Solving Gauge Theories Using Supersymmetry

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Strong Impression

What is important in physics is to come up with a good zeroth order approximation

Kiyoshi Higashijima

- Non-perturbative dynamics is very difficult to understand
- can we come up with a good zeroth order approximation?

Main point

- very few methods to study strongly coupled systems, e.g., QCD
- non-perturbative effects are essential $e^{-8\pi^2/g^2\hbar}$
- supersymmetry makes exact analytic studies of non-perturbative effects possible
- small anomaly-mediated SUSY breaking still allows for exact solutions

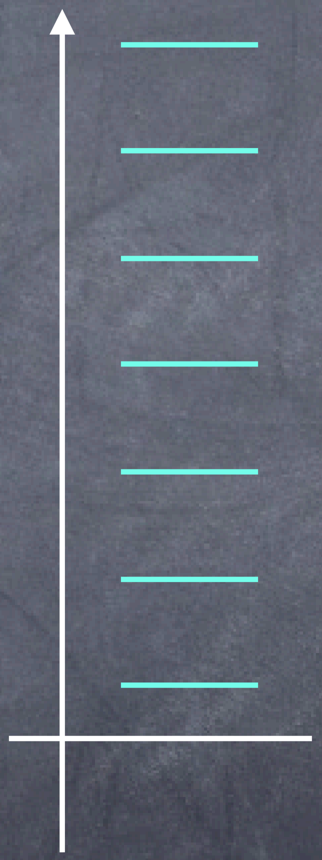
Prototype of SUSY
Landau level with g=2

Landau levels

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2$$

$$E = \hbar\omega_c \left(n + \frac{1}{2} \right)$$

$$\omega_c = \frac{eB}{m}$$



- states are degenerate by $\frac{eBA}{2\pi\hbar}$
- zero-point energy

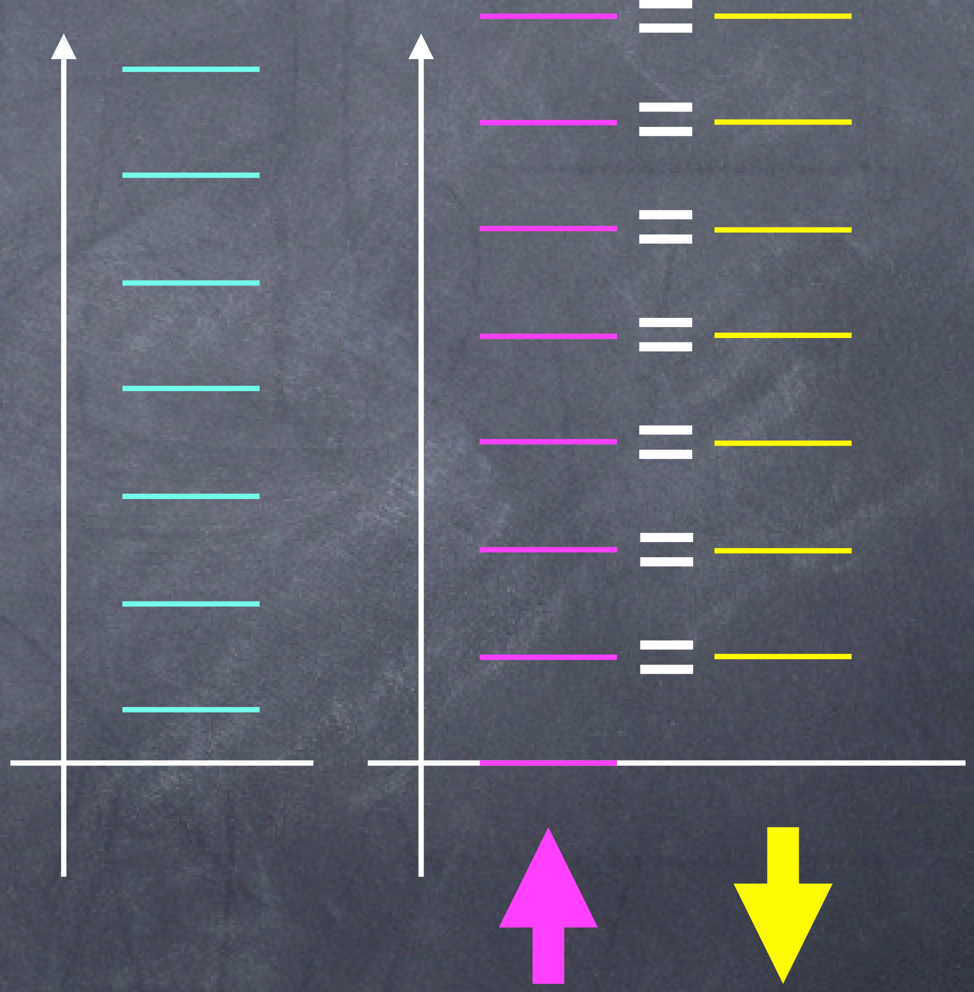
Landau levels with g=2

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 - g\frac{e}{2m}s_z B$$

$$E = \hbar\omega_c\left(n + \frac{1}{2}\right) \mp g\frac{eB}{2m}\frac{\hbar}{2}$$

$$\omega_c = \frac{eB}{m}$$

- states are degenerate by $\dfrac{eBA}{2\pi\hbar}$
- ground state $E_0=0$



SUSY spectrum

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 - g\frac{e}{2m}s_z B = Q^2$$

$$Q = \frac{1}{\sqrt{2m}}\vec{\sigma}\cdot(\vec{p} - e\vec{A})$$

$$\langle\psi|H|\psi\rangle = |Q|\psi\rangle|^2 \geq 0$$

- ground state $Q|0\rangle = 0 \;\Rightarrow H|0\rangle = 0$
- excited states paired $Q|F\rangle = \sqrt{E}|B\rangle$

  $Q|B\rangle = \sqrt{E}|F\rangle$
- ground state not paired

General

$$H = Q^2$$

$$Q : \text{hermitian operator}$$

$$\langle\psi|H|\psi\rangle = |Q|\psi\rangle|^2 \geq 0$$

- ground state $Q|0\rangle = 0 \;\Rightarrow H|0\rangle = 0$
- excited states paired $Q|F\rangle = \sqrt{E}|B\rangle$

  $Q|B\rangle = \sqrt{E}|F\rangle$
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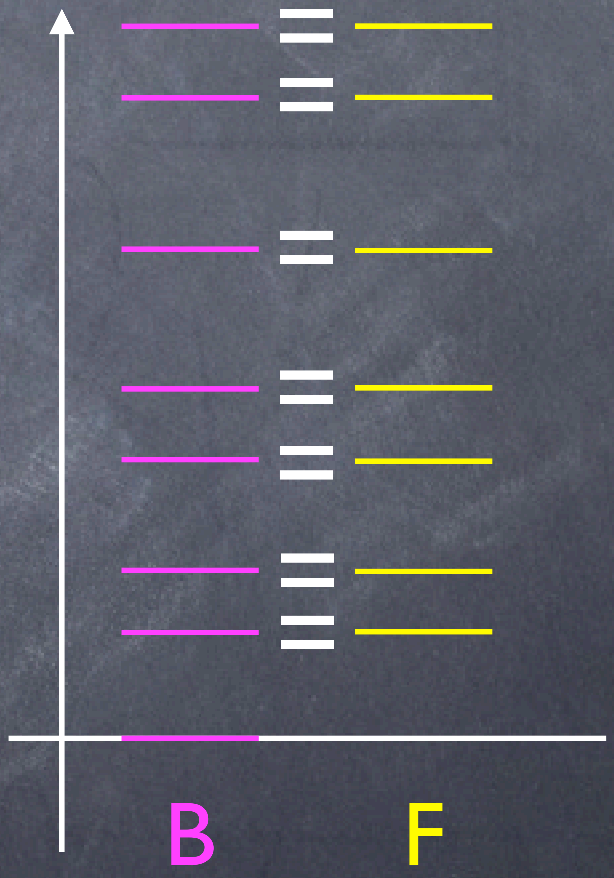
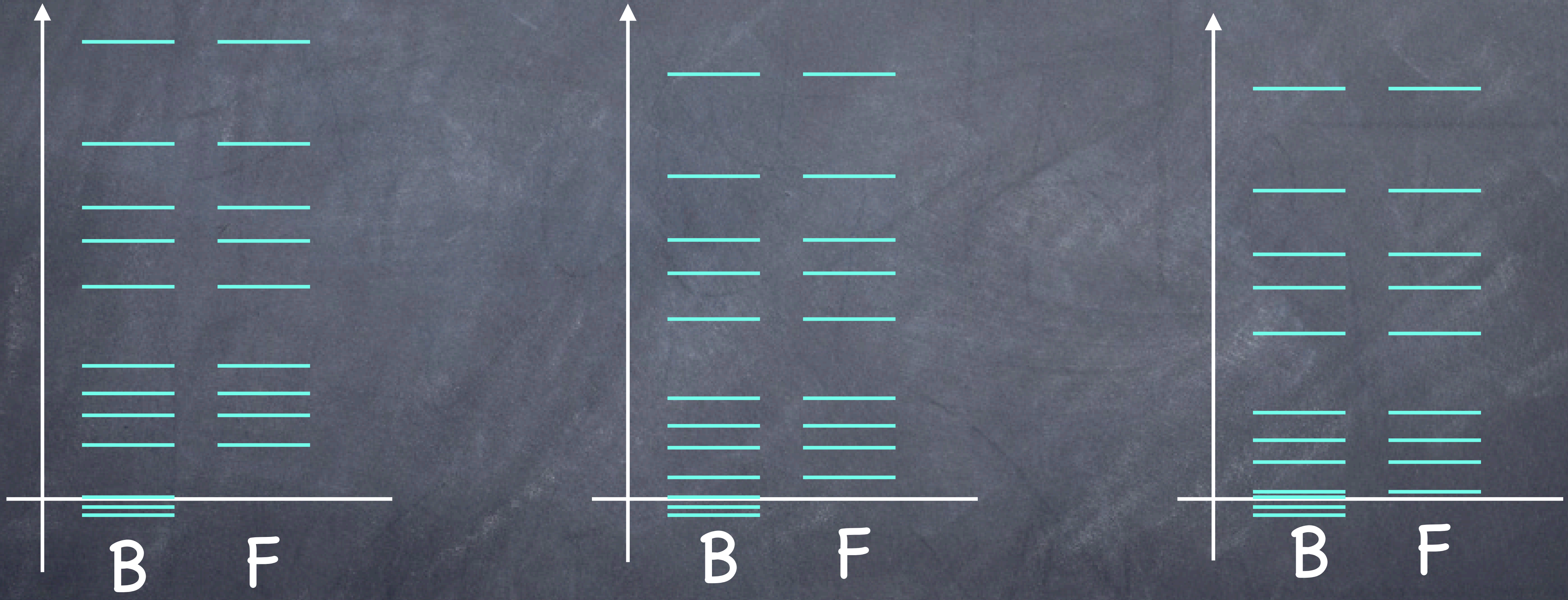


topological invariant



Witten index

$$\mathrm{Tr}(-1)^F e^{-\beta H} = n_{0B} - n_{0F}$$

topological invariant

moduli space

- in many systems, SUSY ground states $E_0=0$ from a continuous space: moduli space
- with SUSY breaking, moduli space gets lifted to isolated ground states

pure Yang-Mills theory

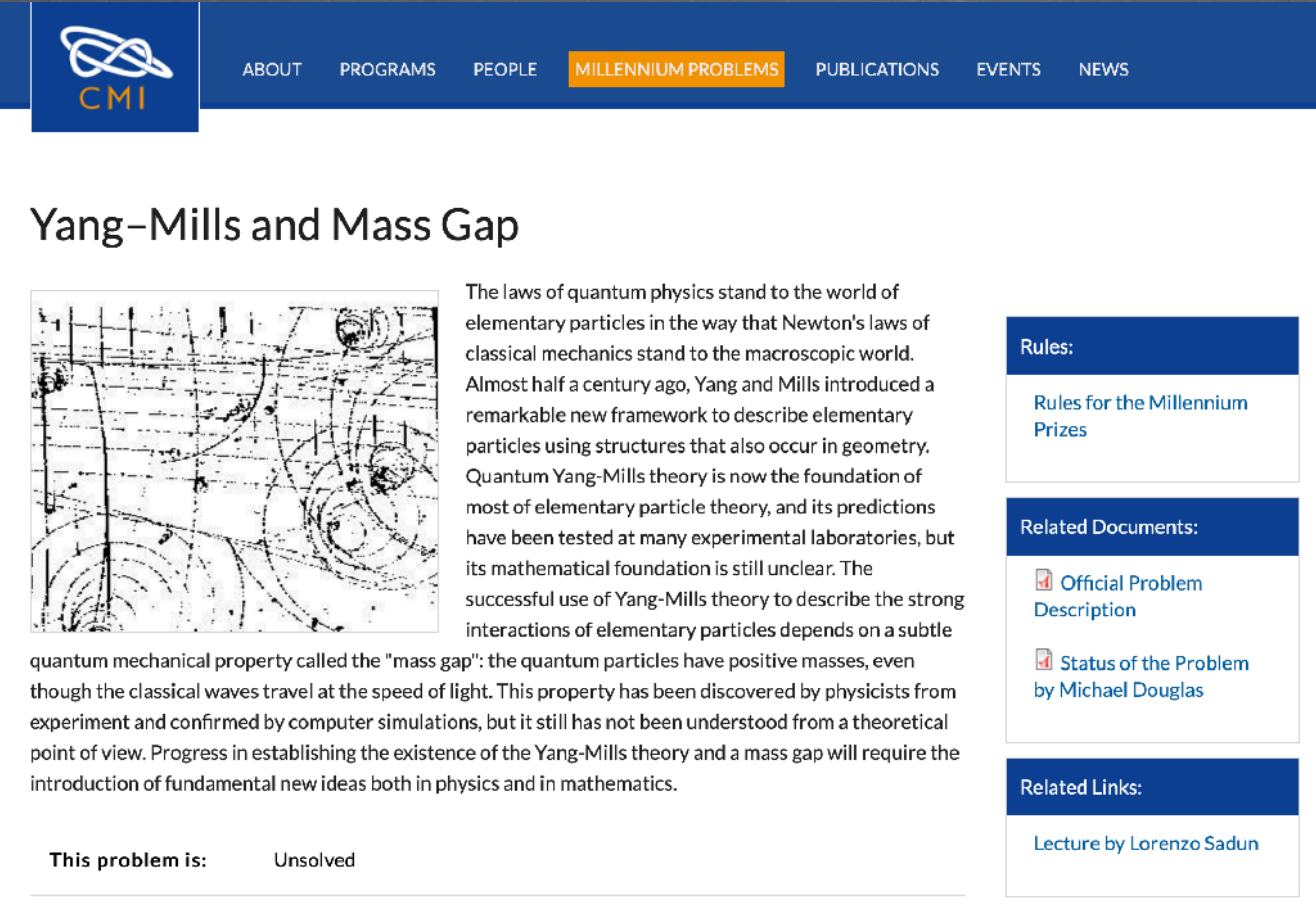
pure Yang-Mills theory

- non-abelian (e.g. SU(N_c)) generalization of Maxwell theory
- only modification is definition of the field strength
- no dimensionful parameter
- yet "believed" to develop a mass gap
- numerical simulation supports it
- similar to 1D anti-ferromagnet with integer spin

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e f^{abc} A^b_\mu A^c_\nu$$

$$m = \mu e^{-8\pi^2/\hbar g^2 3N_c}$$



Yang–Mills and Mass Gap

The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum mechanical property called the "mass gap": the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics.

