カイラル感受率とU(1)アノマリー

Hidenori Fukaya (Osaka U.)

for JLQCD collaboration

[S. Aoki, Y. Aoki, HF, S. Hashimoto, I.Kanamori,

T. Kaneko, Y. Nakamura, K. Suzuki and D. Ward]

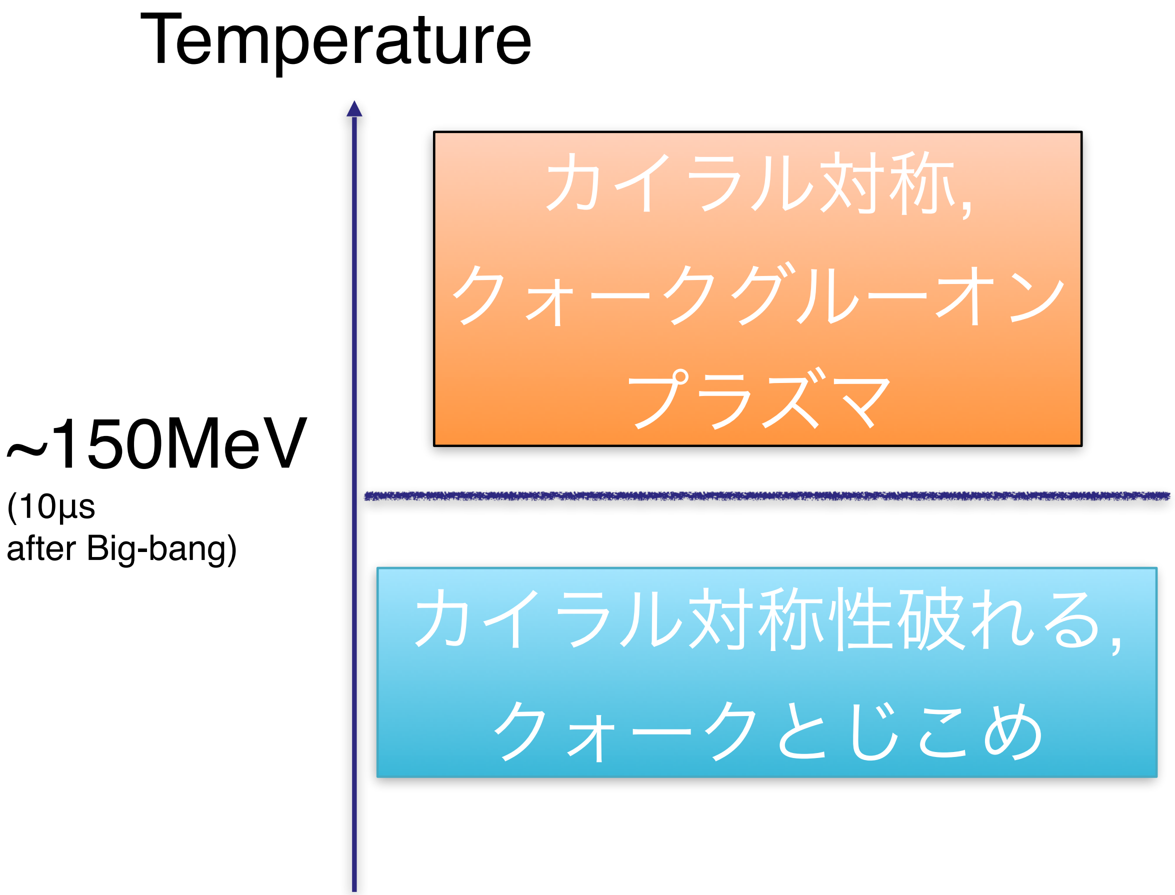
Nf=2 simulation updates from

S. Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki,

PTEP 2022 (2022) 2, 023B05 [2103.05954 [hep-lat]]

and preliminary Nf-2+1 QCD results

QCD相転移



Chiral condensate (at m=0) probes SU(2)$_L$xSU(2)$_R$ symmetry breaking/ restoration :

