Physics of multi-critical point principle

Kiyoharu Kawana (Seoul National Univ. → KIAS)

Related works: Hamada, Kawai, KK, Oda, Yagyu, 2102.04617, 2202.04221
Kawai, KK, 2107.10720
Hamada, Kawai, KK, Oda, Yagyu, in preparation

8/29-9/2 2022 @ YITP

Purpose of This talk

- I want to explain basic concept and idea of Multi-critical point principle (MPP)
- will show a few theoretical approaches and (phenomenological) applications
- See also Kawai-san's past talks (KEK-TH, Corfu, …) for more UV theoretical aspects (Matrix model, 2d gravity, string field theory, etc)

＊ Kawai-san will also give a talk in Corfu summer institute (9/7)

MPP以外のこともやっています

- 一次相転移残存物の崩壊による新しいPBH生成機構 [Ke-Pan, KK, arXiv:2106.00111]
- 一次相転移における偽真空残存物の初期profile計算 [Ke-Pan, KK, Philip, arXiv:2202.03439]
- 偽真空残存物の時間発展、終状態の考察 [Ke-Pan, KK, Philip, arXiv:2206.09923]

Ke-Pan

Philip

Introduction

Q: What is quantum mechanics ?

$$\int \mathscr{D}\phi\, e^{iS[\phi,\partial\phi]/\hbar} \quad \longleftrightarrow \quad i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle \ ,$$

Path integral

Hamiltonian picture

- We usually start from canonical picture, i.e. fixed $\hbar$ and coupling constants

  → Fine-tuning problems ! Mass terms m^2, cosmological constant ρ, ・・・

- Why ? Is canonical picture the only way to define quantum theory ?

Lesson from Statistical mechanics

- There are a few equivalent ways

$$\exp(S[E,V,N]) = \sum_n \delta(E_n - E) \quad \text{(Micro-canonical)}$$

$$\exp(-\beta F[T,V,N]) = \sum_n e^{-\beta E_n} \quad \text{(Canonical)}$$

$$\exp(-\beta \Xi[T,V,\mu]) = \sum_{N=0}^{\infty} \sum_n e^{-\beta E_n + \beta \mu N} \quad \text{(Grand-canonical)}$$

All equivalent in thermodynamic limit $V \to \infty$

Lesson from Statistical mechanics

Cont'd

- Equivalence between micro-canonical ↔ canonical

$$\exp(-\beta F[T,V,N]) = \sum_n e^{-\beta E_n} = \int dE e^{-\beta E} \sum_n \delta(E - E_n) = \int dE e^{-\beta E + S}$$

$$= \int dE e^{-V(\beta\epsilon + s)} \,, \quad \epsilon = E/V \,,\ s = S/V \ \text{(densities are fixed)}$$

$$\underset{V\to\infty}{\sim} e^{-(\beta E_* + S)} \,, \quad \left.\frac{dS}{dE}\right|_{E=E_*} = -\beta$$

$$\therefore \ \beta F[T,V,N] = -\beta E_* + S[E_*, V, N]$$

Legendre transformation in thermodynamics !

Example: Van-der Waals Liquid

- Simple (toy) model which describes Liquid ↔ Vapor transition

- Helmhortz free energy per unit particle (N is always fixed in the following)

$$f(n,T) = -\frac{T}{N}\log Z \simeq -T\left[\log(n^{-1} - \underline{b}) + \underline{\frac{\alpha n}{T}} + \cdots\right] \ , \quad n = \text{density}$$

Finite volume effects of molecules — attractive interaction

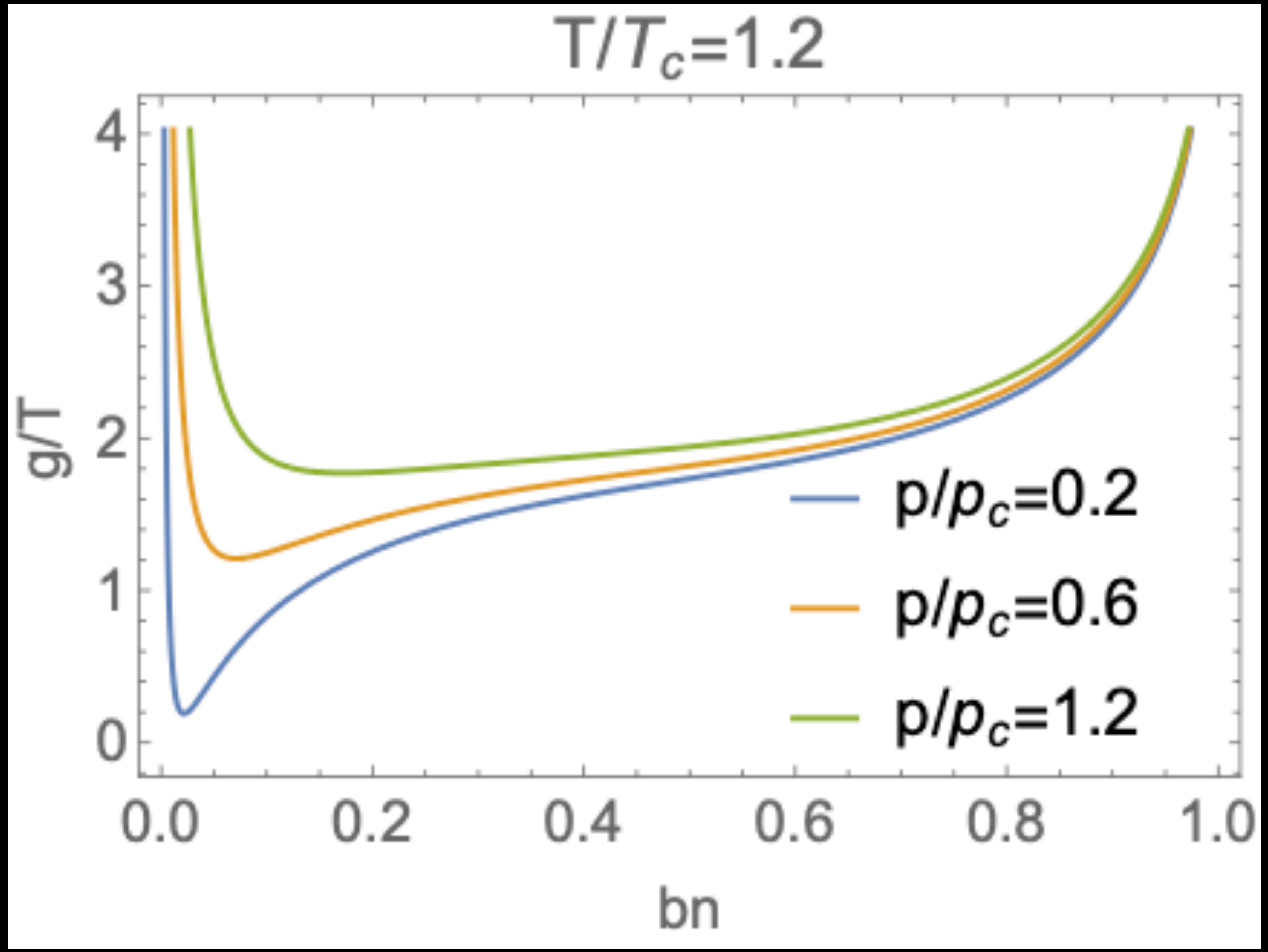
$G = F + pV$

$$\longrightarrow \quad g(n,p) = -T\left[\log(n^{-1} - b) + \frac{\alpha n}{T}\right] + pn^{-1}$$ (Off-shell Gibbs free energy)

- In QFT, this corresponds to effective potential

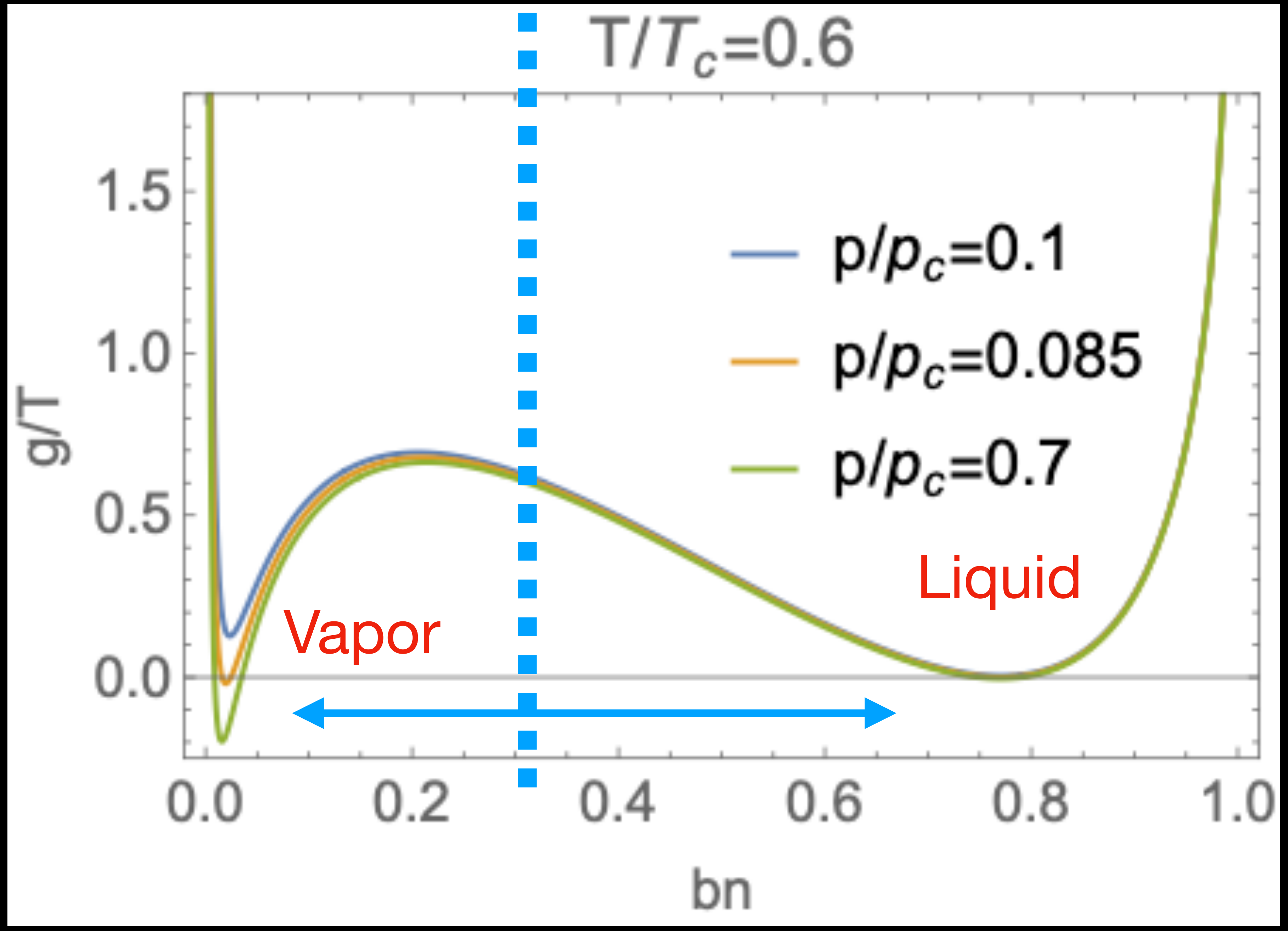
$$n \ \leftrightarrow \ \phi \ , \quad g(n,p) \ \leftrightarrow \ U(\phi,\lambda)$$

$T > T_c$



Unique minimum

$T < T_c$



two minima

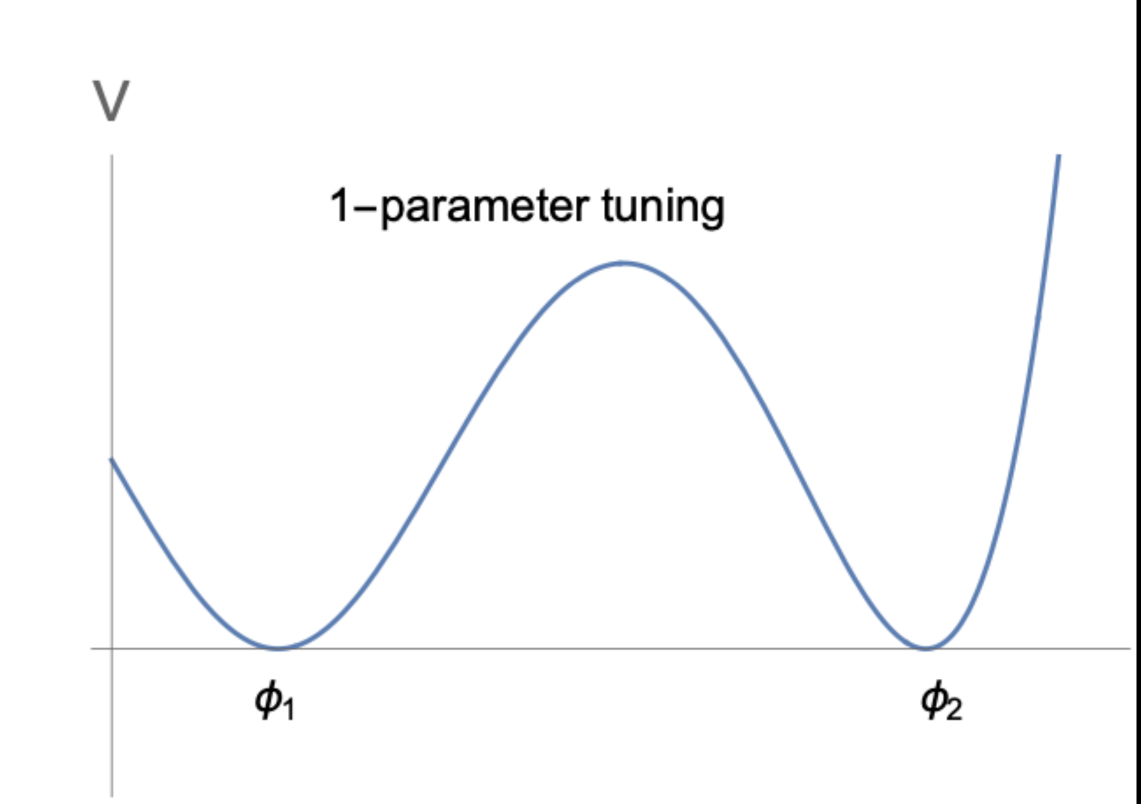
- Potential changes as we vary pressure p (coupling λ) ↔ Same behavior as QFT potential
- In the intermediate density region (blue dotted line), the system is coexisting phase
  → (First-order) phase transition point is most likely realized and p is automatically tuned

Idea of Multi-critical point principle

- Intensive variables (示強変数) are likely to be fixed at the point where different phases (state) of a system degenerate = Multi-critical point
- In QFT, intensive variables are coupling constants, and phase of a system is determined by effective potential, $U(\phi, \lambda)$ c.f. $g(n, p)$
- Considering this, it is natural to think that coupling constants are likely to be fixed at multi-critical (degenerate) point when we start from micro-canonical picture

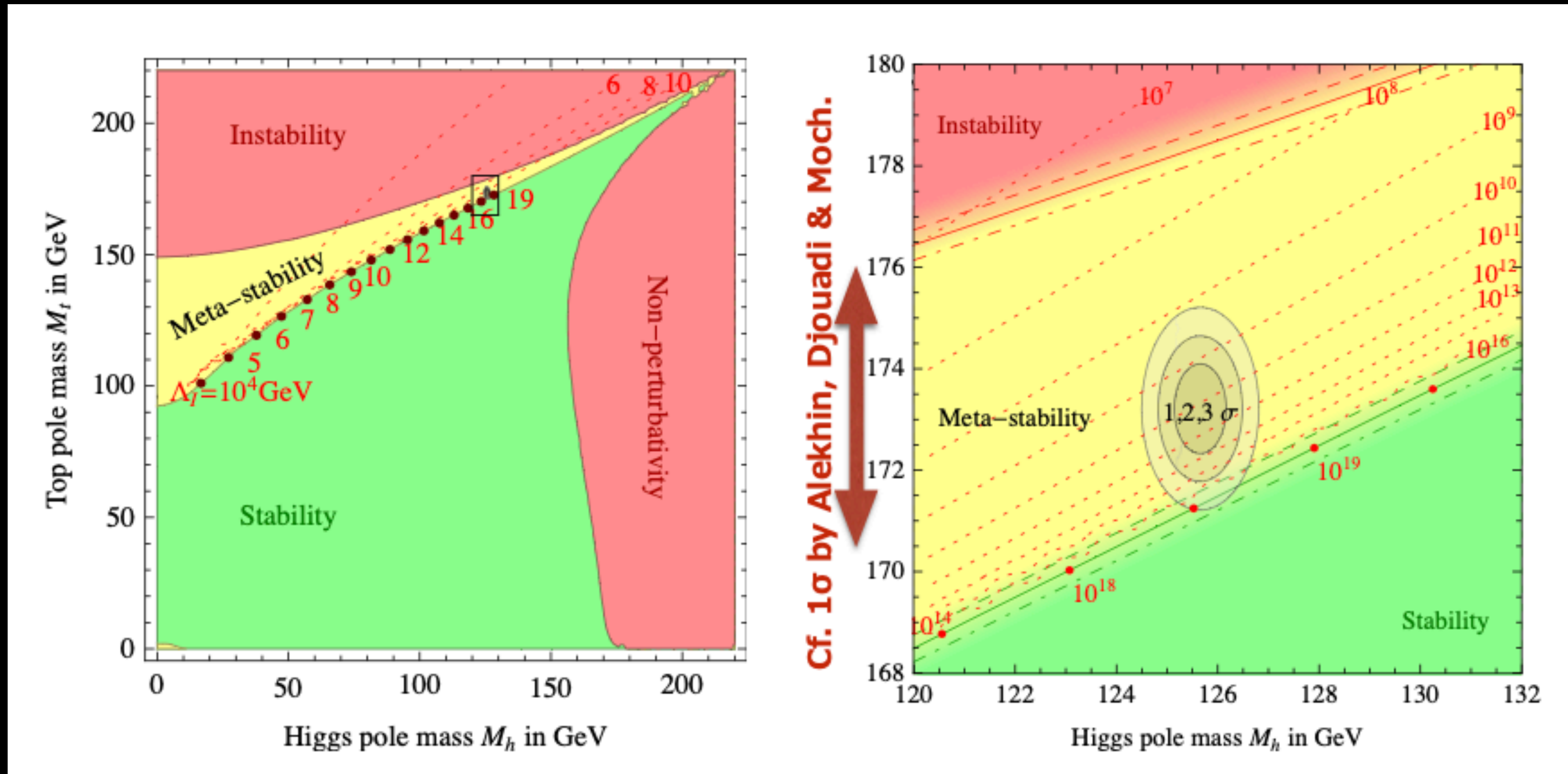
$$T, p, \mu \quad \leftrightarrow \quad g_i$$