This problem is: Unsolved

Rules:

Rules for the Millennium Prizes

Related Documents:

Official Problem Description

Status of the Problem by Michael Douglas

Related Links:

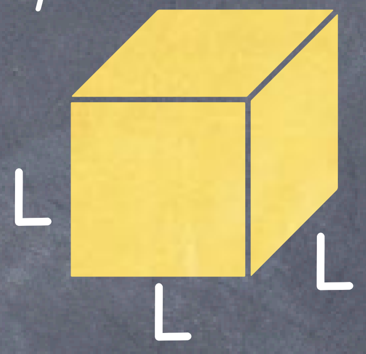
Lecture by Lorenzo Sadun

pure SUSY YM theory

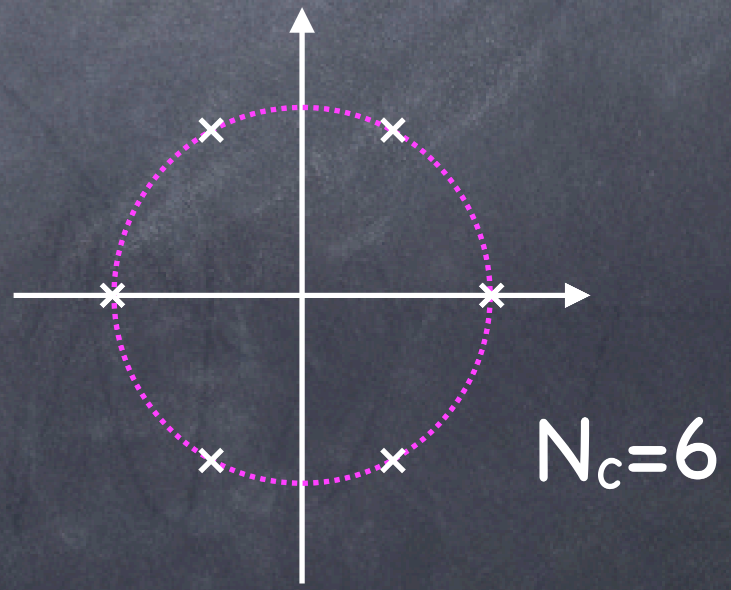
Edward Witten, Nuclear Physics B202 (1982) 253-316

- introduce “gaugino” i.e. Weyl fermions in the adjoint rep
- Witten showed his index can be computed by adiabatically putting the system in small box L→0
- all non-zero momentum modes have energies 1/L
- just keep the zero modes
- reduces QFT to QM!
- index could be worked out
- also a mass gap!

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{\lambda}^a i\gamma^\mu (D_\mu\lambda)^a$$



$$\mathrm{Tr}(-1)^F e^{-\beta H} = n_{0B} - n_{0F} = N_c$$



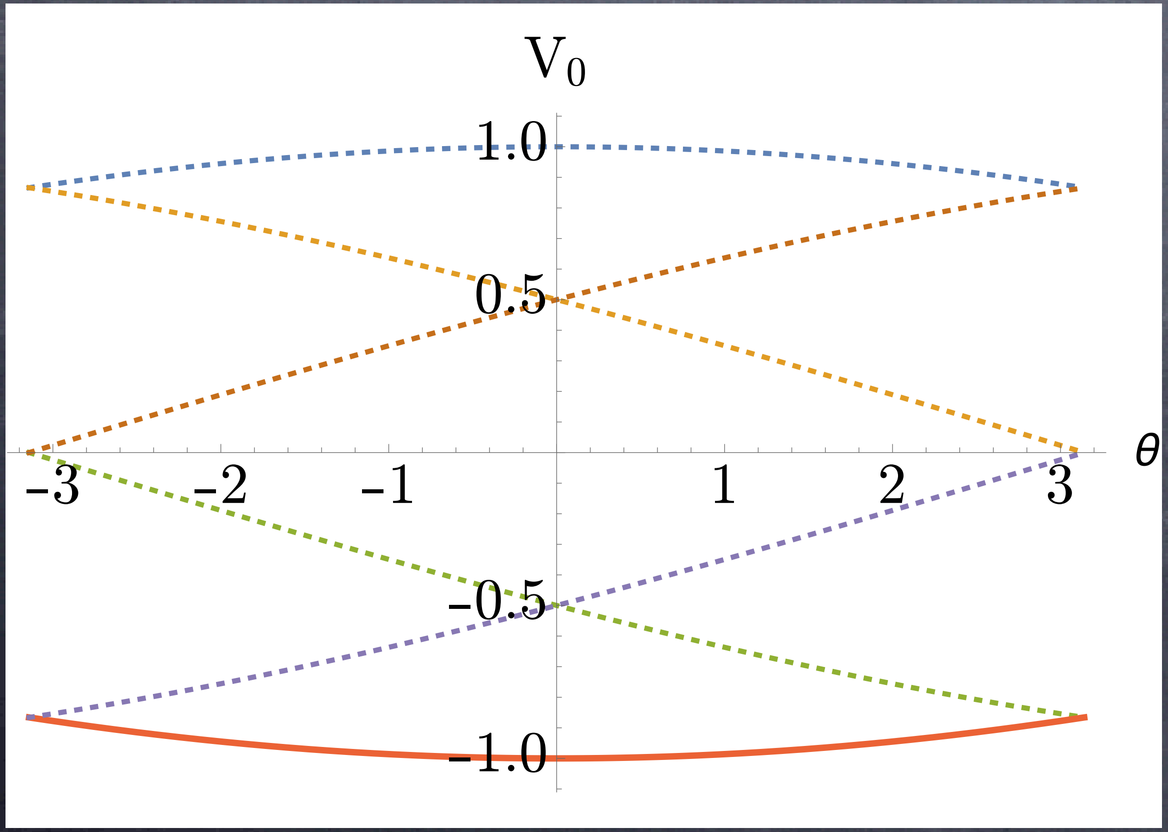
$$\langle\lambda^a\lambda^a\rangle = e^{2\pi i/N_c} e^{-8\pi^2/\hbar g^2 N_c}$$

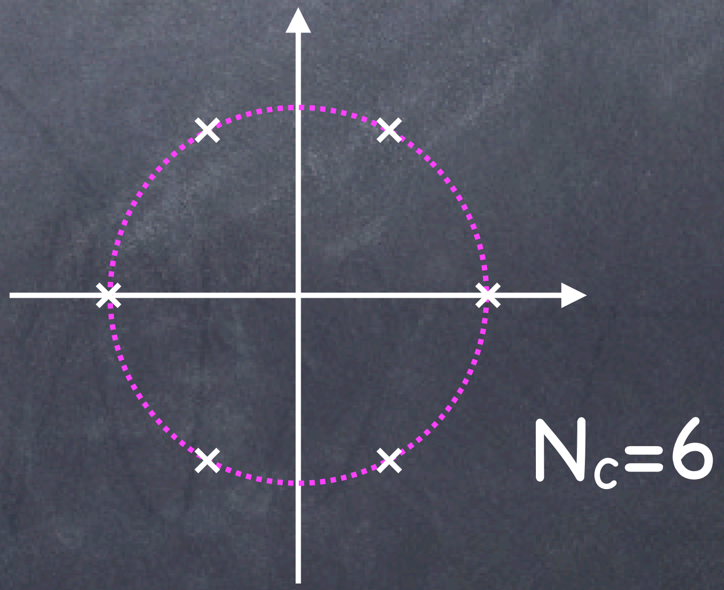
break SUSY

- introduce mass for gaugino
- now topological term also present
- degeneracy of N$_c$ ground states lifted
- This is a special case of anomaly mediation

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{\lambda}^a i\gamma^\mu (D_\mu\lambda)^a$$

$$-\frac{1}{2}m\lambda^a\lambda^a + c.c. + \frac{\theta}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

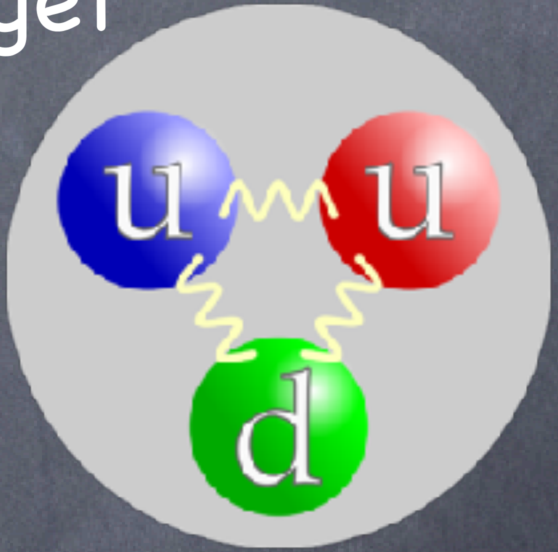




QCD: theory of strong nuclear force

Can we solve QCD?

- When we first learn about quarks, we get told we can never see them
  - Internet Scam?



Dear friend,
I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds ($11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
http://news.bbc.co.uk/1/hi/world/europe/859479.stm
I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

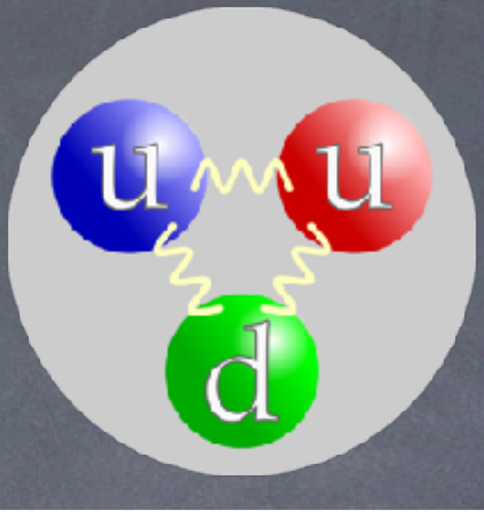
This fund ($11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.
Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

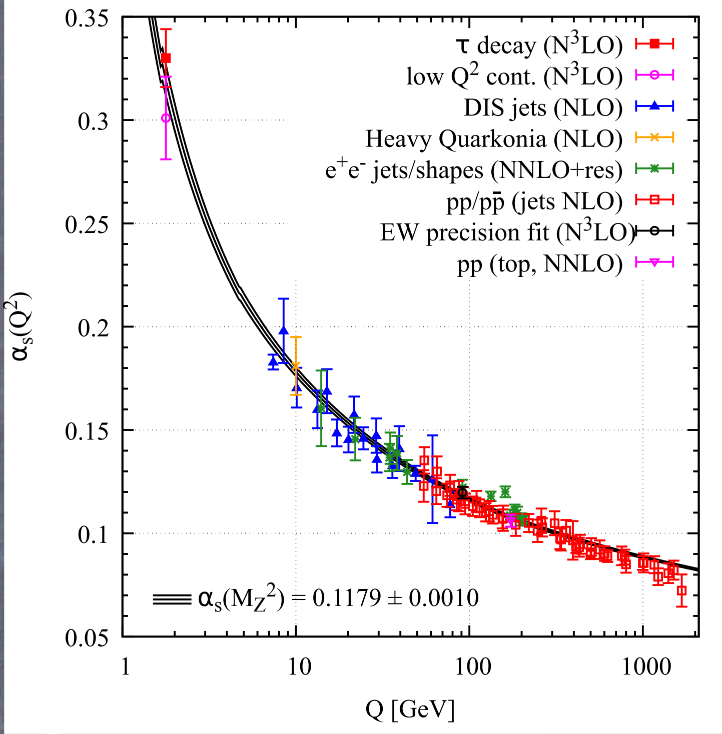
Kindly update me with the
following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name........................
2. Your age/sex ....
3. your occupation ....
4. Your residential address ....
5. Your nationality ....
6. Your private phone number ....
7. Your fax number ....

