隠れたカイラル対称性の破れとスケール生成

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MA, J. Kubo, and J. Yang, JCAP01 (2022)

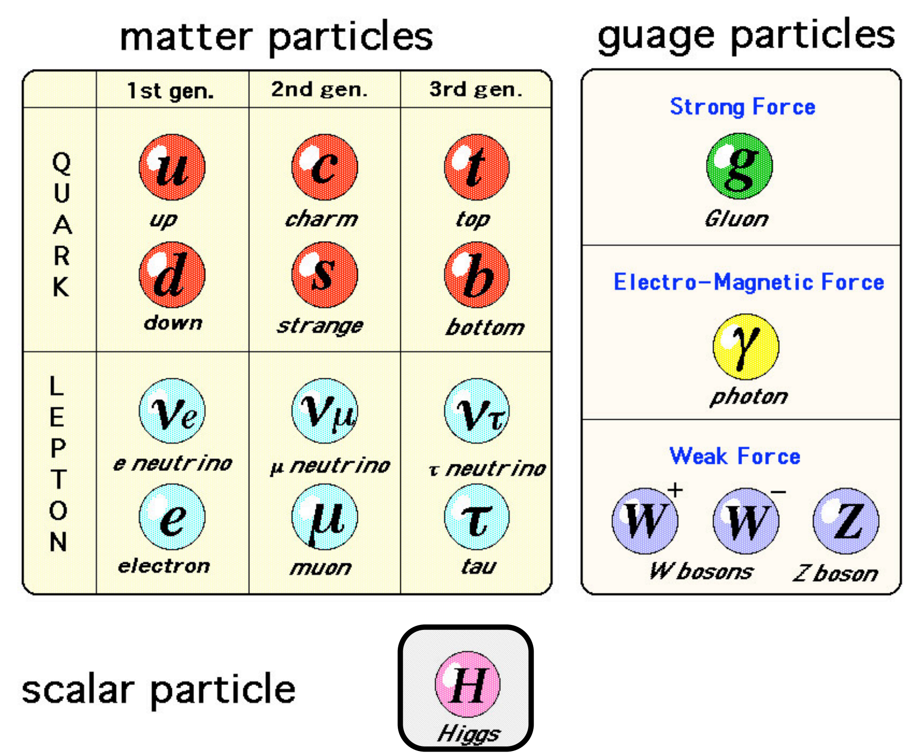
Introduction

❖ **Open Questions :**

Neutrino mass
Dark matter
Baryon asymmetry
Inflation
Dark energy
:

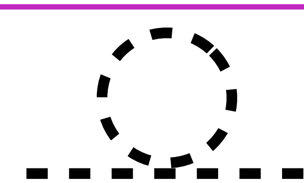
Hierarchy problem
:

Standard Model



Hierarchy problem

$$V_{\rm SM} = m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 \quad \rightarrow$$



$$\sim \lambda_H \Lambda^2 \sim \delta m_H^2$$

$$m_h^2 \ll \Lambda^2$$

$$m_h^2 = m_H^2 - \delta m_H^2 \simeq (10^{16}\ {\rm GeV})^2 - (10^{16}\ {\rm GeV})^2 = (125\ {\rm GeV})^2$$

fine tuning !!

Introduction

❖ Classical scale invariance

$$V_{\rm SM} = m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$

- Boundary condition @ Planck scale

Bardeen, FERMILAB-CONF-95-391 (1995)

  - The RG equation of Higgs mass

$$\frac{dm_H^2}{d\log\mu} = \frac{m_H^2}{16\pi^2}\left(12\lambda_H + 6y_t^2 - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + \cdots\right)$$

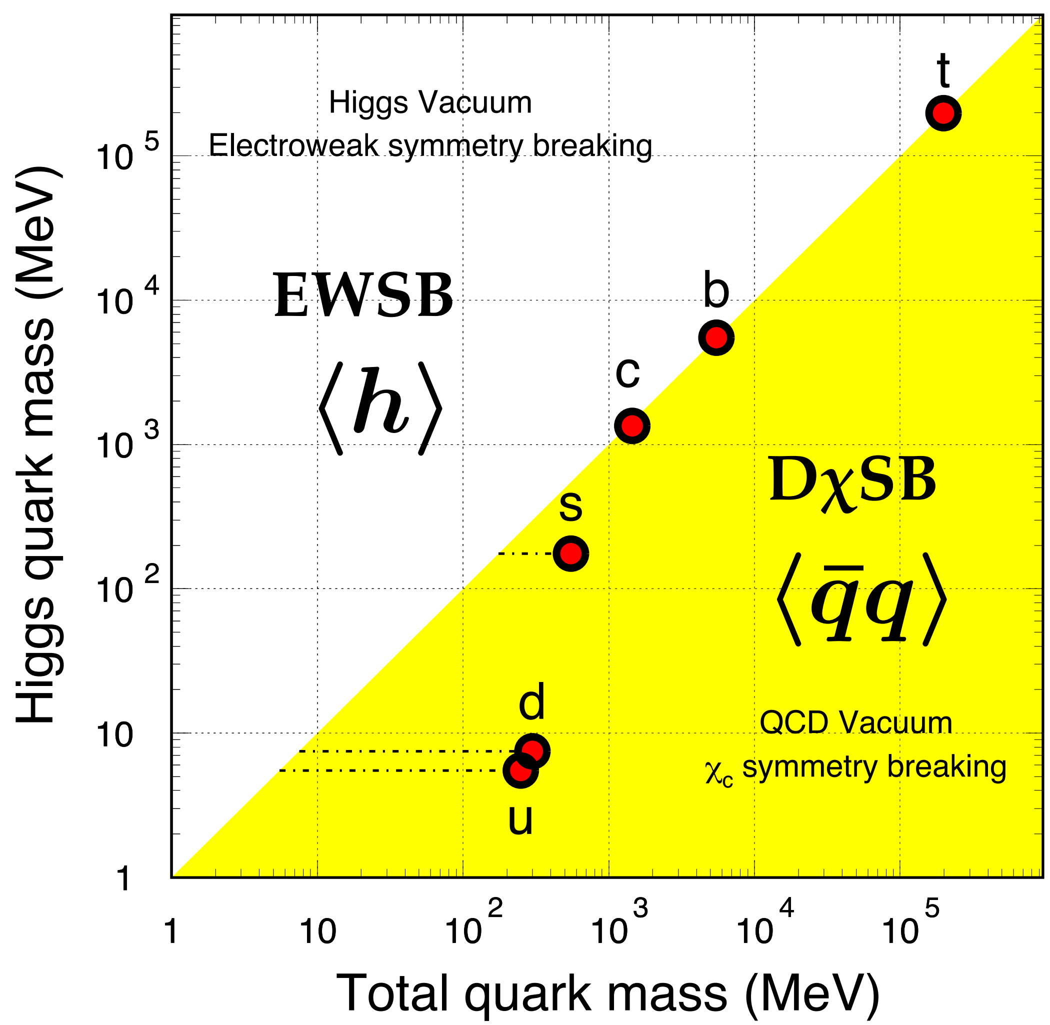
  - In $m_H^2(\Lambda_{\rm pl}) = 0$, the mass does not run.

- **Scale invariant theory + dimensional transmutation**
  - Coleman-Wienberg mechanism
  - Dynamical chiral symmetry breaking

Introduction

❖ Ordinary Matter

Higgs EWSB **2%** + **98%** QCD Dynamical chiral symmetry breaking (DχSB)



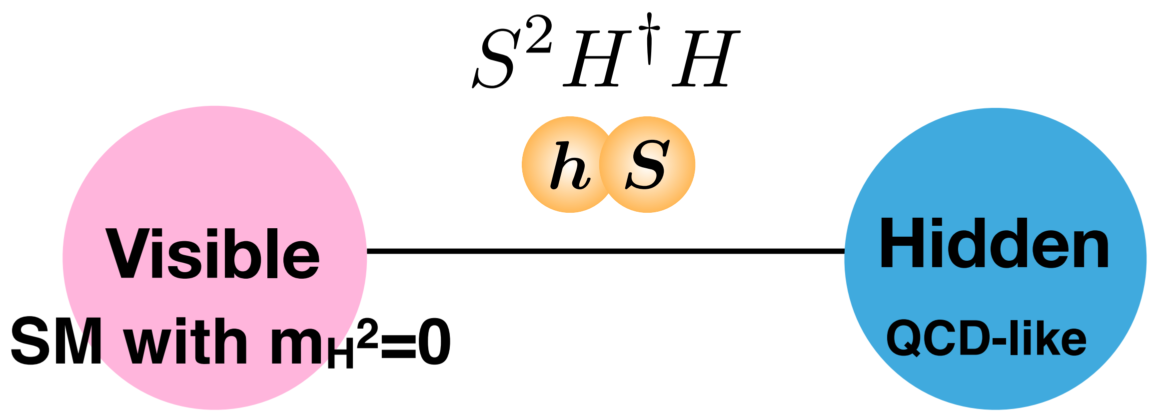
X. Zhu et. al PLB647 (2007)

Introduction

Hur, Jung, Ko, Lee, PLB106 (2011)

Holthausen et al, JHEP1312 (2013)

❖ **Model I**



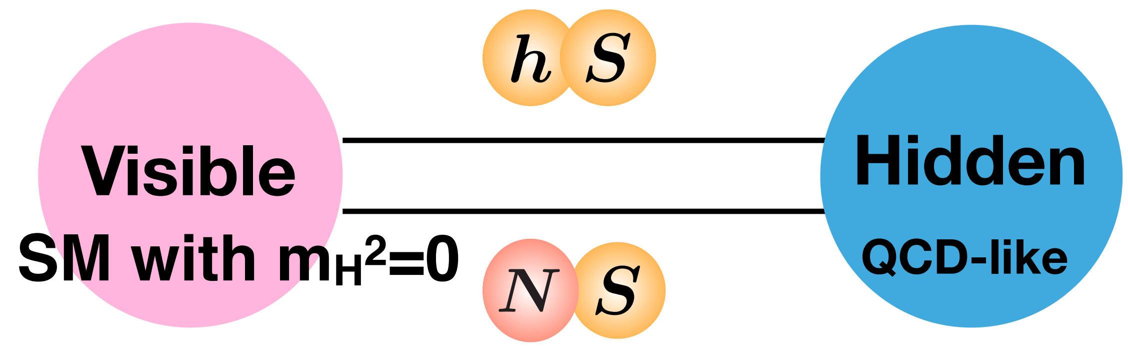
DM : Dark meson

✓ Higgs mass parameter

✓ DM mass

Ametani, MA, Goto, Kubo, PRD91 (2015)
MA, Goto, Kubo, PRD96 (2017)
MA, Kubo, JCAP04 (2020)

❖ **Model II**



MA, Brdar, Kubo, PRD102 (2020)

MA, Kubo, Yang, JCAP01 (2022)

Neutrino Option

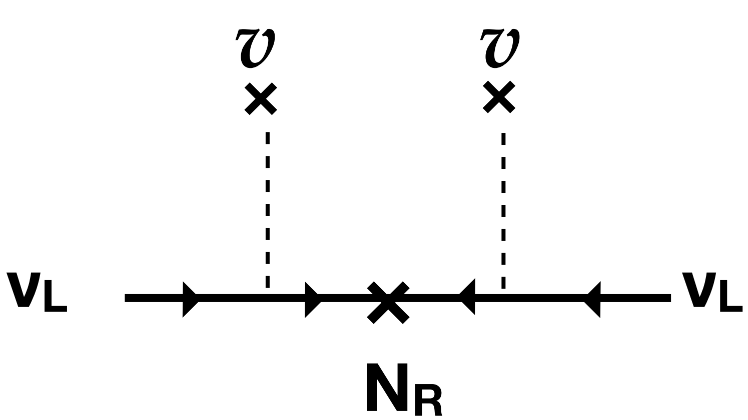
Neutrino Option

- **Neutrino option :**

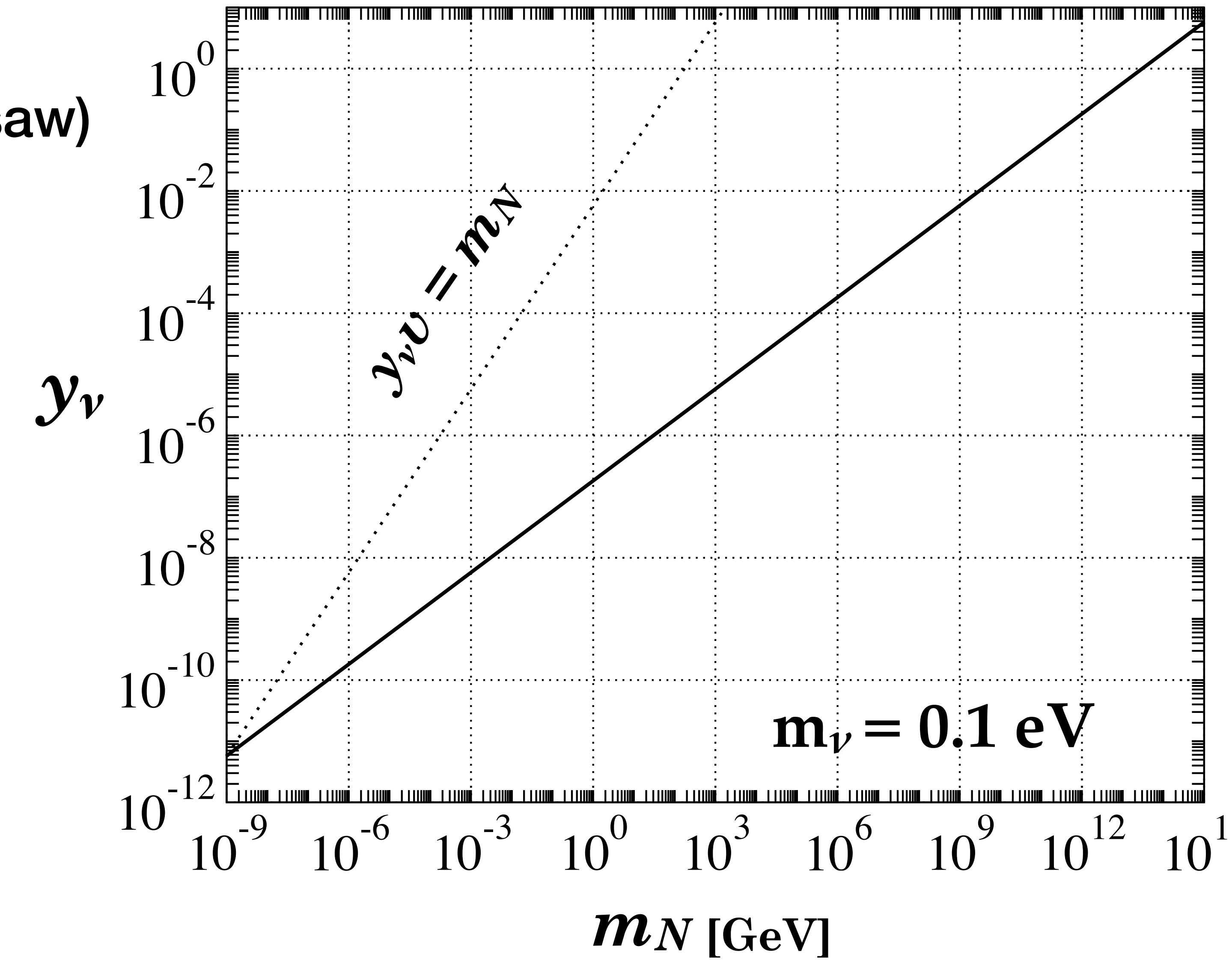
Brivio, Trott, PRL119 (2017)

$$\mathcal{L} \supseteq -\frac{1}{2} m_N \bar{N}_R N_R - \left(y_\nu \bar{N}_R \tilde{H}^\dagger \ell_L + \text{h.c.} \right)$$

