Coleman-Weinberg Abrikosov-Nielsen-Olesen strings

Yu Hamada (KEK)

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Introduction

Topological Soliton

- **Non-perturbative object in field theories**
    - monopole, vortex string, skyrmion, instanton, etc..

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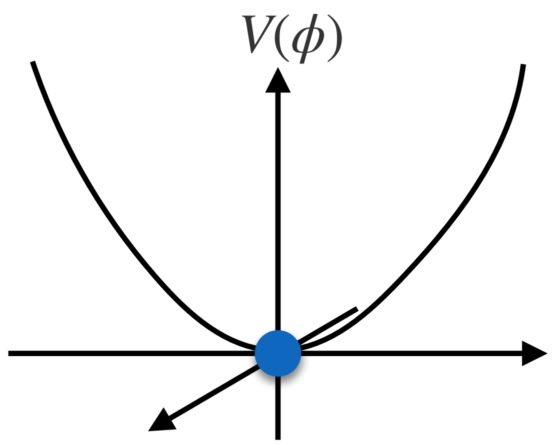
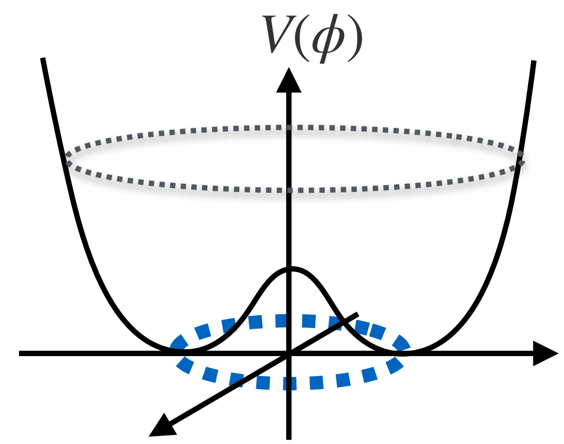
- It appears if **vacuum has non-trivial topology.**



trivial

(point)



non-trivial

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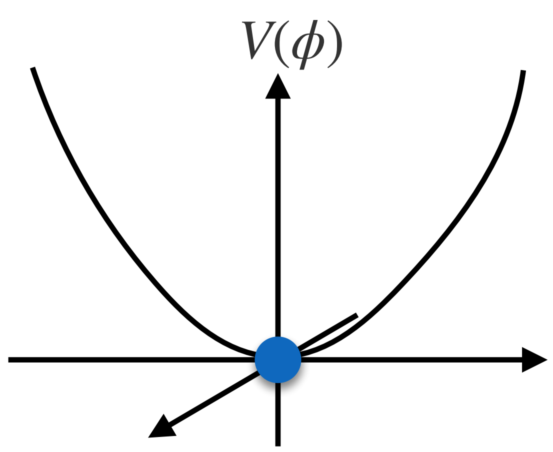

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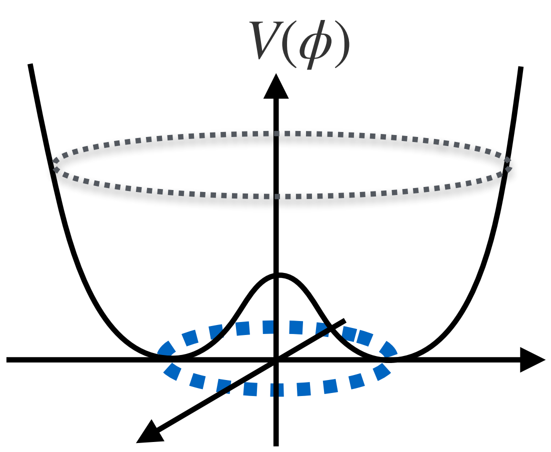

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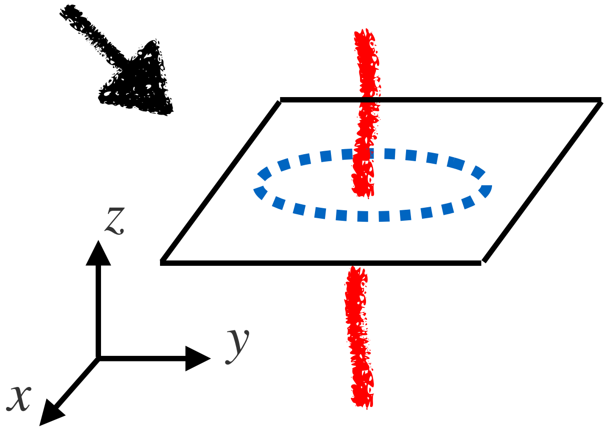

vortex string

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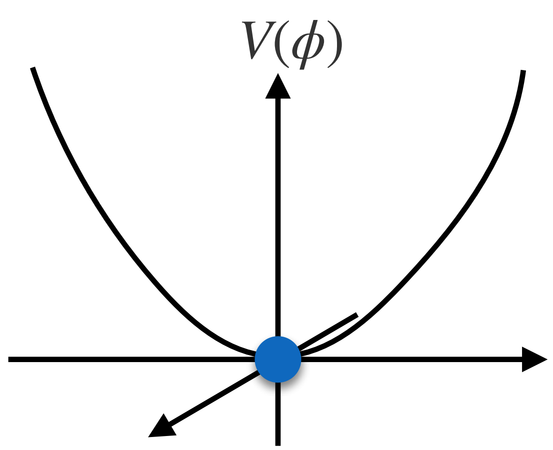

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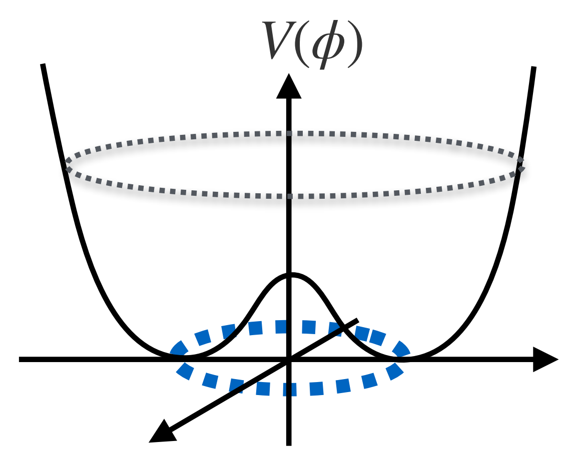

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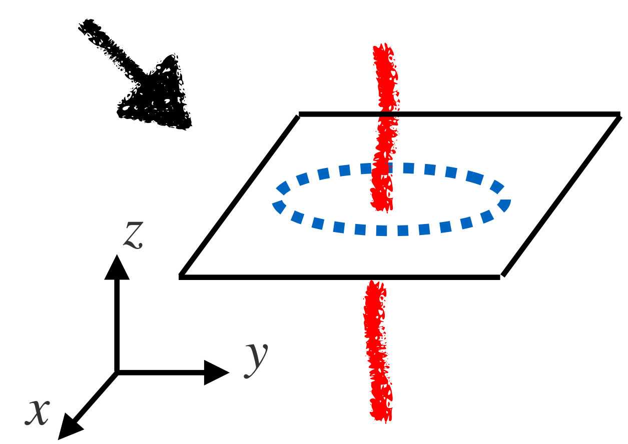


vortex string

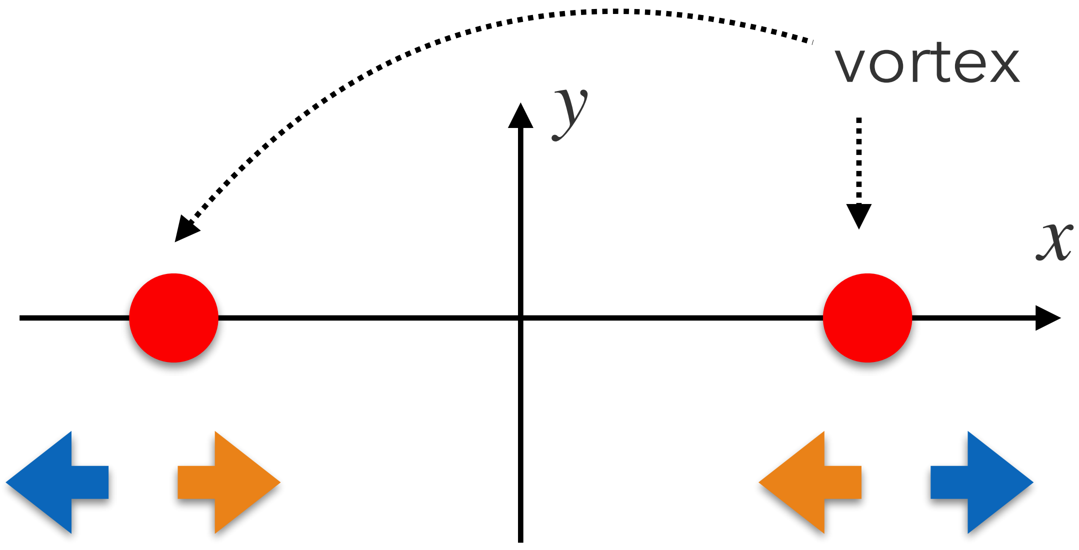
- **Vortex string appears in many systems:**
  - cosmic string, superconductor, neutron star, etc.

Interaction of Vortex Strings

The (most) important question:

interaction between vortex strings

two parallel vortex strings on 2D slice:



repulsive? **attractive?**

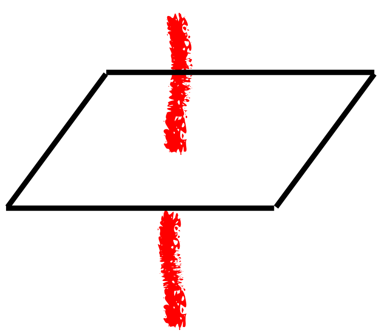
cf.) vortex-antivortex is always attractive

Eg.) Abrikosov-Nielsen-Olesen string

[Abrikosov '58]
[Nielsen-Olesen '73]

- 3+1 D Abelian-Higgs model $\langle\phi\rangle = v \rightarrow \cancel{U(1)}$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$



- vortex string w/ mag. flux → called **ANO string**

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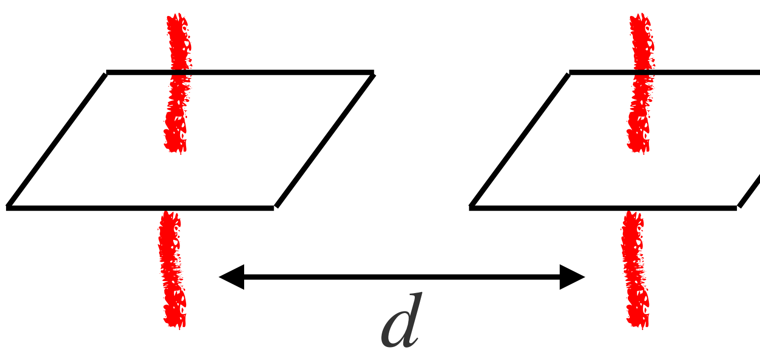
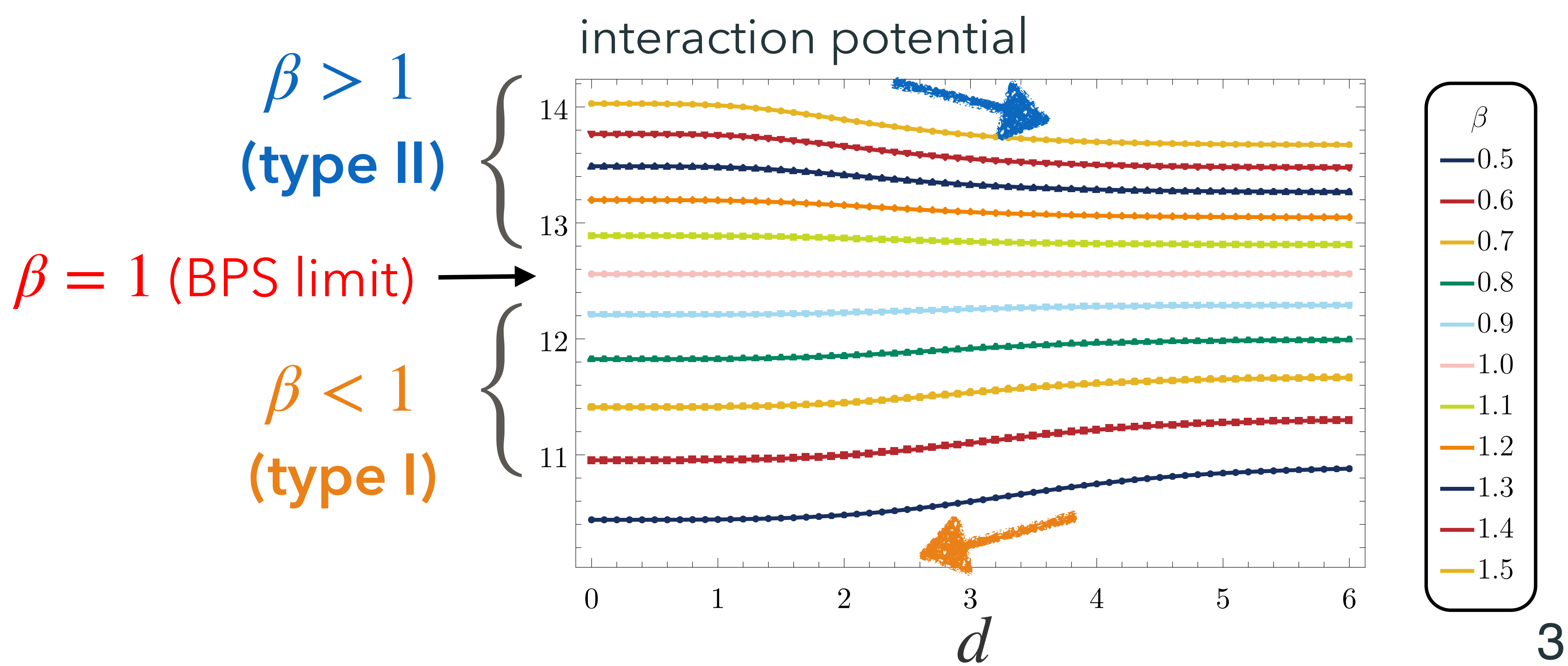


- vortex string w/ mag. flux → called **ANO string**

- Interaction force is **repulsive** or **attractive** depending on $\beta \equiv \frac{m_\phi^2}{m_A^2}$



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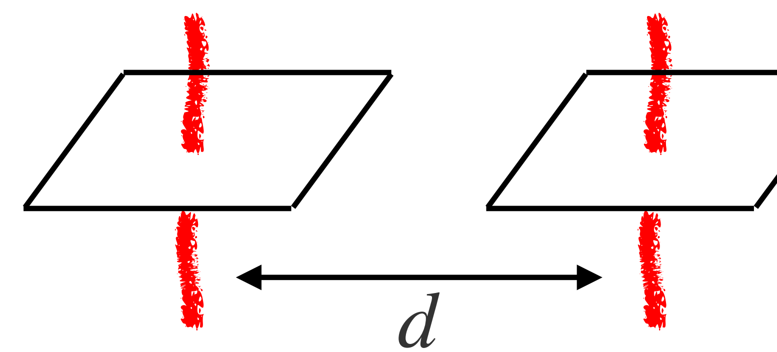

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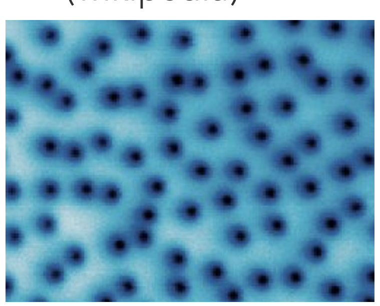
(wikipedia)