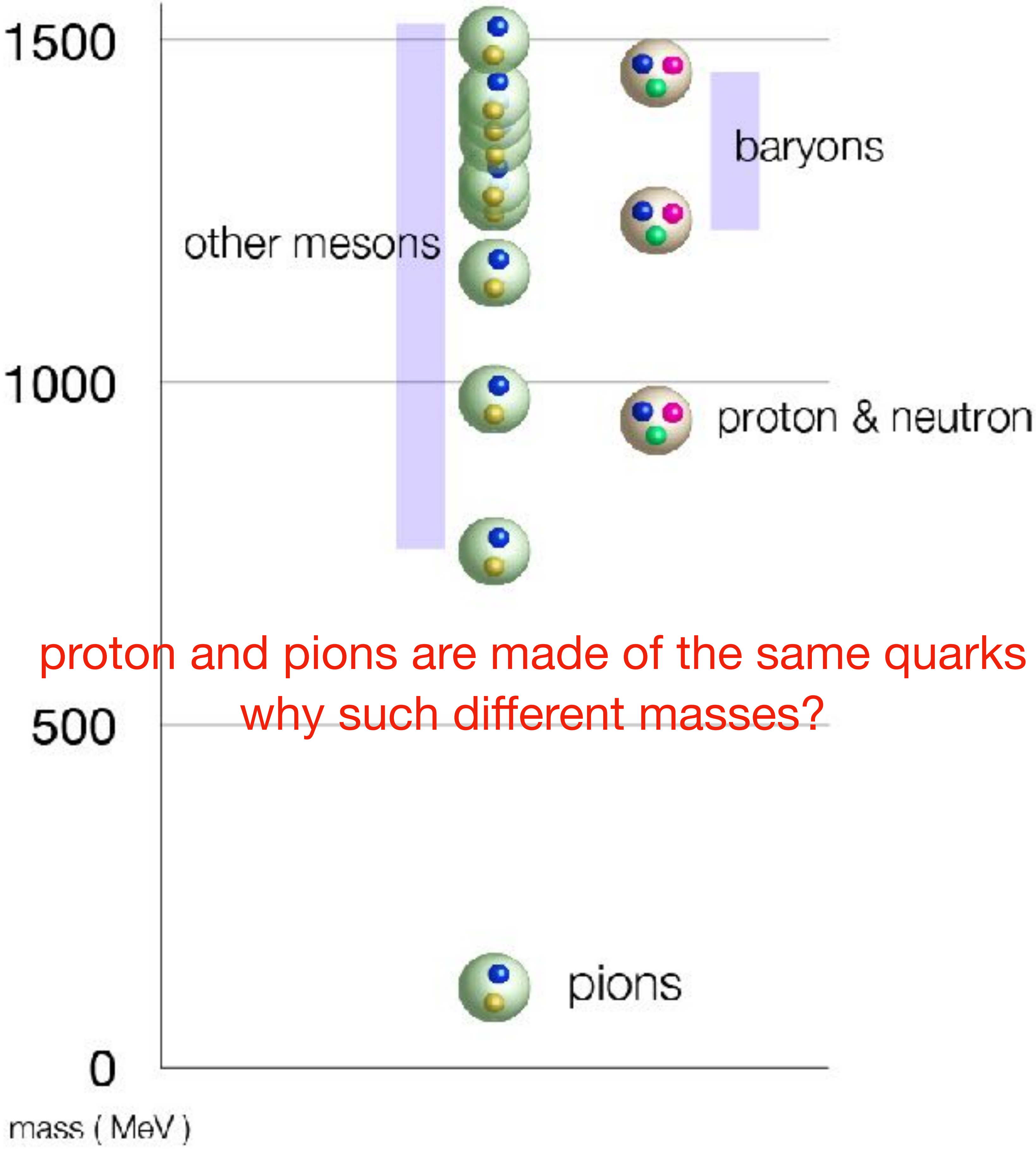
I will be waiting for your response.
Thanks for your understanding.
Have a great day.
Yours.
Andre Ouedraogo

Can we solve QCD?



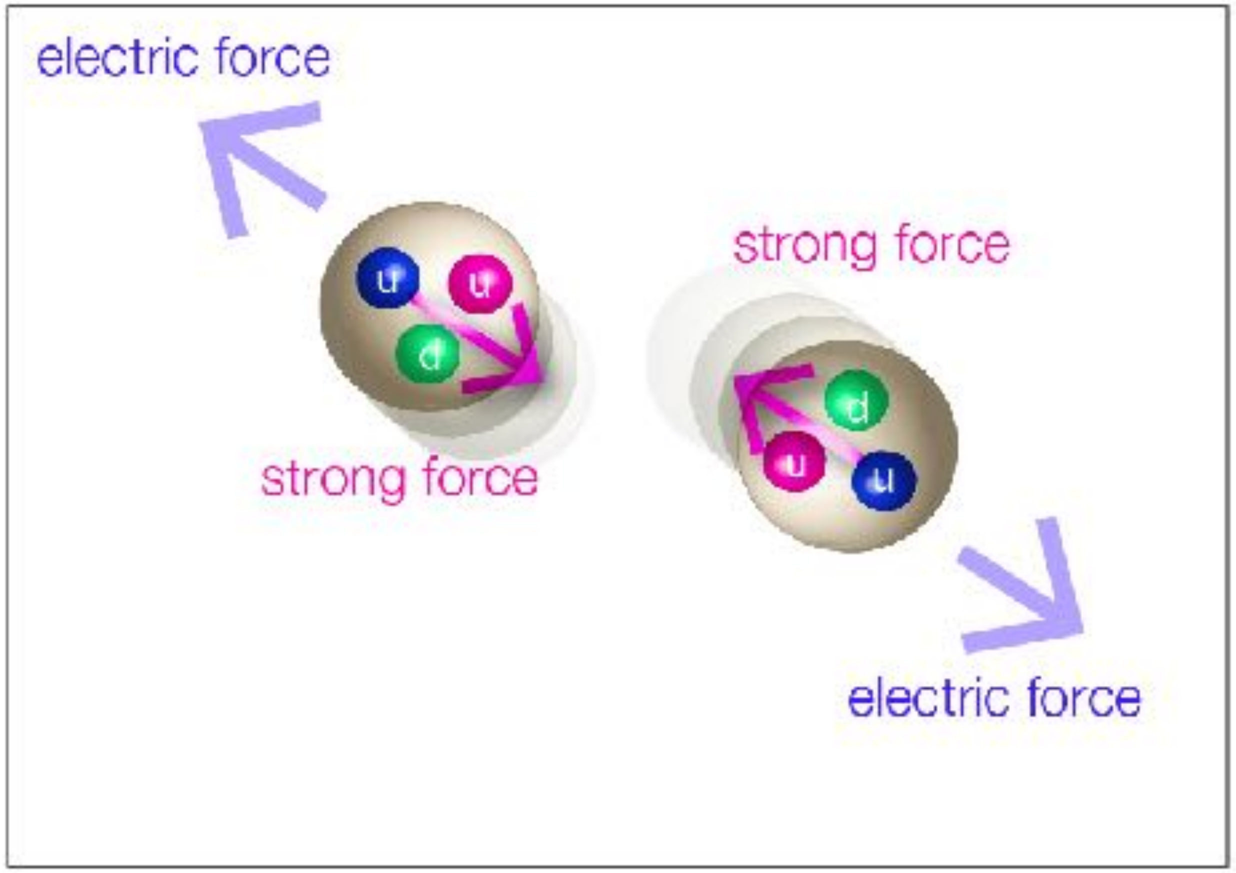


- When we first learn about quarks, we get told we can never see them
  - Internet Scam?
  - Confinement!
  - $\beta<0$ and asymptotic freedom
  - only suggestive, doesn't prove confinement
- Another puzzle: proton and pion are made of same quarks
  - why pion ≈ massless ≪ proton?
- very mysterious!

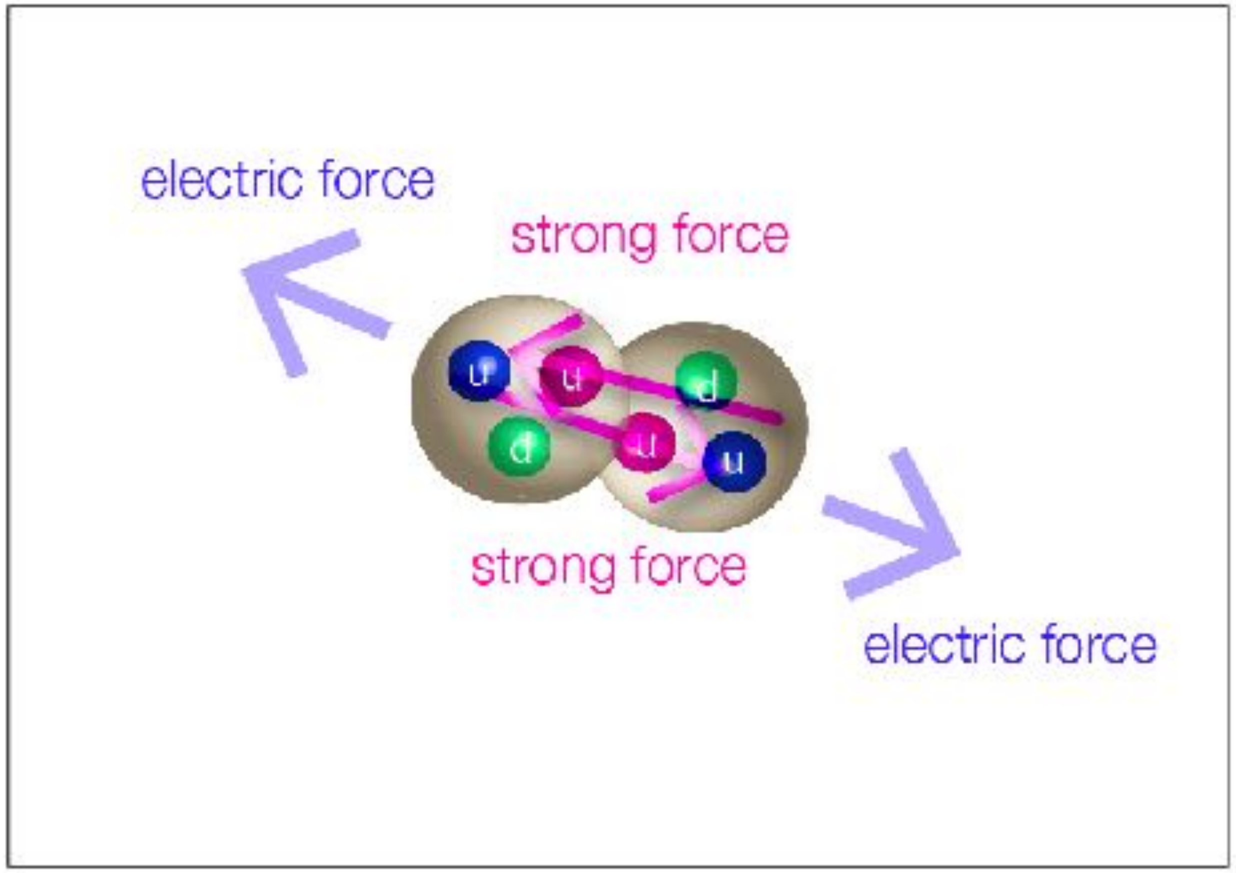


proton and pions are made of the same quarks
why such different masses?

Credit: Kavli IPMU



If pions are heavy



With real light-weight pions

Credit: Kavli IPMU

elements in the Universe

元素の周期表
The Periodic Table

周期＼族	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H 水素 Hydrogen 1.00798																	2 He ヘリウム Helium 4.0026
2	3 Li リチウム Lithium 6.968	4 Be ベリリウム Beryllium 9.01218											5 B 硼(ホウ)素 Boron 10.814	6 C 炭素 Carbon 12.0106	7 N 窒素 Nitrogen 14.0069	8 O 酸素 Oxygen 15.9994	9 F 弗(フッ)素 Fluorine 18.9984	10 Ne ネオン Neon 20.1797
3	11 Na ナトリウム Sodium 22.9898	12 Mg マグネシウム Magnesium 24.306											13 Al アルミニウム Aluminium 26.9815	14 Si 珪(ケイ)素 Silicon 28.085	15 P 燐(リン) Phosphorus 30.9738	16 S 硫黄 Sulfur 32.068	17 Cl 塩素 Chlorine 35.452	18 Ar アルゴン Argon 39.948
4	19 K カリウム Potassium 39.0983	20 Ca カルシウム Calcium 40.078	21 Sc スカンジウム Scandium 44.9559	22 Ti チタン Titanium 47.867	23 V バナジウム Vanadium 50.9415	24 Cr クロム Chromium 51.9961	25 Mn マンガン Manganese 54.938	26 Fe 鉄 Iron 55.845	27 Co コバルト Cobalt 58.9332	28 Ni ニッケル Nickel 58.6934	29 Cu 銅 Copper 63.546	30 Zn 亜鉛 Zinc 65.38	31 Ga ガリウム Gallium 69.723	32 Ge ゲルマニウム Germanium 72.630	33 As 砒(ヒ)素 Arsenic 74.9216	34 Se セレン Selenium 78.971	35 Br 臭素 Bromine 79.904	36 Kr クリプトン Krypton 83.798
5	37 Rb ルビジウム Rubidium 85.4678	38 Sr ストロンチウム Strontium 87.62	39 Y イットリウム Yttrium 88.9058	40 Zr ジルコニウム Zirconium 91.224	41 Nb ニオブ Niobium 92.9064	42 Mo モリブデン Molybdenum 95.95	43 Tc テクネチウム Technetium [99]	44 Ru ルテニウム Ruthenium 101.07	45 Rh ロジウム Rhodium 102.906	46 Pd パラジウム Palladium 106.42	47 Ag 銀 Silver 107.868	48 Cd カドミウム Cadmium 112.414	49 In インジウム Indium 114.818	50 Sn 錫(スズ) Tin 118.710	51 Sb アンチモン Antimony 121.760	52 Te テルル Tellurium 127.60	53 I 沃(ヨウ)素 Iodine 126.904	54 Xe キセノン Xenon 131.293
6	55 Cs セシウム Caesium 132.905	56 Ba バリウム Barium 137.327	※1	72 Hf ハフニウム Hafnium 178.49	73 Ta タンタル Tantalum 180.948	74 W タングステン Tungsten 183.84	75 Re レニウム Rhenium 186.207	76 Os オスミウム Osmium 190.23	77 Ir イリジウム Iridium 192.217	78 Pt 白金(プラチナ Platinum 195.084	79 Au 金 Gold 196.967	80 Hg 水銀 Mercury 200.592	81 Tl タリウム Thallium 204.384	82 Pb 鉛 Lead 207.2	83 Bi ビスマス Bismuth 208.980	84 Po ポロニウム Polonium [210]	85 At アスタチン Astatine [210]	86 Rn ラドン Radon [222]
7	87 Fr フランシウム Francium [223]	88 Ra ラジウム Radium [226]	※2	104 Rf ラザホージウム Rutherfordium [267]	105 Db ドブニウム Dubnium [268]	106 Sg シーボーギウム Seaborgium [271]	107 Bh ボーリウム Bohrium [272]	108 Hs ハッシウム Hassium [277]	109 Mt マイトネリウム Meitnerium [276]	110 Ds ダームスタチウム Darmstadtium [281]	111 Rg レントゲニウム Roentgenium [280]	112 Cn コペルニシウム Copernicium [285]	113 Nh ニホニウム Nihonium [278]	114 Fl フレロビウム Flerovium [289]	115 Mc モスコビウム Moscovium [289]	116 Lv リバモリウム Livermorium [293]	117 Ts テネシン Tennessine [293]	118 Og オガネソン Oganesson [294]