For T>Tc,

$$\langle \bar{q}q \rangle = 0$$

For T<Tc,

$$\langle \bar{q}q \rangle \neq 0$$

カイラル感受率

QCD partition function

A : gluon fields

$$Z(m) = \int [dA] \det(D(A) + m)^{N_f} e^{-S_G(A)}$$

chiral condensate

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m)$$

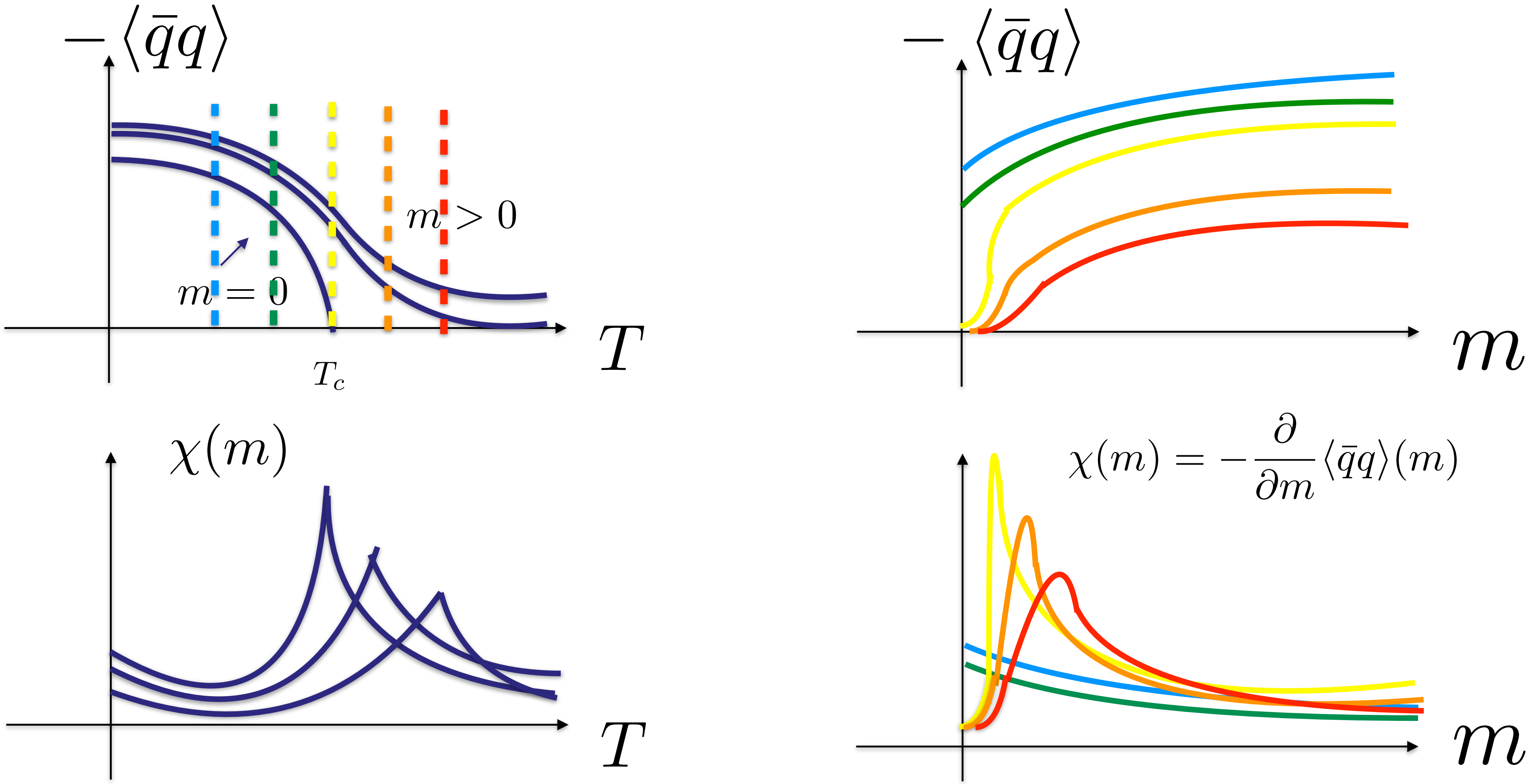
chiral susceptibility

$$\chi(m) = -\frac{\partial}{\partial m} \langle \bar{q}q \rangle(m)$$

