Naohiro Osamura (Nagoya U. M2)

Contribution of Weinberg operator to atomic EDM

JHEP 06 (2022) 072 in collaboration with
P. Gubler and N. Yamanaka

PPP2022 / August 29, 2022

Contents

CP violation

$$C \times P = CP$$

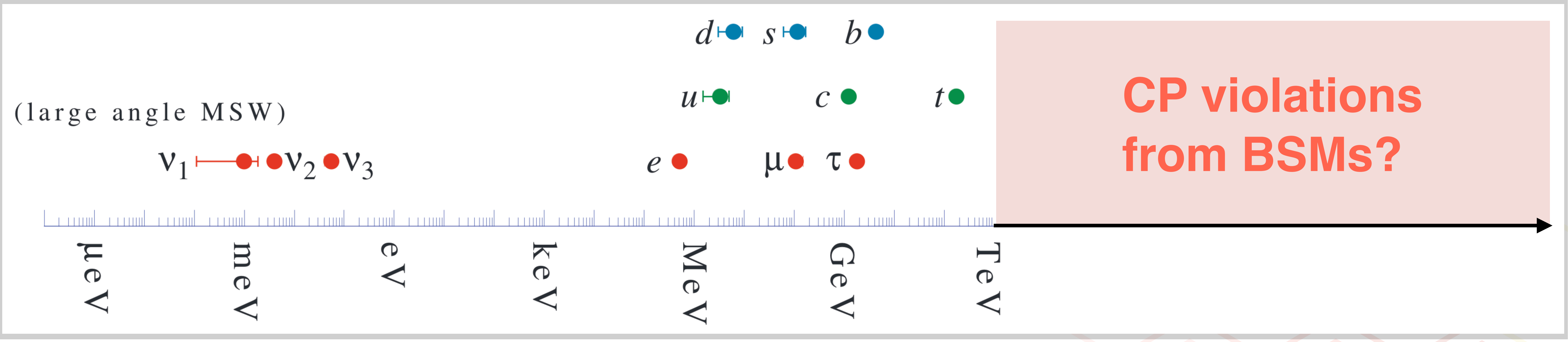
C: $q_{L(R)} \to \bar{q}_{L(R)}$

P: $q_{L(R)} \to q_{R(L)}$

CP: $q_{L(R)} \to \bar{q}_{R(L)}$

Why is the CP violation studied?

i.



ii.

Sakharov conditions to produce a matter-antimatter asymmetry

A. D. Sakharov, Pisma Zh. Eksp. Thor. Fiz. **5** (1967) 32

1. Baryon number must be violated.
2. C and CP must be violated. **SM is insufficient ($1/10^{10}$)!**
3. There must have been some departure from thermal equilibrium.

Electric Dipole Moment

Electric Dipole Moment (EDM)

$$\mathcal{L}_5 = -d\frac{i}{2}\bar{\psi}\sigma^{\mu\nu}\gamma^5\psi F_{\mu\nu} \rightarrow H_{\text{T,P-odd}} = -d\vec{E}\cdot\frac{\vec{S}}{S}$$

	$\vec{S}$	$\vec{E}$	
P	$+$	$-$	$-$
T	$-$	$+$	$-$

- T-odd → CP-odd under $CPT = 1$
- CP violating observable
- sensitive to BSM (SM prediction << experimental result)

$$\left(d^{\text{CKM}}_{\text{neutron}} \simeq 10^{-32} e\,\text{cm}, \quad d^{\text{exp.}}_{\text{neutron}} < 1.8\times10^{-26} e\,\text{cm}\right)$$

Experiment

electron EDM	$d_e < 1.1\times10^{-29} e$ cm	V. Andreev *et al.*, Nature 562, 355 (2018)
neutron EDM	$d_n < 1.8\times10^{-26} e$ cm	C. Abel *et al.*, Phys. Rev. Lett. 124, 0818803 (2020)
^{199}Hg EDM	$d_{\text{Hg}} < 7.4\times10^{-30} e$ cm	B. Graner *et al.*, Phys. Rev. Lett. 116, 161601 (2016)

Hadronic aspects in EDM

beyond SM
- SUSY
- Higgs doublet
- Left-Right
- Leptoquark
- ...

$\mathcal{L}_{\mathrm{SMEFT}}$
- θ-term
- quark EDM
- chromo EDM
- Weinberg op.
- ...

Nucleon EDMs

N-N interactions

Schiff moment

Diamagnetic atom EDM

E ← TeV — EW/QCD — nuclear — atomic

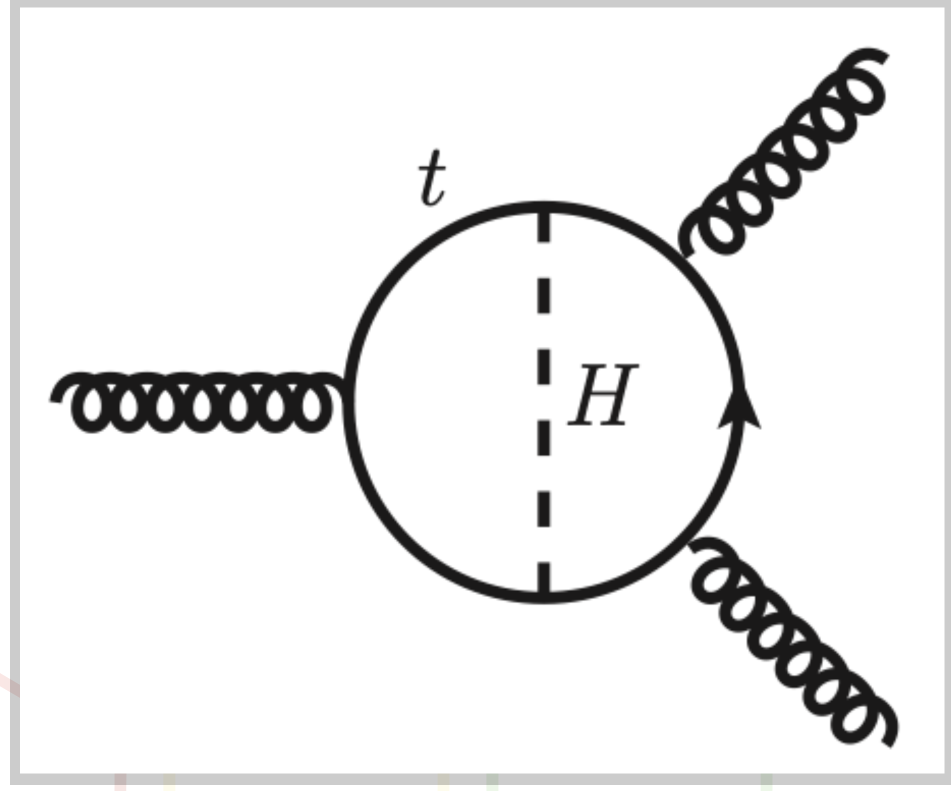
Weinberg operator

CP-odd dimension six gluonic operator

dim-4 | dim-5

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{g_s^2}{32\pi^2}\bar{\theta}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{i}{2}\sum_{i=u,d,s,e,\mu} d_i\bar{\psi}_i\sigma^{\mu}F_{\mu\nu}\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\bar{\psi}_i g_s\sigma^{\mu\nu}G^a_{\mu\nu}\tau^a\gamma_5\psi_i$$
$$+ \frac{2}{3!}wf^{abc}G^a_{\mu\nu}\tilde{G}^{b\nu\rho}G^{c\,\mu}_{\rho} + \sum_{i,j} C_{ij}\left(\bar{\psi}_i\psi_i\right)\left(\bar{\psi}_j i\gamma_5\psi_j\right) + \cdots$$

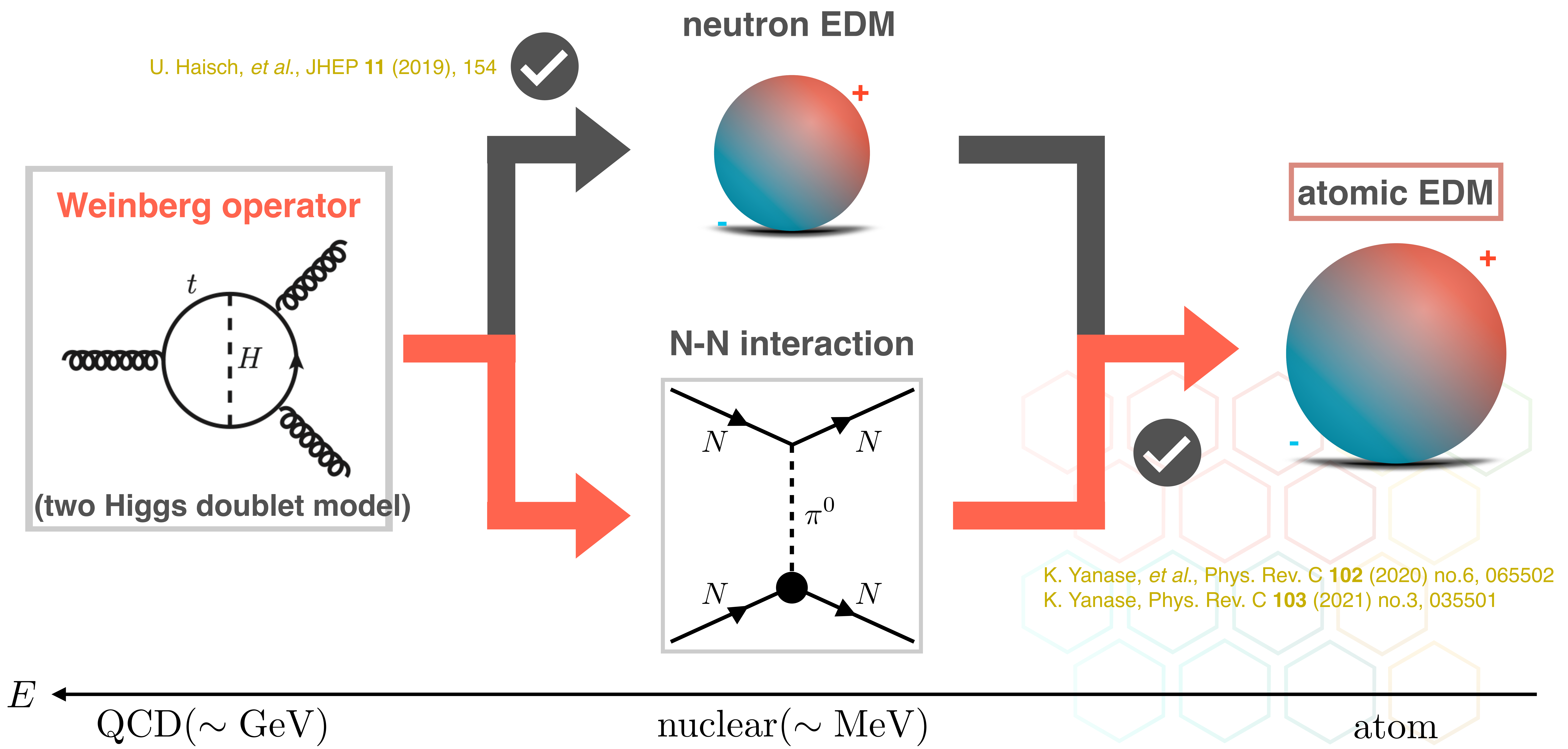
dim-6



BSM

- 2-Higgs doublet model — S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989)
- Minimal supersymmetric standard model — J. Dai *et al.*, Phys. Lett. B 237, 216 (1990)
- Vectorlike quark model — K. Choi *et al.*, Phys. Lett. B 760, 666 (2016)
- Model independent evaluation — T. Abe *et al.*, JHEP 09 (2018)