- Neutrino masses (Type-I seesaw)



$$m_\nu \simeq \frac{y_\nu^2 v^2}{m_N} \lesssim 0.1 \text{ eV}$$



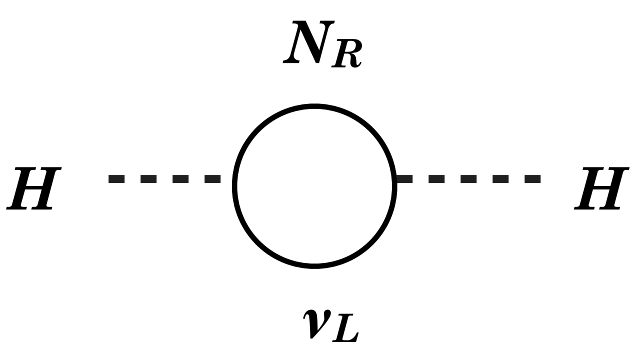
Neutrino Option

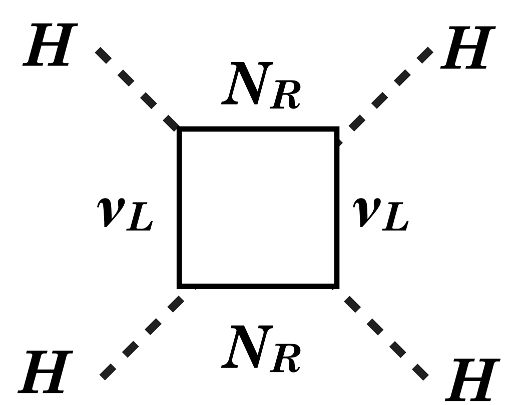
- **Neutrino option :**

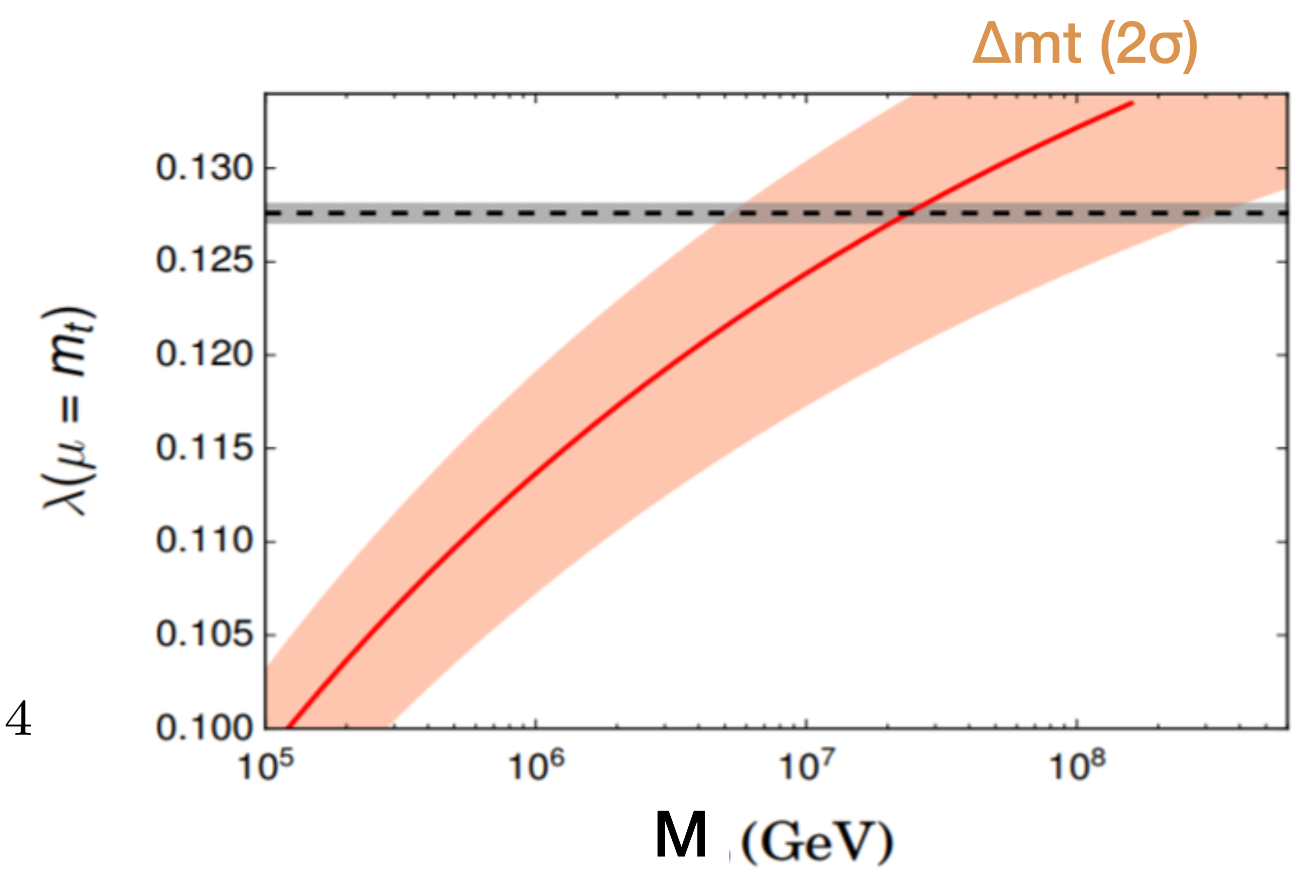
Brivio, Trott, PRL119 (2017)

The right-handed neutrinos can significantly contribute to the correction to the Higgs mass term.

$$\mathcal{L} \supseteq -\frac{\mu_H^2}{2}(H^\dagger H) + \lambda (H^\dagger H)^2$$







$$\Delta\mu_H^2 \sim \frac{|y_\nu|^2}{8\pi^2} m_N^2 \qquad \Delta\lambda \sim -\frac{5}{32\pi^2}|y_\nu|^4$$

Type-I seesaw and

$$\mu_H^2 \simeq \Delta\mu_H^2 \quad \rightarrow \quad m_N \sim 10^{7-8}\ \text{GeV} \ \ \text{with} \ \ y_\nu \lesssim 10^{-4}$$

Neutrino Option

- **Neutrino option based on Classical Scale invariance:**

  Introduce a new scalar to generate right-handed neutrino masses by Yukawa coupling.

$$m_N = y_M \langle S \rangle$$

Brdar et al, PRD99 (2019)

- **Model II :**

  Dynamical chiral symmetry breaking in a QCD-like hidden sector

MA, Brdar, Kubo, PRD102 (2020)

✓ Higgs mass parameter
✓ DM mass
✓ Right-handed neutrino masses
✓ Left-handed neutrino masses

- Require classical scale invariant for gravitational degree of freedom.

MA, Kubo, Yang, JCAP01 (2022)

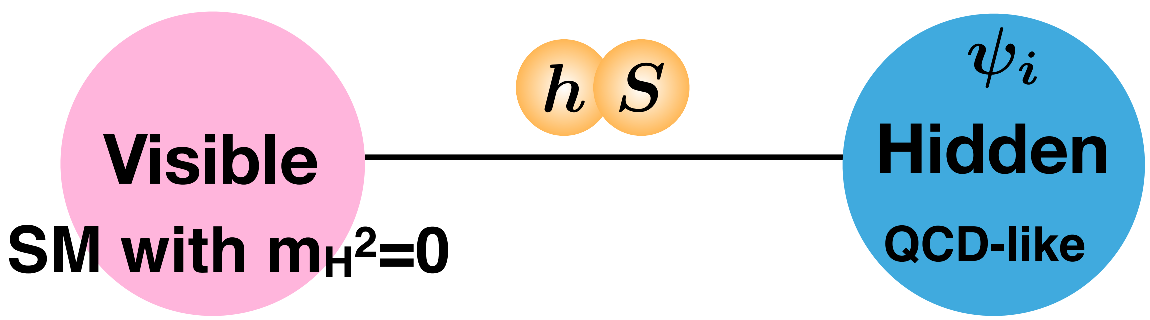
✓ Planck mass $M_{\rm PL} \simeq \sqrt{\beta}\,\langle S \rangle$

Contents

- Introduction
- Model I / Hidden QCD sector
- Model II
  - Inflation
  - Dark matter
- Summary

Classical Scale Invariant Model I

- **Gauge Symmetry :** $G_{\rm SM} \times {\rm SU}(3)_{\rm H}$



S **real singlet scalar**

ψ_i **hidden vector-like fermions (i=1-3)**

- **Scalar potential for vis. sector:** $V_{{\rm SM}+S} = \lambda_H (H^\dagger H)^2 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H) + \frac{1}{4}\lambda_S S^4$
- **Hidden sector :** $\mathcal{L}_{\rm H} = -\frac{1}{2}{\rm Tr}\ F^2 + {\rm Tr}\ \bar{\psi}\,(i\partial\!\!\!/ + g_{\rm H} G\!\!\!/ - yS)\,\psi$
- **The DχSB in the hidden sector triggers the EW symmetry breaking.**

$${\rm SU}(3)_L \times {\rm SU}(3)_R \ \rightarrow\ {\rm SU}(3)_V$$

Hidden QCD scale $\Lambda_{\rm H}$

$\Lambda_{\rm H}$	Dynamical χSB	$\langle \bar{\psi}\psi \rangle \neq 0$	$\langle \bar{\psi}\psi \rangle \sim \Lambda_{\rm H}^3$
	Explicit χSB	$\langle S \rangle \neq 0$	$yS \langle \bar{\psi}\psi \rangle$
EW	EWSB	$\langle h \rangle \neq 0$	$\lambda_{HS} \langle S \rangle^2 (H^\dagger H)$

Classical Scale Invariant Model I

- **Nambu—Jona-Lasinio Model (NJL) Lagrangian :**

$$\mathcal{L}_{\rm H} \longrightarrow \quad \mathcal{L}_{\rm NJL} = {\rm Tr}\ \bar\psi(i\partial\!\!\!/ + g' Q B\!\!\!/ - yS)\psi + 2G{\rm Tr}\ \Phi^\dagger\Phi + G_D(\det\ \Phi + h.c)$$

current mass   4-Fermi   6-Fermi

$$(\Phi)_{ij} = \bar\psi_i(1-\gamma_5)\psi_j$$

  - G and G_D are dimensional parameters.

