# クォーコニウム物理量の4次補正計算の現状

隅野行成
(東北大学)

共同研究者：三嶋、高浦

# ☆Plan of talk

1. Physics Motivation

2. Theory

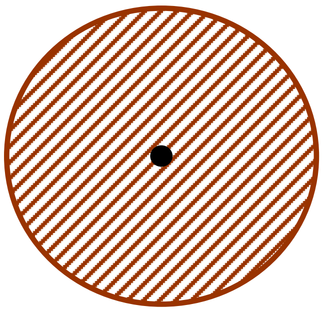
   (a) Expansions-by-regions technique

   (b) EFT: potential-NRQCD

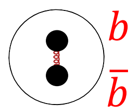
3. 4次補正計算の目標、現状、課題

4. まとめ

# 1. Physics Motivation



$B$




$\Upsilon(1S)$

クォーコニウムは短波長のgluonのみが
束縛状態形成のdynamicsに関与:

$$\lambda_g \ll \Lambda_{\mathrm{QCD}}^{-1} \sim 1\ \mathrm{fm}$$

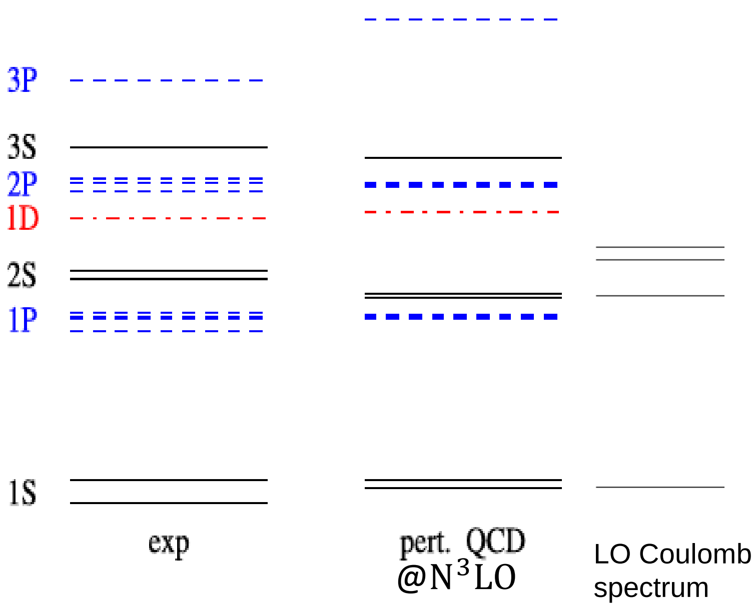
束縛dynamicsの主要部分を摂動QCDで記述できる
唯一の実在するハドロン系

# 現状　摂動QCDに基づく系統的な高次補正計算

3次補正計算は完成

## Bottomonium spectrum

大域的スペクトル構造を再現




Kiyo, YS

## Charmonium/bottomonium level splittings

実験値とconsistent ($< 1$ or $2\sigma_{th}$)

理論の(相対)誤差が一部大きい

| Level splitting | Exp. | Pert. QCD based (Recksiegel,YS 02,03) |
|---|---|---|
| $\chi_{c1}(1P)-\chi_{c0}(1P)$ | 95 | 56 ±34 |
| $\chi_{c2}(1P)-\chi_{c1}(1P)$ | 46 | 43 ±24 |
| $J/\Psi-\eta_c(1S)$ | 113 | 88 ±26 |
| $\Psi(2S)-\eta_c(2S)$ | 53 ✓ | 38 ±36 |
| $\chi_c^{cog}(1P)-h_c(1P)$ | $-0.1 \pm 0.3$ | $-0.8 \pm 0.8$ |
| $\Upsilon(1S)-\eta_b(1S)$ | $62.3 \pm 3.2$ ✓ | 44±11 |
| $\Upsilon(2S)-\eta_b(2S)$ | $24 \pm 5$ ☆ | 21± 8 |
| $\Upsilon(3S)-\eta_b(3S)$ | – | 12± 9 |
| $\chi_b^{cog}(1P)-h_b(1P)$ | $0.6 \pm 1.0$☆ | $-0.4 \pm 0.2$ |
| $\chi_b^{cog}(2P)-h_b(2P)$ | $0.5 \pm 1.0$☆ | $-0.2 \pm 0.1$ |
| $\chi_{b1}(1P)-\chi_{b0}(1P)$ | $33 \pm 1$ | 23±?? |
| $\chi_{b2}(1P)-\chi_{b1}(1P)$ | 19 | 18±?? |
| $\chi_{b1}(2P)-\chi_{b0}(2P)$ | $24 \pm 1$ | 14±?? |
| $\chi_{b2}(2P)-\chi_{b1}(2P)$ | 13 | 11±?? |

} 高次補正の一部を取り入れた場合

✓ 当時(03)から大幅に実験値が動いたもの

☆ 当時(03)には実験値がなかったもの

## 応用例

・基礎物理定数の決定:　$m_c, m_b, (m_t), \alpha_s, |V_{cb}|$

⇨ cf.  清, 三嶋, 高浦, 林, 他との共同研究

・その他にも色々な現象論。

## 2. Theory

(a) Expansion-by-regions technique

(b) EFT

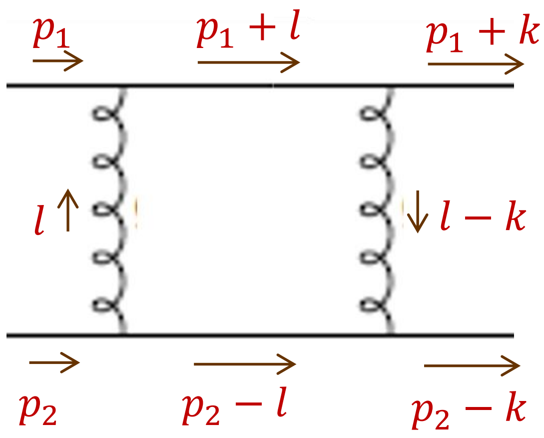
} 両輪

## 2(a) Expansion-by-regions Technique

Beneke, Smirnov

EFTがfull theoryのループ積分をどのように分解しているかの
理論的基礎付けを与える。

# Expansion-by-regions of $Q\bar{Q}$ scattering diagram




c.m. frame

$$p_1 = (\sqrt{|\vec{p}|^2 + m^2}, \vec{p})$$

$$p_2 = (\sqrt{|\vec{p}|^2 + m^2}, -\vec{p})$$

$$k = (0, \vec{k})$$

Master integral

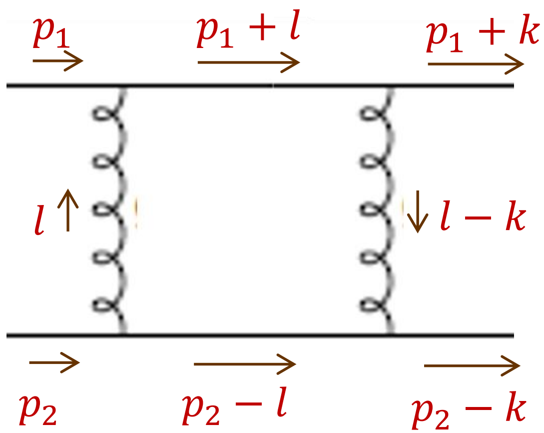
$$D = 4 - 2\epsilon$$

$$I = \int \frac{d^D l}{(2\pi)^D} \frac{1}{(2p_1 \cdot l + l^2 + i0)(-2p_2 \cdot l + l^2 + i0)(l^2 + i0)[(l-k)^2 + i0]}$$