$$e^{-H/T+\mu N} \quad \leftrightarrow \quad e^{ig_i S_i}$$

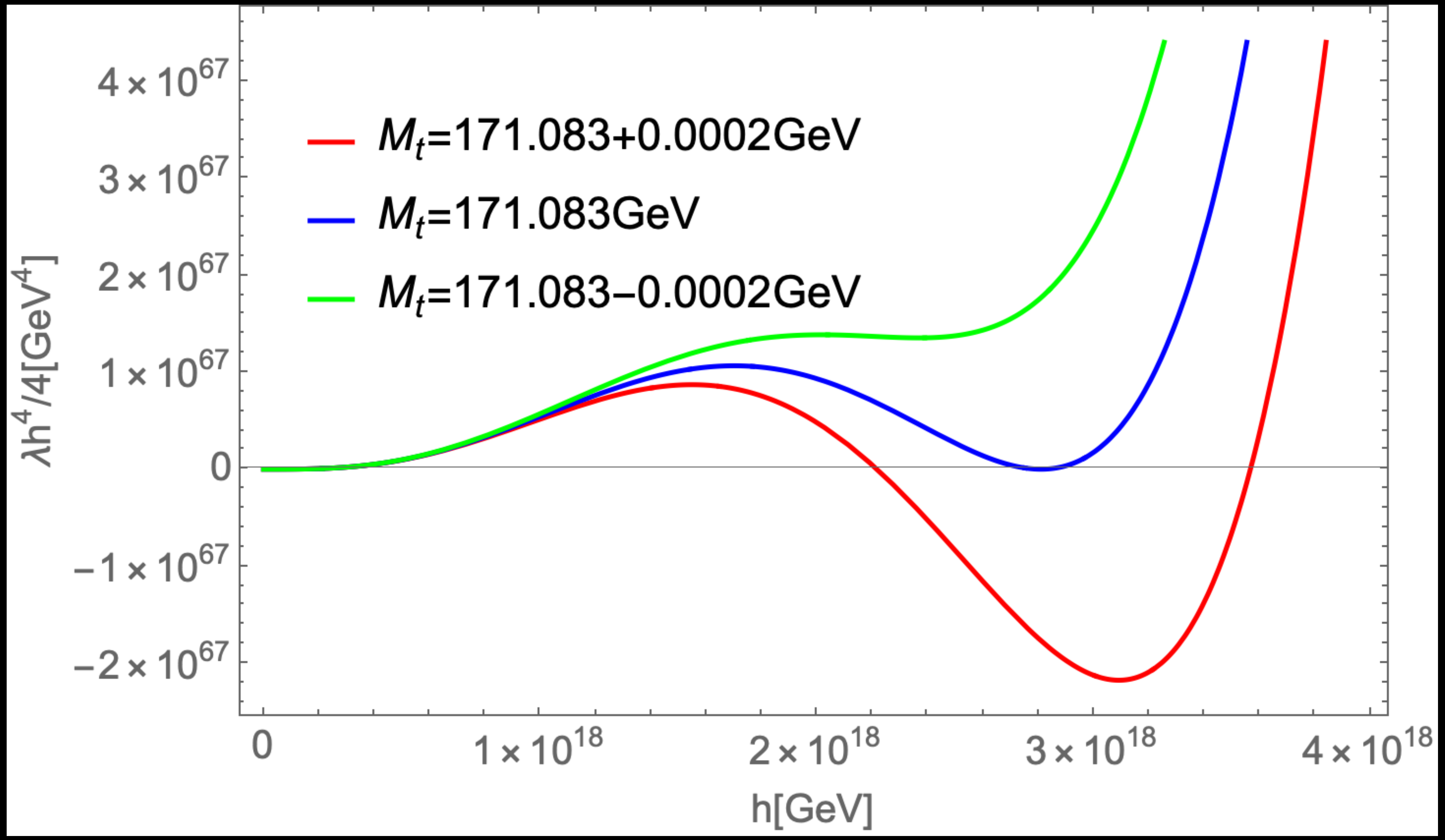


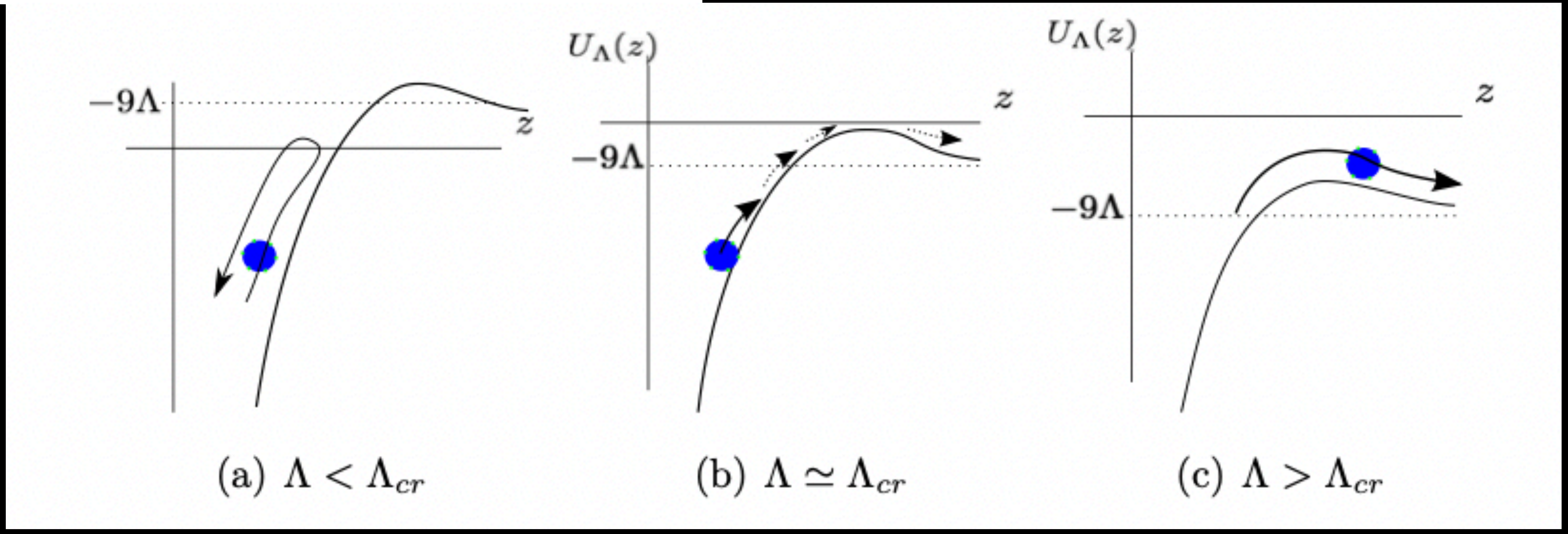
← degenerate vacua

メッセージ 1: 自然は”(多重)臨界点”(キワキワ)を選びたがる

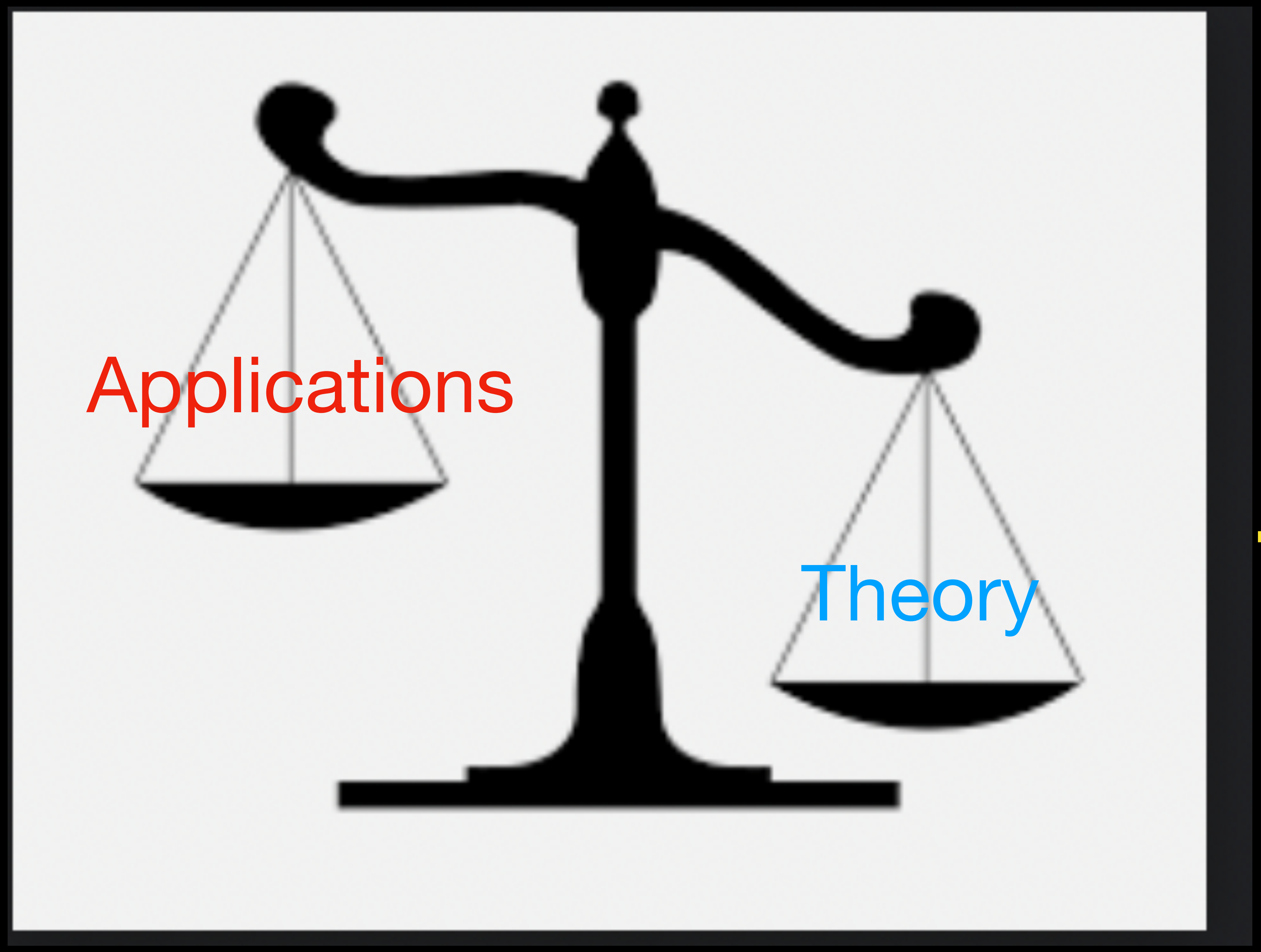


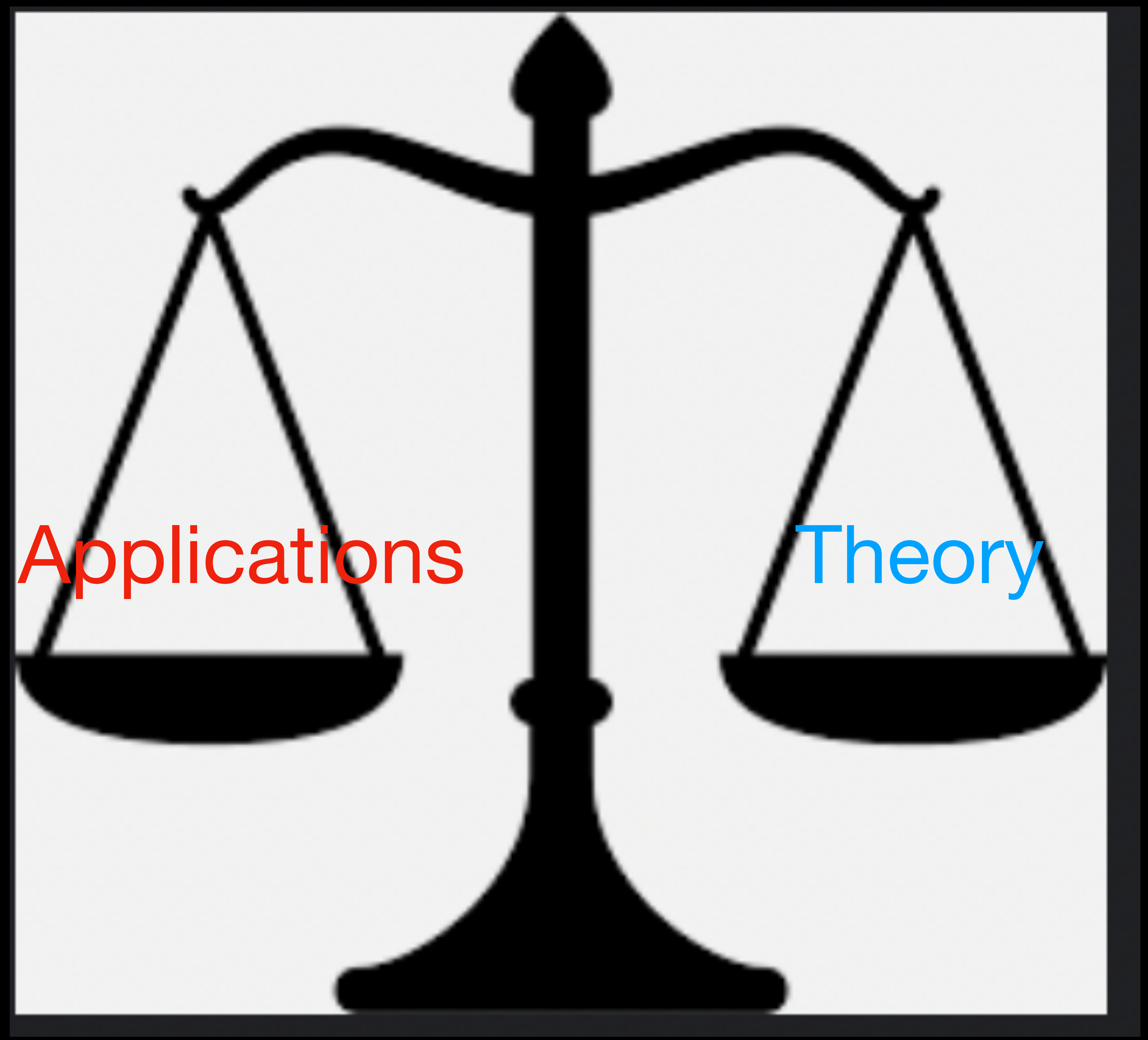
[Buttazzo et al. 1307.3536]





メッセージ2: MPPはまだまだ未熟 (だからこそ面白い)





これまでみんな"MPP"が正しければ
現象論、宇宙論的に何が言えるか?
にフォーカスしてきた

より理論的な側面に力をいれるべき

Outline

1. Various approaches to Micro-canonical QFT
2. Implications of MPP
3. Summary

Various approaches

Non-locality is important

- Micro-canonical QFT [A. Struminger ('82); Froggatt, Nilesen, Bennet, ('95)]
- Wormhole effects (α-parameter) [Hawking ('87),Giddings-Strominger ('88); Coleman ('88)]
- Coleman's baby universe theory [S. Coleman ('88): H. Kawai and T. Okada ('11)]
- Multi-local theory [H.Kawai, Y. Asano, A. Tsuchiya ('12); Kawai ('13)]

They are resemble each other in that they all predict ensemble average of coupling constant

$$\Omega = \int d\lambda f(\lambda) \int \mathscr{D}\phi e^{i\lambda S}$$

A. Strominger (1983)

Microcanonical Quantum Field Theory

ANDREW STROMINGER

The Institute for Advanced Study, Princeton, New Jersey 08540

Received May 24, 1982; revised September 27, 1982

Euclidean quantum field theory is equivalent to the equilibrium statistical mechanics of classical fields in 4 + 1 dimensions at temperature $\hbar$. It is well known in statistical mechanics that the theory of systems at fixed temperature is embedded within the more general and fundamental theory of systems at fixed energy. We therefore develop, in precise analogy, a fixed action (microcanonical) formulation of quantum field theory. For the case of ordinary renormalizable field theories, we show (with one exception) that the microcanonical is entirely equivalent to the canonical formulation. That is, for some particular fixed value of the total

A. Strominger (1983)

[MICROCANONICAL QUANTUM FIELD THOERY, Annals phys. 146, 419]

- He studied the equivalence between canonical partition function $Z(\hbar)$ and micro-canonical partition function $\Omega(A)$ with fixed total action

$$Z(\hbar) = \int \mathscr{D}\phi e^{-S_E/\hbar} \quad \longleftrightarrow \quad \Omega(A) = \int \mathscr{D}\phi \delta(S_E - A)$$

equivalent

total action is fixed

- Formally, we can write $Z(\hbar)$ as

$$Z(\hbar) = \int \mathscr{D}\phi e^{-S_E/\hbar} \int dA \delta\left(S_E - A\right) = \int dA \exp(-\hbar^{-1}A + \ln \Omega(A))$$

Extremum is given by $\hbar^{-1} = \dfrac{\partial \ln \Omega(A)}{\partial A}$

- As an example, let’s consider free scalar theory.

$$S_E = \frac{1}{2} \sum_{i=F/2}^{F/2} (p_i^2 + m^2) |\tilde{\phi}_i|^2$$

Fourier mode, F+1=dof

$$\Omega(A) = \int \mathscr{D}\phi \, \delta\left(\frac{1}{2} \sum_{i=F/2}^{F/2} (p_i^2 + m^2) |\tilde{\phi}_i|^2 - A \right)$$

$$a_i := (p_i^2 + m^2)^{1/2} \tilde{\phi}_i$$

$$= \prod_i \frac{1}{\sqrt{p_i^2 + m^2}} \int \prod da_i \delta\left(\frac{1}{2} \sum_{i=F/2}^{F/2} |\tilde{a}_i|^2 - A \right)$$

← Surface of F-dim sphere !

$$\propto \frac{A^{F/2}}{\Gamma(\frac{F+1}{2})} \longrightarrow \hbar^{-1} = \frac{\partial \ln \Omega}{\partial A} = \frac{F/2}{A}$$

∴ Finite $\hbar$ is allowed in thermodynamic limit $F \sim A \to \infty$

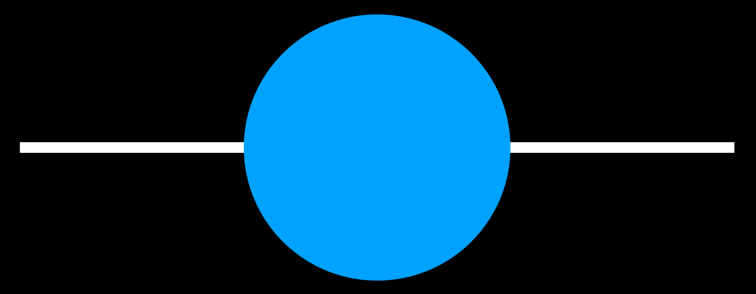
- Gaussian integral around the extremum is

$$Z(\hbar) = e^{-\hbar^{-1}A_* + \ln \Omega(A_*)} \int dA \exp\left(\frac{1}{2} \frac{\partial^2 \ln \Omega(A)}{\partial A^2}\bigg|_{A=A_*} (A - A_*)^2 + \cdots \right) \quad \text{where} \quad \frac{\partial^2 \ln \Omega(A)}{\partial A^2}\bigg|_{A=A_*} = -\frac{2}{F\hbar^2},$$

$$\sim e^{-\hbar^{-1}A_* + \ln \Omega(A_*)} (F\hbar^2)^{1/2}$$

$$\therefore \lim_{F\to\infty} \frac{1}{F} \ln Z(\hbar) = -\lim_{F\to\infty} \left[-\hbar^{-1}\frac{A_*}{F} + \frac{1}{F}\ln \Omega(A_*) + \mathcal{O}\left(\cancel{\frac{\ln F}{F}}\right) \right]$$

✳ In the paper, he also discussed the equivalence in ϕ^4 theory by assuming spectral representation of 2-point function and unitarity

 $= \int_0^\infty d\mu^2 \frac{f(\mu^2)}{p^2 + \mu^2}$ where $\int_0^\infty d\mu^2 f(\mu^2) = 1$