Outline



neutron EDM
U. Haisch, *et al.*, JHEP **11** (2019), 154
Weinberg operator
t
H
(two Higgs doublet model)
N-N interaction
N
N
π^0
N
N
atomic EDM
K. Yanase, *et al.*, Phys. Rev. C **102** (2020) no.6, 065502
K. Yanase, Phys. Rev. C **103** (2021) no.3, 035501
E
QCD($\sim$ GeV)
nuclear($\sim$ MeV)
atom

Isovector CP-odd pion-nucleon interaction

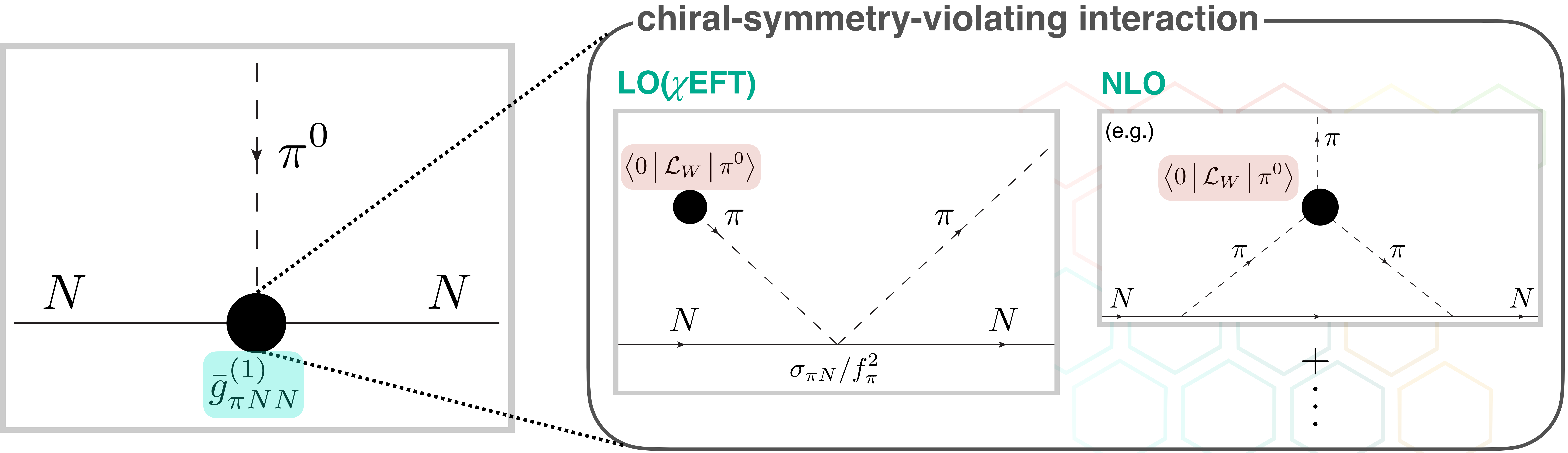
atomic EDM (e.g. Hg): $d^{\mathrm{Hg}} = -6.4 \times 10^{-4} d_n - 2.3 \times 10^{-4} \bar{g}^{(1)}_{\pi NN} e\mathrm{fm}$

neutron EDM $>>$ **pion-nucleon interaction**

B. K. Sahoo *et al.*, Phys. Rev. Lett. 120(2018) no.20, 203001
K. Yanase *et al.*, PRC 102, 065502 (2020)
K. Yanase, PRC 103, 035501 (2021)

chiral-symmetry violation
(quark-mass suppression)

$\bar{g}^{(1)}_{\pi NN} \propto m_{u,d}$



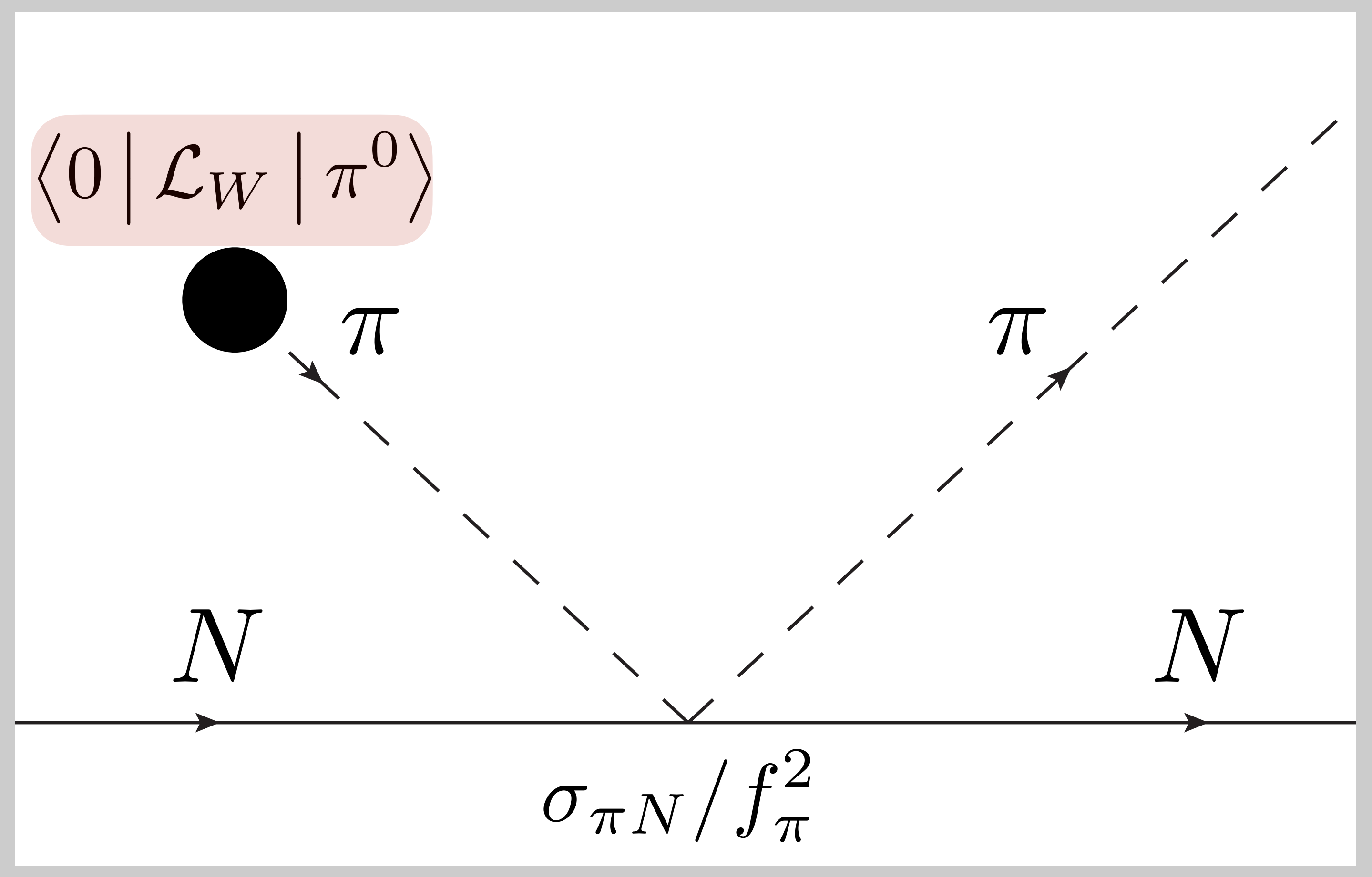
Isovector CP-odd pion-nucleon interaction

atomic EDM (i.e. Hg): $d^{\mathrm{Hg}} = -6.4 \times 10^{-4} d_n - 2.3 \times 10^{-4} \bar{g}^{(1)}_{\pi NN} e\mathrm{fm}$

neutron EDM > > pion-nucleon interaction

B. K. Sahoo *et al.*, Phys. Rev. Lett. 120(2018) no.20, 203001
K. Yanase *et al.*, PRC 102, 065502 (2020)
K. Yanase, PRC 103, 035501 (2021)

