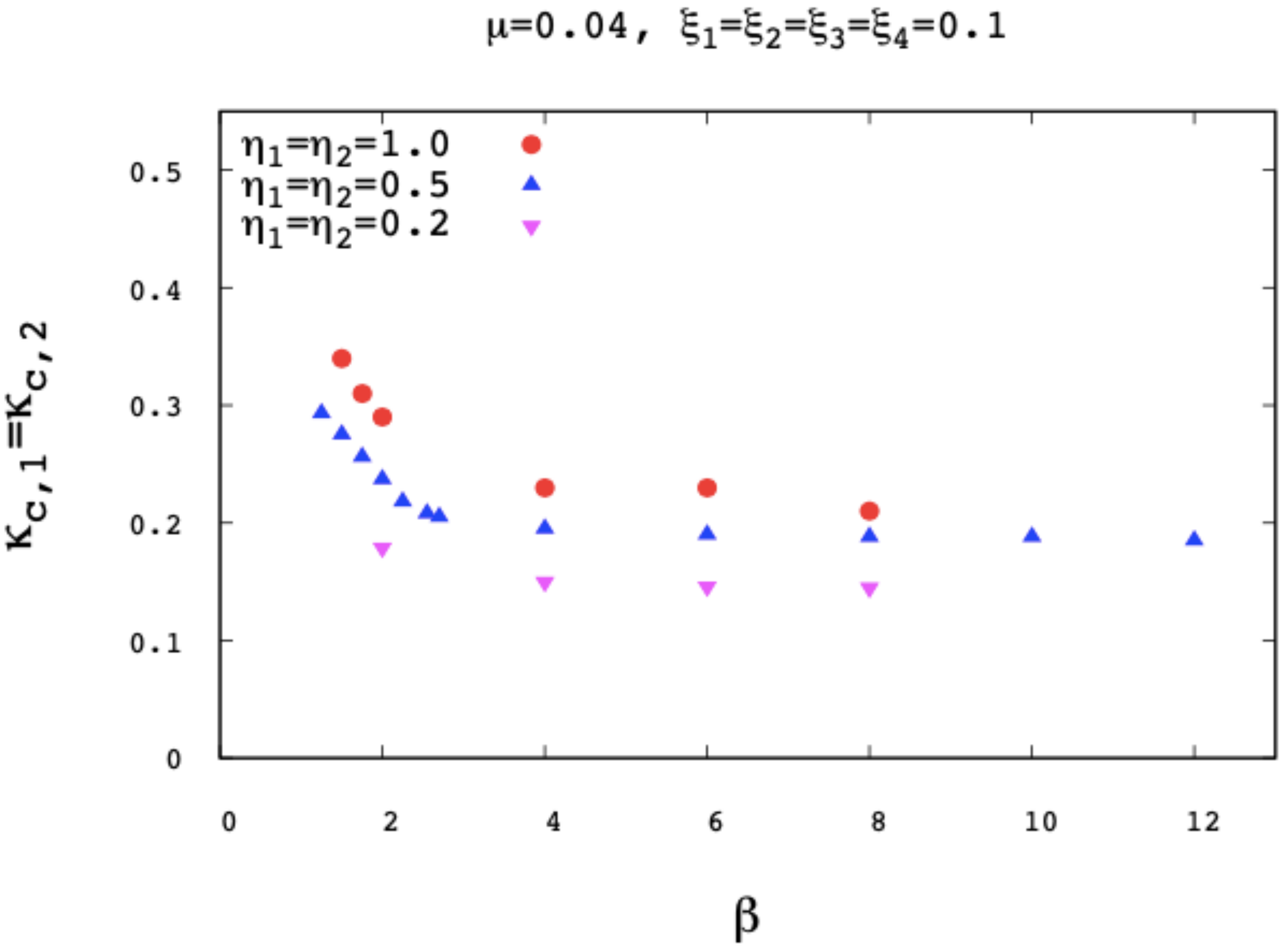






Preliminary results

- phase diagram



Future works

Dimensionless quantities

Ratio of Higgs mass and W boson mass

Renormalized gauge coupling $V(R) = \text{const.} - \frac{3g_R}{4R}e^{-am_W R}$

→ Line of constant physics

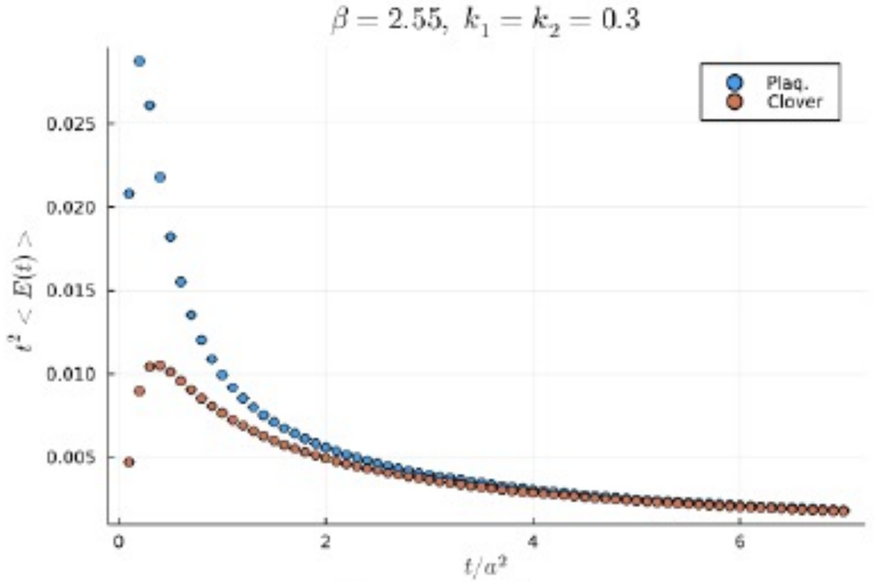
Preliminary results