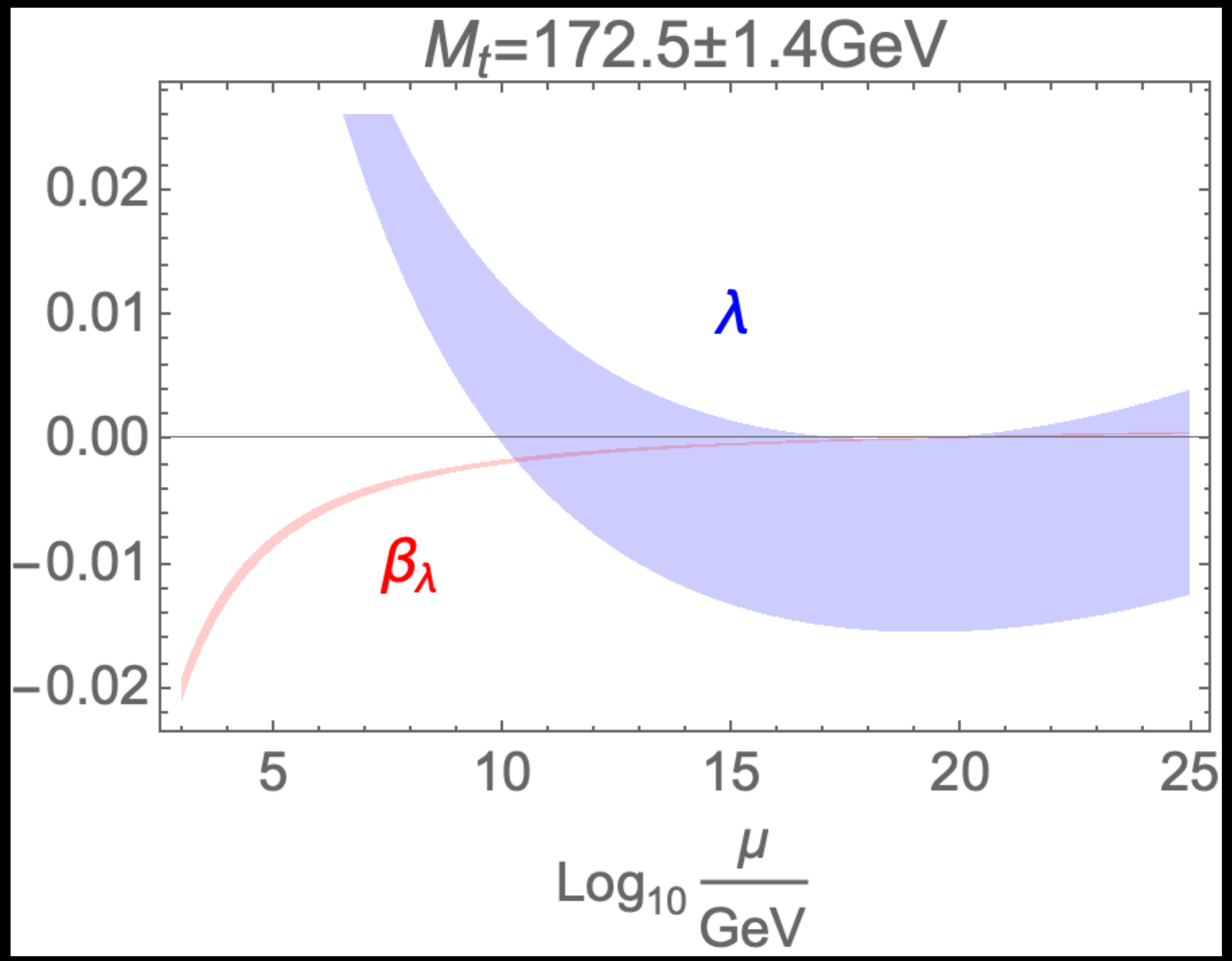
Froggatt-Nielsen (1996)

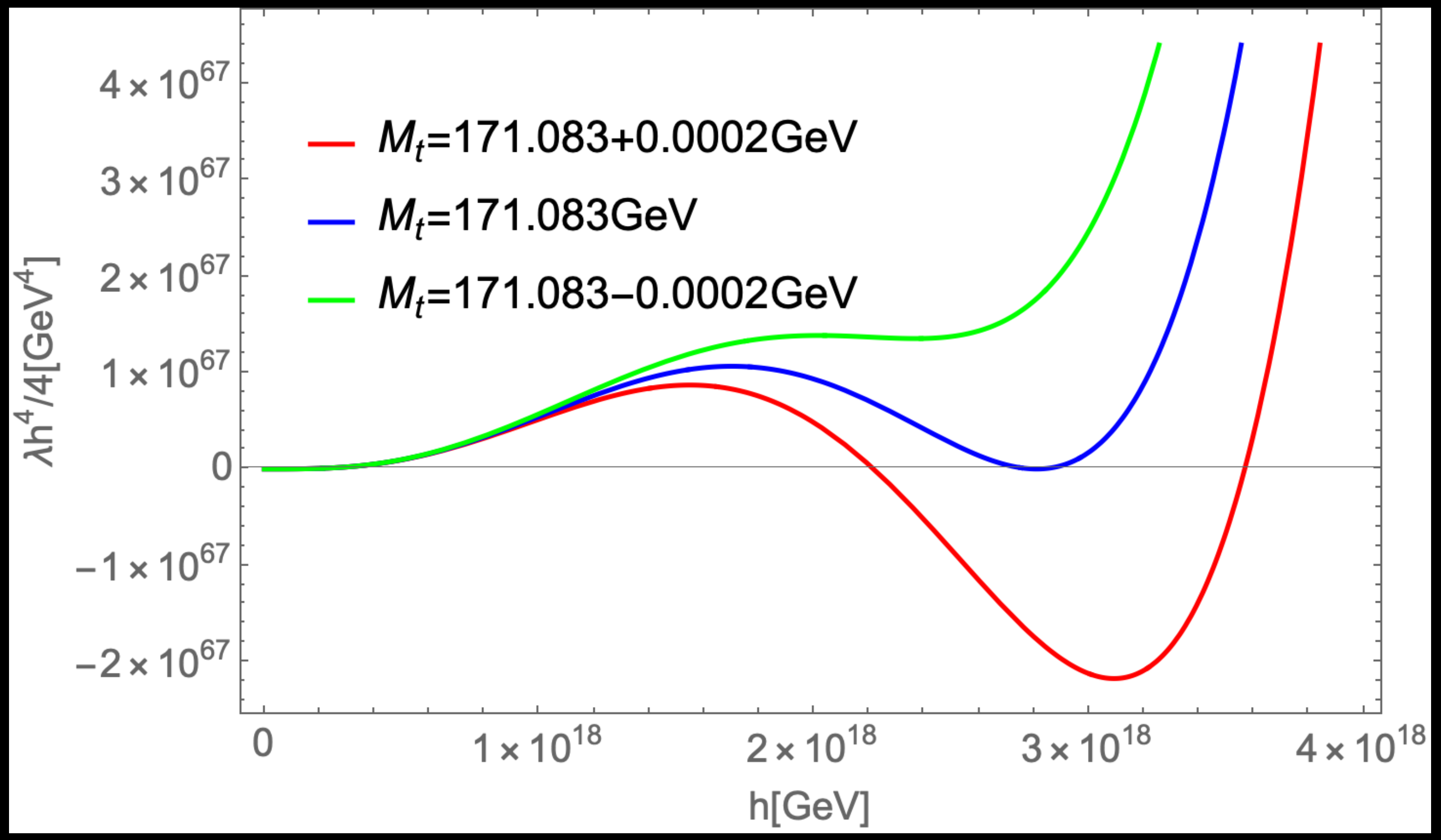
[Phys.Lett. B368 (1996)]

- Generalization of Strominger's idea to general coupling constants

$$Z(g) = \int \mathcal{D}\phi \, e^{i\sum g_i S_i} \longleftrightarrow \Omega(A) = \int \mathcal{D}\phi \prod_i \delta(A_i - S_i)$$

- Motivated by Standard Model Higgs potential





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Froggatt-Nielsen (1996)

[Phys.Lett. B368 (1996)]

I = some constant given by God

- They started from partition function with fixed mass action

$$\Omega(I) = \int \mathscr{D}\phi e^{iS(\phi)} \delta\left(I - \int d^4x\phi^2\right) = \int dm^2 \underline{\int \mathscr{D}\phi \exp\left(iS(\phi) + im^2\left(\int d^4x\phi^2 - I\right)\right)}$$

$$= \int dm^2 \exp(\ln Z(m^2))$$

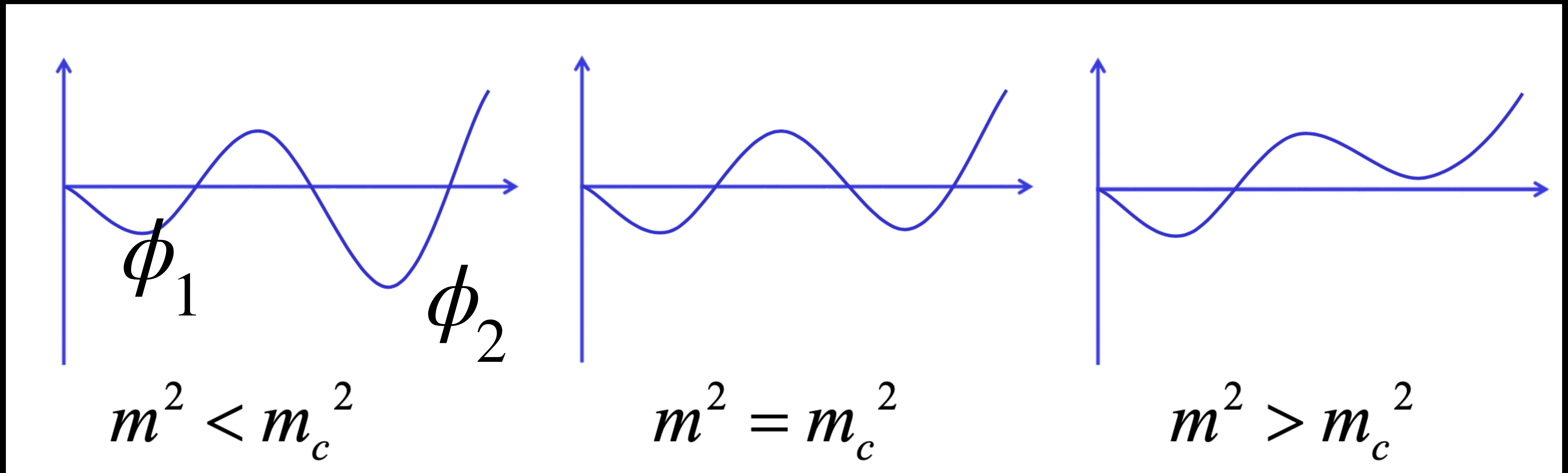
Ordinary partition function in QFT

- Where is the extremum of $\ln Z(m^2)$?

$$\frac{d \ln Z(m^2)}{dm^2} = \frac{1}{Z(m^2)} \frac{dZ(m^2)}{dm^2} = \frac{1}{Z(m^2)} \int \mathscr{D}e^{iS(\phi)} \left(\int d^4x\phi(x)^2 - I\right) = \left\langle \int d^4x\phi(x)^2 \right\rangle - I = 0 \ ,$$

Q: Where is m^2 fixed ?

- Let us further assume that the effective potential has two minima $\phi_1,\ \phi_2$, and they degenerate at some critical value $m^2 = m_c^2$



- And also assume the inequality $\phi_1^2 < I/V_4 < \phi_2^2$

In this case, $m^2 = m_c^2$ corresponds to the extremum point $\left\langle \int d^4x\phi(x)^2 \right\rangle - I = 0$

$\therefore$ The extremum corresponds to the critical point $m^2 = m_c^2$!

Cont’d

Standard model criticality prediction top mass 173 ± 5 GeV and Higgs mass 135 ± 9 GeV

C.D. Froggatt [a], H.B. Nielsen [b]

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[b] *The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

Received 4 November 1995
Editor: P.V. Landshoff

Abstract

Imposing the constraint that the Standard Model effective Higgs potential should have two degenerate minima (vacua), one of which should be – order of magnitude wise – at the Planck scale, leads to the top mass being 173 ± 5 GeV and the Higgs mass 135 ± 9 GeV. This requirement of the degeneracy of different phases is a special case of what we call the multiple point criticality principle. In the present work we use the Standard Model all the way to the Planck scale, and do not introduce supersymmetry or any extension of the Standard Model gauge group. A possible model to explain the multiple point criticality principle is lack of locality fundamentally.

- Based on this principle, Froggatt and Nielsen predicted the Higgs and top masses

$$m_H = 135 \pm 9 \text{ GeV}, \quad m_t = 173 \pm 5 \text{ GeV}$$

Strong predictability of MPP !

- Their discussion seems convincing. But it is not enough because they just checked that the extremum corresponds to the degenerate point….

$$Z(m^2) = \int \mathscr{D}\phi e^{iS(\phi)+im^2\int d^dx\phi^2} \overset{?}{\longleftrightarrow} \Omega(I) = \int \mathscr{D}\phi e^{iS(\phi)}\delta\left(I - \int d^4x\phi^2\right)$$

We need to check the equivalence more explicitly

→ Ongoing work with Kawai-san, Oda-san, Yagyu-san

[Kawai, Oda, K.K, Yagyu, in preparation]

- Free scalar theory with fixed quadratic mass term

$$\Omega(A) = \int \mathscr{D}\phi e^{-S_E} \delta\left(A - \frac{1}{2}\int d^d x \phi^2(x)\right) \qquad S_E = \frac{1}{2}\int d^d x \phi\left(-\partial^2 + \mu^2\right)\phi$$

$$= \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi} \int \mathscr{D}\phi \exp\left(-S_E + i\kappa\left(A - \frac{1}{2}\int d^d x \phi^2(x)\right)\right)$$

renormalized mass

$$= \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi} \exp\left[V_d(i\kappa a - f(\kappa))\right], \qquad a = A/V_d, \quad m^2 = \mu^2 + i\kappa$$

where

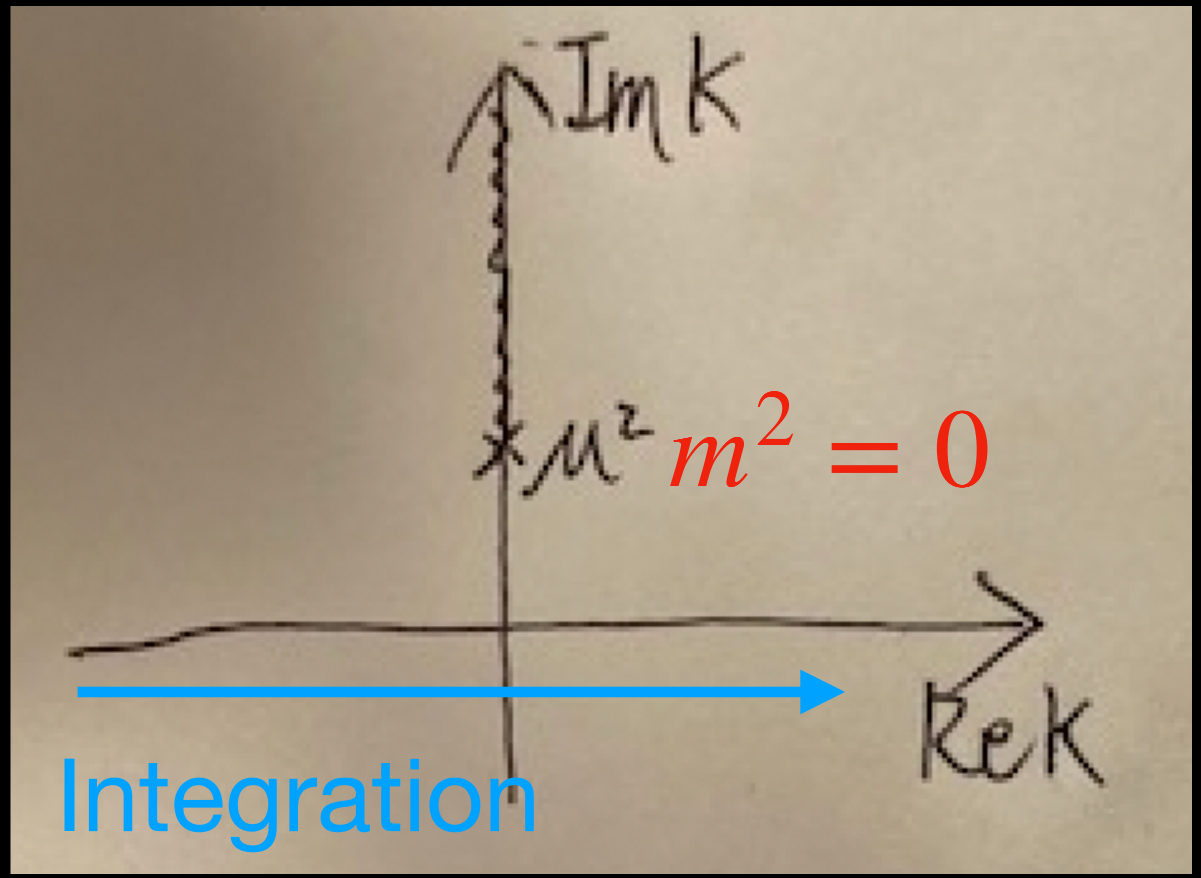
$$f(\kappa) = \frac{1}{2}\int \frac{d^d p}{(2\pi)^d} \log(p^2 + \mu^2 + i\kappa) \underset{d=4}{=} \frac{c_4}{2}\Lambda^4 \left[2\frac{m^2}{\Lambda^2} + 2\frac{m^4}{\Lambda^4}\log m^2 + 2\left(1 - \frac{m^4}{\Lambda^4}\right)\log(\Lambda^2 + m^2)\right]$$

cut off regularization

Cont'd

$$(\text{exponent}) = i\kappa a + \frac{c_d}{2}\Lambda^4\left[2\frac{m^2}{\Lambda^2} + 2\frac{m^4}{\Lambda^4}\log m^2 + 2\left(1 - \frac{m^4}{\Lambda^4}\right)\log(\Lambda^2 + m^2)\right], \quad m^2 = \mu^2 + i\kappa$$

- κ plane



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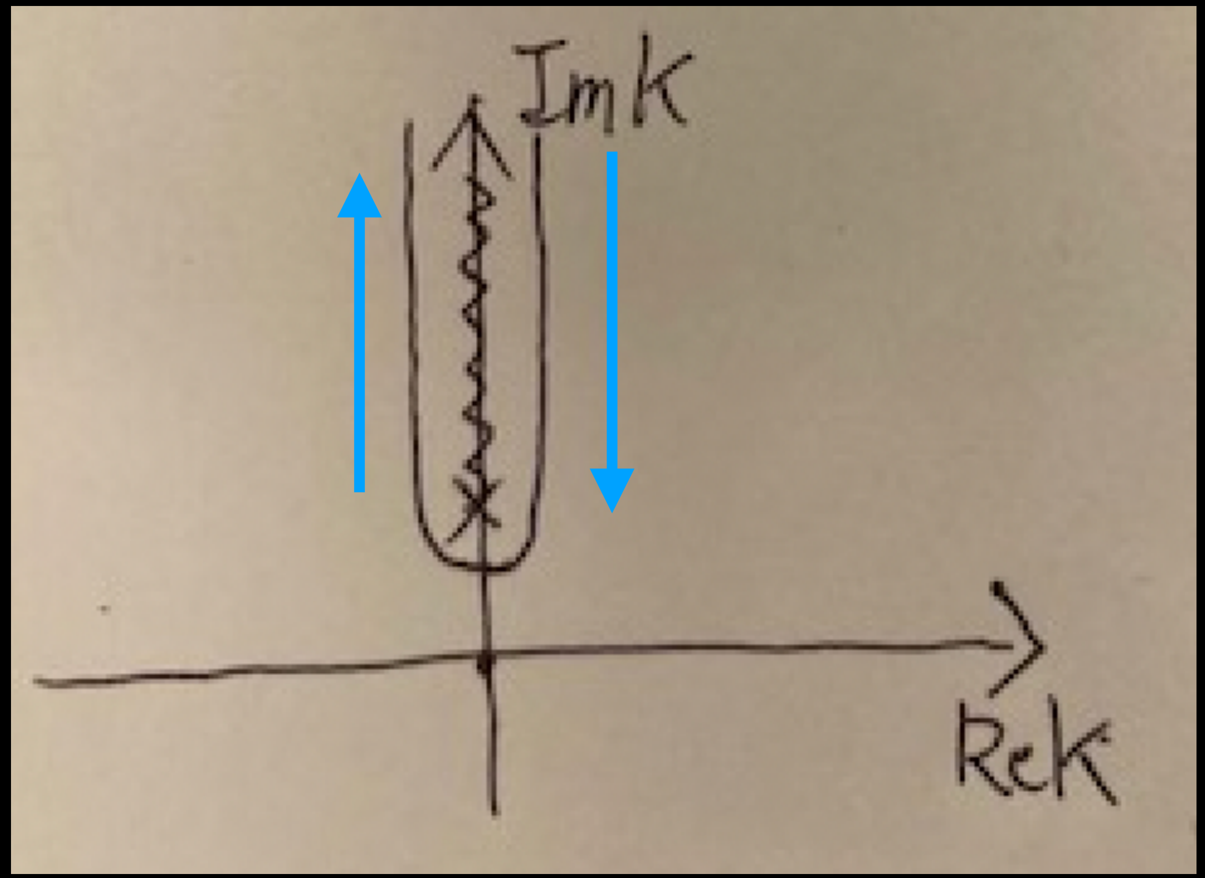