※1 ランタノイド系	57 La ランタン Lanthanum 138.905	58 Ce セリウム Cerium 140.116	59 Pr プラセオジム Praseodymium 140.908	60 Nd ネオジム Neodymium 144.242	61 Pm プロメチウム Promethium [145]	62 Sm サマリウム Samarium 150.36	63 Eu ユウロピウム Europium 151.964	64 Gd ガドリニウム Gadolinium 157.25	65 Tb テルビウム Terbium 158.925	66 Dy ジスプロシウム Dysprosium 162.500	67 Ho ホルミウム Holmium 164.930	68 Er エルビウム Erbium 167.259	69 Tm ツリウム Thulium 168.934	70 Yb イッテルビウム Ytterbium 173.045	71 Lu ルテチウム Lutetium 174.967
※2 アクチノイド系	89 Ac アクチニウム Actinium [227]	90 Th トリウム Thorium 232.038	91 Pa プロトアクチニウム Protactinium 231.036	92 U ウラン Uranium 238.029	93 Np ネプツニウム Neptunium [237]	94 Pu プルトニウム Plutonium [239]	95 Am アメリシウム Americium [243]	96 Cm キュリウム Curium [247]	97 Bk バークリウム Berkelium [247]	98 Cf カリホルニウム Californium [252]	99 Es アインスタイニウム Einsteinium [252]	100 Fm フェルミウム Fermium [257]	101 Md メンデレビウム Mendelevium [258]	102 No ノーベリウム Nobelium [259]	103 Lr ローレンシウム Lawrencium [262]

Unless pions are light, we wouldn't exist!

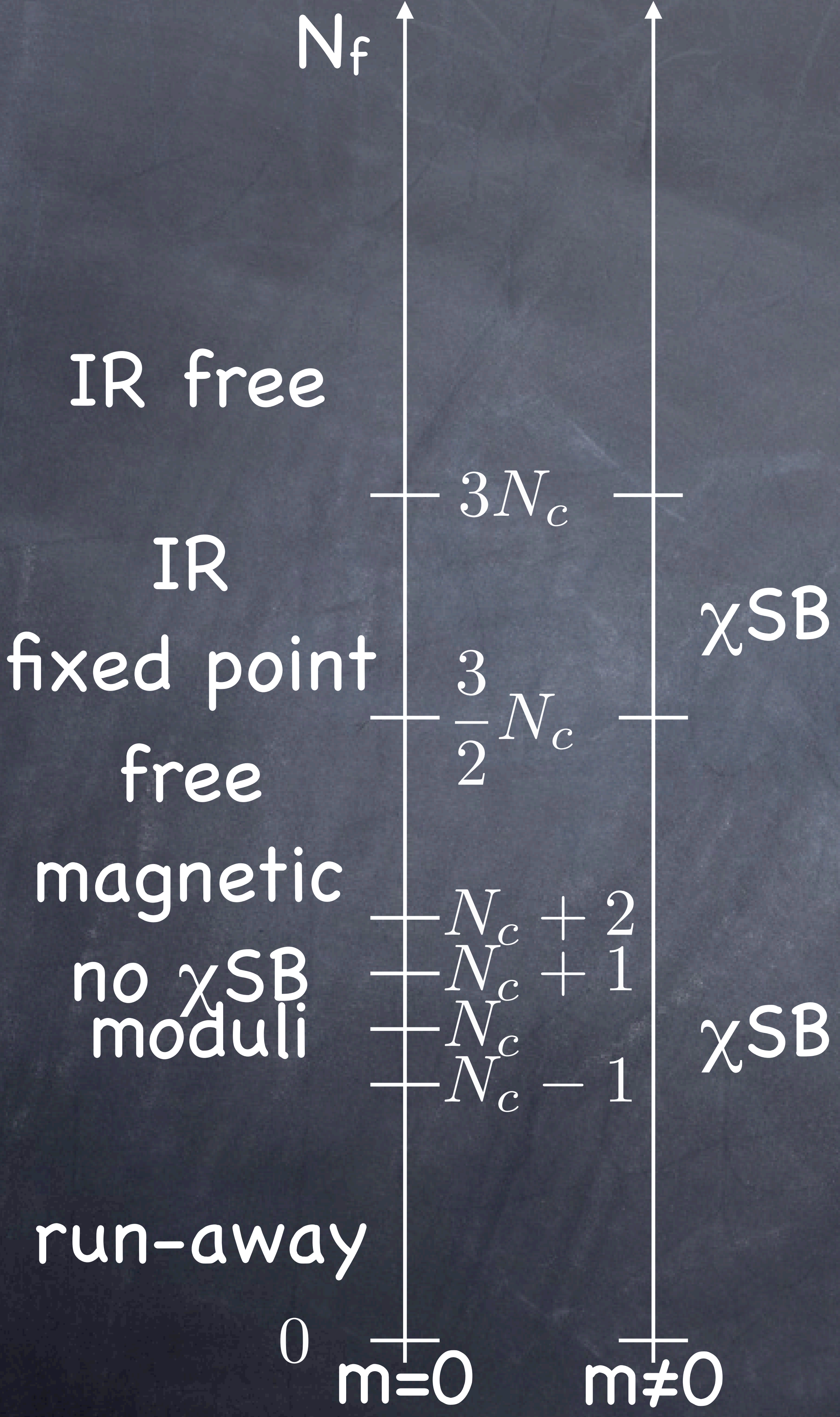
Credit: Wikimedia Commons

Feeling better

- Qualitative picture makes us feel better
- Confinement
  - dual Meißner effect (Mandelstam)
  - assume monopole condensation
  - quarks confined by electric flux tube
- Chiral symmetry breaking (Nambu)
  - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
  - assume broken to $SU(N_f)_V \times U(1)_B$
  - pion = Nambu-Goldstone boson = massless
- but still not derived from QCD!

Feeling even better but not there yet

- Progress in understanding QCD
- Confinement (Seiberg-Witten)
  - N=2 SU(2) SYM a triplet field = vacua
  - SU(2) → U(1): magnetic monopoles!
  - special points = massless monopole/dyon
  - N=1 perturbation: monopole condensation!
- Chiral symmetry breaking
  - N=2 doesn't have the chiral symmetry
  - N=1 (Seiberg) has too unusual phases



N_f

IR free

$\frac{11}{2}N_c$ ← Banks-Saks fixed point

Λ

0

m=∞

Renormalization

Nima Arkani-Hamed and HM, arXiv:hep-th/9707133

- Konishi anomaly $\int \mathcal{D}\phi_i = \int \mathcal{D}(e^{\sigma}\phi_i)e^{-\int d^2\theta T_F^i \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- rescaling anomaly $\int \mathcal{D}V = \int \mathcal{D}(e^{\sigma}V)e^{+\int d^2\theta C_A \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- first rescale matter fields

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i \right) W_\alpha W^\alpha + Z_i^{-1/2} Z_j^{-1/2} Z_k^{-1/2} \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- then rescale the gauge field V→g_cV

$$\frac{1}{g_c^2} = \frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i - C_A \frac{1}{8\pi^2} \ln g_c^2$$

$$\int d^2\theta \frac{1}{g_c^2} (\bar{D}^2 e^{-g_c V} D_\alpha e^{g_c V})^2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots$$

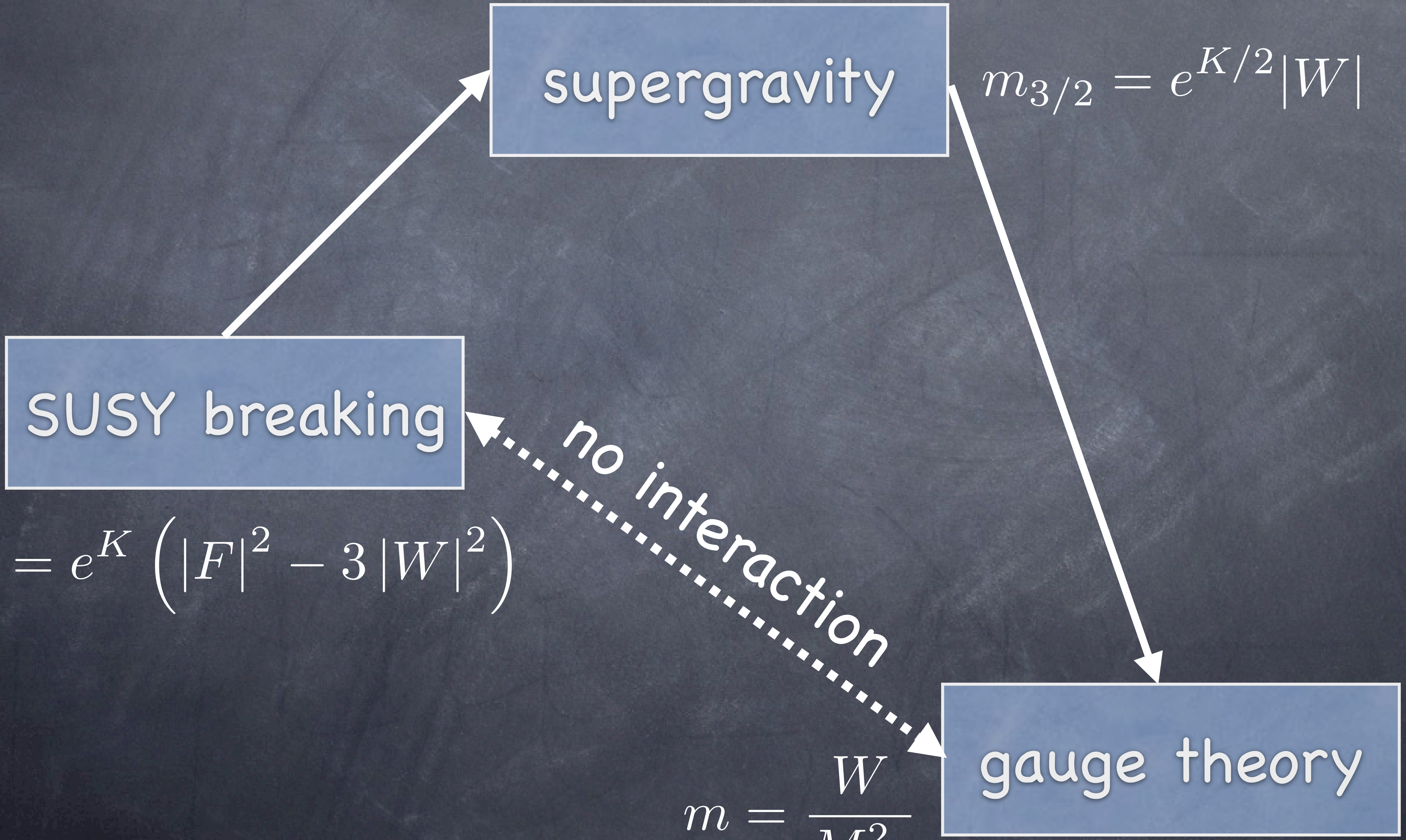
Anomaly-Mediated Supersymmetry Breaking

Randall, Sundrum (1998)
Giudice, Luty, HM, Rattazzi (1998)

Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
  - decouple gauginos and squarks!
  - SUSY breaking m_λ and $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- In particular, we need to know signs of their mass-squared to understand symmetry breaking patterns and universality classes

Sequestering



supergravity

$$m_{3/2} = e^{K/2}|W|$$

SUSY breaking

$$V = e^K \left(|F|^2 - 3\,|W|^2 \right)$$

no interaction

gauge theory

$$m = \frac{W}{M_{Pl}^2}$$

Anomaly Mediation of SUSY Breaking (AMSB)

- Tree-level piece on dimensionful parameters

$$V_{\mathrm{AMSB}} = -m\left(\phi\frac{\partial W}{\partial \phi} - 3W\right)$$

- loop-level piece from running

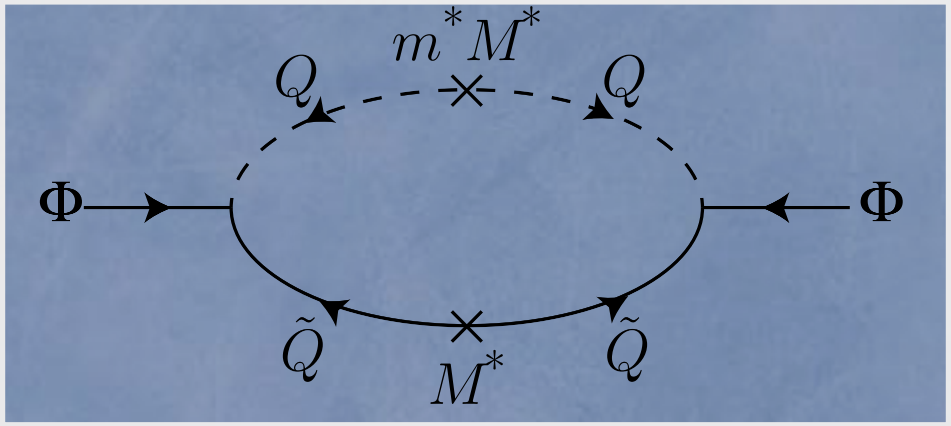
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- Surprising result: AMSB depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass



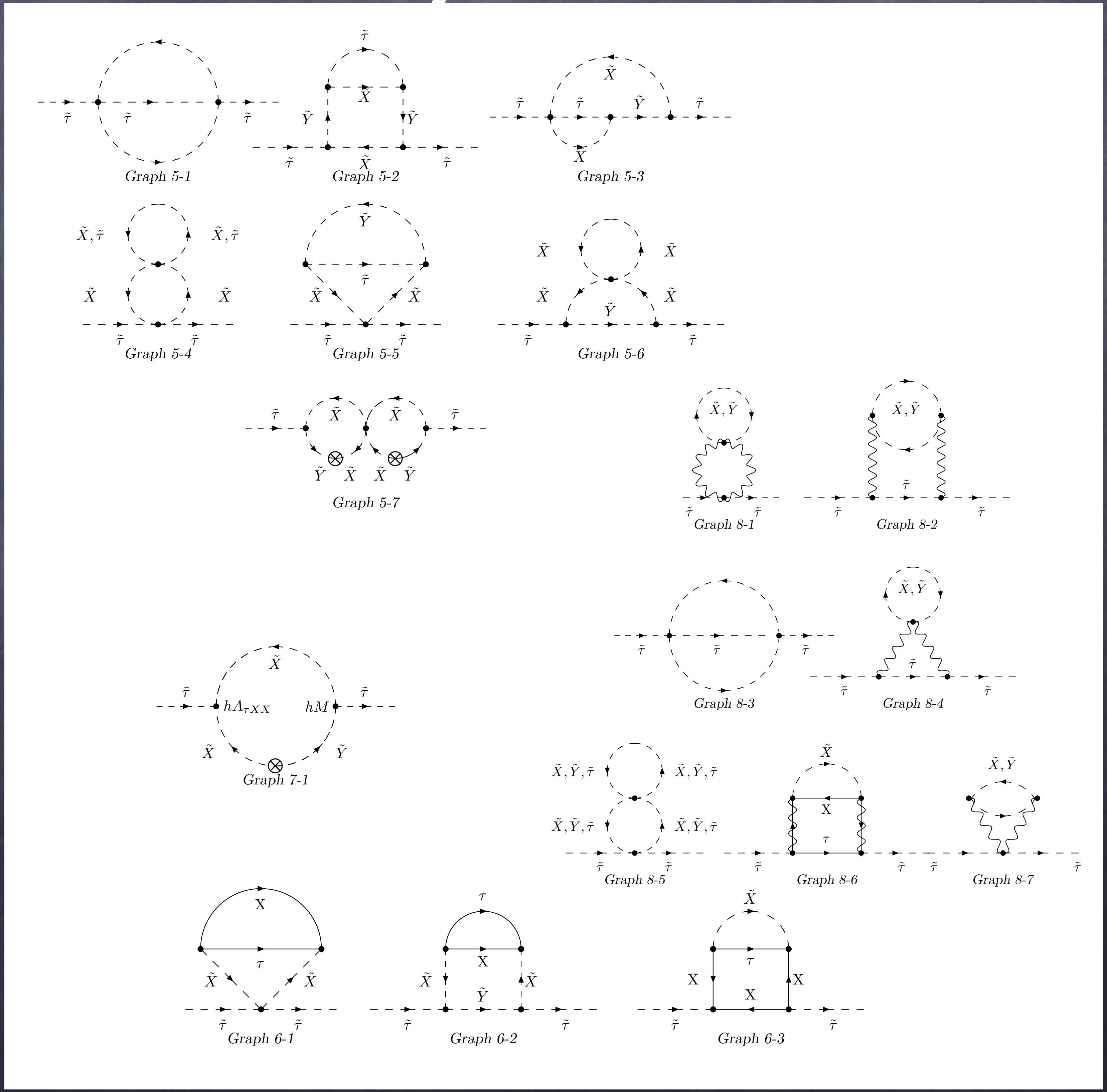
UV insensitivity cont.

- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}$$

$$\begin{aligned} m_i^2 &= -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \\ A_{ijk} &= -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2} \end{aligned}$$

Boyda, HM, Pierce 2001



It is sometimes convenient to introduce a complex gravitino mass, defined as

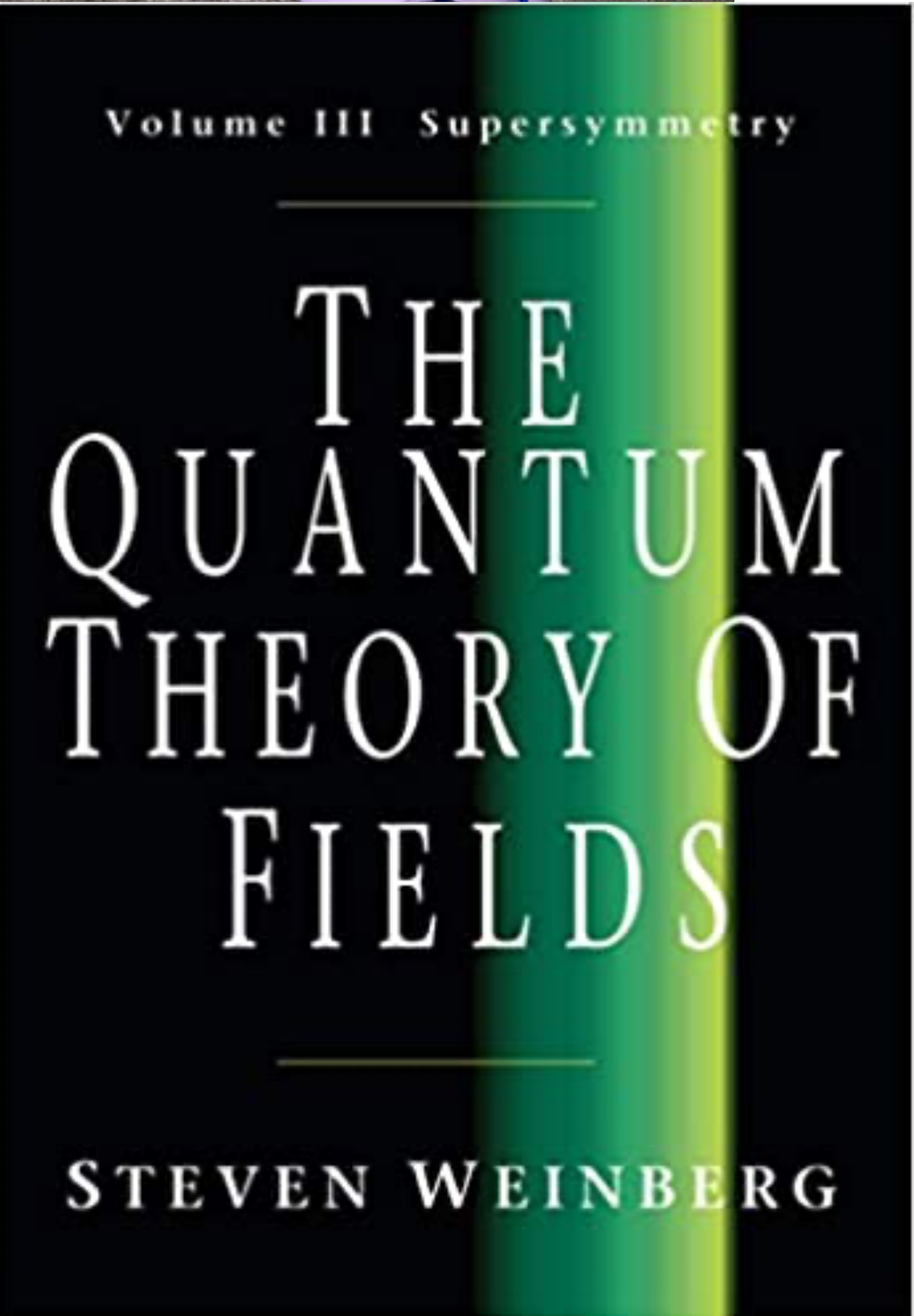
$$\tilde{m}_g \equiv \frac{2\kappa}{3}\Big(\langle s\rangle + i\langle p\rangle\Big)\,, \tag{31.3.20}$$

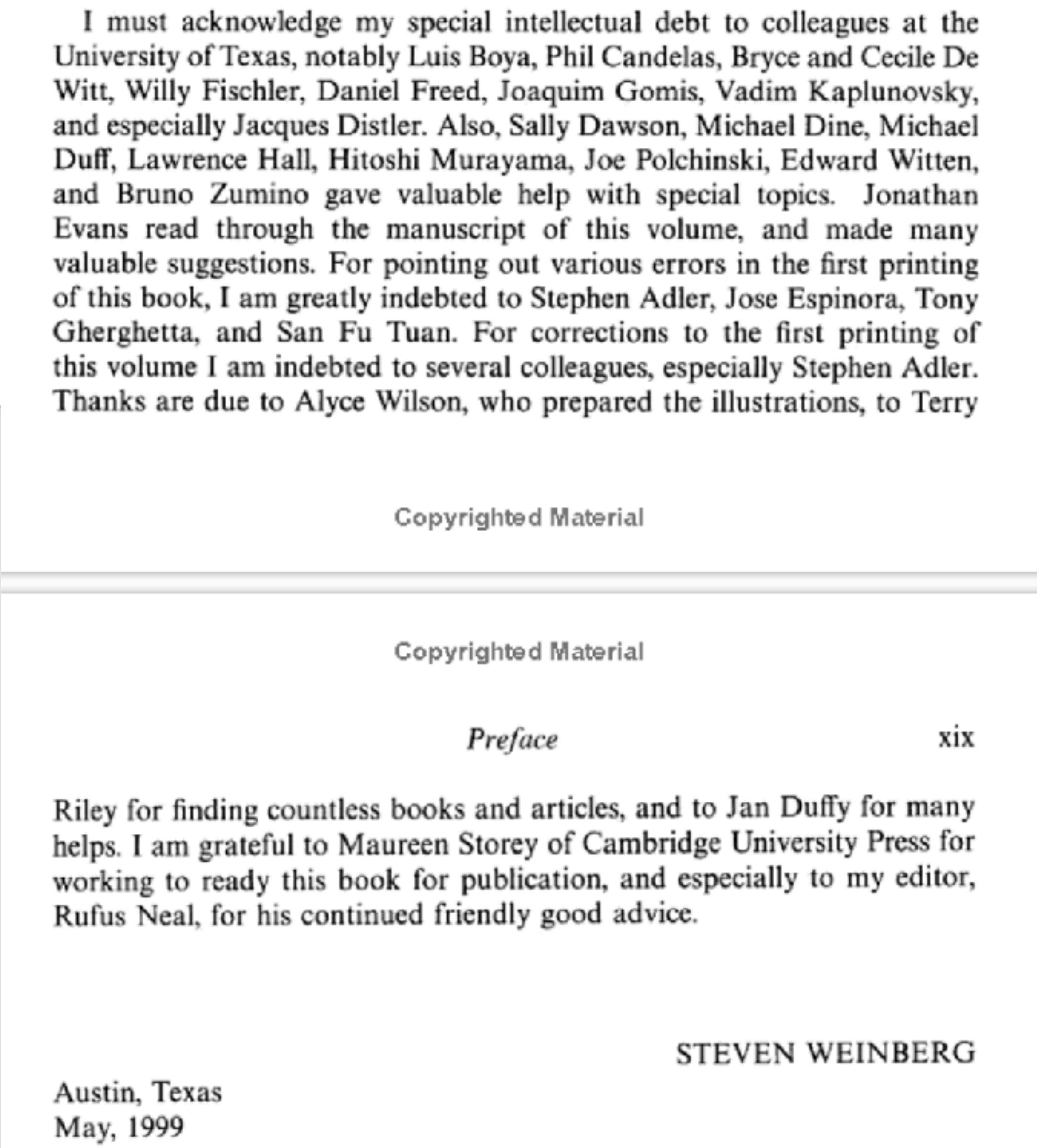
whose absolute magnitude is the physical gravitino mass (31.3.19).

31.4 Anomaly-Mediated Supersymmetry Breaking

In Section 28.3 the possibility was raised that supersymmetry may be broken in some sort of hidden sector of superfields that do not carry the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the standard model, and communicated to observable particles gravitationally. In this section we will deal with one class of supersymmetry-breaking effects in the minimum supersymmetric standard model, those of first order in $\kappa \equiv \sqrt{8\pi G}$. This includes the gaugino masses and the parameters A_{ij} and B in the Lagrangian density (28.4.1). Other supersymmetry-breaking effects such as squark and slepton squared masses are of second order in κ, and will be taken up in Section 31.7, when we consider gravity-mediated supersymmetry breaking using the general supergravity formalism described in Section 31.6.

