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Lorentzian description of vacuum decay

大下　翔誉 (Naritaka OSHITA)

理化学研究所　数理創造プログラム (RIKEN iTHEMS)

Takumi Hayashi, Kohei Kamada, **N.O.**, Jun'ichi Yokoyama, arXiv: 2112.09284

N.O., Yutaro Shoji, Masahide Yamaguchi, arXiv: 2112.10736

RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

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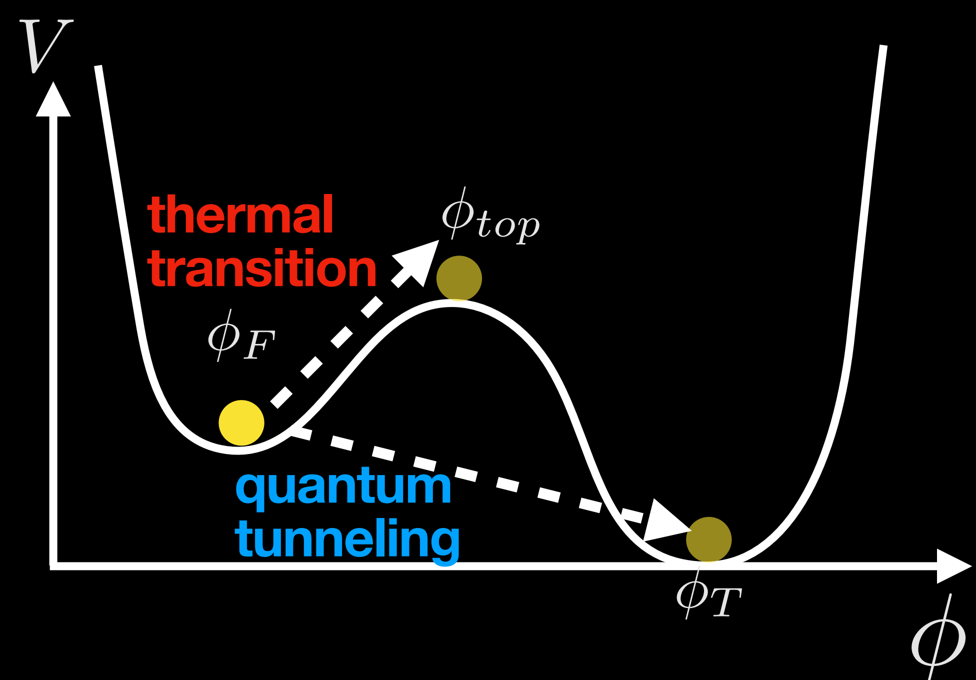
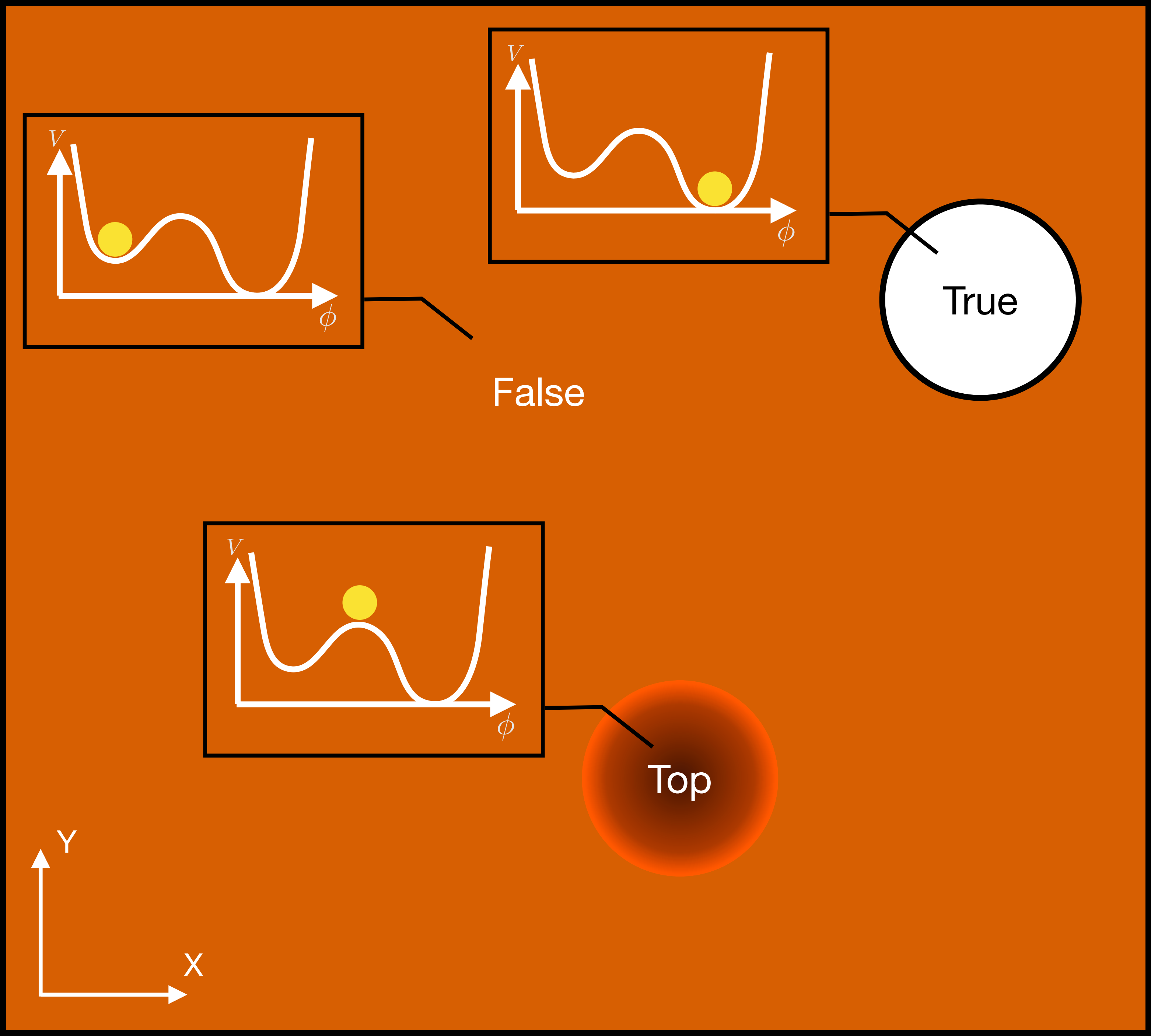
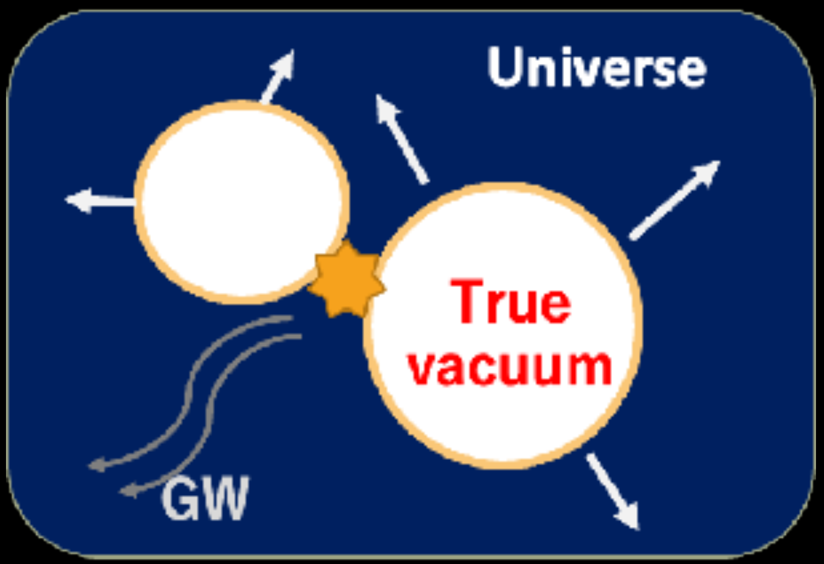
Vacuum phase transition

quantum tunneling or thermal transition of matter fields





Vacuum bubbles



picture: T. Hayashi

- GW emission caused by bubble collisions
- Higgs metastability in (thermal) early Universe
- Vacuum decay seeded by black holes → constraints on PBH parameters or the parameters of Higgs potential

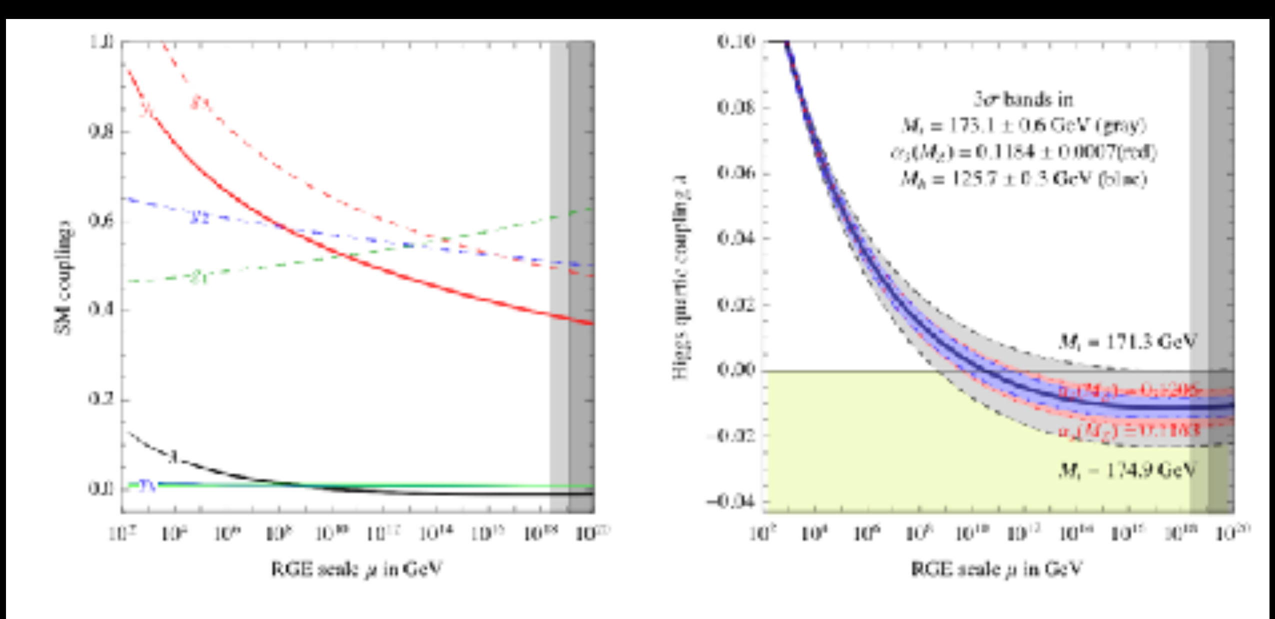
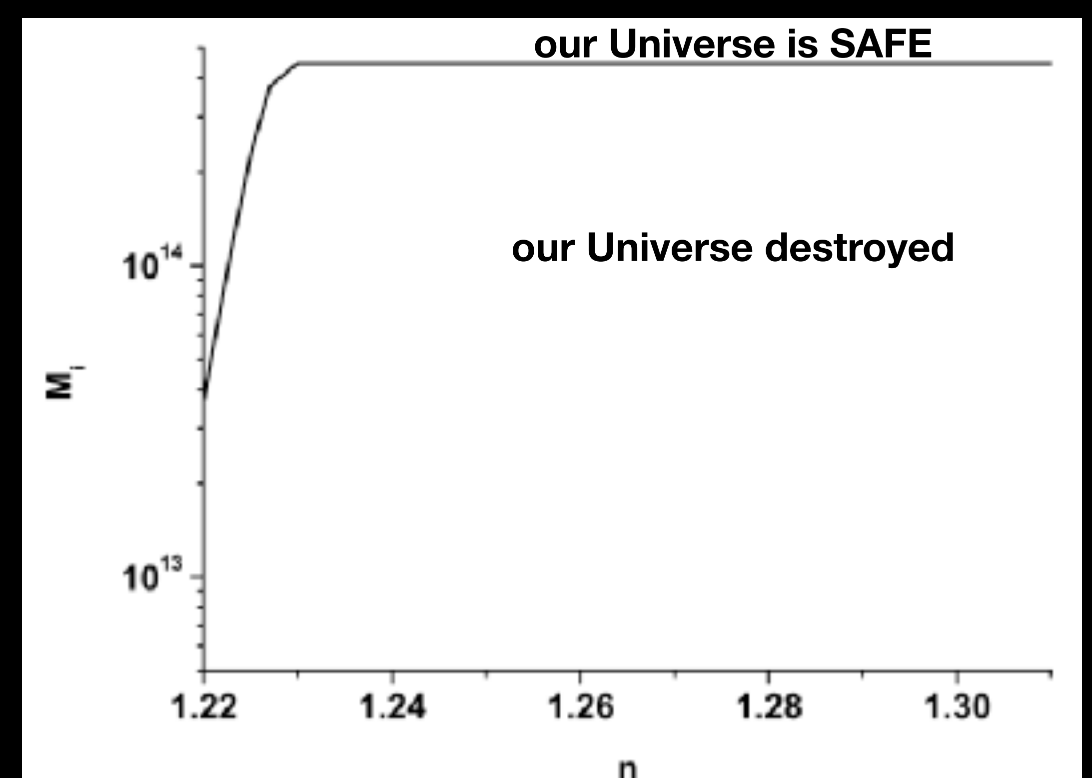


Figure 1: **Left**: *SM RG evolution of the gauge couplings $g_1 = \sqrt{5/3}g'$, $g_2 = g$, $g_3 = g_s$, of the top and bottom Yukawa couplings (y_t, y_b), and of the Higgs quartic coupling λ. All couplings are defined in the $\overline{\text{MS}}$ scheme. The thickness indicates the ±1σ uncertainty.* **Right**: *RG evolution of λ varying M_t, M_h and α_s by ±3σ.*

Degrassi et al. (2013)

(Mass of PBH)



(spectral index) Dai+ (2019)

Standard formulation of vacuum bubbles

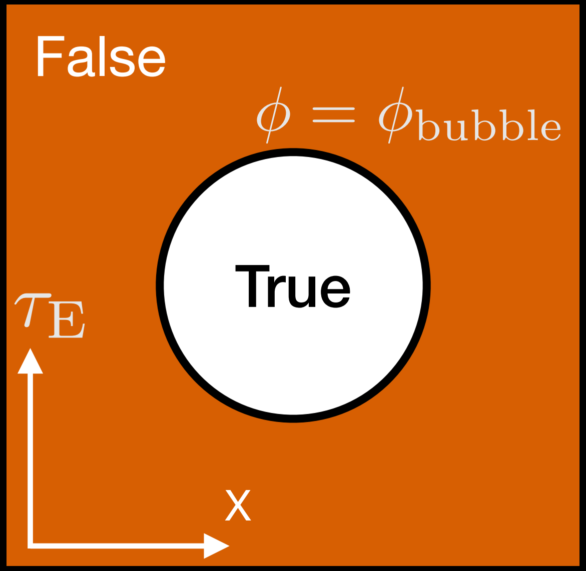
Euclidean path integral

Coleman (1977)
Callan and Coleman (1977)

$$\int \mathcal{D}\phi e^{iS[\phi]} \to \int \mathcal{D}\phi e^{-S_{\rm E}[\phi]} \sim e^{-S_{\rm E}[\phi_F]+iVTAe^{-B}}$$

$t \to -i\tau$
(Wick rotation)