$\bar{g}^{(1)}_{\pi NN,\mathrm{LO}} =$



$$= \langle 0 | \mathcal{L}_W | \pi^0 \rangle \frac{1}{m_\pi^2} \langle \pi^0 N | \mathcal{L}_{\mathrm{QCD}} | \pi^0 N \rangle$$

$$m_q \times 1/m_q \times m_q \propto m_q$$

Isovector CP-odd pion-nucleon interaction

atomic EDM (i.e. Hg): $d^{\mathrm{Hg}} = -6.4\times 10^{-4} d_n - 2.3\times 10^{-4}\bar{g}^{(1)}_{\pi NN}\, e\mathrm{fm}$

neutron EDM $\gtrsim$ **pion-nucleon interaction**

B. K. Sahoo *et al*., Phys. Rev. Lett. 120(2018) no.20, 203001
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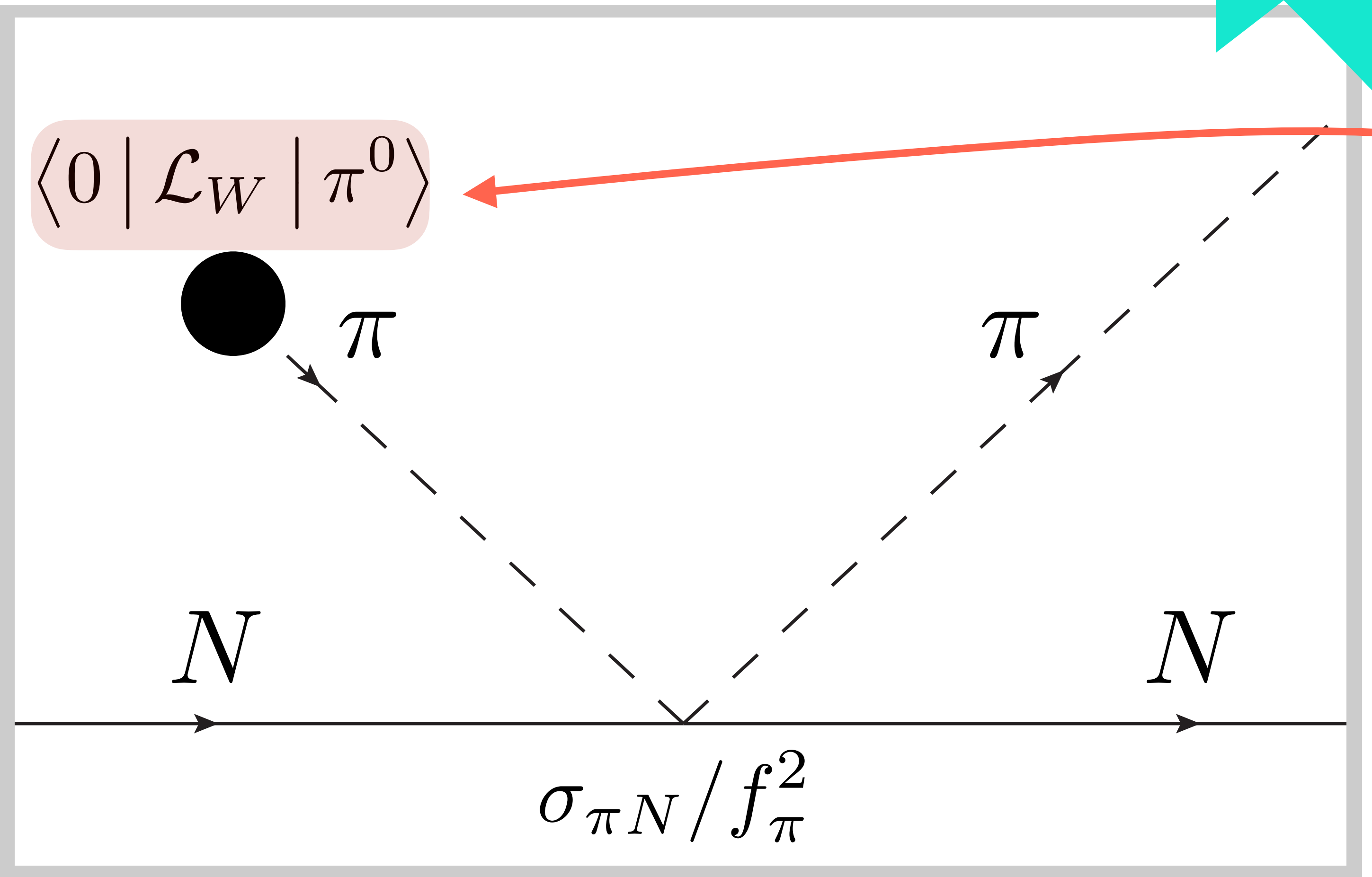
$\bar{g}^{(1)}_{\pi NN,\mathrm{LO}} =$



$= \langle 0\,|\,\mathcal{L}_W\,|\,\pi^0\rangle \dfrac{1}{m_\pi^2} \langle \pi^0 N\,|\,\mathcal{L}_{\mathrm{QCD}}\,|\,\pi^0 N\rangle$

Low-energy constant, QCD sum rules

$m_q \times 1/m_q \times m_q \propto \sigma_{\pi N}$

$m_q \to \sigma_{\pi N}$

enhancement!

$$\langle \pi^0 N\,|\,\mathcal{L}_{\mathrm{QCD}}\,|\,\pi^0 N\rangle \approx \frac{-1}{f_\pi^2}\langle N\,|\,m_u\bar{u}u + m_d\bar{d}d\,|\,N\rangle \approx \frac{\sigma_{\pi N}}{f_\pi^2}, \qquad m_q \sim O(1\mathrm{MeV}) \to \sigma_{\pi N} \approx [30, 60]\mathrm{MeV}$$

[lattice, phen]

QCD sum rules

correlation function $\Pi_{\text{phen}}\left(q^2\right) = i\int d^4x e^{-iq\cdot x}\langle\Omega\,|T\left[\mathcal{L}_W(x)\mathcal{L}_W(0)\right]|\,\Omega\rangle = \Pi_{\text{OPE}}\left(q^2\right)$

read out from OPE

$$\Pi_{\text{phen}}(q^2) = \left(\text{pion-pole, } \left|\langle 0\,|\,\mathcal{L}_W\,|\,\pi^0\rangle\right|^2\right) + (\text{excited and multi-particle state})$$

$$\Pi_{\text{OPE}}(q^2) = \sum_d C_{d;\mu}(q^2)\,\langle O_d\rangle_\mu$$

- high energy
- perturbative

- low energy
- non-perturbative

To improve the accuracy of evaluating $\langle 0\,|\,\mathscr{L}_W\,|\,\pi^0\rangle$

- **The Borel transformation suppresses the continuum. (ordinary)**

$$\mathcal{B}\left[F\left(Q^2\right)\right] = \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=M^2=\text{const.}}} \frac{\left(Q^2\right)^n}{(n-1)!}\left(-\frac{d}{dQ^2}\right)^n F\left(Q^2\right)$$

$Q^2 := -q^2$

M^2: **Borel mass (unphysical)**

- $(m_u - m_d)^2 \equiv m_-^2,\ d/dm_-^2$ **(the unique point)**

$$\left|\langle 0\,|\,\mathcal{L}_W\,|\,\pi^0\rangle\right|^2 \propto m_-^2$$

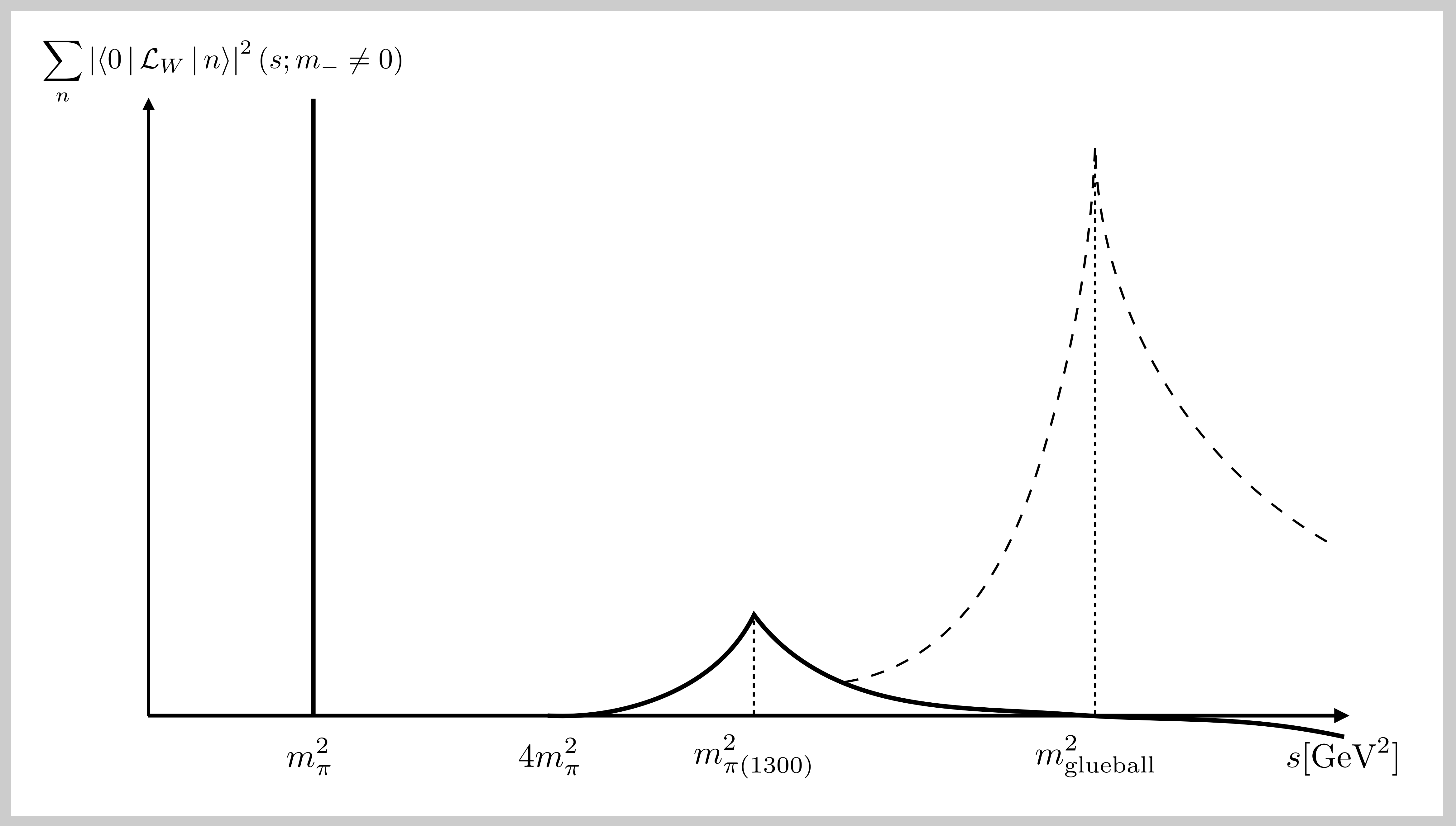
vanish: $|\langle 0\,|\,\mathcal{L}_W\,|\,\text{glueball}\rangle|^2 \not\propto m_-^2$