In this talk, $N_f = 2 \quad (m_u = m_d = m)$

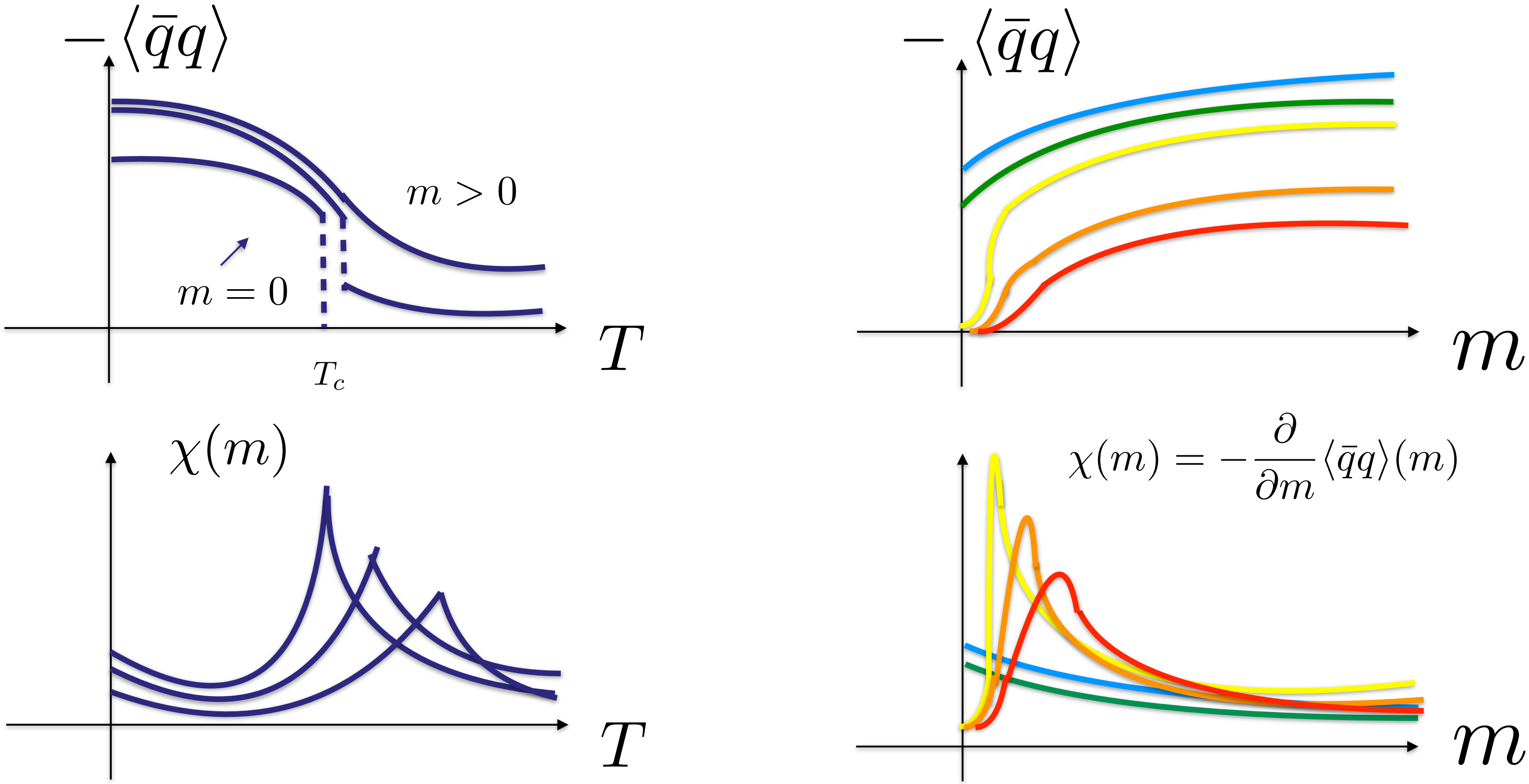
* strange quark is just a spectator.

温度(T) 、質量(m) 依存性


$-\langle\bar{q}q\rangle$
$m > 0$
$m = 0$
T_c
T
$-\langle\bar{q}q\rangle$
m
$\chi(m)$
T
$\chi(m) = -\frac{\partial}{\partial m}\langle\bar{q}q\rangle(m)$
m

When the transition is 1st order

* But finite V effect makes the transition not sharp.



