李政道研究所
TSUNG-DAO LEE INSTITUTE

# Muon Electric Dipole Moment as a Probe of Flavor-Diagonal CP Violation

Yoshihiro Shigekami (TDLI)

with Yuichiro Nakai (TDLI & SPA, SJTU)
Ryosuke Sato (Osaka U.)

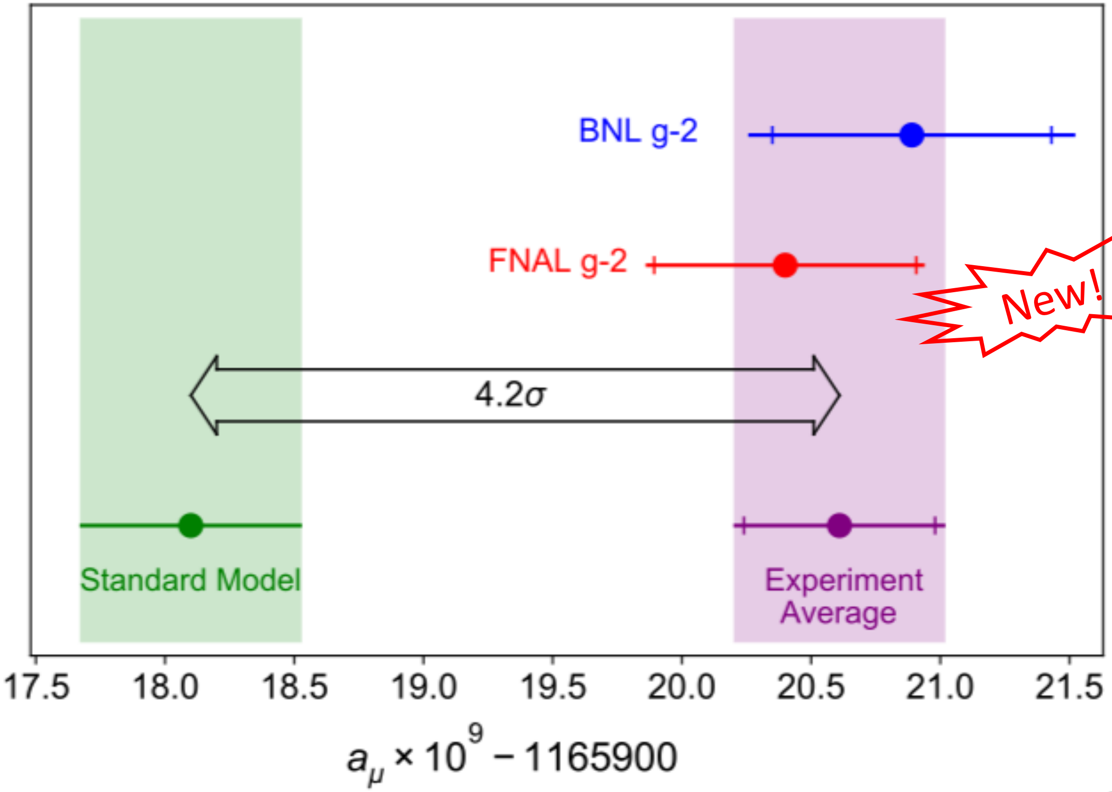
Based on PLB831(2022)137194 [2204.03183]

基研研究会 素粒子物理学の進展 2022 (8月29日 ～ 9月2日)

# Introduction

- Muon anomalous magnetic moment (muon *g*-2)

Muon g-2 collab., PRL126(2021)141801



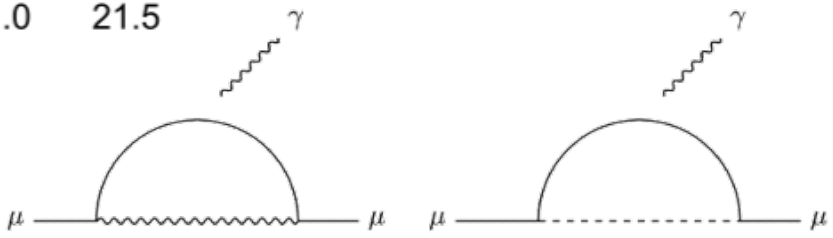
New!

New physics hints

- 2HDM
- MSSM
- U(1) extension
- …

✓ CP conserving process

1-loop diagrams



# Introduction

- The same diagrams will have contributions to CP violating processes → electric dipole moment (EDM)

$$\mathcal{H} = -\mu \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{B} - \textcircled{d} \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{E}$$

⇩ Relativistic

$$\text{Time reversal}: \quad \mathbf{S} \to -\mathbf{S} \quad \mathbf{B} \to -\mathbf{B} \quad \mathbf{E} \to +\mathbf{E}$$

$$\text{Parity}: \quad \mathbf{S} \to +\mathbf{S} \quad \mathbf{B} \to +\mathbf{B} \quad \mathbf{E} \to -\mathbf{E}$$

$$\mathcal{L} \supset -a_\mu \frac{e}{4m_\mu} \left(\bar{\mu}\sigma^{\alpha\beta}\mu\right) F_{\alpha\beta} - d_\mu \frac{i}{2} \left(\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu\right) F_{\alpha\beta}$$

- EDM ($d_\mu$): same type of operator as *g*-2

  if model has CP violating source, EDM is predicted!