- **Mean-field approximation:**

$SU(3)_V$ limit : $\langle\Phi\rangle = -\dfrac{1}{4G}\left({\rm diag.}(\sigma,\sigma,\sigma) + i(\lambda^a)^T\phi^a\right) \qquad (a = 1,\cdots,8)$

CP-even : $\sigma = -4G\left\langle\bar\psi\psi\right\rangle$,

Chiral condensate

CP-odd : $\phi_a = -2iG\left\langle\bar\psi\gamma_5\lambda^a\psi\right\rangle$

Hidden mesons (Massive NG bosons)

Dark matter candidates

  - Lagrangian:

$$\mathcal{L}_{\rm MFA} = {\rm Tr}\ \bar\psi(i\partial\!\!\!/_\mu - M)\psi - i{\rm Tr}\ \bar\psi\gamma_5\phi\psi - \frac{1}{8G}\left(3\sigma^2 + 2\sum_{a=1}^{8}\phi_a\phi_a\right)$$
$$+\frac{G_D}{8G^2}\left(-{\rm Tr}\ \bar\psi\phi^2\psi + \sum_{a=1}^{8}\phi_a\phi_a{\rm Tr}\ \bar\psi\psi + i\sigma{\rm Tr}\ \bar\psi\gamma_5\phi\psi + \frac{\sigma^3}{2G} + \frac{\sigma}{2G}\sum_{a=1}^{8}(\phi_a)^2\right)$$

Classical Scale Invariant Model I

- **One-loop effective potential from MFA:**

$$V_{\rm NJL}(\sigma, S; \Lambda_H) = \frac{3}{8G}\sigma^2 - \frac{G_D}{16G^3}\sigma^3 - 9I_0(M;\Lambda_H)$$

  - Constituent fermion mass : $M = \sigma + yS - \frac{G_D}{8G^2}\sigma^2$
  - G and G_D : determined by scaling-up the values for the real hadrons.

$$G^{QCD\,1/2}\Lambda^{QCD} = 1.82, \quad (-G_D^{QCD})^{1/5}\Lambda^{QCD} = 2.29$$

$$\rightarrow \quad G^{1/2}\Lambda_H = 1.82, \quad (-G_D)^{1/5}\Lambda_H = 2.29$$

Parameter	$(2G^{\rm QCD})^{-1/2}$	$(-G_D^{\rm QCD})^{-1/5}$	$\Lambda^{\rm QCD}$
Value (MeV)	361	406	930

- **Potential :**

$$V_{{\rm SM}+S} = \lambda_H(H^\dagger H)^2 - \frac{1}{2}\lambda_{HS}S^2(H^\dagger H) + \frac{1}{4}\lambda_S S^4$$

$$V_{\rm eff} = V_{\rm SM+S} + V_{\rm NJL}$$

  - **Free parameters :** $\lambda_{HS},\ \lambda_S,\ y\ ,\ \lambda_H,\ \Lambda_H$

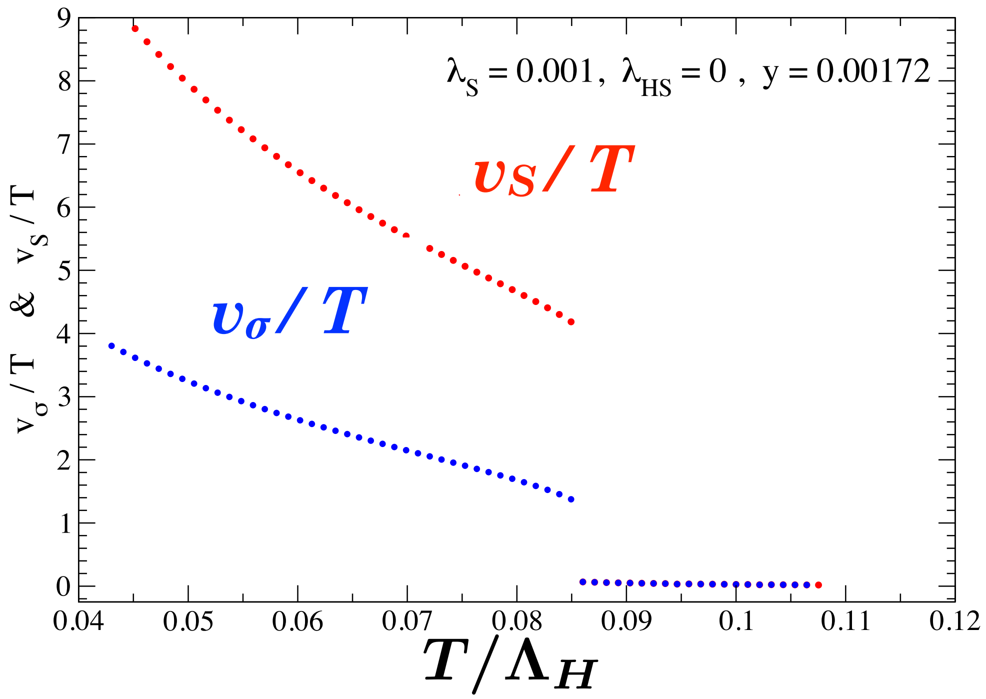
chosen to satisfy $m_h = 126$ GeV, $v = 246$ GeV

Classical Scale Invariant Model I

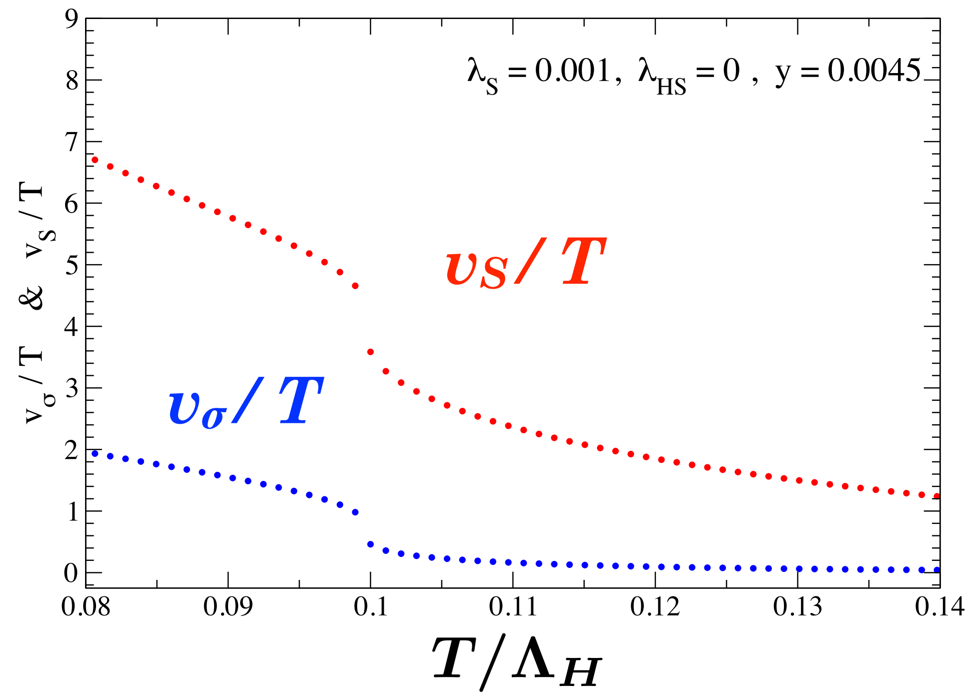
❖ **Hidden Chiral Phase Transition**

MA, Kubo, JCAP04 (2021)

$\lambda_S = 0.001,\ \lambda_{HS} = 0,\ y = 0.00172$



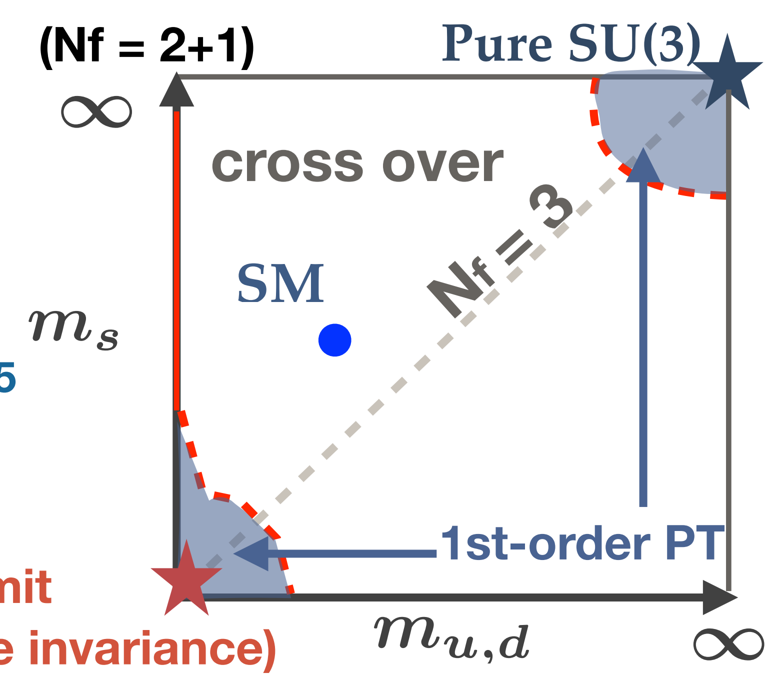
$\lambda_S = 0.001,\ \lambda_{HS} = 0,\ y = 0.0045$



- The chiral PT in the hidden sector becomes first order for $y \lesssim \mathcal{O}(10^{-3})$.

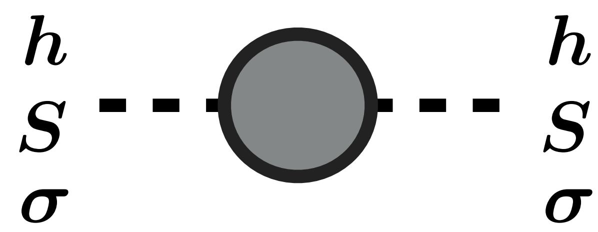
- **QCD PT**

Ding et al. arXiv: 1111.0185

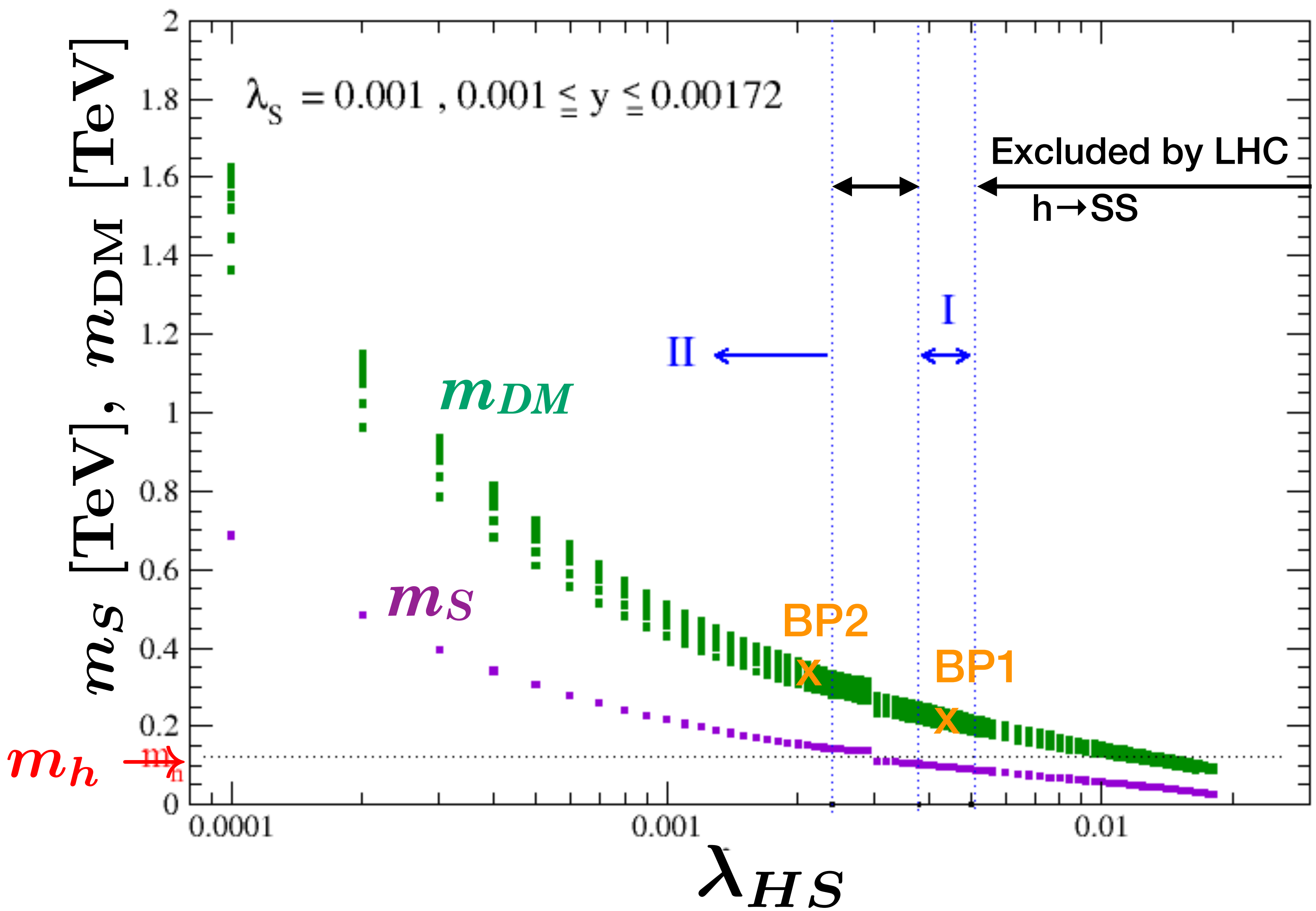


Classical Scale Invariant Model I

- **Mass spectrum**



$$\lambda_S = 0.001 \qquad y = 0.001 - 0.00172$$

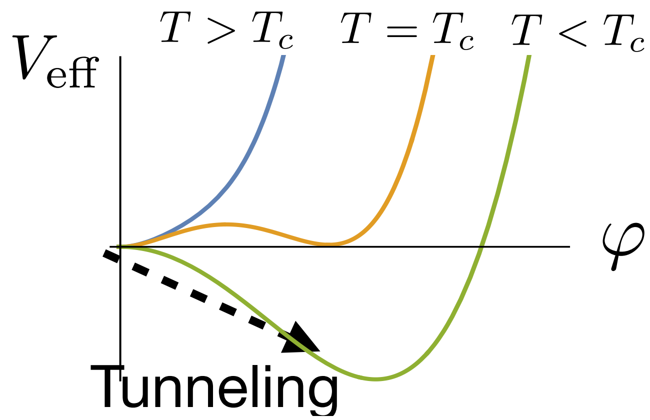


BP1:
$m_s = 90$ GeV
$\Lambda_H = 4.3$ TeV

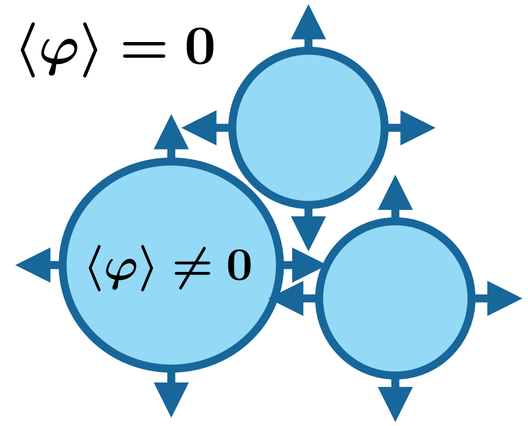
BP2 :
$m_s = 150$ GeV
$\Lambda_H = 6.6$ TeV