カイラル凝縮は
どの対称性の破れを見ているのか？

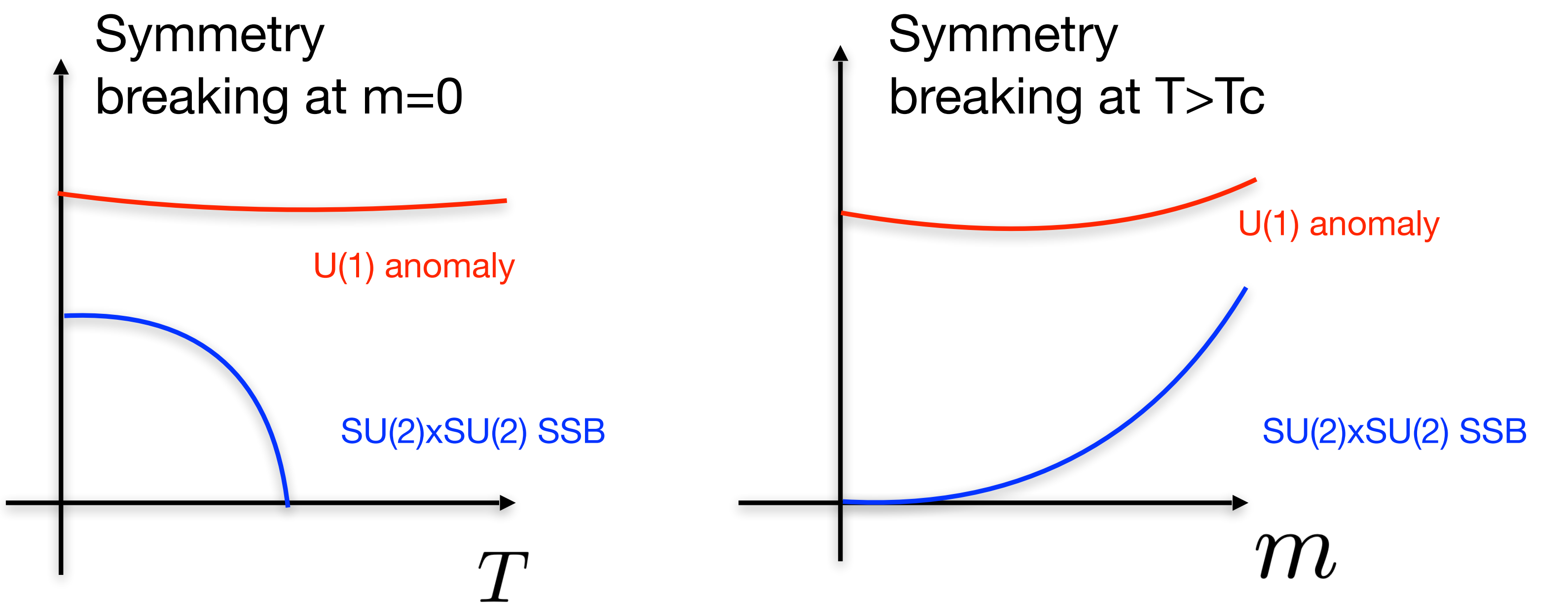
Chiral condensate probes
$SU(2)_L x SU(2)_R$ symmetry breaking/restoration :

For T<Tc, $\langle \bar{q}q \rangle \neq 0$ For T>Tc, $\langle \bar{q}q \rangle = 0$

But $\langle \bar{q}q \rangle$ also breaks $U(1)_A$ symmetry.

Question:
How much does $U(1)_A$ (anomaly) contribute to the transition?

Naive expectation: U(1) anomaly はどのスケールでも存在する(そんなに変化しないはず)



カイラル凝縮の温度、質量依存性はU(1) anomaly　ではなく $SU(2)_L x SU(2)_R$ に起因すると考えるのが自然。

でも初期のQCDでは、、、

70’s and 80’s のQCD創始者たちの考えは

インスタントン → U(1) 量子異常 → SU(2)xSU(2) の破れ

Callan, Dashen & Gross 1978:

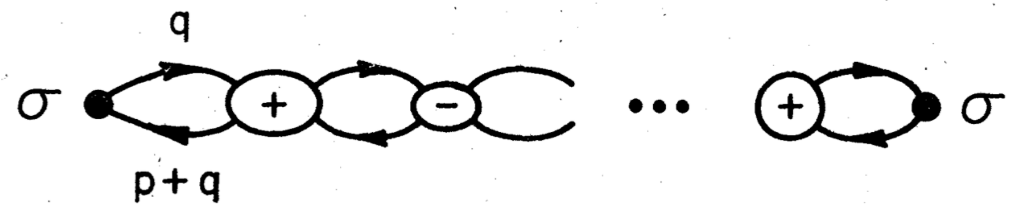


FIG. 9. The structure of the diagrams that produce a tachyon in the σ channel. The + (−) blobs refer to the effective determinantal four-fermion interaction induced by instantons (anti-instantons).

もし逆も正しければ

インスタントンの消失→ U(1)量子異常の消失→ SU(2)xSU(2) の回復.