- Lepton EDMs constrain models with CPV

# Introduction

- Experimental status of EDMs

electron Severe!

$$d_e < 1.1 \times 10^{-29}\, e\,\mathrm{cm} \ \ (90\%\,\mathrm{C.L.})$$

ACME collab., Nature 562 (2018) 355

muon

$$d_\mu < 1.8 \times 10^{-19}\, e\,\mathrm{cm} \ \ (95\%\,\mathrm{C.L.})$$

Muon (*g*-2) collab., PRD80(2009)052008

- Indirect bounds

  - Minimal flavor violation

NPB292(1987)93, PRL65(1990)2939, PLB500(2001)161, NPB645(2002)155, JHEP08(2014)019

$$|d_\mu| = \left|\frac{m_\mu}{m_e} d_e\right| < 2.3 \times 10^{-27}\, e\,\mathrm{cm}$$

→ Severe constraint for MFV models

  - EDM of heavy atoms

PRL128(2022)131803, arXiv:2207.01679

$$|d_\mu| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}$$

→ One order smaller than direct bound

# Introduction

- Future prospects

| | $\lvert d_\mu \rvert$ [$e\,$cm] | Ref. |
|---|---|---|
| Fermilab ($g-2$) exp. | $10^{-21}$ | [1] |
| J-PARC | $\mathcal{O}(10^{-21})$ | [2] |
| PSI | $6 \times 10^{-23}$ | [3-5] |
| J-PARC (dedicated exp.) | $10^{-24}$ | [6] |

[1] EPJ Web Conf. 118 (2016) 01005
[2] PTEP2019(2019)5, 053C02
[3] 2102.08838 [hep-ex]
[4] 2201.06561 [hep-ex]
[5] PoS NuFact2021 (2022) 136
[6] PRL93(2004)052001

Muon EDM is also important obs. for NP search!

Note: similar contributions to muon *g*-2 and EDM predict

$$|d_\mu| \sim \frac{e}{2m_\mu}\Delta a_\mu \sim 2.34 \times 10^{-22}\, e\,\mathrm{cm}$$

# Introduction

- What kind of models can approach muon EDM?

  - flavor off-diagonal CPV PRD82(2010)093015, PRD94(2016)055019, JHEP06(2019)142, JHEP01(2022)092

    ✓ No muon mass suppression, large $d_\mu$ will be predicted

    ⇔ It may suffer from severe LFV constraints…

    ✓ muon EDM is not unique probe of the model

  - flavor diagonal CPV (free from LFV!)

    ✓ $d_\mu$ will be small, may not have enough sensitivity…

    ⇨ There is interesting discussion on $g$-2 and EDM of muon

    e.g.) PRD98(2018)113002

**This is our main focus!**

Note: still, no simple model without ad hoc assumptions

# Introduction

- We give a complete benchmark model for muon EDM

muon specific two Higgs doublet model (2HDM)

Abe, Sato, Yagyu, JHEP07(2017)012

  - One of the solutions of the muon *g*-2 anomaly

- CP violation → scalar potential

$$V_\Phi \supset -m_{12}^2 \Phi_1^\dagger \Phi_2 + \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \mathrm{h.c.}$$

complex parameters

Relative phase is physical

- This model can suppress electron EDM, but give contributions to muon EDM

→ enough size of muon EDM!

$$|d_\mu| \sim \mathcal{O}(10^{-23})\, e\, \mathrm{cm}$$

# Model Setup

- Muon specific 2HDM Ref: Abe, Sato, Yagyu, JHEP07(2017)012

| | $q_L^a$ | $u_R^a$ | $d_R^a$ | $\ell_L^e$ | $\ell_L^\tau$ | $\ell_L^\mu$ | $e_R$ | $\tau_R$ | $\mu_R$ | $\Phi_1$ | $\Phi_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $SU(3)_C$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $SU(2)_L$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{2}$ |
| $U(1)_Y$ | $1/6$ | $2/3$ | $-1/3$ | $-1/2$ | $-1/2$ | $-1/2$ | $-1$ | $-1$ | $-1$ | $1/2$ | $1/2$ |
| $Z_4$ | $1$ | $1$ | $1$ | $1$ | $1$ | $i$ | $1$ | $1$ | $i$ | $-1$ | $1$ |

$Z_4$ sym.: only muon couples to $\Phi_1$

2 scalar doublets

- Yukawa couplings:

$$\mathcal{L}_Y = -\bar{q}_L \widetilde{\Phi}_2 Y_u u_R - \bar{q}_L \Phi_2 Y_d d_R - \sum_{E=e,\tau} y_E \bar{\ell}_L^E \Phi_2 E_R - y_\mu \bar{\ell}_L^\mu \Phi_1 \mu_R + \mathrm{h.c.}$$

Note: $\bar{\ell}_L^e \Phi_2 \tau_R, \bar{\ell}_L^\tau \Phi_2 e_R$ is allowed, but can take zero by field rotations

# Model Setup

- Scalar potential:

softly break $Z_4$

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{h.c.} \right] + \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2$$

$$+ \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left[ \frac{\lambda_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \mathrm{h.c.} \right]$$

✓ $m_{12}^2, \lambda_5$ can be complex, relative phase is physical one

- Potential analysis is 2HDM with CP violation

minimization conditions:

$$\langle \Phi_\alpha^0 \rangle = v_\alpha, \tan\beta = v_2/v_1, v^2 = v_1^2 + v_2^2$$

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5$$

$$\mathrm{Im}\lambda_5 = \frac{2}{s_\beta c_\beta} \frac{\mathrm{Im} m_{12}^2}{v^2}$$

$$m_{11}^2 = -\frac{1}{2}\lambda_1 v^2 c_\beta^2 - \frac{1}{2}\lambda_{345} v^2 s_\beta^2 + \mathrm{Re} m_{12}^2 t_\beta$$

$$m_{22}^2 = -\frac{1}{2}\lambda_2 v^2 s_\beta^2 - \frac{1}{2}\lambda_{345} v^2 c_\beta^2 + \mathrm{Re} m_{12}^2 t_\beta^{-1}$$

In this model, we can take large tanβ ($\sim$ O(1000)!)