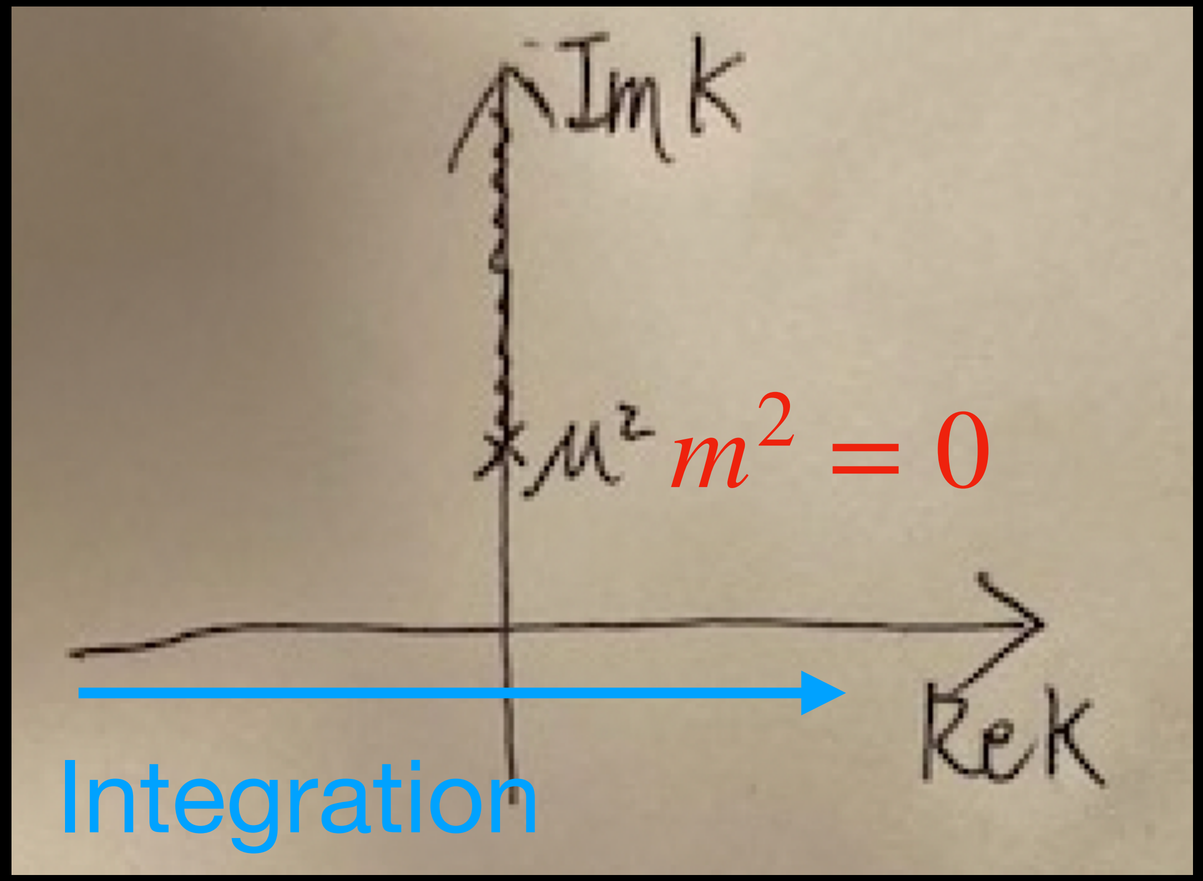
$$f(\kappa) = \frac{c_4\Lambda^4}{2}\log(i\kappa) + \cdots \qquad \text{for } |\kappa| \to \infty$$

$$e^{i\kappa a - f(\kappa)} < e^{-aR - \frac{1}{2}\log R} < e^{-aR} \underset{R\to\infty}{\longrightarrow} 0$$

Integration on half circle is negligible for $R \to \infty$

Cont'd

- κ plane



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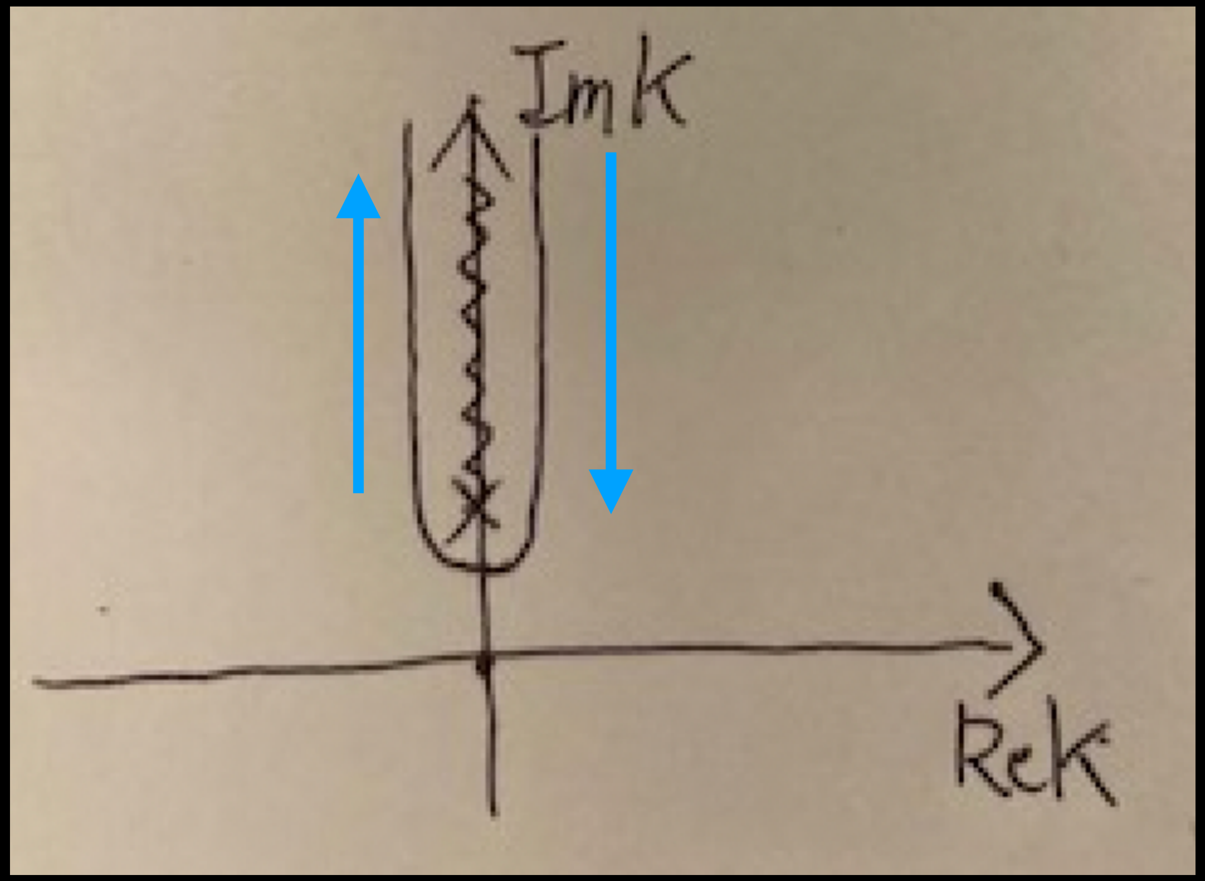


- Integration

$x = -m^2/\Lambda^2$

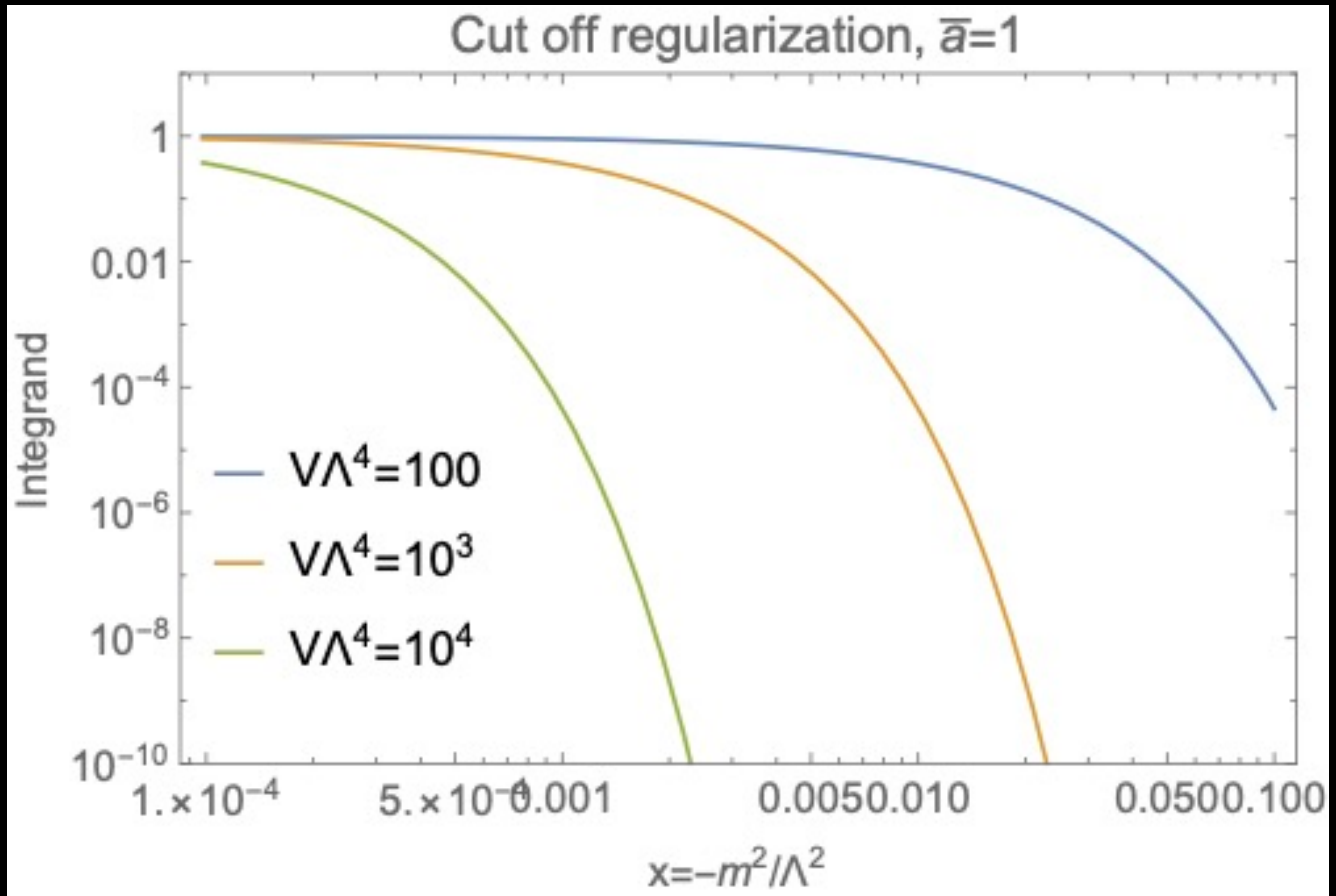
Normalized by Λ^2

$$\Omega(A) = \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi} \exp\left[V_d(i\kappa a - f(\kappa))\right] = \Lambda^2 \int_0^{\infty} \frac{dx}{2\pi} \exp\left[-\bar{V}_4 \times (x\bar{a} - \mathrm{Re}\bar{f})\right] \left(1 - \cancel{e^{i\pi V_4 \cdots}}\right)$$

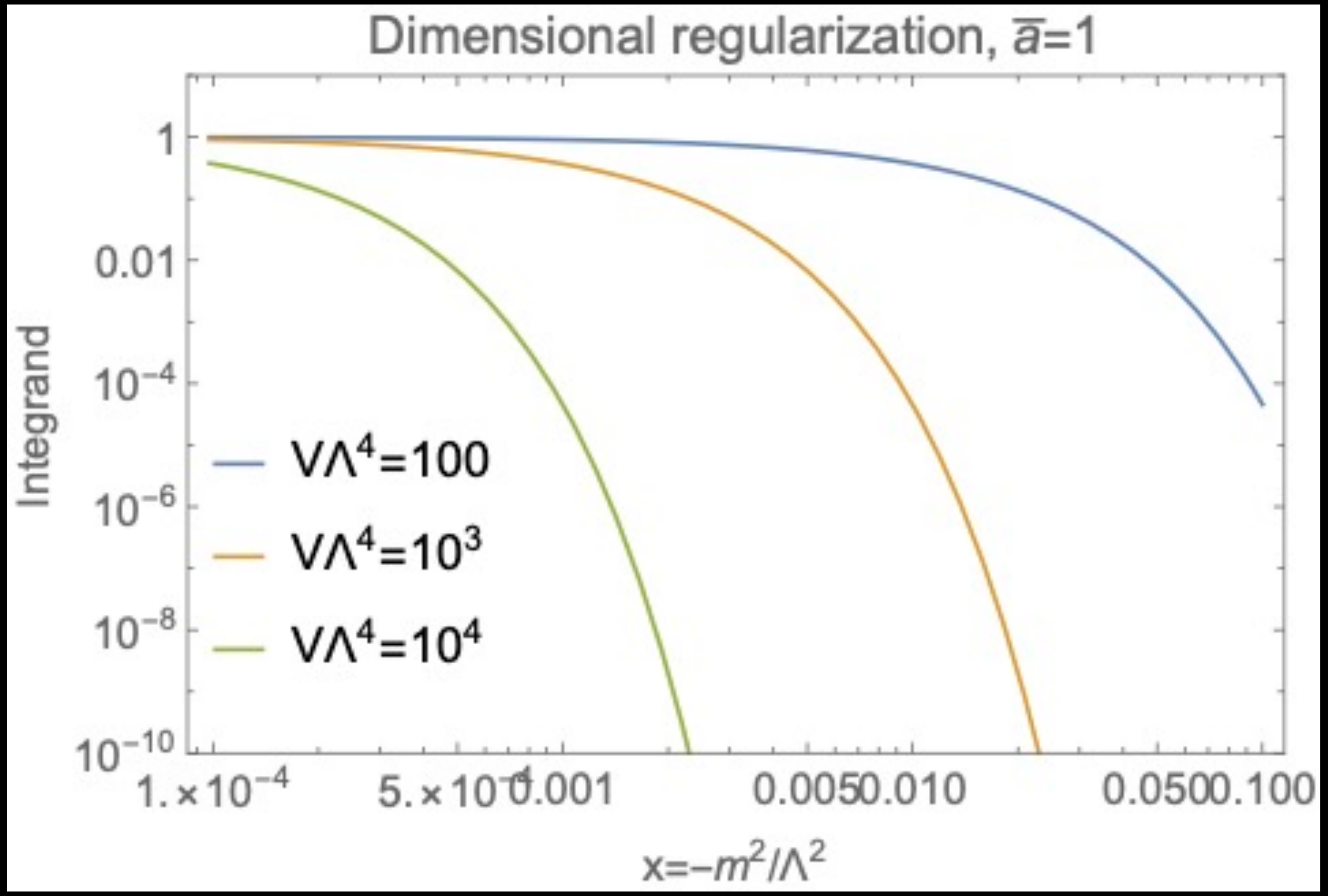
$$= \Lambda^2 \int_0^{\infty} \frac{dx}{2\pi} e^{-\bar{V}_4 \times (\bar{a} - c_4)x - \cdots}$$

$\therefore$ Integration is dominated by the small region, $x \lesssim 1/\bar{V}_4 \;\leftrightarrow\; m^2 \lesssim \Lambda^2/V_4$

Integrand as a function of renormalized mass



Cut-off regularization



Dimensional regularization

- More explicitly,

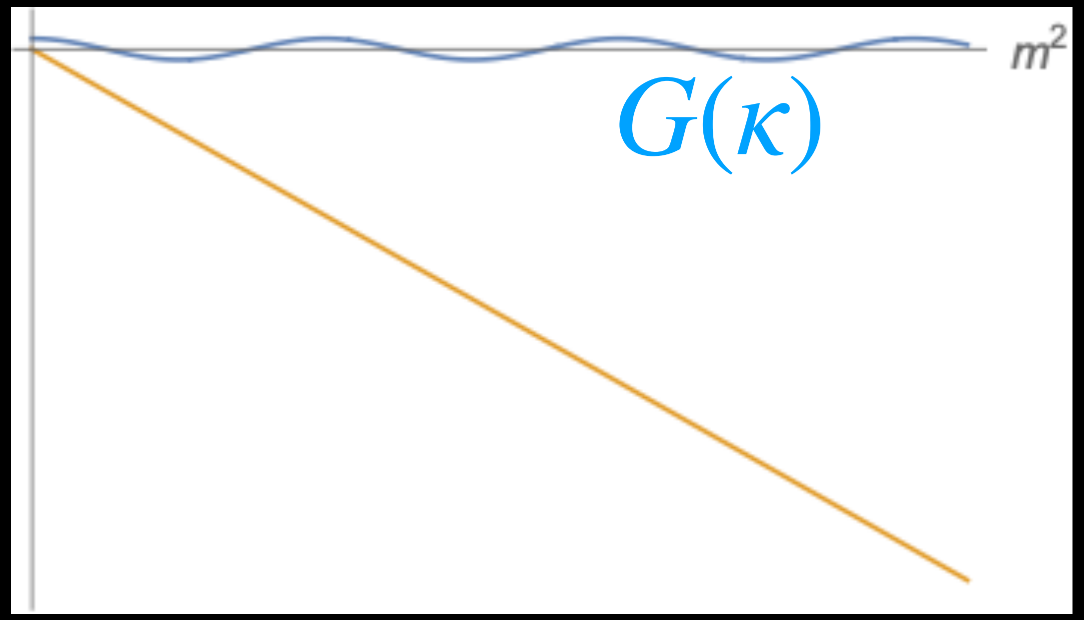
$$\lim_{V\to\infty}\frac{1}{V}\ln\Omega(A) = \lim_{V\to\infty}\frac{1}{V}\ln Z(m^2=0) + \mathcal{O}\left(\frac{\ln V}{V}\right) \qquad \lim_{V\to\infty} e^{-Vx} = \frac{1}{V}\delta(x)\,,$$

- How about correlation functions ? → Introducing source term

$$\Omega[A,J] = \int_{-\infty}^{\infty}\frac{d\kappa}{2\pi}\underline{e^{iVa\kappa}} \times \underbrace{Z(\kappa)\exp\left(\int d^dx_E J(-\Box+m^2)^{-1}J\right)}_{:=\,G(\kappa)}$$