$$= \frac{i}{8\pi^2 t}\left(\frac{1}{\epsilon} - \log(-t/m^2)\right)\frac{1}{\sqrt{s(s-4m^2)}}\log\left(\frac{\sqrt{s-4m^2} - \sqrt{s}}{\sqrt{s-4m^2} + \sqrt{s}}\right) + O(\epsilon)$$ exact

$$= \frac{1}{32\pi\, m|\vec{p}|\left|\vec{k}\right|^2}\left(\frac{1}{\epsilon} - \log(\left|\vec{k}\right|^2/m^2)\right)\left(1 - \frac{|\vec{p}|^2}{2m^2} + \frac{3|\vec{p}|^4}{8m^4} + \cdots\right)$$ $1/m$ expansion

# Expansion-by-regions of $Q\bar{Q}$ scattering diagram




$p_1 = (\sqrt{|\vec{p}|^2 + m^2}, \vec{p})$

$p_2 = (\sqrt{|\vec{p}|^2 + m^2}, -\vec{p})$

$k = (0, \vec{k})$

$$I = \int \frac{d^D l}{(2\pi)^D} \frac{1}{(2p_1 \cdot l + l^2 + i0)(-2p_2 \cdot l + l^2 + i0)(l^2 + i0)[(l-k)^2 + i0]}$$

$$= \frac{1}{32\pi\, m|\vec{p}|\left|\vec{k}\right|^2}\left(\frac{1}{\epsilon} - \log(\left|\vec{k}\right|^2/m^2)\right)\left(1 - \frac{|\vec{p}|^2}{2m^2} + \frac{3|\vec{p}|^4}{8m^4} + \cdots\right)$$

$1/m$ expansion

4 regions　外線運動量 $|\vec{p}|, |\vec{k}| \sim \beta$　($m \sim 1$, $\beta$ = $Q, \bar{Q}$の重心系での速度 $\ll 1$)

Hard region: $l^0 \sim 1,\ |\vec{l}| \sim 1$

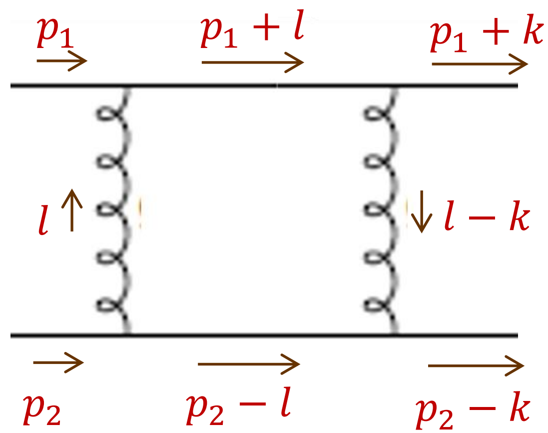
Soft region: $l^0 \sim \beta,\ |\vec{l}| \sim \beta$

Potential region: $l^0 \sim \beta^2,\ |\vec{l}| \sim \beta$

Ultrasoft region: $l^0 \sim \beta^2, |\vec{l}| \sim \beta^2$

それぞれのregionからの寄与の和が全体($I$)の1/m展開を与える。

# Expansion-by-regions of $Q\bar{Q}$ scattering diagram




$$p_1 = (\sqrt{|\vec{p}|^2 + m^2}, \vec{p})$$

$$p_2 = (\sqrt{|\vec{p}|^2 + m^2}, -\vec{p})$$

$$k = (0, \vec{k})$$

$$I = \int \frac{d^D l}{(2\pi)^D} \frac{1}{(2p_1 \cdot l + l^2 + i0)(-2p_2 \cdot l + l^2 + i0)(l^2 + i0)[(l-k)^2 + i0]}$$

$$= \frac{1}{32\pi\, m|\vec{p}|\left|\vec{k}\right|^2}\left(\frac{1}{\epsilon} - \log(\left|\vec{k}\right|^2/m^2)\right)\left(1 - \frac{|\vec{p}|^2}{2m^2} + \frac{3|\vec{p}|^4}{8m^4} + \cdots\right)$$

$1/m$ expansion

4 regions　外線運動量 $|\vec{p}|, |\vec{k}| \sim \beta$　($m \sim 1,\ \beta$ = $Q, \bar{Q}$の重心系での速度 $\ll 1$)

Hard region: $l^0 \sim 1,\ |\vec{l}| \sim 1$

Soft region: $l^0 \sim \beta,\ |\vec{l}| \sim \beta$

Potential region: $l^0 \sim \beta^2,\ |\vec{l}| \sim \beta$

ex. ……$2p_1 \cdot l + l^2 \approx 2\sqrt{\vec{p}^2 + m^2}\, l^0 - 2\vec{p} \cdot \vec{l} + (l^0)^2 - \vec{l}^2$

($\beta^2$, $\beta^2$, $\beta^4$, $\beta^2$)

$O(\beta^2)$ の同次式

Ultrasoft region: $l^0 \sim \beta^2, |\vec{l}| \sim \beta^2$

# 2(b) EFT：Potential-NRQCD

Brambilla, Pineda, Soto, Vairo

EFT of a $Q\bar{Q}$ bound state and Ultrasoft gluons.

$\alpha_s \sim \beta$

EFTのdynamicalな自由度:

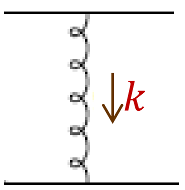
$Q, \bar{Q}$： $p^0 - m, \bar{p}^0 - m \lesssim \beta^2 m,\ |\vec{p}| \lesssim \beta m$ Potential quark, antiquark

$g$： $k^0, |\vec{k}| \lesssim \beta^2 m$ Ultrasoft gluon

他の自由度はintegrate out

Hard $Q, \bar{Q}, g$
Soft $Q, \bar{Q}, g$
Potential $g$

はintegrate out




$k^0 \sim \beta^2$
$|\vec{k}| \sim \beta$

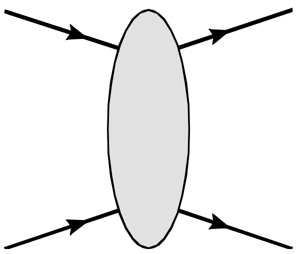
Potential gluon

## Potential-NRQCD

$$Z = \int \mathcal{D}A_\mu \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger \exp\left[ i \int dt\, d^3\vec{x}_1 d^3\vec{x}_2\, \psi^\dagger \{ iD_t - \hat{H}(\vec{x}_1, \vec{x}_2, \vec{p}_1 - ig\vec{A}, \vec{p}_2 - ig\vec{A}) \} \psi \right]$$

## Potential-NRQCD

$$Z=\int \mathcal{D}A_\mu \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger \exp\left[i\int dt\, d^3\vec{x}_1 d^3\vec{x}_2\, \psi^\dagger\{iD_t-\hat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1-ig\vec{A},\vec{p}_2-ig\vec{A})\}\psi\right]$$




On-shell散乱振幅を full pert. QCD と pNRQCD で matching.