QCD sum rules -Phenomenology-

$$i\int d^4x e^{-iq\cdot x}\langle\Omega|T[\mathcal{L}_W(x)\mathcal{L}_W(0)]|\Omega\rangle = \Pi_{\rm phen}(q^2) = \frac{\lambda_\pi^2}{m_\pi^2-q^2} + \frac{\lambda_{\rm glueball}^2}{m_{\rm glueball}^2-q^2} + \frac{1}{\pi}\int_{s_{\rm th}}^\infty ds\frac{{\rm Im}\,\Pi_{\rm OPE}(s)}{s-q^2}$$

one-particle pole　　　continuum

$$\mathcal{B}[\Pi_{\rm phen}](M^2) = \frac{\lambda_\pi^2}{M^2}e^{-m_\pi^2/M^2} + \frac{1}{\pi}\int_{s_{\rm th}}^\infty ds\frac{e^{-s/M^2}}{M^2}{\rm Im}\,\Pi_{\rm OPE}(s; m_- \neq 0)$$



- Borel transformation → continuum ↓
- $\frac{d}{dm_-^2}$ → ~~glueball resonance~~

uncertainty

- Borel mass M
- threshold parameter $s_{\rm th}$　sign not determined

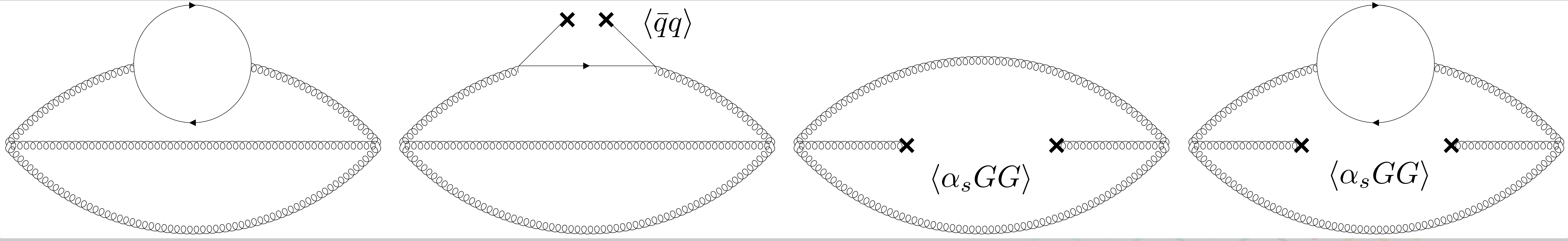
$$\left(\lambda_\pi^2 = |\langle 0|\mathcal{L}_W|\pi^0\rangle|^2\right)$$

QCD sum ruels -Operator Product Expansion-

$$i\int d^4x e^{-iq\cdot x}\langle\Omega\,|T\left[\mathcal{L}_W(x)\mathcal{L}_W(0)\right]|\,\Omega\rangle = \Pi_{\mathrm{OPE}}\left(q^2\right) = \sum_d C_d(q;\mu)\,\langle O_d\rangle\,(\mu)$$

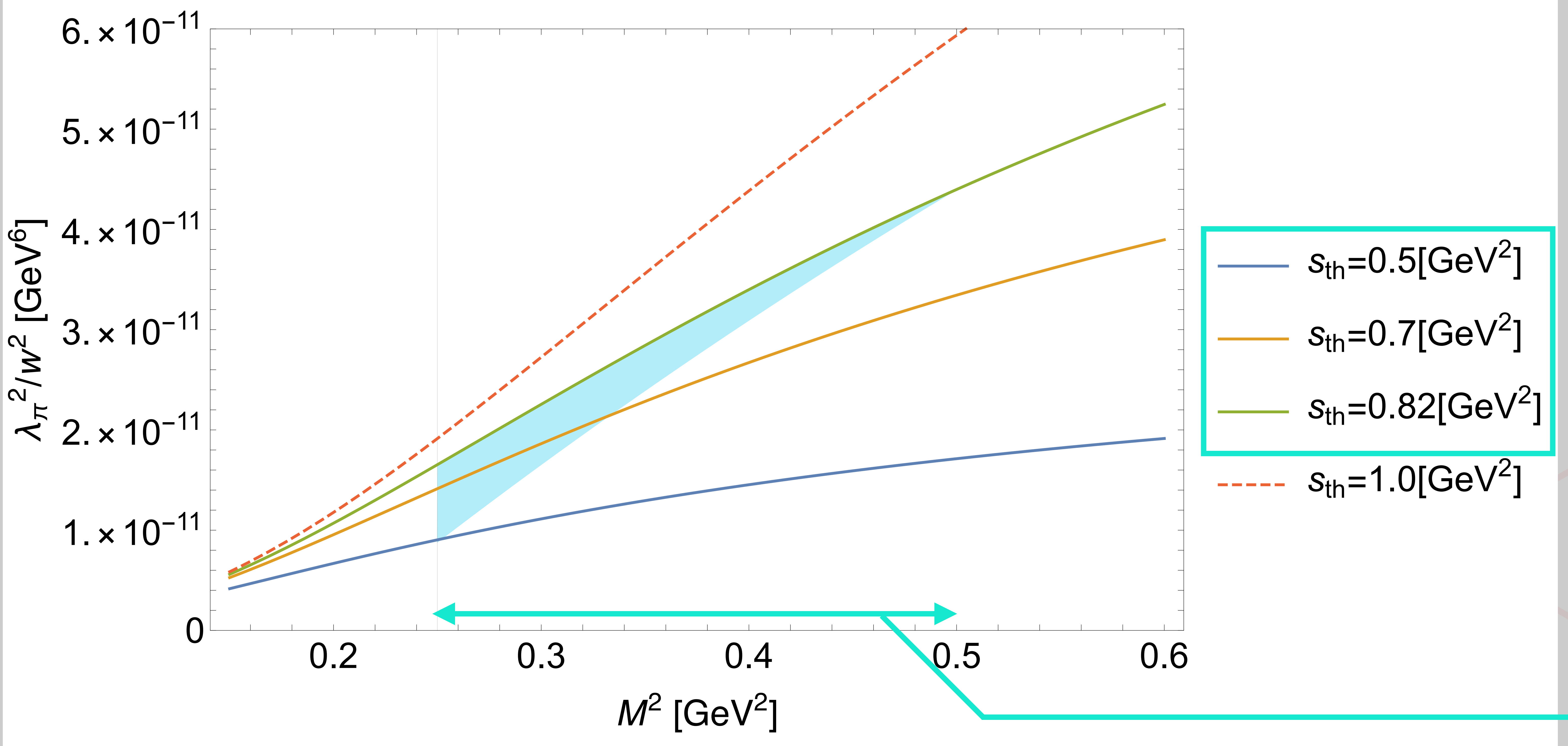
$$= \Pi_{\mathrm{OPE}}\left(q^2; m_- = 0\right) + \Pi_{\mathrm{OPE}}\left(q^2; m_- \neq 0\right)$$

vanish by $\frac{d}{dm_-^2}$



$$\mathcal{B}\left[\Pi_{\mathrm{OPE}}\left(q^2; m_- \neq 0\right)\right] = w^2\left[-\frac{3!m_-^2\alpha_s}{2^7\pi^4}M^6 - \frac{2!m_-^2 C_q\alpha_s}{2^4\pi^3}M^4\right]$$

Numerical analysis



uncertainty

- threshold parameter s_{th} (phenomenology)
- Borel mass M (Borel tr.)

$$\left|\langle 0 \,|\mathcal{L}_W|\, \pi^0 \rangle\right| \in w \cdot [2.3, 8.3] \times 10^{-6}\mathrm{GeV}^5$$

sign not determined

Numerical analysis

our result

atomic EDM

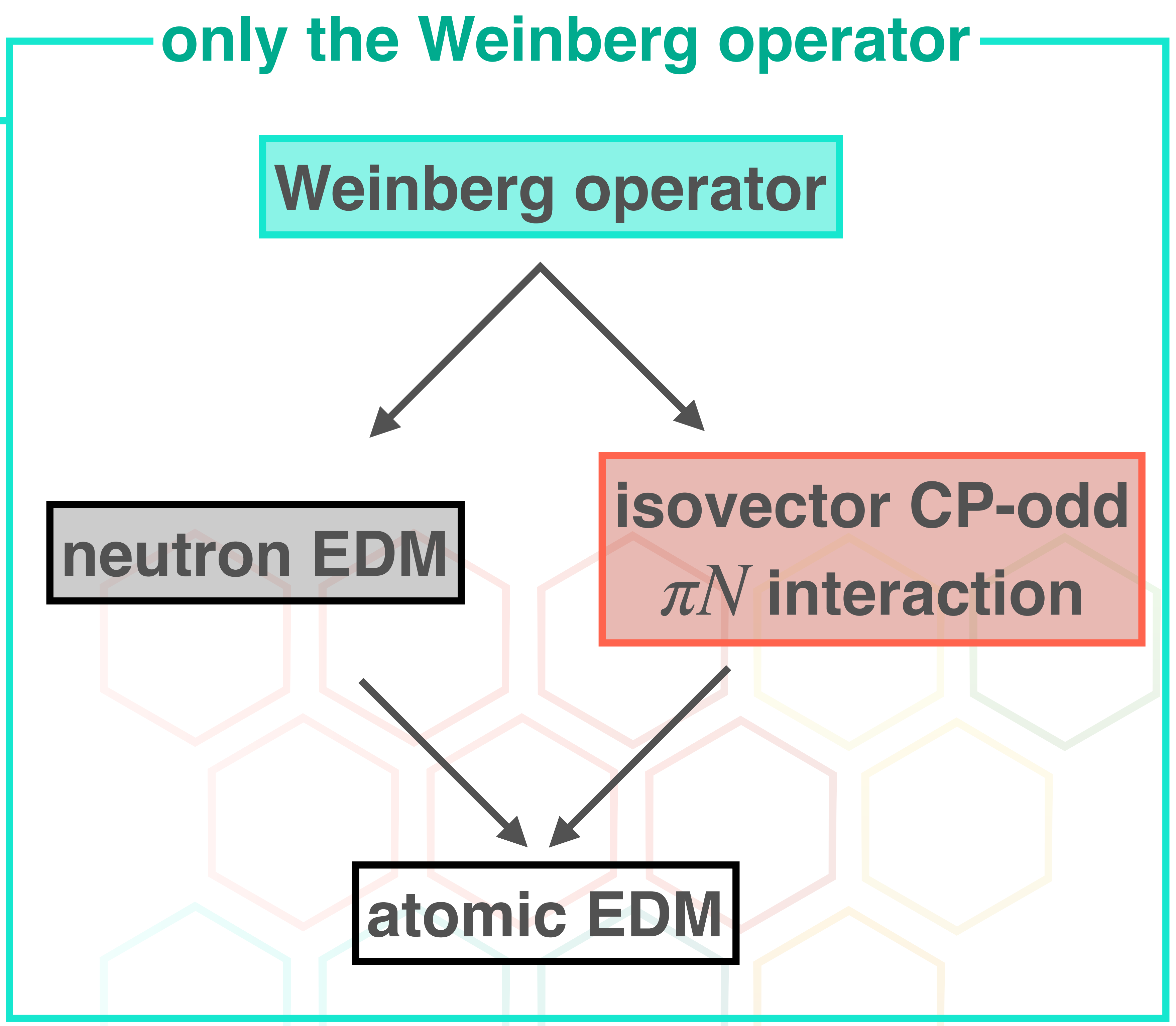
$$d^{\rm atom} = ({\rm prefactor})d_n + ({\rm prefactor})\bar{g}^{(1)}_{\pi NN} e\,{\rm fm}$$