?

$$\simeq Z(m^2)\exp\left(\int d^dx_E J(-\Box+m^2)^{-1}J\right)\Bigg|_{m^2=0}$$



$\ast$ Still work in progress. But, this would hold as long as $J(x)$ is finite supported

Cont'd

More things to do

- In free scalar theory, mass is fixed at zero (at least in partition function)

  = Realization of clasical conformality ! [Bardeen (95); Iso, Orikasa (09)]

- What happens if we add interaction $\lambda\phi^4$? → Seems m^2 is still fixed at 0 (Wait for our paper)
- Rederivation of original Froggatt-Nielsen MPP
- Fermion, Gauge theory, Gravity, $\cdots$
- Other naturalness problems

For now, let's accept MPP and see its implications for particle physics and cosmology

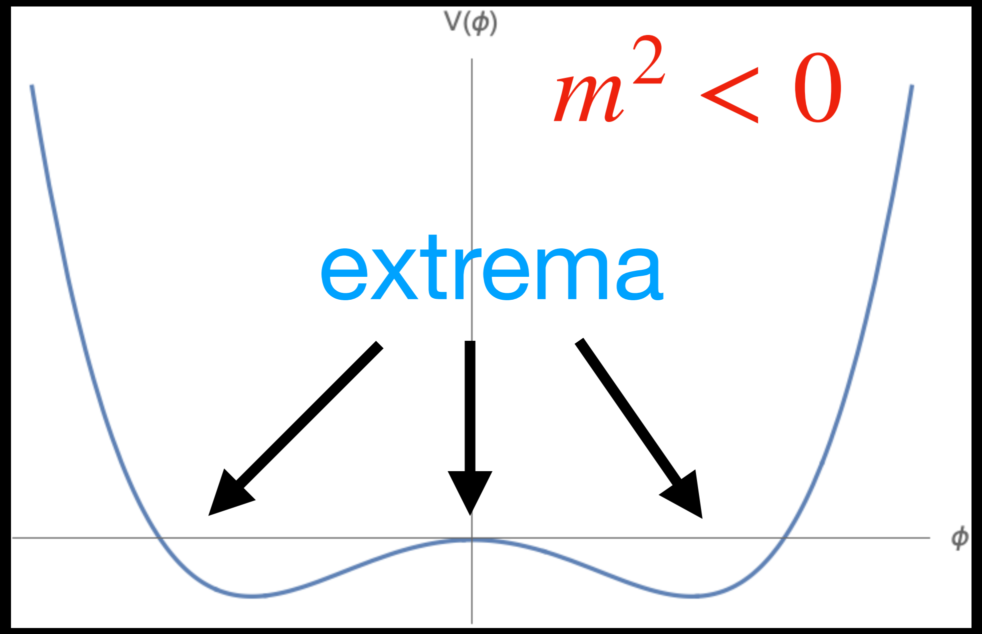
Outline

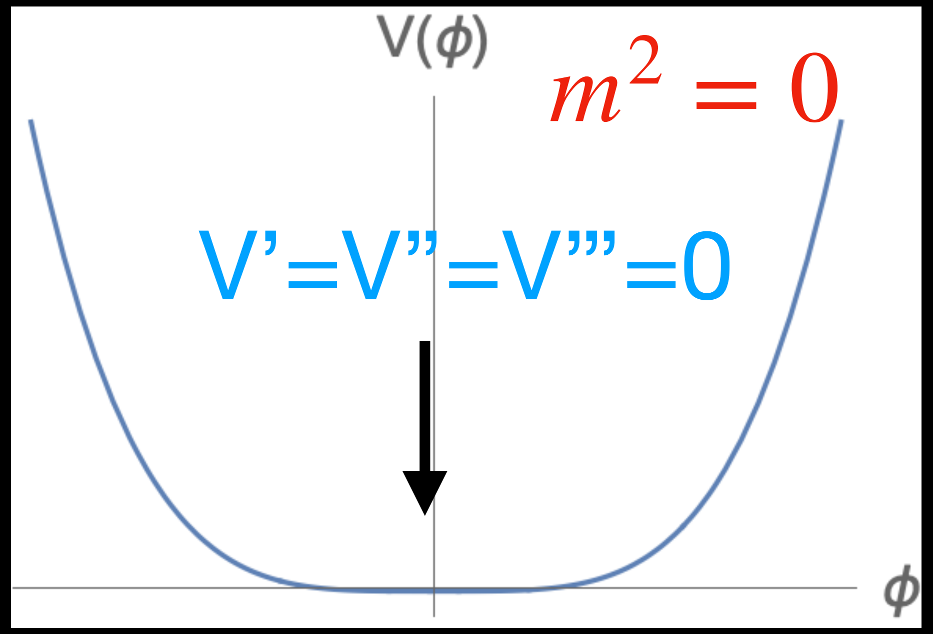
1. Various approaches to Micro-canonical QFT
2. Implications of MPP
3. Summary

1. Generalization of Classical Conformality

H. Kawai, K.K ('21), PTEP 2022 1, 013B11, arXiv:2107.10720

- Classical Conformality can be interpreted as the degeneracy between multiple extrema

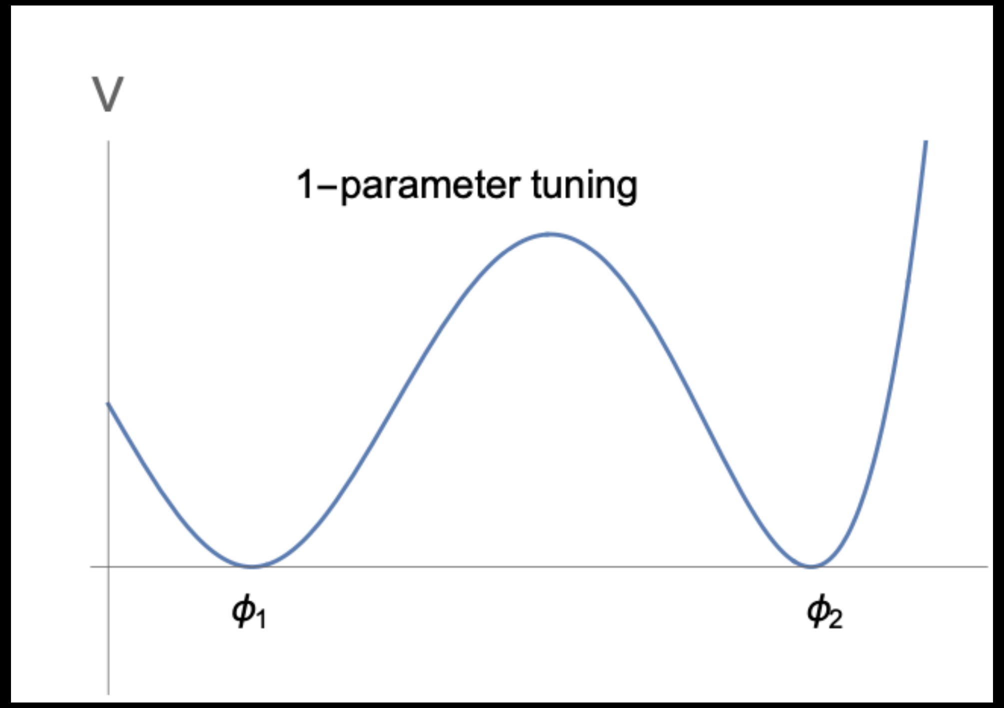




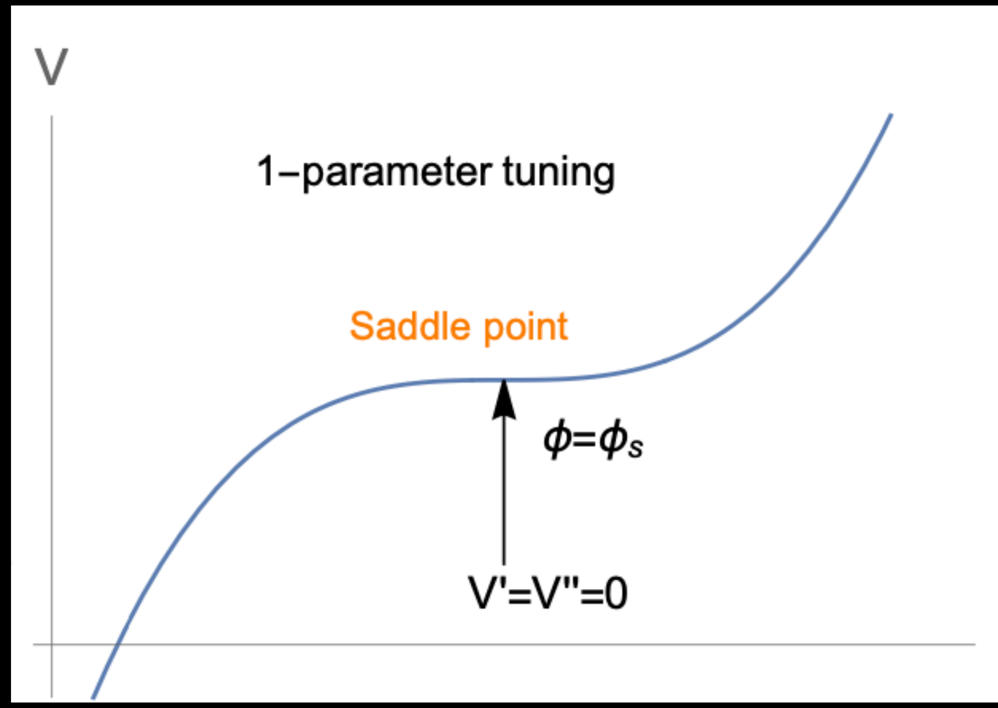
- More generally, coupling constants would be fixed at the point where different extrema become degenerate = multi-critical point

e.g.)

Degeneracy between two minima → Original MPP by Froggatt- Nielsen



Degeneracy between two adjacent extrema →



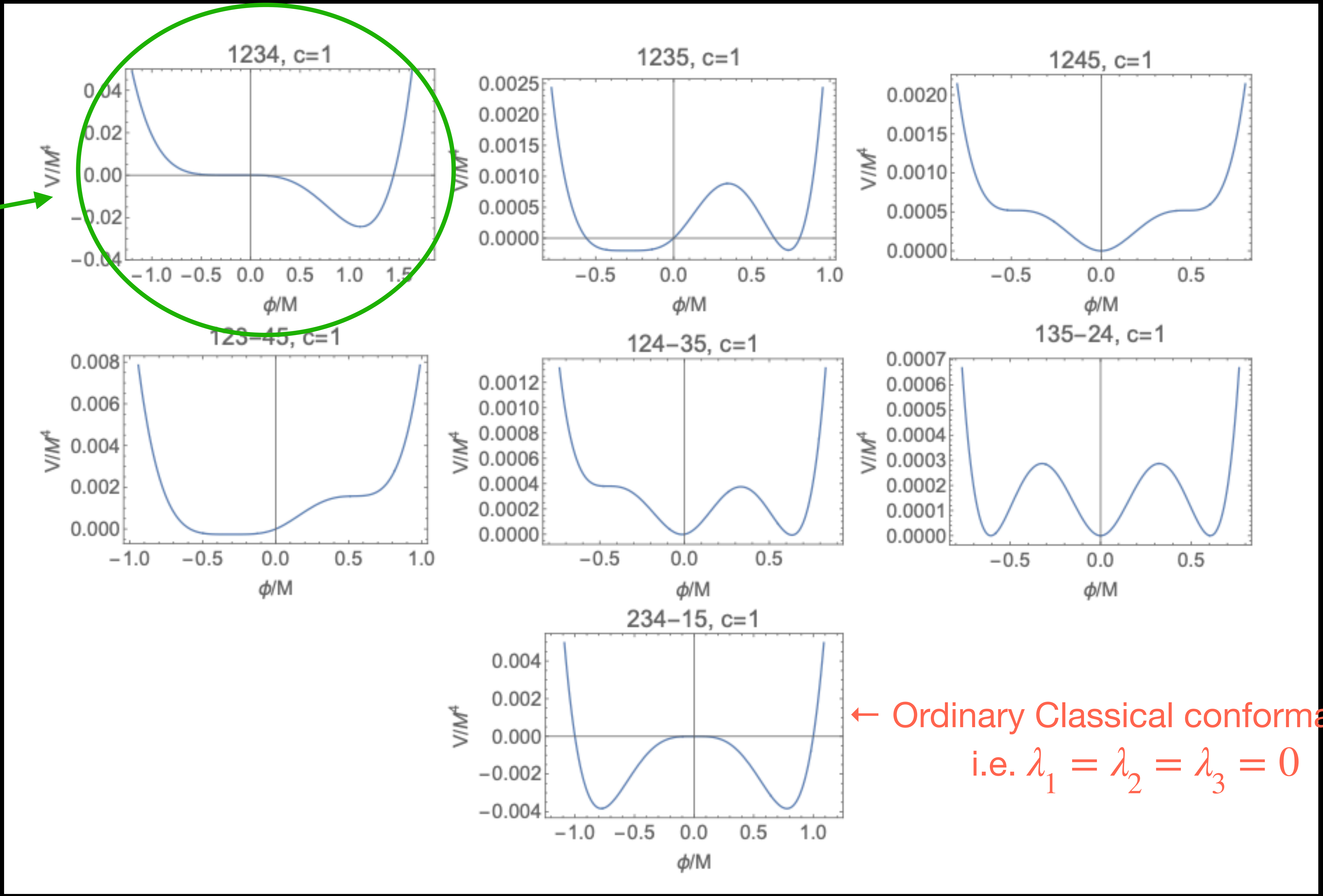
One-loop Results

$$V(\phi) = \lambda_1 \phi + \frac{\lambda_2}{2}\phi^2 + \frac{\lambda_3}{3!}\phi^3 + \frac{c}{2\cdot 4!}\phi^4 \log \frac{\phi^2}{M_{\rm CW}^2},$$

H. Kawai, K.K ('21)

$V' = V'' = V''' = V'''' = 0$

Can predict first order phase transition
→ GWs !



← Ordinary Classical conformal case
i.e. $\lambda_1 = \lambda_2 = \lambda_3 = 0$

$$V_\phi(\phi) = cM^4 \left[-\frac{\bar{\phi}}{18e^{25/4}} - \frac{\bar{\phi}^2}{8e^{25/6}} - \frac{\bar{\phi}^3}{6e^{25/12}} + \frac{\bar{\phi}^4}{48} \ln \bar{\phi}^2 \right],$$

Effective potential determined by MPP

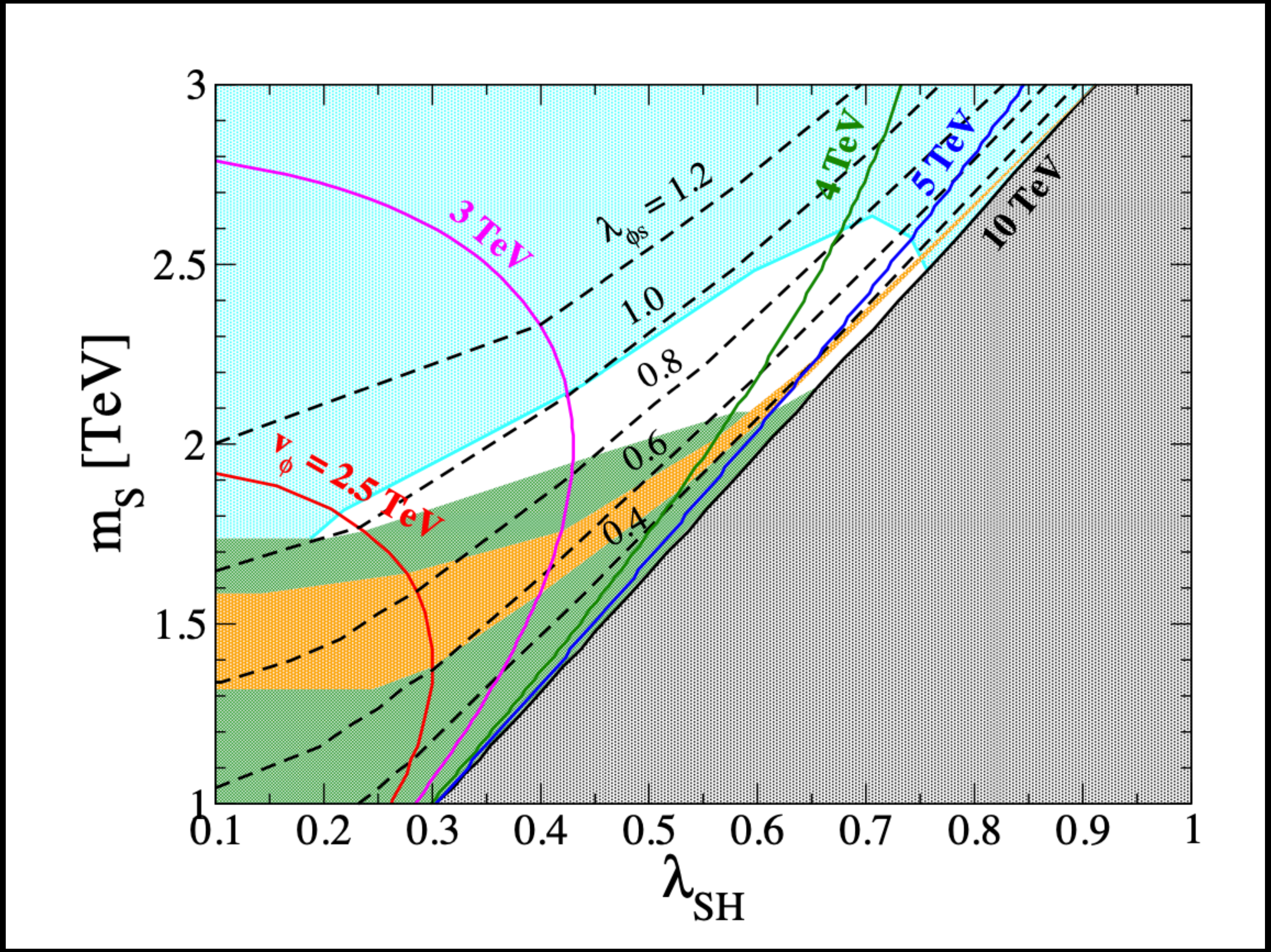
- As a concrete model, let's consdider SM + two additional real scalars, ϕ , S

[Y. Hamada, H. Kawai, K.K, K. Oda, K.Yagyu, ('21)]