$\alpha_s \sim \beta$ の展開の各次で.

量子力学の運動方程式に従う束縛状態とUS gluonの相互作用を表す

$$\left[i\frac{\partial}{\partial t}-\hat{H}\right]\psi(\vec{x}_1,\vec{x}_2;t)=0$$

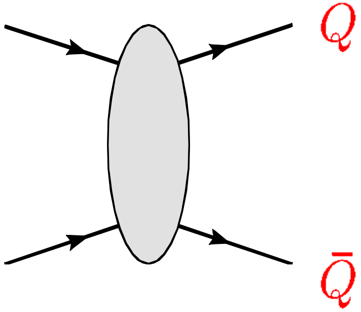
$$\hat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1,\vec{p}_2)=\frac{\vec{p}_1^{\,2}}{2m}+\frac{\vec{p}_2^{\,2}}{2m}-\frac{C_F\alpha_s}{|\vec{x}_1-\vec{x}_2|}+\frac{C_1}{m^2}\delta^3(\vec{x}_1-\vec{x}_2)+\frac{C_2}{m^2r^3}\vec{L}\cdot\vec{S}+\cdots$$

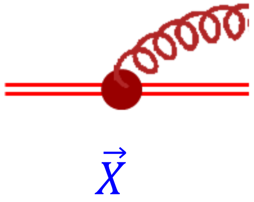
$$=\sum_i \frac{1}{m^{n_i}}\hat{O}_i(\vec{x}_1,\vec{x}_2,\vec{p}_1,\vec{p}_2)$$ ⇦ $\frac{1}{m}$ 展開 = non-rel.展開 = $\beta$展開

## Potential-NRQCD

$$Z=\int \mathcal{D}A_\mu \int \mathcal{D}\psi\,\mathcal{D}\psi^\dagger\ \exp\left[i\int dt\, d^3\vec{x}_1 d^3\vec{x}_2\,\psi^\dagger\{iD_t-\underline{\widehat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1-ig\vec{A},\vec{p}_2-ig\vec{A})}\}\psi\right]$$

ゲージ場を多重極展開




On-shell散乱振幅を full pert. QCD と pNRQCD で matching.

$\alpha_s \sim \beta$ の展開の各次で.

⇨ $\mathcal{L}_{\mathrm{pNRQCD}}$ is now known up to NNNLO.

Titard, Yndurain; Pineda, Yndurain
Kniehl, Penin, Smirnov, Steinhauser
Kiyo, Anzai, YS; Smirnov, Steinhauser

$$\hat{H}_0 = \frac{\vec{p}^{\,2}}{m} - C_F \frac{\alpha_S}{r},$$

$$\hat{H}_1 = - C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right) \cdot \left\{\beta_0 \log(\mu'^2 r^2) + a_1\right\},$$

$$\begin{aligned}\hat{H}_2 = & -\frac{\vec{p}^{\,4}}{4m^3} - C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right)^2 \cdot \left\{\beta_0^2 \left[\log^2(\mu'^2 r^2) + \frac{\pi^2}{3}\right] + (\beta_1 + 2\beta_0 a_1)\log(\mu'^2 r^2) + a_2\right\} \\ & + \frac{\pi C_F \alpha_S}{m^2}\delta^3(\vec{r}) + \frac{3 C_F \alpha_S}{2m^2 r^3}\vec{L}\cdot\vec{S} - \frac{C_F \alpha_S}{2m^2 r}\left(\vec{p}^{\,2} + \frac{1}{r^2} r_i r_j p_j p_i\right) - \frac{C_A C_F \alpha_S^2}{2mr^2} \\ & - \frac{C_F \alpha_S}{2m^2}\left\{\frac{S^2}{r^3} - 3\frac{(\vec{S}\cdot\vec{r})^2}{r^5} - \frac{4\pi}{3}(2S^2 - 3)\delta^3(\vec{r})\right\},\end{aligned}$$

$\widehat{H}_3 =$ known

$\widehat{H}_4 =$ largely unknown

## 当面の目標

$\mathrm{N}^4\mathrm{LO}$ハミルトニアン $\hat{H}_4$ のうち2ループまでで計算できる部分を計算

実用価値： level splittingの高次補正計算に使える。

## 現状

- 2-loop $Q\bar{Q}$ scattering diagram生成
- スピノル基底での展開 ⇒ スカラー積分
- IBP idによる簡約化 ⇒ 150個のマスター積分

## 今後

- マスター積分の1/m展開による評価
- Expansion-by-regionsとのconsistency check
- くりこみ
- ハミルトニアンの決定(matching/exp.-by-regions)

## 当面の目標

$\mathrm{N^4LO}$ハミルトニアン $\hat{H}_4$ のうち2ループまでで計算できる部分を計算

実用価値： level splittingの高次補正計算に使える。

## 課題

・Soft と potential regionの境界の不定性

pinch singularity, single-pole potential contr.

・Offshell operator の同定

vanish for onshell tree-level amplitude

くり返しの効果を見極めないとExp.-by-regionsを壊すように見える。

1-loopでは解決済み。
2-loopではどうなるか。

# まとめ

1. Heavy quarkoniumの物理のmotivation: 現状、応用

2. Theory

   (a) Expansion-by-regions technique
   (b) EFT potential-NRQCD

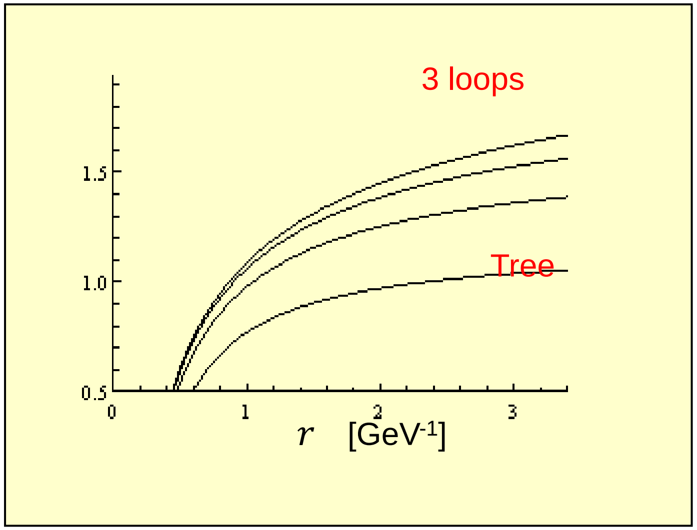
   } 両輪

$$\mathcal{L}_{\mathrm{pNRQCD}} = \psi^\dagger(\vec{x}_1,\vec{x}_2;t)\left\{iD_t - \widehat{H}\left(\vec{x}_1,\vec{x}_2,\vec{p}_1 - ig\vec{A},\vec{p}_2 - ig\vec{A}\right)\right\}\psi(\vec{x}_1,\vec{x}_2;t)$$