GW Spectrum

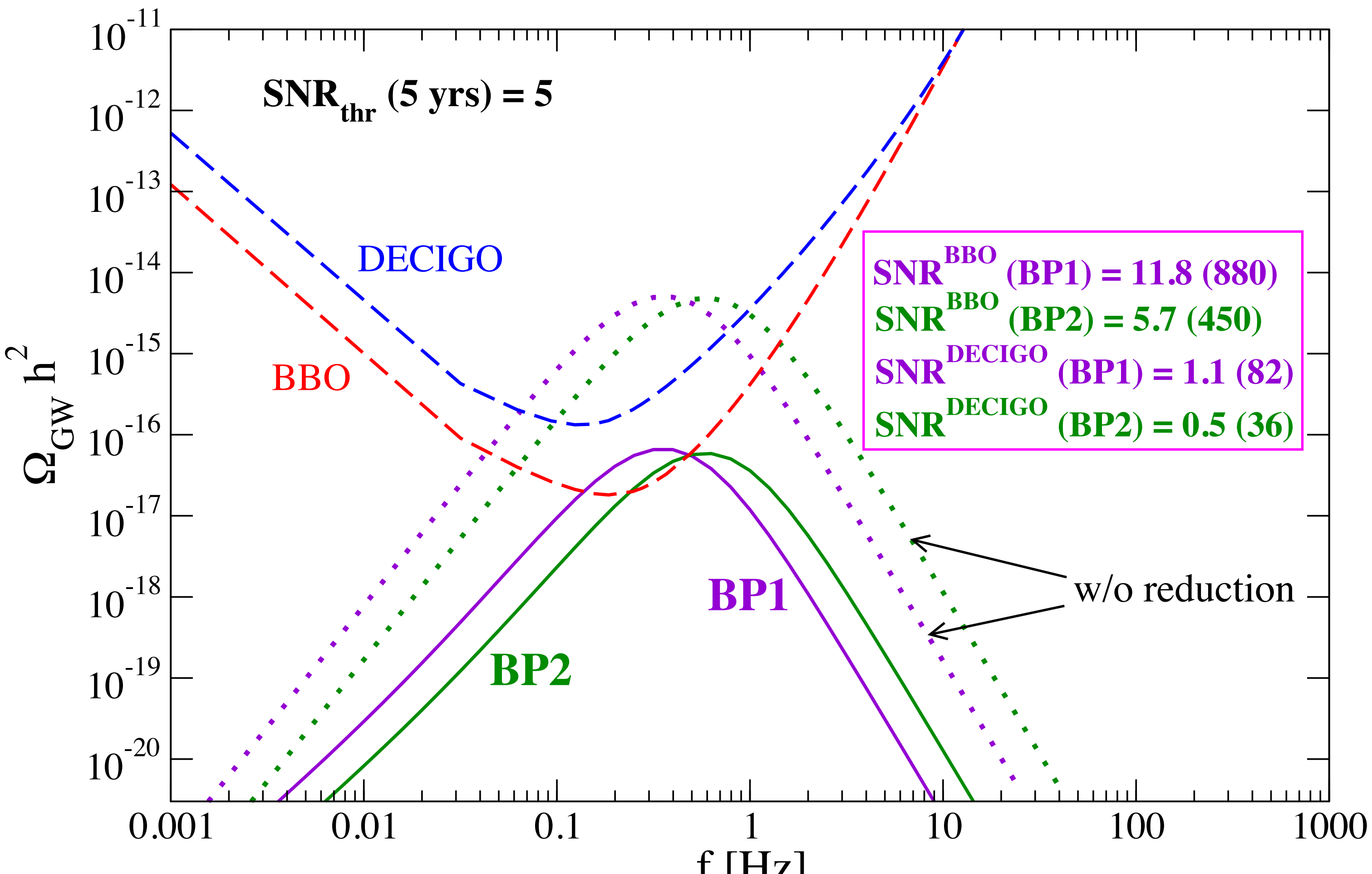
- First order phase transition :



- The nucleation of bubbles of the broken phase.



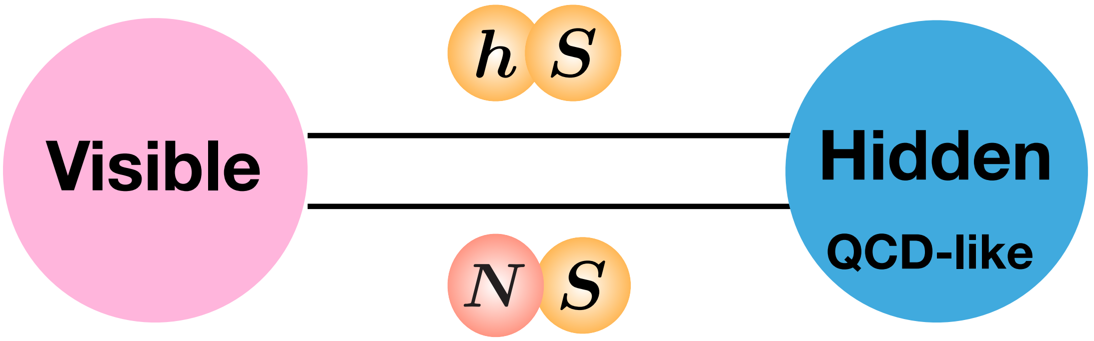
- **Sources of GWs :**
  - scalar field contribution
  - sound waves
  - magnetohydrodynamic turbulence



MA, Kubo, JCAP04 (2021)

- LISA 2035 launch
- DECIGO 2030s (?)

Classical Scale Invariant Model II



Classical Scale Invariant Model II

❖ No part of the model should contain any dimensionful parameter at the classical level.

► **Hidden QCD :**

$$\frac{\mathcal{L}_H}{\sqrt{-g}} = -\frac{1}{2}\mathrm{Tr}\ F^2 + \mathrm{Tr}\ \bar{\psi}(i\not{D} - \boldsymbol{y}S)\psi$$

► **SM interactions :**

$$\frac{\mathcal{L}_{\mathrm{SM}+S}}{\sqrt{-g}} = \mathcal{L}_{\mathrm{SM}}|_{\mu_H=0} + \frac{1}{2}g^{\mu\nu}\partial_\mu S\partial_\nu S - \frac{1}{4}\lambda_{HS}S^2H^\dagger H - \frac{1}{4}\lambda_S S^4$$

► **Heavy right-handed neutrinos :**

$$\frac{\mathcal{L}_N}{\sqrt{-g}} = \frac{i}{2}\bar{N}_R\not{\partial}N_R - \frac{1}{2}y_M S N_R^T C N_R - \left(y_\nu \bar{L}\tilde{H}\,\frac{1+\gamma_5}{2}N_R + \mathrm{h.c.}\right)$$

► **Gravity :**

$$\frac{\mathcal{L}_{\mathrm{G}}}{\sqrt{-g}} = -\frac{\beta}{2}S^2\,R + \gamma\,R^2 + \kappa\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}$$

R : Ricci curvature scalar

$W_{\mu\nu\alpha\beta}$: Weyl tensor

Classical Scale Invariant Model II

- **Effective potential :**

$$U_{\rm eff}(S,\sigma,R) = V_{\rm NJL}(S,\sigma) + U_S(S,R) - U_0$$

  - **One-loop effective potential :**

$$U_S(S,R) = \frac{1}{4}\lambda_S S^4 + \frac{1}{64\pi^2}\left(\tilde{m}_s^4 \ln[\tilde{m}_s^2/\mu^2]\right), \qquad \tilde{m}_s^2 = 3\lambda_S S^2 + \beta R$$

  - **U_0 is the zero-point energy density.**

$$U_{\rm eff}(S = v_S, \sigma = v_\sigma, R = 0) = 0$$

- **We assume that** $\beta R < 3\lambda_S S^2$ **.**

- **Expansion in powers of βR :**

$$U_S(S,R) = U_{\rm CW}(S) + U_{(1)}(S)\,R + U_{(2)}(S)R^2 + O(R^3)$$

Classical Scale Invariant Model II

- **Planck mass :** $M_{\rm Pl} = 1/\sqrt{8\pi G}$ Reduced Planck mass

$$M_{\rm Pl} = v_S \left(\beta + \frac{2U_{(1)}(v_S)}{v_S^2}\right)^{1/2} = \sqrt{\beta}\, v_S \left(1 + \frac{3\lambda_S}{16\pi^2}\ln[3\lambda_S]\right)^{1/2} \sim \sqrt{\beta} v_S$$

  - **β ~ O(10³) for a successful inflation.**

    → v_S is few orders of magnitude smaller than $M_{\rm Pl}$.

- **Right-handed Neutrino mass :**
  - **For the neutrino option to work,**

$$m_N = y_M v_S \sim 10^7 \quad \rightarrow \quad y_M \sim \sqrt{\beta}\, 10^{-11}$$

  - **The small y_M is not unnatural, because in its absence the lepton number is conserved.**

Inflation

Inflation

Effective Lagrangian for inflation

Jordan frame

$$\frac{\mathcal{L}_{\rm eff}}{\sqrt{-g_J}} = -\frac{1}{2}M_{\rm Pl}^2 B(S) R_J + G(S) R_J^2 + \frac{1}{2} g_J^{\mu\nu}\left(\partial_\mu S \partial_\nu S + Z_\sigma^{-1}(S,\sigma)\partial_\mu\sigma\partial_\nu\sigma\right) - U(S,\sigma)$$

$$\begin{aligned}
B(S) =& \frac{\beta\, S^2}{M_{\rm Pl}^2}\left(1 + \frac{3\lambda_S}{16\pi^2}\ln[3\lambda_S S^2/v_S^2]\right) , \\
G(S) =& \gamma - \frac{\beta^2}{64\pi^2}\left(1 + \ln[3\lambda_S S^2/v_S^2]\right) , \\
U(S,\sigma) =& V_{\rm NJL}(S,\sigma) + \frac{\lambda_S}{4}S^4 + \frac{9\lambda_S^2 S^4}{64\pi^2}\left(-\frac{1}{2} + \ln[3\lambda_S S^2/v_S^2]\right) - U_0
\end{aligned}$$

- **Auxiliary field :** χ $\quad G(S)R_J^2 \;\rightarrow\; 2G(S)R_J\chi - G(S)\chi^2$
- **Weyl rescaling of the metric :** $g_{\mu\nu} = \Omega^2\, g_{\mu\nu}^J$

$$\Omega^2(S,\chi) = B(S) - \frac{4\,G(S)\chi}{M_{\rm Pl}^2}$$

Inflation

❖ **Einstein frame :**

$$\frac{\mathcal{L}^{E}_{\rm eff}}{\sqrt{-g}} = -\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi + \frac{1}{2}e^{-\Phi(\varphi)} g^{\mu\nu}\left(\partial_\mu S\,\partial_\nu S + Z_\sigma^{-1}(S,\sigma)\partial_\mu\sigma\,\partial_\nu\sigma\right) - V(S,\sigma,\varphi)$$

- scalaron : $\varphi = \sqrt{\frac{3}{2}}M_{\rm Pl}\ln\left|\Omega^2\right|$

❖ **Potential :**

$$V(S,\sigma,\varphi) = e^{-2\,\Phi(\varphi)}\left[U(S,\sigma) + \frac{M_{\rm Pl}^4}{16\,G(S)}\left(B(S) - e^{\Phi(\varphi)}\right)^2\right], \qquad \Phi(\varphi) = \sqrt{\frac{2}{3}}\frac{\varphi}{M_{\rm pl}}$$

Three-field system : $S,\ \sigma,\ \varphi$

❖ **The valley approximation :**

- We only have to deal with a single-field inflaton system.

$$\left.\frac{\partial V(S,\sigma,\varphi)}{\partial\varphi}\right|_{\varphi=\varphi_{\rm v}} = 0$$

- Double-field system potential :

$$\tilde{V}(S,\sigma) = V(S,\sigma,\varphi_{\rm v}) = \frac{U(S,\sigma)M_{\rm Pl}^4}{16G(S)\,U(S,\sigma) + B^2(S)\,M_{\rm Pl}^4}$$

Inflation

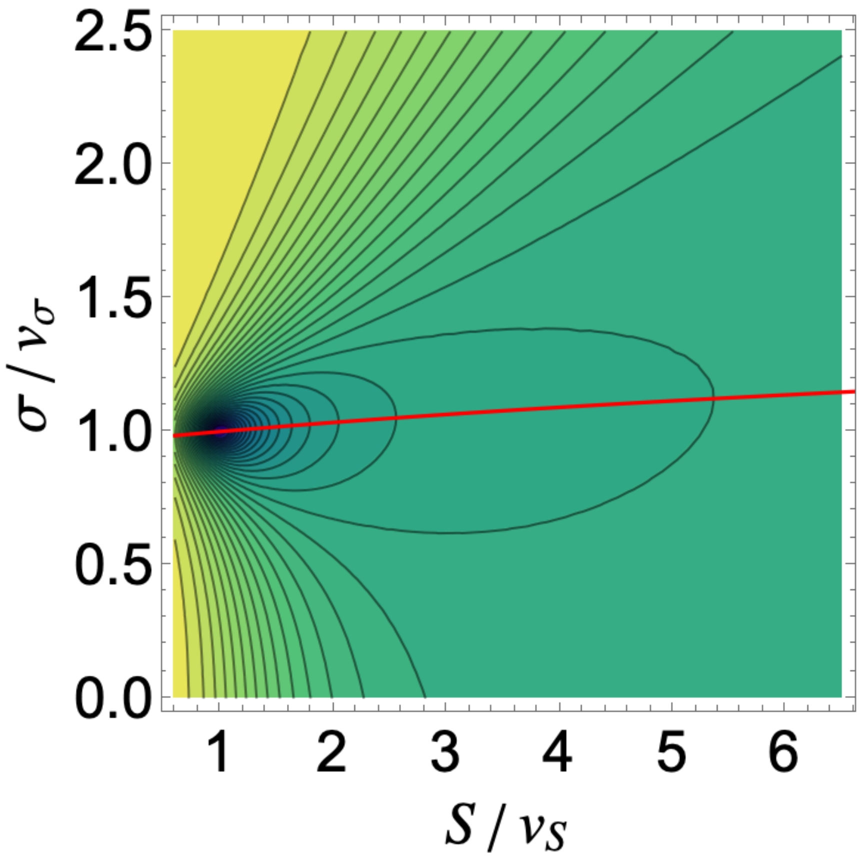
$$y, \quad \lambda_S, \quad \beta, \quad \gamma$$