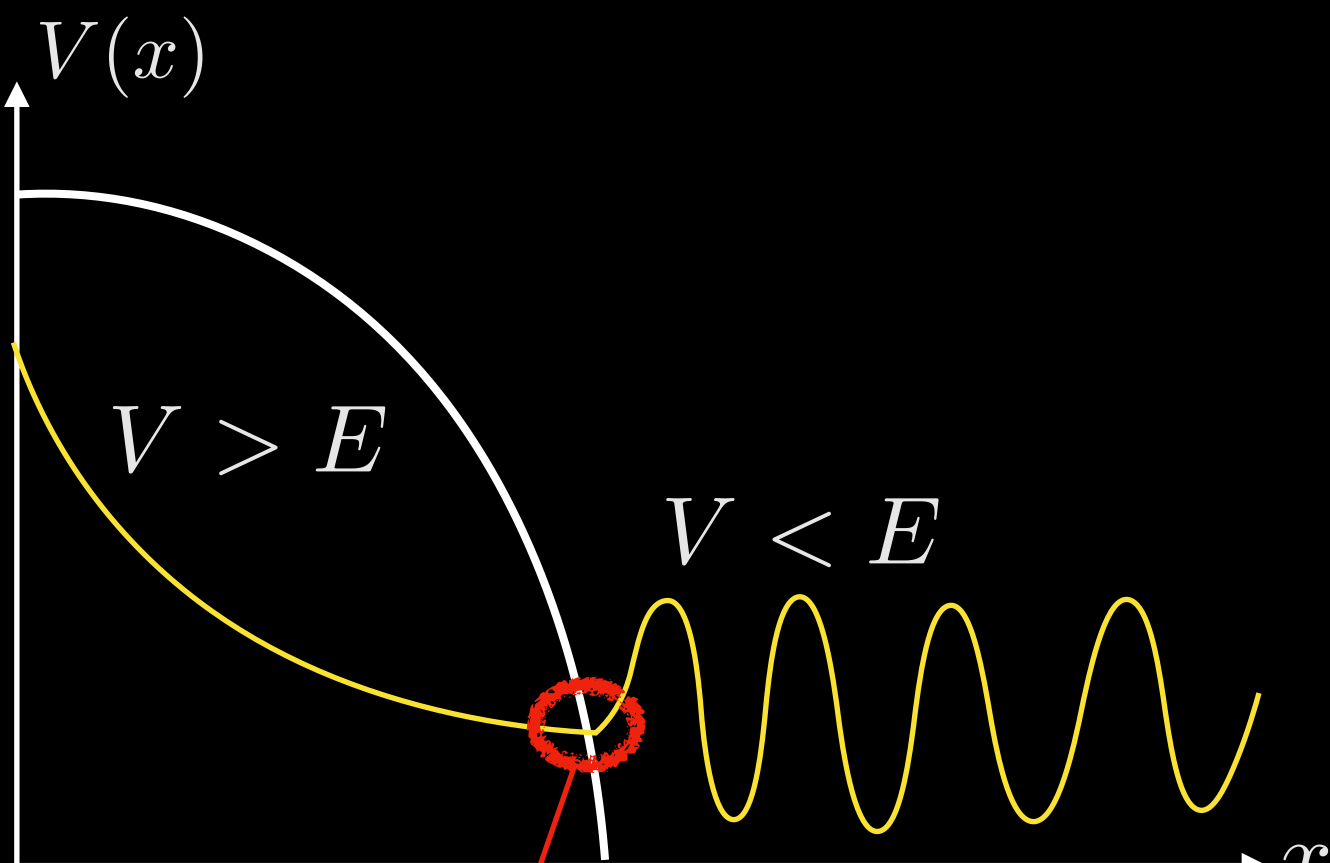
$$B \equiv S_{\rm E}[\phi_{\rm bubble}]$$



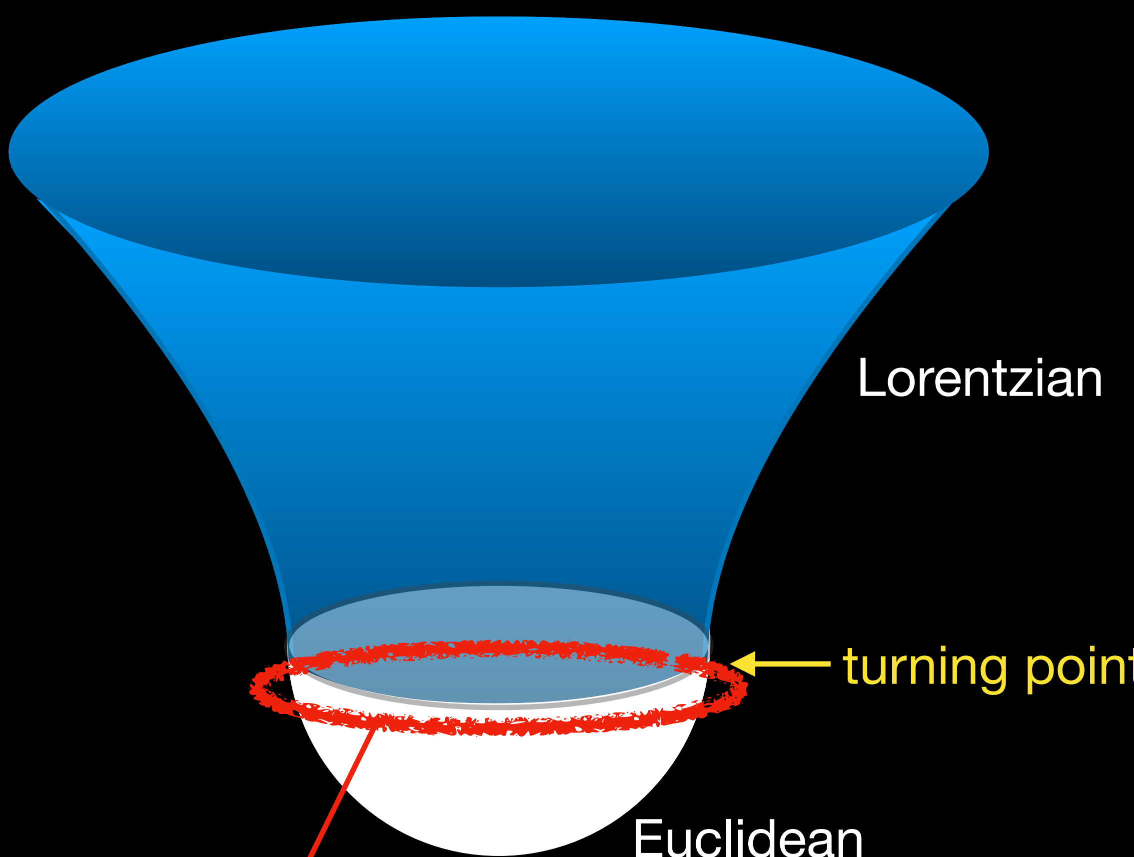
$$\mathrm{Im}E_{\rm fv} = -VAe^{-B}$$

Why do we need the Lorentzian path integral?

1. Is the junction of Euclidean and Lorentzian regions at the turning point (dR/dt = 0) verified?



WKB is not a good approximation at the turning point in general.



Is this okay??

2. negative mode problem

$$\int \mathcal{D}\phi e^{-S_{\rm E}[\phi]} \sim e^{-S_{\rm E}[\phi_F] + iVTAe^{-B}}$$

negative mode

Infinite number of negative modes could appear in the existence of gravity
(sensitive to the gauge choice)

e.g., G. V. Lavrelashvili, et al. (1985)
T. Tanaka and M. Sasaki (1992)
H. Lee and E. J. Weinberg (2014)
R. Jinno and R. Sato (2020)

Imaginary part of the energy eigenvalue of the false vacuum is uncertain.

Thermal transition of a false vacuum is driven by thermal excitation.

Do we need to care the number of negative modes?

(de Sitter)
S. W. Hawking and I. G. Moss (1982)

(Schwarzschild-de Sitter)
R. Gregory, I. G. Moss, **N. O.** (2020)

Lorentzian description of thermal transition (stochastic inflation)

A. A. Starobinsky (1982)
A. Linde (1991)
A. A. Starobinsky and J. Yokoyama (1994)

3. Euclidean path integral in the presence of gravity leads to unbounded action.

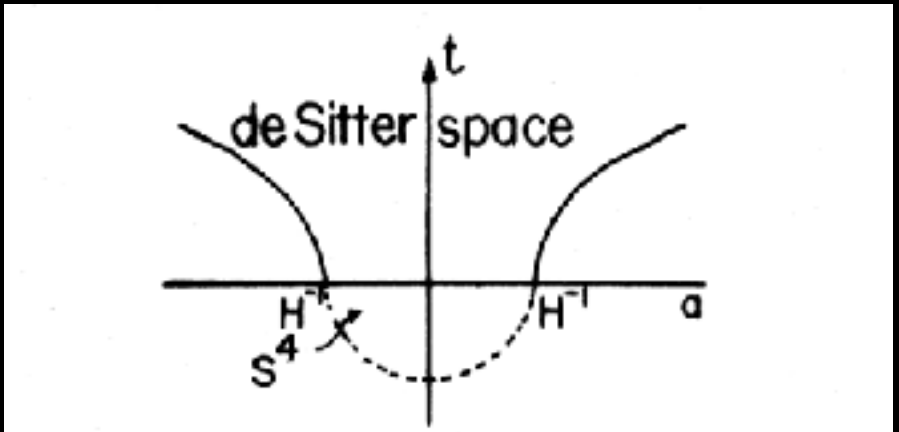


FIG. 3. A schematic representation of the nucleation of the inflationary universe.

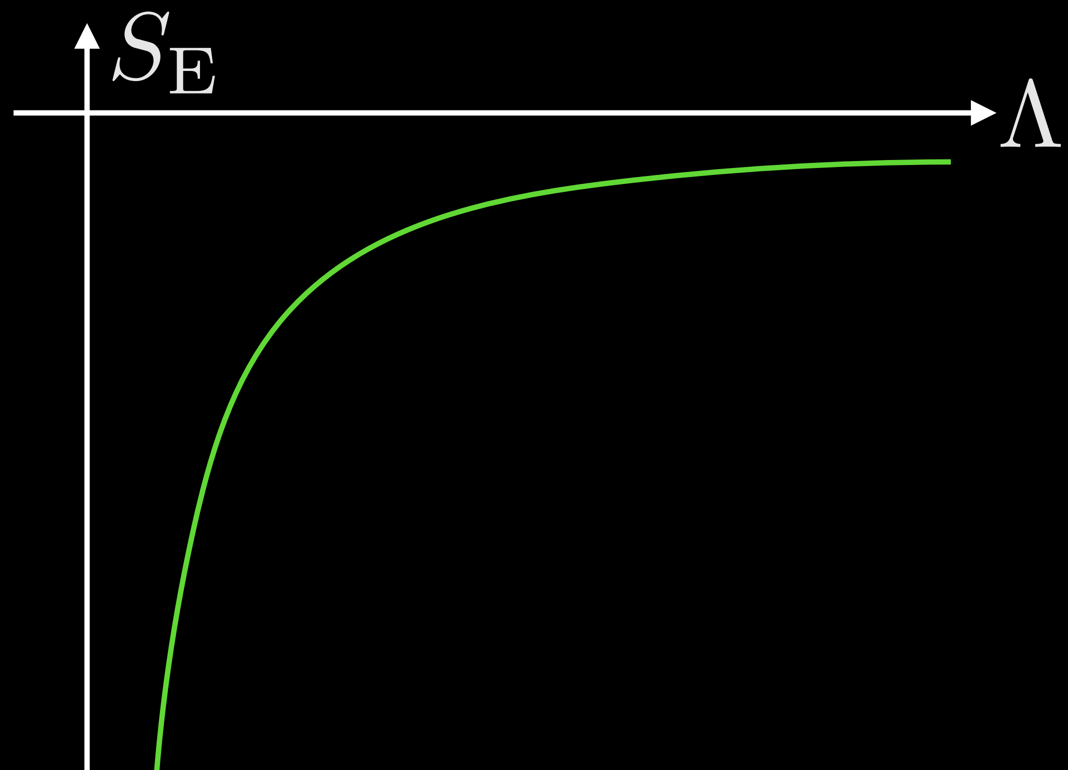
A. Vilenkin (1983)

Wavefunction of the de Sitter universe with a cosmological constant Λ

$$\Gamma = |\Psi|^2 \simeq e^{-S_{\mathrm{E}}} = e^{+\frac{3\pi}{G\Lambda}}$$

J. B. Hartle and S. W. Hawking (1983)

c.f. A. Vilenkin (1983)



4. Is the Wick rotation with gravity $t \to -i\tau$ or $+i\tau$?

$$\mathcal{H}_{\rm tot} = \mathcal{H}_{\rm M} + \mathcal{H}_{\rm G} = 0$$ Hamiltonian constraint

Hamiltonian of matter (positive energy)

Hamiltonian of gravity (negative energy)

action of matter fields is bounded below for

$$t \to -i\tau$$

action of gravity is bounded below for

$$t \to +i\tau$$

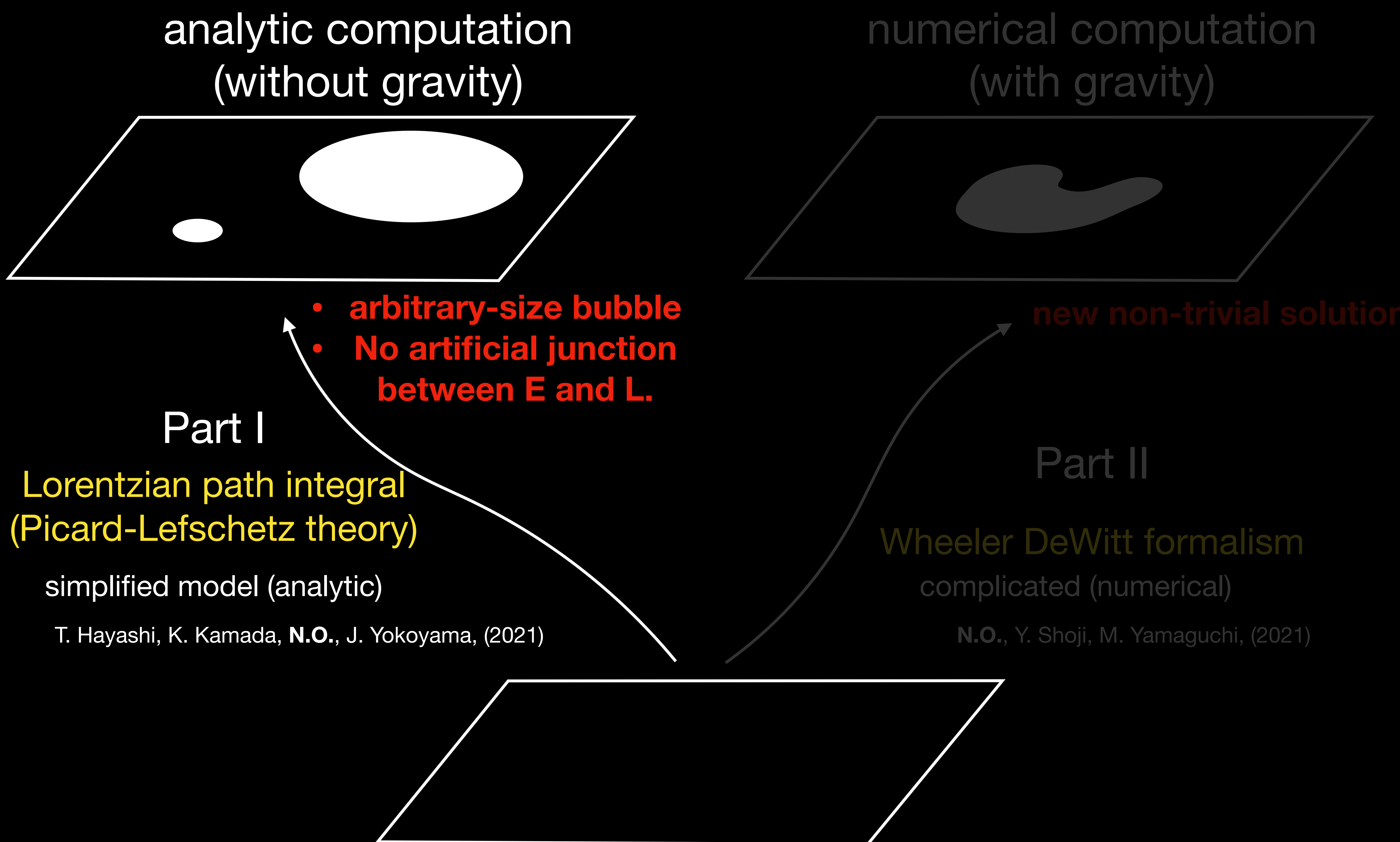
(3) $$S(a) = -\frac{1}{2}\int d\eta \left[\left(\frac{da}{d\eta}\right)^2 - a^2 + \frac{\Lambda}{3}a^4\right]\frac{3\pi M_{\rm P}^2}{2},$$

where η is the conformal time, $\eta = \int dt/a(t)$. Λ is the cosmological constant, $\Lambda = 8\pi V(\varphi)/M_{\rm P}^2$ for a slowly changing field φ. Equation (3) implies that the energy of «excitations» of the scale factor a near $a = 0$ is negative, $E_n \leqslant 0$. This is related to the fact that the total energy of a closed universe is zero, being a sum of the positive energy of matter and the negative energy of the scale factor a. In such a case, as far as the evolution of the field φ can be neglected, to obtain $\Psi_0(a, \varphi)$ by means of eq. (1) one should rotate t not to $-i\tau$, but to $+i\tau$, which leads to [11]

(4) $$\Psi_0(a, \varphi) \sim \exp[S_{\rm E}(a, \varphi)] \sim \exp\left[-\frac{3M_{\rm P}^4}{16V(\varphi)}\right].$$

A. D. Linde (1984)

Do we need to assign different sign to matter's and gravity's time?