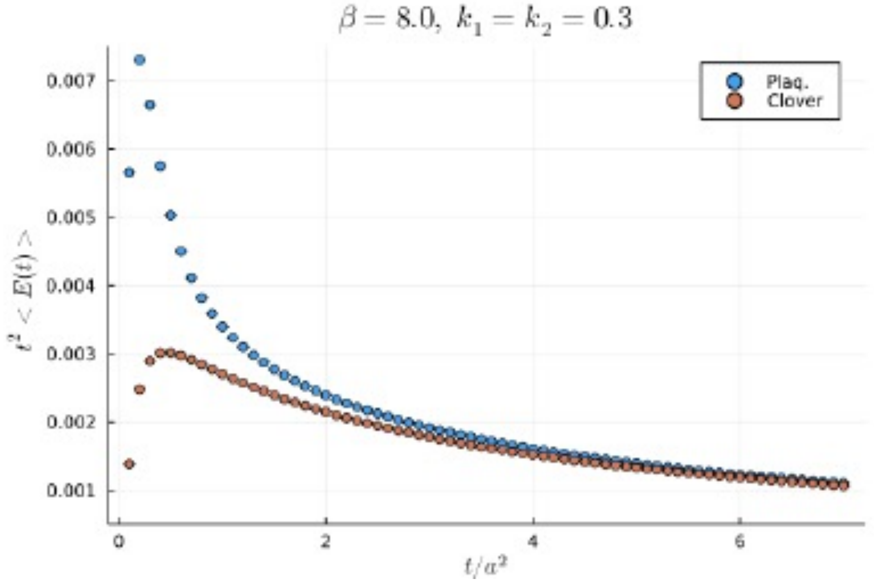
- **Scale setting with gradient flow**

Renormalized coupling at the scale of W boson mass
$g^2 \equiv 4\pi\alpha_W \simeq 0.5$

choice for scale setting

$$t^2 \langle E(t) \rangle = \frac{9g^2}{128\pi^2}(1 + \mathcal{O}(g^2))$$
$$\simeq 0.0036\Big|_{\mu=1/\sqrt{8t}=m_W}$$