Abrikosov lattice

stable superconductor

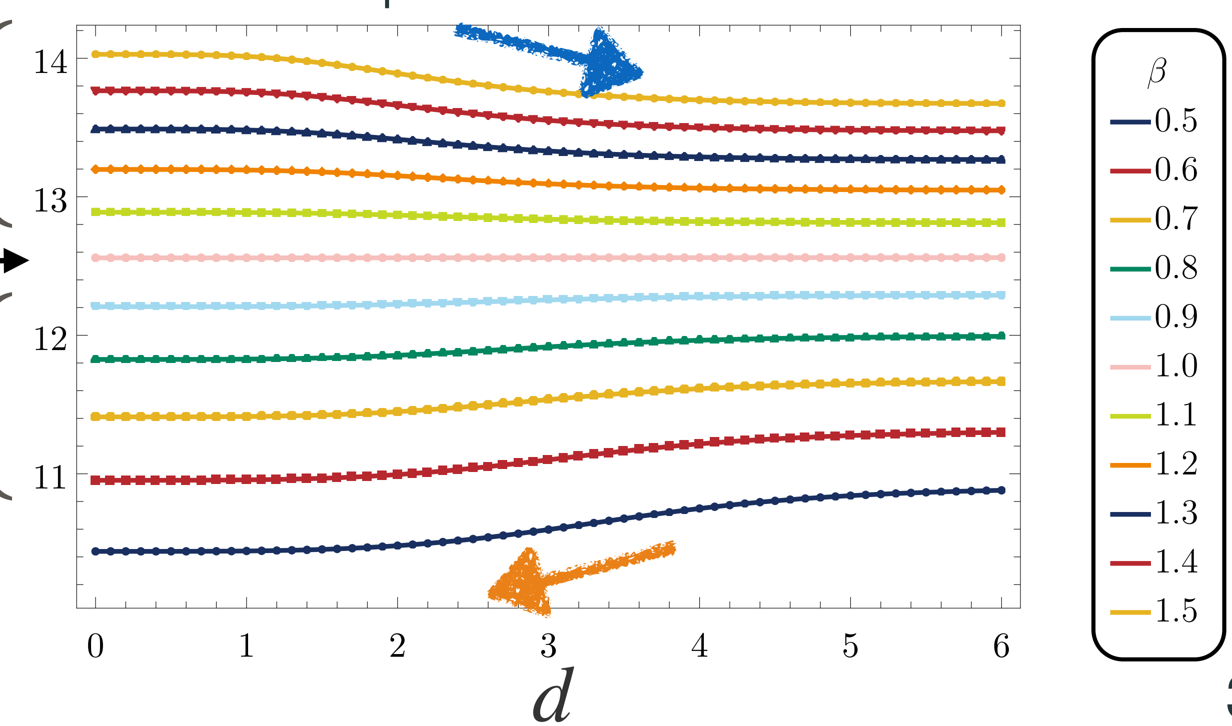
$\beta > 1$ **(type II)**

$\beta = 1$ (BPS limit)

$\beta < 1$ **(type I)**

unstable supercond.
($U(1)$ restored)

interaction potential



Coleman-Weinberg potential

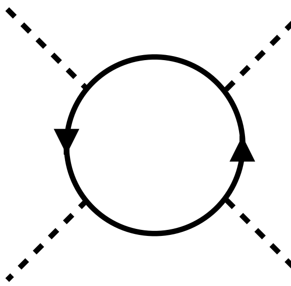
[Coleman-Weinberg '73]

- **Coleman-Weinberg potential w/o quadratic term:**

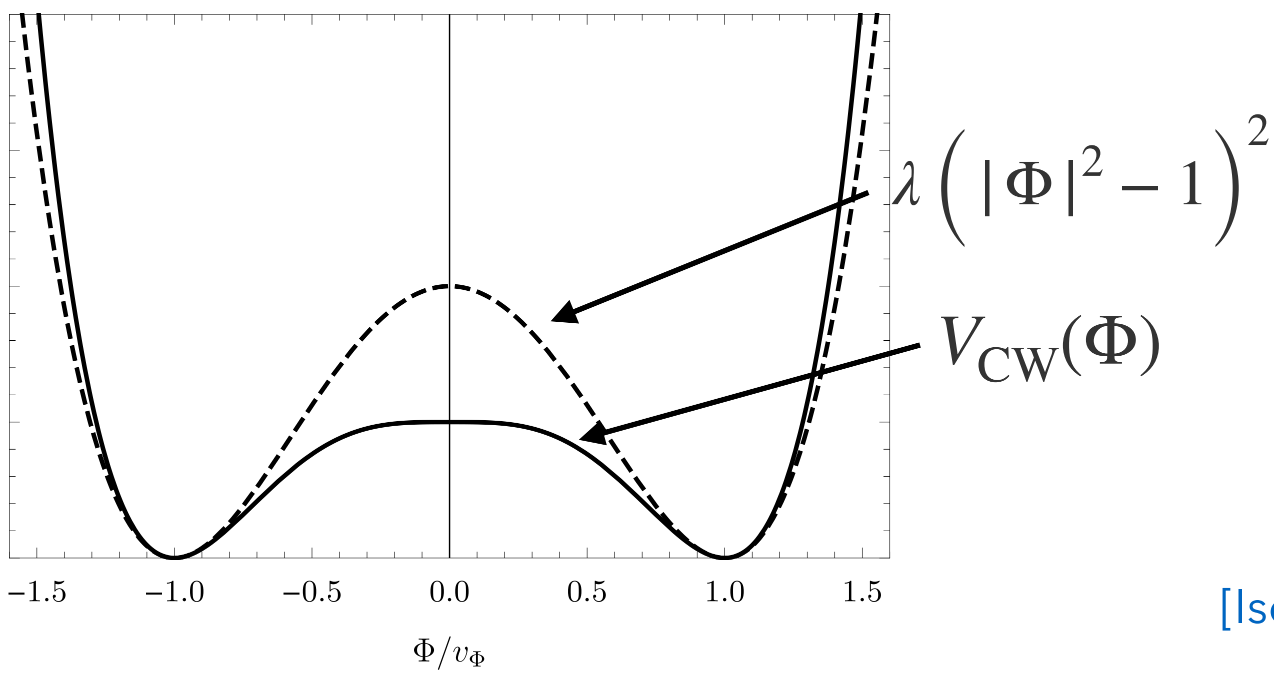
$$V_{\rm CW}(\Phi) = \lambda(\Phi)\,|\,\Phi\,|^4$$

$\lambda(\Phi)$: **running quartic coupling**

- tree level → λ=const., **scale invariant, no SSB**
- quantum effects → $\lambda(\Phi) = \lambda_{\rm CW}\left(\log\frac{|\Phi|^2}{v^2} - \frac{1}{2}\right)$, **triggers SSB**



depend on underlying d.o.f.



- flatter structure around origin
- scale is induced by quantum effect

→possible solution of hierarchy problem

[Iso-Okada-Orikasa '09] [Iso-Orikasa '12] [Chway+ '13]

Coleman-Weinberg potential

[Coleman-Weinberg '73]

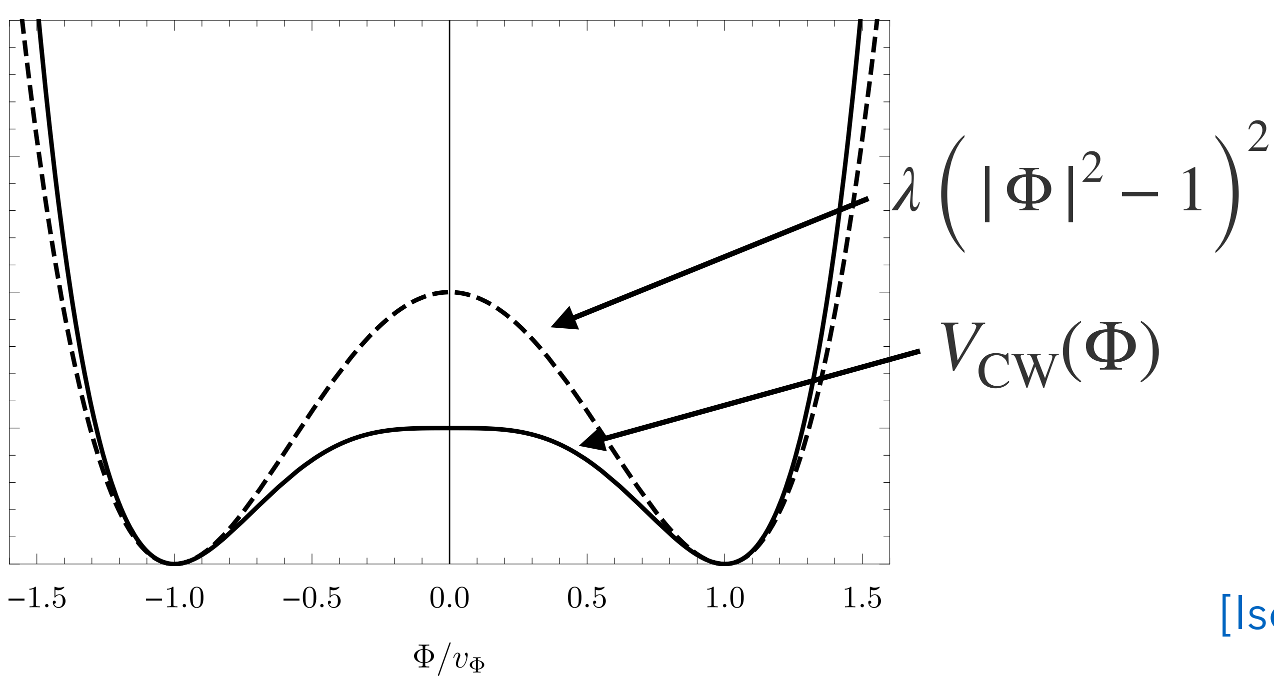
- **Coleman-Weinberg potential w/o quadratic term:**

$$V_{\rm CW}(\Phi) = \lambda(\Phi)\,|\Phi|^4$$

$\lambda(\Phi)$: running quartic coupling

Does this potential affect interaction of vortices? → Yes!!

depend on underlying d.o.f.



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[Iso-Okada-Orikasa '09] [Iso-Orikasa '12] [Chway+ '13]

Plan of talk

- Introduction ←Done
- CW-ANO string
- Interaction of CW-ANO string
- Summary

CW-ANO string

Model

- **3+1 D Abelian-Higgs model w/ two types of potential**

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) \right] \qquad D_\mu = \partial_\mu + igA_\mu$$

- **usual Quadratic-Quartic**

$$V(\Phi) = \lambda \left(|\Phi|^2 - v^2 \right)^2$$

- **Coleman-Weinberg**

$$V(\Phi) = \lambda \left(\log \frac{|\Phi|^2}{v^2} - \frac{1}{2} \right) |\Phi|^4$$

- Both models spontaneously break $U(1)$ sym and have vortex strings.
  - Quadratic-Quartic → conventional ANO string
  - Coleman-Weinberg → **CW-ANO string!** (main interest)

Model

- It is convenient to introduce rescaling: $A_\mu \to A_\mu / g \qquad \Phi \to \Phi / g$

$$S = \frac{1}{g^2} \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V_\beta(\Phi) \right] \qquad D_\mu = \partial_\mu + i A_\mu$$

$$V_\beta(\Phi) = \frac{\beta}{2} \left(|\Phi|^2 - 1 \right)^2 \quad \textbf{(QQ)}$$

$$V_\beta(\Phi) = \frac{\beta}{2} \left(\log |\Phi|^2 - \frac{1}{2} \right) |\Phi|^4 \quad \textbf{(CW)}$$

$$\beta \equiv \frac{m_\phi^2}{m_A^2} = \frac{2\lambda}{g^2}$$

- Tension (=energy per unit length of string):

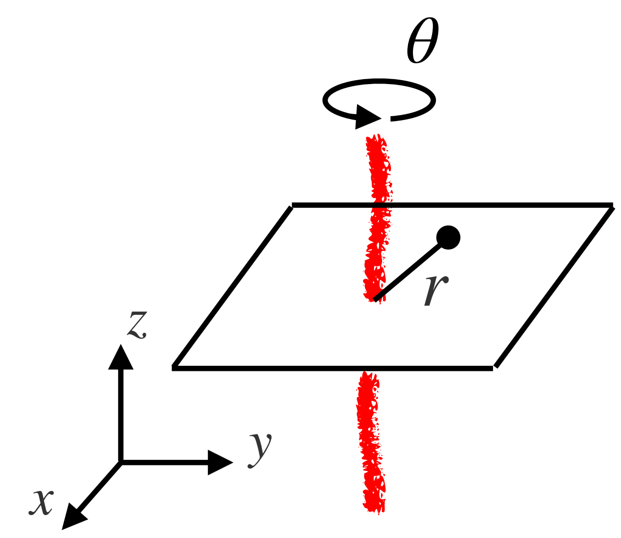
$$T = \frac{dE}{dz} = \int d^2x \left[\frac{1}{2} (\partial_i A_j)^2 + |D_i \Phi|^2 + V_\beta(\Phi) \right]$$

(assuming static and Coulomb gauge)

Axisymmetric string

- **Field configuration:**

$$\Phi(x) = f(r)e^{i\theta} \qquad\qquad A_\theta(x) = a(r)$$



$\longrightarrow$ winding # = 1 & magnetic flux $\int d^2x\, B = 2\pi$

- **classical EOMs for $f(r)$ and $a(r)$:**

$$f'' + \frac{1}{r}f' - \frac{n^2(1-a)^2}{r^2}f - \frac{1}{2}\frac{\partial V}{\partial f} = 0$$

$$a'' - \frac{1}{r}a' + 2(1-a)f^2 = 0$$

- **boundary conditions:**

$$f(0) = a(0) = 0 \qquad\qquad f(\infty) = a(\infty) = 1$$

Axisymmetric string

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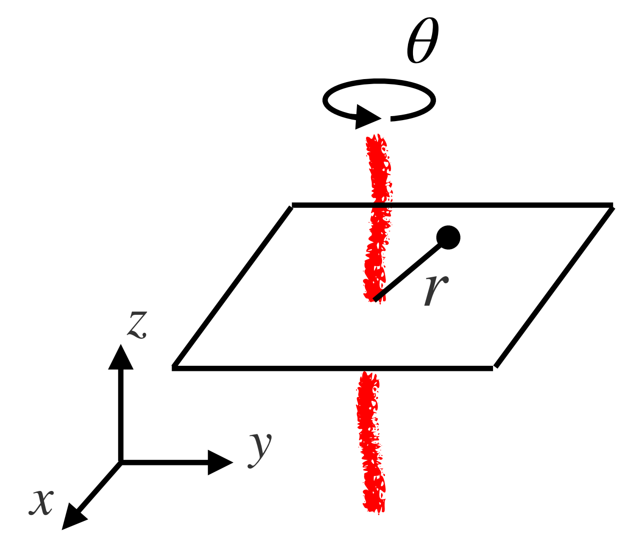
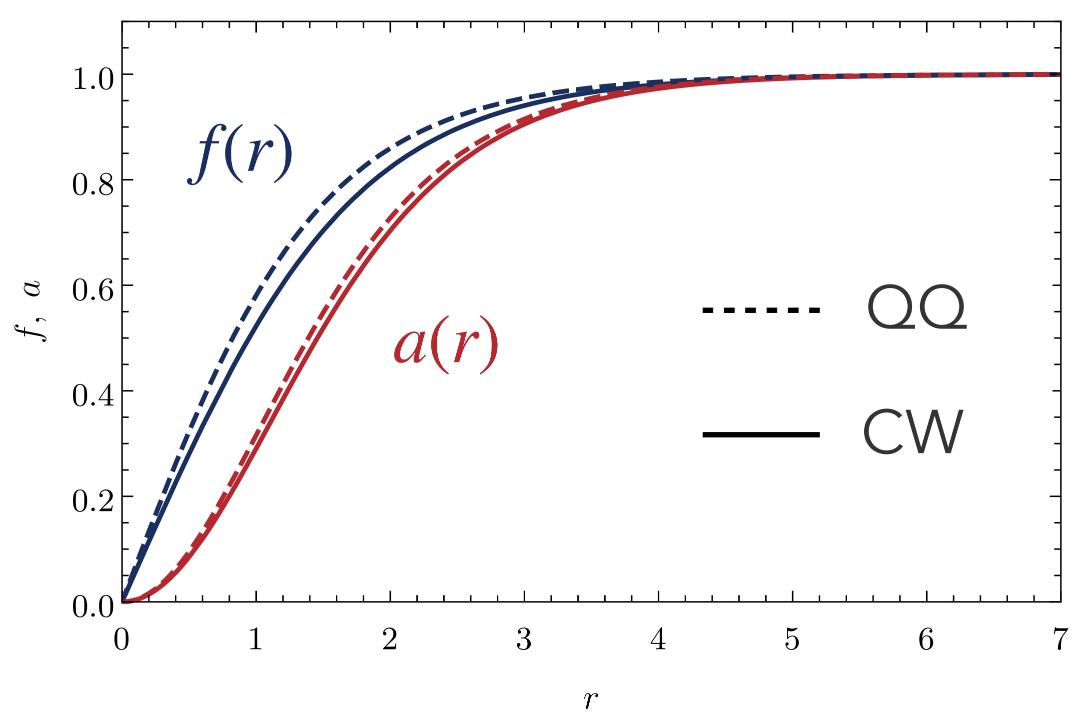
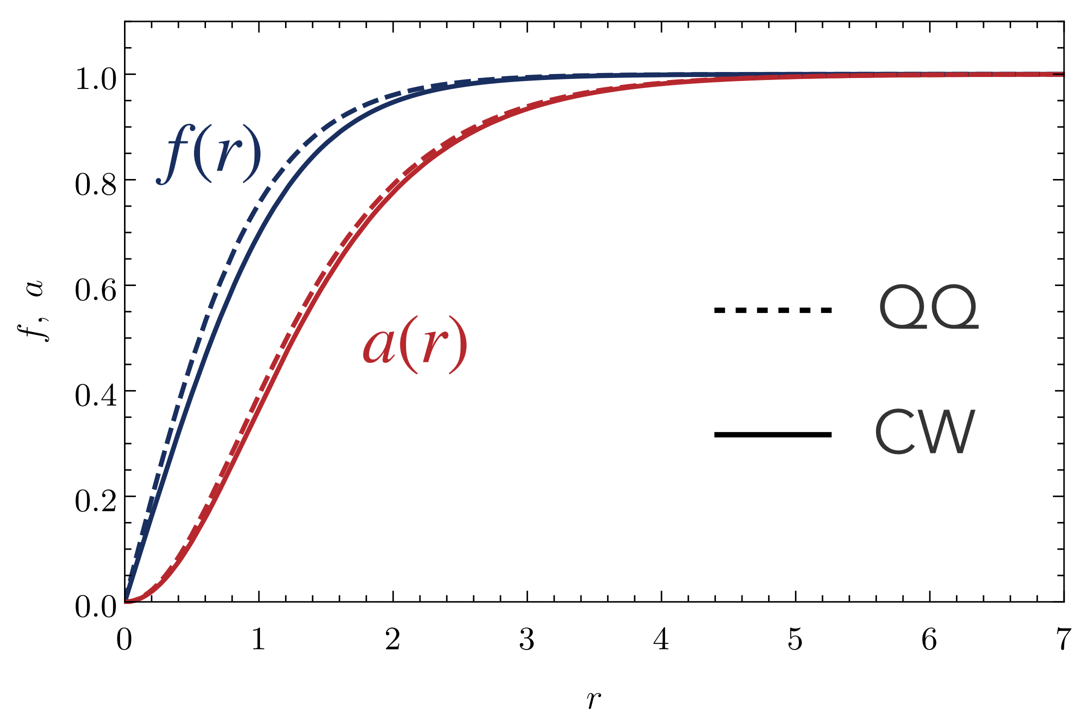