analytic computation
(without gravity)
numerical computation
(with gravity)
• arbitrary-size bubble
• No artificial junction
between E and L.
new non-trivial solution
Part I
Lorentzian path integral
(Picard-Lefschetz theory)
simplified model (analytic)
T. Hayashi, K. Kamada, N.O., J. Yokoyama, (2021)
Part II
Wheeler DeWitt formalism
complicated (numerical)
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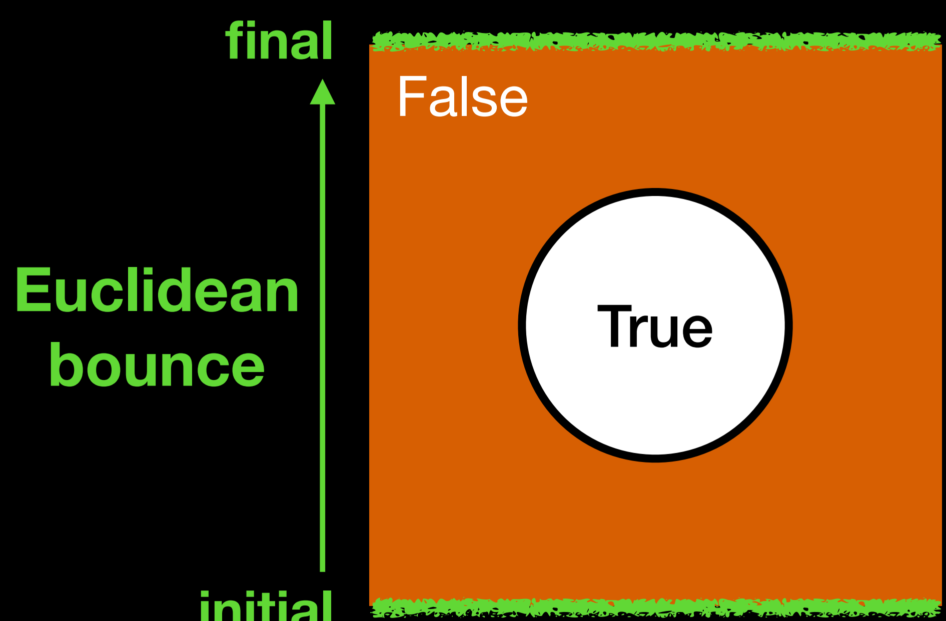
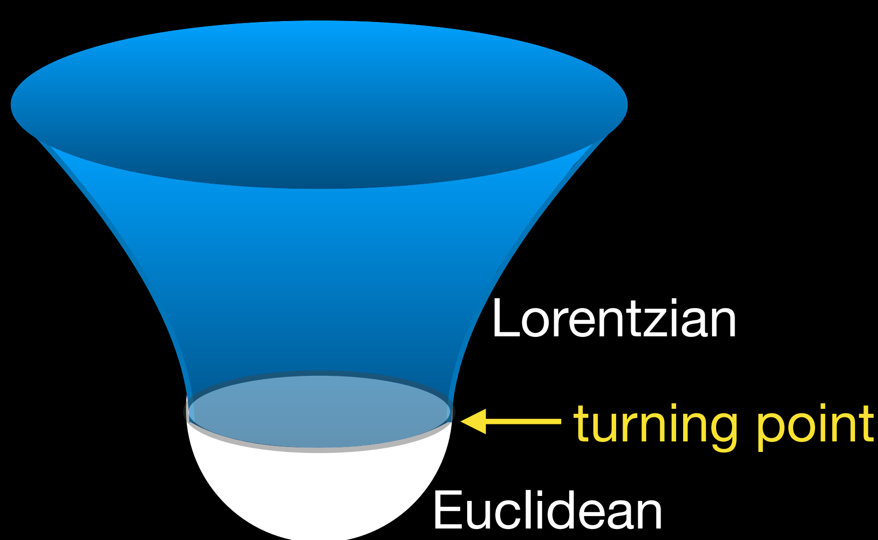
Standard



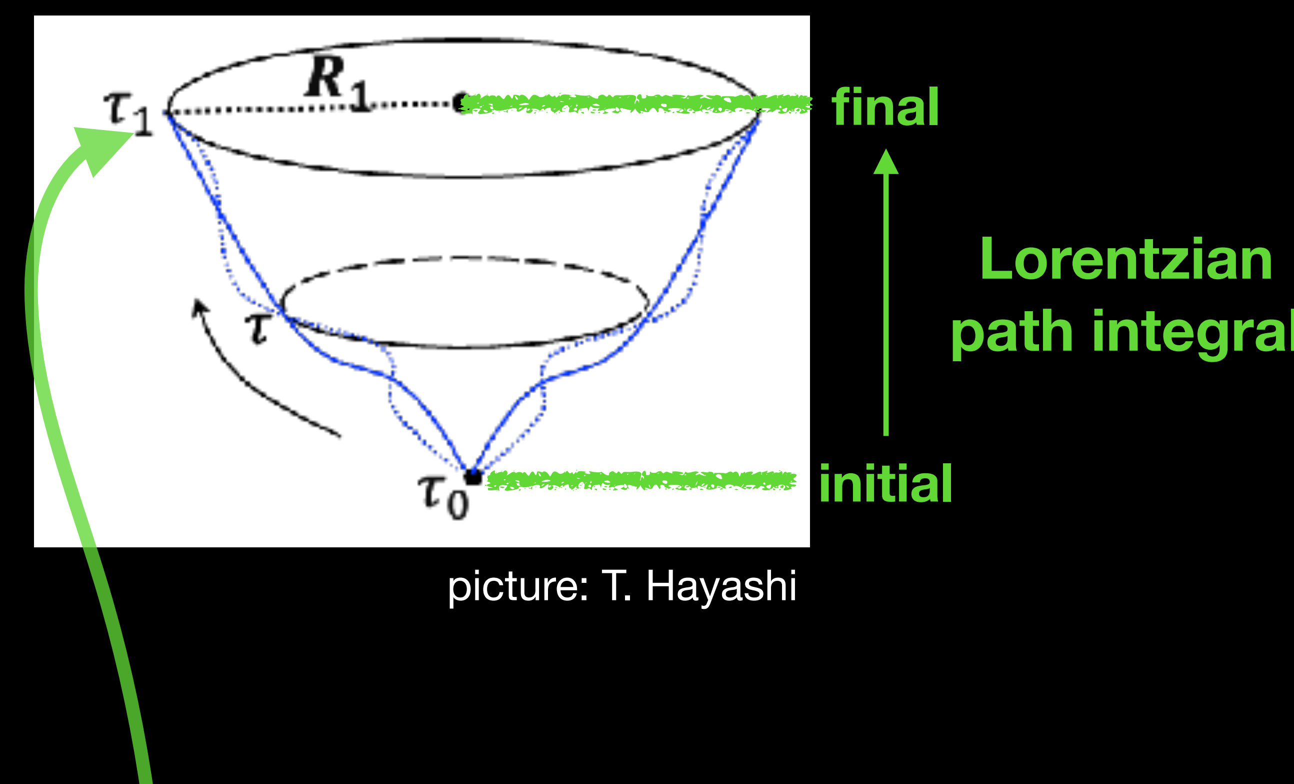
Coleman (1977)

Callan and Coleman (1977)



Ours

T. Hayashi, K. Kamada, **N.O.**, J. Yokoyama, (2021)



picture: T. Hayashi

Set a bubble of arbitrary size
as the final state
(No need to join Euclidean sol. to Lorentzian one.)

No subtlety of the WKB at the turning point!

Setup

1. Assumption (for simplification)

- thin wall

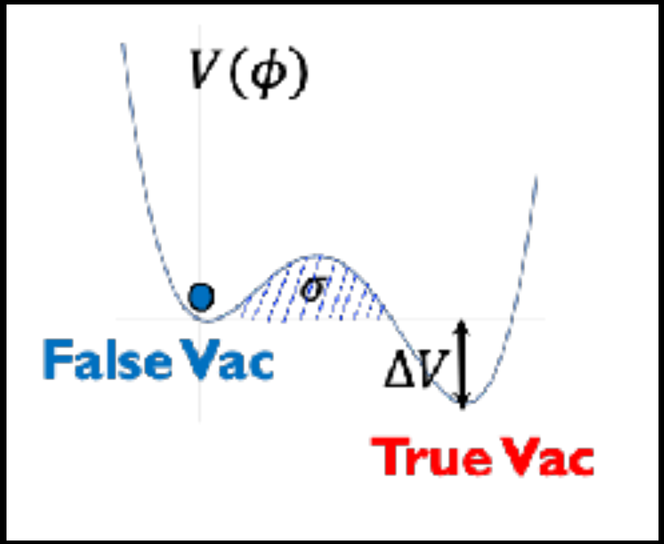
  details of a potential barrier → energy density of bubble surface

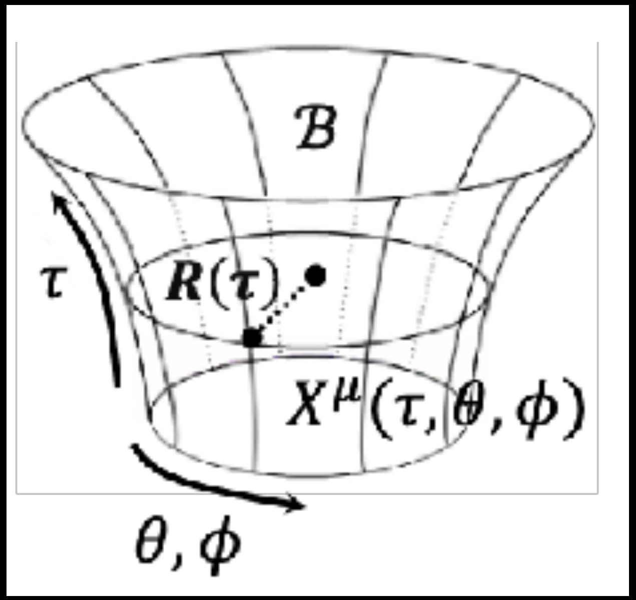
- spherical symmetry

  dynamics of a bubble wall → 1-dim dynamics

- no gravity

  To perform analytic computation





picture: T. Hayashi

Setup

2. Formalism

- action of a thin wall bubble (Polyakov action)

$$S_{\rm P}[X^\mu, \gamma^{ab}] = \underbrace{-\sigma \int_{\partial\mathcal{B}} d^3x \sqrt{-\gamma}\,\frac{1}{2}\left[\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - 1\right]}_{\text{bubble wall}} + \underbrace{\Delta V \int_{\mathcal{B}} d^4X \sqrt{-g}}_{\text{bulk (bubble interior)}}$$

bubble wall

bulk (bubble interior)

- spherical bubble

$$\{X^\mu(x^a)\} \to \{T(\tau), R(\tau), \theta, \varphi\} \qquad \gamma_{ab}dx^a dx^b = -N^2 d\tau^2 + R^2(\tau)d\Omega^2$$

τ :proper time on the wall

gauge fixing : $dN/d\tau = 0$

$$S_{\rm P}[T, R, N] = 4\pi\sigma \int \frac{d\tau}{2\pi}\left\{\frac{1}{2}R^2\left[N^{-1}\left(-f\dot{T}^2 + f^{-1}\dot{R}^2\right) - N\right] + \rho_0^{-1}R^3\dot{T}\right\}$$

$$\rho_0 \equiv \frac{3\sigma}{\Delta V}$$

$$G(R=0, R=R_1) = \int_0^\infty dN \int_{R(\tau=0)=0}^{R(\tau=1)=R_1} \mathcal{D}T\mathcal{D}R \exp(iS_{\mathrm{P}}[T,R,N])$$

take into account every proper duration

quantum-mechanical amplitude from a zero-size bubble to finite one

$$\int_0^\infty dN \, A(N) \exp\left(iS_{\mathrm{P}}[\bar{T},\bar{R},N]\right)$$

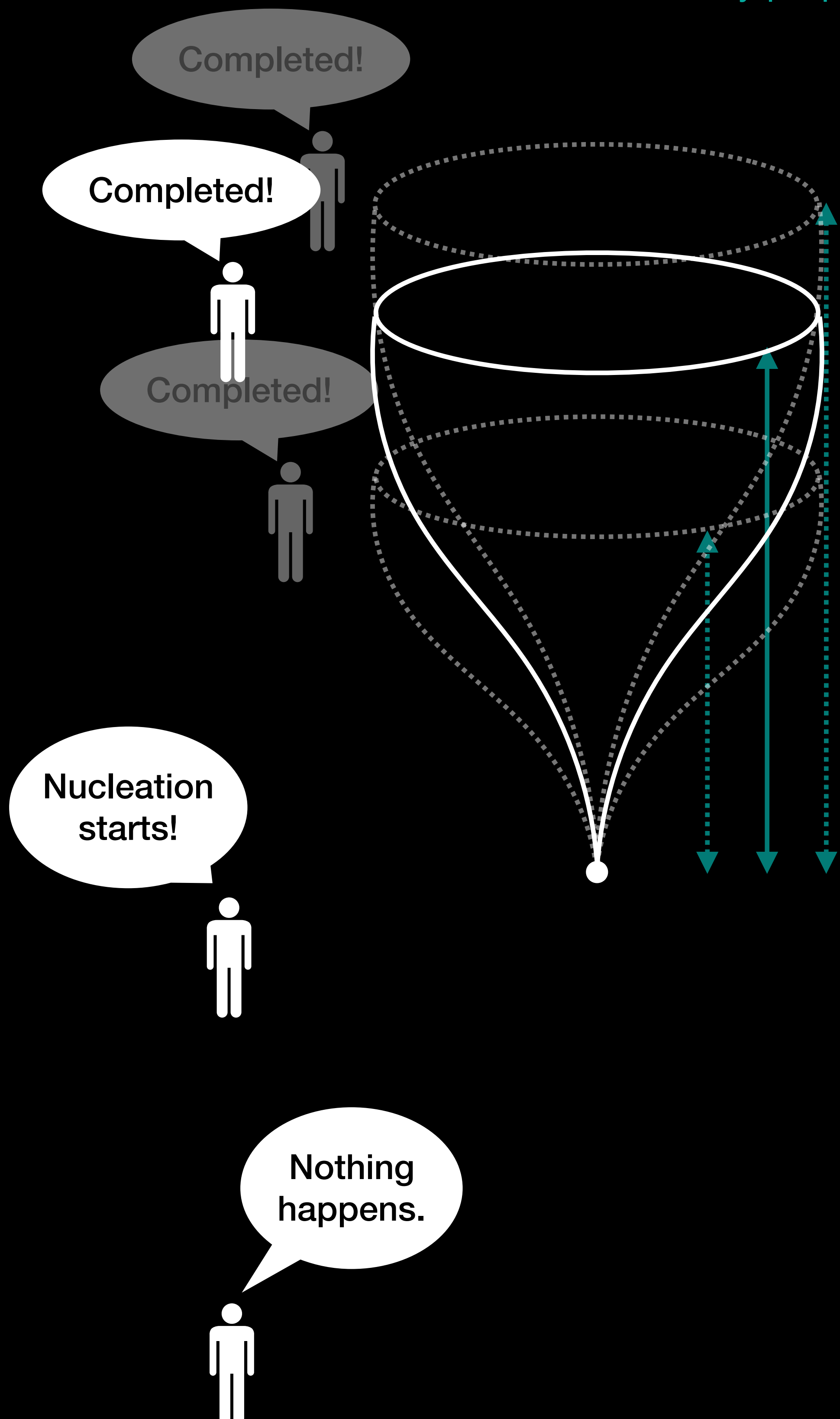
quantum corrections +zero modes

leading contribution (exponential suppression)

oscillatory integrand

$T = \bar{T}$ and $R = \bar{R}$

sol.of the E.O.M.



lapse N controls the proper duration

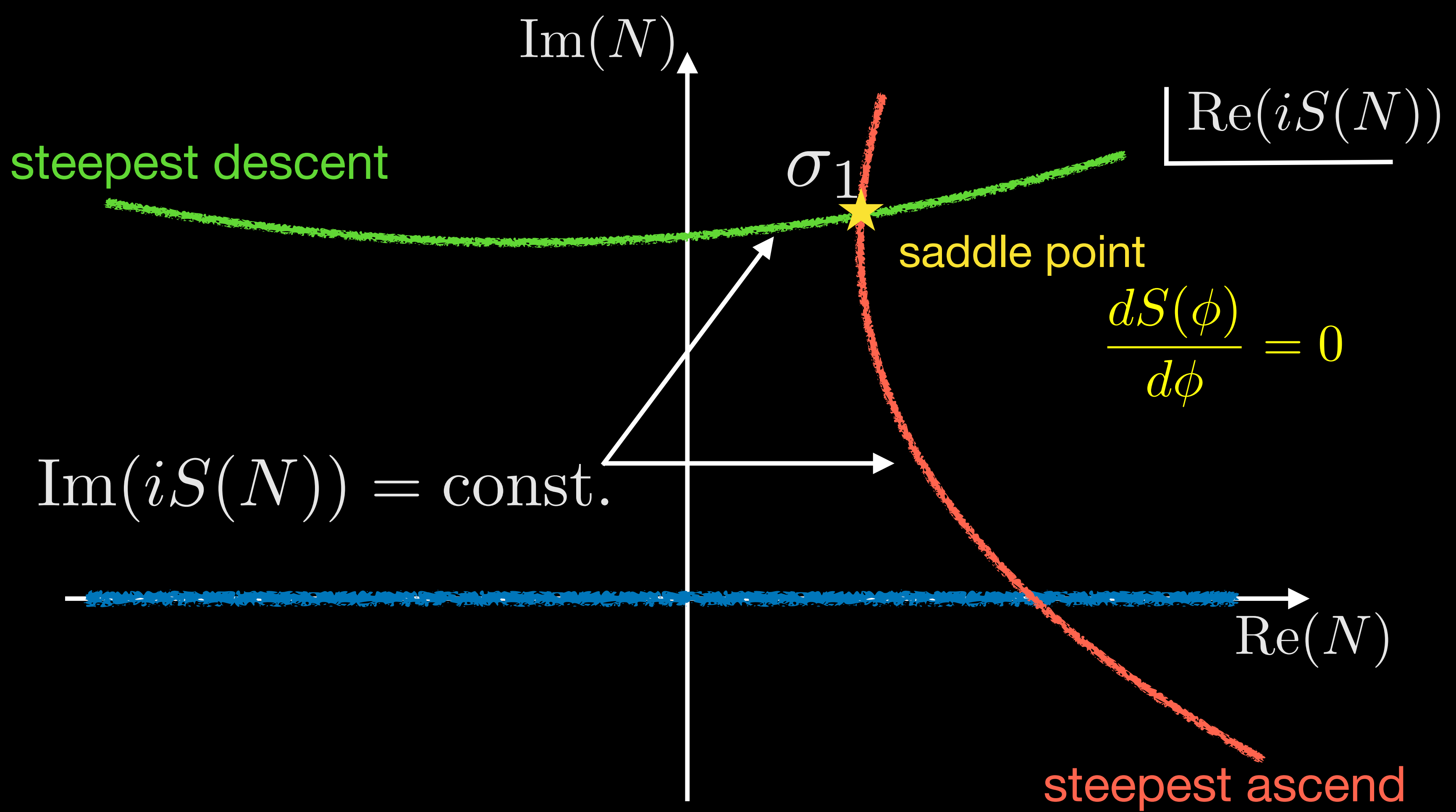
$$\Delta t = N \Delta \tau$$

$$\Delta \tau = 1$$

Picard-Lefschetz theory

$$\int_{\mathbb{R}} dN \exp\left(iS(N)\right) \longrightarrow \int_{\mathcal{C}} dN \exp\left(iS(N)\right)$$

absolutely convergent!!
(No oscillatory integrand!!)