Results (Adjoint Higgs)

SU(2) + Scalar fields

- Fundamental representation
- Adjoint representation

$$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2}\boldsymbol{F}_{\mu\nu}\cdot\boldsymbol{F}_{\mu\nu} + |\partial_\mu \boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_x|^2 + |\partial_\mu \boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_y|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$$

$$V(\boldsymbol{\mathcal{H}}_{x,y}) = s\left(\boldsymbol{\mathcal{H}}_x^*\cdot\boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^*\cdot\boldsymbol{\mathcal{H}}_y\right) + u_0\left(\boldsymbol{\mathcal{H}}_x^*\cdot\boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^*\cdot\boldsymbol{\mathcal{H}}_y\right)^2 \cdot \frac{u_1}{4}\phi^2 + \frac{u_2}{2}\left(|\Phi_x|^2 + |\Phi_y|^2\right) + u_3\left(|\Phi_+|^2 + |\Phi_-|^2\right)$$

($u_1 = u_2 = u_3 \to$ global $O(4)$ symmetry)

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

Gauge-invariant bilinears

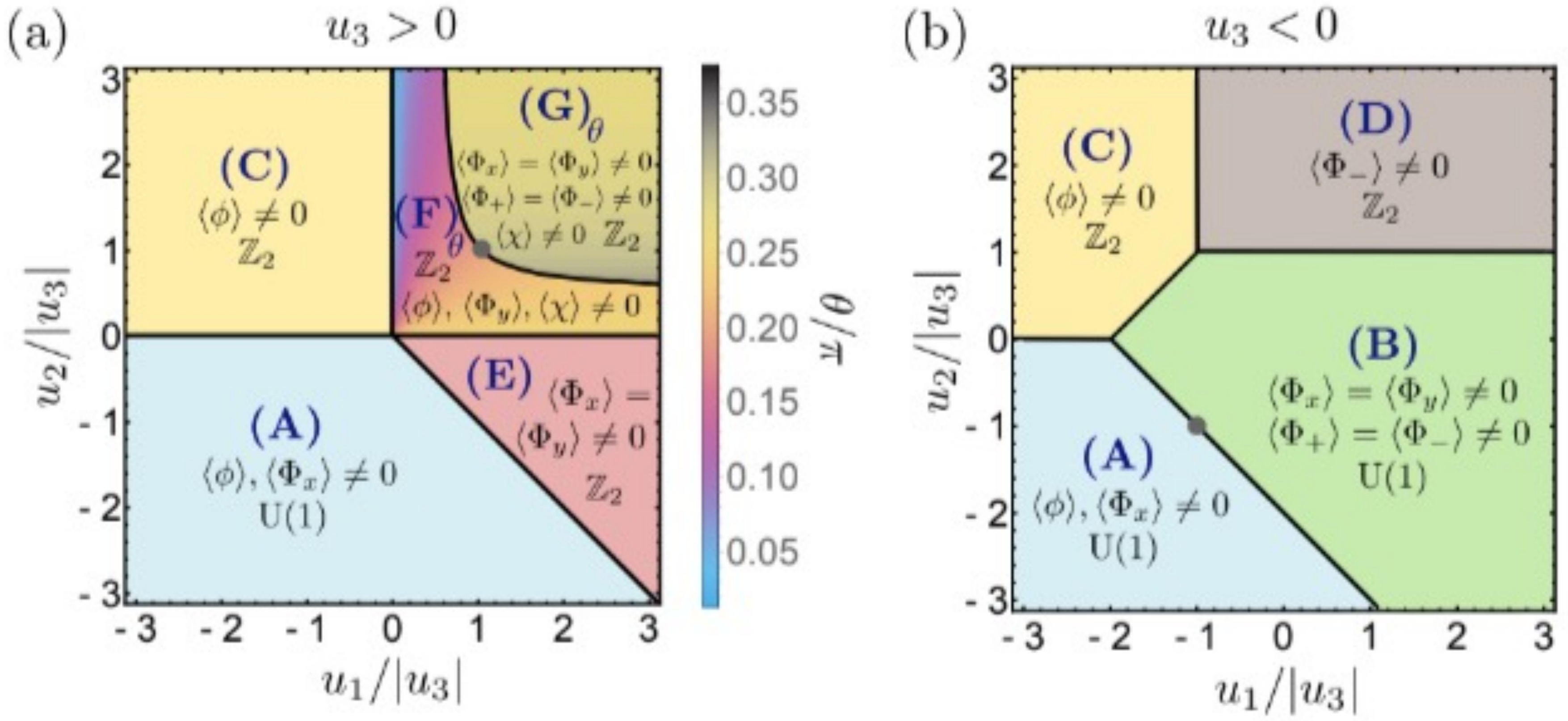
$$\phi = |\boldsymbol{\mathcal{H}}_x|^2 - |\boldsymbol{\mathcal{H}}_y|^2\,,$$
$$\Phi_x = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_x\,,\ \Phi_y = \boldsymbol{\mathcal{H}}_y\cdot\boldsymbol{\mathcal{H}}_y\,,$$
$$\Phi_+ = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_y\,,\ \Phi_- = \boldsymbol{\mathcal{H}}_x\cdot\boldsymbol{\mathcal{H}}_y^*$$