まだ未解決の問題

解析的手法:

インスタントンによる半古典近似で全て記述できるほど低エネルギーQCDは甘くなかった。

従来の格子QCD 数値計算は格子間隔誤差だらけ:

Staggered fermions explicitly breaks
$SU(2)_L x SU(2)_R x U(1)_A \rightarrow U(1)_{A'}$
Wilson fermion explicitly breaks
$SU(2)_L x SU(2)_R x U(1)_A \rightarrow SU(2)_V$

Moreover, we found that
lattice artifacts are enhanced at high temperature
(even for domain-wall fermions)
[JLQCD 2015, 2016]

私たちの研究

カイラル対称性を精密に保つドメインウォール/オーバーラップフェルミオンを用いて 2- and 2+1-flavor QCD を大規模シミュレーション、

カイラル感受率における軸性U(1)の破れの寄与を、理論的に厳密な方法で分離して抽出、定量評価。

Acknowledgements

私たちがお世話になっている計算機資源:

- 富岳 (hp200130, hp210165, hp210231, hp220279)
- Oakforest-PACS [JCAHPC(最先端共同HPC基盤施設)]

  HPCI projects : hp170061, hp180061, hp190090, hp200086, hp210104,

  MCRP in CCS, U. Tsukuba : xg17i032 and xg18i023
- Wisteria/BDEC-01 [HPCI: hp220093, hp230070, MCRP: wo22i038]
- Polarie/Grand Chariot (hp200130)
- Flow at Nagoya U.
- SQUID at Osaka U.
- Program for Promoting Researches on the Supercomputer Fugaku, Simulation for basic science: from fundamental laws of particles to creation of nuclei Joint
- Institute for Computational Fundamental Science (JICFuS)

Contents

✔ 1. Introduction

カイラル対称性を保つDirac演算子を用いて、カイラル相転移における axial U(1) アノマリーの役割を探ろう。

2. $U(1)_A$ contribution to chiral susceptibility

3. Numerical results

4. Summary

Dirac eigenmode decomposition

$$Z(m) = \int [dA] \det(D(A)+m)^{N_f} e^{-S_G(A)} = \int [dA] \prod_{\lambda} (\underline{i\lambda(A)}+m)^{N_f} e^{-S_G(A)}$$

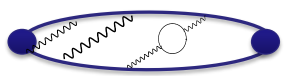
O(100) eigenvalues can be computed on the lattice.

$$-\langle \bar{q}q \rangle = \frac{1}{N_f V} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{V} \left\langle \sum_{\lambda} \frac{1}{i\lambda(A)+m} \right\rangle,$$

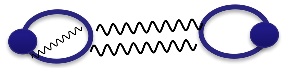
chiral susceptibility

$$\chi(m) = \frac{1}{N_f V} \frac{\partial^2}{\partial m^2} \ln Z(m) = \chi^{con.}(m) + \chi^{dis.}(m),$$

$$\chi^{con.}(m) = -\frac{\partial}{\partial m_{valence}} \langle \bar{q}q \rangle \Big|_{m_{valence}=m}$$



$$\chi^{dis.}(m) = -\frac{\partial}{\partial m_{sea}} \langle \bar{q}q \rangle \Big|_{m_{sea}=m}$$



Chiral rotations (with angle π)

$S^0(x) = \bar{q}q(x)$ $\xleftrightarrow{SU(2)_L \times SU(2)_R}$ $P^a(x) = \bar{q}i\gamma_5\tau^a q(x)$

$\exp(i\pi\gamma_5\tau_a/2)$

$U(1)_A$ $\updownarrow$ $\exp(i\pi\gamma_5/2)$

$P^0(x) = \bar{q}i\gamma_5 q(x)$ $\longleftrightarrow$ $S^a(x) = \bar{q}\tau^a q(x)$

isospin singlet

isospin triplet

Relation to scalar susceptibility

$$L_{\mathrm{QCD}} = \left[\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}\left(\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) + m\right)q\right]$$

$$\chi(m) = \frac{1}{N_f V}\frac{\partial^2}{\partial m^2}\ln Z(m, \theta = 0)$$

$$= -\sum_x \langle S^0(x) S^0(0)\rangle - V\langle S^0\rangle^2$$

$$S^0(x) = \bar{q}q(x)$$

Relation to pseudoscalar susceptibility

$$Z(m,\theta) = \int [dA]\, \det(D(A)+m)^{N_f} e^{-S_G(A)+i\theta Q(A)}$$

$$= \int [dA]\, \det(D(A)+me^{i\gamma_5\theta/N_f})^{N_f} e^{-S_G(A)}$$

← U(1)A rotation

$$\chi_{\text{top.}}(m) = -\frac{1}{N_f V}\frac{\partial^2}{\partial\theta^2}\ln Z(m,\theta)|_{\theta=0} = m\left[\frac{\partial}{\partial\theta}\langle\bar{q}i\gamma_5 e^{i\gamma_5\theta/N_f}q\rangle\right]|_{\theta=0}$$