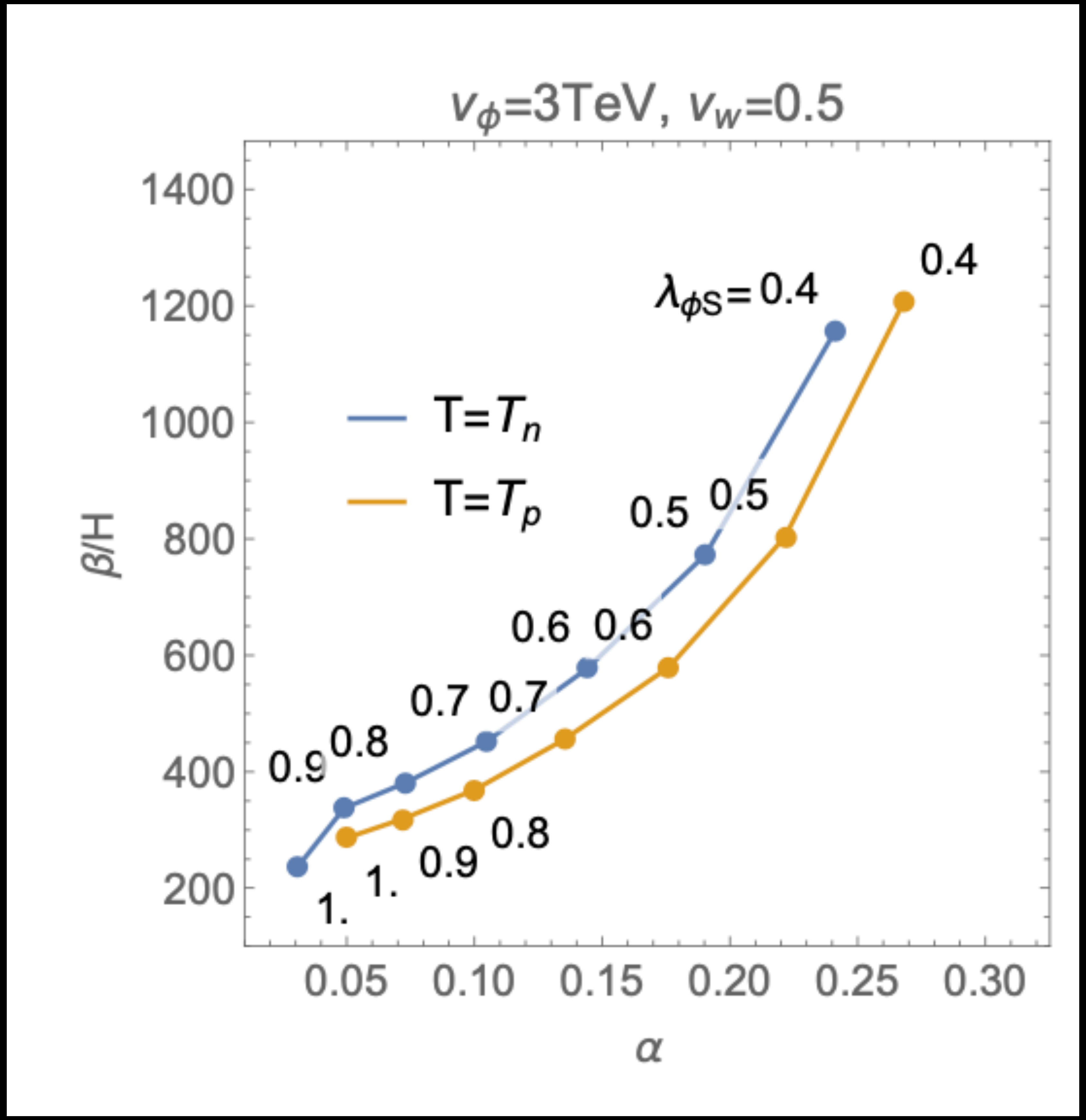
$$V(H,\phi,S) = \frac{\lambda_H}{4}|H|^4 + V_\phi(\phi) + \frac{\lambda_S}{4!}S^4 + \frac{\lambda_{HS}}{2}|H|^2 S^2 + \frac{\lambda_{H\phi}}{2}|H|^2 \phi^2$$

Can realize

- Coleman Weinberg Mechanism
- DM relic abundance by S
- (EW) First-order phase transition



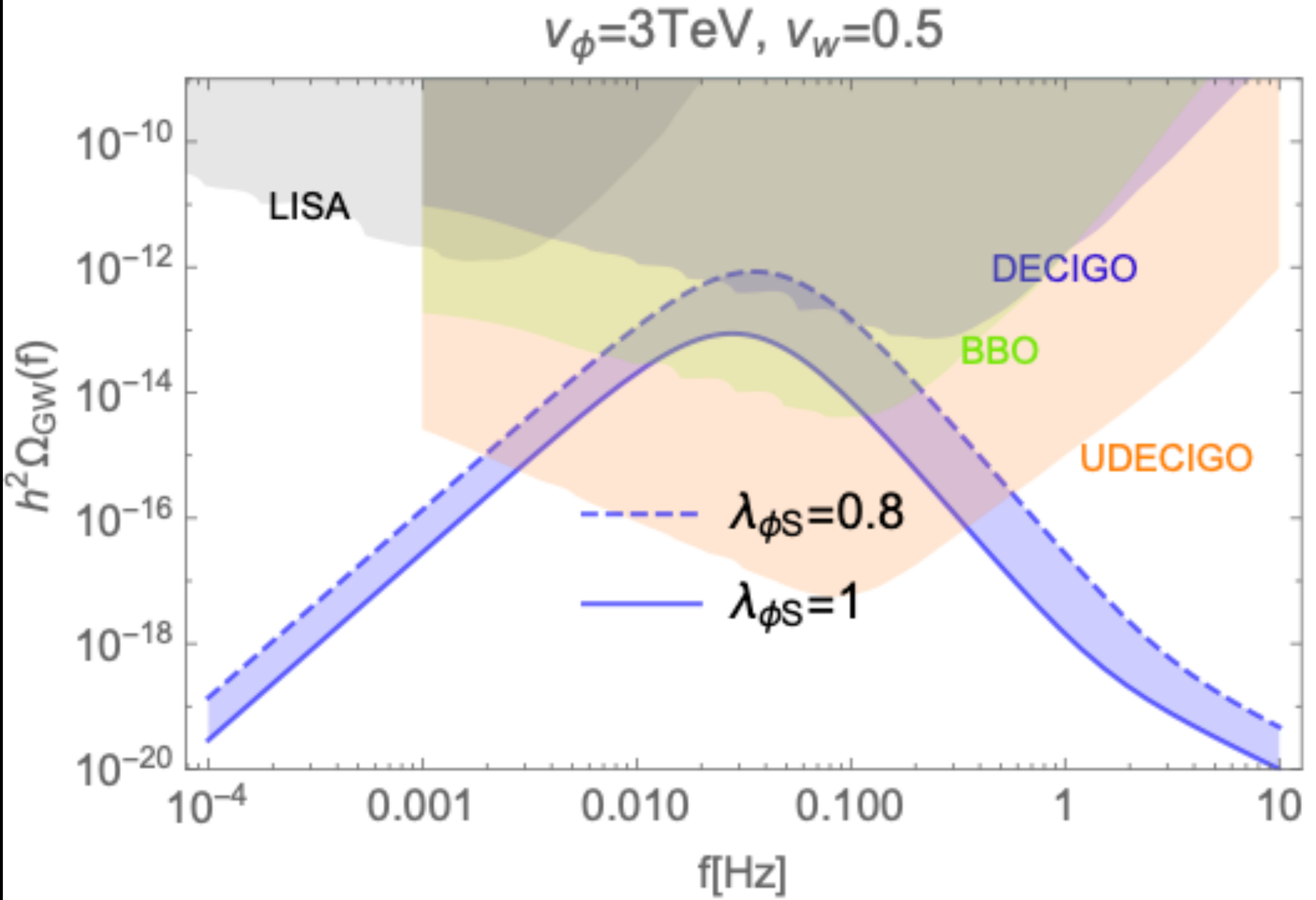
Allowed parameter region



Strength parameters, α and β

Gravitational Waves

[Y. Hamada, H. Kawai, K.K, K. Oda, K.Yagyu, ('21)]



Model
SM + two real scalars ϕ, S

Dominant contribution is
Sound wave

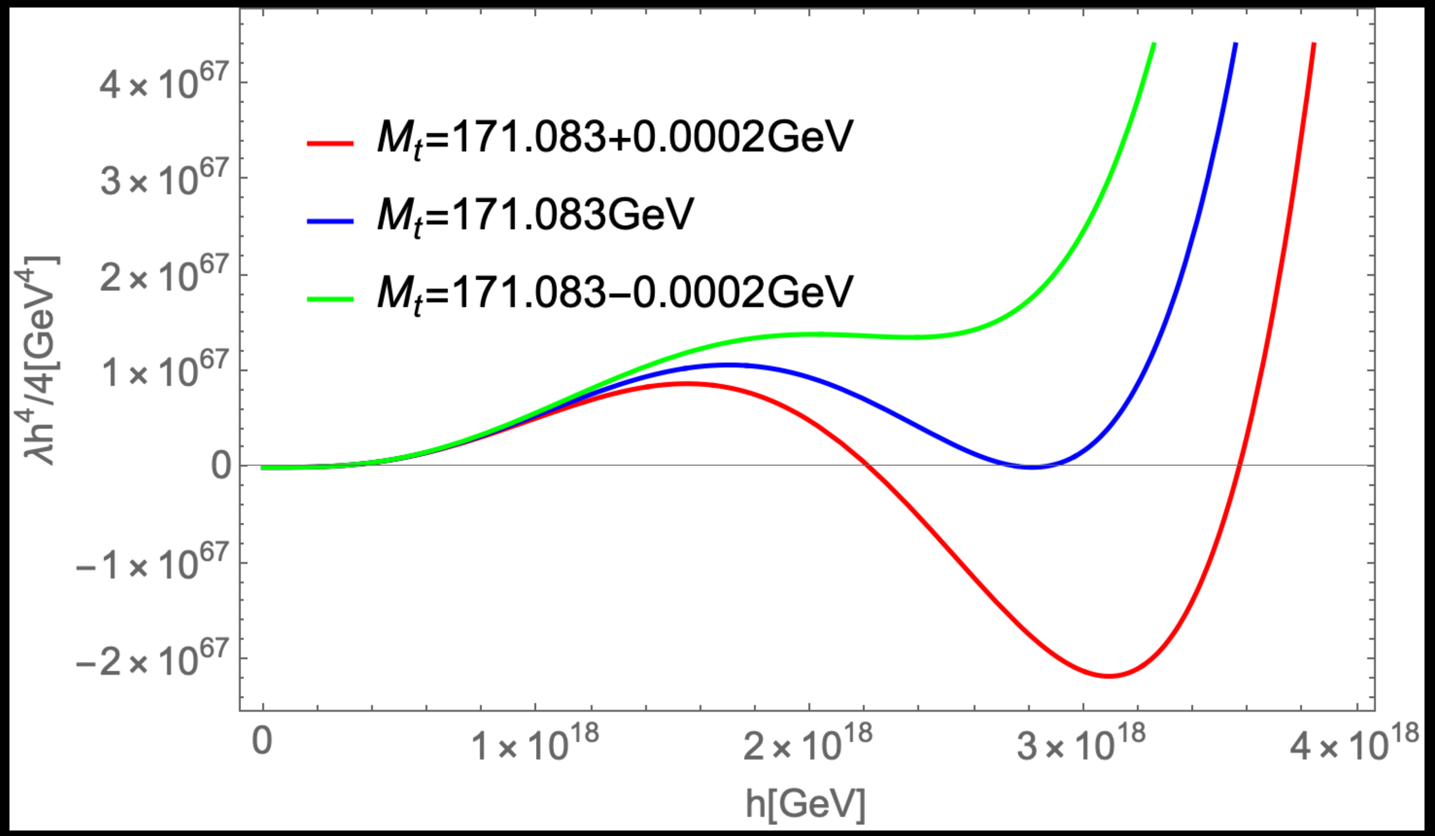
2. Critical Higgs Inflation

[Hamada, Kawai, Oda, Park ('14)]

- In the conventional Higgs inflation ($\lambda \sim 0.1$), the nonminimal coupling $\xi h^2 R$ has to be very large

CMB amplitude: $A_s = 2.2 \times 10^{-9}$ ⟷ $\frac{\lambda}{\xi^2} \sim \left(\frac{50}{N}\right)^2 \times 10^{-10}$

- However, MPP predicts the degenerate vacua or inflection point



$\lambda \sim \beta_\lambda \sim 0$ around the Planck scale

→ Small ξ is allowed

* Small ξ is also preferable from the point of unitarity

[Ema, Jinnno, Mukaida, nakayama ('16)]

Minimal model for EW scale, Neutrino mass, Dark Matter, and critical Higgs inflation

[Y. Hamada, H. Kawai, K.K, K. Oda, K.Yagyu, (21)]

- SM + two real scalars ϕ, S + right-handed neutrinos N_i
- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu S)^2 - \lambda_H (H^\dagger H)^2 - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_{\phi S}}{4}\phi^2 S^2 - \frac{\lambda_S}{4!}S^4 + \frac{\lambda_{\phi H}}{2}\phi^2 (H^\dagger H)$$
$$- \frac{\lambda_{SH}}{2} S^2 (H^\dagger H) - \frac{\mu_\phi}{3!}\phi^3 + \frac{1}{2}\sum_{i=1}^{3} \overline{\nu_{Ri}}\gamma^\mu i\partial_\mu \nu_{Ri} - \frac{1}{2}\sum_{i=1}^{3} M_{Ri}\overline{\nu_{Ri}{}^c}\nu_{Ri}$$
$$- \sum_{i,j=1}^{3}\left(y_{\nu ij}\overline{L_i}H^c \nu_{Rj} + \frac{y_{ij}^\phi}{2}\phi\,\overline{\nu_{Ri}{}^c}\nu_{Rj} + \text{h.c}\right), \quad (1)$$

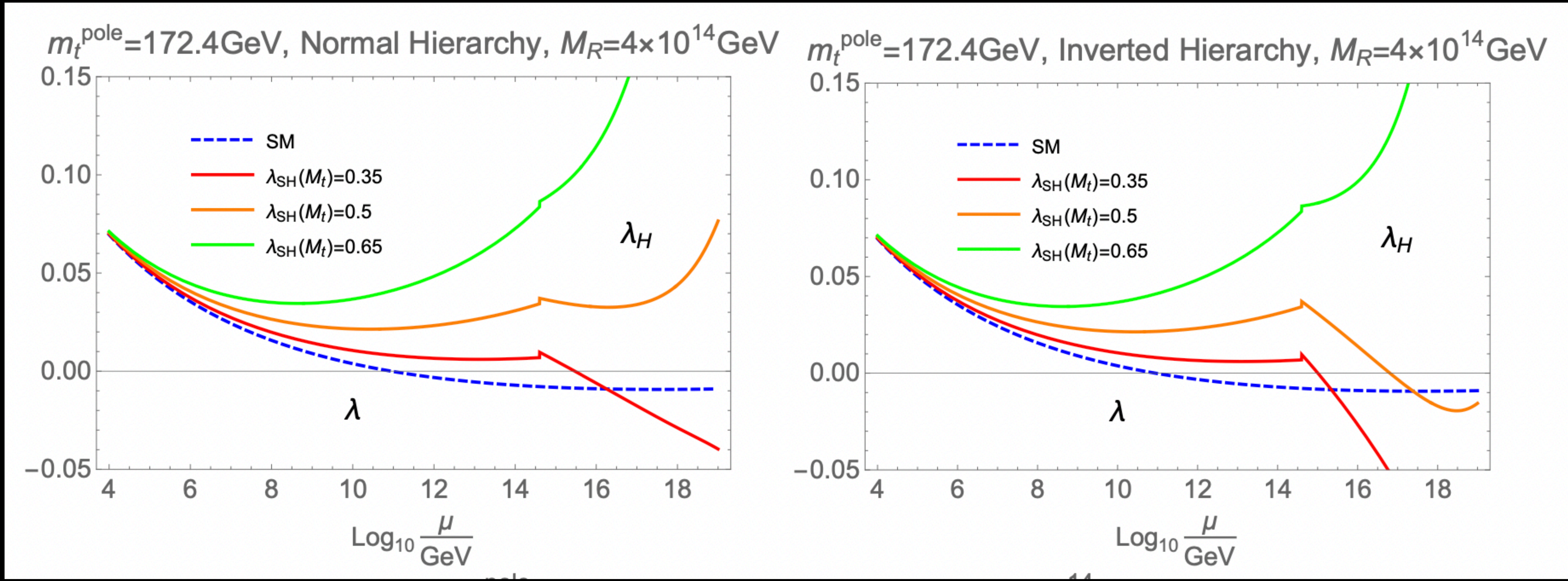
How the existence of new particles changes the Higgs potential ?

Cont'd

- Additional contribution to β_λ

$$\beta_\lambda = \beta_\lambda^{\rm SM} + \frac{1}{16\pi^2}\left(\frac{\lambda_{\phi H}^2}{2} - 2n_\nu y_\nu^4 + \cdots\right)$$