S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)

Previous results

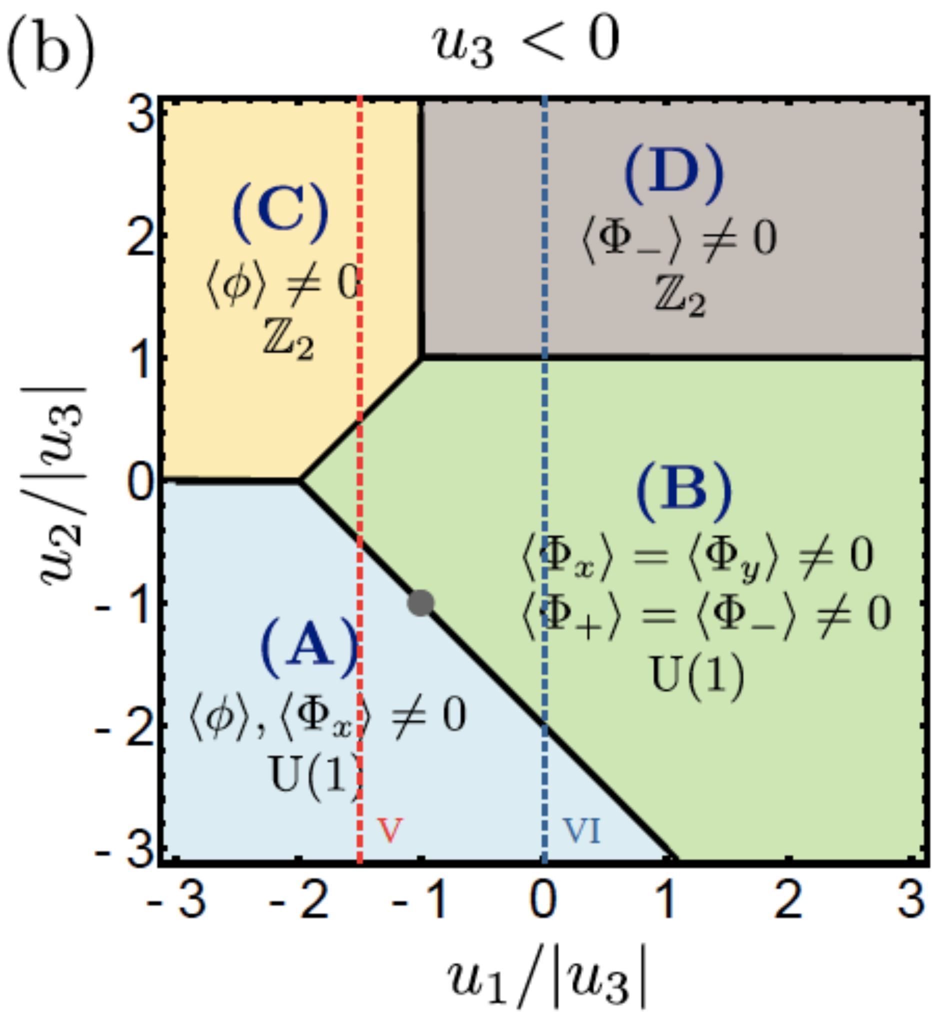
- **Mean field results**

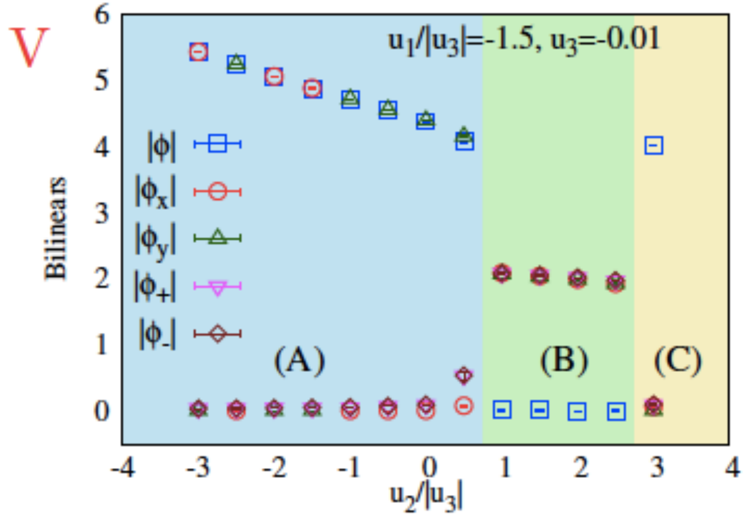
S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)

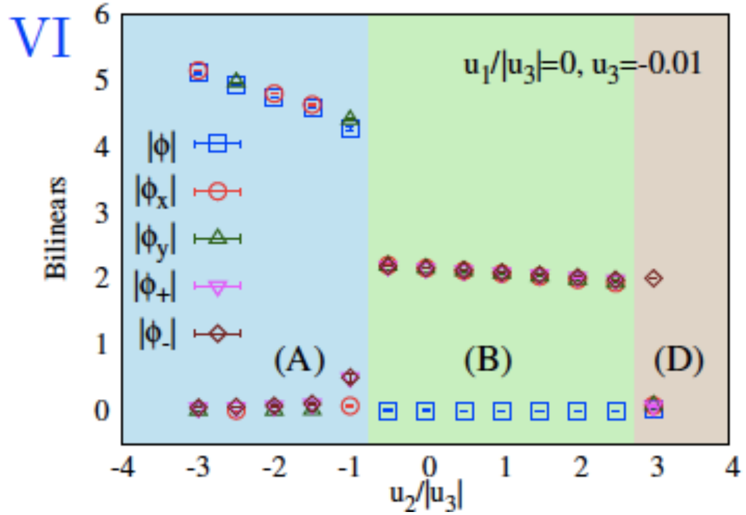


Preliminary results

Lattice results







Summary

☆Two Higgs doublet model on the lattice

- First-time study of complete potential with real coupling in 4d
- Preliminary study of phase diagram and scale setting

➡ To do scale setting, mass measurement code currently being implemented.

☆3d SU(2) gauge with theory with 4 adjoint Higgs

- First-time study of full Higgs potential revealing different broken phase
- Preliminary numerical results qualitatively confirm the mean-field expectations

➡ The critical points and topological order