- **no significant difference for the string solutions**

$\beta = 0.5$



$\beta = 1.5$



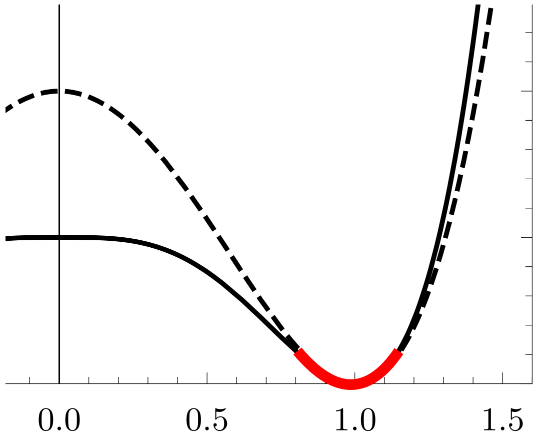
Asymptotics of CW-ANO string

- Asymptotic behavior at $r \to \infty$ can be derived analytically.

- Since $f(r) \simeq 1$ and $a(r) \simeq 1$ at $r \to \infty$, it is useful to write down linearized EOM w.r.t. $\delta f \equiv 1 - f$ and $\delta a \equiv 1 - a$

$$\delta f'' + \frac{1}{r}\delta f' - 2\beta\delta f = \mathcal{O}((\delta f)^2, (\delta a)^2)$$

$$\delta a'' - \frac{1}{r}\delta a' - 2\delta a = \mathcal{O}((\delta f)^2, (\delta a)^2)$$



Only curvature around vac is relevant.

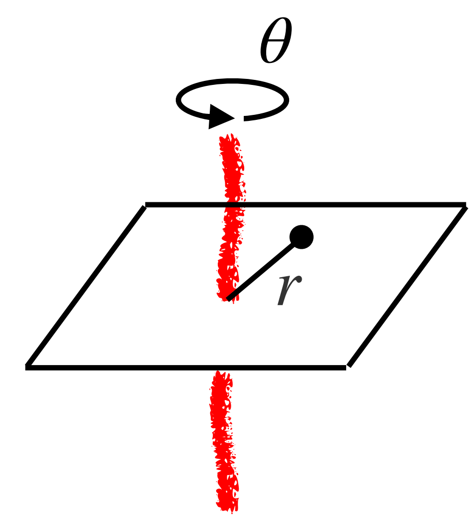
- Asymptotic behavior doesn't depend on the potential shapes.

$$\delta f \simeq r^{-1/2} \exp\left[-\sqrt{2\beta} r\right] \qquad \delta a \simeq r^{1/2} \exp\left[-\sqrt{2} r\right]$$

Higher winding

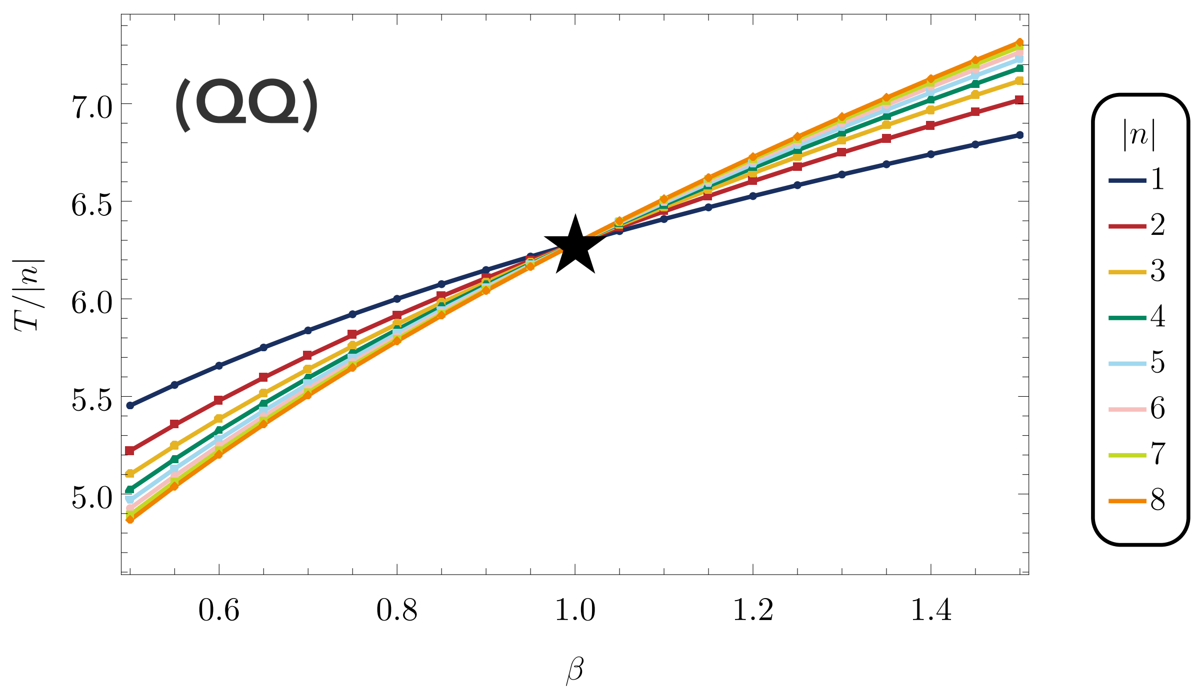
- **Field configuration:**

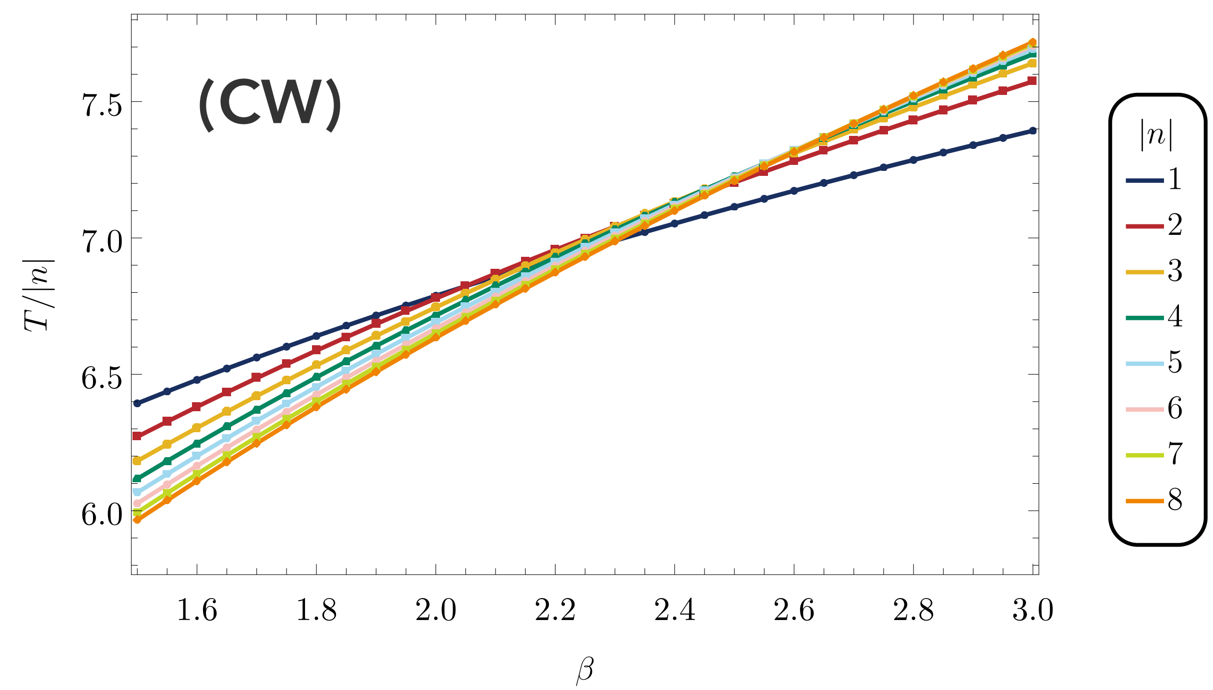
$$\Phi(x) = f(r)e^{in\theta} \qquad A_\theta(x) = n\,a(r)$$



$\longrightarrow$ winding # $= n$ & magnetic flux $\int d^2x\, B = 2\pi n$

- **String tension for QQ and CW cases:**





- **For QQ case, all lines cross at $\beta = 1$ (BPS state) while it doesn't happen for CW case** (next slide).

BPS state

[Bogomol'nyi '76]

[Prasad-Sommerfield '75]

- In Quadratic Quartic case, the energy can be rewritten by completion of square:

$$
\begin{aligned}
T &= 2\pi|n| \\
&+ 2\pi\int_0^\infty \mathrm{d}r\, r\left[\left(f' + |n|\frac{a-1}{r}f\right)^2 + \frac{n^2}{2r^2}\left(a' + \frac{r}{|n|}(f^2-1)\right)^2 + \frac{1}{2}(\beta-1)(f^2-1)^2\right]
\end{aligned}
$$

- For $\beta = 1$, the last term vanishes and the EOMs reduce to

$$f' + |n|\frac{a-1}{r}f = 0 \qquad a' + \frac{r}{|n|}(f^2-1) = 0$$

BPS equations

$$\longrightarrow \quad \frac{T}{|n|} = 2\pi$$

But, CW case doesn't have this property due to the log-potential.

Plan of talk

- Introduction ←Done
- CW-ANO string ←Done
- Interaction of CW-ANO string
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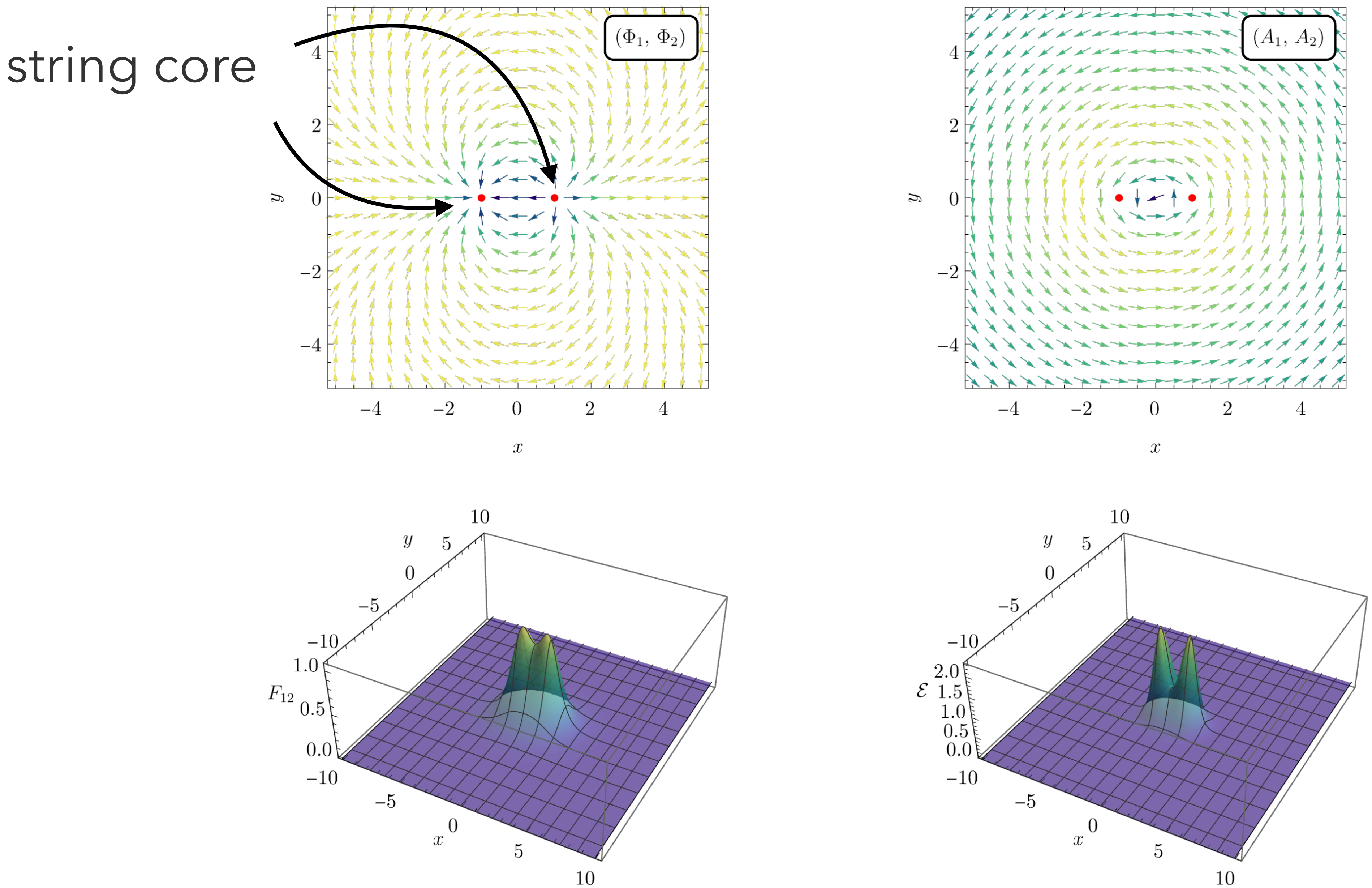
Interaction of CW-ANO string

Two string system

- put two strings orthogonal to xy plane w/ distance d.