$$\int_0^\infty dN\, A(N) \exp\left(iS_{\mathrm{P}}[\bar{T}, \bar{R}, N]\right)$$

oscillatory integrand

↓

$$\int_{\mathcal{C}} dN\, A(N) \exp\left(iS_{\mathrm{P}}[\bar{T}, \bar{R}, N]\right)$$

Lefschetz thimbles

~ Gaussian!!

$$\exp\left(iS_{\rm P}[\bar{T}, \bar{R}, N]\right)$$

Integrated E.O.M.

$$-2\pi\sigma R^2\left(N^{-2}(f\dot{T}^2 - f^{-1}\dot{R}^2) - 1\right) = H$$

Integration constant

fix H so that B.C. is satisfied

$$4\pi\sigma R^2 f(R)\left(\frac{\dot{T}}{N} - \frac{R}{\rho_0 f(R)}\right) = E$$

bubble's total energy

set to zero

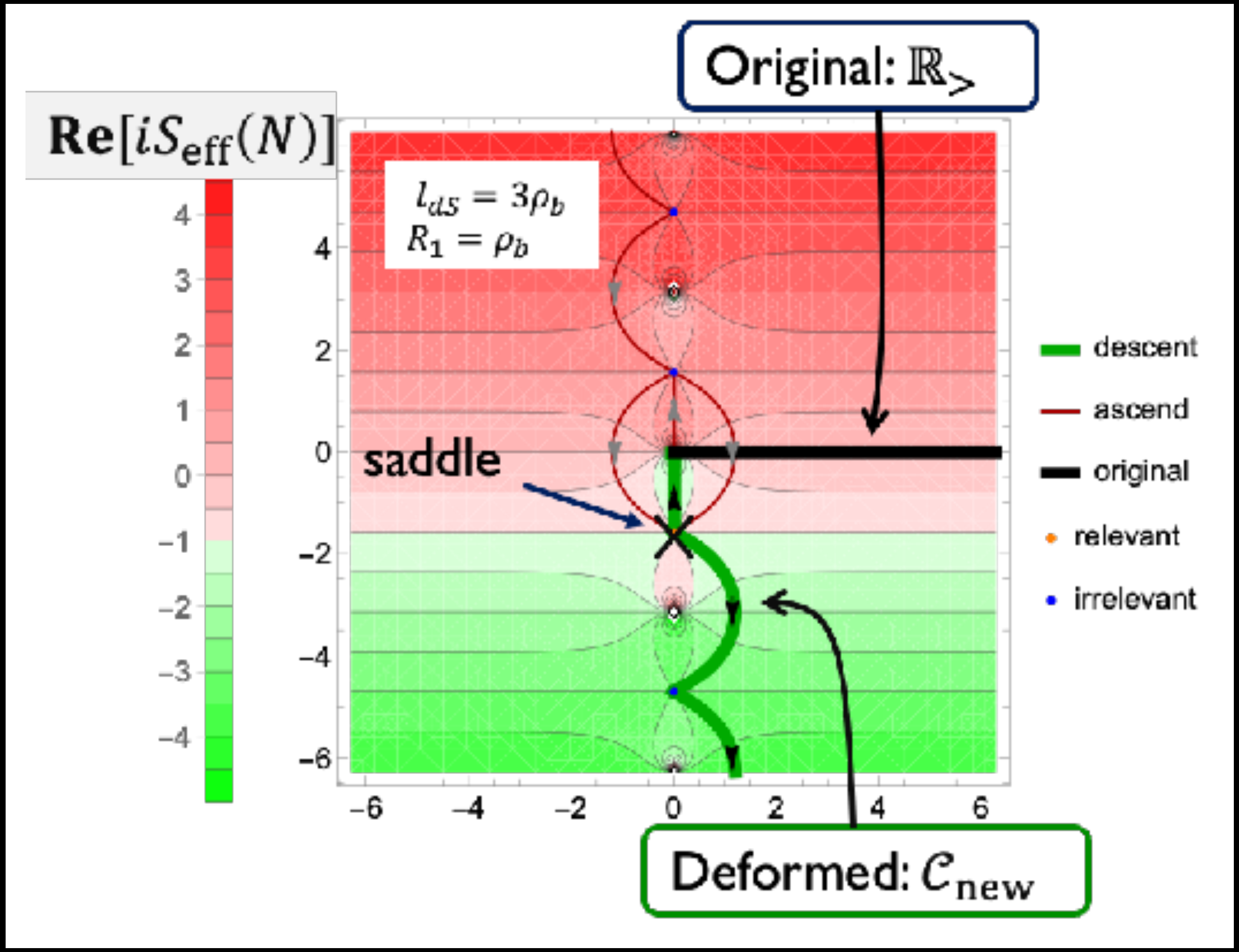
boundary condition on $R = \bar{R}$ (and $T = \bar{T}$)

Coleman's bubble	$R(\tau = 0) = 0$ and $R(1) = \rho_b$
large bubble	$R(0) = 0$ and $R(1) > \rho_b$
small bubble	$R(0) = 0$ and $R(1) < \rho_b$

boundary condition changes the structure of $S_{\rm P}(\bar{T}, \bar{R}, N)$

e.g. position of the saddle points in N-space

Coleman's bubble



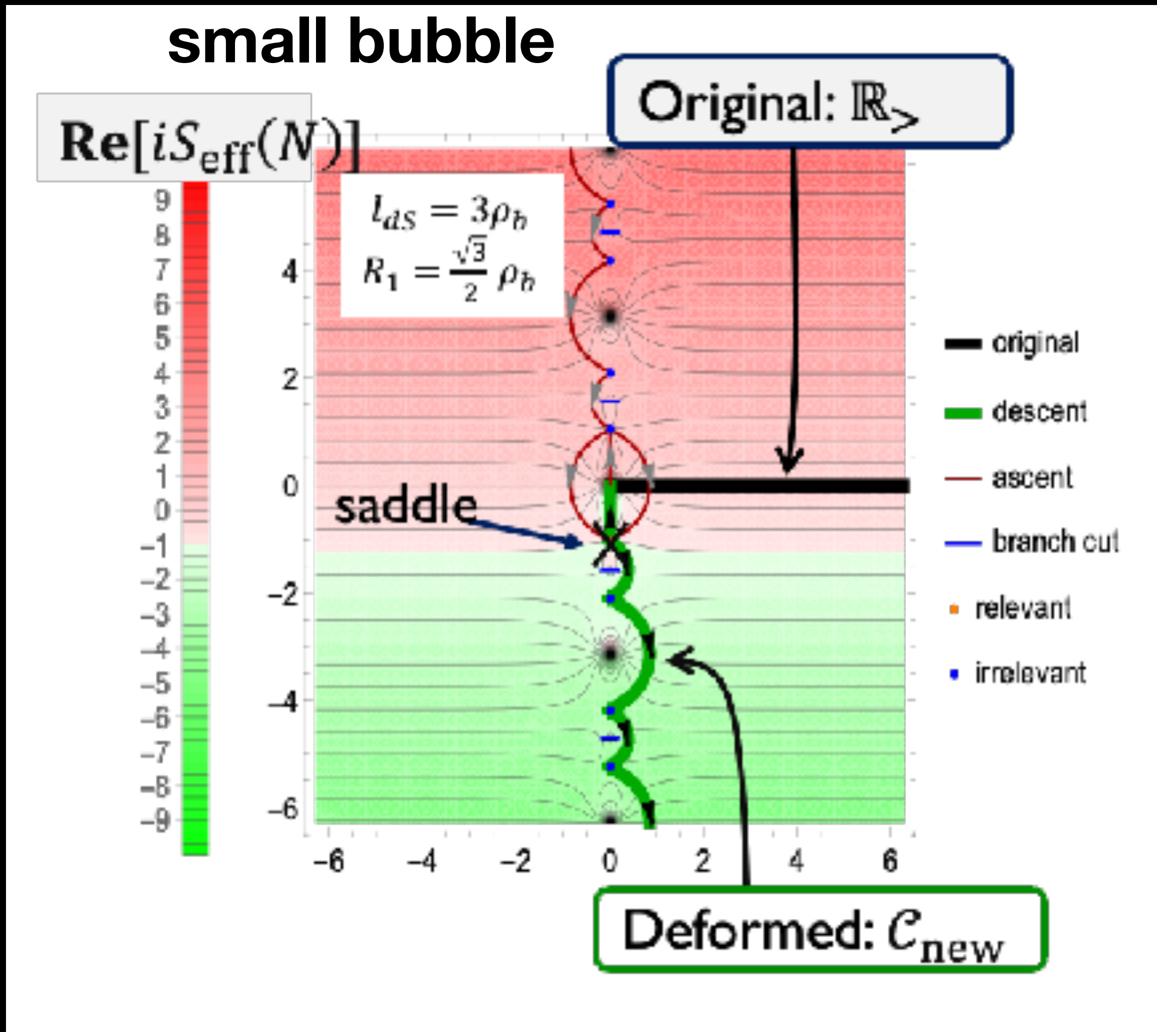
$$S_{\rm P}[\bar{T}, \bar{R}, N] = \frac{2\pi\sigma\rho_b^3}{(1+\rho_b/\rho_0)^2}\left[\coth\frac{N}{\rho_b} - \frac{N}{\rho_b}\right]$$

$$G(R=0, R=\rho_b) \sim \int_0^\infty dN \exp\left(iS_{\rm P}(N)\right) = \int_{\mathcal{C}_{\rm new}} dN \exp\left(iS_{\rm P}(N)\right) \sim \exp\left(-\frac{\pi^2\sigma\rho_b^3}{(1+\rho_b/\rho_0)^2}\right)$$

$$P_{\rm Decay} = |G(R=0, R=\rho_b)|^2 \sim \exp\left(-B_{\rm Coleman}\right)$$

$\frac{B_{\rm Coleman}}{2}$

consistent with the standard result!



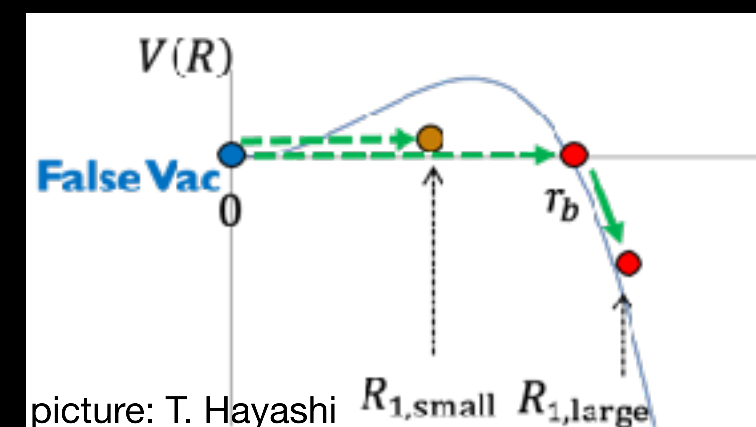
$$G(R=0, R=R_{\rm bubble})$$

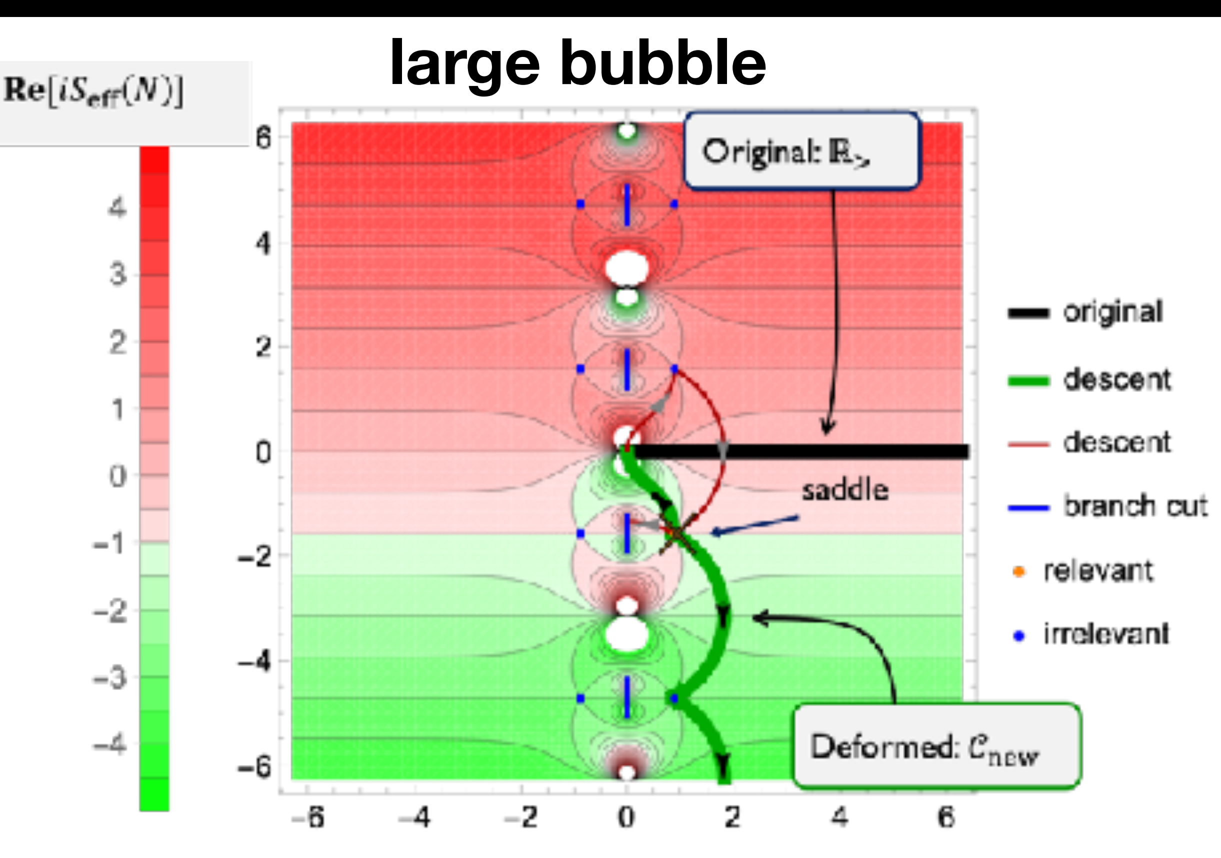
$$R_{\rm bubble} < \rho_b$$

$$\sim \int_0^\infty dN \exp\left(iS_{\rm P}(N)\right) \sim \exp\left(-B_{\rm small}/2\right)$$

$$B_{\rm small} < B_{\rm Coleman}$$

A small bubble may have a chance to expand due to the external source
e.g. thermal radiation inside the bubble, external gravitational force etc..