⇨

$$\frac{N_f}{m^2}\chi_{\text{top.}}(m) = -\sum_x \langle P^0(x)P^0(0)\rangle - \frac{\langle\bar{q}q\rangle(m)}{m}.$$

$$P^0(x) = \bar{q}i\gamma_5 q(x)$$

$$*N_f = 2$$

Connected/disconnected pseudoscalar susceptibilities

From a Ward-Takahashi identity $0 = \langle \delta^a_{SU(2)} P^a(0) \rangle - \langle \delta^a_{SU(2)} S P^a(0) \rangle,$

we have

$$m \sum_x \langle P^a(x) P^a(0) \rangle + \langle S^0 \rangle = 0.$$

Therefore,

$$\frac{N_f}{m^2} \chi_{\text{top.}}(m) = -\sum_x \langle P^0(x) P^0(0) \rangle - \frac{\langle S(0) \rangle}{m}$$

$$= \sum_x \langle P^a(x) P^a(0) \rangle - \sum_x \langle P^0(x) P^0(0) \rangle$$

Symmetry structure of scalar/pseudoscalar susceptibilities

$\chi(m)$

$SU(2)_L \times SU(2)_R$

$$\sum_x \langle S^0(x) S^0(0) \rangle \longleftrightarrow \sum_x \langle P^a(x) P^a(0) \rangle$$

iso-singlet $\quad U(1)_A \quad\quad U(1)_A \quad$ iso-triplet

$$\sum_x \langle P^0(x) P^0(0) \rangle \longleftrightarrow \sum_x \langle S^a(x) S^a(0) \rangle$$

$SU(2)_L \times SU(2)_R$

$$-\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) - \frac{-\langle \bar{q}q \rangle(m)}{m}$$

See also LLNL/RBC Collaboration 2014, Nicola & Elvira 2018, Nicola 2020.

Separating U(1)A breaking part

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V}}_{\text{U(1)A breaking contribution}} \underbrace{- \frac{-\langle \bar{q}q \rangle_{\mathrm{sub.}}(m)}{m}}_{\text{mixed}}$$

* quadratic divergence is subtracted using the data at reference quark mass mref=0.005.

$$\chi^{\mathrm{dis.}}(m) = \overbrace{\frac{N_f}{m^2}\chi_{\mathrm{top.}}(m)}^{\text{U(1)A breaking contribution}} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{\text{SU(2)xSU(2) breaking}}$$

where $\Delta_{U(1)}(m) \equiv \sum_x \langle P^a(x)P^a(0) - S^a(x)S^a(0) \rangle$ is axial U(1) susceptibility

$$\Delta^{(1)}_{SU(2)}(m) \equiv \sum_x \langle S^0(x)S^0(0) - P^a(x)P^a(0) \rangle \qquad \Delta^{(2)}_{SU(2)}(m) \equiv \sum_x \langle S^a(x)S^a(0) - P^0(x)P^0(0) \rangle$$

Lattice formulas

Using

$$\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$$

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2\right].$$

Remark.1 eigen functions do not matter.

Remark.2 chiral symmetry is essential for this decomposition.