# Model Setup

- Mass matrix for neutral scalar (1 NG mode)

$$M^2 \equiv \frac{1}{s_\beta c_\beta}\mathrm{Re}m_{12}^2$$

$$\widetilde{\mathcal{M}}^2 = \begin{pmatrix} \delta_{\tilde{h}\tilde{h}} & \delta_{\tilde{h}\tilde{H}} & \delta_{\tilde{h}\tilde{A}} & 0 \\ \delta_{\tilde{h}\tilde{H}} & M^2 + \delta_{\tilde{H}\tilde{H}} & \delta_{\tilde{H}\tilde{A}} & 0 \\ \delta_{\tilde{h}\tilde{A}} & \delta_{\tilde{H}\tilde{A}} & M^2 + \delta_{\tilde{A}\tilde{A}} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\delta_{\tilde{h}\tilde{A}}, \delta_{\tilde{H}\tilde{A}} \propto \mathrm{Im}\lambda_5$$

diagonalize

$$R^T \widetilde{\mathcal{M}}^2 R = \mathrm{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2, 0)$$

SM Higgs mass

For later convenience, we define

$$\Delta m_H^2 \equiv m_{H_2}^2 - m_{H_1}^2$$

- (Physical) Charged Higgs mass

$$m_{H^\pm}^2 = M^2 - \frac{v^2}{2}(\lambda_4 + \mathrm{Re}\lambda_5) \equiv m_{H_1}^2 + \Delta m_\pm^2$$

# Model Setup

- Higgs quartic couplings → related to theoretical conditions

parameterize $M^2 \equiv m_H^2 + s_\theta^2 \Delta m_H^2 - 2\frac{\delta_{\tilde{h}\tilde{H}}}{t_\beta} - v^2 \frac{X}{t_\beta^2}$

$$\lambda_1 v^2 \simeq m_h^2 + X v^2$$

$$\lambda_2 v^2 \simeq m_h^2$$

$$\lambda_3 v^2 \simeq m_h^2 - 2s_\theta^2 \Delta m_H^2 + 2\Delta m_\pm^2 - \delta_{\tilde{h}\tilde{H}} t_\beta$$

$$\lambda_4 v^2 \simeq \Delta m_H^2 - 2\Delta m_\pm^2$$

$$\mathrm{Re}\lambda_5 v^2 \simeq -c_{2\theta} \Delta m_H^2$$

$$\mathrm{Im}\lambda_5 v^2 \simeq s_{2\theta} \Delta m_H^2$$

θ: related to CP violating parameter

θ = 0 → CP conserving limit

Roughly, required to be:

$$X \sim \mathcal{O}(1)$$

$$\delta_{hH} \sim \mathcal{O}(v^2/t_\beta)$$

$$\Delta m_H^2 \sim v^2$$

$$\Delta m_\pm^2 \sim v^2$$

Independent parameters of the model:

$$\boxed{m_H^2,\ \Delta m_H^2,\ \Delta m_\pm^2,\ t_\beta,\ \theta,\ X,\ \delta_{\tilde{h}\tilde{H}}}$$

# Model Setup

- Muon physics → dominated by heavy neutral scalars, $H_{1,2}$

*g*-2

$$\Delta a_\mu \simeq \frac{m_\mu^4}{8\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 c_{2\theta}\log\left(\frac{m_H^2}{m_\mu^2}\right)$$

EDM

$$d_\mu \simeq -\frac{e\, m_\mu^3}{32\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 s_{2\theta}\log\left(\frac{m_H^2}{m_\mu^2}\right)$$

- Both enhance large tanβ and $\Delta m_H^2$

- But different dependence of θ

  θ → 0 (or π/2): $\Delta a_\mu$ ⤴, $d_\mu$ ⤵ (CP conserving limit)
  θ → π/4: $\Delta a_\mu$ ⤵, $d_\mu$ ⤴ (maximal CP violation)

θ → π/8 ($s_\theta$ → 0.35) will be important both for $\Delta a_\mu$ and $d_\mu$

# Model Setup

- Parameter scan strategy – we have 7 parameters

$m_H^2,\ \Delta m_H^2,\ \Delta m_\pm^2,\ t_\beta,\ \theta,\ X,\ \delta_{\tilde{h}\tilde{H}}$ We can optimize these: $\begin{cases} -\dfrac{m_h^2}{v^2} < X \lesssim 10 \\ |\delta_{\tilde{h}\tilde{H}}| \lesssim 10 \times \dfrac{v^2}{t_\beta} \end{cases}$

Sub-dominant for $\Delta a_\mu$, we can fix it as $\Delta m_\pm^2 = \Delta m_H^2/2 \Rightarrow \lambda_4 \simeq 0$

**Higher cutoff $\Lambda_{\text{cutoff}}$ can be obtained!**

satisfy all perturbative unitarity and vacuum stability conditions at this scale

- We focus on $m_H \geqq 650$ GeV to avoid LHC constraint

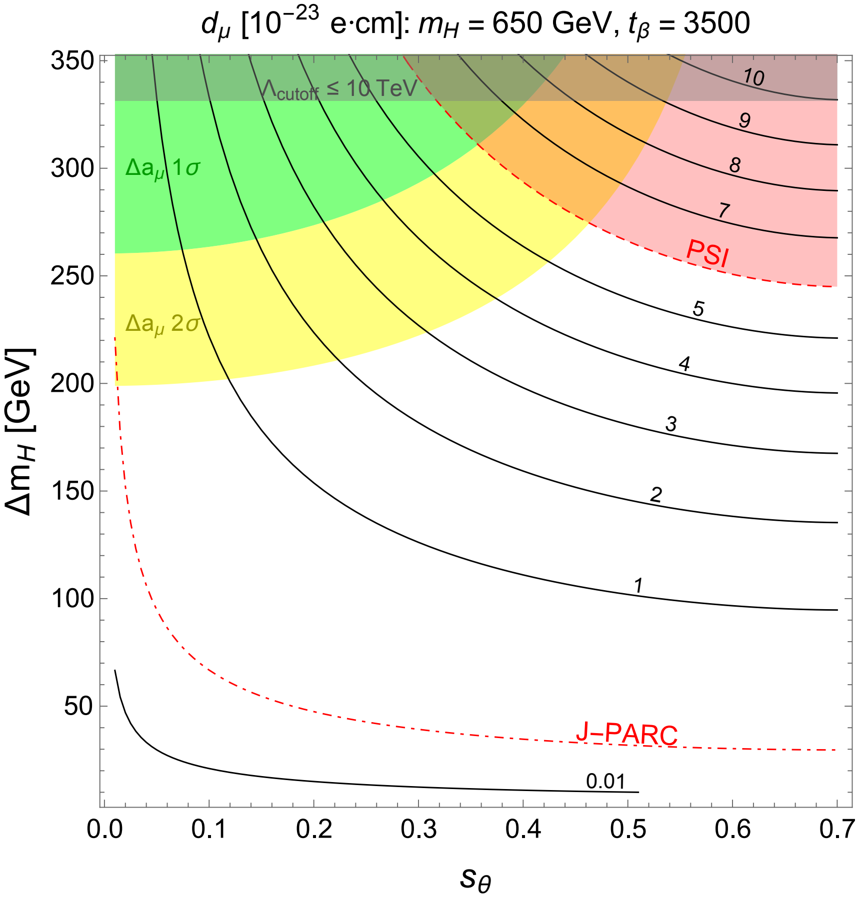
Abe, Sato, Yagyu, JHEP07(2017)012

- Find parameter space for ($s_\theta$, $\Delta m_H$) and ($s_\theta$, tanβ) planes