$$G(R=0, R=R_{\rm bubble})$$

$$R_{\rm bubble} > \rho_b$$

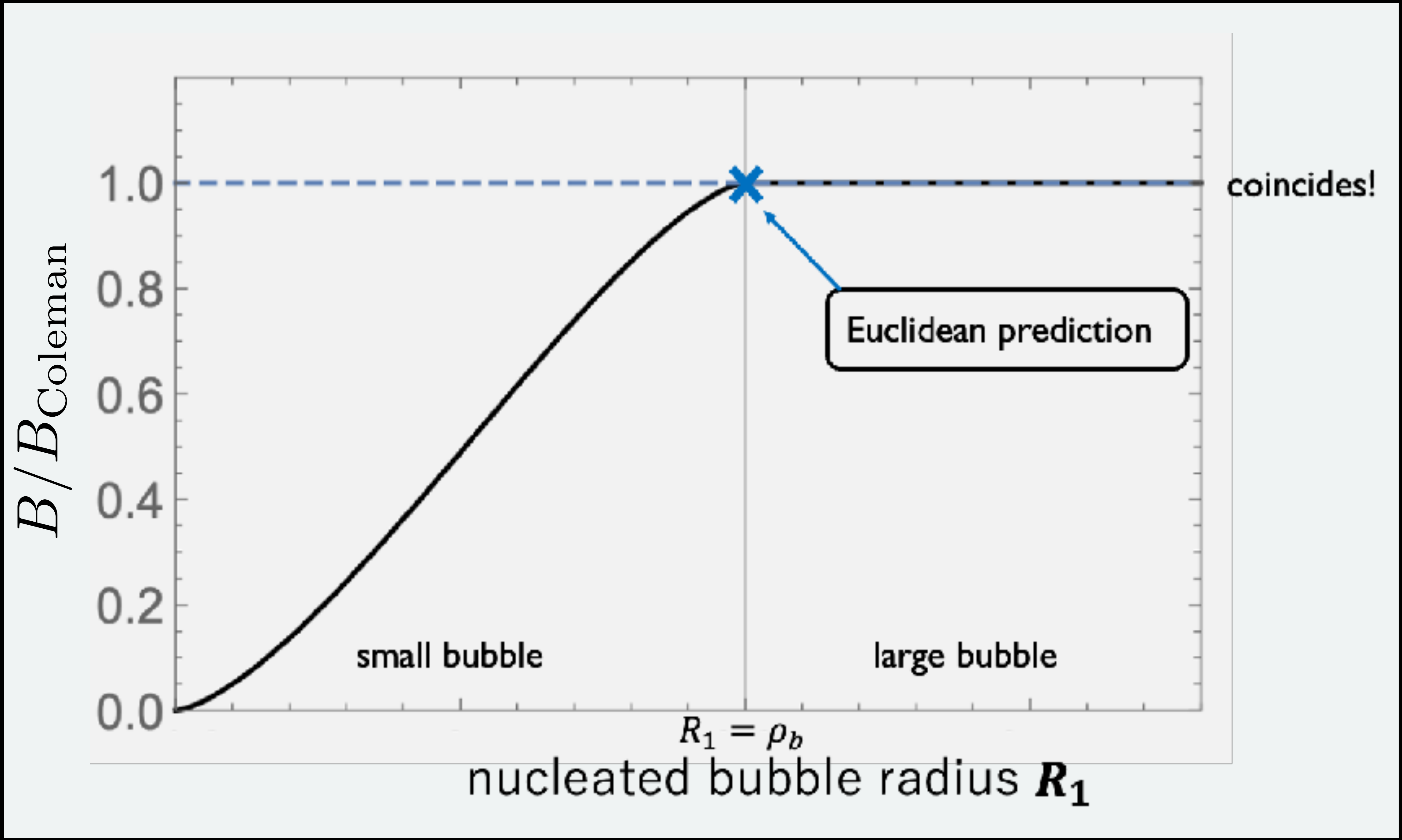
$$\sim \int_0^\infty dN \exp\left(iS_{\rm P}(N)\right) \sim \exp\left(-B_{\rm large}/2 + i\theta(\rho_b, R_{\rm bubble})\right)$$

$$B_{\rm large} = B_{\rm Coleman}$$

phase rotation caused by the classical bubble expansion

$$R = \rho_b \to R = R_{\rm bubble}$$

$$\Gamma \sim A e^{-B}$$



Summary for PART I

We formulated the nucleation of a vacuum bubble in the Lorentzian path integral.

We demonstrated the nucleation of a bubble of arbitrary size and found:

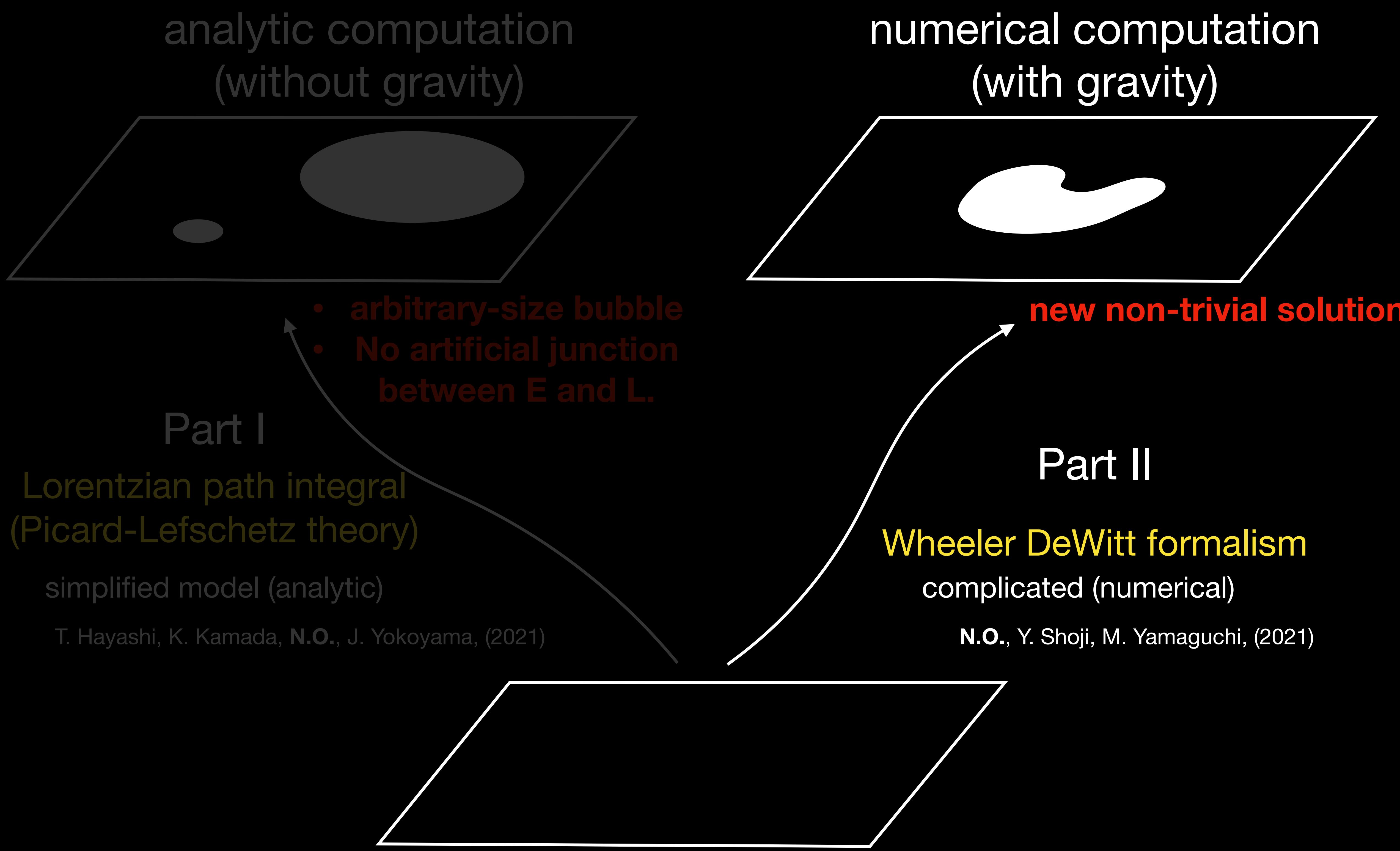
- exponent of the nucleation rate of a bubble of *standard size*
  agrees with the Coleman's result.

- large bubble nucleation can be interpreted as
  nucleation of standard-size bubble + classical expansion

- small bubble nucleates with higher probability

Our strategy is useful to search for other vacuum decay processes that are more probable than the known process.
An arbitrary vacuum bubble can be set as the final state unlike the bounce calculation.

Still, there are some restrictions: no gravity and thin wall approximation.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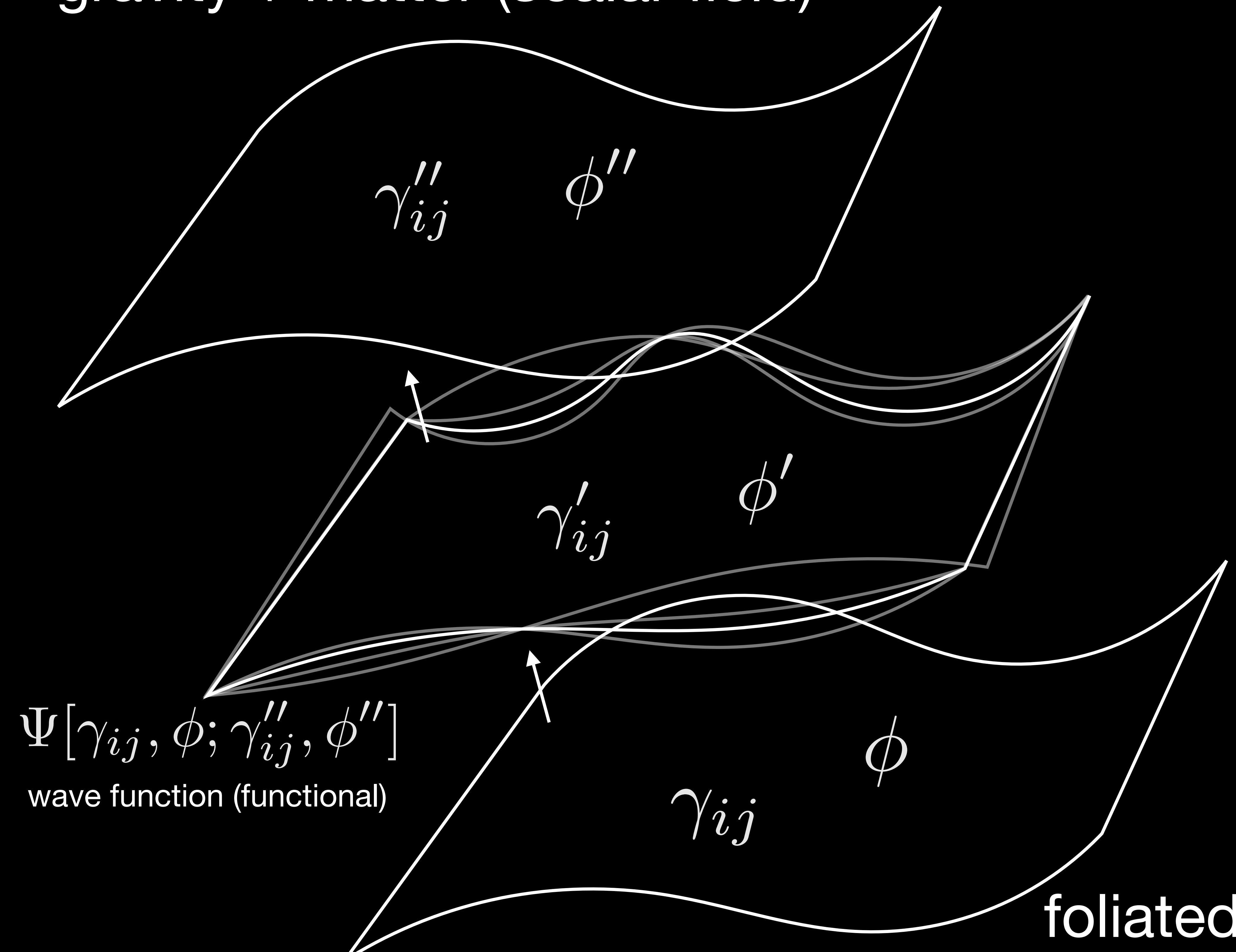
Wheeler-DeWitt formalism

gravity + matter (scalar field)



foliated spacetime

3+1decomposition

"NO TIME" in the WdW formalism

Physical states should satisfy

$$\mathcal{H}\Psi[\gamma_{ij},\phi]=0$$

$$\mathcal{H}^i\Psi[\gamma_{ij},\phi]=0$$

$$\pi\Psi[\gamma_{ij},\phi]=0$$

$$\pi^i\Psi[\gamma_{ij},\phi]=0$$

time development

$$\gamma_{ij}(x^0,\mathbf{x})=e^{iHx^0}\gamma_{ij}(0,\mathbf{x})e^{-iHx^0}$$

$$\Psi^\dagger\gamma_{ij}(x^0,\mathbf{x})\Psi=\Psi^\dagger\gamma_{ij}(0,\mathbf{x})\Psi$$

NO dynamics (NO time development) ??

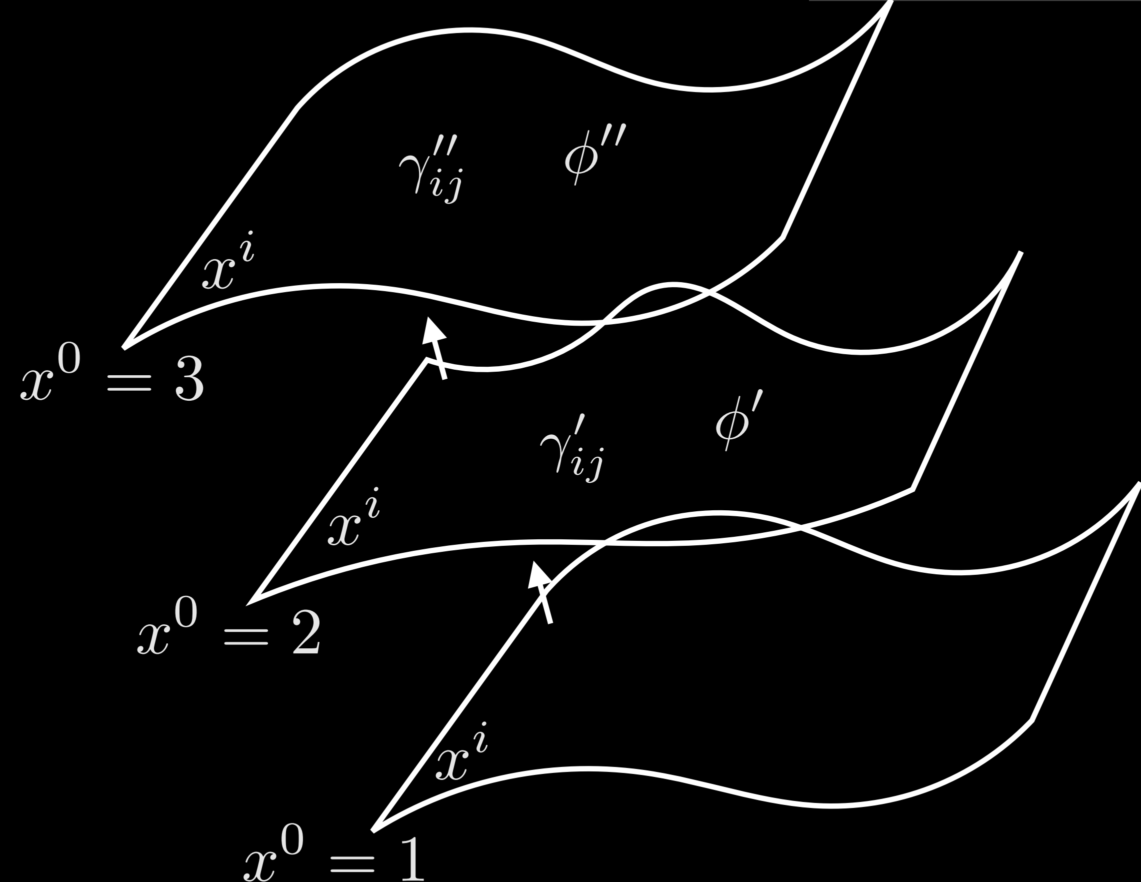
How can we appreciate the “dynamics” of the contents of the Universe?