Calculation of interaction potential

1. **put two strings w/ distance d**
2. **fix distance d (pinning string cores)**
3. **minimize the energy of the system**

   → minimum-energy configuration w/ fixed d
4. **do 1~3 for various d**

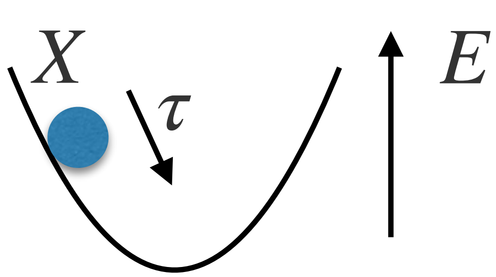
   → interaction potential as a function of d

- minimization is performed by the relaxation method (gradient flow):

$$\partial_\tau X = -\frac{\delta E}{\delta X}$$

$X = \Phi$ or A_i

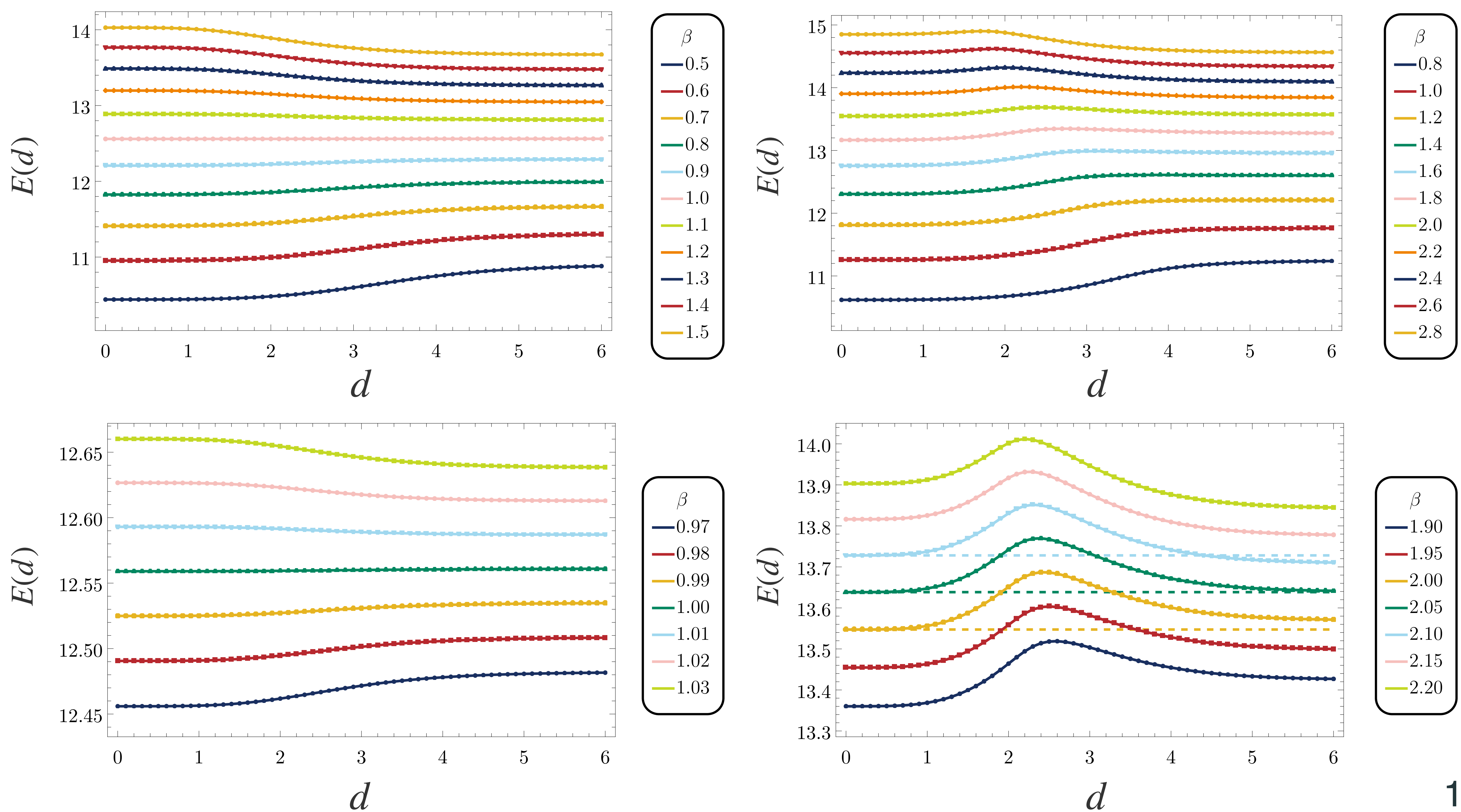
τ: fictitious time



Result

- Interaction potential as a function of d for different β

Quadratic-Quartic   Coleman-Weinberg



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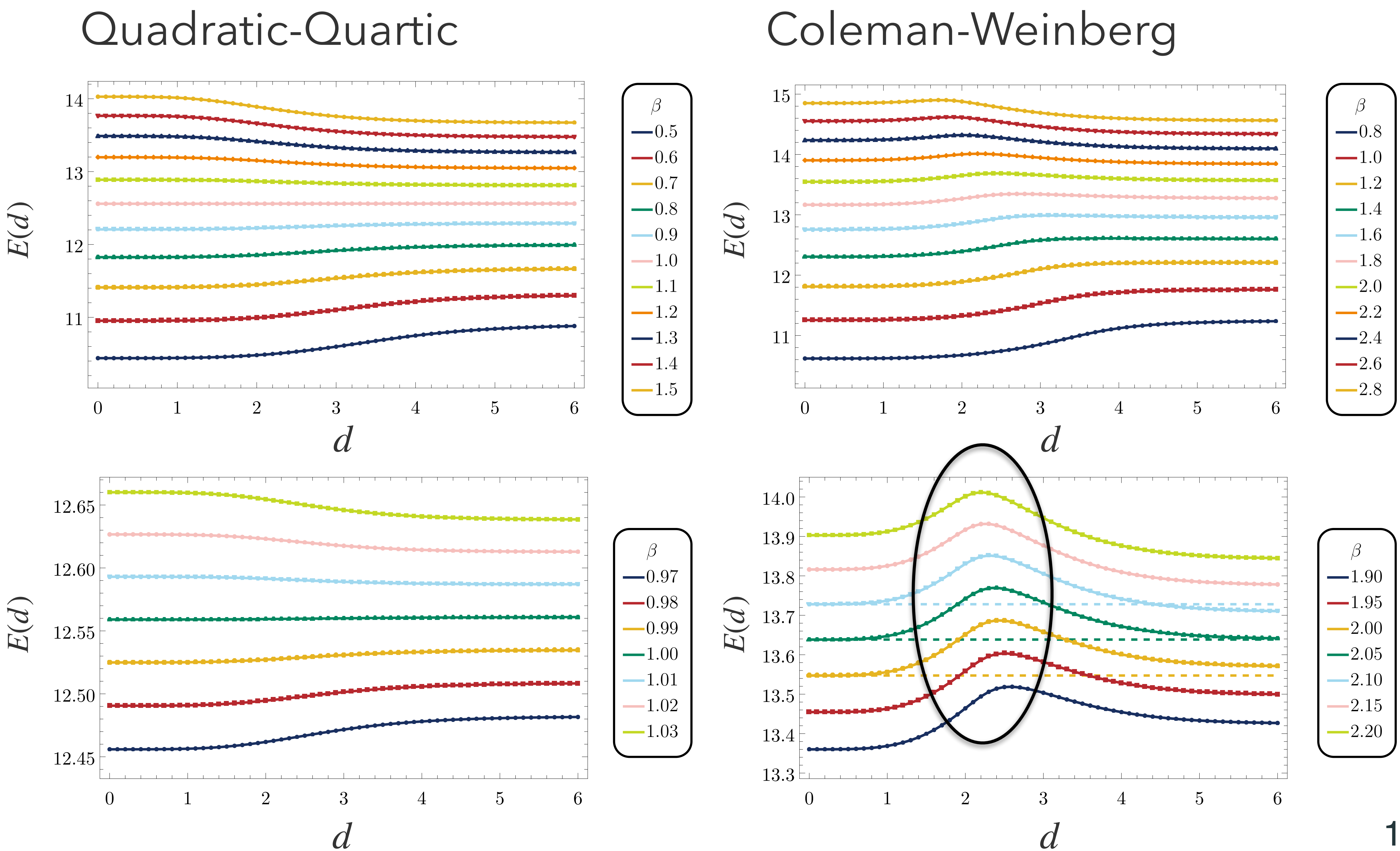
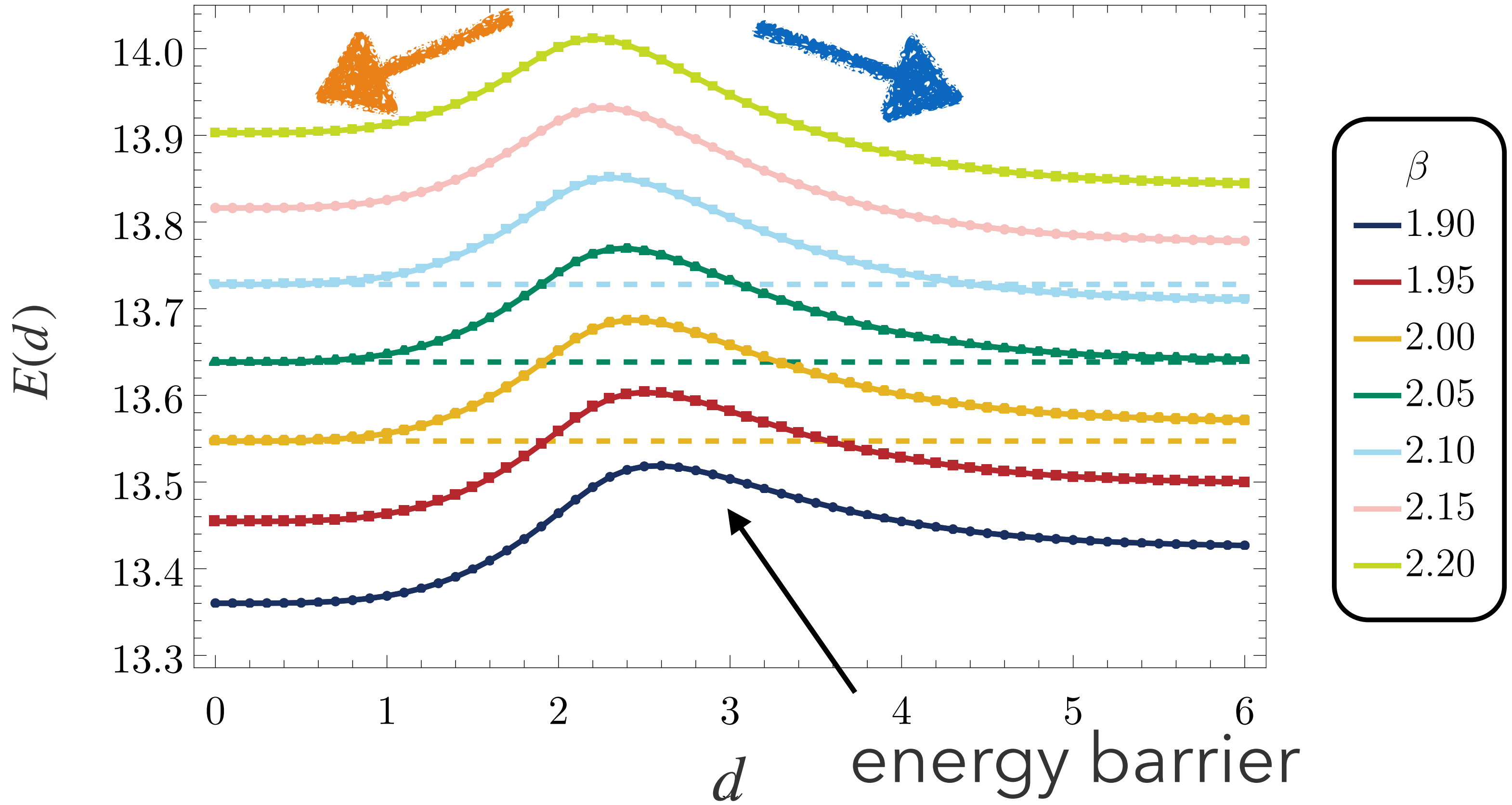


Result: Energy barrier

- Interaction potential as a function of d for different β

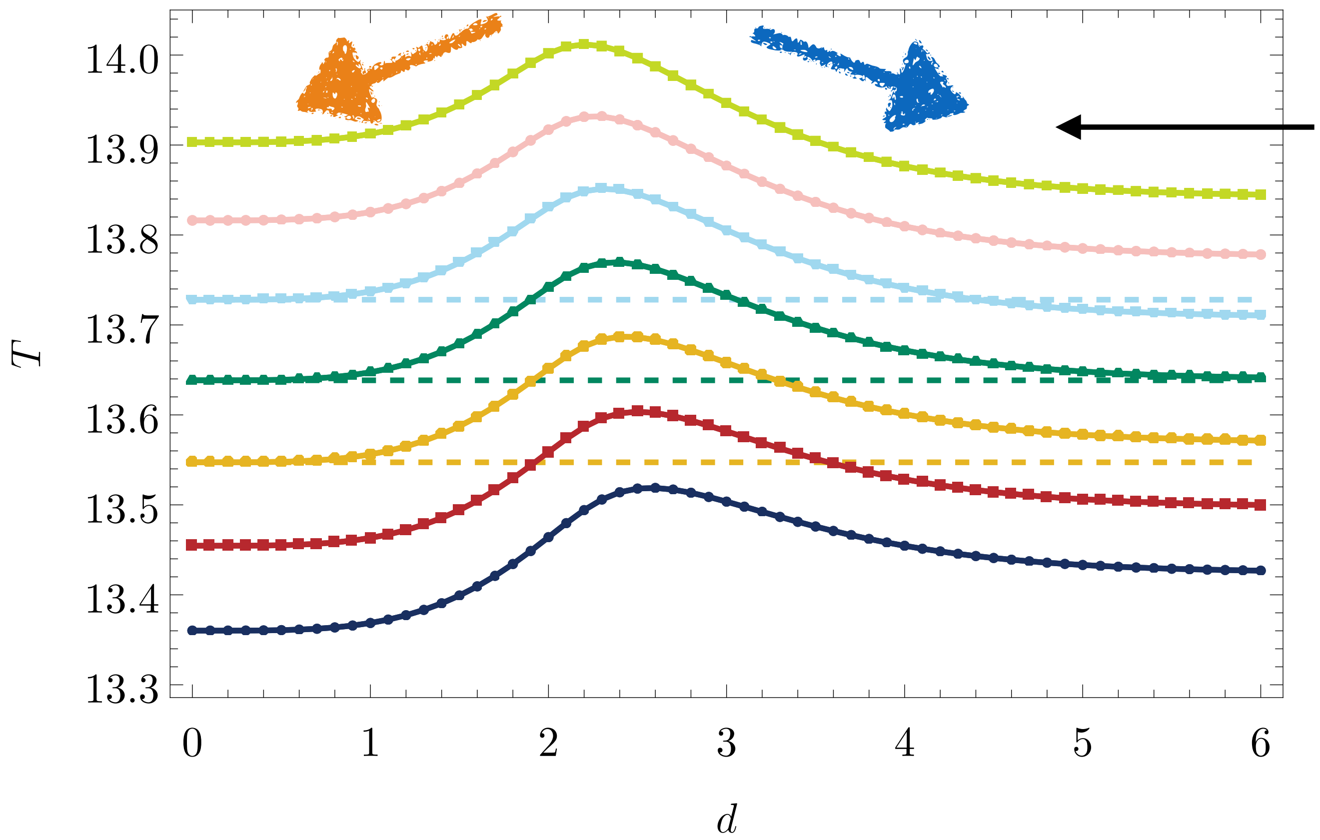
Coleman-Weinberg



- **Energy barrier** appears in CW case for $\beta > 1$!!

$\left\{\begin{matrix}\text{attractive} \\ \text{repulsive}\end{matrix}\right\}$ **for** $\left\{\begin{matrix}\text{short} \\ \text{large}\end{matrix}\right\}$ **distance**

Reason?



This repulsive behavior is easy to understand.

Each vortex has the asymptotic behavior:

$$\begin{cases} \delta f \simeq r^{-1/2} \exp\left[-\sqrt{2\beta} r\right] \\ \delta a \simeq r^{1/2} \exp\left[-\sqrt{2} r\right] \end{cases}$$

The gauge field is dominant at large d for $\beta > 1$.

→ **The gauge field mediates the repulsive force.**

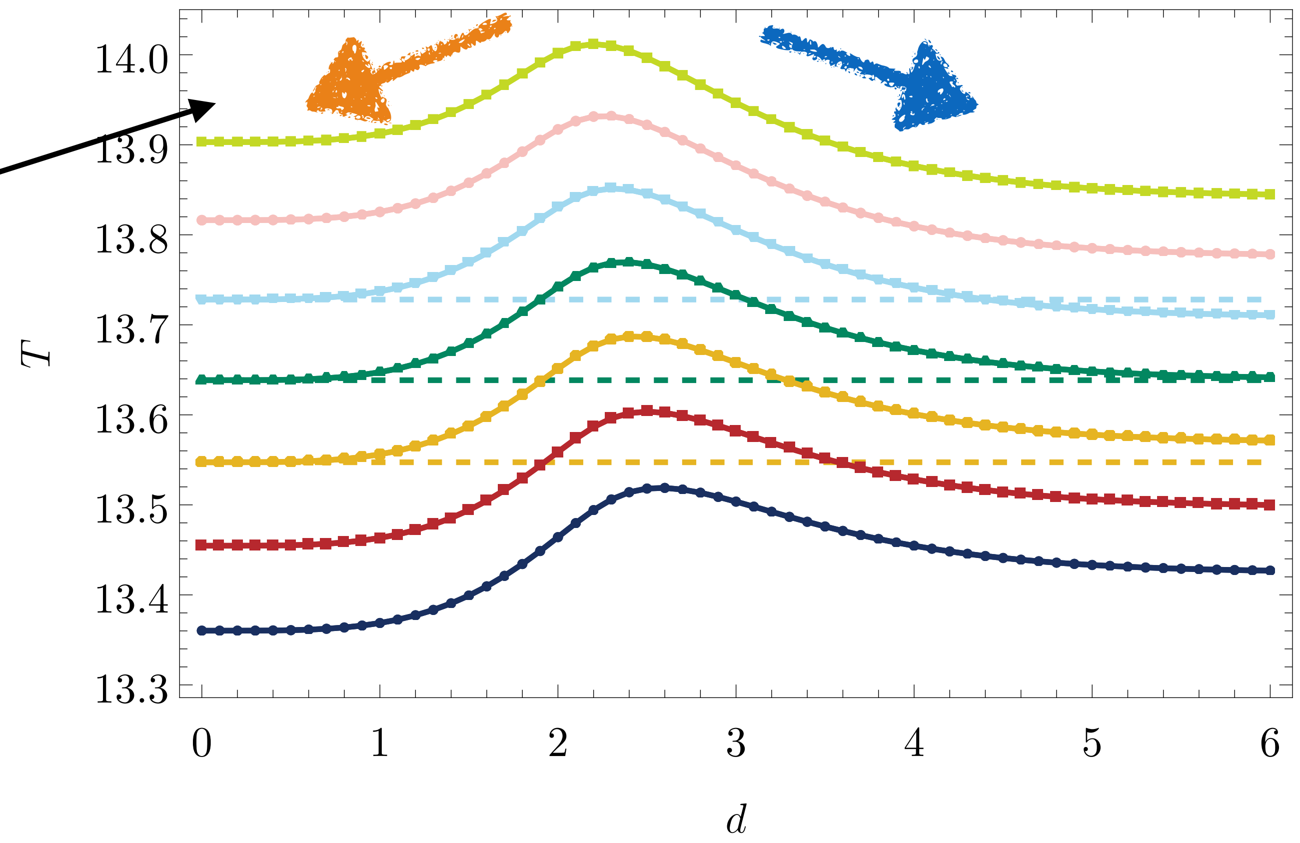
Reason?