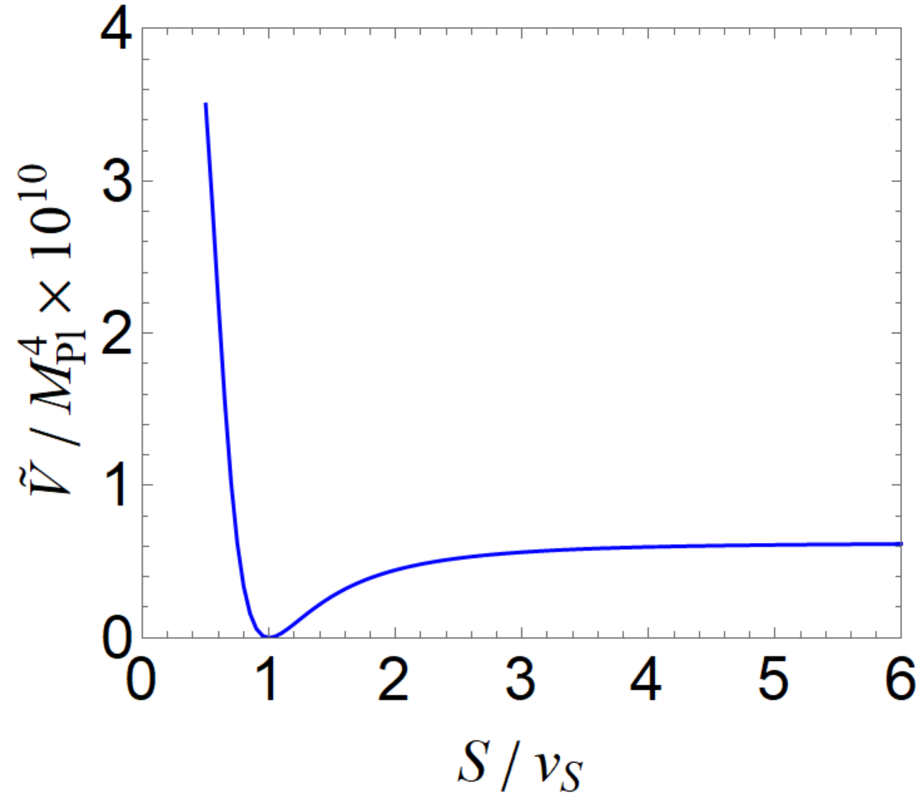
Benchmark :

$$y = 4.00 \times 10^{-3}\,, \quad \lambda_S = 1.14 \times 10^{-2}\,, \quad \beta = 6.31 \times 10^{3}\,, \quad \gamma = 1.26 \times 10^{8}\,.$$

$$v_\sigma = 8.86 \times 10^{-3} M_{\rm Pl}\,, \quad v_S = 1.26 \times 10^{-2} M_{\rm Pl}\,, \quad U_0 = -7.02 \times 10^{-10} M_{\rm Pl}^4\,, \quad \Lambda_H = 5.33 \times 10^{-2} M_{\rm Pl}\,.$$

$$\tilde{V}(S,\sigma) = V(S,\sigma,\varphi_{\rm v})$$





- **The single-field inflaton system :**

$$\sigma \;\to\; \sigma_{\rm v}(S), \quad \varphi \;\to\; \varphi_{\rm v}(S) \qquad\qquad V_{\rm inf}(S) = V(S,\sigma_{\rm v}(S),\varphi_{\rm v}(S))$$

Inflation

❖ **Slow roll parameters :**

$$\varepsilon(S) = \frac{M_{\rm Pl}^2}{2\,F^2(S)}\left(\frac{V'_{\rm inf}(S)}{V_{\rm inf}(S)}\right)^2 , \qquad \eta(S) = \frac{M_{\rm Pl}^2}{F^2(S)}\left(\frac{V''_{\rm inf}(S)}{V_{\rm inf}(S)} - \frac{F'(S)}{F(S)}\frac{V'_{\rm inf}(S)}{V_{\rm inf}(S)}\right)$$

$$e^{-\Phi(\varphi_{\rm v}(S))}\, g^{\mu\nu}\left[\partial_\mu S\partial_\nu S + Z_\sigma^{-1}(S,\sigma_{\rm v})\partial_\mu\sigma_{\rm v}(S)\,\partial_\nu\sigma_{\rm v}(S)\right] + g^{\mu\nu}\partial_\mu\varphi_{\rm v}(S)\partial_\nu\varphi_{\rm v}(S) = F(S)^2 g^{\mu\nu}\partial_\mu S\partial_\nu S$$

❖ **Number of e-folds :**

$$N_e = \int_{S_{\rm end}}^{S_*} dS\,\frac{F^2(S)}{M_{\rm Pl}^2}\frac{V_{\rm inf}(S)}{V'_{\rm inf}(S)}$$

S_* : the value of S at the time of CMB horizon exit

$S_{\rm end}$: the end of inflation $\quad \varepsilon(S = S_{\rm end}) = 1$

❖ **Amplitude of the scalar perturbation :**

$$A_s = \frac{V_{\rm inf\,*}}{24\pi^2\,\varepsilon_*\,M_{\rm Pl}^4}$$

- We constrain the parameter space scanned by

$$y, \quad \lambda_S, \quad \beta, \quad \gamma$$

such that $\ln(10^{10}A_s) \simeq 3.044, \quad 50 \lesssim N_e \lesssim 60$ are satisfied.

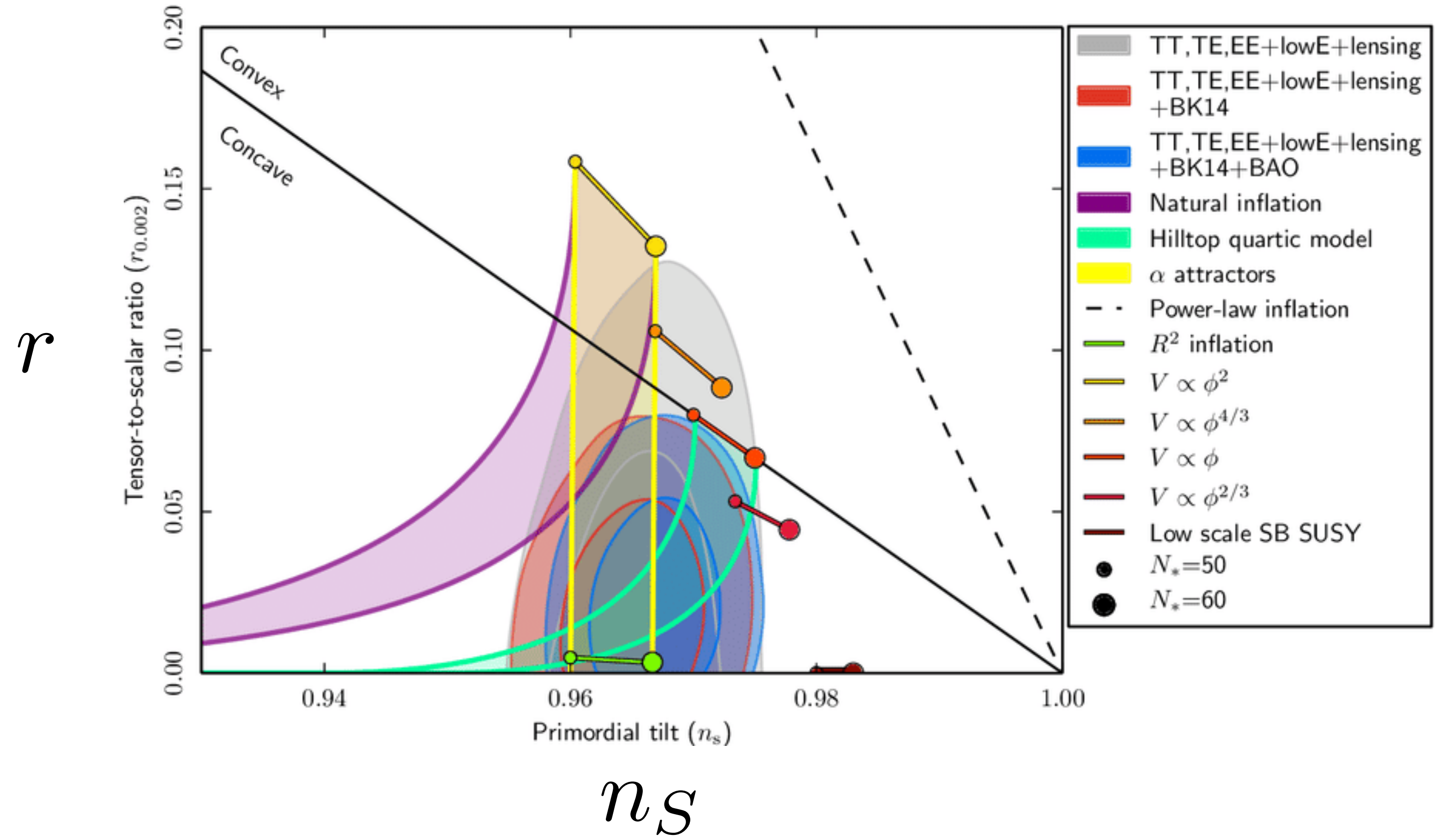
Inflation

❖ **Scalar spectral index :**

$$n_s = 1 + 2\,\eta_* - 6\,\varepsilon_*$$

❖ **Tensor-to-scalar ratio :**

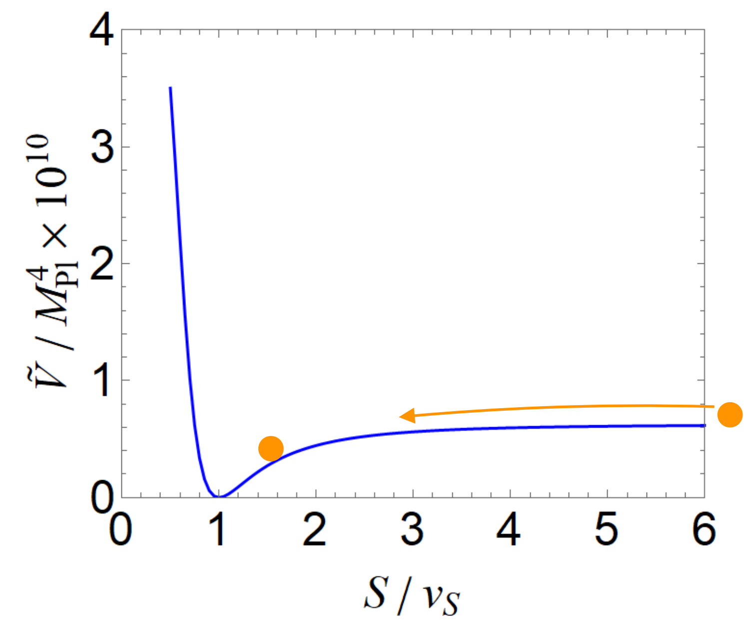
$$r = 16\,\varepsilon_*$$



✤ **Benchmark :**

$$y = 4.00 \times 10^{-3}\,, \quad \lambda_S = 1.14 \times 10^{-2}\,, \quad \beta = 6.31 \times 10^{3}\,, \quad \gamma = 1.26 \times 10^{8}\,.$$

$$\ln(10^{10} A_s) = 3.04\,,\ N_e = 55.5 \ \rightarrow \ n_s = 0.964\,, \ \ r = 2.00 \times 10^{-3}\,.$$