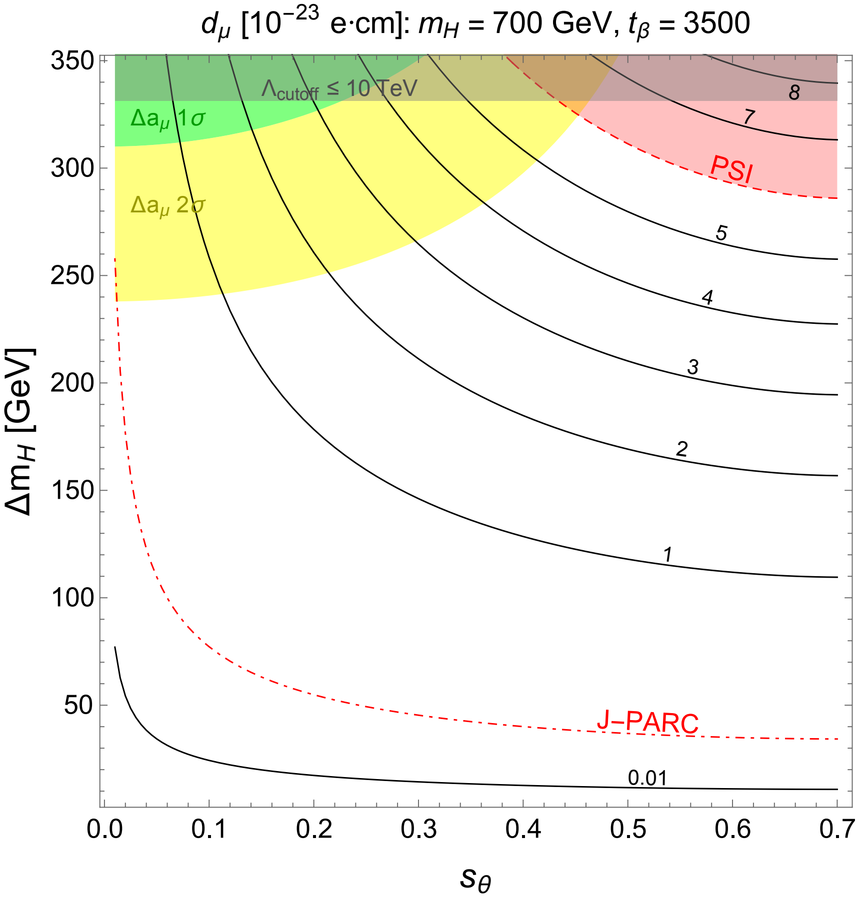
we assume $\Lambda_{\text{cutoff}} \geqq 10$ TeV is viable parameter space