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3. Numerical results

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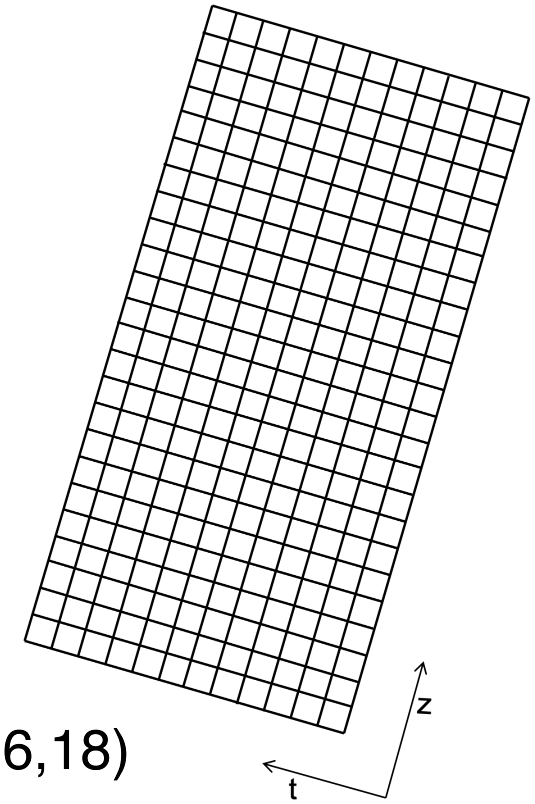
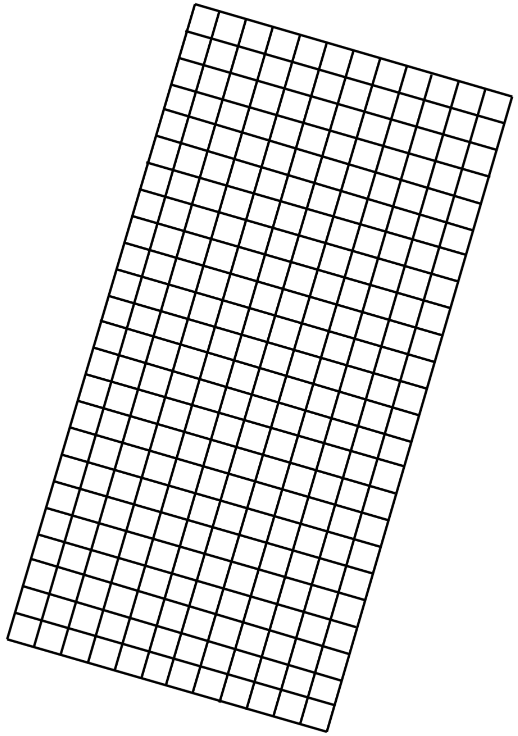
Simulation setup (Nf=2)



Nf=2 flavor QCD
1/a = 2.6 GeV (0.075fm)
Symanzik gauge action
L=24,32,40,48 [1.8-3.6fm] (at T=220MeV)
Mobius domain-wall fermions with m_{res}<1MeV
(and reweighted overlap fermion)
Quark mass from 3MeV (< phys. pt. ~4MeV) to 30MeV.
T=147, 165 (~Tc), 195, 220, 260, 330 MeV (Lt=8,10,12,14,16,18)
Tc is estimated to be around 175MeV (from Polyakov loop)

Simulation codes : Irolro++ (https://github.com/coppolachan/Irolro)
Grid (https://github.com/paboyle/Grid)
Bridge++(https://bridge.kek.jp/Lattice-code/)

Simulation setup (Nf=2+1)



Nf=2+1 flavor QCD

1/a = 2.453GeV

L=32 (2.58 fm), 40 (3.22 fm), 48(3.9fm)

Mobius domain-wall fermion with m_{res}<1MeV

(and reweighted overlap fermion)

up-down quark mass from

phys. pt. ~4MeV to 30MeV.

strange quark mass at phys.pt.

T=136, 153(~Tc), 175, 220 MeV

Overlap/domain-wall reweighting

The fermion action can be changed

AFTER simulation.

$$\langle O \rangle_{overlap} = \frac{\int dAO[\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA[\det D_{\rm ov}(m)]^2 e^{-S_G}}$$

$$= \frac{\int dAOR[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}}{\int dAR[\det D^{\rm 4D}_{\rm DW}(m)]^2 e^{-S_G}}$$

$$= \frac{\langle OR \rangle_{domain-wall}}{\langle R \rangle_{domain-wall}}$$

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D^{\rm 4D}_{\rm DW}(m)]^2}$$

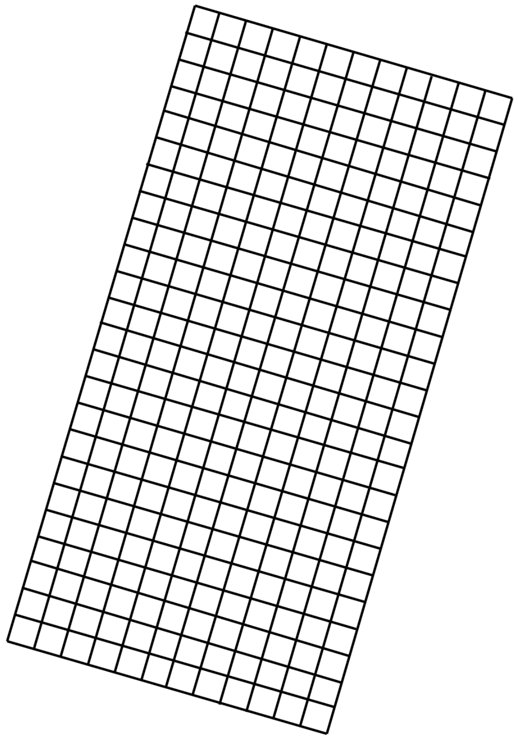
can be numerically computable.