& ゲージ場を多重極展開 $A_\mu(\vec{X} \pm \frac{\vec{r}}{2},t) \to A_\mu(\vec{X},t) \pm \frac{\vec{r}}{2}\cdot\nabla A_\mu(\vec{X},t) + \cdots$

3. $\widehat{H}_4$ のうち2ループ以下の部分の計算

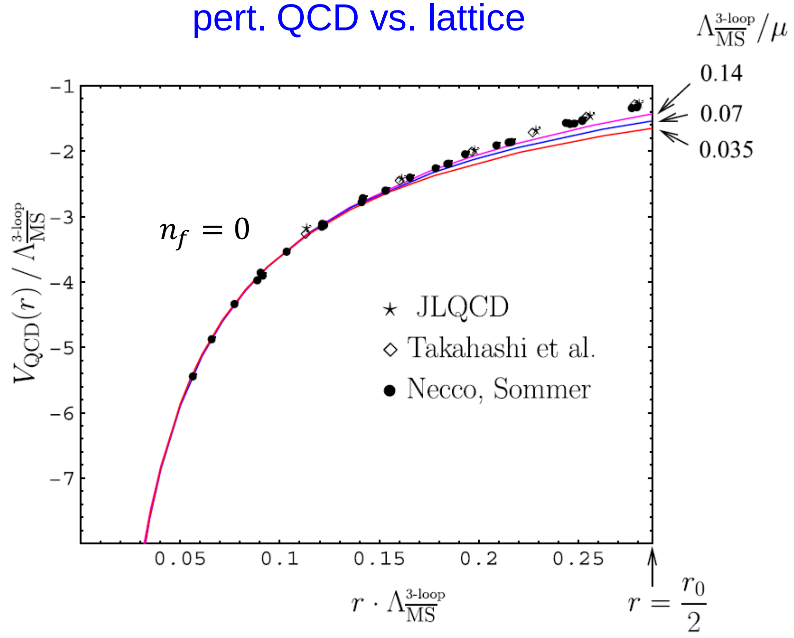
   現状、今後、課題




✓

# QCD Potential

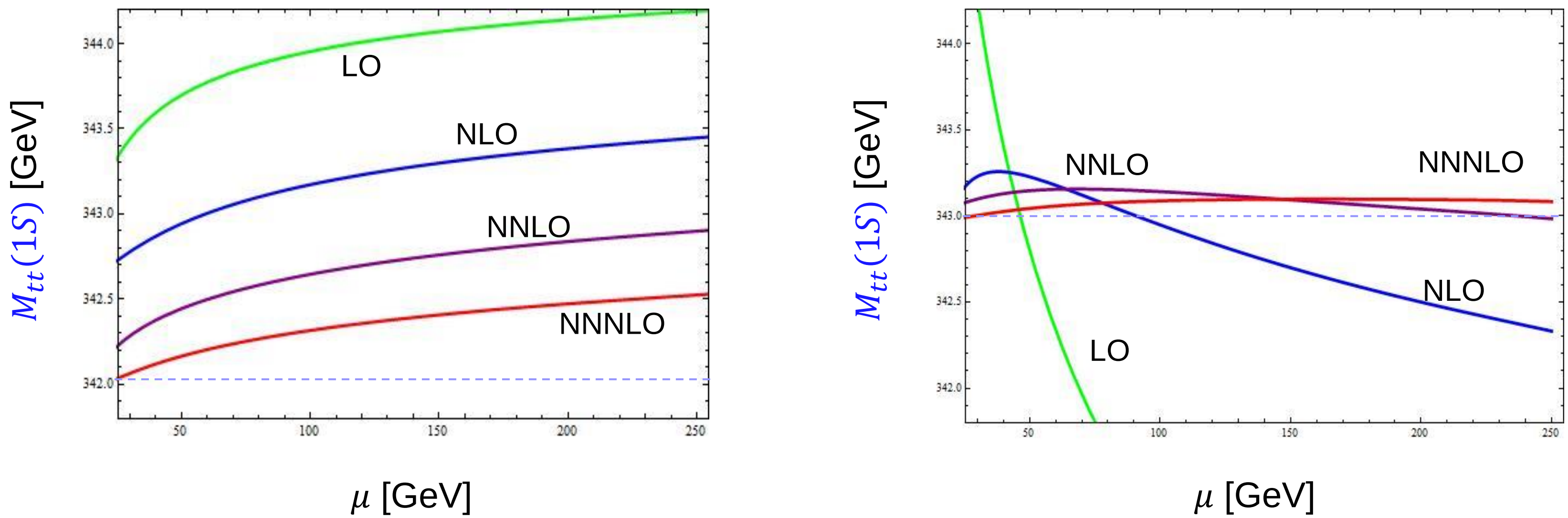
pert. QCD vs. lattice




Anzai, Kiyo, YS

Smirnov, Smirnov, Steinhauser

# μ dependence and convergence of $M_{Q\bar{Q}}(1S)$




- $\Upsilon(1S)$ : $M_{\Upsilon(1S)}$ $= \mathbf{9.94 - 0.10 - 0.15 - 0.20 - 0.26}$ GeV (Pole-mass scheme)

  $= \mathbf{8.43 + 0.72 + 0.25 + 0.07 - 0.02}$ GeV ($\overline{\text{MS}}$ scheme)

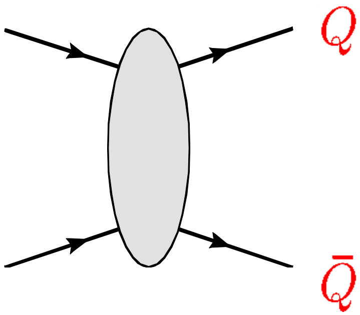
- $\Upsilon(2S)$ : $M_{\Upsilon(2S)}$ $= \mathbf{9.94 - 0.06 - 0.11 - 0.22 - 0.41}$ GeV (Pole-mass scheme)

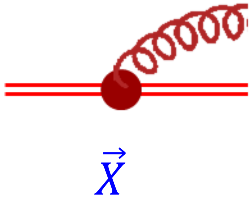
  $= \mathbf{8.43 + 1.17 + 0.26 + 0.10 - 0.04}$ GeV ($\overline{\text{MS}}$ scheme)

## Potential-NRQCD

$$Z=\int \mathcal{D}A_\mu \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger \,\exp\left[ i\int dt\, d^3\vec{x}_1 d^3\vec{x}_2\, \psi^\dagger\{ i\underline{D_t} - \hat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1 - ig\underline{\vec{A}}, \vec{p}_2 - ig\underline{\vec{A}})\}\psi \right]$$

ゲージ場を多重極展開




On-shell散乱振幅を full pert. QCD と pNRQCD で matching.

$\alpha_s \sim \beta$ の展開の各次で.

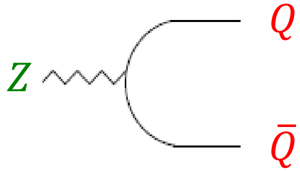
⇨ $\mathcal{L}_{\rm pNRQCD}$ is now known up to NNNLO.