We can find the effects of gravity-mediated supersymmetry breaking to first order in κ by simply replacing the component fields of the gravitational supermultiplet in the interaction (31.1.34) with their expectation values. The only ones of these component fields that can acquire non-vanishing vacuum expectation values from the spontaneous breakdown





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STEVEN WEINBERG

Austin, Texas
May, 1999

SU(N_c) QCD ($N_c \geq 3$)

HM, 2104.01179

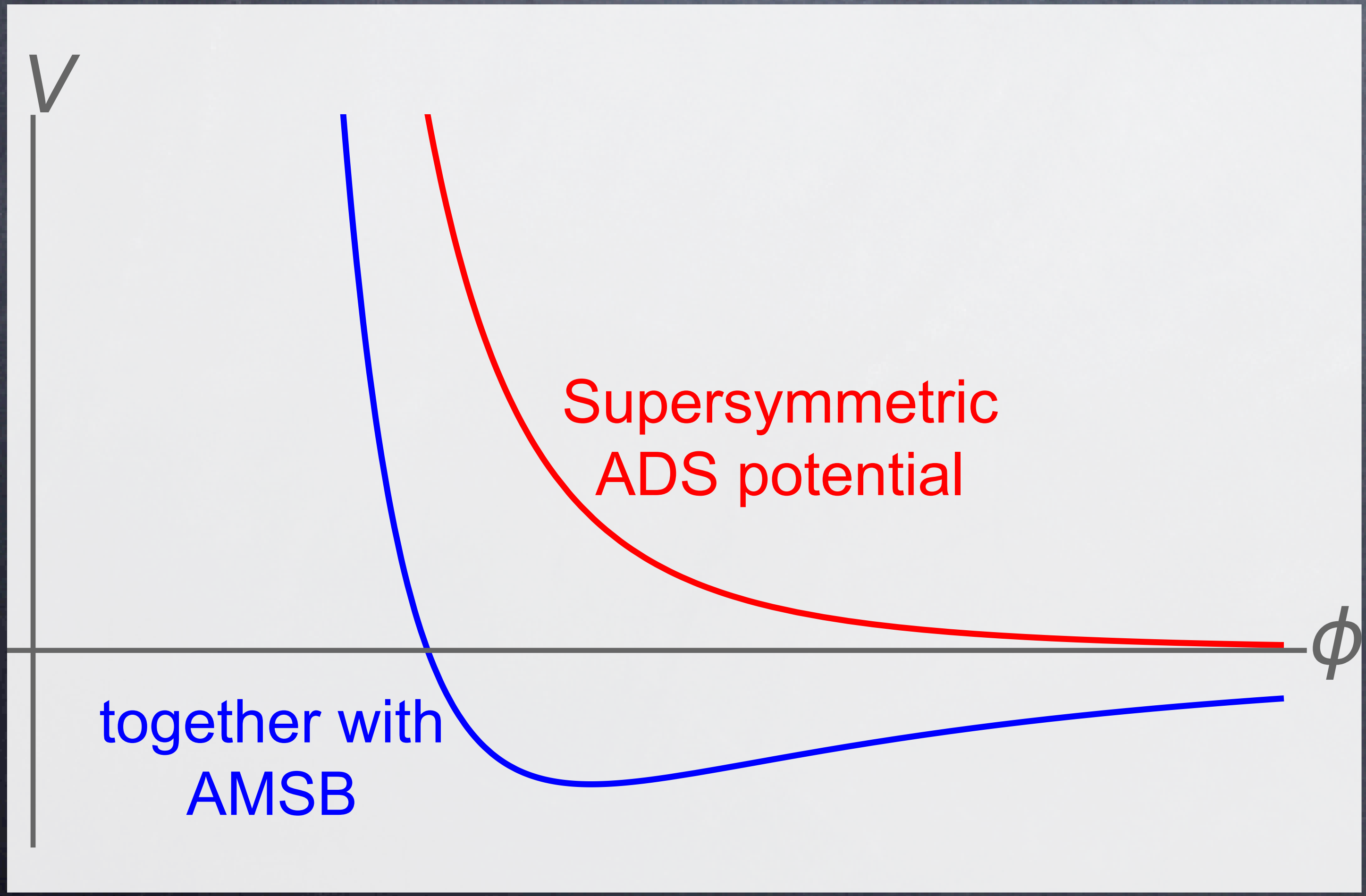
Phys.Rev.Lett. 126 (2021) 25, 251601

$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$

$$V = \left|2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$



$$M_{ij} = \Lambda^2\left(\frac{4N_f(N_c+N_f)}{3N_c-N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c}\delta_{ij}$$

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

χSB! Proving Nambu

mesino loop→WZW term

$N_f=1$ special

no NGB, gapped

$N_f = N_c+1$

- "Confinement without χSB"

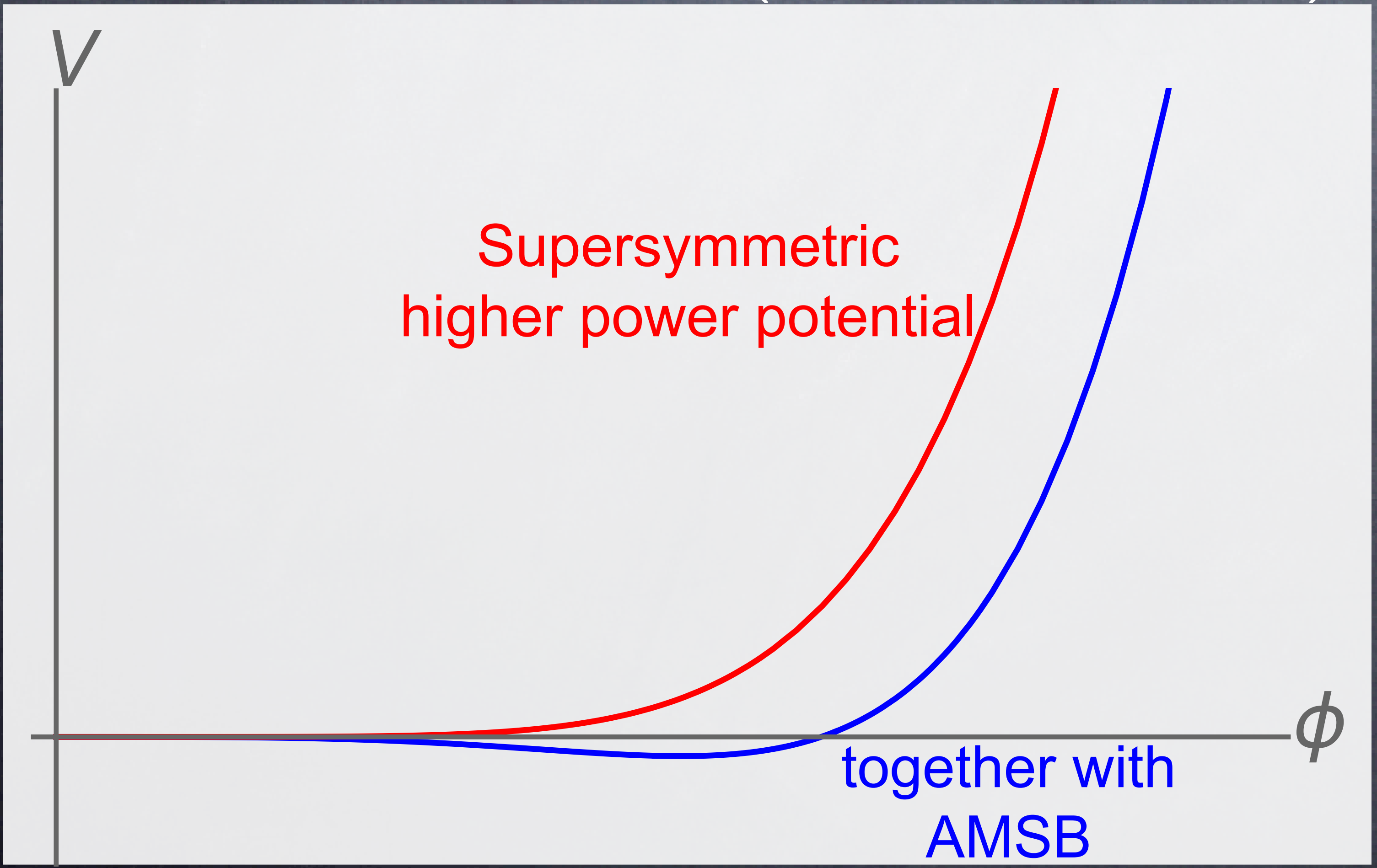
$$W = \frac{\det M - \tilde{B}MB}{\Lambda^{2N_c-1}}$$

$$W = \lambda\frac{\det M}{\Lambda^{N_f-3}} - \kappa\tilde{B}MB$$

$$B_i = \epsilon_{ij_1\cdots j_{N_c}} Q^{j_1}\cdots Q^{j_{N_c}}$$

$$V = N_f\lambda^2\frac{|\phi|^{2N_f-2}}{\Lambda^{2N_f-6}} - \lambda(N_f-3)m\phi^{N_f} + c.c.$$

$$\phi = \kappa^{-1}\Lambda\left(\frac{2N_f-3N_c}{N_c}\frac{m}{\Lambda}\right)^{(N_f-N_c)/(2N_c-N_f)} \ll \Lambda$$



$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

- χSB!
- massless pions
- massive baryons

$m_B = \kappa f_\pi \approx 4\pi f_\pi$

$N_c+2 \leq N_f < 3N_c/2$

- "free magnetic phase"
- SU(N_c) becomes strong, binds baryons, which break up into dual quarks

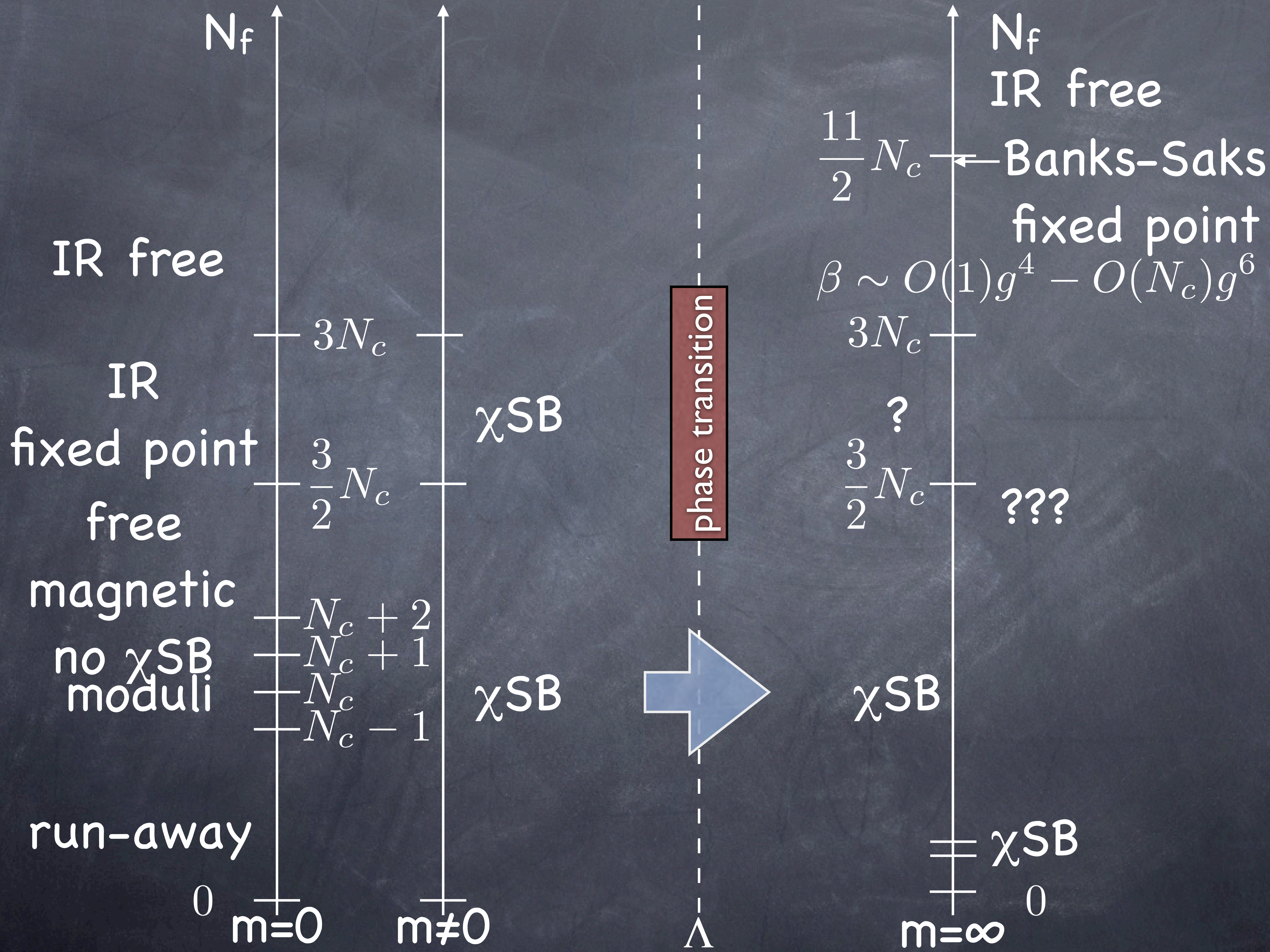
$$B^{i_1,\cdots i_{N_c}} = Q^{i_1}_{\alpha_1}\cdots Q^{i_{N_c}}_{\alpha_{N_c}}\epsilon^{\alpha_1\cdots\alpha_{N_c}} = \epsilon^{i_1\cdots i_{N_c} i_{N_c+1}\cdots i_{N_f}} b_{i_{N_c+1}\cdots i_{N_f}}$$

$$b_{i_{N_c+1}\cdots i_{N_f}} = q^{\beta_1}_{i_{N_c+1}}\cdots q^{\beta_{N_f-N_c}}_{i_{N_f}}\epsilon_{\beta_1\cdots\beta_{N_f-N_c}}$$

- "magnetic" IR-free SU(N_f-N_c) gauge theory

$$W = \frac{1}{\mu}M^{ij}q_i\tilde{q}_j \to \lambda\tilde{M}^{ij}q_i\tilde{q}_j$$

- λ dimensionless, only loop-level AMSB



N_f

IR free

IR

fixed point

free

magnetic

no χSB

moduli

run-away

$3N_c$

$\frac{3}{2}N_c$

N_c+2

N_c+1

N_c

N_c-1

0

m=0

χSB

χSB

m≠0

phase transition

Λ

N_f

IR free

$\frac{11}{2}N_c$

Banks-Saks

fixed point

$\beta \sim O(1)g^4 - O(N_c)g^6$

$3N_c$

?

$\frac{3}{2}N_c$

???