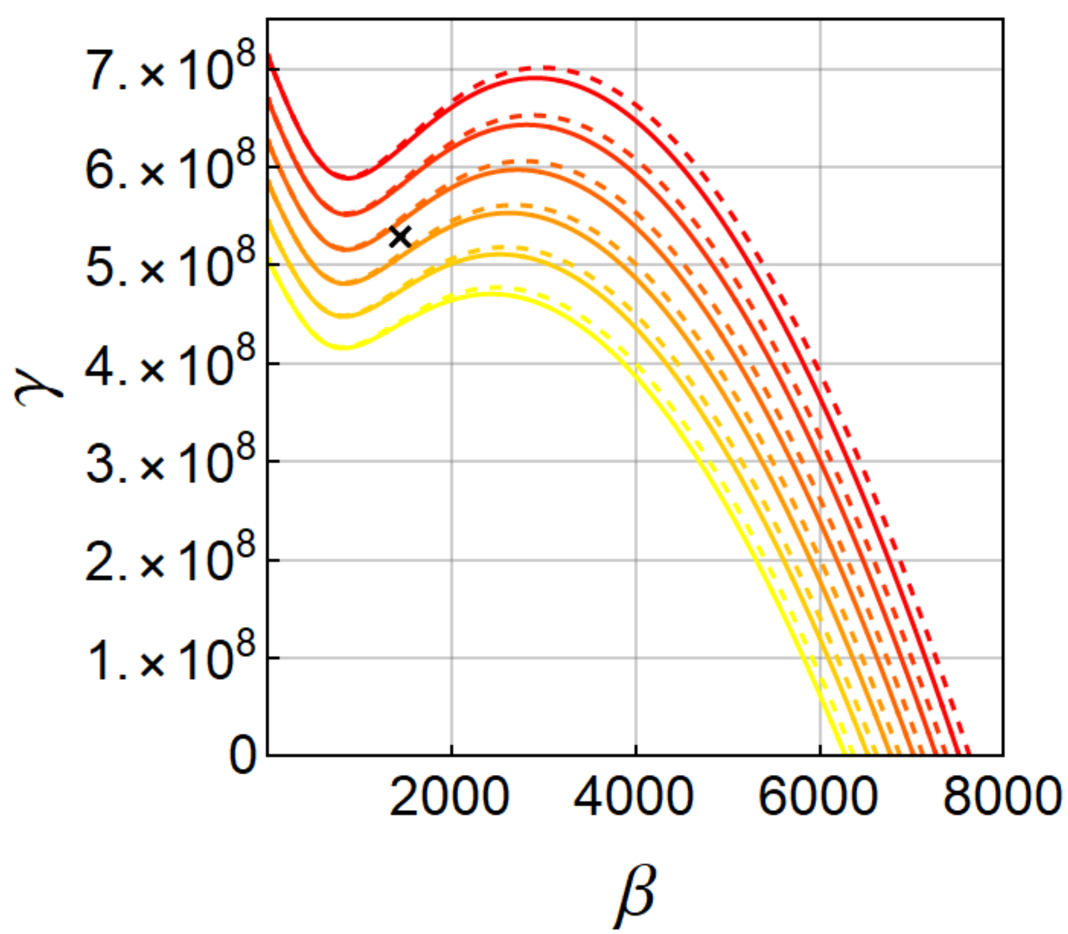


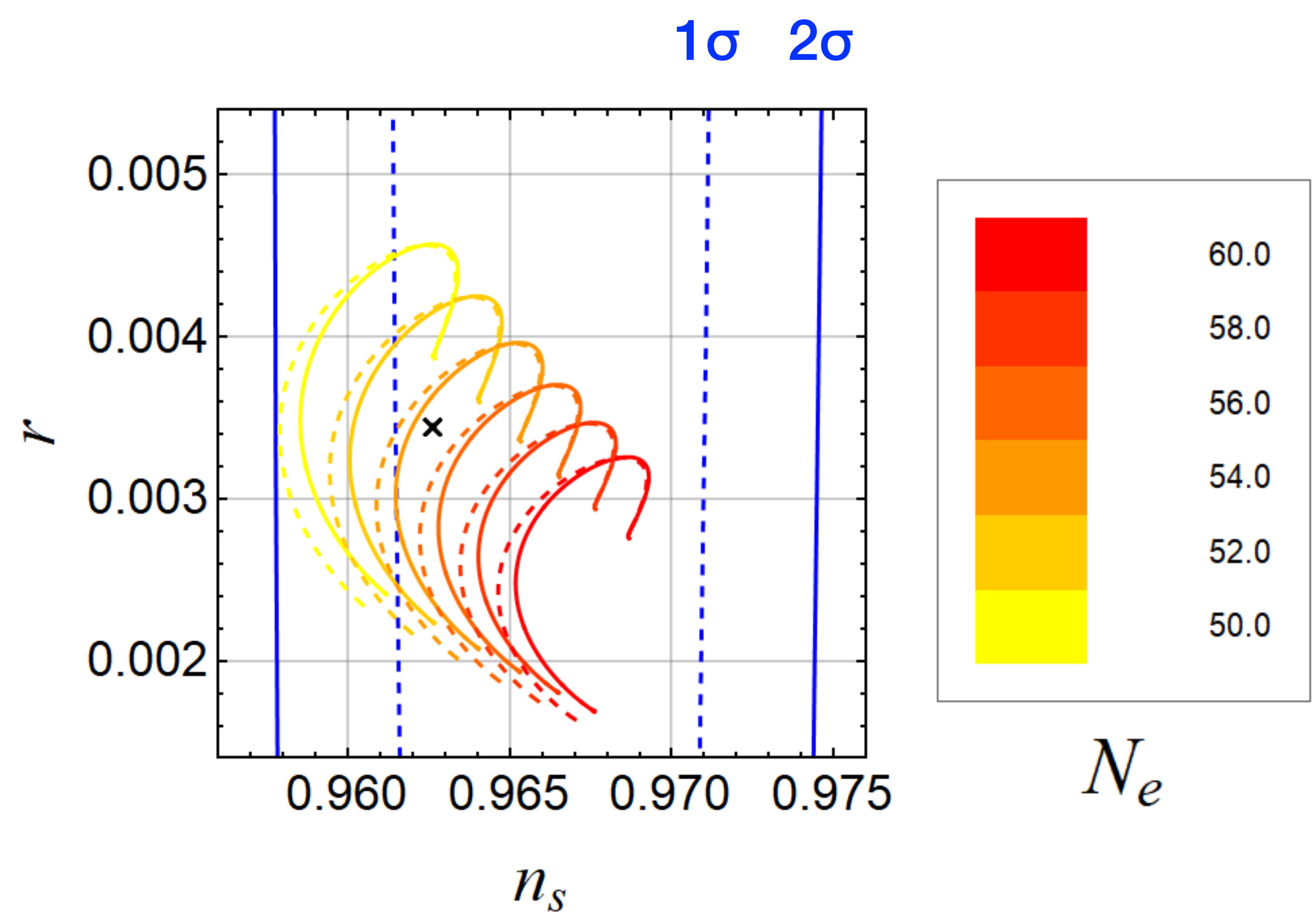
Inflation

$\lambda_S = 1.20 \times 10^{-2}$ $y = 4 \times 10^{-3}$ (solid), 4×10^{-4} (dashed)

x : benchmark point

$\ln(10^{10} A_s) = 3.044$

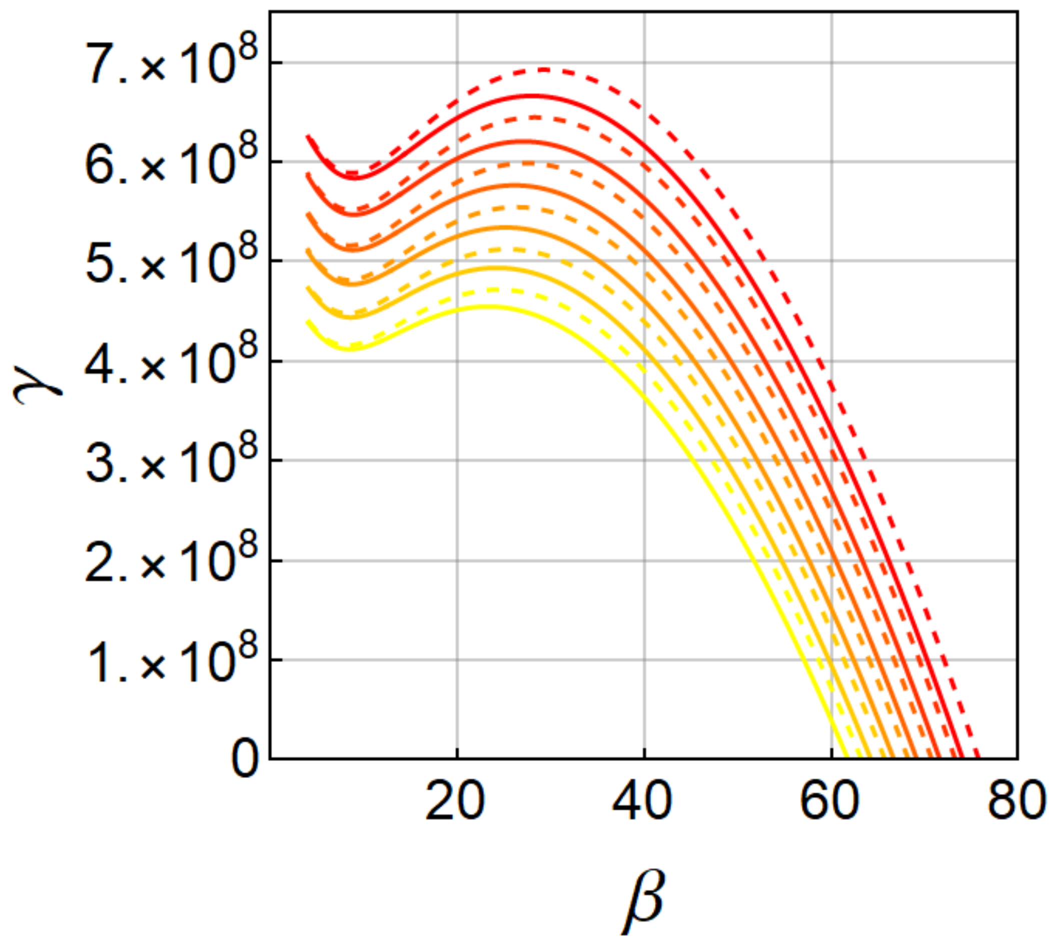


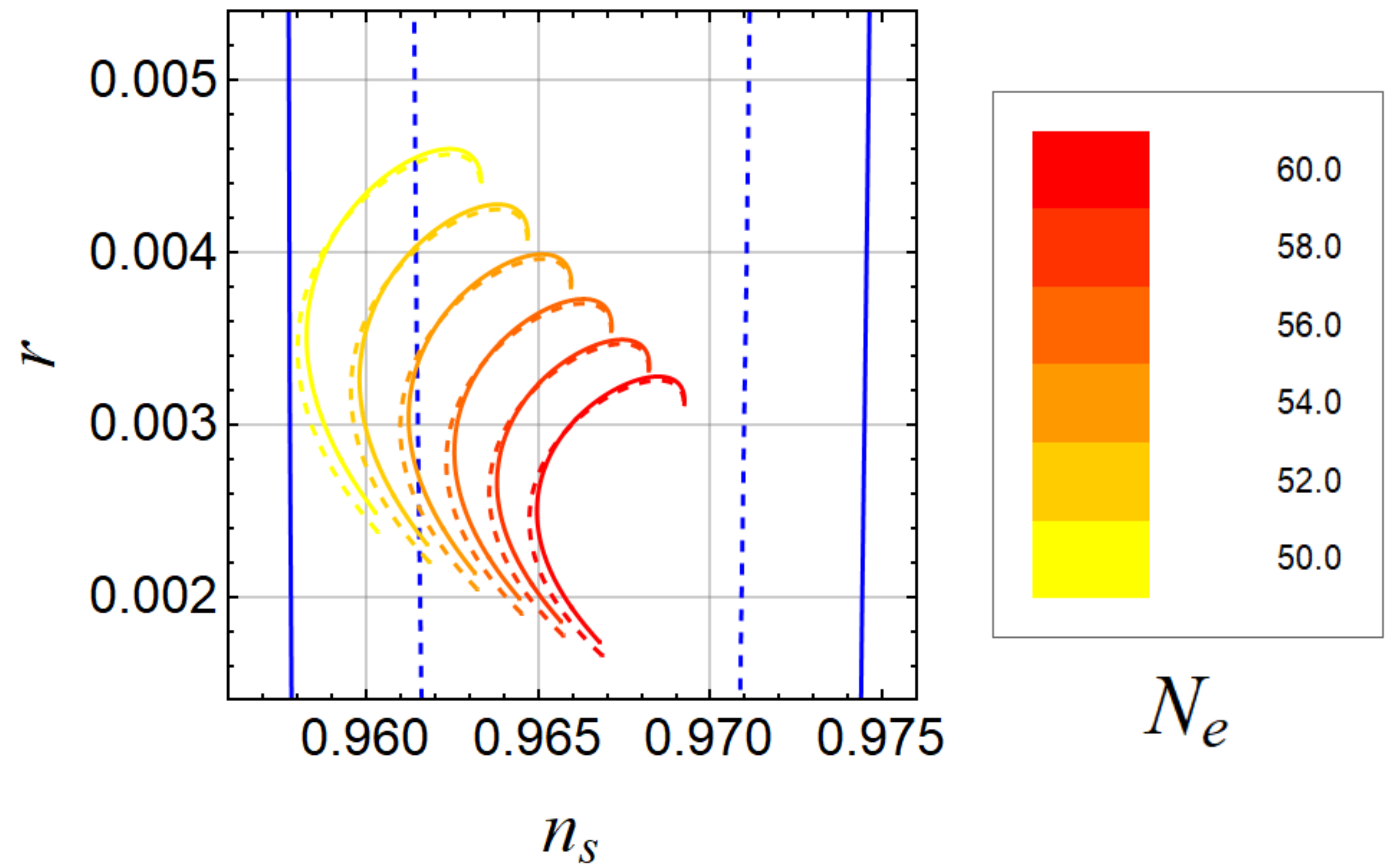


Inflation

$$\lambda_S = 1.20 \times 10^{-6} \qquad y = 4 \times 10^{-3}\ \text{(solid)},\quad 4 \times 10^{-4}\ \text{(dashed)}$$