昔はsystematicな展開は出来なかった。

---

補足

$\psi$ を量子化する意義




$$\int d^4x\ \vec{Z}(x)\cdot\vec{\sigma}_{ij}\ \psi^\dagger(\vec{x},\vec{x};t)_{ikjl}$$

大部分では$Q$, $\bar{Q}$数が保存するので量子力学系と等価。

# EFTのDynamicalな自由度:

$Q,\bar{Q}$ :  $p^0 - m, \bar{p}^0 - m \lesssim \beta^2 m (\sim \alpha_s^2 m)$

$g$ :  $k^0, |\vec{k}| \lesssim \beta^2 m$

他は積分

量子力学の運動方程式に従う2体系

$$\left[i\frac{\partial}{\partial t} - \hat{H}\right]\psi(\vec{x}_1,\vec{x}_2;t) = 0$$

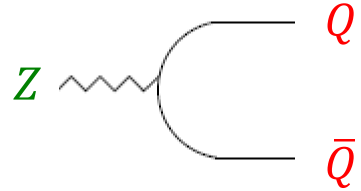
$$\hat{H} = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} - \frac{C_F\alpha_s}{|\vec{x}_1 - \vec{x}_2|} + \frac{C_1}{m^2}\delta^3(\vec{x}_1 - \vec{x}_2) + \frac{C_2}{m^2 r^3}\vec{L}\cdot\vec{S} + \cdots$$

$$= \sum_i \frac{1}{m^{n_i}}\hat{O}_i(\vec{x}_1,\vec{x}_2,\vec{p}_1,\vec{p}_2)$$

⇦ $\dfrac{1}{m}$ 展開 = non-rel.展開

$$Z = \int \mathcal{D}A_\mu \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger\ \exp\left[i\int dt\, d^3\vec{x}_1 d^3\vec{x}_2\, \psi^\dagger\{iD_t - \hat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1 - ig\vec{A},\vec{p}_2 - ig\vec{A})\}\psi\right]$$

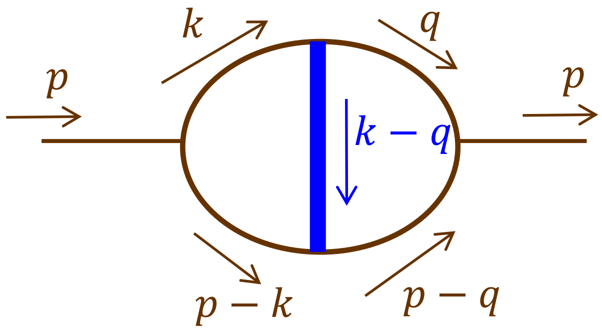
$\psi$ を量子化する意義 ⟶




$$\int d^4x\ \vec{Z}(x)\cdot \mathrm{tr}[\vec{\sigma}\,\psi^\dagger(\vec{x},\vec{x};t)]$$

それ以外だと粒子数が保存するので
量子力学系と等価。

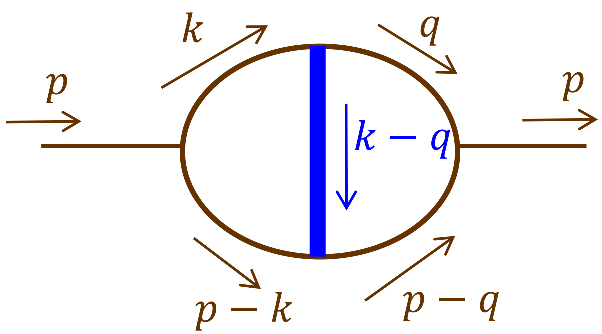
# Asymptotic expansion of a diagram and Wilson coeffs in EFT




$$= \int d^D k \, d^D q \, \frac{1}{k^2 (p-k)^2 [(k-q)^2 + M^2] \, q^2 (p-q)^2}$$

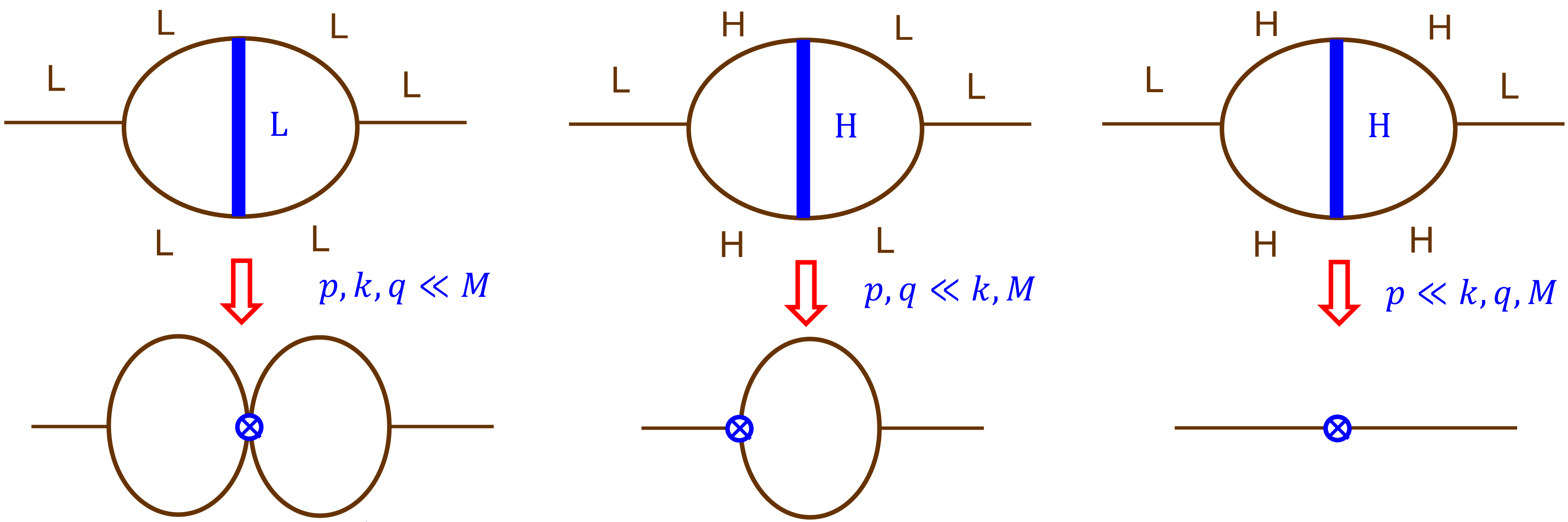
in the case $p^2 \ll M^2$

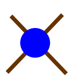
# Asymptotic expansion of a diagram and Wilson coeffs in EFT

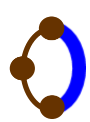


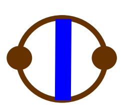

$$= \int d^D k \, d^D q \, \frac{1}{k^2 (p-k)^2 [(k-q)^2 + M^2] \, q^2 (p-q)^2}$$

in the case $p^2 \ll M^2$




$\otimes$ =  $= \dfrac{1}{M^2}$

$\otimes$ =  $= \displaystyle\int \frac{d^D k}{k^4 [k^2 + M^2]}$

$\otimes$ =  $= \displaystyle\int \frac{d^D k \, d^D q}{k^4 [(k-q)^2 + M^2] \, q^4}$