$n_\nu = 1$ (2) for normal (inverted) hierarchy



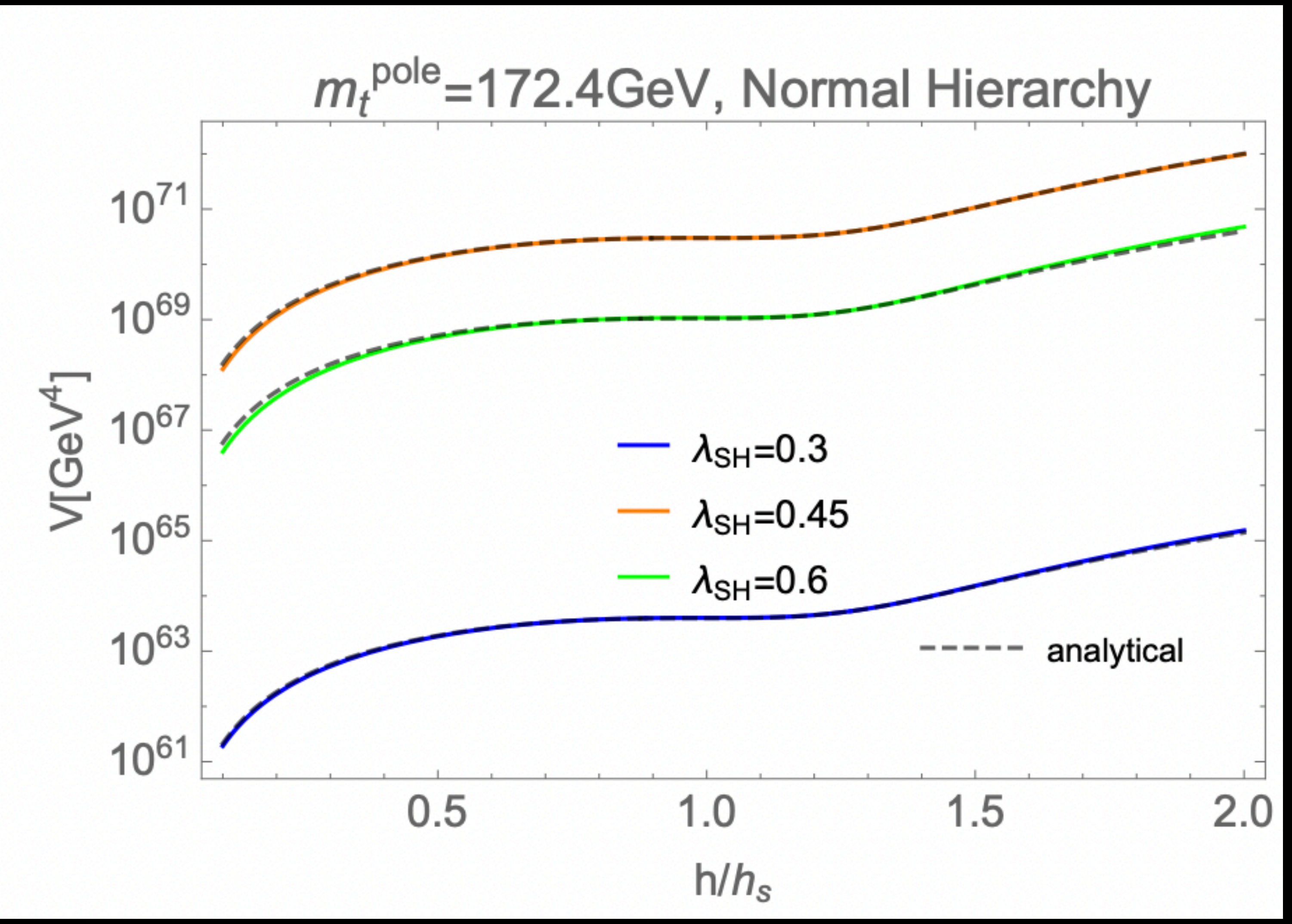
Normal Inverted

Cont'd

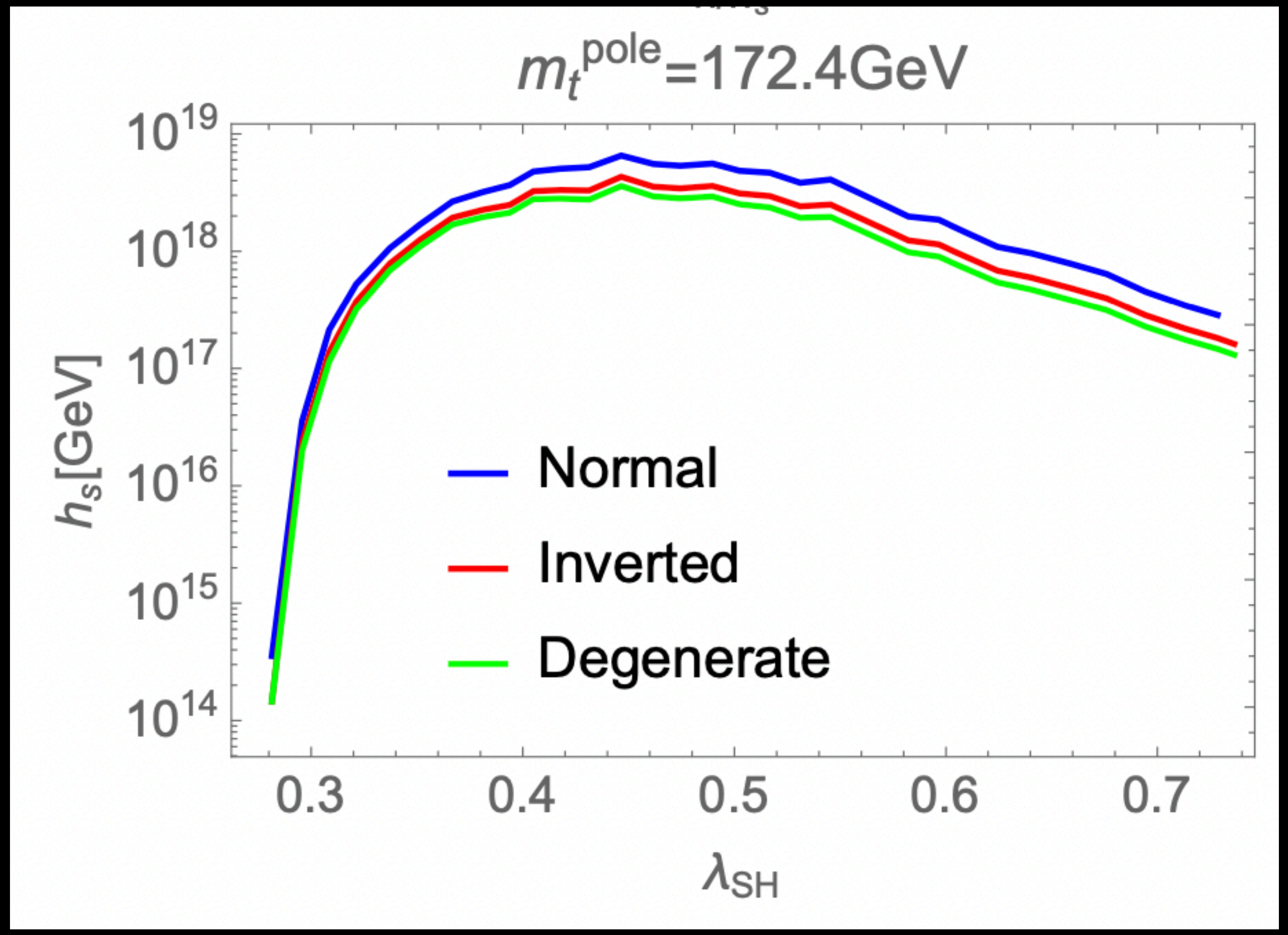
- Conditions for saddle point $$V' = V'' = 0$$

$$V = \frac{\lambda(\mu = h)}{4} h^4 \longrightarrow \lambda + \frac{1}{4}\beta_\lambda = 0 \ , \quad \lambda + \frac{2}{3}\beta_\lambda + \frac{d\beta_\lambda}{d \ln h} = 0$$

We can realize these conditions by tuning λ_{SH} and y_ν



Higgs potential

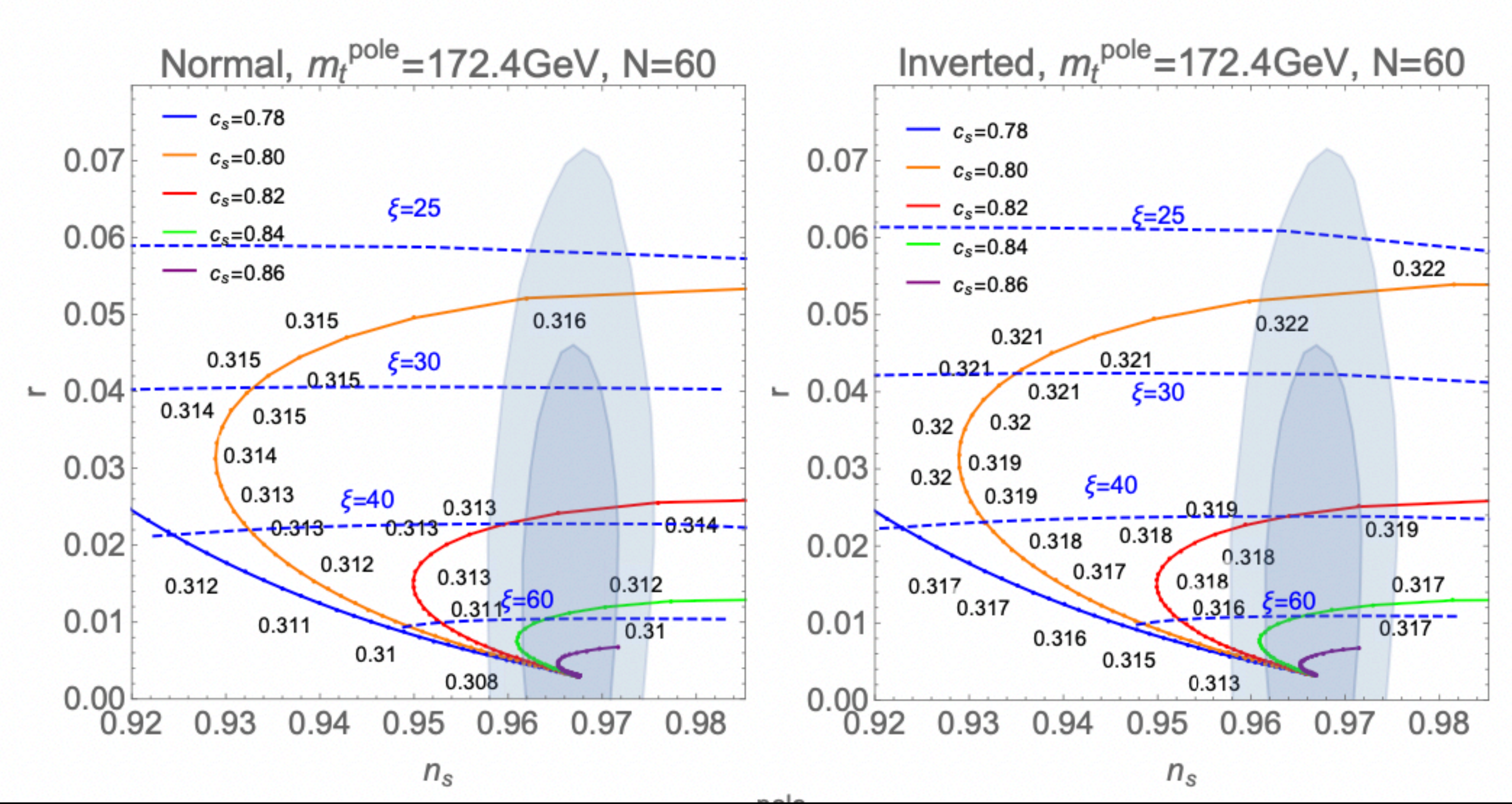


Position of saddle point

Cont'd

Inflation Predictions

$$c_s = \frac{h_s}{M_{\rm Pl}/\sqrt{\xi}}$$

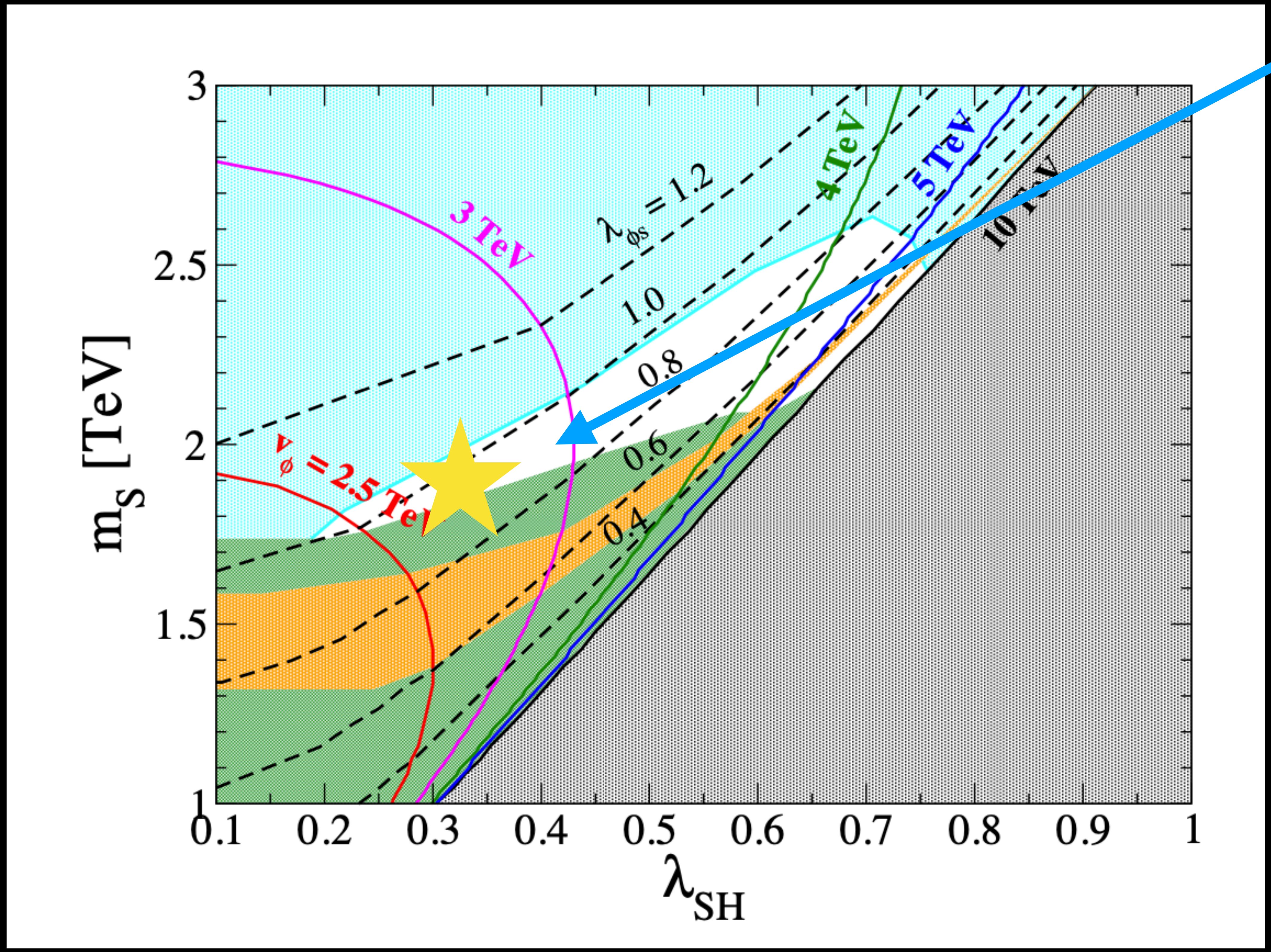


$\xi \sim 30$ is allowed

Planck 2018

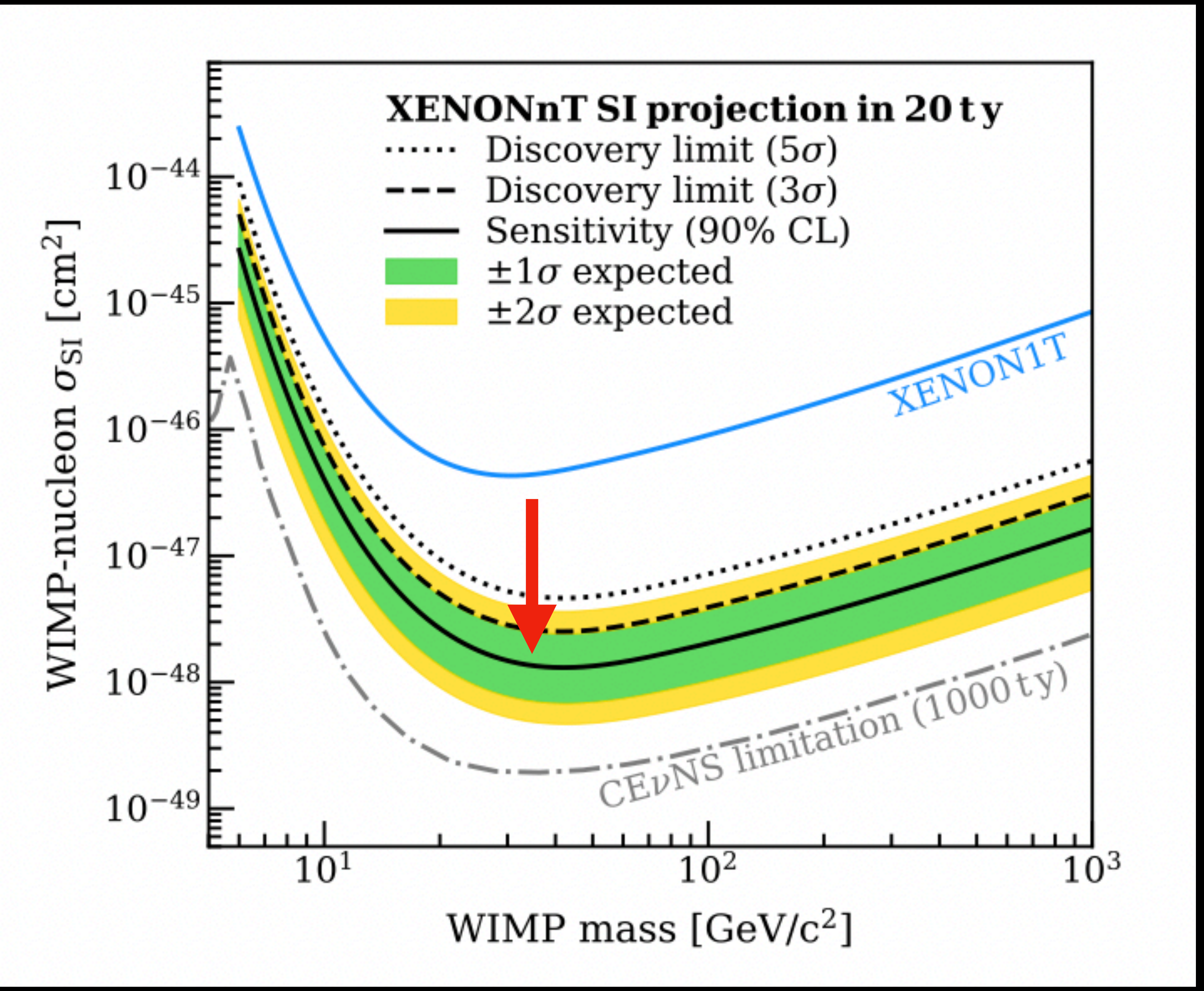
Allowed parameter region

Cont'd



EW scale, neutrino masses, Dark Matter, Inflation are all explained !

XENON ('20)



$$\sigma_{SI} \propto \frac{1}{m_S^2}$$

But this parameter space would be soon killed by XENONnT

3. Maximum entropy principle

[Kawai, Okada '12; Hamada, KK, Kawai '16]

- Consider compact Friedman Universe

$$ds^2 = -N^2(t)dt^2 + a^2(t)ds^2_{\text{spatial}} \ ,$$

- The Lagrangian can be written as $(M_{\text{Pl}} = 1)$

$$L = \frac{\dot{a}p_a}{a} - N(t)H \ , \quad H = -\frac{p_a^2}{2a} + \frac{a^2}{6}\rho(a) \ , \quad p_a = a\dot{a} \ , \quad \rho(a) : \text{energy density}$$

- At classical level, variation of $N(t)$ gives Friedman equation

$$H = 0 \quad \leftrightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho(a)}{3}$$

Cont'd

- At quantum level, we have to consider path-integral

$$Z = \int_{-\infty}^{\infty} dT \int da \exp\left(\int_0^1 dt\left(\dot{a}p_a/a - TH\right)\right)$$

$$= \int_{-\infty}^{\infty} dT \langle f | e^{-iT\hat{H}} | i \rangle = 2\pi \langle f | \delta(\hat{H}) | i \rangle ,$$

Gauge fixing: $N(t) = T$

Moduli

$t = t'/T$

$$= \langle f | \mathrm{WD} \rangle \langle \mathrm{WD} | i \rangle ,$$ where $$\hat{H} | \mathrm{WD} \rangle = 0$$ Wheeler-Dewitt state

- As well as original Coleman's theory, we assume multiverse state

$$|\,\mathrm{Multiverse}\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} |\,\mathrm{WD}\rangle \otimes \cdots \otimes |\,\mathrm{WD}\rangle ,$$

When we focus on the observables in our universe, we should trace out other universes

Cont'd

$$\rho(a_f, a_i) = \sum_{n=0}^{\infty} \phi^*_{\rm WD}(a_f)\phi_{\rm WD}(a_i)\frac{1}{n!}\left(\int_0^\infty da\,|\phi_{\rm WD}(a)|^2\right)^n$$

$$= \sum_{n=0}^{\infty} \phi^*_{\rm WD}(a_f)\phi_{\rm WD}(a_i)\exp\left(\int_0^\infty da\,|\phi_{\rm WD}(a)|^2\right)$$

depends on coupling constants and history of universe

- Micro-canonical reduced density density matrix is

$$\rho_{\rm mic}(a_f, a_i) = \int \prod d\lambda_j \rho(a_f, a_i)$$

∴ Probability distribution of coupling constant λ_i is

$$P(\{\lambda_j\}) \sim \exp\left(\int_0^\infty da\,|\phi_{\rm WD}(a)|^2\right)$$

- Naively $$\int_0^\infty da\,|\phi_{\mathrm{WD}}(a)|^2 = \langle \mathrm{WD}\,|\,\mathrm{ED}\rangle = \delta(0)$$

- Within WKB solution $$|\phi_{\mathrm{WD}}(a)|^2 \sim \frac{a}{p_a} \sim \frac{1}{\dot{a}}$$

$$\therefore \quad \int_0^\infty da\,|\phi_{\mathrm{WD}}(a)|^2 \sim \int_0^\infty \frac{da}{\dot{a}} \sim (\text{life time of universe}) := \mathrm{T}_{\mathrm{life}}$$

∗ Same as the derivation of Fermi's golden rule $$\delta(E)^2 \to \frac{t}{2\pi}\delta(E)$$