$$\ln(10^{10} A_s) = 3.044$$

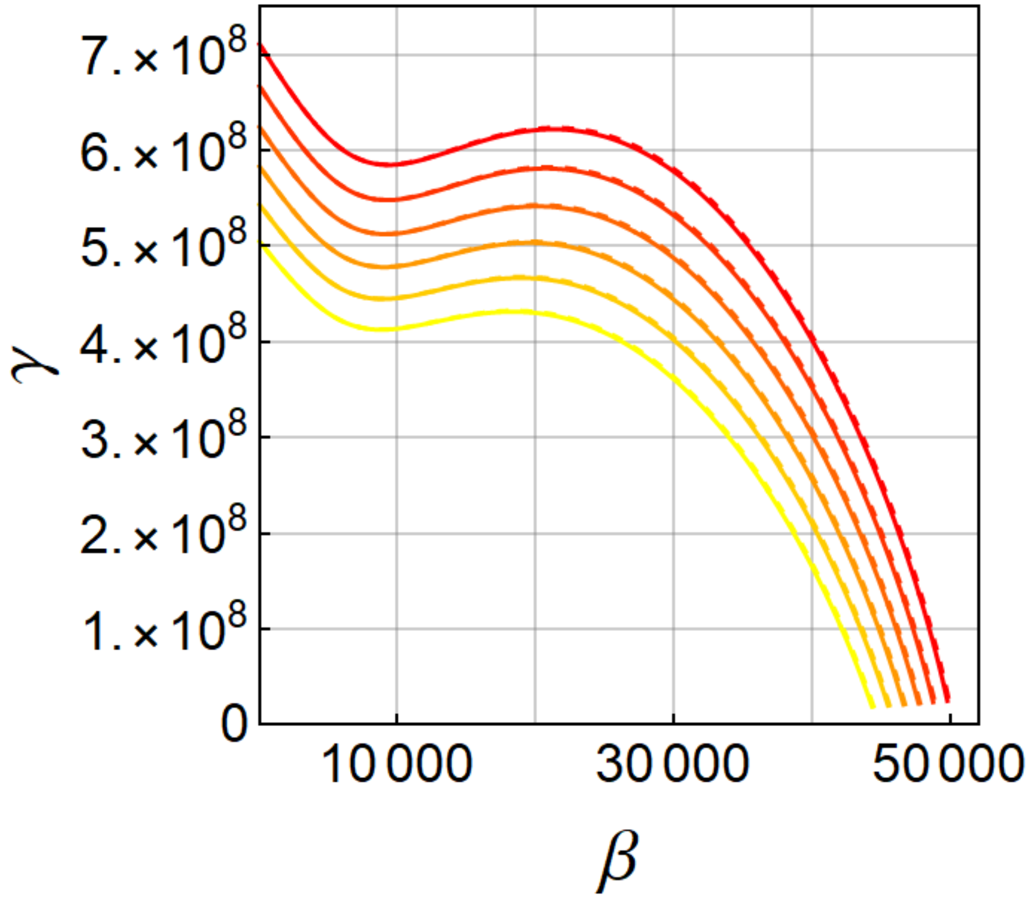


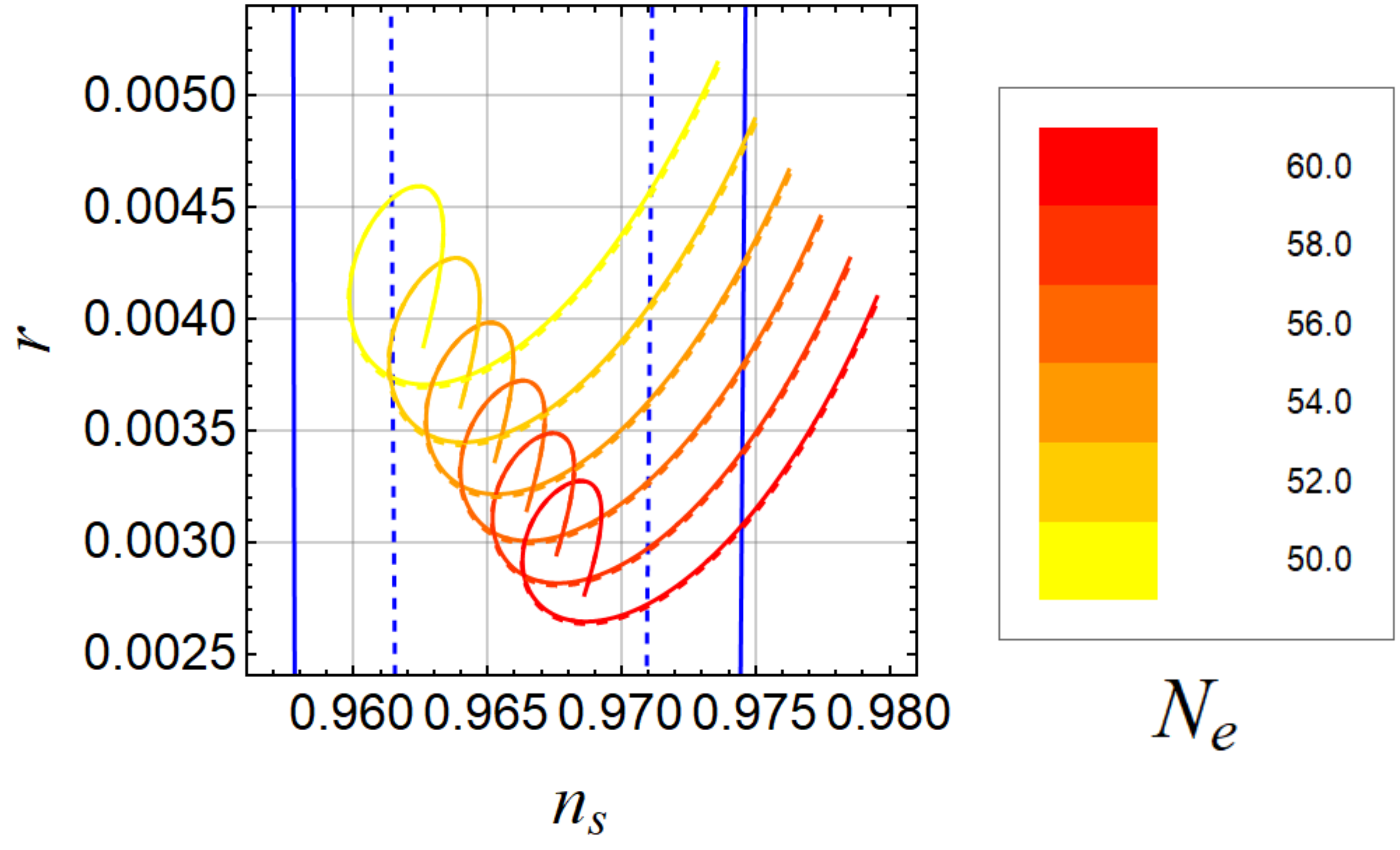


Inflation

$\lambda_S = 1.2$ $\qquad$ $y = 4 \times 10^{-3}$ (solid), 4×10^{-4} (dashed)

$\ln(10^{10} A_s) = 3.044$





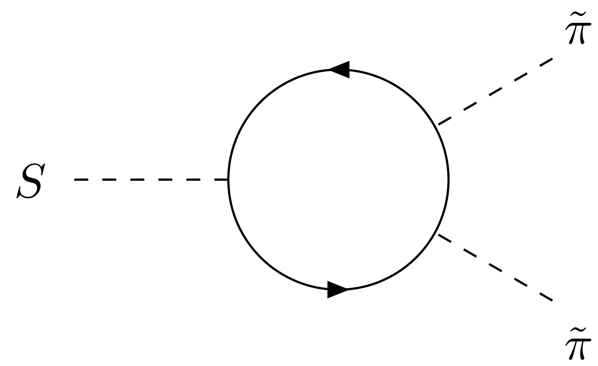
Dark Matter

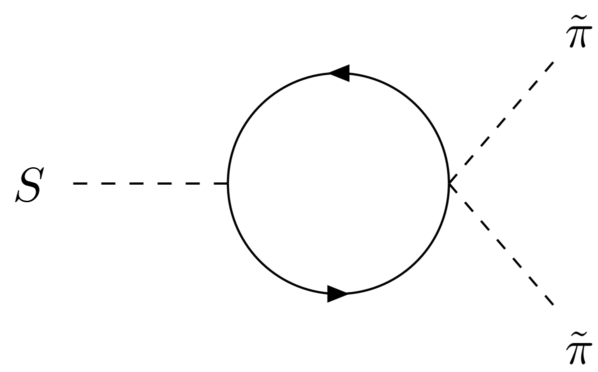
Dark Matter

❖ DM production:

Dark mater can be produced during or after the reheating phase.

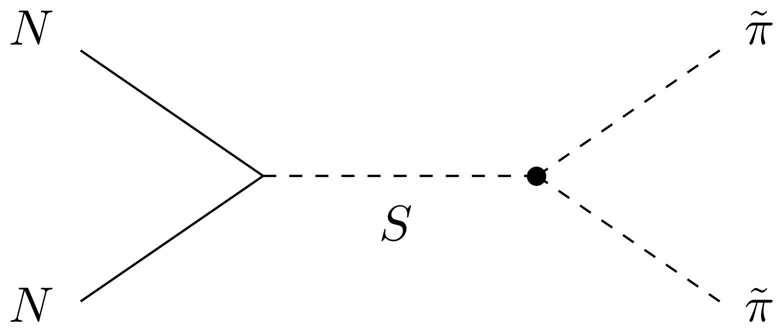
- **S decay :**





$$G_{\tilde{\pi}\tilde{\pi}S}/\Lambda_H \simeq -0.012\, y_1 \text{ for } y_1 \lesssim 5 \times 10^{-4}.$$

- **N annihilation :**

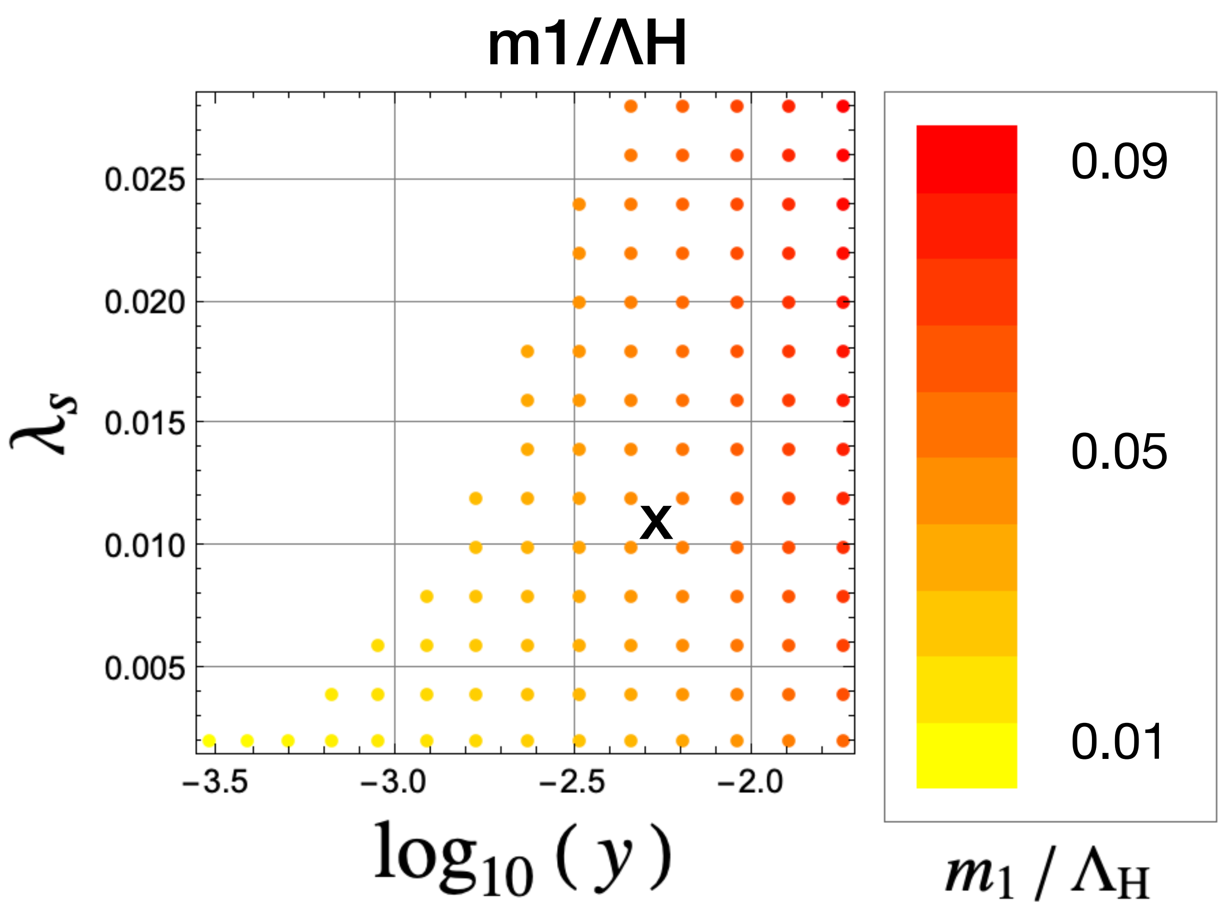


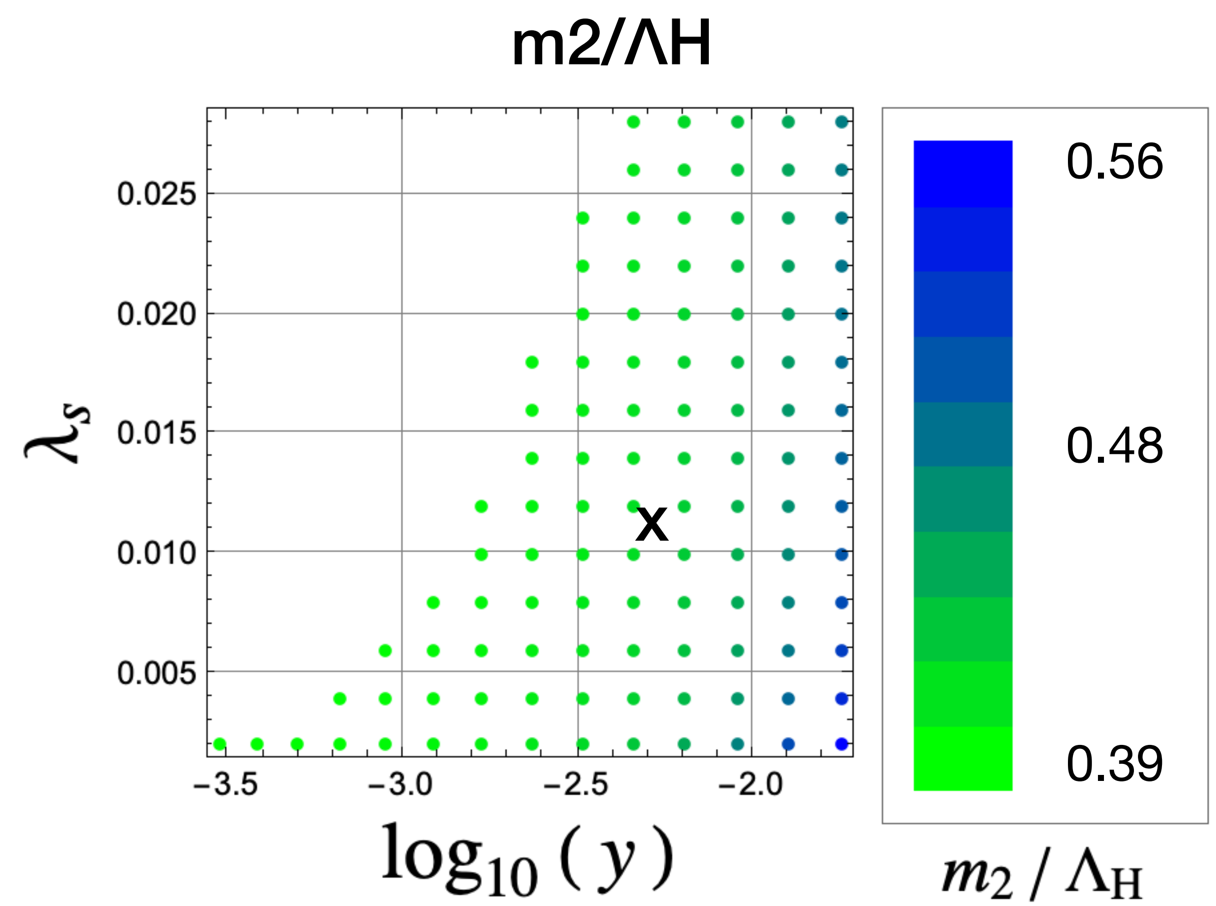
$$y_M^2 \sim m_N^2/v_S^2 \sim 10^{-18}$$