On the other hand, this attractive behavior is difficult.

Numerical simulation saids $T(d)$ increases w/

$$T(d) \propto d^4$$

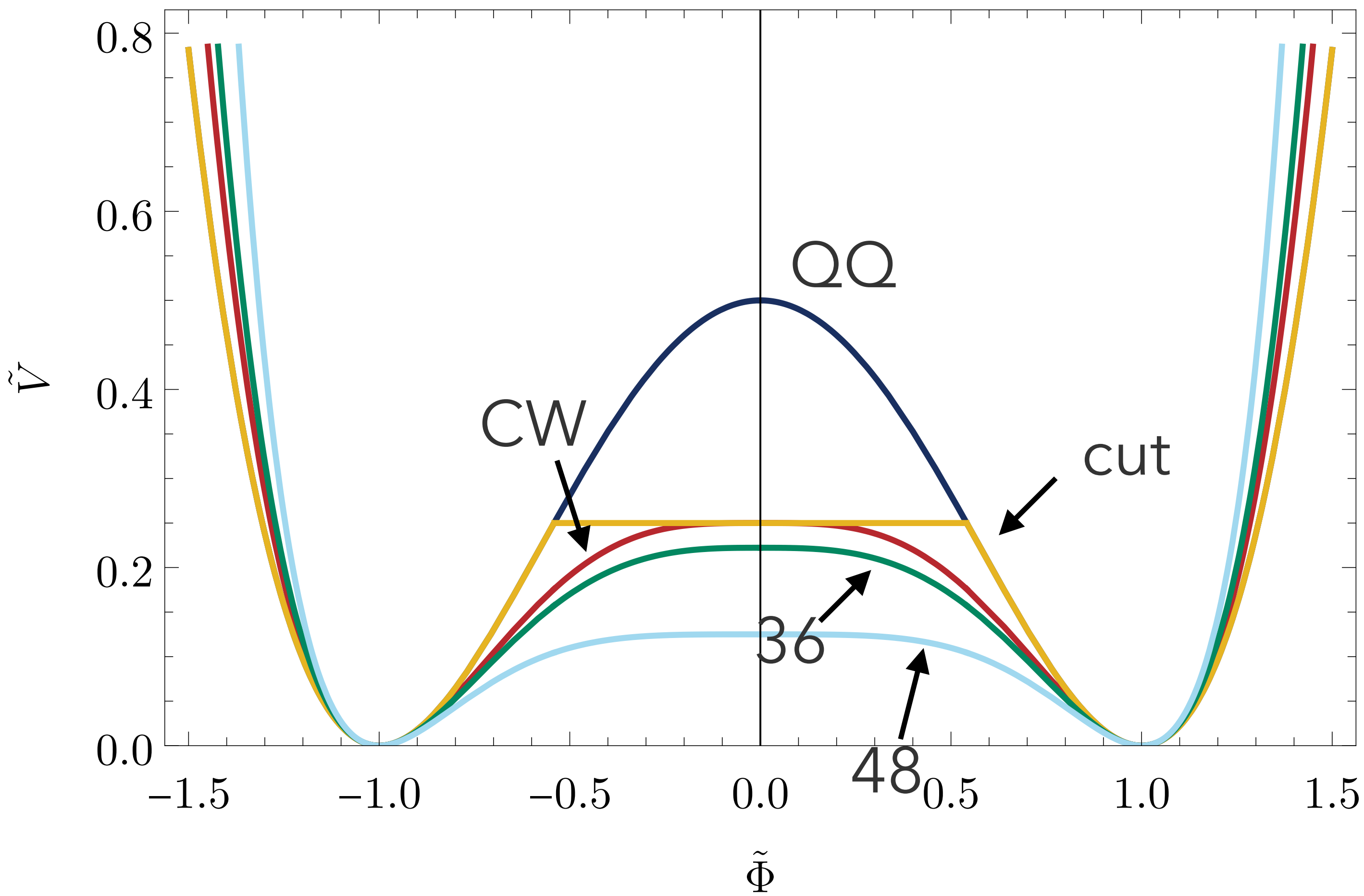
for small d independently of β.



But it is difficult to understand analytically!

As shown later, the flatter structure of the potential seems crucial...

Other potentials



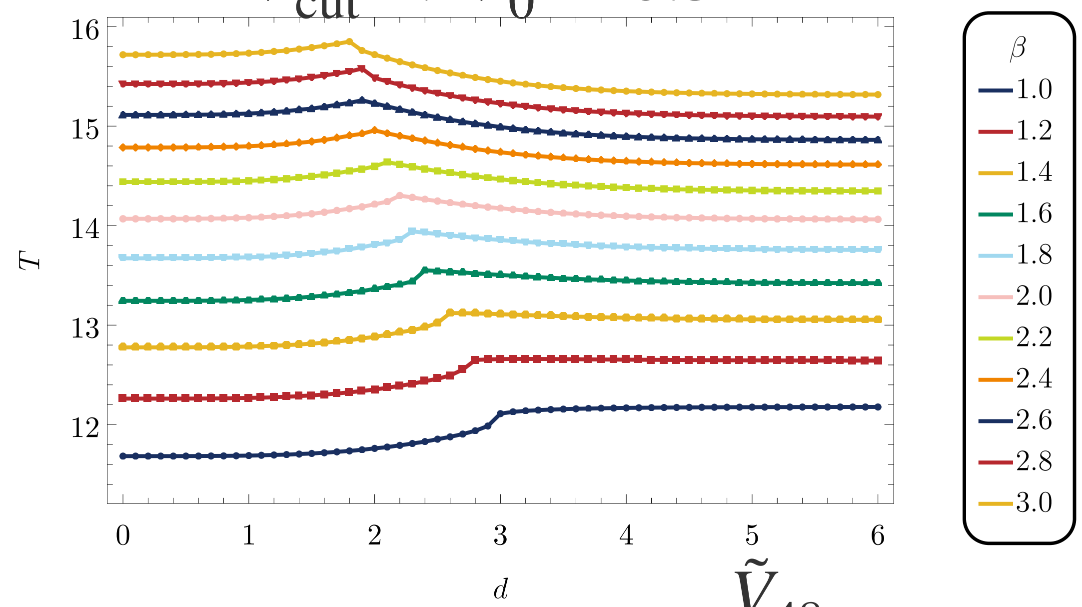
$$\tilde{V}_{\mathrm{AH-cut}} = \begin{cases} \dfrac{\beta}{2}\tilde{V}_0 & \left(|\tilde{\Phi}| < \sqrt{1-\sqrt{\tilde{V}_0}}\right) \\ \dfrac{\beta}{2}\left(|\tilde{\Phi}|^2 - 1\right)^2 & \left(|\tilde{\Phi}| > \sqrt{1-\sqrt{\tilde{V}_0}}\right) \end{cases}$$

$$\tilde{V}_{\mathrm{AH-36}} = \frac{2\beta}{9}\left(|\tilde{\Phi}|^3 - 1\right)^2,$$

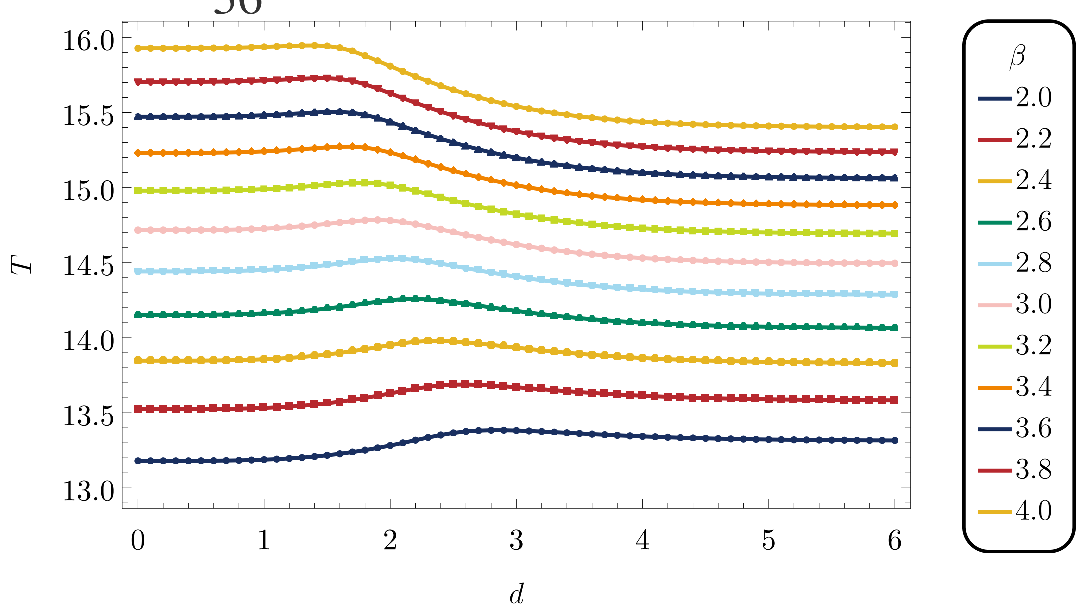
$$\tilde{V}_{\mathrm{AH-48}} = \frac{\beta}{8}\left(|\tilde{\Phi}|^4 - 1\right)^2.$$

Other potentials

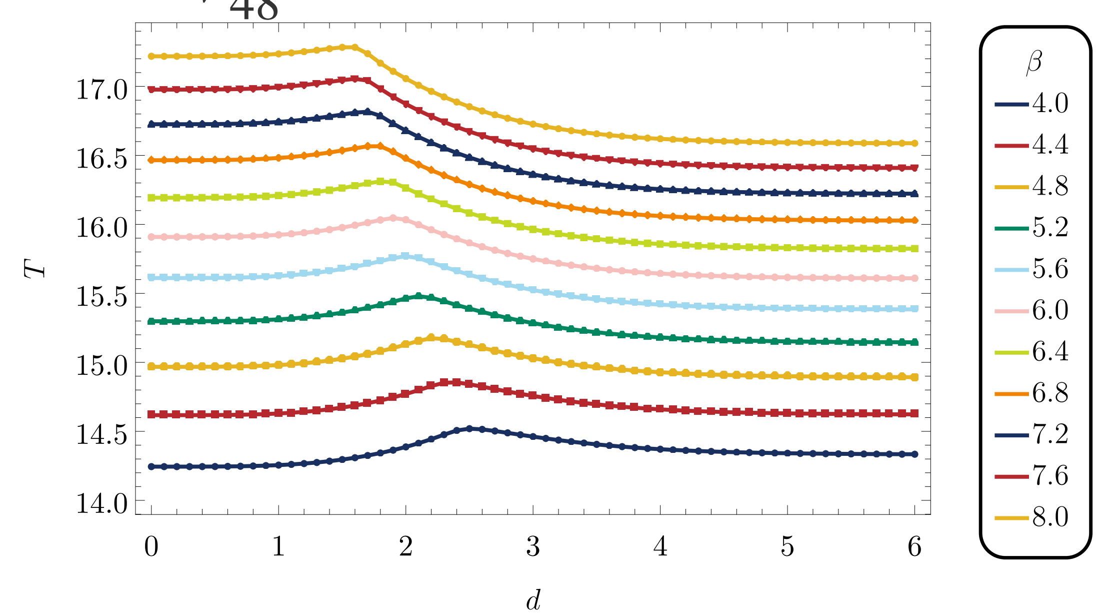
$\tilde{V}_{\rm cut}$ w/ $\tilde{V}_0 = 0.5$



$\tilde{V}_{36}$



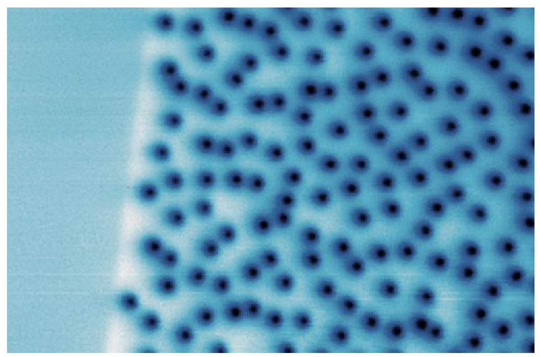
$\tilde{V}_{48}$



→ **It seems that the energy barrier is universal for flatter potential than Quadratic-Quartic one.**

Discussion

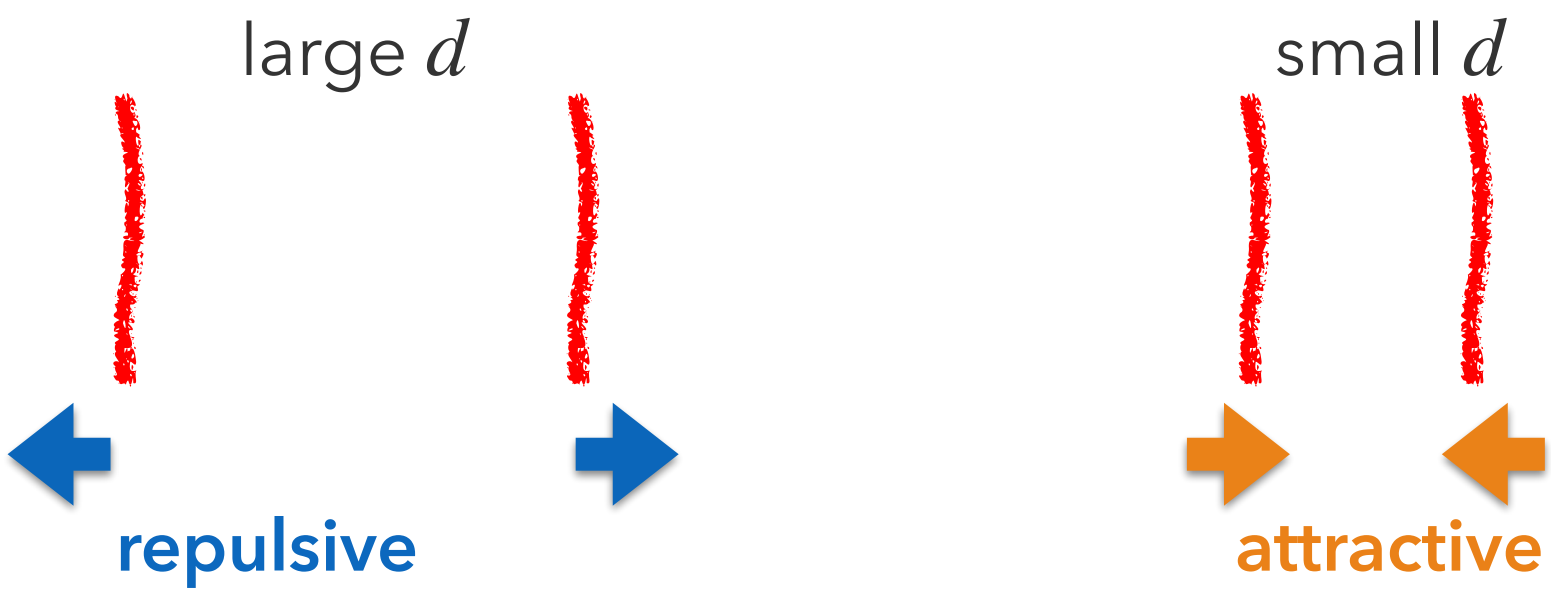
- **Formation of Abrikosov-like lattice in superconductor?**



dilute → lattice-like structure

dense → gather and merge!