- As a result

$$P(\{\lambda_j\}) \sim e^{T_{\mathrm{life}}}$$

Coupling constants are tuned at the point that maximizes life time of universe

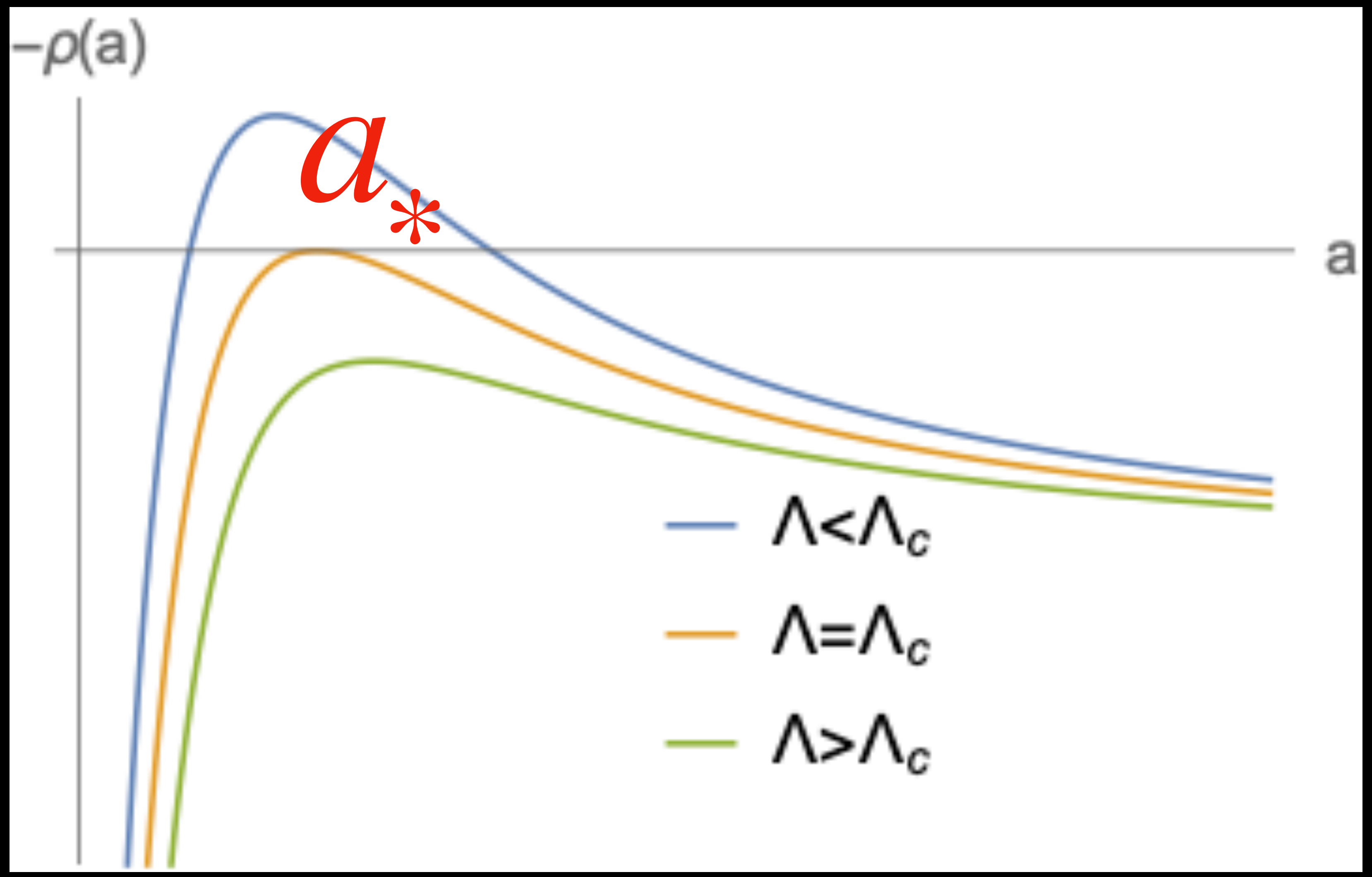
Cont'd

- Now let's consider the history of the universe.

- After long time evolution, all matter would decay to radiation (photon, graviton, etc)

$$\rho(a) = \frac{S}{a^4} - \frac{1}{a^2} + \Lambda \,, \quad S : \text{total entropy of radiation}$$

— curvature



Maximum exists at $\frac{S}{a^4} \sim \frac{1}{a^2} \to a_*^2 \sim S$

Moreover, the maximum becomes zero when

$$\frac{1}{a_*^2} \sim \Lambda \to \Lambda_c \sim S^{-1} \ll 1$$

Very tiny cosmological constant !

Cont'd

- More explicitly, the contribution around $a = a_*$ is

$$T_{\mathrm{life}} \sim \int^{a_*-\epsilon} da\,|\phi_{\mathrm{WD}}(a)|^2 \sim S^{3/4} \int^{a_*-\epsilon} da \frac{1}{|a-a_*|} \sim S^{3/4} \log|\epsilon|\ ,$$

$\therefore\ T_{\mathrm{life}}$ is an increasing function of S !

→ Parameters are tuned at the point where S is maximized

(Maximum entropy principle)

[Kawai, Okada '12; Hamada, KK, Kawai '16]

- We can repeat the same calculations in the case of matter dominated universe

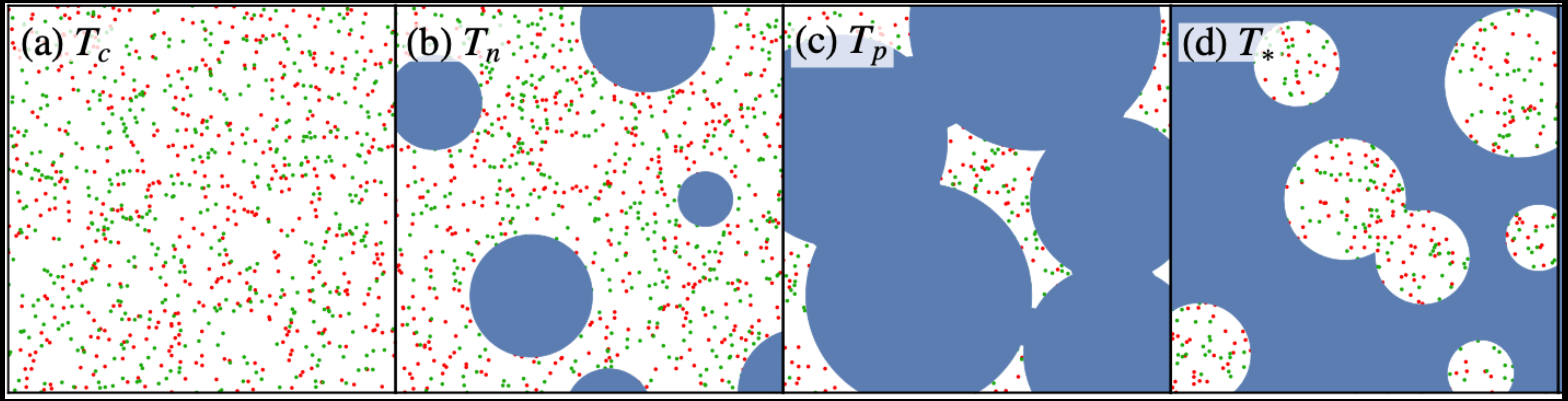
$$T_{\mathrm{life}} \sim \int^{a_*-\epsilon} da\,|\phi_{\mathrm{WD}}(a)|^2 \sim M^{3/2} \int^{a_*-\epsilon} da \frac{1}{|a-a_*|} \sim M^{3/2} \log|\epsilon|\ ,$$

Parameters are tuned at the point where M is maximized

(Maximum matter principle)

4. Implication for First-order phase transition

[Kawana, Kawai, Hamada, work in progress]



- When first-order phase transition (FOPT) occurs in the early Universe, there is additional entropy production of radiation.

$$\Delta S = V_3 \times \frac{\partial \Delta V(T)}{\partial T} + (\text{reheating})$$

Q: Does MEP prefer strong FOPT ?

- As a very crude estimation, let's assume that all the vacuum energy goes to radiation

$$\delta S = \Delta V \times a^4(t_*) = \frac{\Delta V}{\rho_{\rm rad}(t_*)} \times \rho_{\rm rad}(t_*) a^4(t_*) = \alpha \times S_{\rm ini}\ , \qquad S_{\rm ini} : \text{initial entropy after reheating}$$

Percolation time (end of FOPT)

Increasing function of α !

- More conservatively, let's assume that the bubble wall energy does not contribute to entropy production (e.g. PBH formation via bubble collapse)

$$\delta S = \alpha(1-\kappa_\phi) S_{\rm ini} = \alpha\left(1 - c\left(1 - \frac{d}{\alpha^{1/2}}\right)\right) S_{\rm ini} \sim \alpha(1-c) S_{\rm ini}\ , \quad 0 < c < 1$$

energy fraction to bubble walls

Still increasing function of α !

[Ellis et al. (19)]

$\therefore$ Coupligs are tuned so that FOPT becomes as strong as possbile via MEP

Many cosmological implications

- Gravitational Waves
- Supercooling, Thermal Inflation, Secondary reheating
- (EW) baryogenesis/leptogenesis
- Remnant formations. e.g. Primordial Black Holes, Q-balls, Fermi-balls, Thermal balls and more

See also [KK, Ke-Pan, Philip, arXiv: 2206.09923

Summary

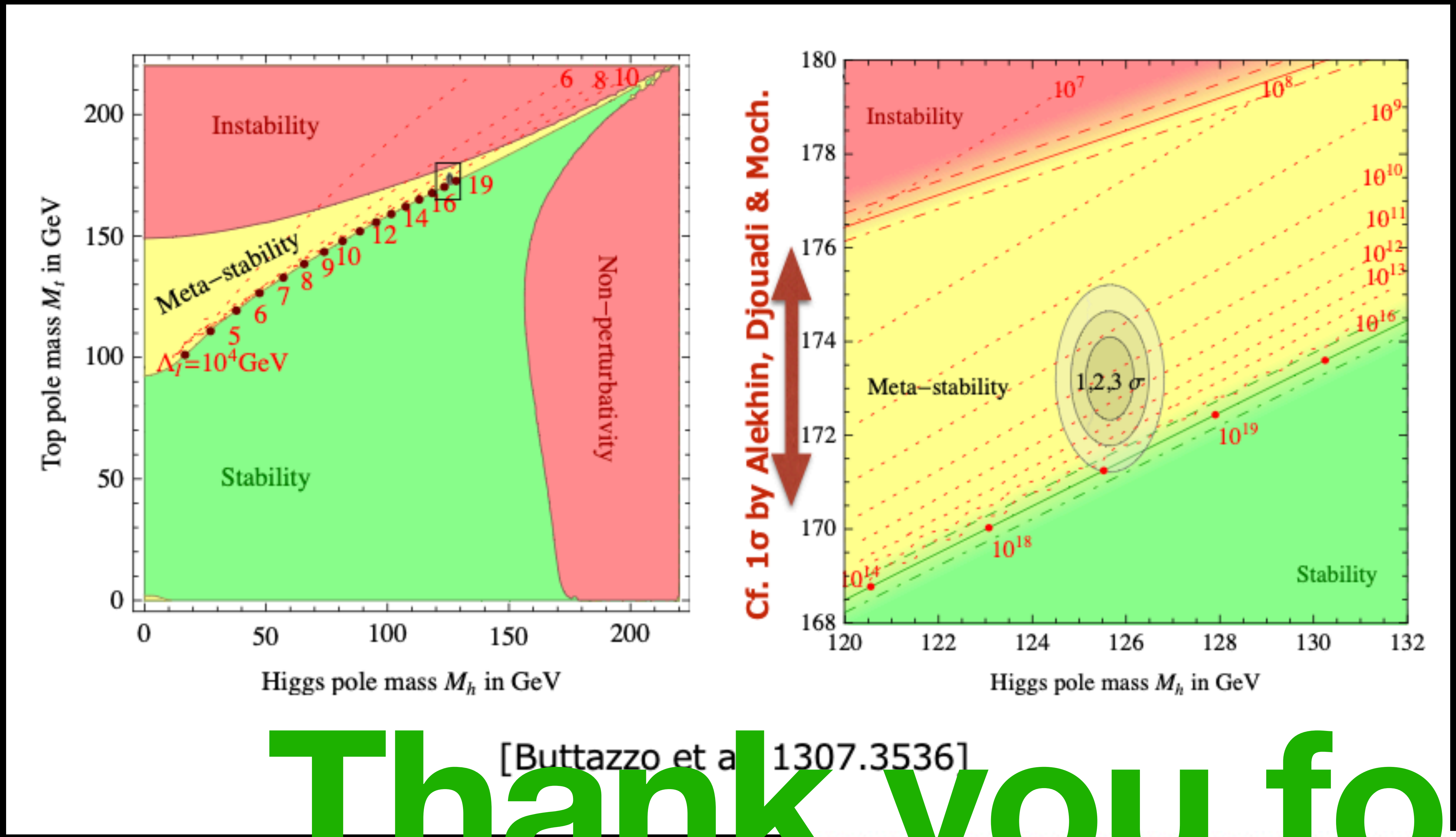
- We have discussed basic idea of Multi-critical point principle and a few theoretical approaches
- Coupling constants are naturally tuned at multi-critical (degenerate) point in micro-canonical picture
- A lot of implications for particle physics and cosmology

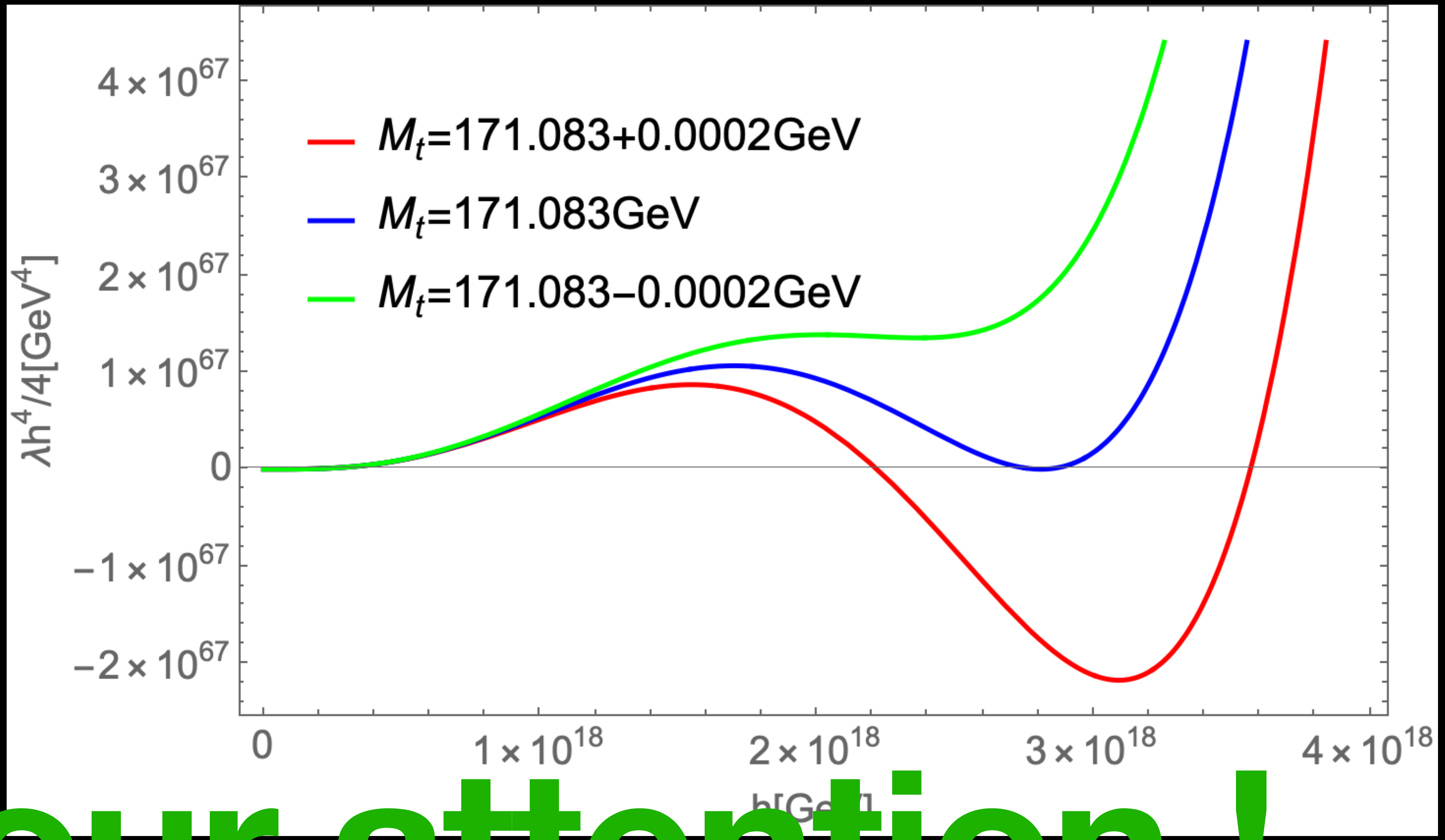
e.g. Naturalness problems, classical conformality, (Higgs) inflation, first-order PT, and more

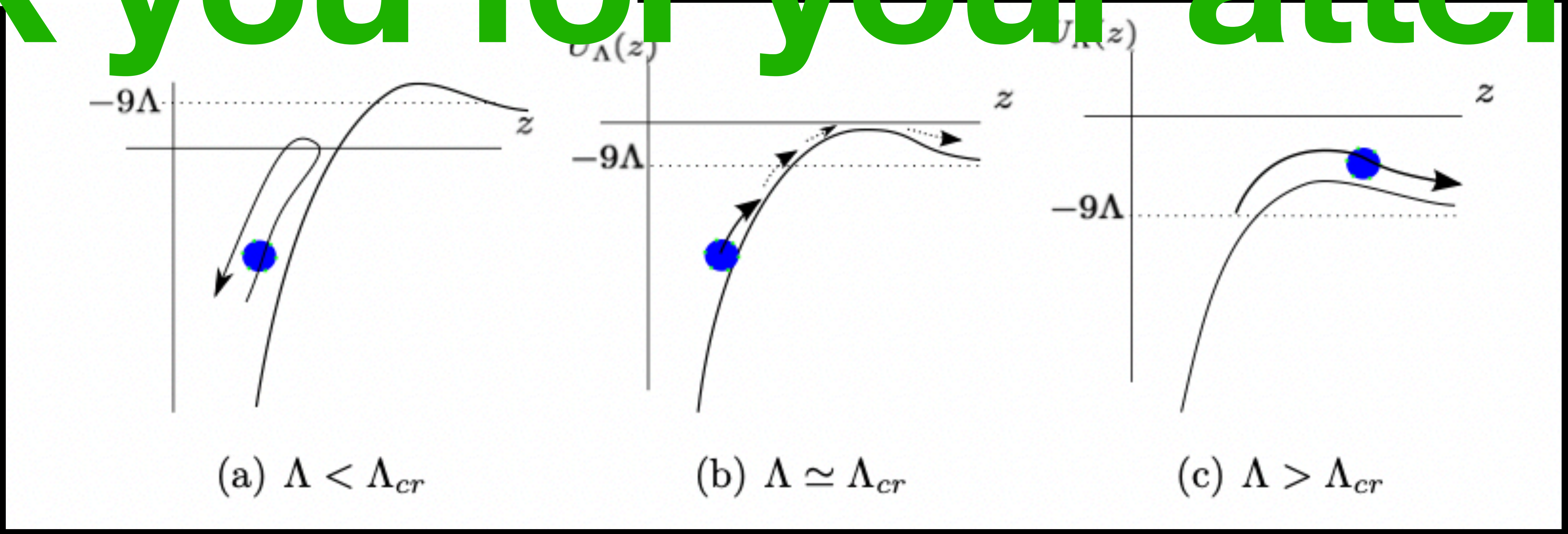
- And there are still many open questions

e.g. Check equivalence more concretely (correlation functions)
More general QFTs (Fermion, gauge fields, …)
Global (PQ) symmetry is favored or not ?
What happens in gravity ? etc

メッセージ: 自然は"(多重)臨界点"(キワキワ)を選びたがる







Thank you for your attention !