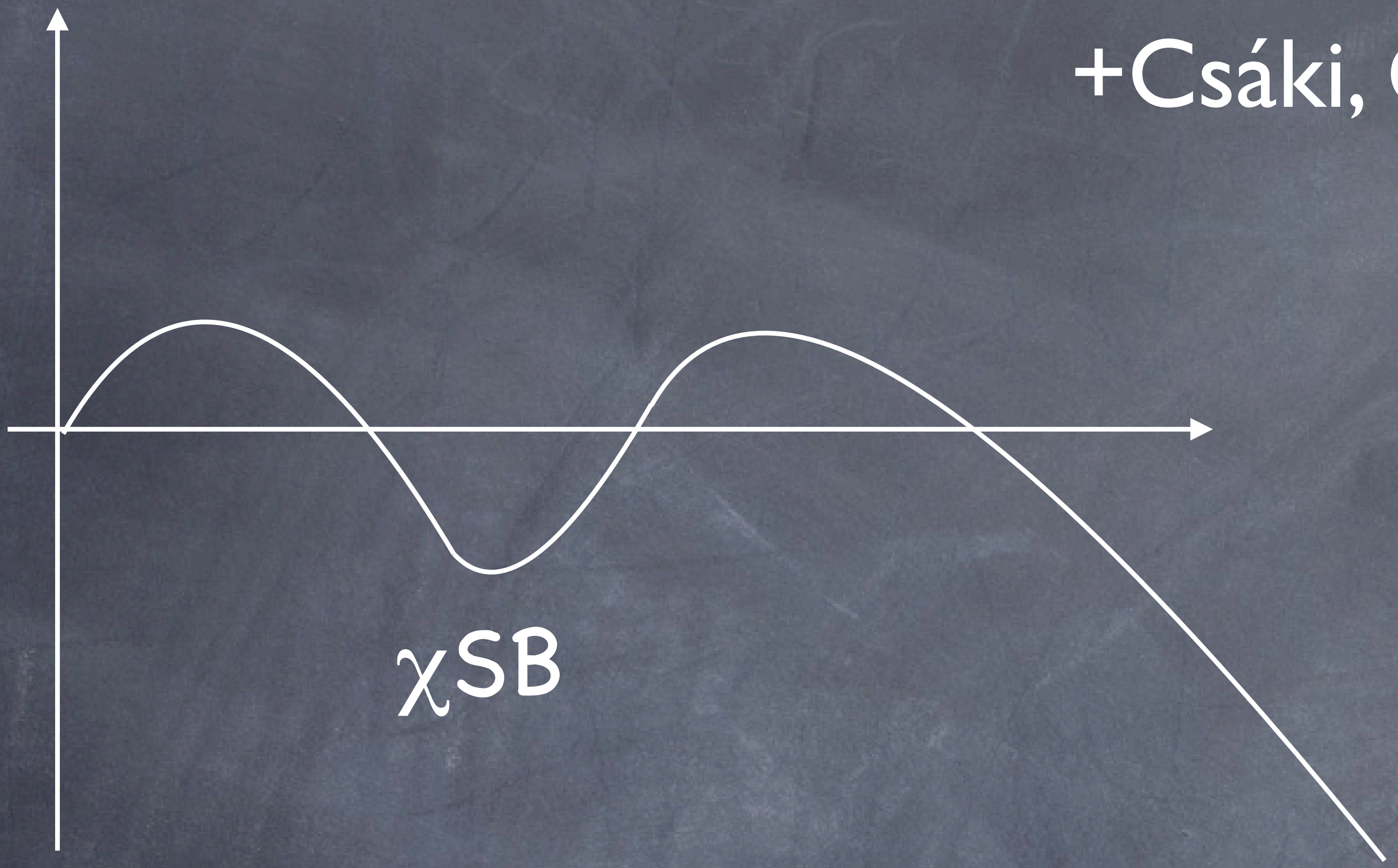
χSB

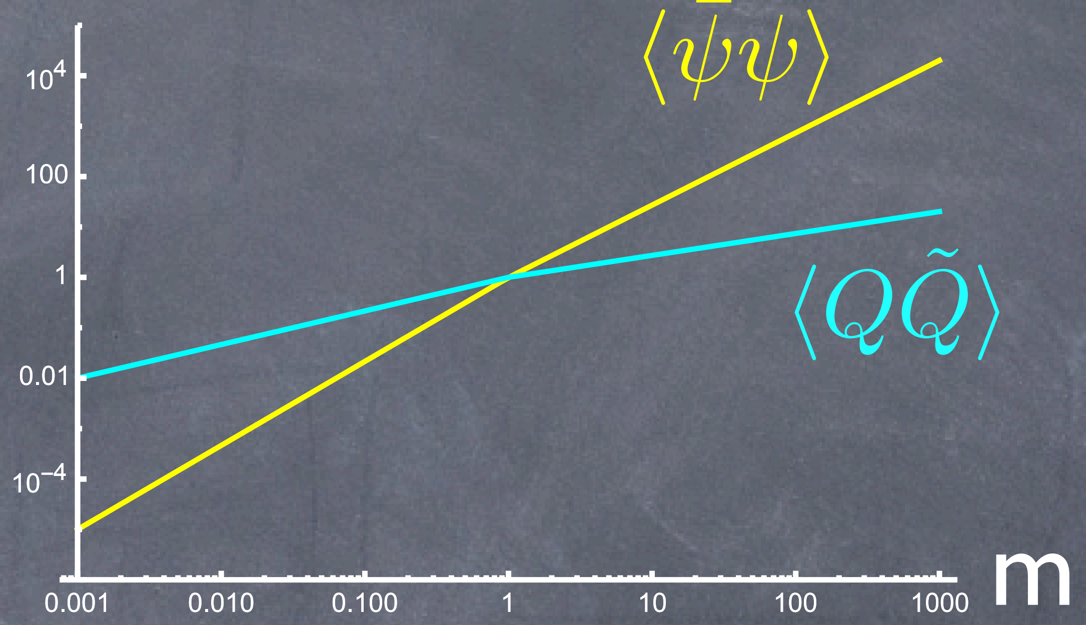
χSB

0

m=∞

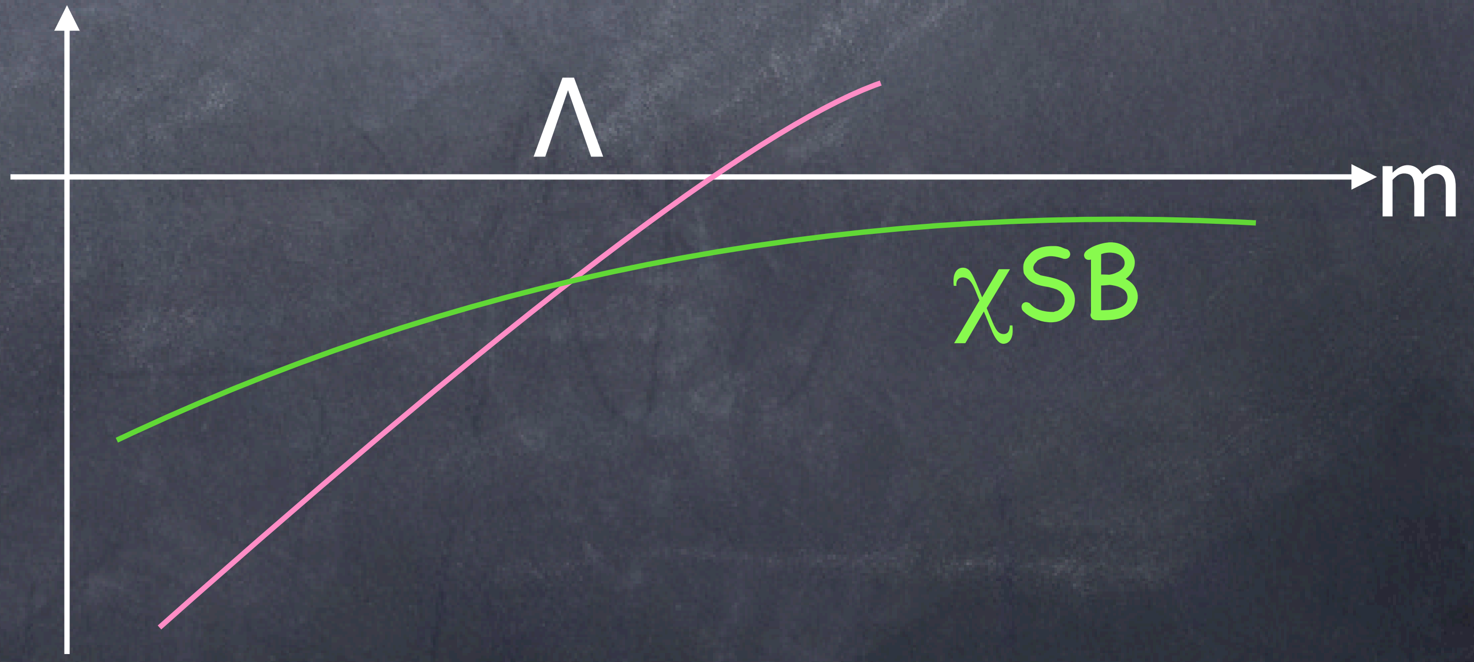
+Csáki, Gomes, Noether, Varier





χSB is a local minimum
but well-defined
and still useful

no similar issues
with SO, Sp

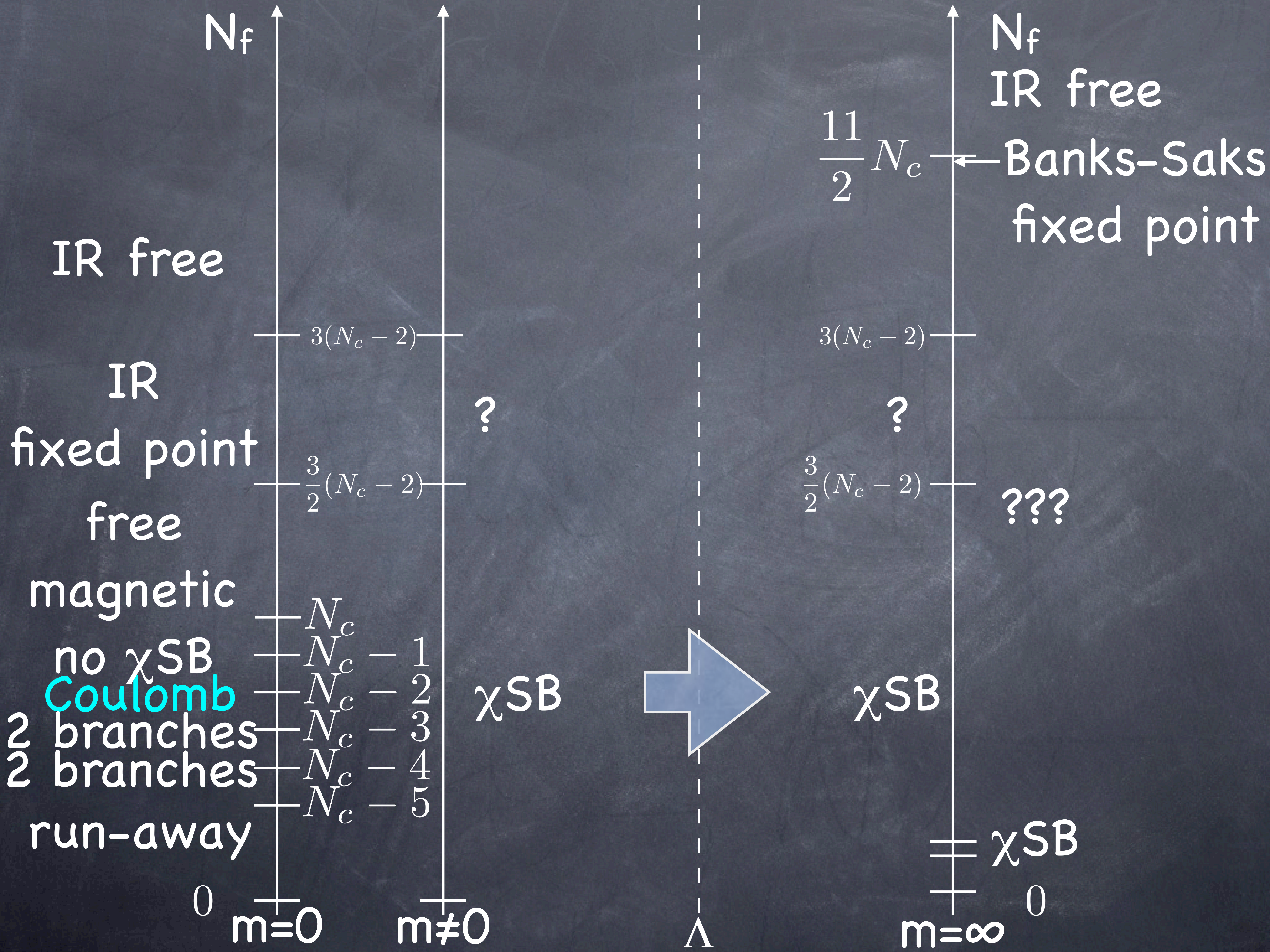


confinement and χSB

Csáki, Gomes, HM, Telem, 2106.10288, 2107.02813

confinement vs screening

- We've derived χSB in SU(N_c) QCD
  - it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law
- SO(N_c) QCD with quarks in vector rep
  - cannot screen Z_2 center (e.g. spinor rep)
  - rigorous definition of confinement
  - can we see an interplay with χSB?



N_f

IR free

$3(N_c-2)$

IR fixed point

$\frac{3}{2}(N_c-2)$

free magnetic

N_c

no χSB

N_c-1

Coulomb

N_c-2

2 branches

N_c-3

2 branches

N_c-4

N_c-5

run-away

0

m=0

?

χSB

m≠0

Λ

N_f

IR free

$\frac{11}{2}N_c$

Banks-Saks fixed point

$3(N_c-2)$

?

$\frac{3}{2}(N_c-2)$

???