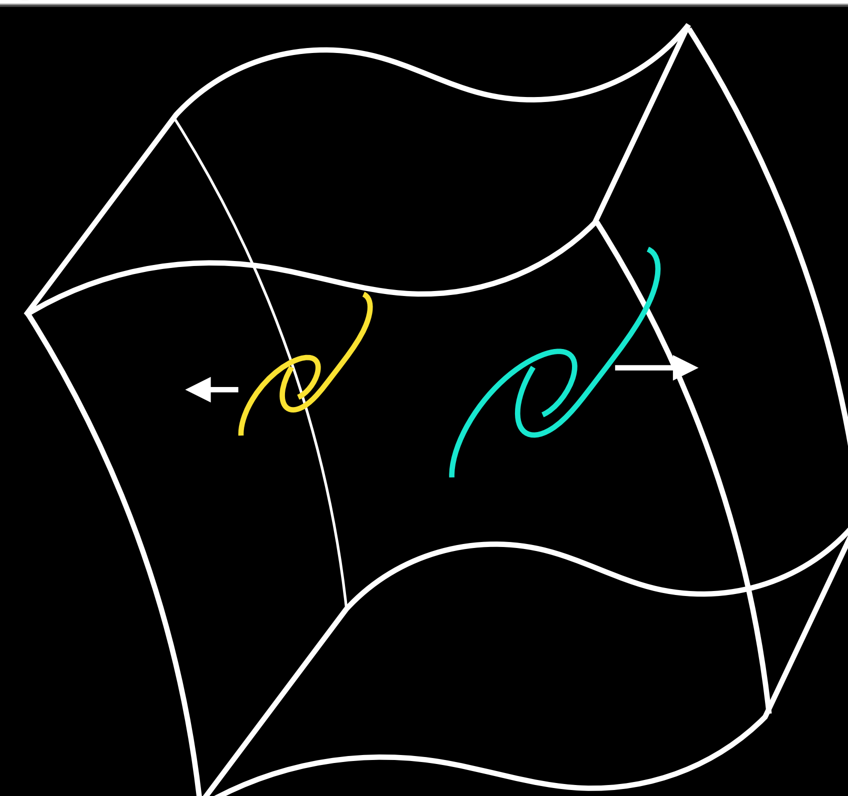
B. S. DeWitt (1967)

$$H\Psi=0, \quad \Psi^\dagger H=0,$$

$$\Psi^\dagger\gamma_{ij}(x^0,\mathbf{x})\Psi=\Psi^\dagger\gamma_{ij}(0,\mathbf{x})\Psi\,.$$

posed by Eq. (4.7). The following procedure will be adopted: Instead of regarding this equation as implying that the universe is static we shall interpret it as informing us that the coordinate labels x^μ are really irrelevant. Physical significance can be ascribed only to the intrinsic dynamics of the world, and for the description of this we need some kind of intrinsic coordinatization based either on the geometry or the contents of the universe.





external parametrization is ***irrelevant***

internal parametrization based on geometry(e.g. cosmic expansion) or matter in the universe is relevant

x^0 time coordinate → S new parameter

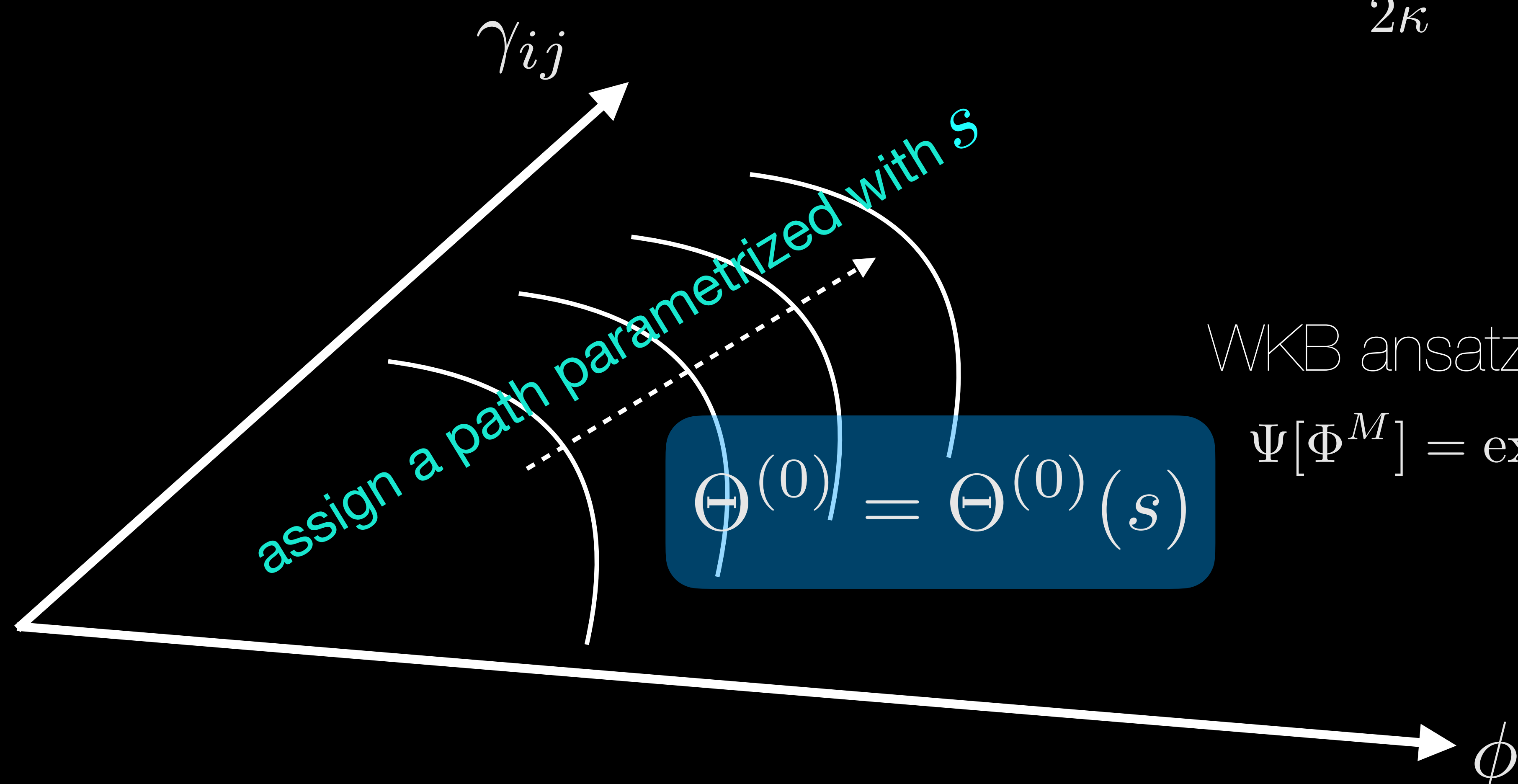
Wheeler DeWitt equation -functional wave equation-

$$\left[-\frac{1}{2\sqrt{\gamma}}\frac{\delta}{\delta\Phi^M(\mathbf{x})}G^{MN}\frac{\delta}{\delta\Phi^N(\mathbf{x})}+\sqrt{\gamma}\mathcal{V}\right]\Psi[\Phi^M]=0$$

$$\Phi^M=(\gamma_{ij},\phi) \qquad G^{\phi\phi}=1,\, G^{(ij)(kl)}=4\kappa\tilde{G}_{ijkl}$$

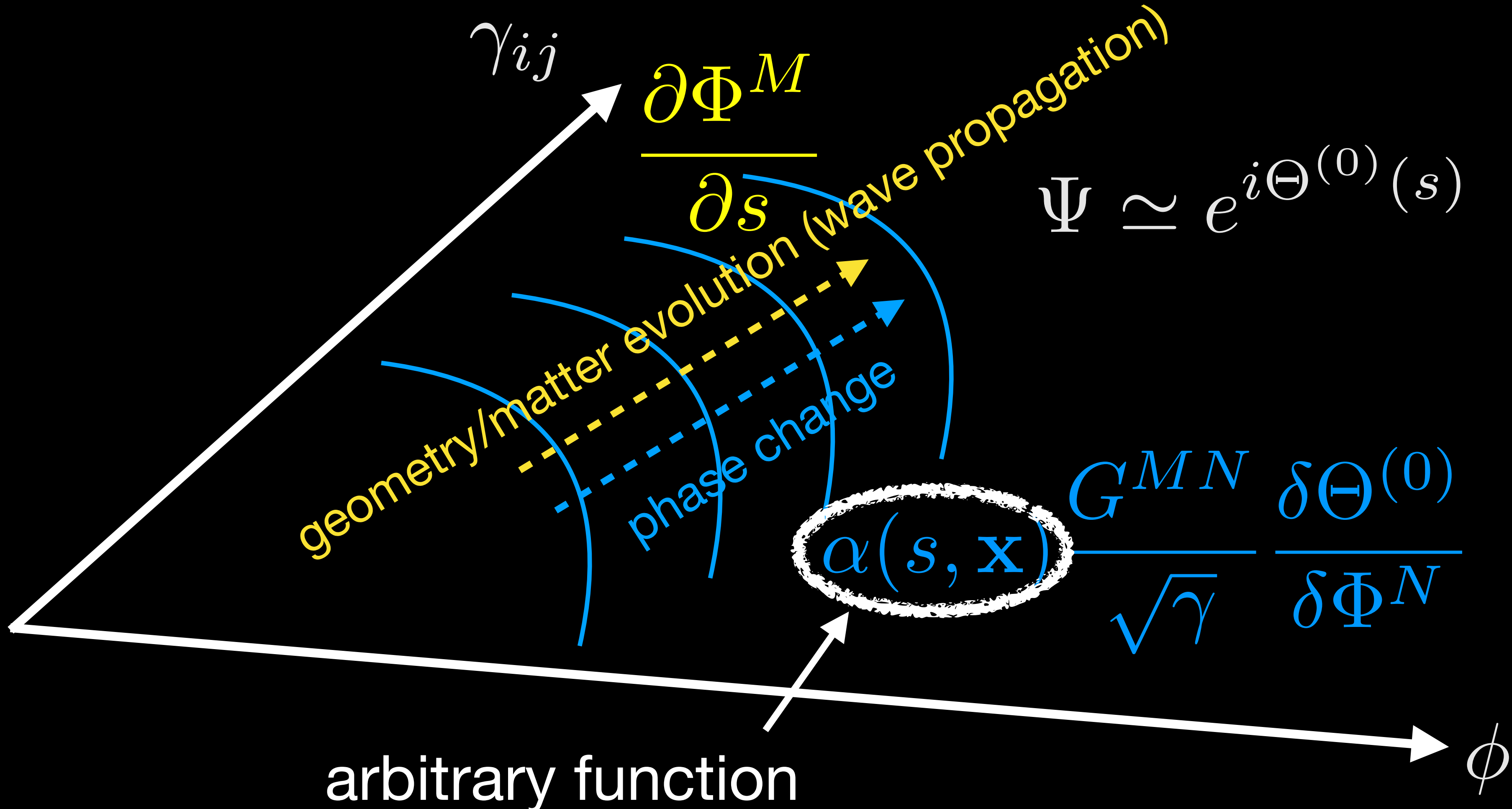
$$\tilde{G}_{ijkl}\equiv\frac{1}{2}\left(\gamma_{ik}\gamma_{jl}+\gamma_{il}\gamma_{jk}-\gamma_{ij}\gamma_{kl}\right)$$

$$\mathcal{V}\equiv-\frac{1}{2\kappa}{}^{(3)}\mathcal{R}+\frac{1}{2}h^{ij}(\partial_i\phi)(\partial_j\phi)+V(\phi)$$



WKB ansatz

$$\Psi[\Phi^M]=\exp\left[i\Theta^{(0)}[\Phi^M]+\Theta^{(1)}[\Phi]+\dots\right]$$



$$\frac{\partial \Phi^M}{\partial s} = \alpha(s, \mathbf{x}) \frac{G^{MN}}{\sqrt{\gamma}} \frac{\delta \Theta^{(0)}}{\delta \Phi^N}$$

phase change

$$\Theta^{(0)}(s_f) - \Theta^{(0)}(s_i) = \int_{s_i}^{s_f} ds \int d^3x \frac{\delta \Theta^{(0)}(s)}{\delta \Phi^M(s, x)} \frac{\partial \Phi^M(s, \mathbf{x})}{\partial s}$$

$$= \int_{s_i}^{s_f} ds \int d^3x \frac{2\sqrt{\gamma}}{\alpha} \mathcal{K}$$

Substituting into the WdW equation

$$\alpha(s, \mathbf{x}) = \frac{\sqrt{\mathcal{K}}}{\sqrt{-\mathcal{V}}}$$

What does it mean?

$$\mathcal{K} \equiv \frac{1}{2} G_{MN} \frac{\partial \Phi^M(s, \mathbf{x})}{\partial s} \frac{\partial \Phi^N(s, \mathbf{x})}{\partial s}$$

What's $\alpha(s, \mathbf{x})$?

$$\alpha(s, \mathbf{x}) = \frac{\sqrt{\mathcal{K}}}{\sqrt{-\mathcal{V}}}$$

square $\longrightarrow$ $0 = \sqrt{\gamma}\left(\frac{\mathcal{K}}{\alpha^2} + \mathcal{V}\right)$

$$= \sqrt{\gamma}\left[\frac{(\partial_s \phi)^2}{2\alpha^2} + \frac{G^{ijkl}}{8\kappa\alpha^2}(\partial_s h_{ij})(\partial_s h_{kl}) + \frac{h^{ij}}{2}(\partial_i \phi)(\partial_j \phi) + V(\phi) - \frac{{}^{(3)}\mathcal{R}}{2\kappa}\right]$$

identification of $\alpha \rightarrow N$ (lapse) and $s \rightarrow x^0$ (time coordinate)

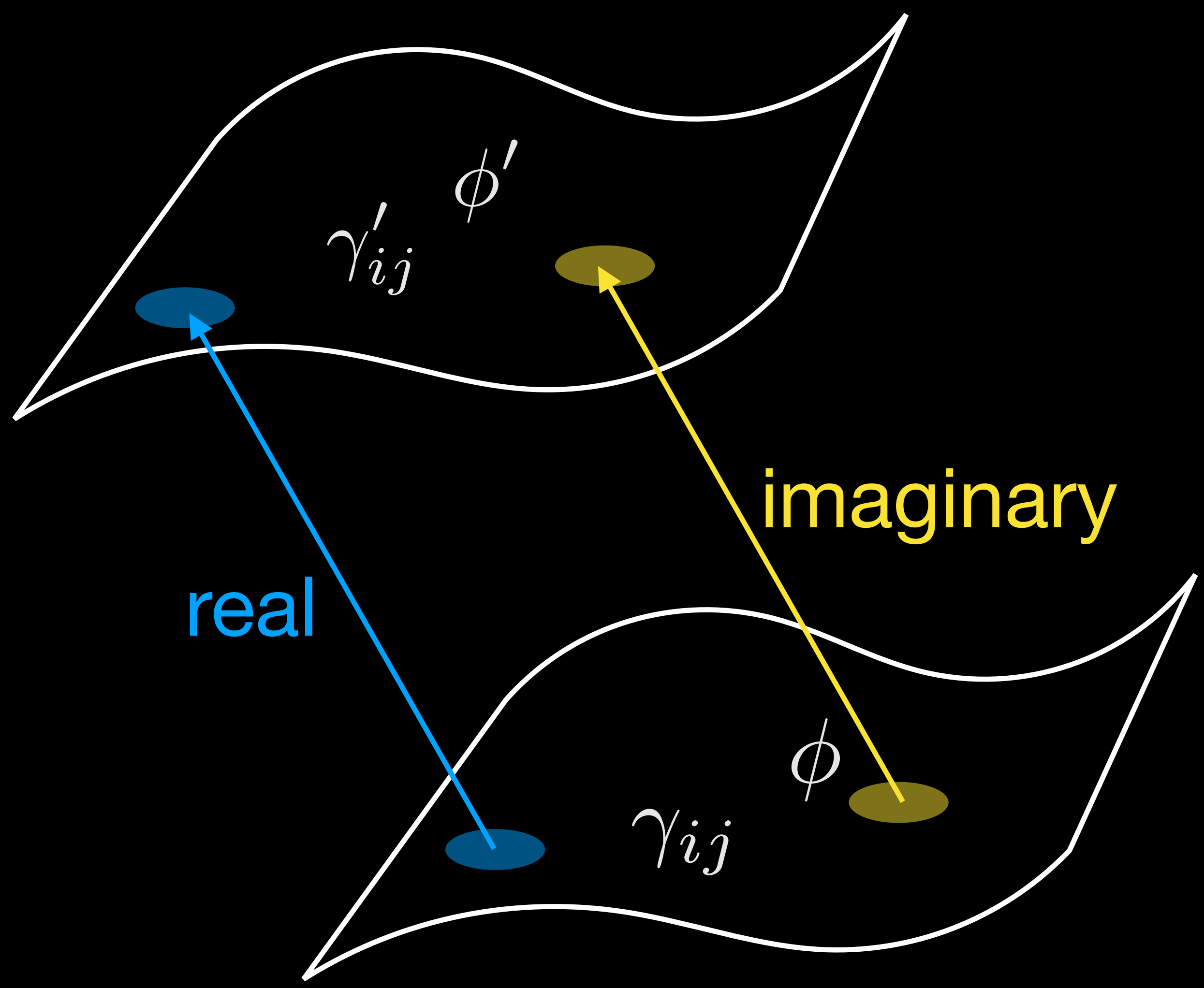
Hamiltonian constraint

Does $\alpha(s,\mathbf{x})$ correspond to $N(s,\mathbf{x})$?

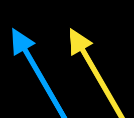
We assume

real/imaginary $\alpha(s,\mathbf{x})$ ⟷ Lorentzian/Euclidean evolution

$$\underset{\textit{local}}{\alpha(s,\mathbf{x})} = \frac{\sqrt{\mathcal{K}}}{\sqrt{-\mathcal{V}}}$$



However, the meaning of

$|\alpha(s,\mathbf{x})\Delta s|$ ~ length of 

is still obscure.