Operators and Wilson coeffs in EFT

# Expansion-by-regions Technique

EFTがfull theoryのloop積分をどのように分解しているのかを説明する。

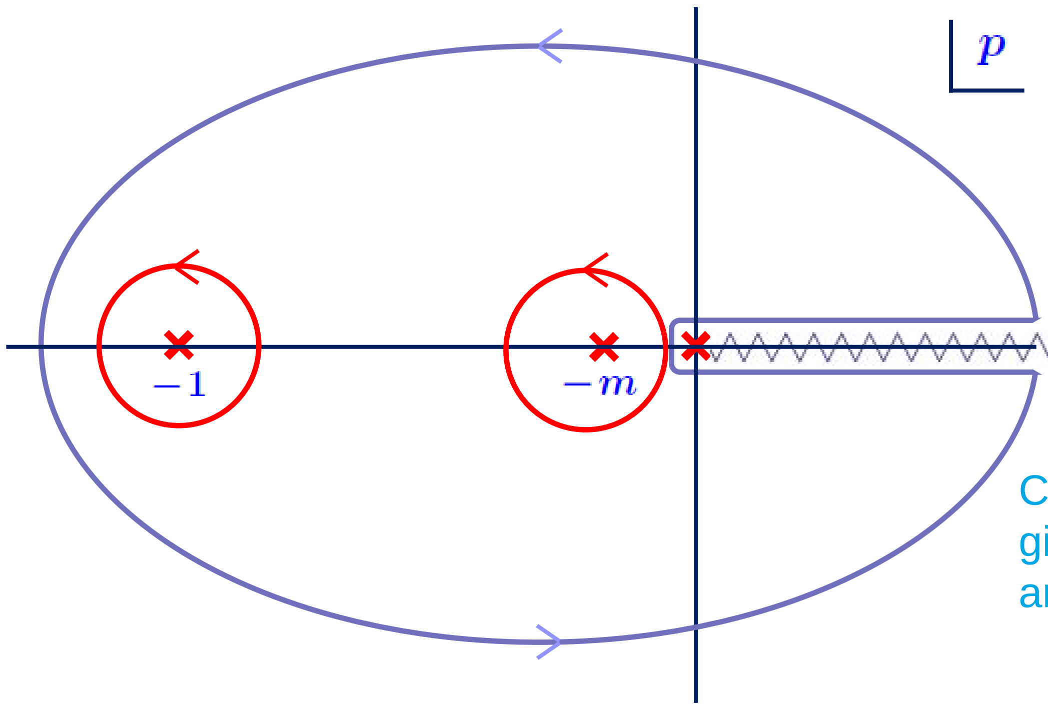
Simplified example:

Beneke, Smirnov

$$\int_0^\infty dp \frac{p^\varepsilon}{(p+m)(p+1)} \qquad m \ll 1 \, (= M)$$

$$= \int_0^\infty dp \frac{p^\varepsilon}{p+m}\left(1 - p + p^2 + \cdots\right) + \int_0^\infty dp \frac{p^\varepsilon}{p+1}\frac{1}{p}\left(1 - \frac{m}{p} + \cdots\right)$$

$p \ll 1 \qquad\qquad p \gtrsim 1 \gg m$




Contribution of each scale given by contour integral around singularity

## 応用例

基礎物理定数の決定： $m_c, m_b, (m_t), \alpha_s, |V_{cb}|$

⇨ Kiyo, Mishima, Takaura, Hayashi 他との共同研究

**Particle Data Group 2016**

**c**

$I(J^P) = 0(\frac{1}{2}^+)$

Charge $= \frac{2}{3}\ e$　　Charm $= +1$

### c-QUARK MASS

The c-quark mass corresponds to the "running" mass $m_c$ ($\mu = m_c$) in the $\overline{\rm MS}$ scheme. We have converted masses in other schemes to the

| VALUE (GeV) | | | DOCUMENT ID | | TECN |
|---|---|---|---|---|---|
| **1.27 ±0.03** | | | **OUR EVALUATION** See the ideogram below. | | |
| 1.246 | ±0.023 | | [1] KIYO | 16 | THEO |
| 1.2715 | ±0.0095 | | [2] CHAKRABOR... | 15 | LATT |
| 1.288 | ±0.020 | | [3] DEHNADI | 15 | THEO |
| 1.348 | ±0.046 | | [4] CARRASCO | 14 | LATT |
| 1.26 | ±0.05 | ±0.04 | [5] ABRAMOWICZ | 13C | COMB |
| 1.24 | ±0.03 | $^{+0.03}_{-0.07}$ | [6] ALEKHIN | 13 | THEO |
| 1.282 | ±0.011 | ±0.022 | [7] DEHNADI | 13 | THEO |
| 1.286 | ±0.066 | | [8] NARISON | 13 | THEO |
| 1.159 | ±0.075 | | [9] SAMOYLOV | 13 | NOMD |
| 1.36 | ±0.04 | ±0.10 | [10] ALEKHIN | 12 | THEO |

**b**

$I(J^P) = 0(\frac{1}{2}^+)$

Charge $= -\frac{1}{3}\ e$　　Bottom $= -1$

### b-QUARK MASS

The first value is the "running mass" $\overline{m}_b(\mu = \overline{m}_b)$ in the $\overline{\rm MS}$ scheme, and the second value is the 1S mass, which is half the mass of the $\Upsilon(1S)$

| $\overline{\rm MS}$ MASS (GeV) | 1S MASS (GeV) | DOCUMENT ID | | TECN |
|---|---|---|---|---|
| **4.18 $^{+0.04}_{-0.03}$** | **OUR EVALUATION** of $\overline{\rm MS}$ Mass. See the ideogram below. | | | |
| **4.66 $^{+0.04}_{-0.03}$** | **OUR EVALUATION** of 1S Mass. See the ideogram below. | | | |
| 4.197±0.022 | 4.671 ± 0.024 | [1] KIYO | 16 | THEO |
| 4.183±0.037 | 4.656 ± 0.041 | [2] ALBERTI | 15 | THEO |
| 4.193 $^{+0.022}_{-0.035}$ | 4.667 $^{+0.024}_{-0.039}$ | [3] BENEKE | 15 | THEO |
| 4.176±0.023 | 4.648 ± 0.026 | [4] DEHNADI | 15 | THEO |
| 4.07 ±0.17 | 4.53 ± 0.19 | [5] ABRAMOWICZ | 14A | HERA |
| 4.201±0.043 | 4.676 ± 0.048 | [6] AYALA | 14A | THEO |
| 4.21 ±0.11 | 4.69 ± 0.12 | [7] BERNARDONI | 14 | LATT |
| 4.169±0.002±0.008 | 4.640 ± 0.002 ± 0.009 | [8] PENIN | 14 | THEO |
| 4.166±0.043 | 4.637 ± 0.048 | [9] LEE | 13O | LATT |
| 4.247±0.034 | 4.727 ± 0.039 | [10] LUCHA | 13 | THEO |
| 4.236±0.069 | 4.715 ± 0.077 | [11] NARISON | 13 | THEO |