- **Cosmic string in universe → reconnection? gravitational waves?**



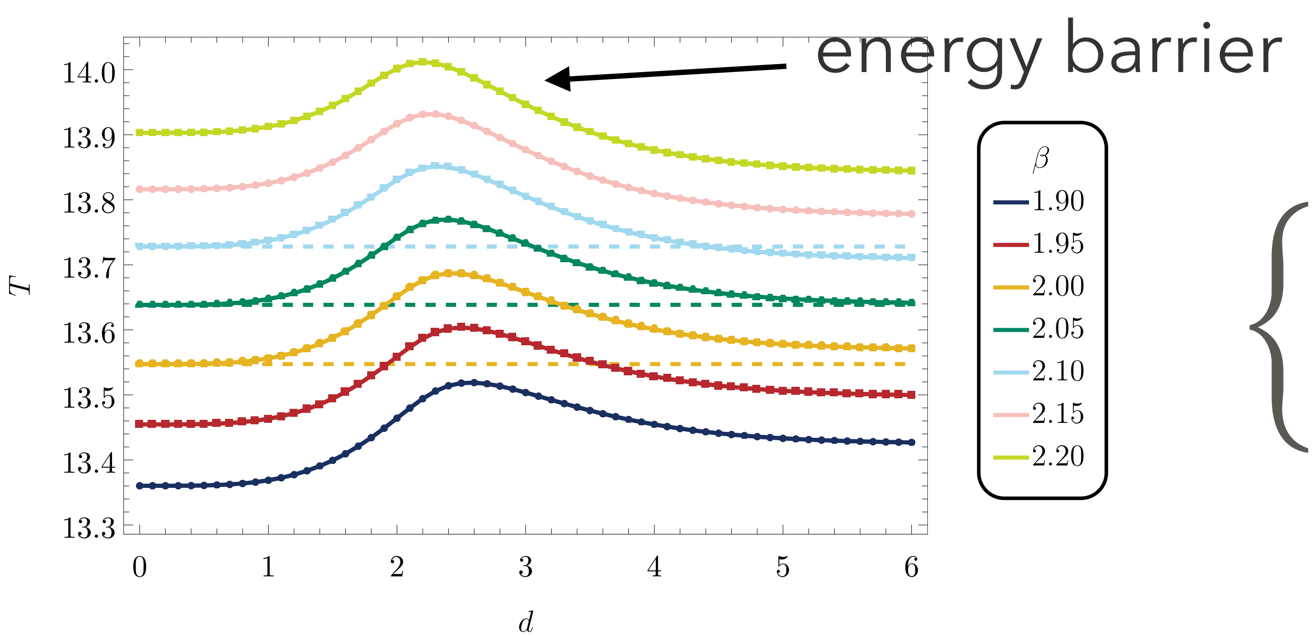
might lead to non-trivial dynamics! (future work)

Summary

- We study vortex strings in U(1) gauged model w/ Coleman-Weinberg potential (called CW-ANO string).

$$V_{\rm CW}(\Phi) = \frac{\beta}{2}\left(\log\frac{|\Phi|^2}{v^2} - \frac{1}{2}\right)|\Phi|^4$$

- In contrast to the conventional ANO string, interaction between the two CW-ANO strings has the energy barrier for $\beta > 1$.

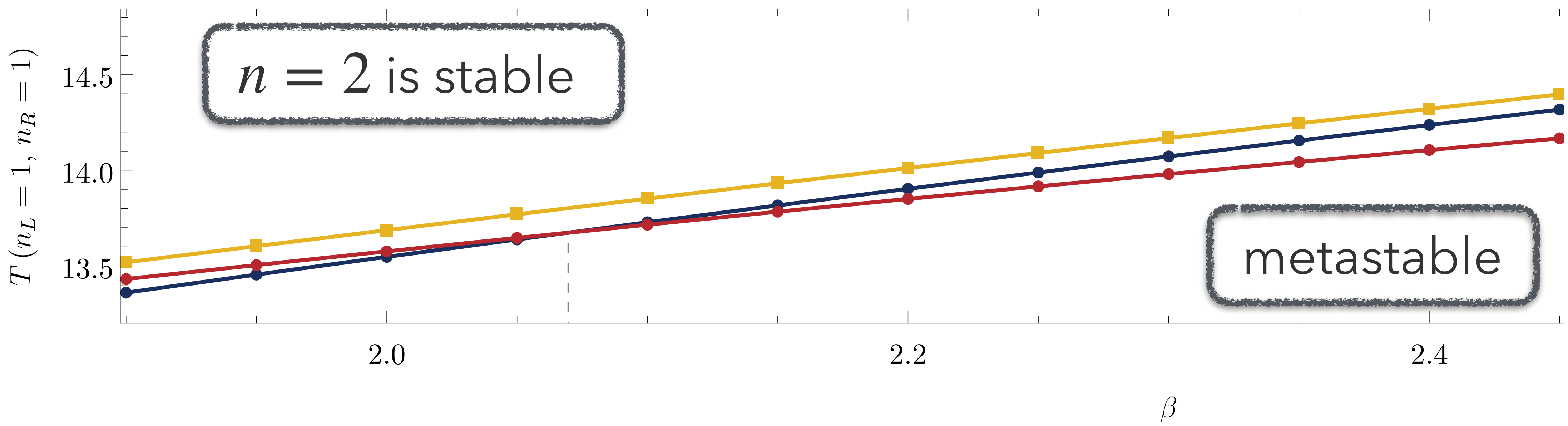


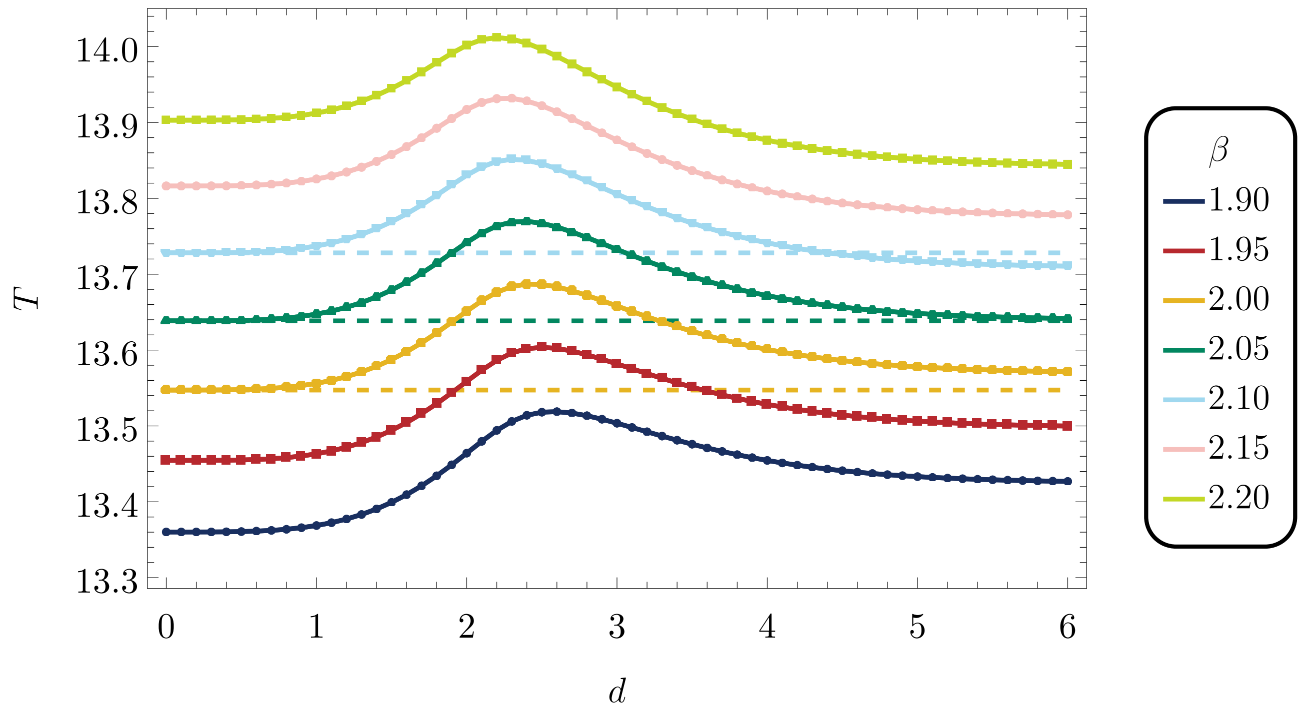
{ attractive / repulsive } for { short / large } distance

Backup

Stability of $n = 2$ state

- We can read off the stability of the vortex of $n = 2$ state.





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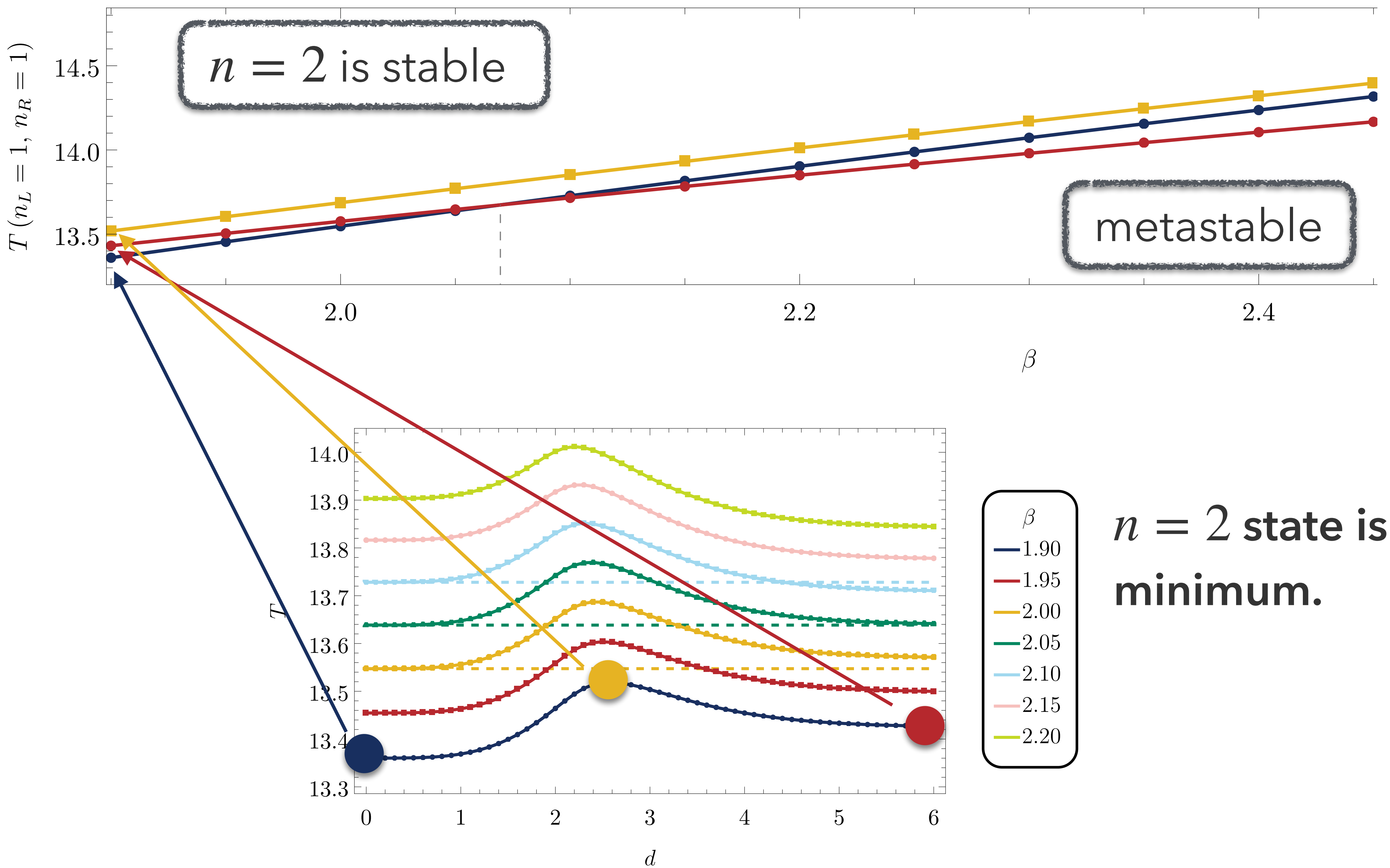

$n = 2$ state is absolute minimum.

Stability of $n = 2$ state

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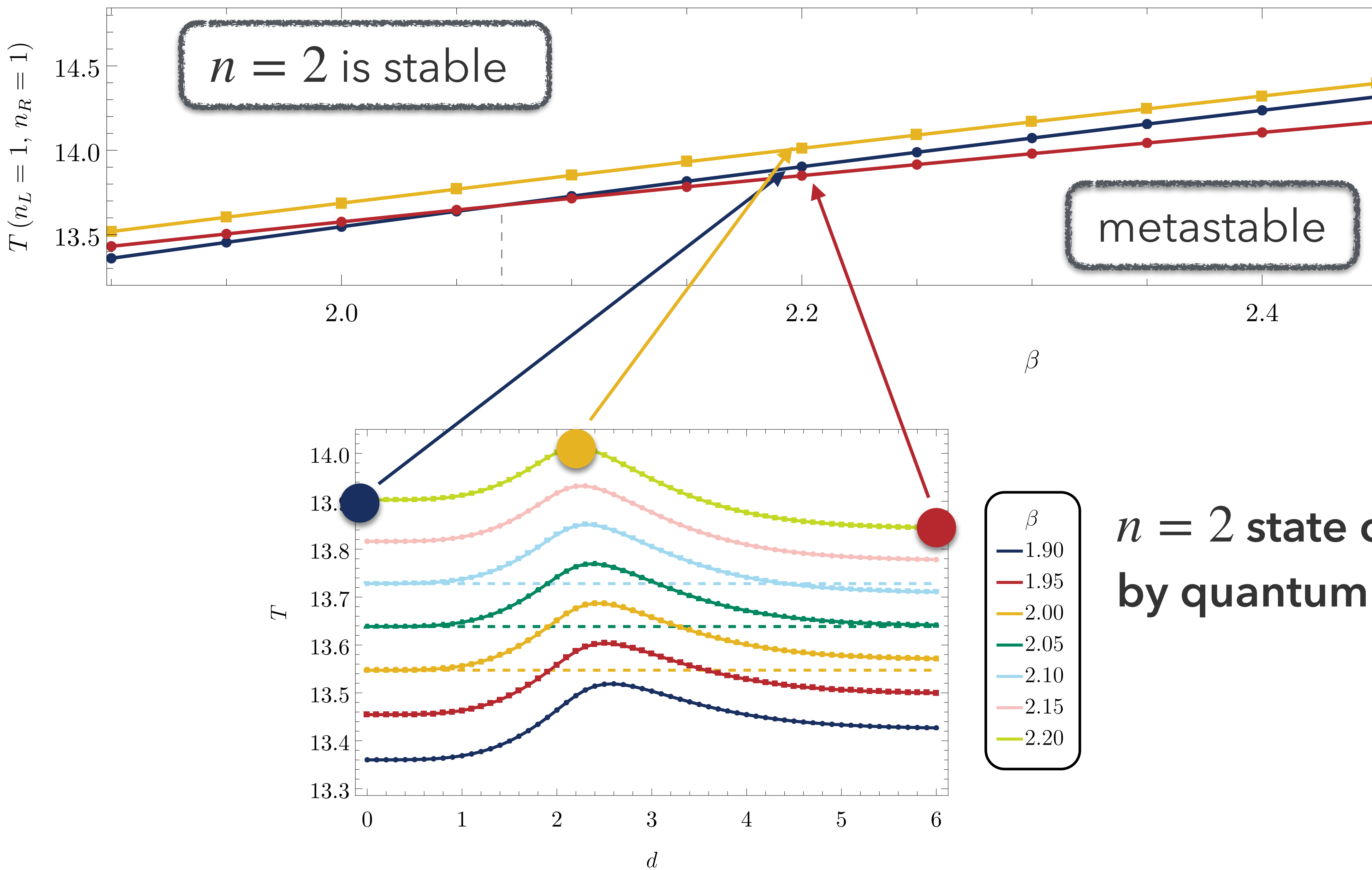
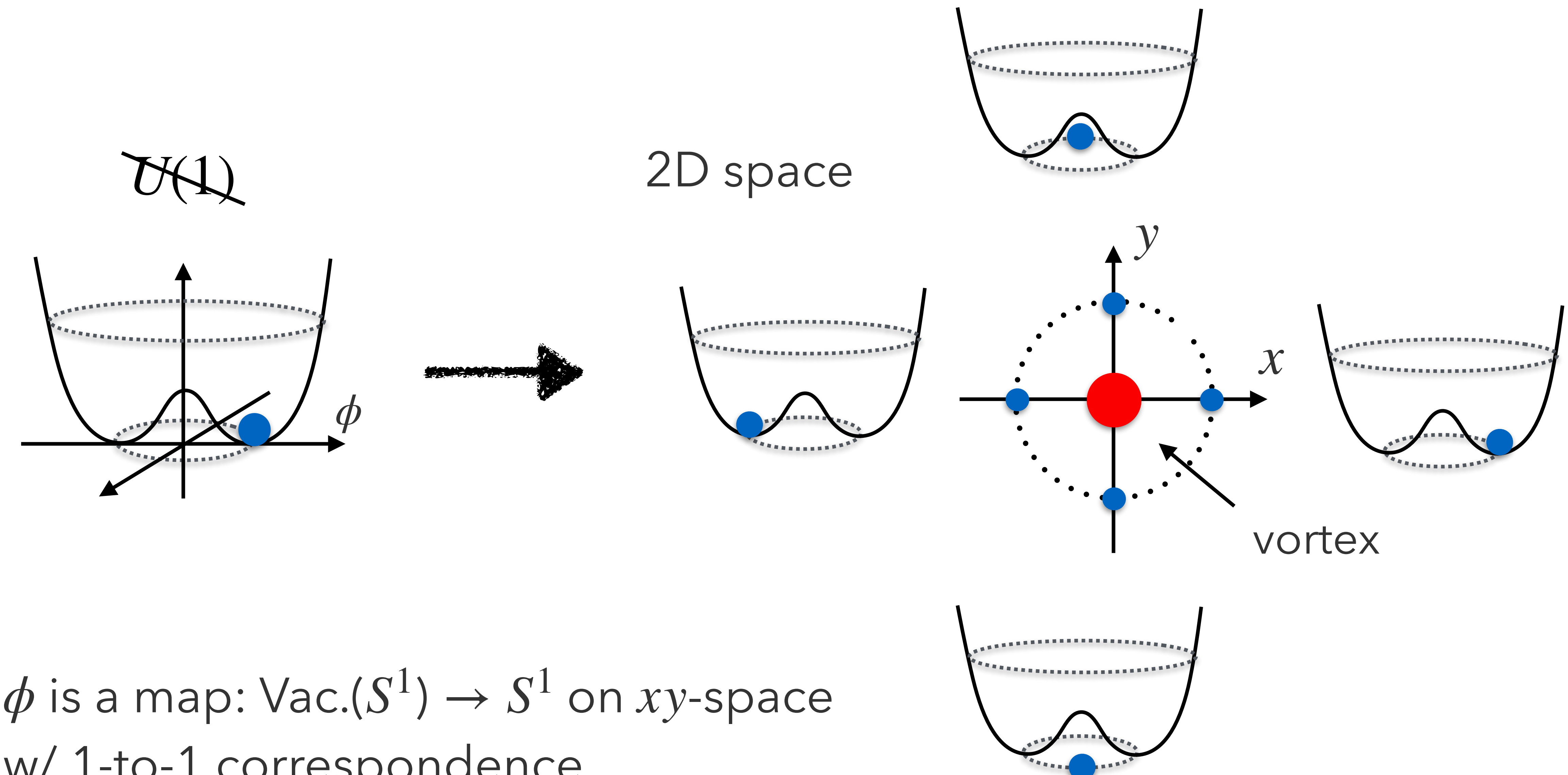