# Results: tanβ = 3500

Note: $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$



$d_\mu$ [$10^{-23}$ e·cm]: $m_H$ = 650 GeV, $t_\beta$ = 3500

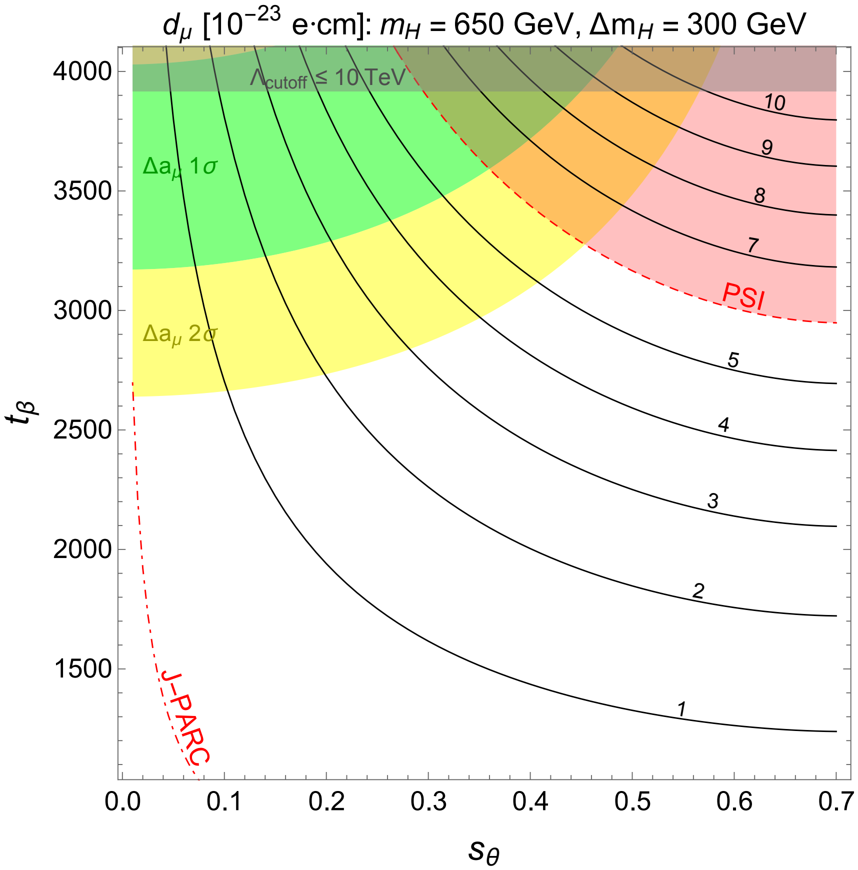


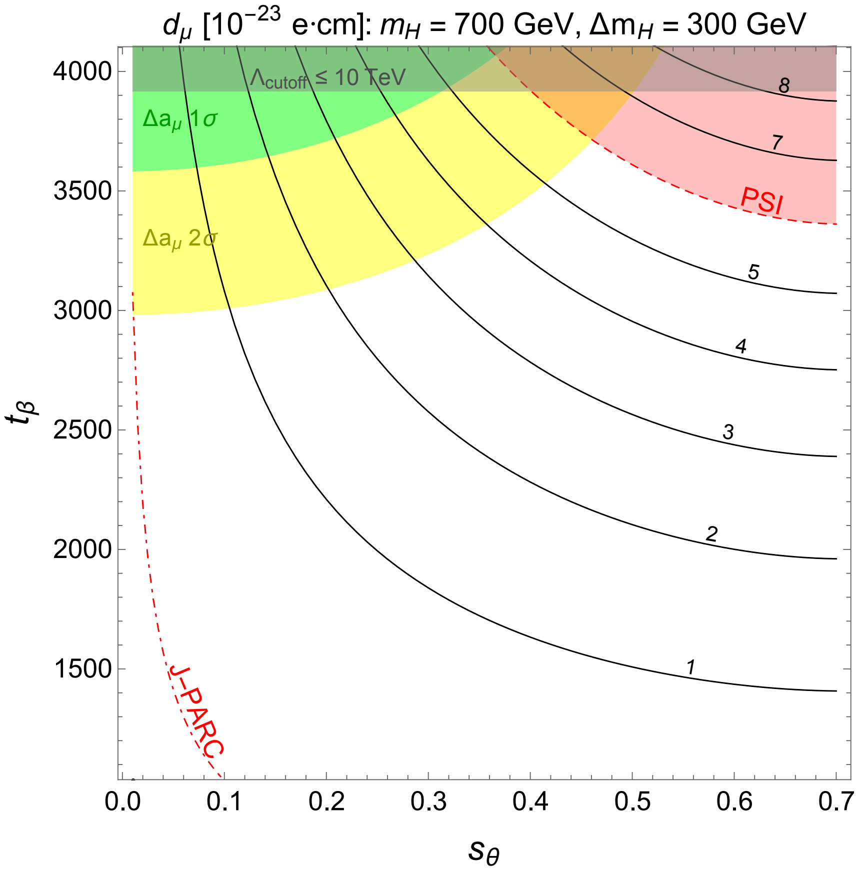
$d_\mu$ [$10^{-23}$ e·cm]: $m_H$ = 700 GeV, $t_\beta$ = 3500

Interesting parameter space: $s_\theta \sim 0.35$, $\Delta m_H \sim 320$ GeV

# Results: $\Delta m_H$ = 300 GeV

Note: $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$





Interesting parameter space: $s_\theta \sim 0.35$, tanβ $\sim 3700$

# Conclusion and discussion

- We focus on flavor-diagonal CP violation of the muon

  one benchmark: muon specific 2HDM

- We find viable parameter space for $\Delta a_\mu$ with large $d_\mu$

$$\boxed{\Delta m_H \sim 320\,\mathrm{GeV}, \quad t_\beta \sim 3700, \quad s_\theta \sim 0.35 \quad \text{for} \quad m_H = 650\,\mathrm{GeV}}$$

$\rightarrow m_{H_2} - m_{H_1} \sim 74\,\mathrm{GeV}$

- LHC can also explore our viable parameter space: $\Delta a_\mu, |d_\mu| \propto \dfrac{1}{m_H^4}$

- $\Delta a_\mu$ will be small (due to Lattice HVP), S. Borsanyi, et. al., Nature 593 (2021) 51

  and more heavy $m_H$ is allowed: $m_H \simeq 760\,\mathrm{GeV}$

  we can choose $s_\theta \sim 0.7$ to suppress $\Delta a_\mu$ which enhances $d_\mu$!

The muon EDM may have important key for the NP!

# Back up

# Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3\left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4\left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

- Mass matrix elements of neutral scalars:

$$\delta_{\tilde{h}\tilde{h}} \equiv \lambda_1 v^2 c_\beta^4 + \lambda_2 v^2 s_\beta^4 + 2\lambda_{345} v^2 s_\beta^2 c_\beta^2 \,,$$

$$\langle \Phi_\alpha^0 \rangle = v_\alpha, \tan\beta = v_2/v_1, v^2 = v_1^2 + v_2^2$$
$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re}\lambda_5$$

$$\delta_{\tilde{h}\tilde{H}} \equiv -(\lambda_1 - \lambda_{345}) v^2 s_\beta c_\beta^3 + (\lambda_2 - \lambda_{345}) v^2 s_\beta^3 c_\beta \,,$$

$$\delta_{\tilde{H}\tilde{H}} \equiv (\lambda_1 + \lambda_2 - 2\lambda_{345}) v^2 s_\beta^2 c_\beta^2 \,,$$

$$\left.\begin{aligned} \delta_{\tilde{h}\tilde{A}} &\equiv -\text{Im}\lambda_5 v^2 s_\beta c_\beta \,, \\ \delta_{\tilde{H}\tilde{A}} &\equiv \frac{1}{2}(-c_\beta^2 + s_\beta^2)\text{Im}\lambda_5 v^2 \,, \end{aligned}\right\}$$

→ 0 for CP conserving models

$$\delta_{\tilde{A}\tilde{A}} \equiv -\text{Re}\lambda_5 v^2 \,.$$

→ CP-odd scalar mass for CP conserving models

# Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

- We consider $M^2 > \delta \sim v^2$

mass matrix is approximately diagonalized:

$R^T \widetilde{\mathcal{M}}^2 R = \text{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2, 0)$ → diagonalizing matrix: $R \equiv R_2 R_3$

$$R_2 \simeq \begin{pmatrix} 1 & \delta_{\tilde{h}\tilde{H}}/M^2 & \delta_{\tilde{h}\tilde{A}}/M^2 & 0 \\ -\delta_{\tilde{h}\tilde{H}}/M^2 & 1 & 0 & 0 \\ -\delta_{\tilde{h}\tilde{A}}/M^2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\theta & s_\theta & 0 \\ 0 & -s_\theta & c_\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Mass eigenvalues: $$\begin{cases} m_h^2 = \delta_{\tilde{h}\tilde{h}} + \mathcal{O}(1/M^2)\,, \\ m_{H_1}^2 = M^2 + \delta_{\tilde{H}\tilde{H}} c_\theta^2 - 2\delta_{\tilde{H}\tilde{A}} s_\theta c_\theta + \delta_{\tilde{A}\tilde{A}} s_\theta^2 + \mathcal{O}(1/M^2)\,, \\ m_{H_2}^2 = M^2 + \delta_{\tilde{H}\tilde{H}} s_\theta^2 + 2\delta_{\tilde{H}\tilde{A}} s_\theta c_\theta + \delta_{\tilde{A}\tilde{A}} c_\theta^2 + \mathcal{O}(1/M^2)\,. \end{cases}$$

# Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \left[ \frac{\lambda_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right]$$

- Expressions for $\lambda_i$ in $M^2 > \delta$ and large tanβ limit

$$\lambda_1 v^2 \simeq m_h^2 + \left( m_H^2 + s_\theta^2 \Delta m_H^2 - M^2 \right) t_\beta^2 - 2\delta_{\tilde{h}\tilde{H}} t_\beta \,,$$

$$\lambda_2 v^2 \simeq m_h^2 + \left( m_H^2 + s_\theta^2 \Delta m_H^2 - M^2 \right) t_\beta^{-2} + 2\delta_{\tilde{h}\tilde{H}} t_\beta^{-1} \,,$$

$$\lambda_3 v^2 \simeq m_h^2 - m_H^2 - s_\theta^2 \Delta m_H^2 - M^2 + 2m_{H\pm}^2 + \delta_{\tilde{h}\tilde{H}} \left( t_\beta^{-1} - t_\beta \right) \,,$$

$$\lambda_4 v^2 \simeq M^2 + m_H^2 + c_\theta^2 \Delta m_H^2 - 2m_{H\pm}^2 \,,$$

$$\mathrm{Re}\lambda_5 v^2 \simeq M^2 - m_H^2 - c_\theta^2 \Delta m_H^2 \,,$$

$$\mathrm{Im}\lambda_5 v^2 \simeq -\frac{s_{2\theta}}{c_\beta^2 - s_\beta^2} \Delta m_H^2 \,.$$

$M^2$ is chosen so that $\lambda_1$ becomes *O*(1) in large tanβ limit

# Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right]$$

- Theoretical conditions

Vacuum stability

$$\lambda_1 > 0\,, \quad \sqrt{\lambda_1\lambda_2} + \lambda_3 > 0\,,$$
$$\lambda_2 > 0\,, \quad \sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0\,.$$

Perturbative unitarity

$$\left|\frac{3(\lambda_1+\lambda_2) \pm \sqrt{9(\lambda_1-\lambda_2)^2 + 4(2\lambda_3+\lambda_4)^2}}{2}\right| < 8\pi\,, \quad \left|\lambda_3 + 2\lambda_4 \pm |\lambda_5|\right| < 8\pi\,,$$
$$\left|\frac{(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}}{2}\right| < 8\pi\,, \quad \left|\lambda_3 \pm \lambda_4\right| < 8\pi\,,$$
$$\left|\frac{(\lambda_1+\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4|\lambda_5|^2}}{2}\right| < 8\pi\,, \quad \left|\lambda_3 \pm |\lambda_5|\right| < 8\pi\,.$$

# Potential analysis for 2HDM with CPV

- 1-loop RGEs for dimensionless couplings $\mu\frac{d}{d\mu}c = \frac{1}{16\pi^2}\beta_c$

  → relevant couplings: $g_i$, $\lambda_i$, $y_t$, $y_\mu$

$$\beta_{g_1} \simeq +7g_1^3\,, \qquad \beta_{g_2} \simeq -3g_2^3\,, \qquad \beta_{g_3} \simeq -7g_3^3\,,$$

$$\beta_{\lambda_1} \simeq +\frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 - 3g_1^2\lambda_1 - 9g_2^2\lambda_1 + 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2 + 4\lambda_1y_\mu^2 - 4y_\mu^4\,,$$

$$\beta_{\lambda_2} \simeq +\frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 - 3g_1^2\lambda_2 - 9g_2^2\lambda_2 + 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2 + 12\lambda_2y_t^2 - 12y_t^4\,,$$

$$\beta_{\lambda_3} \simeq +\frac{3}{4}g_1^4 - \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4 + (2y_\mu^2 + 6y_t^2 - 3g_1^2 - 9g_2^2 + 6\lambda_1 + 6\lambda_2 + 4\lambda_3)\lambda_3 + 2\lambda_1\lambda_4 + 2\lambda_2\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2\,,$$

$$\beta_{\lambda_4} \simeq +3g_1^2g_2^2 + 8|\lambda_5|^2 + (2y_\mu^2 + 6y_t^2 - 3g_1^2 - 9g_2^2 + 2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 4\lambda_4)\lambda_4\,,$$

$$\beta_{\lambda_5} \simeq +(2y_\mu^2 + 6y_t^2 - 3g_1^2 - 9g_2^2 + 2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 12\lambda_4)\lambda_5\,,$$

$$\beta_{y_t} \simeq +\frac{9}{2}y_t^3 + \left(-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)y_t\,, \qquad \boxed{\beta_{y_\mu} \simeq +\frac{5}{2}y_\mu^3 - \frac{3}{4}\left(5g_1^2 + 3g_2^2\right)y_\mu\,.}$$

- We have large $y_\mu$ : $y_\mu = \frac{\sqrt{2}m_\mu}{v}\sqrt{1+t_{\beta^2}} \simeq 0.6\times\left(\frac{t_\beta}{1000}\right)$

# 1-loop integrals for $\Delta a_\mu$ and $d_\mu$

- Neutral scalar contribution

$\Delta a_\mu$:

$$I_S(r) \equiv \int_0^1 dx \frac{x^2(2-x)}{rx^2 - x + 1}$$ (CP-even)

$$I_P(r) \equiv \int_0^1 dx \frac{-x^3}{rx^2 - x + 1}$$ (CP-odd)