However, the meaning of

$|\alpha(s, \mathbf{x})\Delta s|$ ~ length of

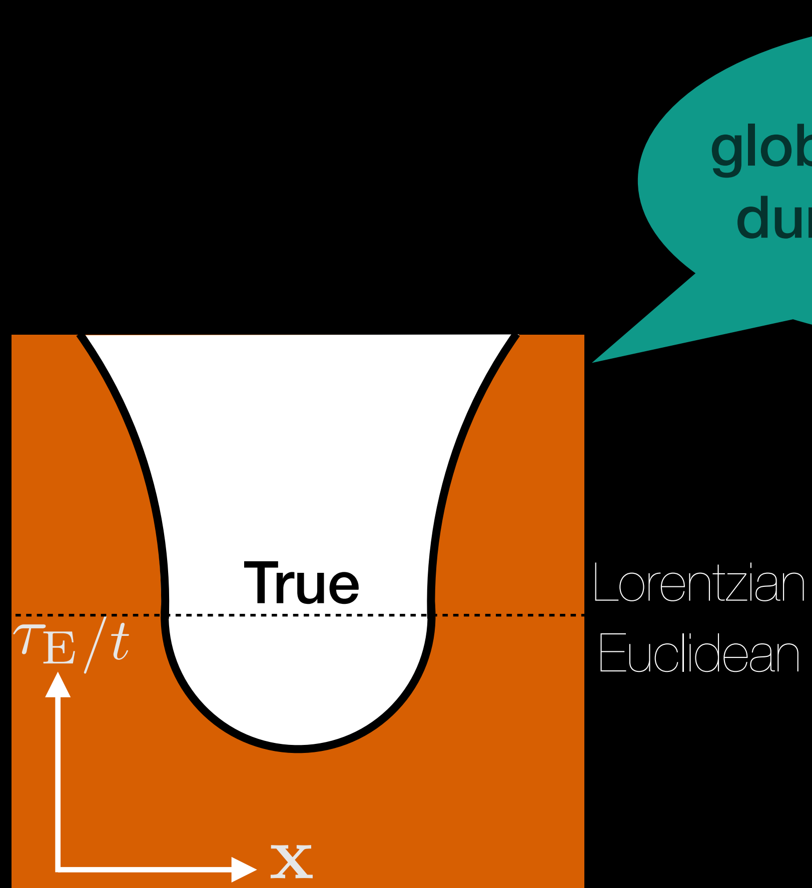
is still obscure.

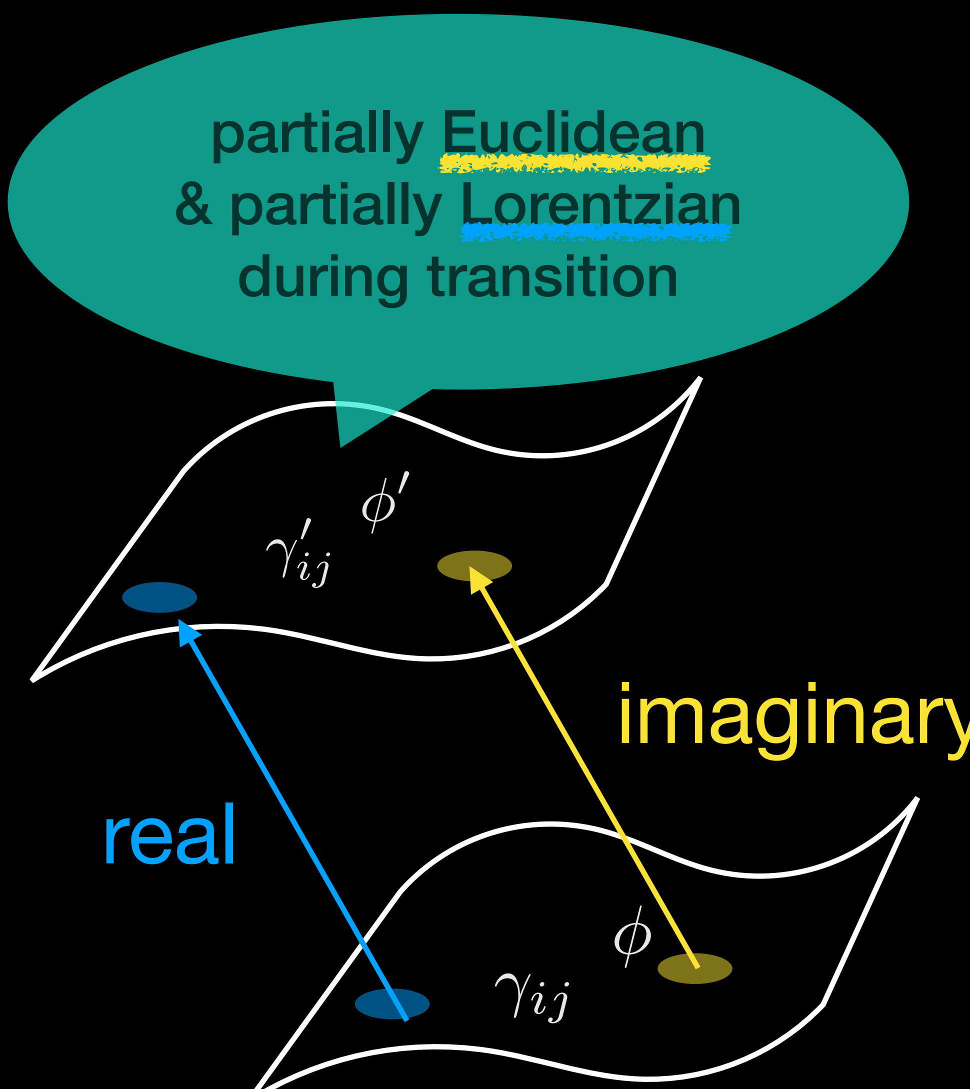
We cannot compute the "rate" of vacuum decay!!

$$\Gamma \sim A e^{-B}$$

exponential suppression
is possible to predict!!

Is the nucleation of Coleman de Luccia bubble most probable?

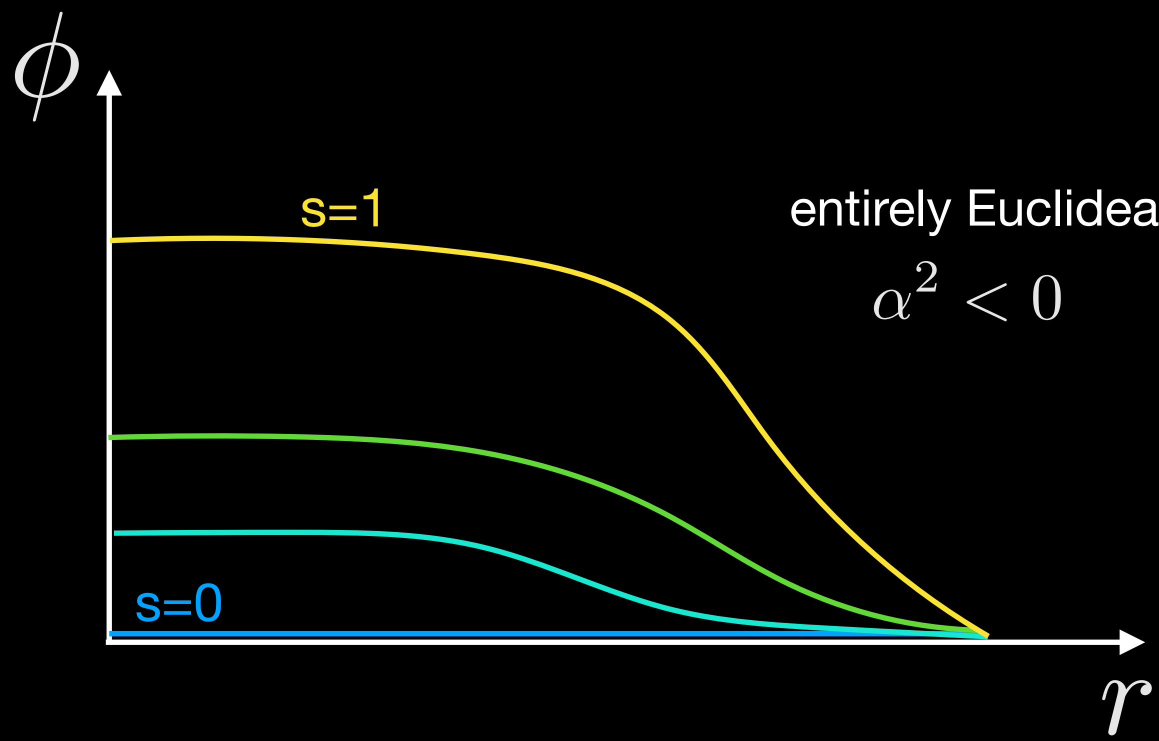




Methodology

$$0 \leq s \leq 1$$

transition starts — transition completed



initial configuration set to CdL solution

$$\phi = \phi(s, r)$$

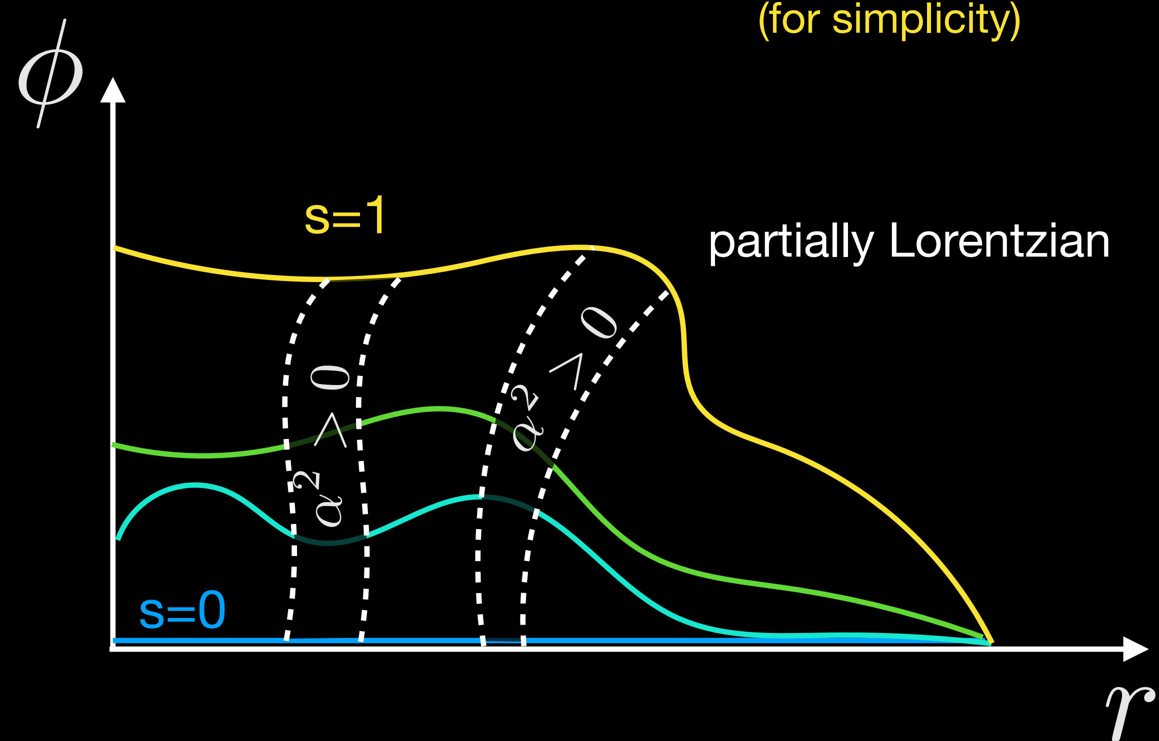
$$\gamma_{ij} dx^i dx^j = e^{\eta(s,r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

$$V(\phi) = \frac{\phi^4}{4} - \frac{k+1}{3}\phi^3 + \frac{k}{2}\phi^2 + V_0$$

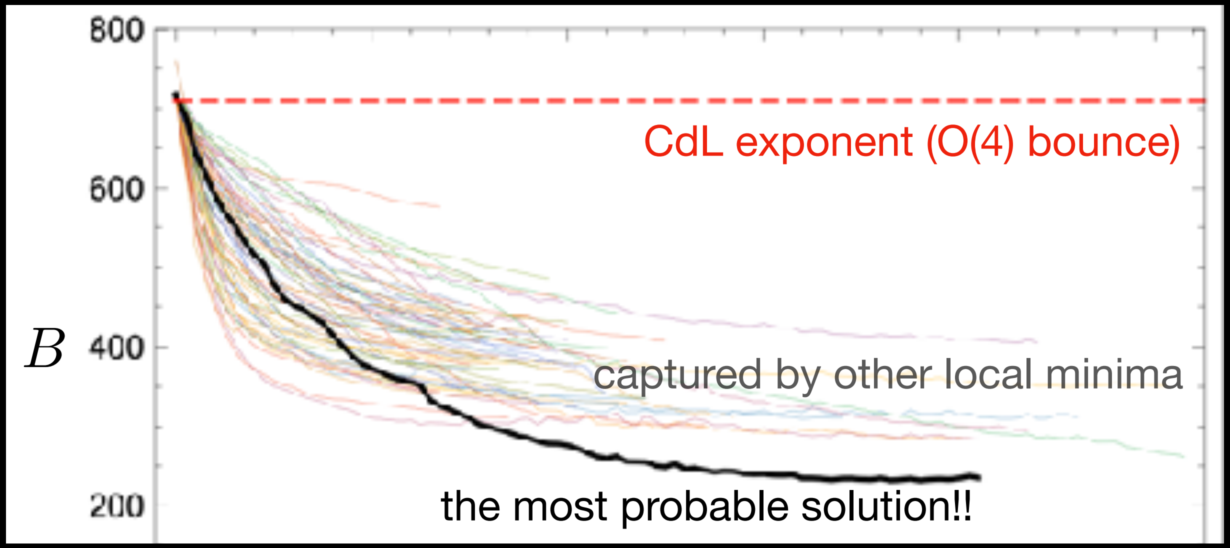
randomly perturb & optimize the config. to find a solution such that:

(Monte Carlo simulation)

- $\dfrac{\delta \Theta^{(0)}}{\delta \Phi^M} \simeq 0$ (stationary point)
- $B < B_{\mathrm{CDL}}$ $\quad B \equiv \mathrm{Im}(2\Theta^{(0)})$ (more probable decay)
- symmetry is restricted to O(3) (for simplicity)