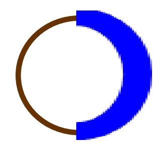
# Expansion-by-regions Technique

Beneke, Smirnov

EFTがfull theoryのループ積分をどのように
分解しているかの理論的基礎付けを与える。

Simple example (2 scales): $m^2 \ll M^2$

$$M^{D-4} f\left(\frac{m^2}{M^2}\right) = \int d^D p \, \frac{1}{(p^2+m^2)(p^2+M^2)}$$



$$= \int d^D p \, \frac{1}{p^2+m^2} \hat{T}_{p^2}\left[\frac{1}{p^2+M^2}\right] + \int d^D p \, \hat{T}_{m^2}\left[\frac{1}{p^2+m^2}\right] \frac{1}{p^2+M^2}$$

$$\frac{1}{M^2}\left(1-\frac{p^2}{M^2}+\frac{p^4}{M^4}-\frac{p^6}{M^6}+\cdots\right) \qquad \frac{1}{p^2}\left(1-\frac{m^2}{p^2}+\frac{m^4}{p^4}-\frac{m^6}{p^6}+\cdots\right)$$

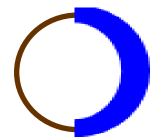
# Expansion-by-regions Technique

Beneke, Smirnov

EFTがfull theoryのループ積分をどのように
分解しているかの理論的基礎付けを与える。

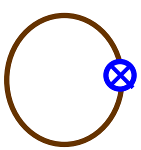
Simple example (2 scales): $m^2 \ll M^2$

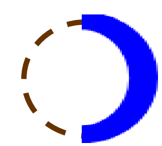
$$M^{D-4} f\left(\frac{m^2}{M^2}\right) = \int d^D p \, \frac{1}{(p^2+m^2)(p^2+M^2)}$$



$p^2 \lesssim m^2 \ll M^2$ $\qquad$ $m^2 \ll M^2 \lesssim p^2$

$$= \int d^D p \, \frac{1}{p^2+m^2} \hat{T}_{p^2}\left[\frac{1}{p^2+M^2}\right] + \int d^D p \, \hat{T}_{m^2}\left[\frac{1}{p^2+m^2}\right] \frac{1}{p^2+M^2}$$





$$\frac{1}{M^2}\left(1 - \frac{p^2}{M^2} + \frac{p^4}{M^4} - \frac{p^6}{M^6} + \cdots\right) \qquad \frac{1}{p^2}\left(1 - \frac{m^2}{p^2} + \frac{m^4}{p^4} - \frac{m^6}{p^6} + \cdots\right)$$

# EFTのdynamicalな自由度:

$Q, \bar{Q}$ :  $p^0 - m, \bar{p}^0 - m \lesssim \beta^2 m, \ |\vec{p}| \lesssim \beta m$

$g$ :  $k^0, \left|\vec{k}\right| \lesssim \beta^2 m$

他はintegrate out

量子力学の運動方程式に従う2体系


$$\left[i\frac{\partial}{\partial t} - \hat{H}\right]\psi(\vec{x}_1, \vec{x}_2; t) = 0$$

$$\hat{H} = \frac{\vec{p}_1^2}{2m} + \frac{\vec{p}_2^2}{2m} - \frac{C_F \alpha_s}{|\vec{x}_1 - \vec{x}_2|} + \frac{C_1}{m^2}\delta^3(\vec{x}_1 - \vec{x}_2) + \frac{C_2}{m^2 r^3}\vec{L}\cdot\vec{S} + \cdots$$

$$= \sum_i \frac{1}{m^{n_i}} \hat{O}_i(\vec{x}_1, \vec{x}_2, \vec{p}_1, \vec{p}_2)$$

⇦ $\frac{1}{m}$ 展開 ≠ $\beta$展開

IR gluonとの結合:  $i\frac{\partial}{\partial t} \to iD_t$ ,  $i\vec{p}_k \to \vec{\nabla}_k - ig\vec{A}(\vec{x}_k, t)$

多重極展開:

$\vec{r} \equiv \vec{x}_1 - \vec{x}_2$ とすると $r \sim p^{-1} \sim (\beta m)^{-1}$ より $rk^0, \ r\left|\vec{k}\right| \sim \beta \ll 1$ で展開。

$$A_\mu\left(\vec{X} \pm \frac{\vec{r}}{2}, t\right) \to A_\mu(\vec{X}, t) \pm \frac{\vec{r}}{2}\cdot\nabla A_\mu(\vec{X}, t) + \frac{1}{8} r_i r_j \partial_i \partial_j A_\mu(\vec{X}, t) + \cdots$$

## Potential-NRQCD

$$Z=\int \mathcal{D}A_\mu \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger \exp\left[ i\int dt\, d^3\vec{x}_1 d^3\vec{x}_2\, \psi^\dagger\{i\underline{D_t} - \hat{H}(\vec{x}_1,\vec{x}_2,\vec{p}_1 - ig\underline{\vec{A}}, \vec{p}_2 - ig\underline{\vec{A}})\}\psi \right]$$

ゲージ場を多重極展開

量子力学の運動方程式に従う2体系

$\vec{X}$

$$\left[ i\frac{\partial}{\partial t} - \hat{H} \right] \psi(\vec{x}_1,\vec{x}_2;t) = 0$$

$$\hat{H} = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} - \frac{C_F\alpha_s}{|\vec{x}_1-\vec{x}_2|} + \frac{C_1}{m^2}\delta^3(\vec{x}_1-\vec{x}_2) + \frac{C_2}{m^2 r^3}\vec{L}\cdot\vec{S} + \cdots$$

$$= \sum_i \frac{1}{m^{n_i}} \hat{O}_i(\vec{x}_1,\vec{x}_2,\vec{p}_1,\vec{p}_2)$$

⇦ $\frac{1}{m}$ 展開 ≠ $\beta$展開

IR gluonとの結合： $i\frac{\partial}{\partial t} \to iD_t\,,\qquad i\vec{p}_k \to \vec{\nabla}_k - ig\vec{A}(\vec{x}_k,t)$

多重極展開：

$\vec{r} \equiv \vec{x}_1 - \vec{x}_2$ とすると $r \sim p^{-1} \sim (\beta m)^{-1}$ より $rk^0,\ r\left|\vec{k}\right| \sim \beta \ll 1$ で展開。

$$A_\mu\left(\vec{X} \pm \frac{\vec{r}}{2}, t\right) \to A_\mu(\vec{X},t) \pm \frac{\vec{r}}{2}\cdot\nabla A_\mu(\vec{X},t) + \frac{1}{8} r_i r_j \partial_i \partial_j A_\mu(\vec{X},t) + \cdots$$

Titard, Yndurain; Pineda, Yndurain
Kniehl, Penin, Smirnov, Steinhauser
Kiyo, Anzai, YS; Smirnov, Steinhauser

$$\hat{H}_0 = \frac{\vec{p}^{\,2}}{m} - C_F \frac{\alpha_S}{r},$$

$$\hat{H}_1 = -C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right) \cdot \left\{\beta_0 \log(\mu'^2 r^2) + a_1\right\},$$

$$\hat{H}_2 = -\frac{\vec{p}^{\,4}}{4m^3} - C_F \frac{\alpha_S}{r} \cdot \left(\frac{\alpha_S}{4\pi}\right)^2 \cdot \left\{\beta_0^2 \,[\log^2(\mu'^2 r^2) + \frac{\pi^2}{3}] + (\beta_1 + 2\beta_0 a_1)\log(\mu'^2 r^2) + a_2\right\}$$
$$+\frac{\pi C_F \alpha_S}{m^2}\delta^3(\vec{r}) + \frac{3 C_F \alpha_S}{2m^2 r^3}\vec{L}\cdot\vec{S} - \frac{C_F \alpha_S}{2m^2 r}\left(\vec{p}^{\,2} + \frac{1}{r^2} r_i r_j p_j p_i\right) - \frac{C_A C_F \alpha_S^2}{2mr^2}$$
$$-\frac{C_F \alpha_S}{2m^2}\left\{\frac{S^2}{r^3} - 3\frac{(\vec{S}\cdot\vec{r})^2}{r^5} - \frac{4\pi}{3}(2S^2 - 3)\delta^3(\vec{r})\right\},$$