$d_\mu$: $$f_0(r) \equiv \int_0^1 dx \frac{x^2}{rx^2 - x + 1}$$

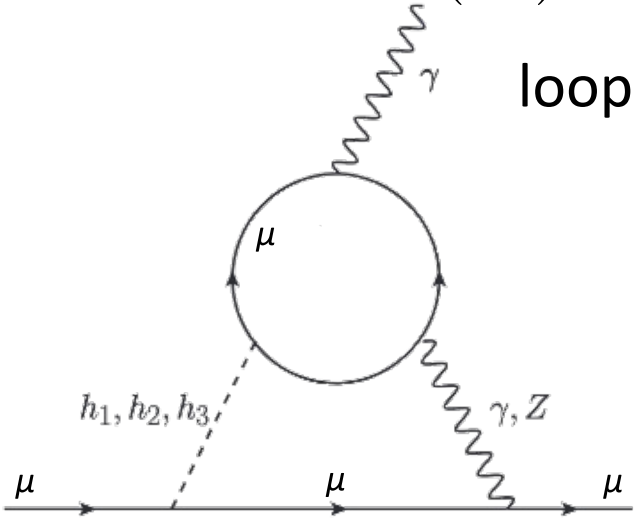
- Charged scalar contribution

$\Delta a_\mu$: $$I_C(r) \equiv \int_0^1 dx \frac{-x(1-x)}{rx + 1 - r}$$

# 2-loop contribution to $d_\mu$

- Dominant one: Barr-Zee type diagram with muon loop

$$d_\mu^{i,\phi_i\text{-}\gamma\text{-}\gamma} = -\frac{e\,m_\mu}{(4\pi)^2 v^2} r_i \left(R_{1i} - R_{2i} t_\beta\right) R_{3i} t_\beta \frac{2\alpha}{\pi} I_\mu(r_i)$$



loop integral: $I_\mu(r) = \int_0^1 dx \frac{1 - x(1-x)}{r - x(1-x)} \ln\left[\frac{x(1-x)}{r}\right]$

JHEP01(2014)106, JHEP08(2017)031, JHEP12(2019)068

✓ 1- and 2-loop contributions are

$$d_\mu^{i,\phi_i\text{-}\gamma\text{-}\gamma} = d_\mu^i \times \frac{2\alpha}{\pi} \frac{I_\mu(r_i)}{f_0(r_i)}$$

- Facts: $\begin{cases} 2\alpha/\pi \sim 4.6 \times 10^{-3} \\ I_\mu(r) \sim -(15\text{-}19) f_0(r) \end{cases}$

2-loop contribution is important!

# *T*-parameter

Peskin and Takeuchi, PRL65(1990)964; PRD46(1992)381

- *ρ*-parameter: $\rho = \dfrac{m_W^2}{m_Z^2 \cos\theta_W^2}$ (= 1 in the SM)
- NP contributions deviate it from 1: $\Delta\rho = \alpha T$

  experimental status (PDG): $|T| \lesssim 0.2$
- In 2HDM, mass differences among $H_{1,2}$ and $H^+$ are crucial

  Grimus, Lavoura, Ogreid, Osland, J.Phys.G35(2008)075001; NPB801(2008)81
- In our model, $\Delta m_H$ is enough small to satisfy

  e.g.) $m_H$ = 650 GeV with $\Delta m_H$ = 320 GeV and $s_\theta$ = 0.35

  → $T$ = -0.03

# $h \rightarrow \mu^+\mu^-$ decay

- Yukawa interaction in mass basis for scalars:

$$\mathcal{L}_Y^{\text{int}} = -\sum_{f \neq \mu} \frac{m_f}{v} \left[ \left( R_{1i} + \frac{R_{2i}}{t_\beta} \right) \bar{f} f + i s_f \frac{R_{3i}}{t_\beta} \bar{f} \gamma_5 f \right] \phi_i - \frac{m_\mu}{v} \left[ \boxed{(R_{1i} - R_{2i} t_\beta)} \bar{\mu} \mu - i R_{3i} t_\beta \bar{\mu} \gamma_5 \mu \right] \phi_i$$

$$+ \left\{ -\frac{\sqrt{2}}{v t_\beta} \sum_{a=1}^{3} \bar{u}^a \left( m_{d^a} P_R - m_{u^a} P_L \right) d^a H^+ \left[ \frac{\sqrt{2}}{v} t_\beta m_\mu \bar{\nu}_\mu P_R \mu - \frac{\sqrt{2}}{v t_\beta} \sum_{\ell \neq \mu} m_\ell \bar{\nu}_\ell P_R \ell \right] H^+ + \text{h.c.} \right\}$$

- Large tan limit, h-*μ*-*μ* coupling may be large

current experimental status (95% C.L.):