$$d^{\rm Hg} = w\left(-1.3\times 10^{-2} \pm [0.71, 4.4]\times 10^{-3}\right) e{\rm MeV}$$

$$\left|\frac{(\text{Isovector CP-odd }\pi N\text{ interaction})_{\rm max}}{(\text{neutron EDM})}\right|_{\rm Hg} = 34\%$$

$$d^{\rm Xe} = w\left(2.7\times 10^{-4} \pm [0.52, 3.2]\times 10^{-4}\right) e{\rm MeV}$$

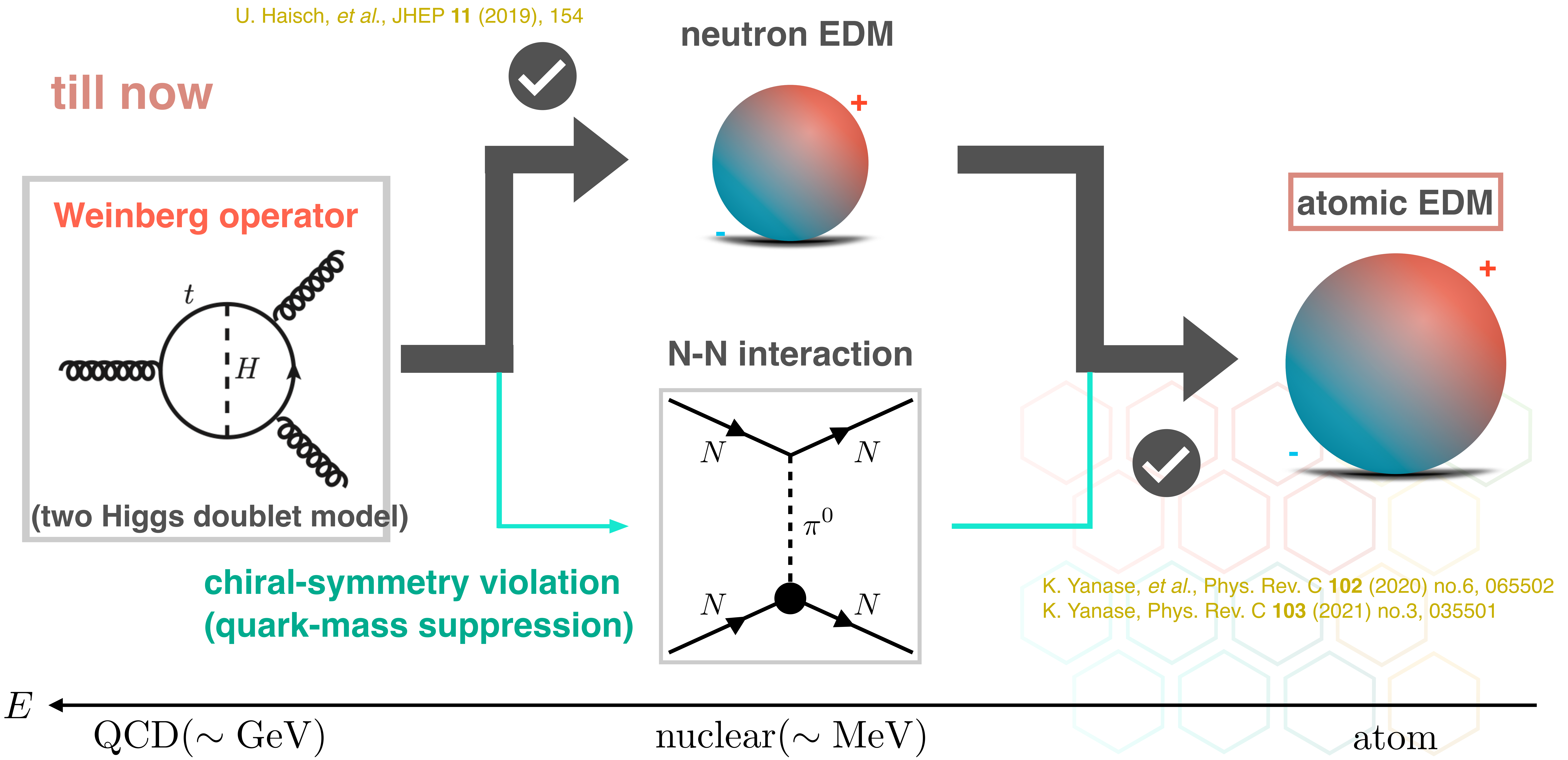
$$\left|\frac{(\text{ Isovector CP-odd }\pi N\text{ interaction })_{\rm max}}{(\text{ neutron EDM})}\right|_{\rm Xe} = 120\%$$



U. Haisch and A. Hala JHEP11(2019)154

N. Yamanaka and E. Hiyama, Phys. Rev. D **103**, 035023 (2021)

Conclusion


U. Haisch, *et al.*, JHEP **11** (2019), 154
till now
Weinberg operator
t
H
(two Higgs doublet model)
chiral-symmetry violation
(quark-mass suppression)
neutron EDM
+
-
N-N interaction
N
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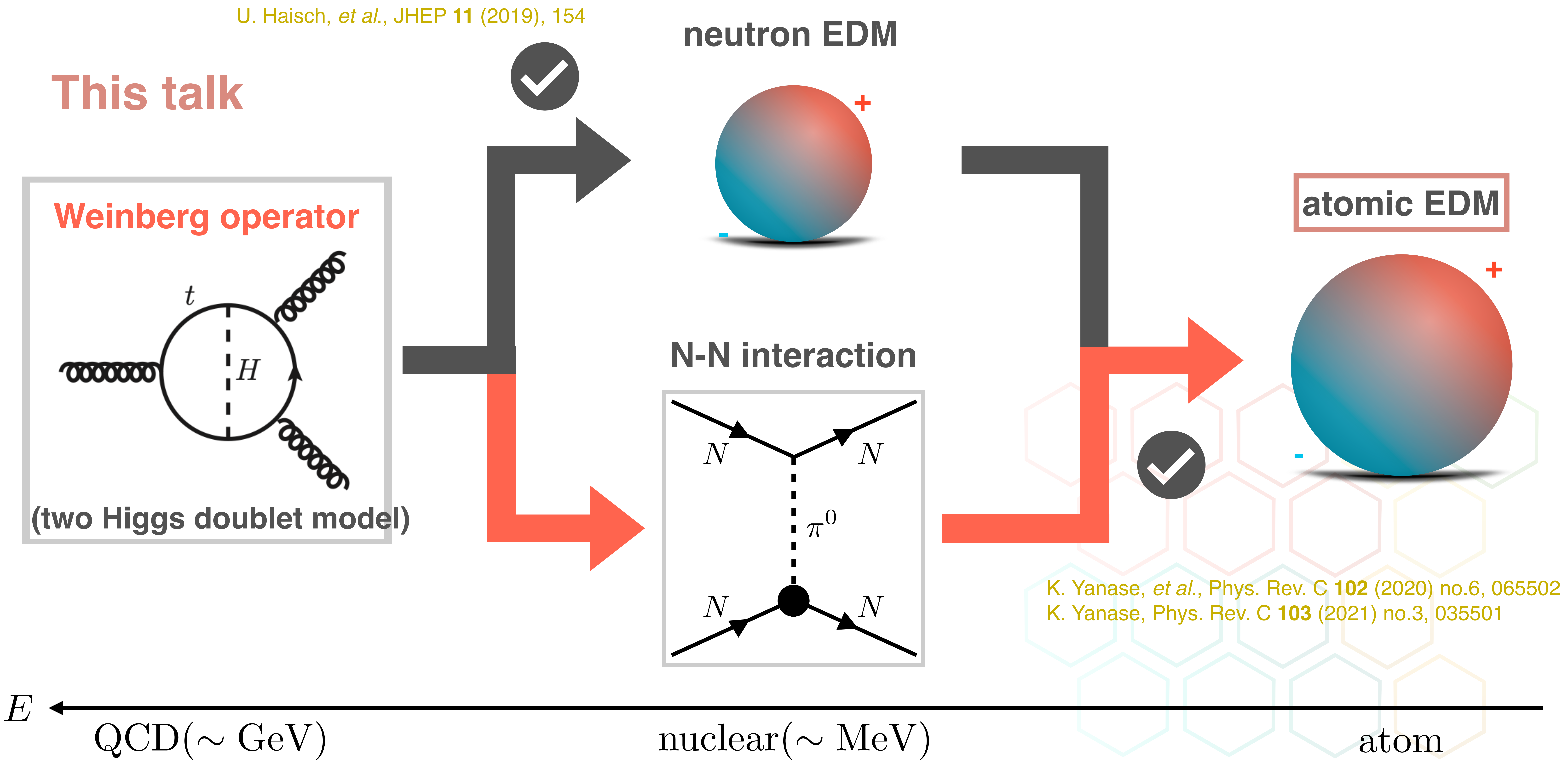
Conclusion