$n = 2$ state can decay by quantum tunneling.

Vortex String

- **Vortex string appears by SSB of $U(1)$**



ϕ is a map: Vac.$(S^1) \to S^1$ on xy-space
w/ 1-to-1 correspondence

→ winding # = 1, vortex is topologically protected

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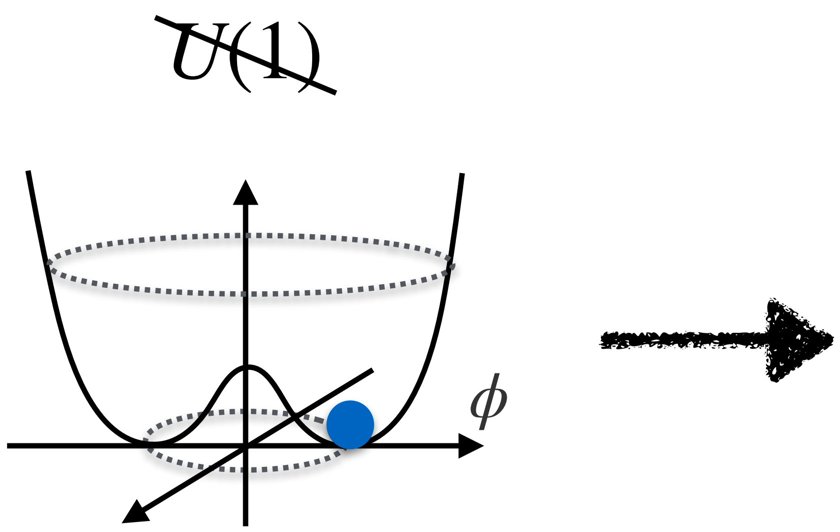

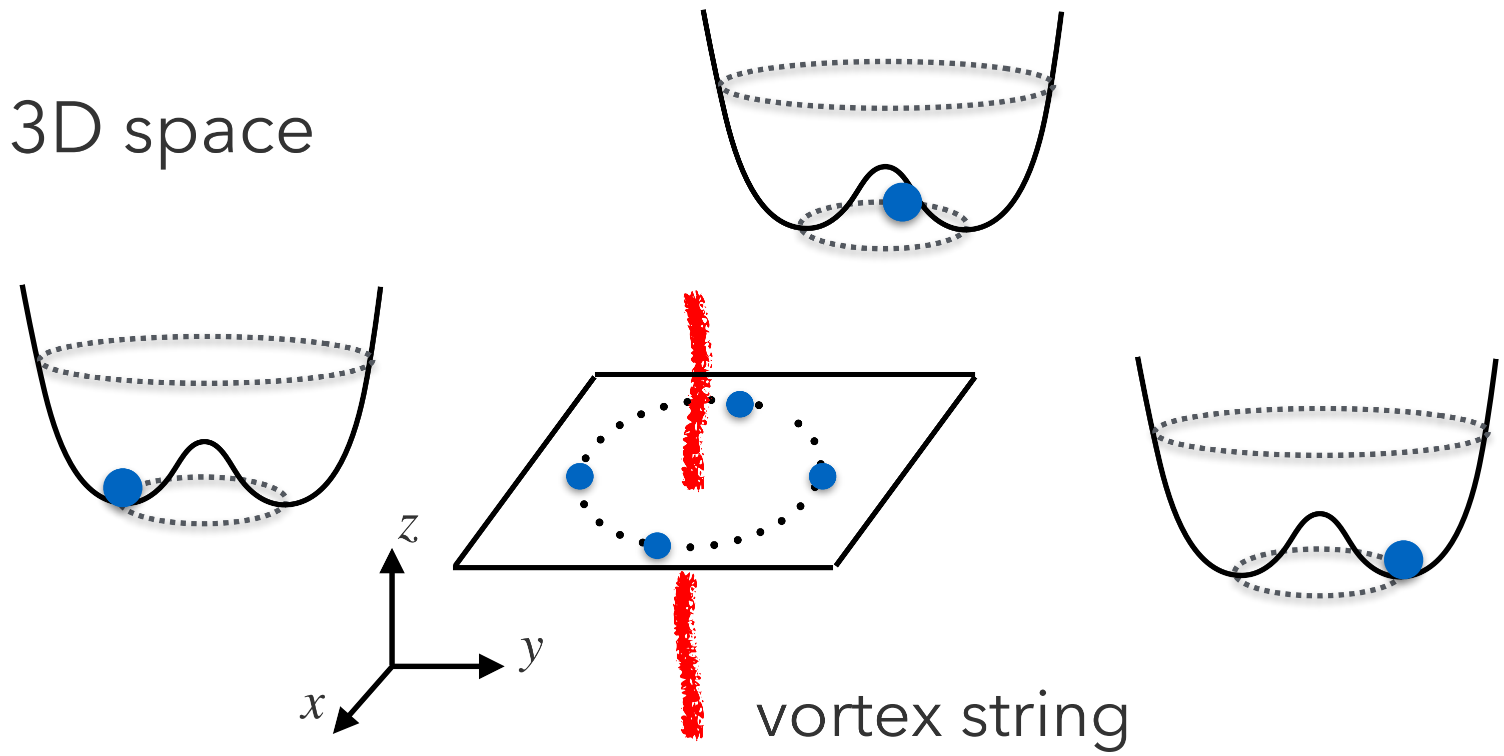

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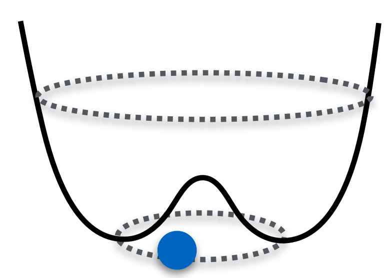


→ winding # = 1, vortex is topologically protected

Bardeen's argument

[Bardeen '95]

- **Conventional potential**

$$V(\Phi) = -m^2|\Phi|^2 + \lambda(\Phi)|\Phi|^4$$

- quantum correction for m^2

$$\delta m^2 = \Lambda^2 + m^2 \log \frac{\mu^2}{\Lambda^2} + \cdots$$

$\Lambda^2 \to 0$

Λ: UV cutoff scale

μ: renormalization scale

- In scale invariant scheme (such as $\overline{MS}$), Λ^2 does not appear.
- In the RG-running sense, this corresponds to a choice of "boundary conditions" at the cutoff scale $\mu = \Lambda$.

⟶ If we adopt a boundary condition that the mass vanishes at $\mu = \Lambda$, then $m^2 = 0$ at all scale. → no naturalness problem

classically scale invariance

Dimensional transmutation

[Coleman-Weinberg '73]

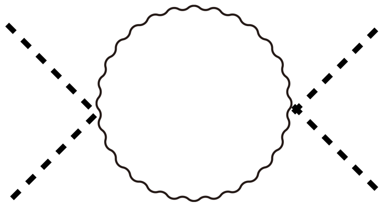
- **QCD:** $\alpha_s(\mu)^{-1} = \alpha_s(\Lambda)^{-1} + \frac{b_0}{2\pi} \log \frac{\mu}{\Lambda}$

$$\frac{\partial \alpha_s}{\partial \log \mu} = -\frac{b_0}{2\pi}\alpha_s$$

$$\alpha_s(\Lambda_{QCD})^{-1} = 0 \Leftrightarrow \Lambda_{QCD} = \Lambda \exp\left(-\frac{2\pi}{b_0 \alpha_s(\Lambda)}\right)$$

Scale Λ_{QCD} is non-perturbatively generated.

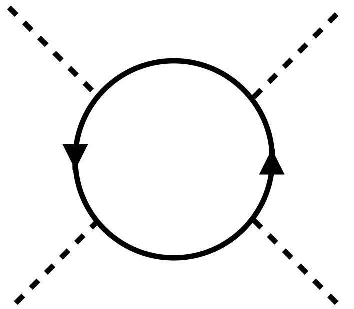
- **Coleman-Weinberg mechanism** (taking unitary gauge)



$$V_{\rm CW}(\phi) = \lambda(\phi)\phi^4$$

$$\lambda(\phi) = b \log \frac{\phi}{\Lambda}$$

$b = \frac{1}{16\pi^2}\left(\#g^4 - \#y^4\right)$: β-func coeff

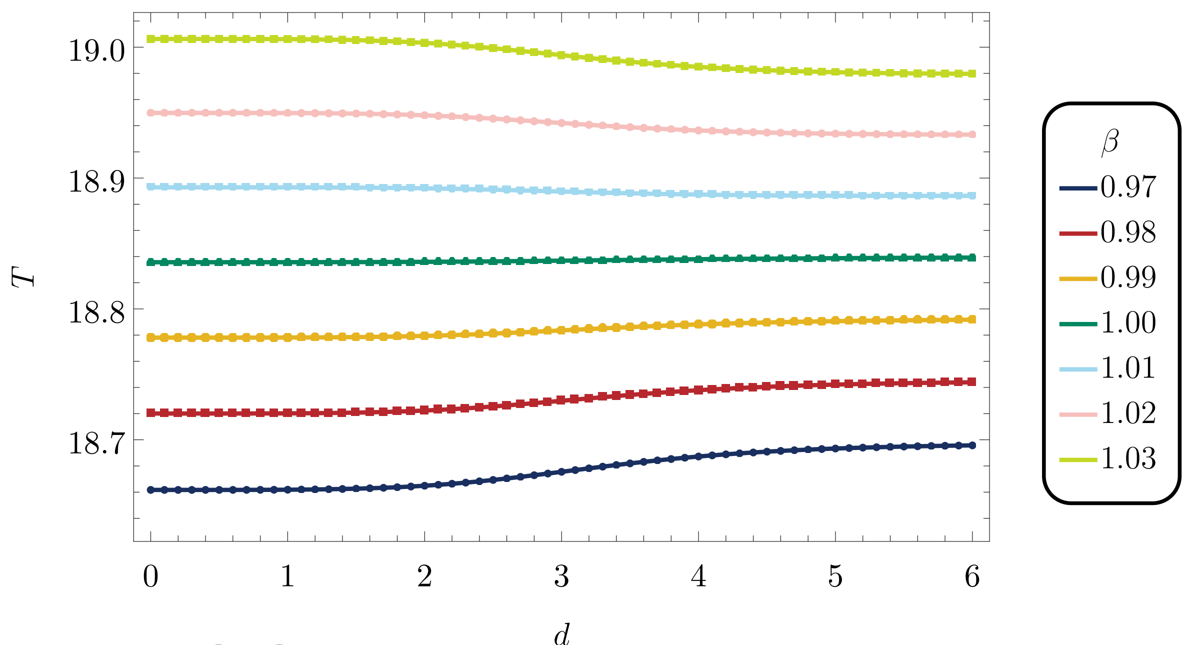


$$V'_{\rm CW}(\phi) = 0 \Leftrightarrow \langle \phi \rangle = \Lambda \exp\left[-\left(4\frac{\lambda(\Lambda)}{b} + 1\right)\right]$$

Potential minimum $\langle \phi \rangle$ is non-perturbatively generated.