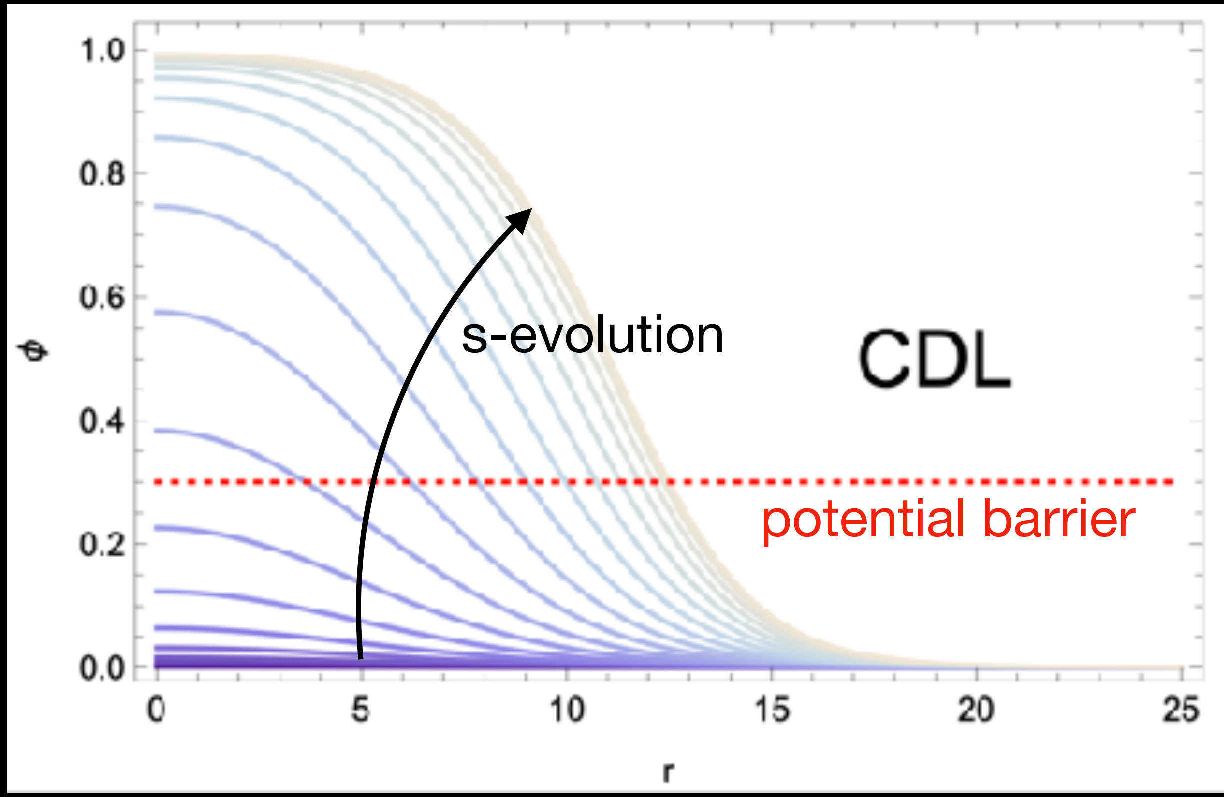
Result



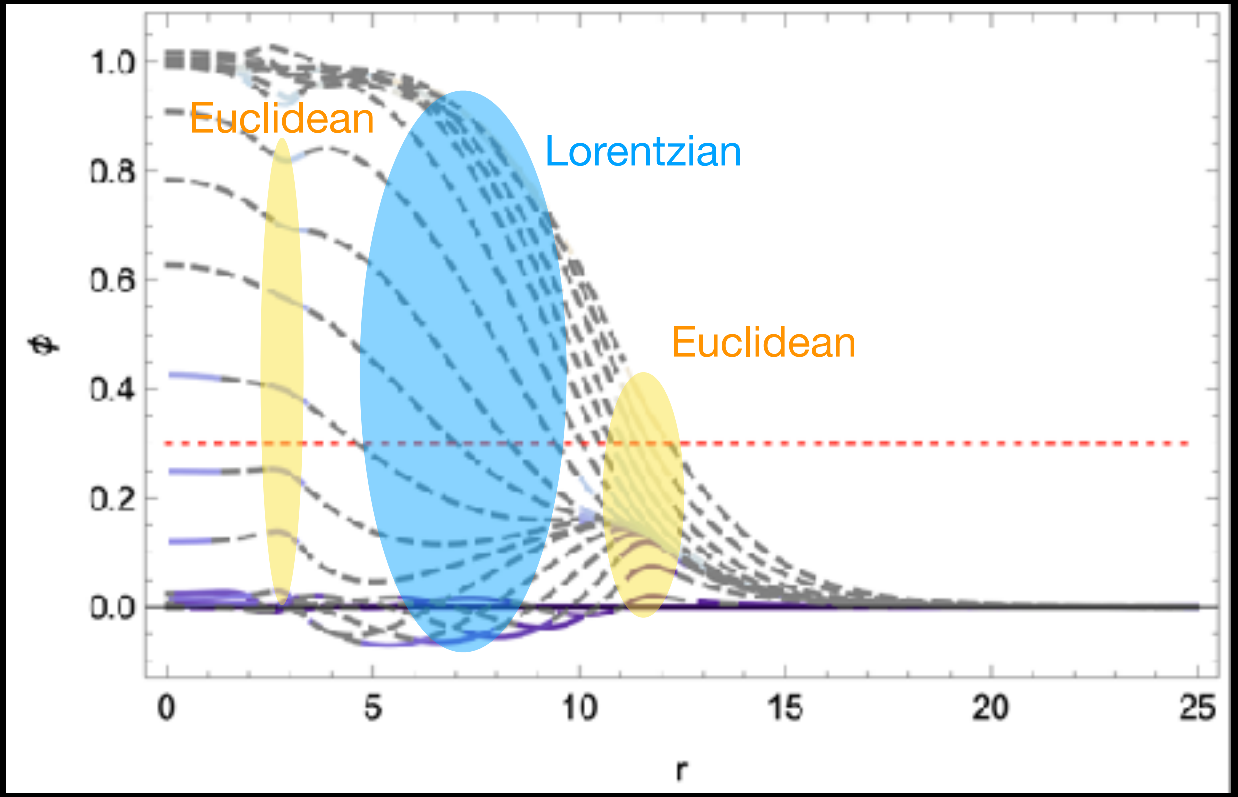
Lorentzian and Euclidean evolution appear simultaneously

→ *Polychronic tunneling*

seed configuration



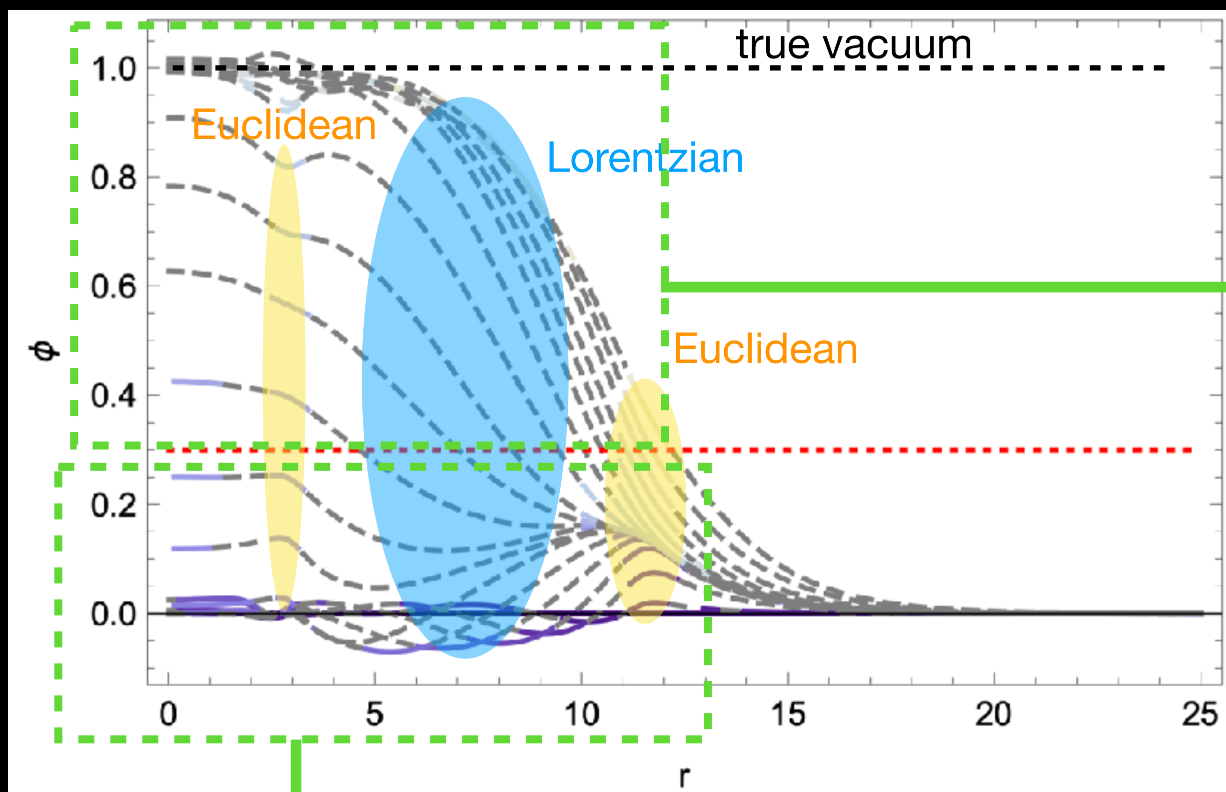
new solution

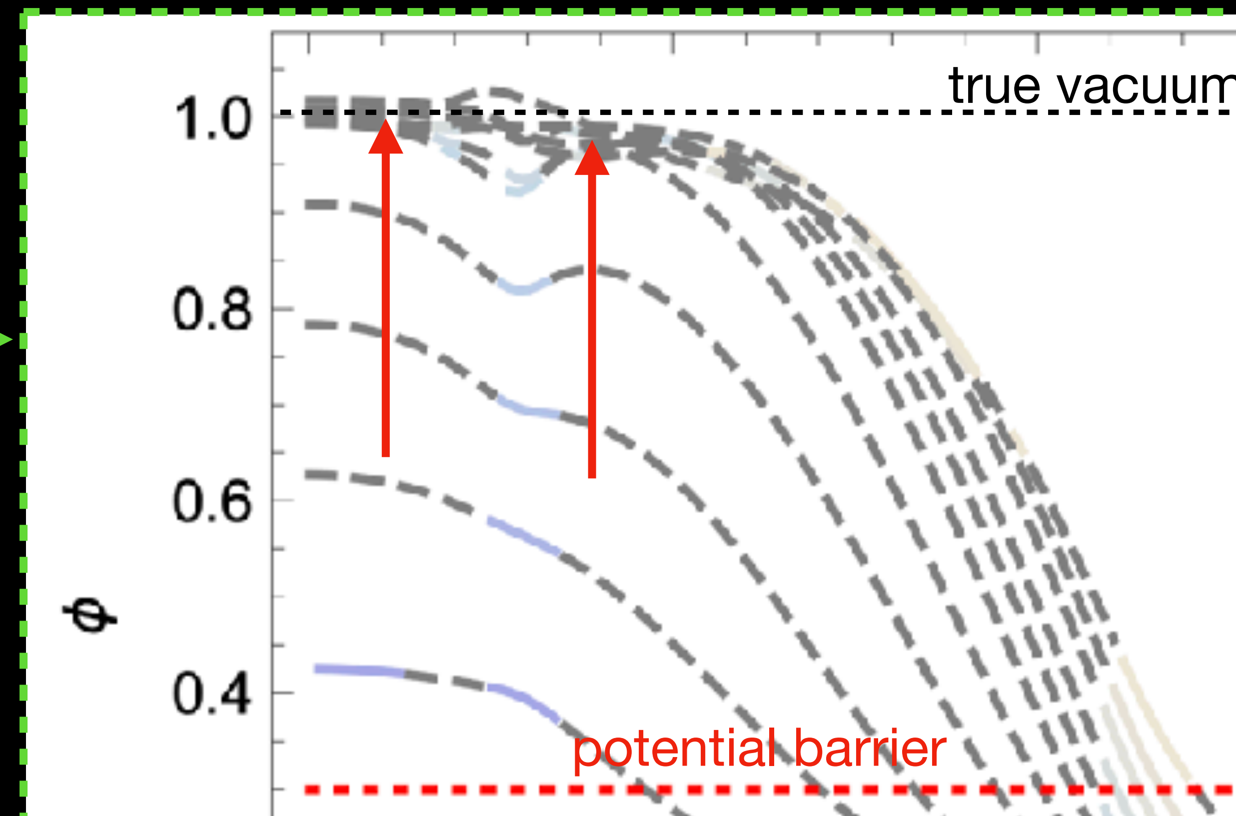


solid: Euclidean dashed: Lorentzian

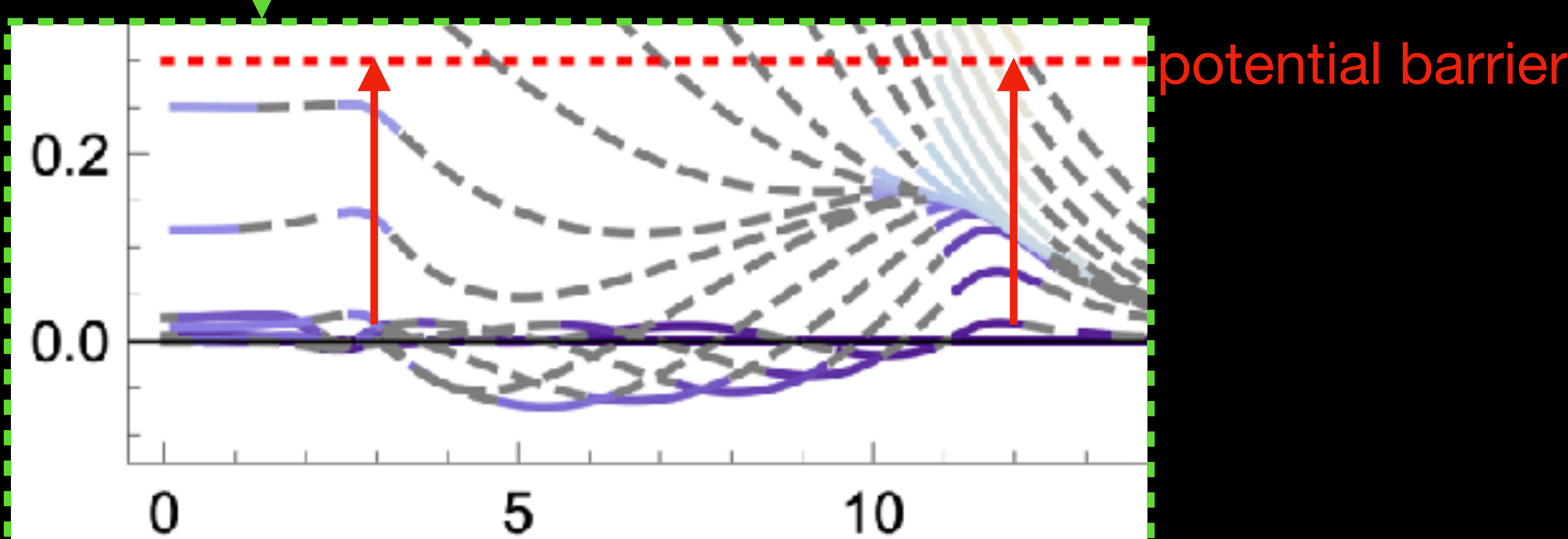
You may think the result is messy, but carefully looking at this…

Carefully looking at this solution…



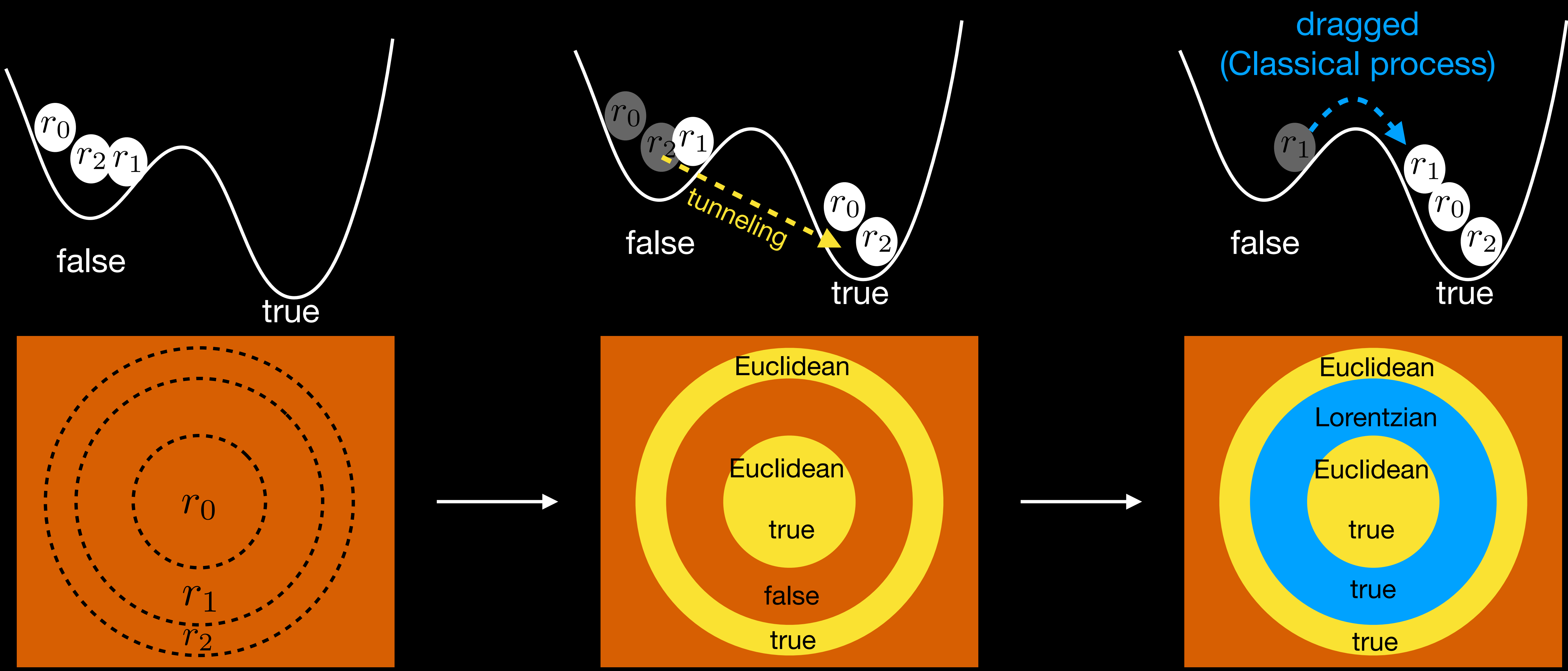


Lorentzian parts precede and Euclidean parts follow



Euclidean parts precede and Lorentzian parts are dragged

How can we interpret the Polychronic tunneling?



Why does this reduce the transition probability?

Smallness of the tunneling region

→ Tunneling rate enhanced!!!

Open questions

- Does this happen even in the thin-wall regime?
- Does it expand after the nucleation?
  (nucleation size is close to that of the CdL bubble)
- Can we construct such a polychronic solution (i.e. Lorentzian + Euclidean processes) in an analytic way?
- What if gravity is turned off?
- Can we formulate thermal transition (i.e. Hawking-Moss) with the WdW formalism?

Summary for PART II

- We formulate the vacuum decay process with the WdW equation while relaxing the O(4) symmetry.
- We performed numerical computation to search for a more probable vacuum decay process.
- We found a new type solution where the Euclidean and Lorentzian processes coexist (polychronic tunneling).
- Its decay probability is higher than the rate predicted by the Coleman-de Luccia solution.
- We may have to consider the Polychronic tunneling more carefully to achieve the most probable vacuum decay process.