U. Haisch, *et al.*, JHEP **11** (2019), 154
This talk
Weinberg operator
t
H
(two Higgs doublet model)
neutron EDM
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N
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K. Yanase, Phys. Rev. C **103** (2021) no.3, 035501
E
QCD(∼ GeV)
nuclear(∼ MeV)
atom

Conclusion

Constraints of the Weinberg operator

- **From neutron EDM** $|w(\mu = 1\mathrm{TeV})| < 4 \times 10^{-10}\mathrm{GeV}^{-2}$ U. Haisch, *et al.*, JHEP **11** (2019), 154
- **From $^{199}\mathrm{Hg}$ EDM** $|w(\mu = 1\mathrm{TeV})| < 4 \times 10^{-10}\mathrm{GeV}^{-2}$ **our result**

comparable!!

future experiment: $|d^{\mathrm{Xe}}| < 10^{-33} e\,\mathrm{cm}$ W. A. Terrano, *et al.*, Quantum Sci. Technol. **7** (2022) no.1, 014001

- **From Xe EDM** $|w(\mu = 1\mathrm{TeV})| < 10^{-11}\mathrm{GeV}^{-2}$

Backup

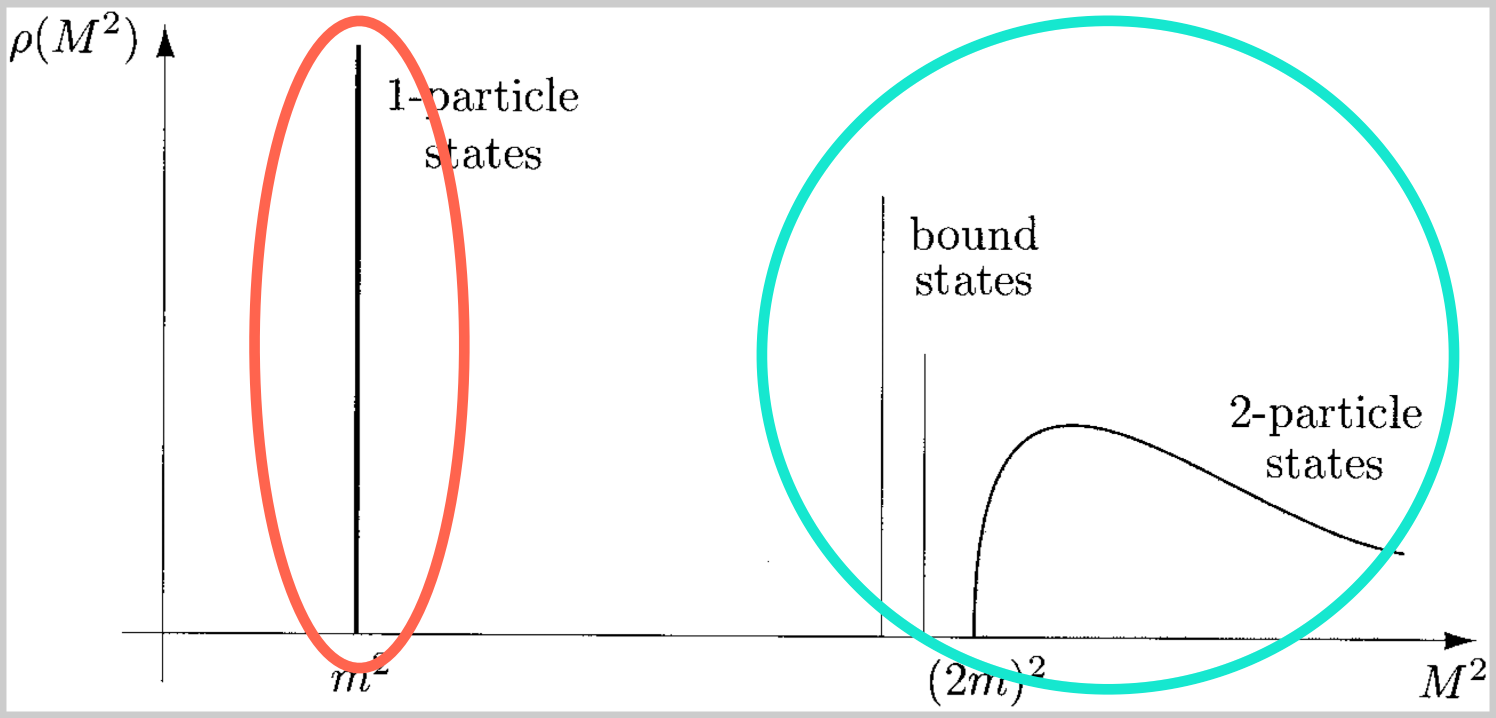
QCD sum rules -Review-

two-point function

$$\Pi_{\rm phen}(q^2) = \int d^4x e^{-iq\cdot x}\langle\Omega|O_1(x)O_2(0)|\Omega\rangle = \Pi_{\rm OPE}(q^2)$$

Hadronic / read out from OPE

$$\Pi_{\rm phen} = (\text{one-particle state}) + (\text{excited and multi-particle state})$$



Borel tr.

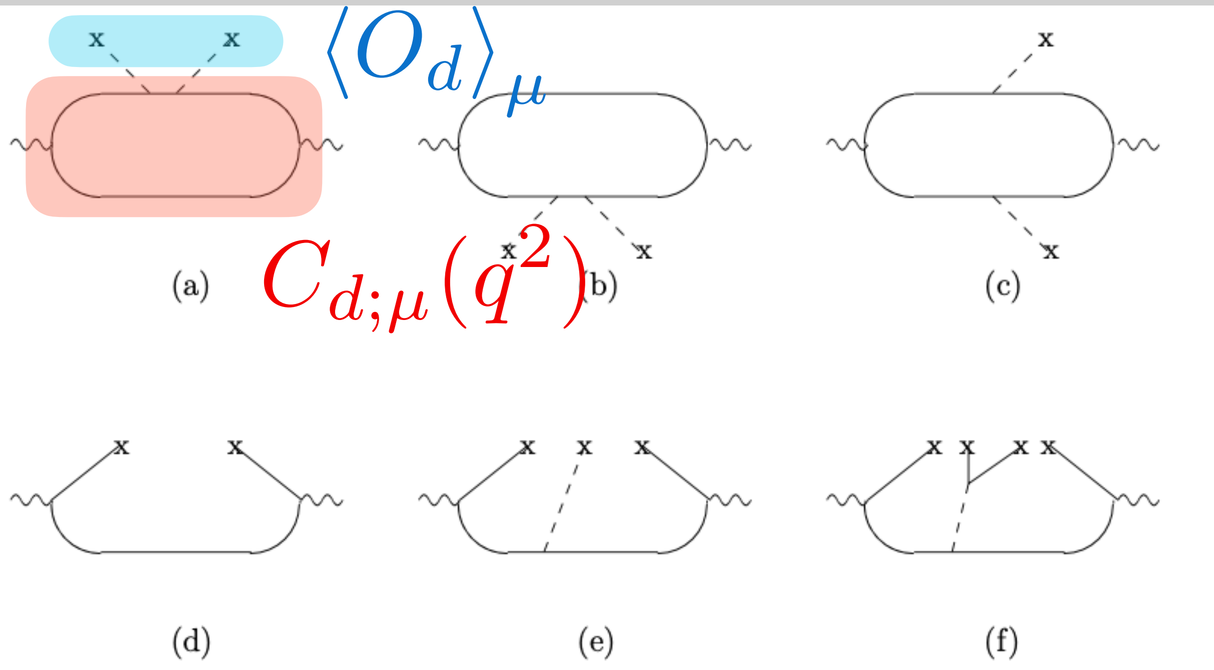
negligible

$$\Pi_{\rm OPE}(q^2) = \sum_d C_{d;\mu}(q^2)\langle O_d\rangle_\mu$$ scale splitting

- higher momentum
- perturbative

- lower momentum
- non-perturbative

e.g.
vector meson



Borel transformation

$$\mathcal{B}\left[F\left(Q^2\right)\right] = \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=M^2=\text{ const.}}} \frac{\left(Q^2\right)^n}{(n-1)!}\left(-\frac{d}{dQ^2}\right)^n F\left(Q^2\right)$$