$$|\kappa_\mu| < 1.47 \quad (\text{ATLAS})$$

$$0.61 < |\kappa_\mu| < 1.44 \quad (\text{CMS})$$

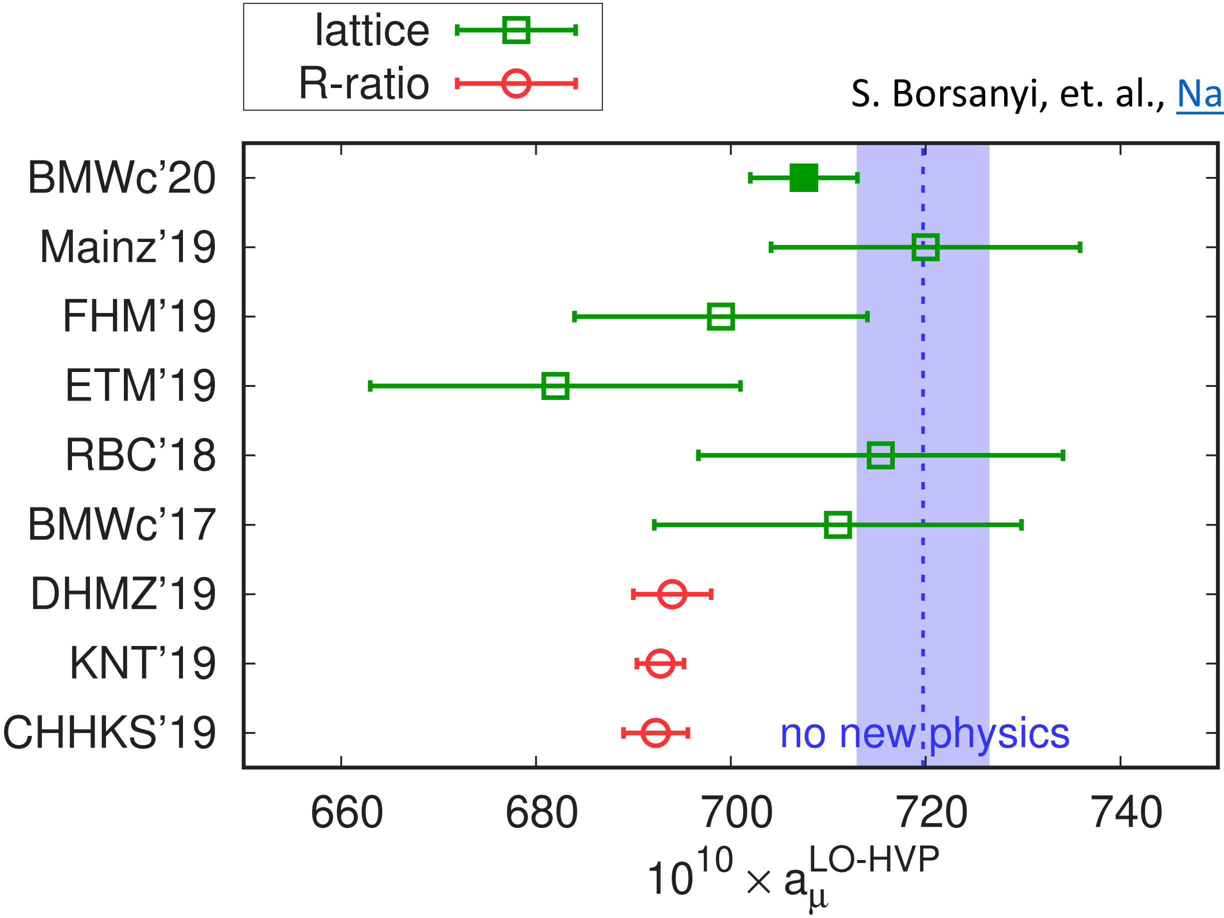
ATLAS collab., PLB812(2021)135980
CMS collab., JHEP01(2021)148

⇔ our model

$$\kappa_\mu = R_{11} - R_{21} t_\beta \simeq 1 + \frac{\delta_{\tilde{h}\tilde{H}}}{M^2} t_\beta \sim 1 + \frac{\mathcal{O}(v^2)}{m_H^2}$$

# Lattice results on HVP

- **Result:** $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0) \times 10^{-10}$



S. Borsanyi, et. al., Nature 593 (2021) 51

# Lattice results on HVP

- Result: $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0) \times 10^{-10}$

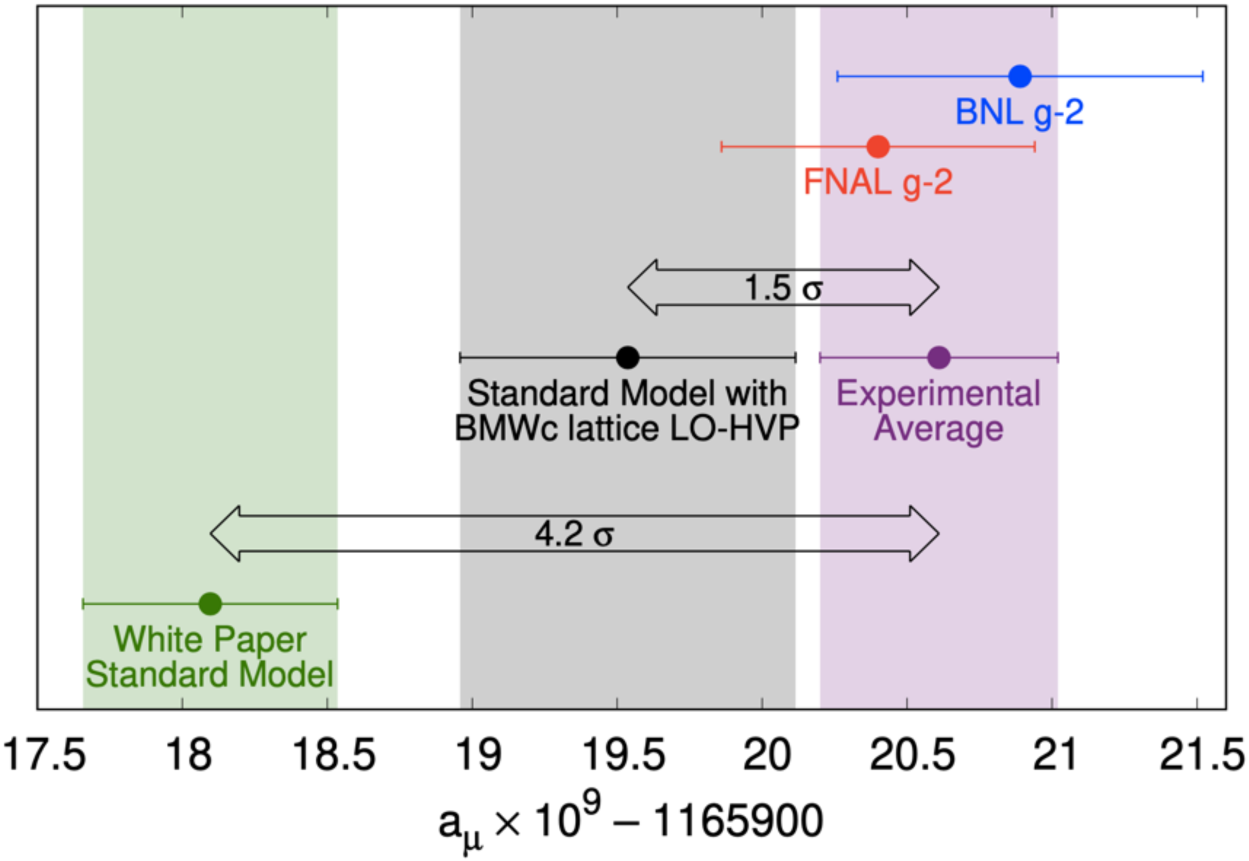


Figure from talk of L. Lellouch (Wits ICPP iThemba Labs seminar)