Higher winding

QQ, $(n_L, n_R) = (2,1)$



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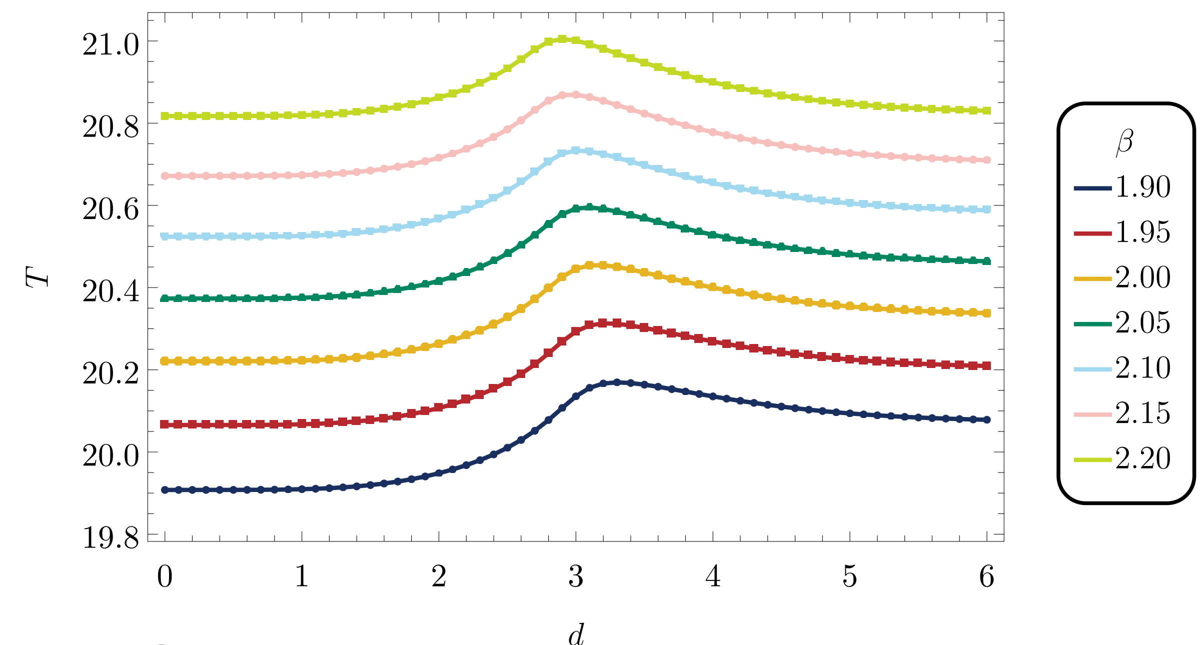
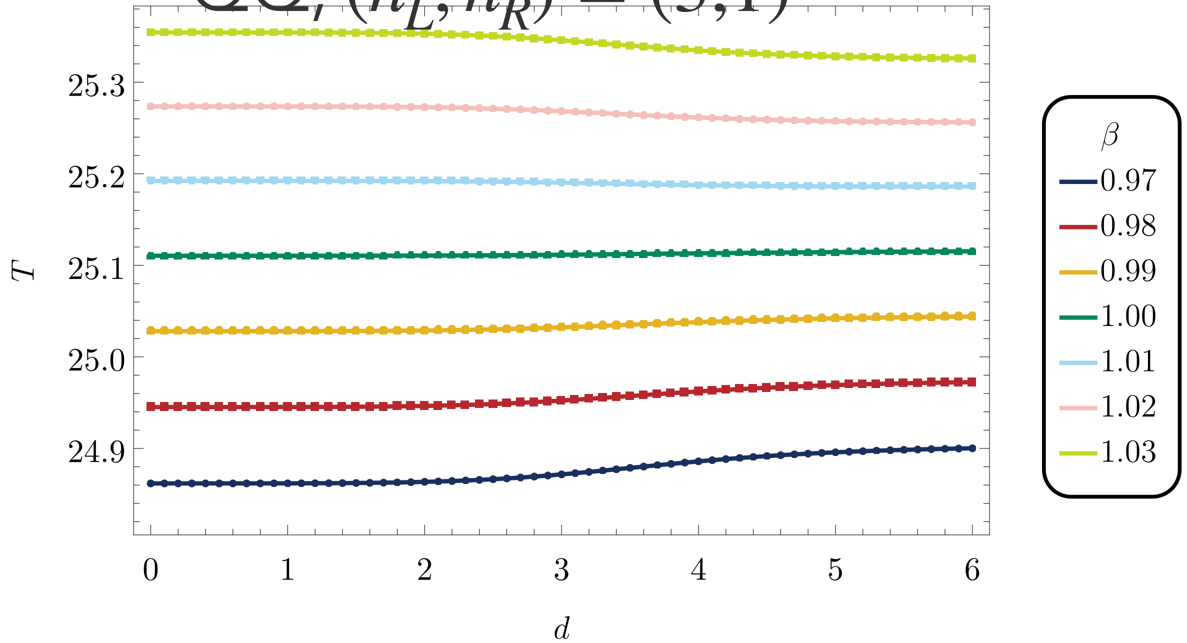
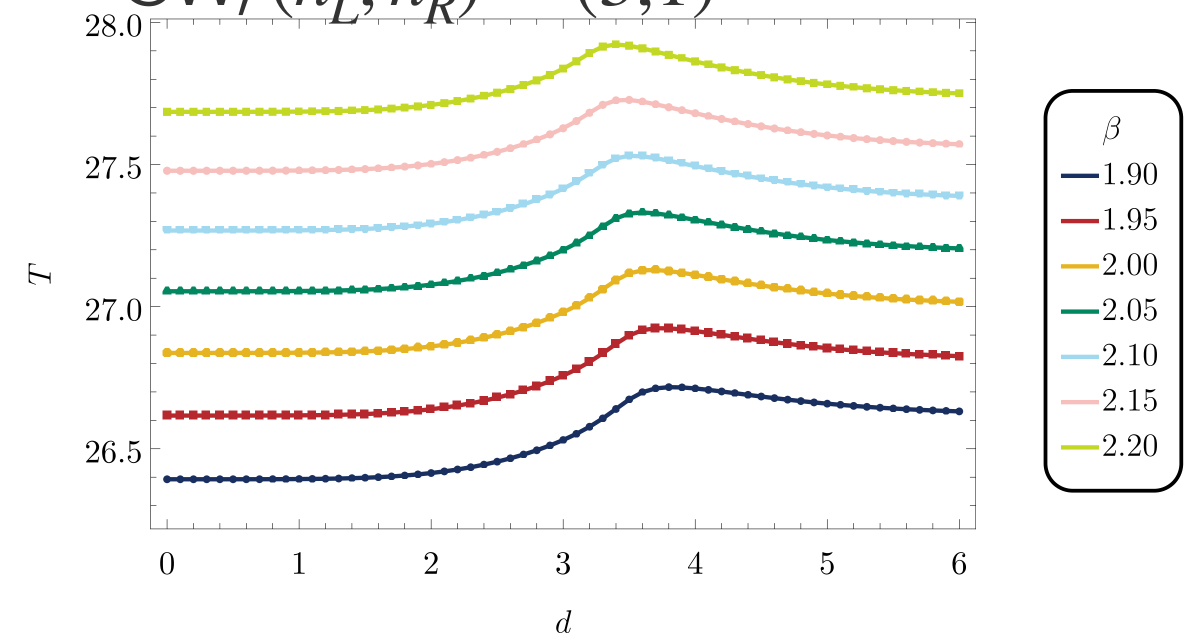


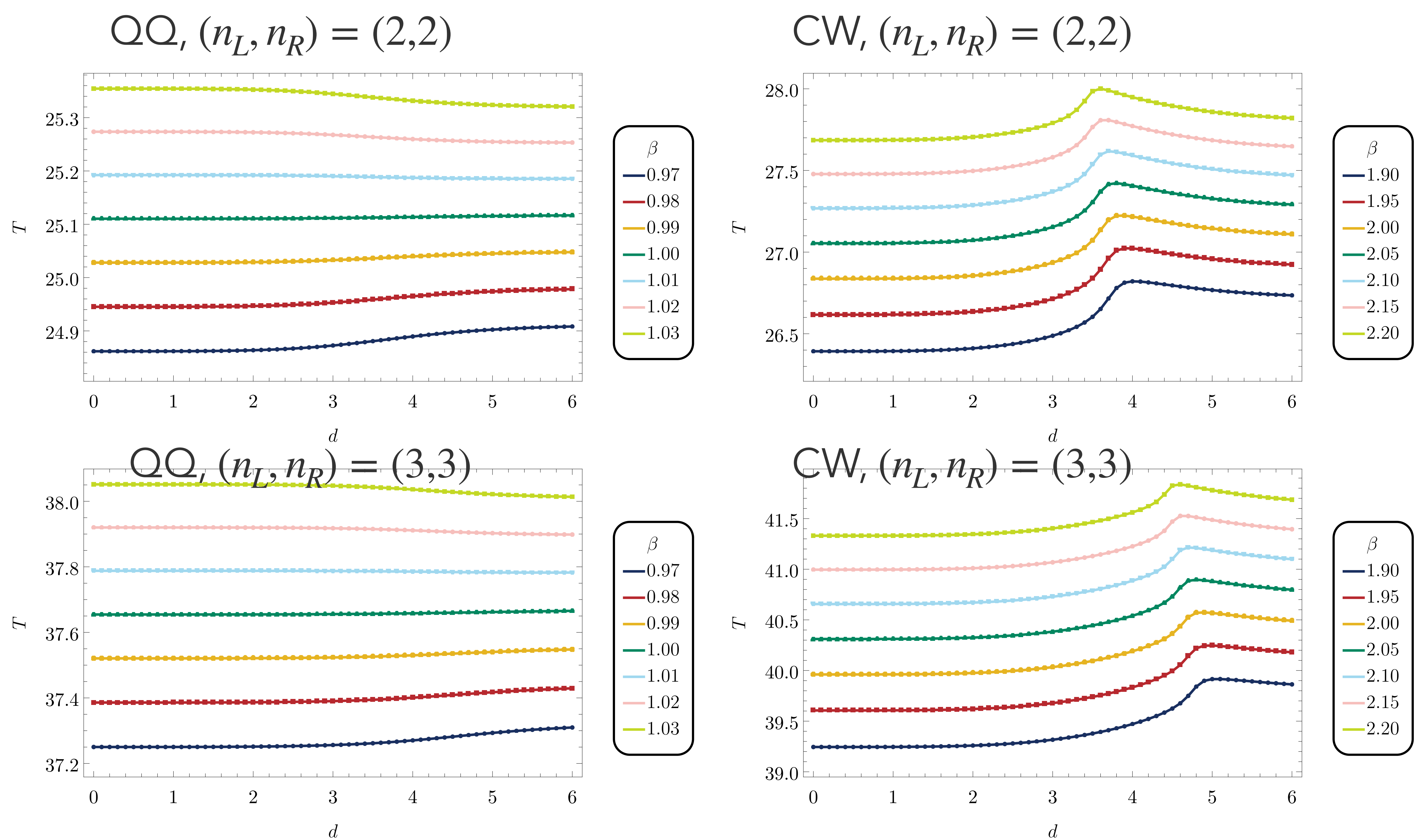
QQ, $(n_L, n_R) = (3,1)$



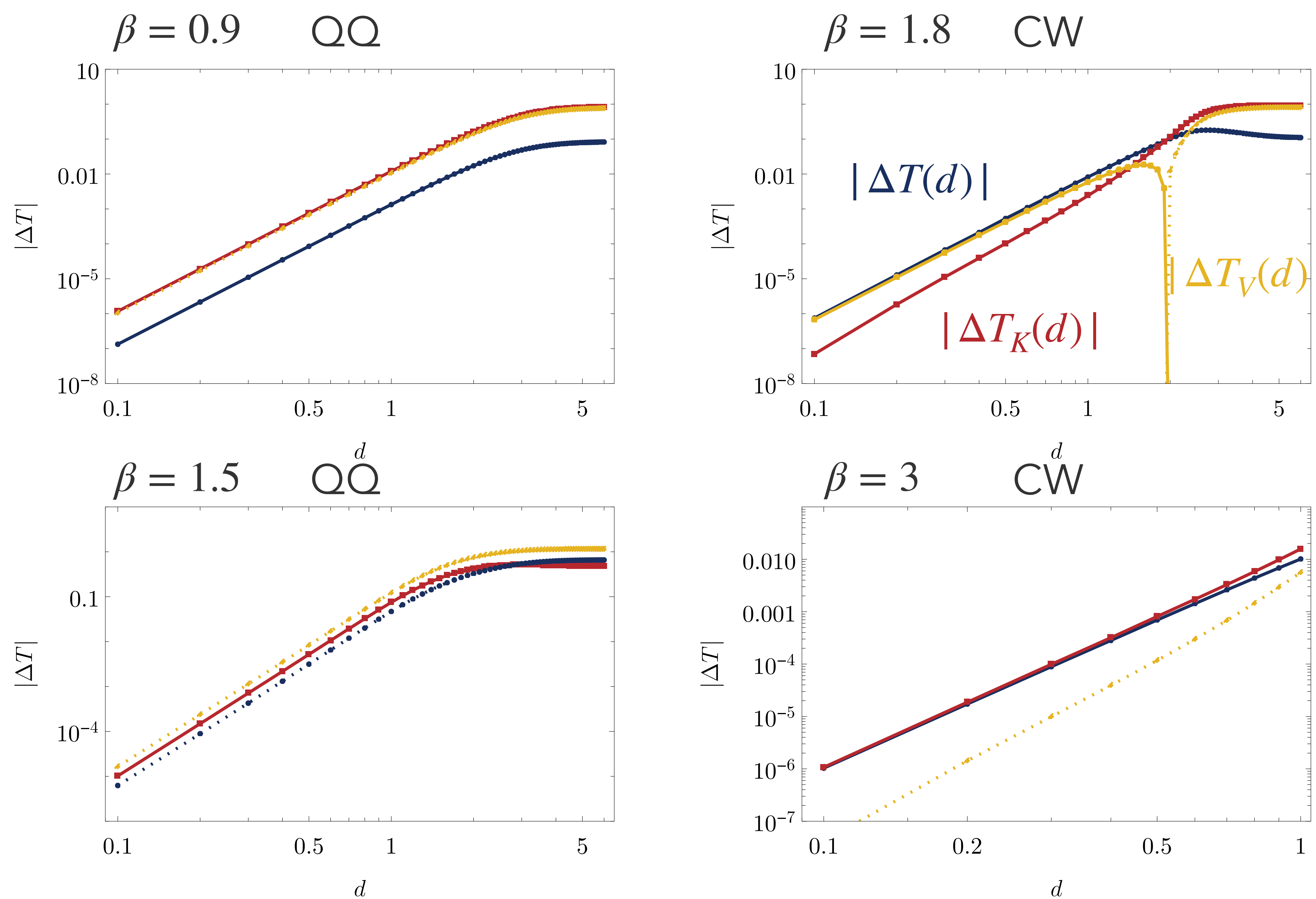
CW, $(n_L, n_R) = (3,1)$



Higher winding (con't)

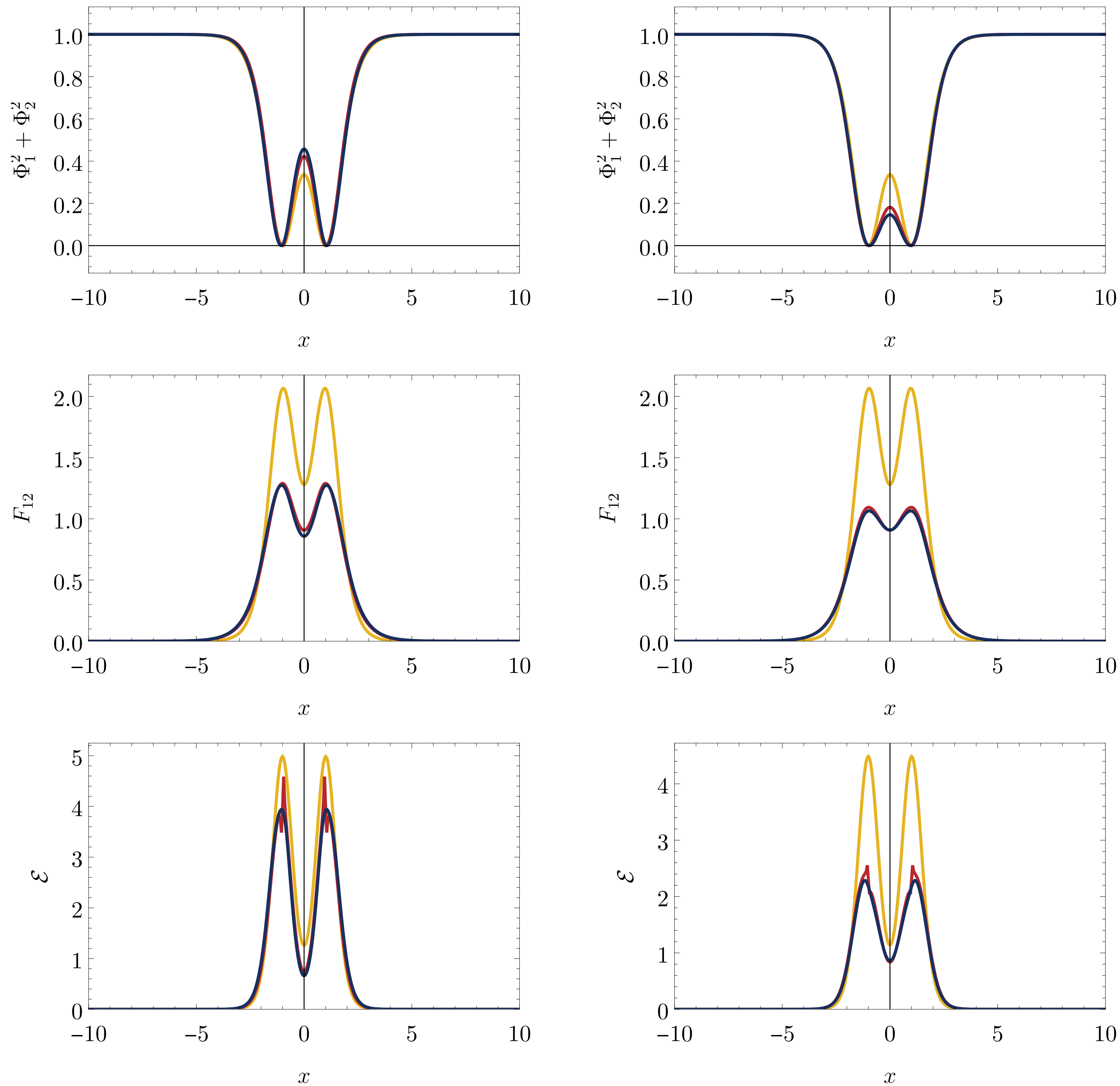


Reason?



$\beta = 0.9$ QQ
10
0.01
10^{-5}
10^{-8}
$|\Delta T|$
0.1
0.5
1
5
d
$\beta = 1.8$ CW
10
0.01
10^{-5}
10^{-8}
$|\Delta T|$
$|\Delta T(d)|$
$|\Delta T_V(d)|$
$|\Delta T_K(d)|$
0.1
0.5
1
5
d
$\beta = 1.5$ QQ
0.1
10^{-4}
$|\Delta T|$
0.1
0.5
1
5
d
$\beta = 3$ CW
0.010
0.001
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$|\Delta T|$
0.1
0.2
0.5
1
d

Relaxation



$\tau = 0, 2, 15$

$y = 0$

$\beta = 1.5$

Energy decomposition

$\beta = 1.5$

n=1

n=2

n=3