$Q^2 := -q^2$

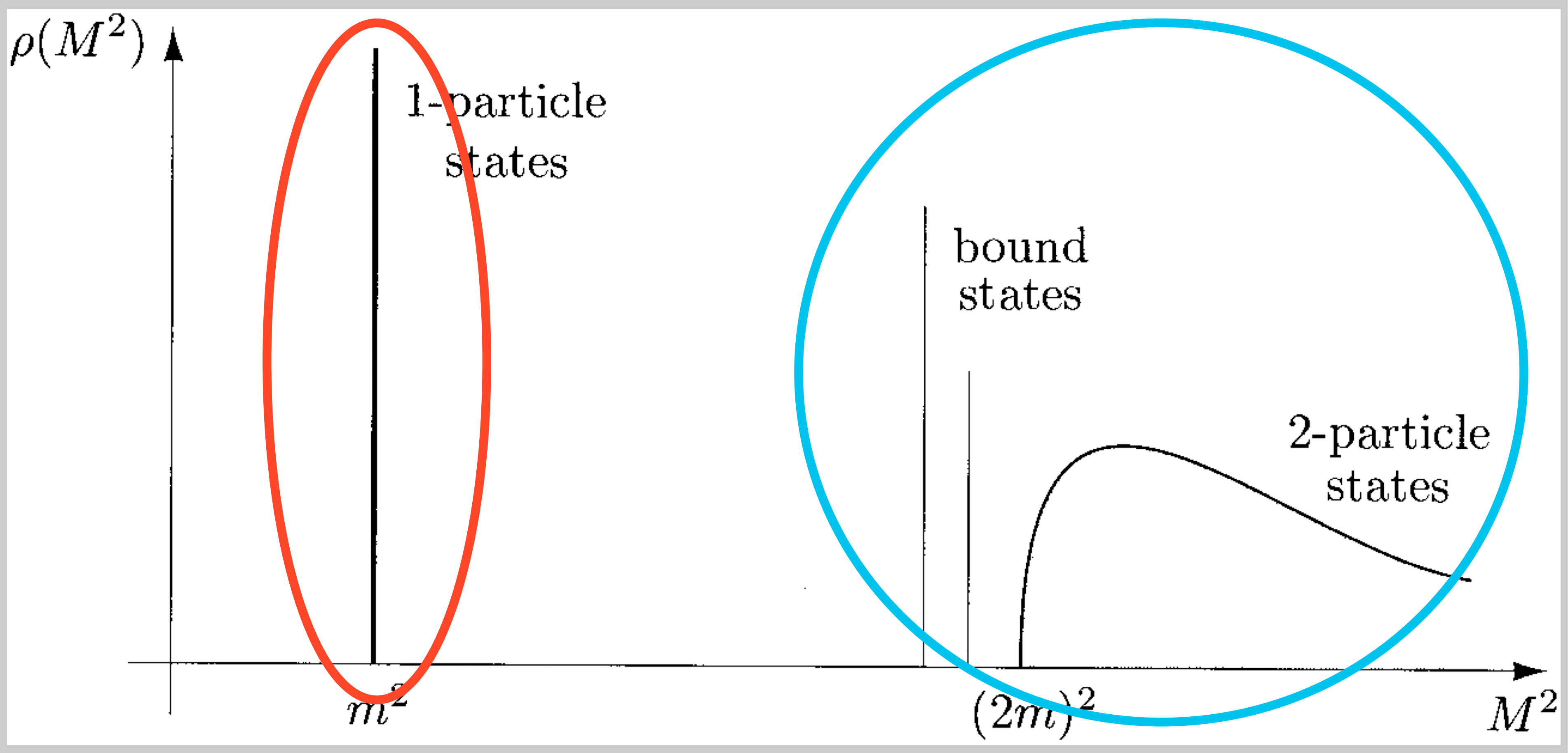
M^2: **Borel mass (unphysical)**

What are QCD sum rules?

two-point function

$$\Pi_{\text{phen}}(q^2) = \Pi_{\text{OPE}}(q^2)$$

one-particle pole



1. Parametrize Π_{phen} from phenomenology.

- **Assumption:** $\rho(s) = f_V^2 \delta(s - m_V^2) + \rho_{\text{conti}}(s)\theta(s - s_{\text{th}})$
- Dispersion relation

$$\Pi_{\text{phen}}\left(q^2\right) = \frac{1}{\pi}\int_{t_{\min}}^{\infty} ds \frac{\text{Im}\Pi(s)}{s - q^2} = \int_{t_{\min}}^{\infty} ds \frac{\rho(s)}{s - q^2} = \frac{f_V^2}{m_V^2 - q^2} + \int_{s_{th}}^{\infty} ds \frac{\rho_{\text{conti}}(s)}{s - q^2}$$

$$\longrightarrow \quad \mathcal{B}\left[\Pi_{\text{phen}}\left(q^2\right)\right] = f_V^2 e^{-m_V^2/M^2} + \int_{s_{\text{th}}}^{\infty} ds \rho_{\text{conti}}(s) e^{-s/M^2}$$

suppressed

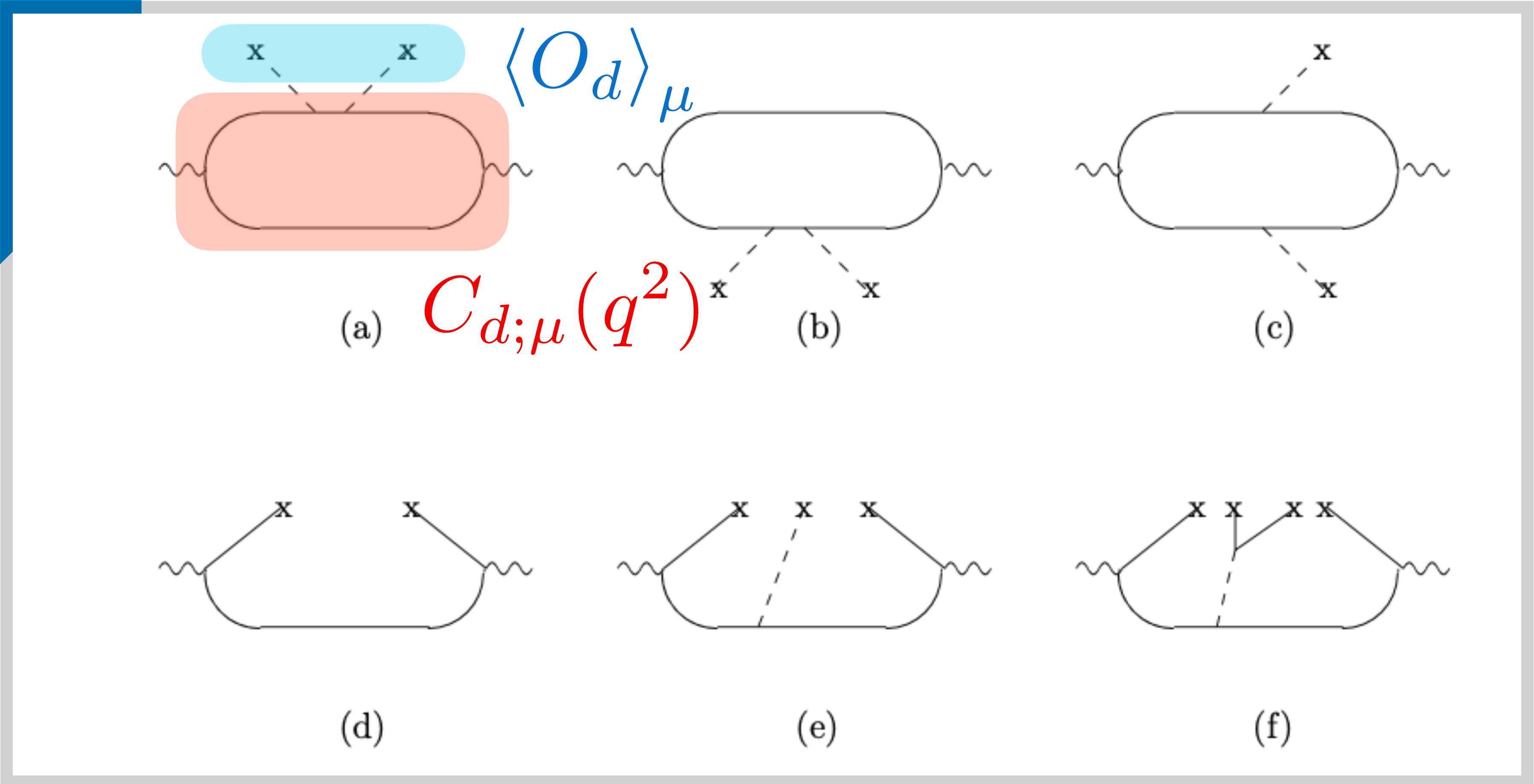
Borel transformation

$$\mathcal{B}\left[F\left(Q^2\right)\right] = \lim_{\substack{Q^2, n \to \infty \\ Q^2/n = M^2 = \text{const.}}} \frac{\left(Q^2\right)^n}{(n-1)!}\left(-\frac{d}{dQ^2}\right)^n F\left(Q^2\right)$$

$Q^2 := -q^2$

M^2: **Borel mass (unphysical)**

What are QCD sum rules?



two-point function

$$\Pi_{\mathrm{phen}}(q^2) = \Pi_{\mathrm{OPE}}(q^2)$$

2. Calculate Π_{OPE} using **operator product expansion (OPE).**

- **OPE:** $O_1(x)O_2(y) = \sum_d C_d(x-y;\mu)O_d((x+y)/2;\mu) \qquad (d = \dim[O_d])$

i. If $x \to y$, $C_d(x-y;\mu) \to 0$ (d:higher dimension) by dimensional analysis.

ii. higher momentum part C_d ⟵ split ⟶ lower one O_d

- **two-point function:** $\langle O_1(x)O_2(0)\rangle = \sum_d C_d(x;\mu)\langle O_d(\mu)\rangle$

perturbative **non-perturbative, but condensate (c-number)**

Example: vector meson channel

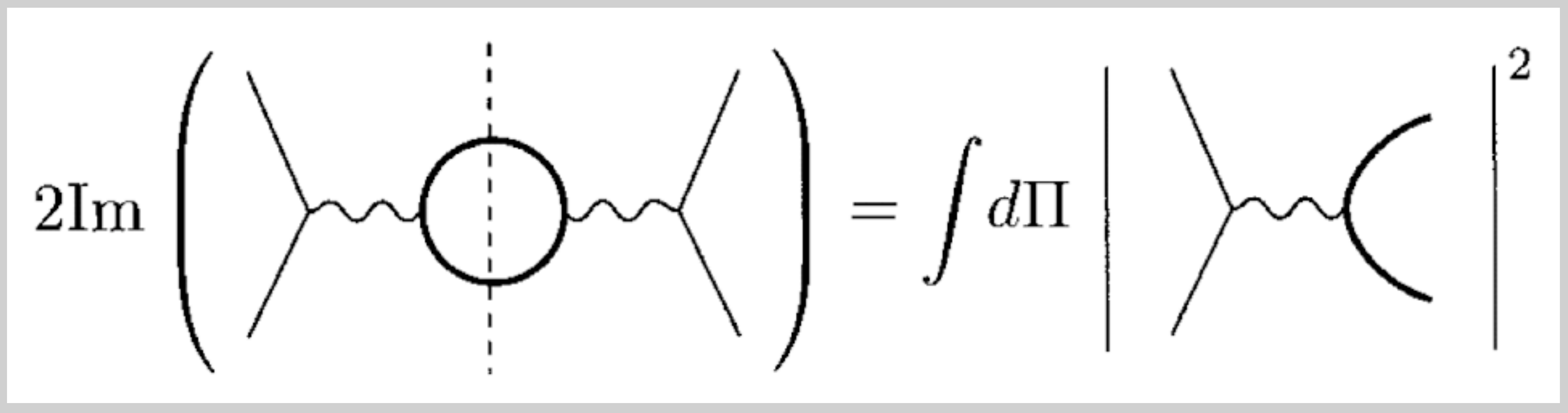
$$\Pi^{\mu\nu}_{\mathrm{OPE}}\left(q^2\right) = i\int d^4x e^{iq\cdot x}\left\langle T\left\{\bar{\psi}\gamma^\mu\psi(x), \bar{\psi}\gamma^\nu\psi(0)\right\}\right\rangle = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi\left(q^2\right)$$

$$\Pi(q^2) = -\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{-q^2}{4m^2} + \frac{2m\langle\bar{\psi}\psi\rangle}{q^4} + \frac{\alpha_s\left\langle G^a_{\mu\nu}G^{a\mu\nu}\right\rangle}{12\pi q^4} + \frac{m^3}{3q^8}\left\langle g_s\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}\psi\right\rangle + \cdots$$

What are QCD sum rules?

two-point function

$$\mathcal{B}\left[\Pi_{\text{phen}}\right]\left(M^2\right) = \mathcal{B}\left[\Pi_{\text{OPE}}\right]\left(M^2\right)$$



3. Combine two approaches.

We can carry out the integral of the continuum spectrum function by quark-hadron duality.