χSB

χSB

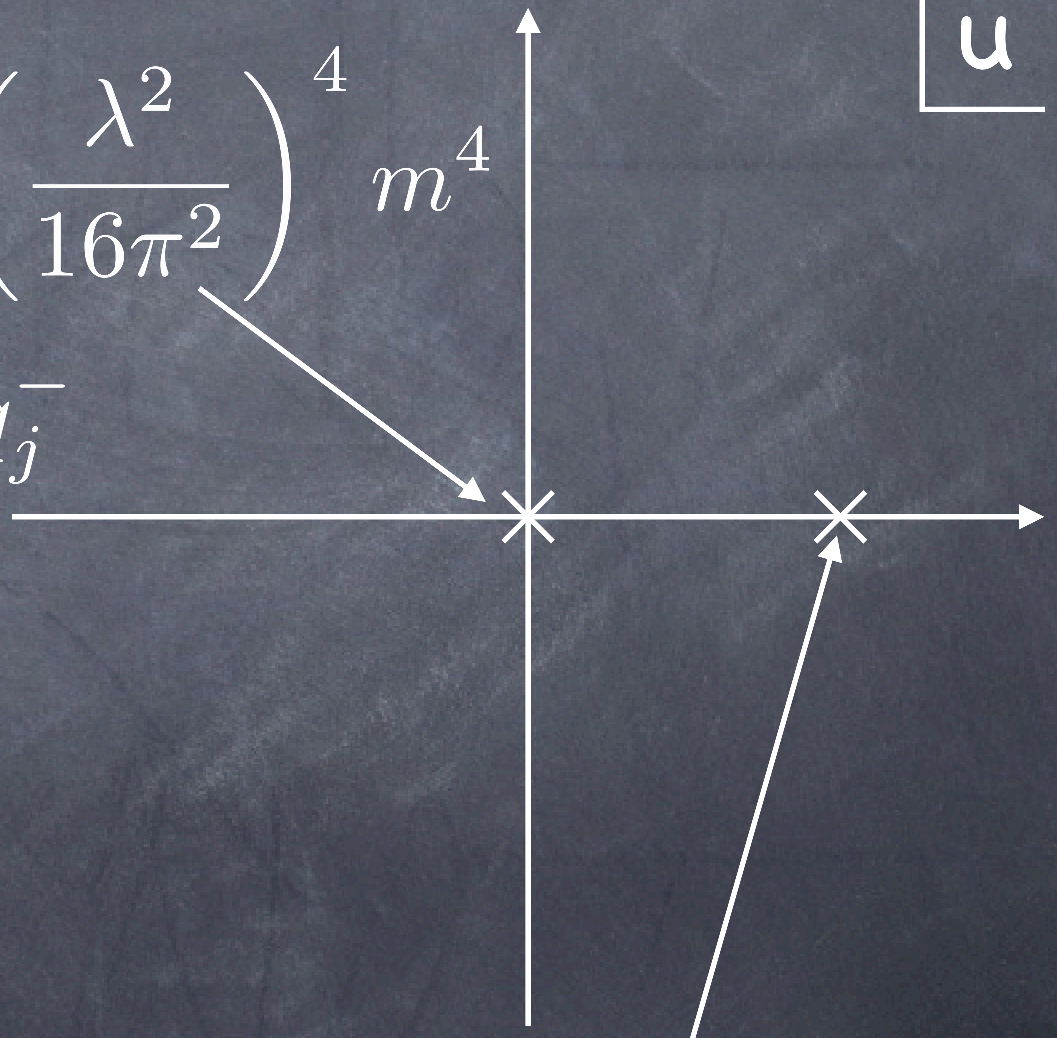
0

m=∞

$N_f = N_c - 2$

- for $M^{ij}=Q^iQ^j\neq 0$ with rank $M=N_f$, $SO(N_c)$ is broken to $SO(2)$
- Coulomb branch $u = \mathrm{det} M$
- two singularities
- $u = \mathrm{det} M = 0$
  - dyons: $q_i^\pm$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$
- $u = \mathrm{det} M = \Lambda^{2N_f}$
  - monopoles: $W = (u - \Lambda^{2N_f}) E^+ E^-$

    $|E^\pm| = (m\Lambda)^{1/2}$
  - both monopoles and meson condense!

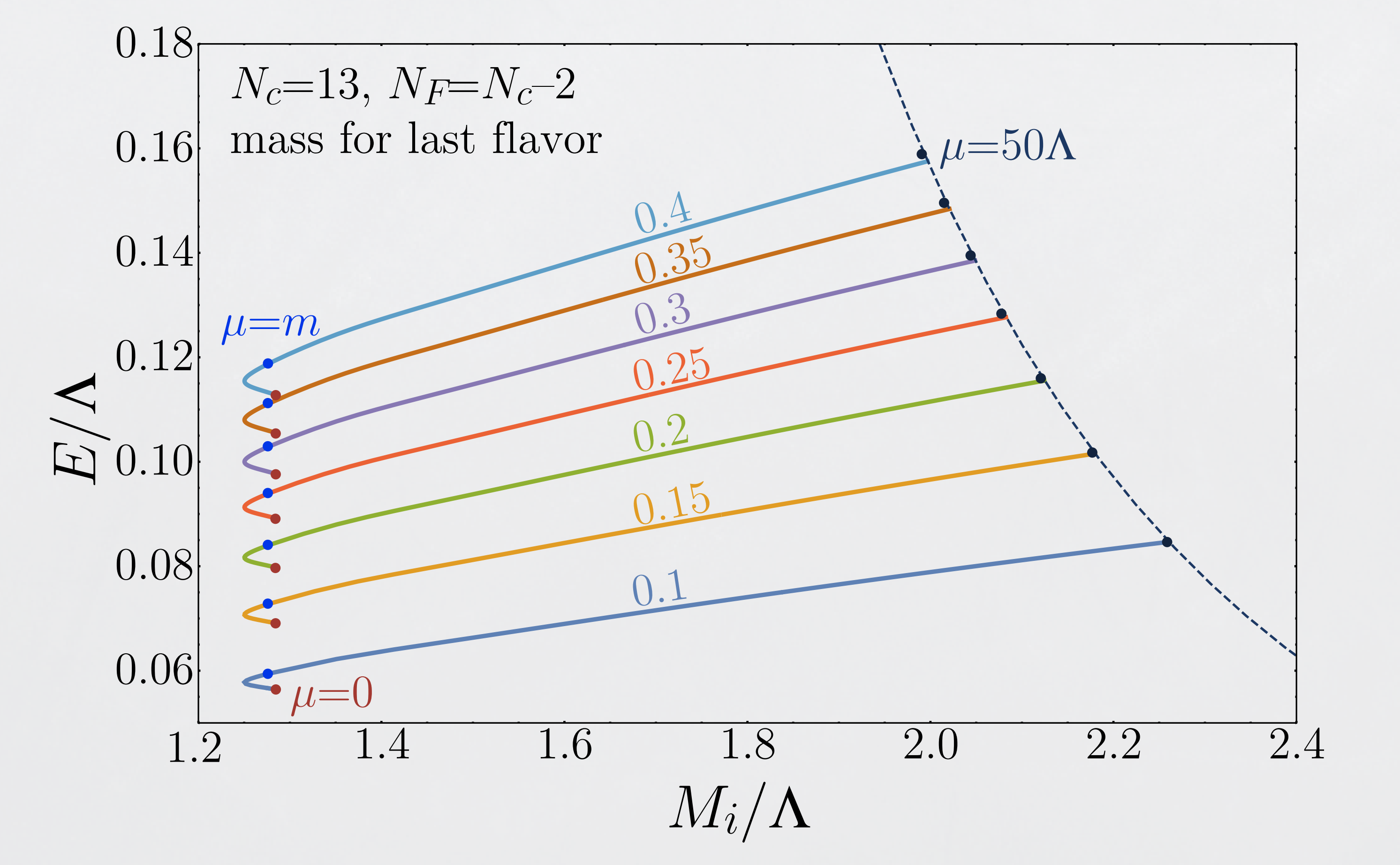


$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$

$$V = -N_f m^2 \Lambda^2$$

$N_f < N_c-2$

- add mass m_q to some of the quarks
- can show monopole VEVs persist $m_q \to \infty$
- demonstration of confinement and chiral symmetry breaking for all $N_f \leq N_c-2$



Chiral gauge theories

Nature is chiral

- Standard model is a chiral gauge theory
- but chiral gauge theories are difficult to regularize
- no lattice simulations yet!
- SO(10): smallest anomaly-free group that admits complex representation
- 16: smallest complex representation of SO(10)
- Kikukawa: probably the best candidate for first lattice simulation of chiral gauge theory
- What do we expect?

SO(10) with N_f 16's

- SU(N_f) global symmetry among 16's
- lowest dimension fermion composite operator
- 120=${}_{10}C_3$ rep $A_{ij}^{\mu\nu\rho} = 16_i^T \mathcal{C}\Gamma_{\mu\nu\rho} 16_j = -A_{ji}^{\mu\nu\rho}$

$$S_{ij;kl} = A_{ij}^{\mu\nu\rho} A_{kl}^{\mu\nu\rho} \qquad \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array}$$

+Kondo, Sylber, in preparation

SO(10) with one 16

- no global symmetry
- dynamically breaks SUSY (HM: hep-th/9505082 "Studying noncalculable models of dynamical supersymmetry breaking")
- no D-flat direction, no weakly coupled limits
- add 10, D-flat direction emerges, breaks SO(10) to SO(7), gauge condensate generates superpotential

$$W = \left(\frac{\Lambda^{21}}{(16\ 16\ 10)^2} \right)^{1/5} + M 10\ 10$$

- breaks SUSY
- gapped, no symmetry breaking

SO(10) with two 16

- SU(2) global symmetry
- $S_{ij;kl} = A_{ij}^{\mu\nu\rho} A_{kl}^{\mu\nu\rho}$ $\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array}$ is SU(2) singlet
- D-flat direction breaks SO(10) to G_2
- gaugino condensate generates superpotential

$$W = \left(\frac{\Lambda^{20}}{S^2}\right)^{1/4}$$

- gapped, no symmetry breaking

SO(10) with three 16

- SU(3) global symmetry
- $S_{ij;kl} = A_{ij}^{\mu\nu\rho} A_{kl}^{\mu\nu\rho}$ is SU(3) symmetric tensor

$$\begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \quad 3^* \times 3^* = 6^*$$

- D-flat direction breaks SO(10) to SU(2)
- gaugino condensate generates superpotential

$$W = \left(\frac{\Lambda^{18}}{\det S}\right)^{1/2}$$

- breaks SU(3) to SO(3)
- low-energy: SU(3)/SO(3) chiral L with WZW
- testable predictions!

Chiral SU(N_c) with A + (N_c-4) F*

Csáki, HM, Telem, 2104.10171

conjectures

- A + (N_c-4)F*: anomaly free chiral gauge th
- SU(N_c-4) x U(1) global symmetry

1. massless composite fermions: $A\bar{F}_{\{i,}\bar{F}_{j\}}$ satisfy 't Hooft anomaly matching conditions without breaking any global symmetries (E. Eichten, R. D. Peccei, J. Preskill, and D. Zeppenfeld (1986))
2. condensate $A\bar{F}_i$ breaks SU(N_c)$_g$xSU(N_c-4)xU(1) to SU(N_c-4)xU(1)xSU(4)$_g$ with massless (½N_c(N_c+1),N_c) fermion $A\bar{F}_{\{i,}\bar{F}_{j\}}$ (S. Dimopoulos, S. Raby, L. Susskind (1980)) "tumbling"

SUSY + AMSB, N_c odd

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 5} \end{array} \right), \qquad \bar{F} = \varphi \left(\begin{array}{c|c} I_{(N-5)} & 0 \\ \hline 0 & 0_{5\times 1} \end{array} \right)$$

$$W = \left(\frac{\Lambda_N^{2N+3}}{(\mathrm{Pf}' A\bar{F}\bar{F})(\mathrm{Pf}' A)} \right)^{3/13}$$

Pouliot (1995)

$$\varphi \approx \Lambda \left(\frac{\Lambda}{m} \right)^{13/(4N-7)} \gg \Lambda$$

- SU(N_c-4)xU(1) broken to Sp(N_c-5)xU(1)
- massless fermions (N_c-4,N_c)

revised tumbling

- $A\bar{F}_i$ breaks SU(N_c)$_g$xSU(N_c-4)xU(1) to SU(4)$_g$xSU(N_c-4)xU(1)
- $\bar{F}_{[i,}\bar{F}_{j]}$ breaks it further to SU(4)$_g$xSp(N_c-5)xU(1)
- SU(4)$_g$ becomes strong and A(4) and F*(4*) condense

SUSY + AMSB, N_c even

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left(\begin{array}{c|c} J_{(N_c-4)} & 0 \\ \hline 0 & 0_{4\times 4} \end{array} \right), \qquad \bar{F} = \varphi \left(\begin{array}{c} I_{(N_c-4)} \\ \hline 0 \end{array} \right)$$

$$W = \left(\frac{\Lambda^{2N+3}}{(\mathrm{Pf} A \bar{F} \bar{F})(\mathrm{Pf} A)} \right)^{1/3}$$

Pouliot
(1995)

$$\varphi \approx \Lambda \left(\frac{\Lambda}{m} \right)^{3/2N_c}$$

- SU(N_c-4)xU(1) broken to Sp(N_c-4)
- no massless fermions

revised tumbling

- $A\bar{F}_i$ breaks SU(N_c)$_g$xSU(N_c-4)xU(1) to SU(4)$_g$xSU(N_c-4)xU(1)
- $\bar{F}_{[i,}\bar{F}_{j]}$ breaks it further to SU(4)$_g$xSp(N_c-4)
- SU(4)$_g$ becomes strong and A(4) and F*(4*) condense

Conclusions

- SUSY + AMSB help us understand quantum dynamics
- obtain results consistent with Yukawa, Nambu
- no obvious signs of phase transition as SUSY breaking is increased
- applied to systems not understood yet, results very different from past conjectures
- can be applied to 2+1D, 1+1D systems (not yet)
- interested in strongly-coupled dark sector