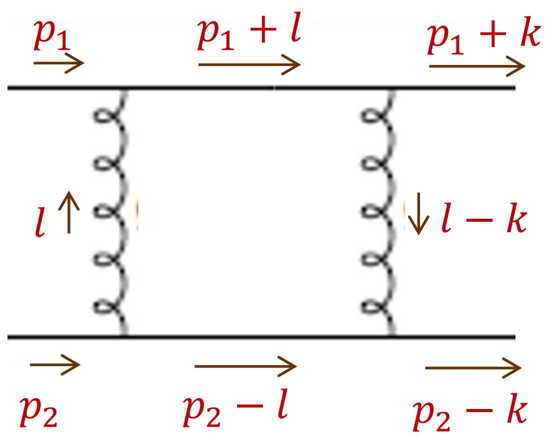
$\widehat{H}_3$ = known

$\widehat{H}_4$ = largely unknown

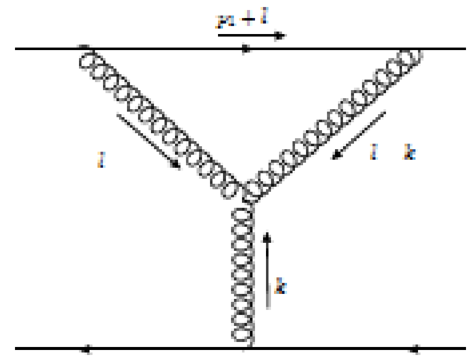
- $\Upsilon(1S)$ : $M_{\Upsilon(1S)} = 8.43 + 0.72 + 0.25 + 0.07 - 0.02$ GeV
- $\Upsilon(2S)$ : $M_{\Upsilon(2S)} = 8.43 + 1.17 + 0.26 + 0.10 - 0.04$ GeV

Kiyo, Mishima, YS

・Soft と potential regionの境界の不定性 (1ループの場合)




pinch singularity



single-pole potential contr.

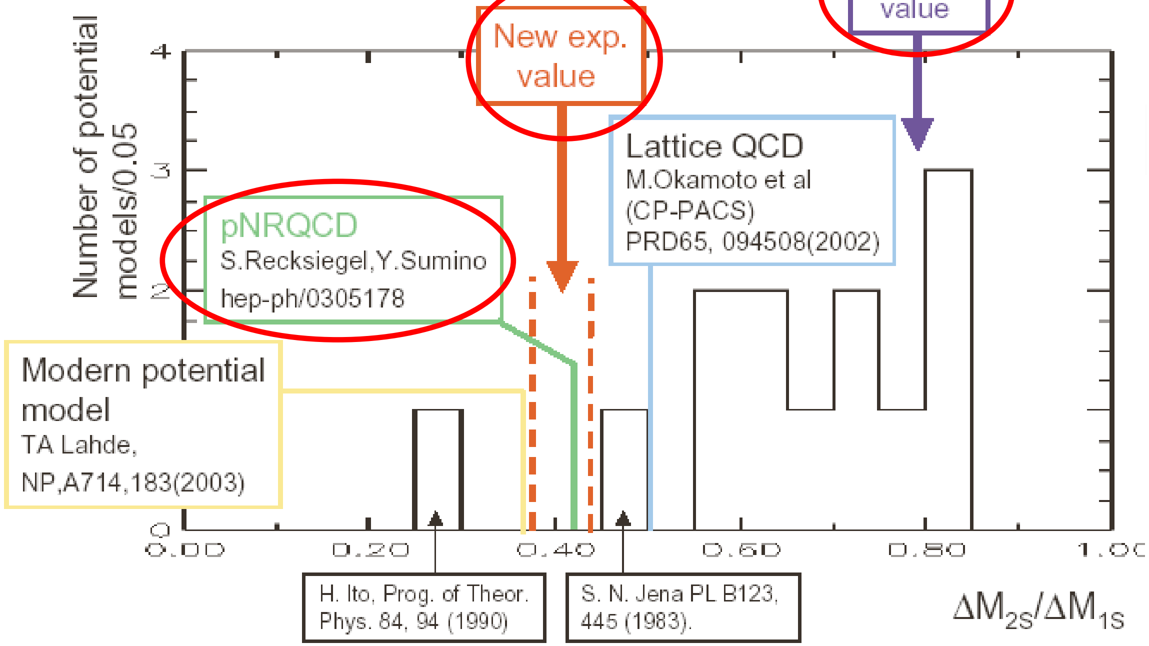
同じ積分でも、ダイアグラムのトポロジーごとに区別して、potential/softの寄与を定義した方が、物理的にはreasonable。（積分ごとに定義することも可能だが、不自然さが不可避。）

Slides from Skwarnicki's plenary talk at Lepton-Photon 2003

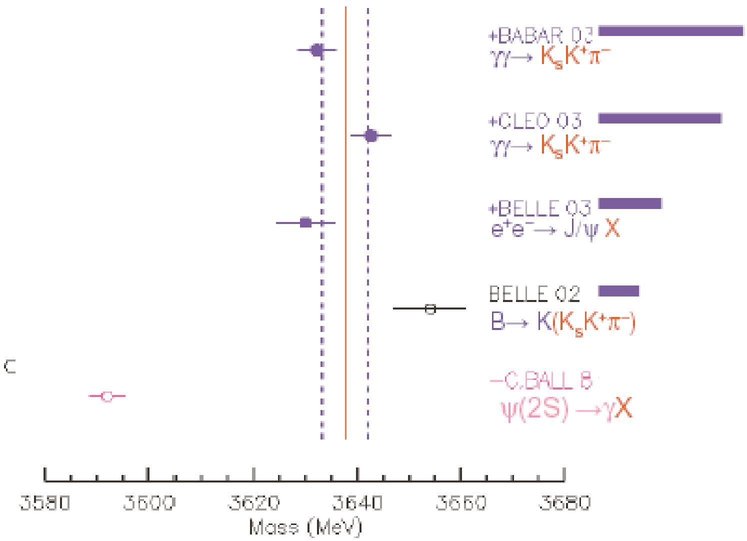


## Predictions for hyperfine splitting ratio

- For 20 years theorists were exposed to the experimental value of $\Delta M_{2S}=M(\psi(2S))-M(\eta_c(2S))$ which was wrong by a factor of 2
- Predictions for $\Delta M_{2S}/\Delta M_{1S}$




3637.7±4.4 MeV




*CL=14% scale factor=1.3*

*New measurements of mass are consistent*