- $$f_V^2 e^{-m_V^2/M^2} + \int_{s_{\text{th}}}^{\infty} ds\,\rho_{\text{conti}}(s) e^{-s/M^2} = \mathcal{B}\left[\sum_d C_d(q;\mu)\left\langle O_d(\mu)\right\rangle\right]$$

$$= \mathcal{B}\left[\sum_{d_a} A_{d_a}\left(-q^2\right)^{d_1+d_2-d_a/2}\log\left(\frac{-q^2}{s_{\text{th}}}\right) + \sum_{d_b} B_{d_b}\frac{1}{(-q^2)^{d_b}}\right]$$

$d_1 := \dim[O_1]$, $d_2 := \dim[O_2]$

$d_a = 0, 1, \ldots, d_1 + d_2$, $d_b = d_1 + d_2, \ldots$

Quark-hadron duality

$$\frac{1}{\pi}\,\text{Im}\,\Pi(s) \to \rho_{\text{conti}}(s)$$

continuum spectrum

Example: vector meson channel

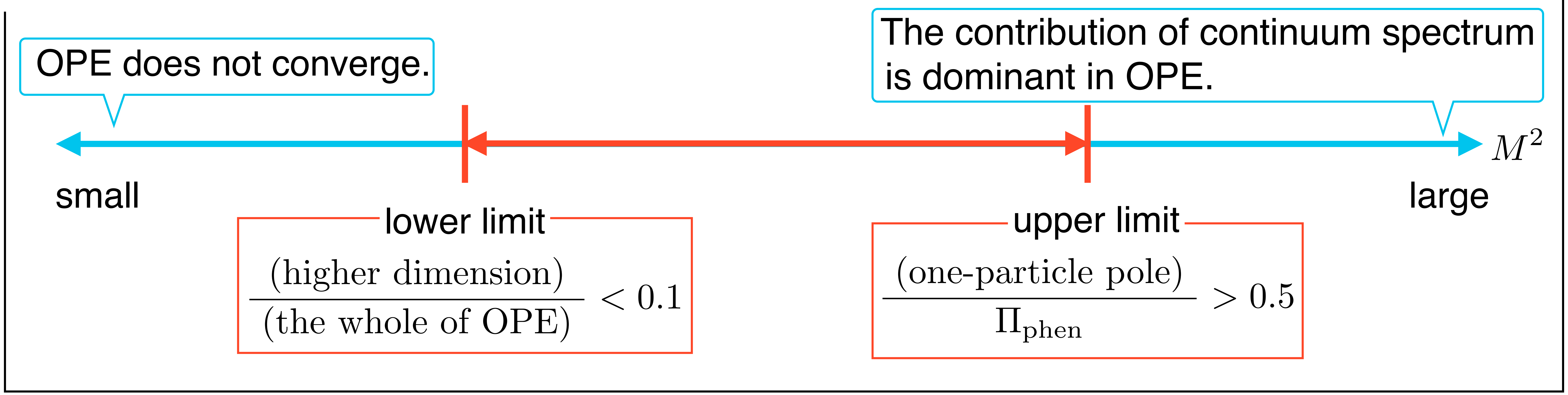
$$f_\rho^2 = M^2 e^{m_\rho^2/M^2}\left[\frac{1}{4\pi^2}\left(1 - e^{-s_0^\rho/M^2}\right)\left(1 + \frac{\alpha_s(M)}{\pi}\right) + \frac{(m_u + m_d)\left\langle\bar{\psi}\psi\right\rangle}{M^4} + \frac{1}{12}\frac{\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle}{M^4} - \frac{112\pi}{81}\frac{\alpha_s\langle\bar{\psi}\psi\rangle^2}{M^6}\right]$$

What are QCD sum rules?

$$f_\rho^2 = M^2 e^{m_\rho^2/M^2}\left[\frac{1}{4\pi^2}\left(1-e^{-s_0^\rho/M^2}\right)\left(1+\frac{\alpha_s(M)}{\pi}\right)+\frac{(m_u+m_d)\langle\bar{\psi}\psi\rangle}{M^4}+\frac{1}{12}\frac{\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle}{M^4}-\frac{112\pi}{81}\frac{\alpha_s\langle\bar{\psi}\psi\rangle^2}{M^6}\right]$$

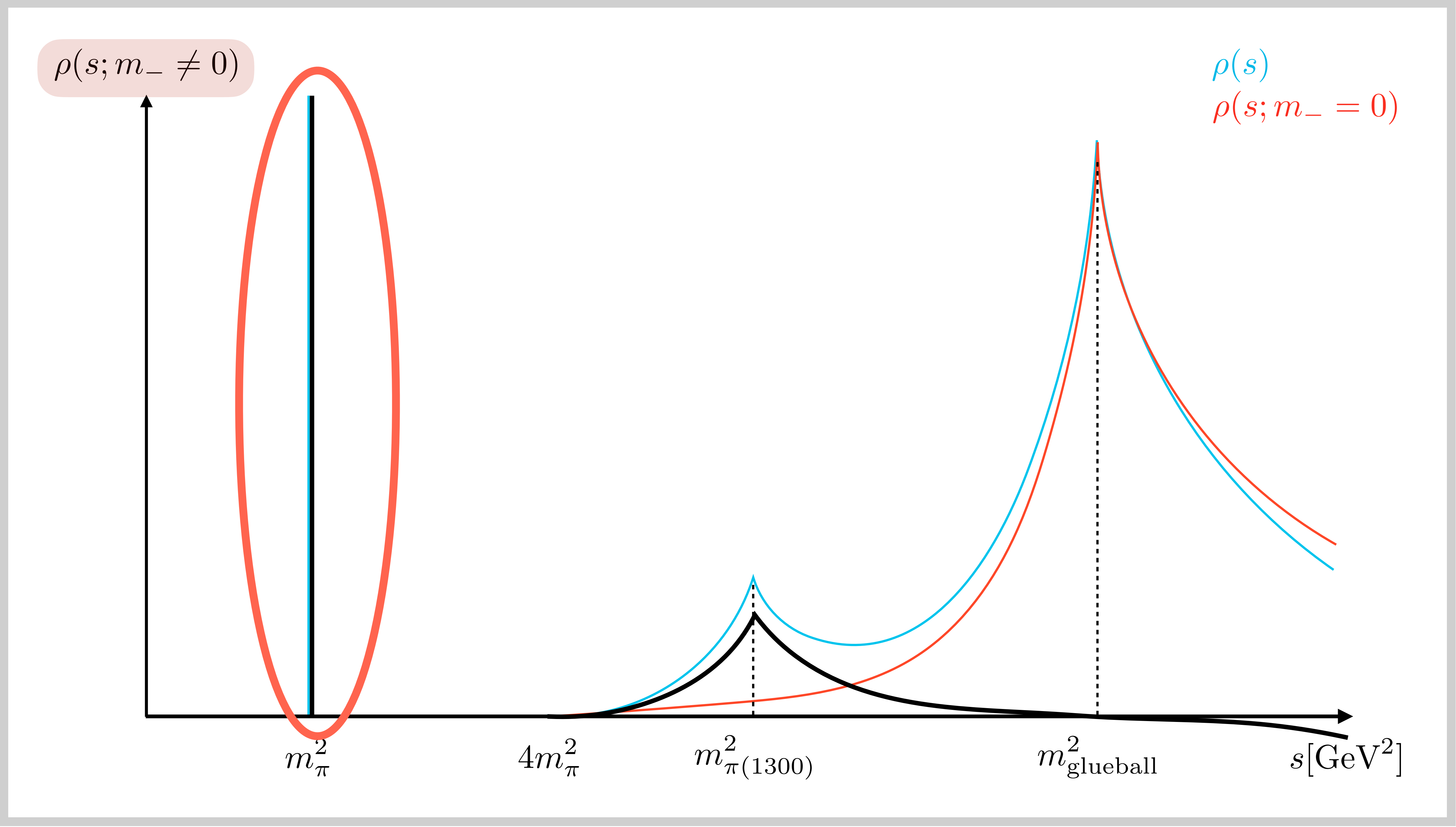
4. Determine remaining parameters: Borel mass and threshold parameter.

- threshold parameter s_0^ρ: model dependence in the way to determine (experiment, by hand, …)
- Borel mass M^2: Borel window



QCD sum rules -Phenomenology-

$$i\int d^4x e^{-iq\cdot x}\langle\Omega|T[\mathcal{L}_W(x)\mathcal{L}_W(0)]|\Omega\rangle = \Pi_{\rm phen}(q^2) = \frac{\lambda_\pi^2}{m_\pi^2-q^2}+\frac{1}{\pi}\int_{s_{\rm th}}^{\infty}ds\frac{{\rm Im}\,\Pi_{\rm OPE}(s)}{s-q^2}$$



$$\rho(s;m_-\neq 0) = \rho(s) - \rho(s;m_-\neq 0)$$

- $\rho(s)$: **the entire spectrum function**

$$\rho(s) = \rho(s;m_-=0) + \rho(s;m_-\neq 0)$$

- $\rho(s;m_-=0)$: **the isospin-symmetric part**

	the pion-pole	the glueball resonance
$\rho(s;m_-\neq 0)$	✔	✘
$\rho(s;m_-=0)$	✘	✔

$$\lambda_\pi = \langle 0|\mathscr{L}_W|\pi^0\rangle \propto m_-$$

Borel window

$$\frac{\frac{\lambda_\pi^2}{M^2} e^{-m_\pi^2/M^2}}{\frac{\lambda_\pi^2}{M^2} e^{-m_\pi^2/M^2} + \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds\, \frac{e^{-s/M^2}}{M^2} \,\mathrm{Im}\,\Pi_{\rm OPE}\left(s; m_- \neq 0\right)} > 0.5$$