Dark Matter

❖ **Mass spectrum**

- CP-even scalar bosons : $(S,\ \sigma) \rightarrow (S_1,\ S_2)$ mass eigenstates
- CP-odd scalar (dark meson) : ϕ_a





- Data points : $m_\phi > \tilde{m}_S$ is satisfied. $(\tilde{m}_S \equiv m_1)$
- Benchmark point : $m_\phi/\Lambda_H \simeq 0.06 > \tilde{m}_S/\Lambda_H$

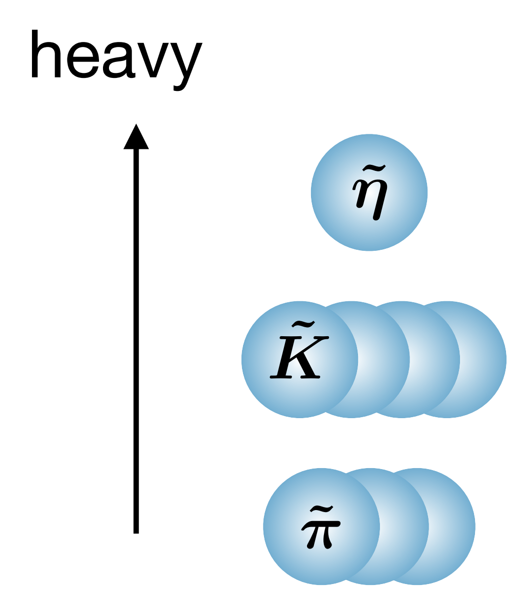
Dark Matter

- **Flavor symmetry :** $\mathcal{L} \supseteq -\bar{\psi} y S \psi$

$SU(3)_V \to SU(2)_V \times U(1)$

$$\boldsymbol{y} = \mathrm{diag.}\,(y_1\,,\, y_1\,,\, y_3\,) \text{ with } y_1 = y_2 < y_3$$

  - $\boldsymbol{y_1 = y_2 < y_3}$



- We assume that the $\tilde{\pi}$ Yukawa couplings y_1 and y_3 are so chosen that the mass hierarchy $2m_{\tilde{\pi}} < \tilde{m}_S < m_{\tilde{K}} < m_{\tilde{\eta}}$ is satisfied.

Dark Matter

- **Coupled Boltzmann equations :**

$$\begin{cases} \dfrac{dn_S}{dt} = -3Hn_S - \Gamma_S n_S\,, & \to\ n_S(a) = \dfrac{\rho_{\rm end}}{\tilde{m}_S}\left[\dfrac{a_{\rm end}}{a}\right]^3 e^{-\Gamma_S(t-t_{\rm end})} \\ & \swarrow \\ \dfrac{dn_{\tilde\pi}}{dt} = -3Hn_{\tilde\pi} + \gamma_{\tilde\pi}\, n_S & \to\ n_{\tilde\pi}(a) = B_{\tilde\pi}\dfrac{\rho_{\rm end}}{\tilde{m}_S}\left[\dfrac{a_{\rm end}}{a}\right]^3\left(1 - e^{-\Gamma_S(t-t_{\rm end})}\right) \end{cases}$$

$$B_{\tilde\pi} = \gamma_{\tilde\pi}/\Gamma_S$$

  - Γ_S : total decay width of S
  - $S \to \tilde\pi\tilde\pi \qquad \gamma_{\tilde\pi} = \dfrac{3\,G^2_{\tilde\pi\tilde\pi S}}{16\pi\tilde{m}_S}\sqrt{1 - \dfrac{4m^2_{\tilde\pi}}{\tilde{m}^2_S}}$
  - at $t_{\rm end}$, $\quad \rho_{\rm end} = \tilde{m}_S n_S(a_{\rm end})$, $\quad n_{\tilde\pi}(a_{\rm end}) = 0$

- **DM relic abundance :**

Allahverdi, Drees, PRL89 (2002)

$$\Omega_{\tilde\pi}h^2 = \frac{m_{\tilde\pi}n_{\tilde\pi}h^2}{\rho(a_0)} = m_{\tilde\pi}\,B_{\tilde\pi}\,\frac{\rho_{\rm end}}{\tilde{m}_S}\left[\frac{a_{\rm end}}{a_0}\right]^3\frac{1}{3M^2_{\rm Pl}(H_0/h)^2} \simeq 2.04\times 10^8\,B_{\tilde\pi}\left(\frac{m_{\tilde\pi}}{\tilde{m}_S}\right)\frac{T_{\rm RH}}{1\,{\rm GeV}}$$

$$a_0 = 1,\ H_0 = h\,2.1332\times 10^{-42}\ {\rm GeV},\ h \simeq 0.674$$

  - **Branching ratio :**

assumption : $1/\Gamma_S$ is the time scale at the end of the reheating phase.

$$1/\Gamma_S \quad \to \quad 1/H(a_{\rm RH}) = \left(3\,M^2_{\rm Pl}/\rho_{\rm RH}\right)^{1/2}$$

Dark Matter

❖ T_{RH} and N_e

Martin, Ringeval, PRD82 (2010)

- It is possible to constrain the reheating phase and hence T_{RH} for a given inflation model without specifying reheating mechanism.

- **Number of e-folds**

$$N_e = \ln\left(\frac{a_{\text{end}}}{a_*}\right)$$

$$= 66.80 - \ln\left(\frac{k_*}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_{\text{inf}\,*}^2}{M_{\text{Pl}}^4\,\rho_{\text{end}}}\right) - \frac{1}{12}\ln\left(\frac{V_{\text{end}}(3-\varepsilon_*)}{(3-\varepsilon_{\text{end}})M_{\text{Pl}}^4}\right) + \frac{1}{3}\ln\left(\frac{T_{\text{RH}}}{M_{\text{Pl}}}\right)$$

- $$\sqrt{3}H_* = \frac{V_{\text{inf}\,*}^{1/2}}{M_{\text{Pl}}}, \quad \rho_{\text{end}} = \frac{V_{\text{end}}(3-\varepsilon_*)}{(3-\varepsilon_{\text{end}})}$$

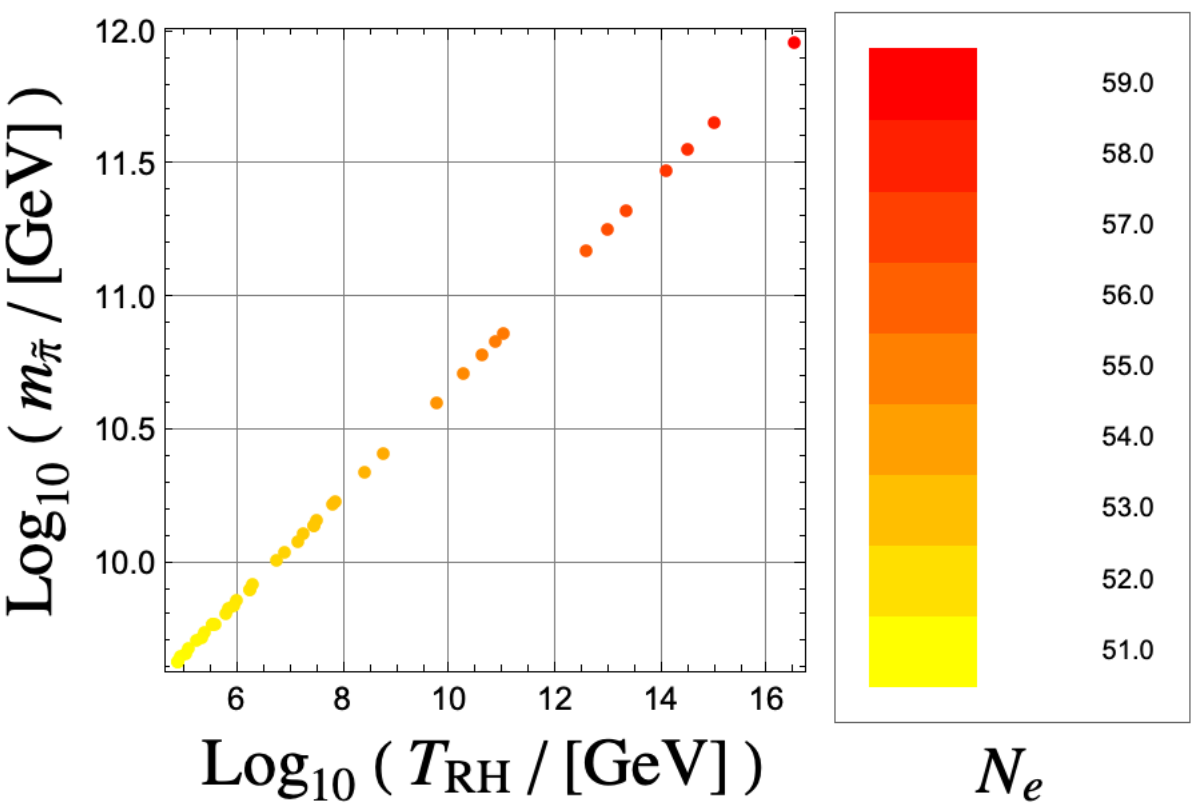
$$V_{\text{end}} = V_{\text{inf}}(S_{\text{end}})\,,\, V_{\text{inf}\,*} = V_{\text{inf}}(S_*),\ \varepsilon_{\text{end}} = \varepsilon(S_{\text{end}}),\ \text{and}\ \varepsilon_* = \varepsilon(S_*).$$

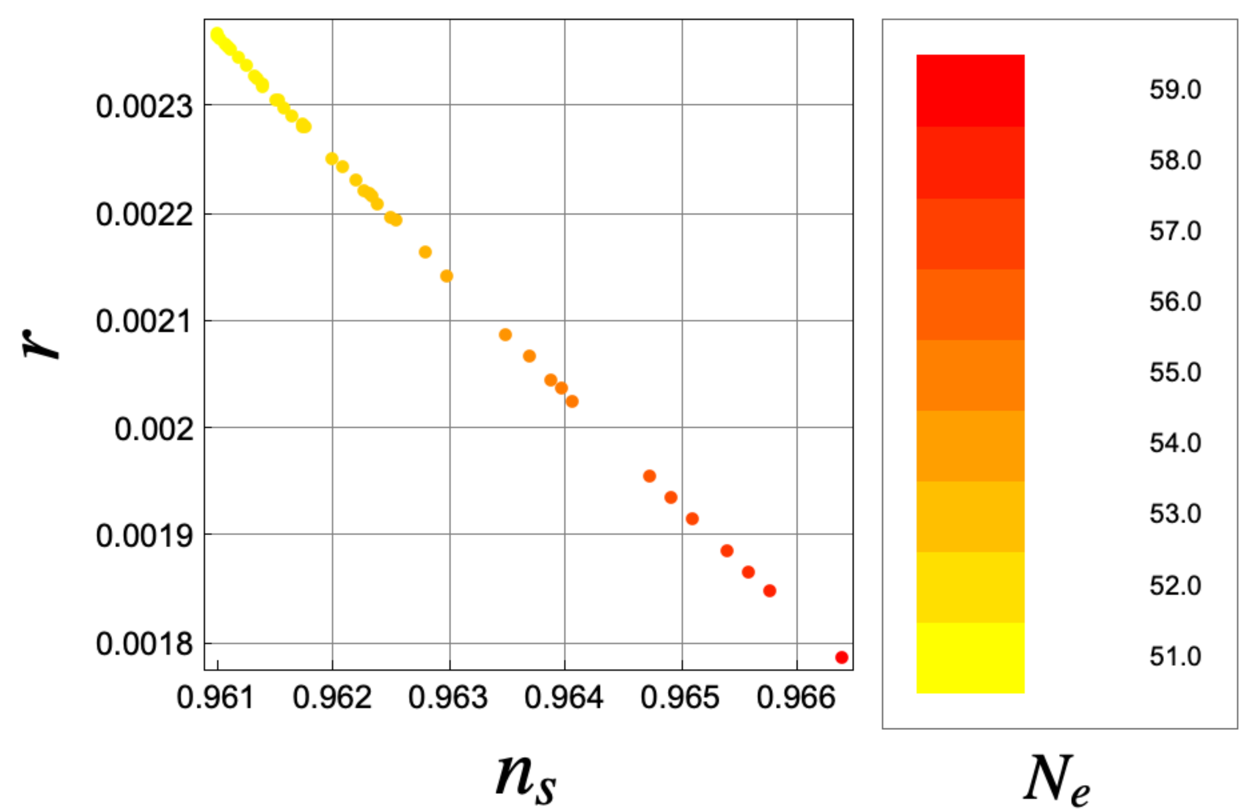
Dark Matter

$$y_3 = 4.00 \times 10^{-3}, \quad \beta = 6.31 \times 10^{3}, \quad \lambda_S = 1.2 \times 10^{-2}$$

Scan : y1 and γ.

$$\Omega_\pi h^2 = 0.1198 \pm 0.0024 \ \ (2\sigma)$$





- The DM relic abundance can become comparable with the observed value for $y_1 \simeq 10^{-13}$.

Summary

❖ We have studied a classical scale invariant model.

Scalegenesis :

The dynamical symmetry breaking by strong dynamics in hidden QCD sector.

❖ Neutrino option + gravity (Model II) :

- ❖ The origin of the dimensionful parameters, i.e. the Planck mass and the electroweak scale including the right-handed neutrino mass, is chiral symmetry breaking in a hidden QCD sector.
- ❖ Inflation :
  - The prediction of the CMB obs. is similar to that of the Higgs inflation or R^2 inflation. (small r)
- ❖ Dark matter : NG bosons
  - Stable by the unbroken vector-like flavor symmetry.
  - The only viable scenario for a realistic DM in our model is the decay from S.
- ❖ The reheating temperature is constrained.