Mass reweighting

さらにchiral limit に近づけるため、質量の異なるdeterminantにreweightingを実施。

Nf=2 : m=0.0002(1/5 physical mass), 0.0005, 0.0015 from m=0.001

Nf=2+1: m=0.001(1/2 physical mass) from m=0.002



Low-mode approximation

In the eigenvalue summations,

$$\Delta_{U(1)}(m) = \frac{1}{V(1-m^2)^2}\left\langle \sum_{\lambda_m} \frac{2m^2(1-\lambda_m^2)^2}{\lambda_m^4} \right\rangle,$$

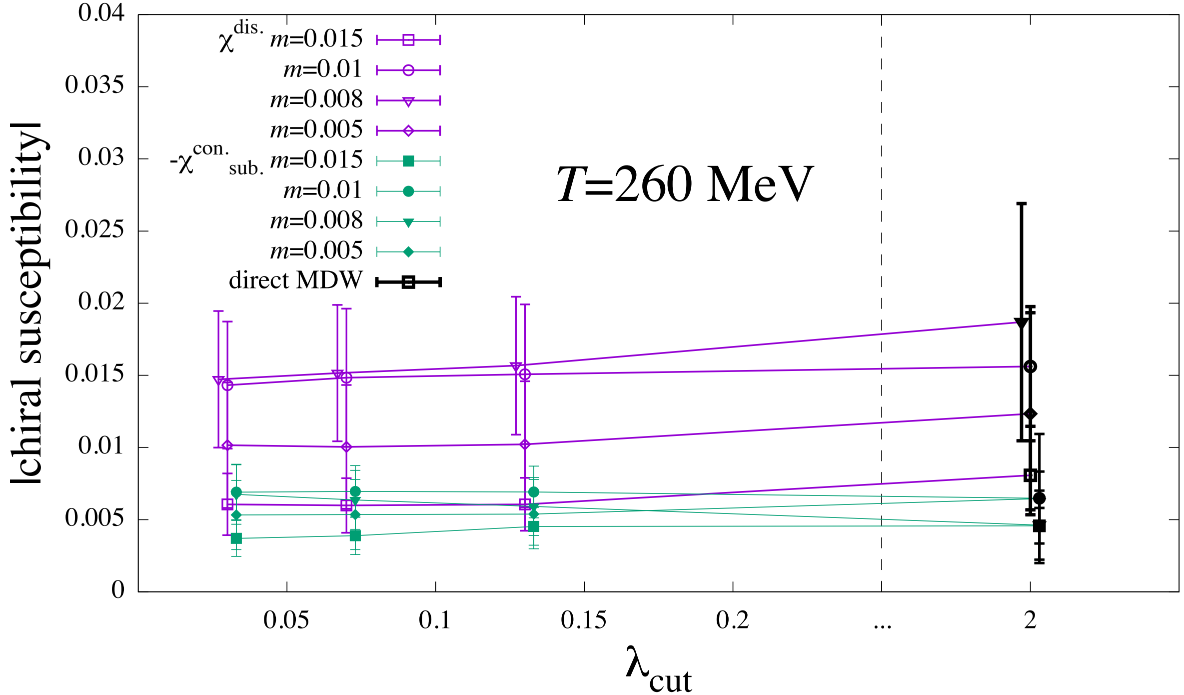
$$-\langle \bar{q}q \rangle = \frac{1}{V(1-m^2)}\left\langle \sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2} \right\rangle.$$

$$\chi^{\text{dis.}}(m) = \frac{N_f}{V}\left[\frac{1}{(1-m^2)^2}\left\langle \left(\sum_{\lambda_m} \frac{m(1-\lambda_m^2)}{\lambda_m^2}\right)^2 \right\rangle - |\langle \bar{q}q \rangle^{lat}|^2 V^2\right].$$

where $\lambda_m = \text{eigenvalues of } H_m = \gamma_5[(1-m)D_{ov} + m]$

we truncate at 30-40th lowest mode ($\lambda_{\text{threshold}} \sim 150\text{–}300\ \text{MeV}$).

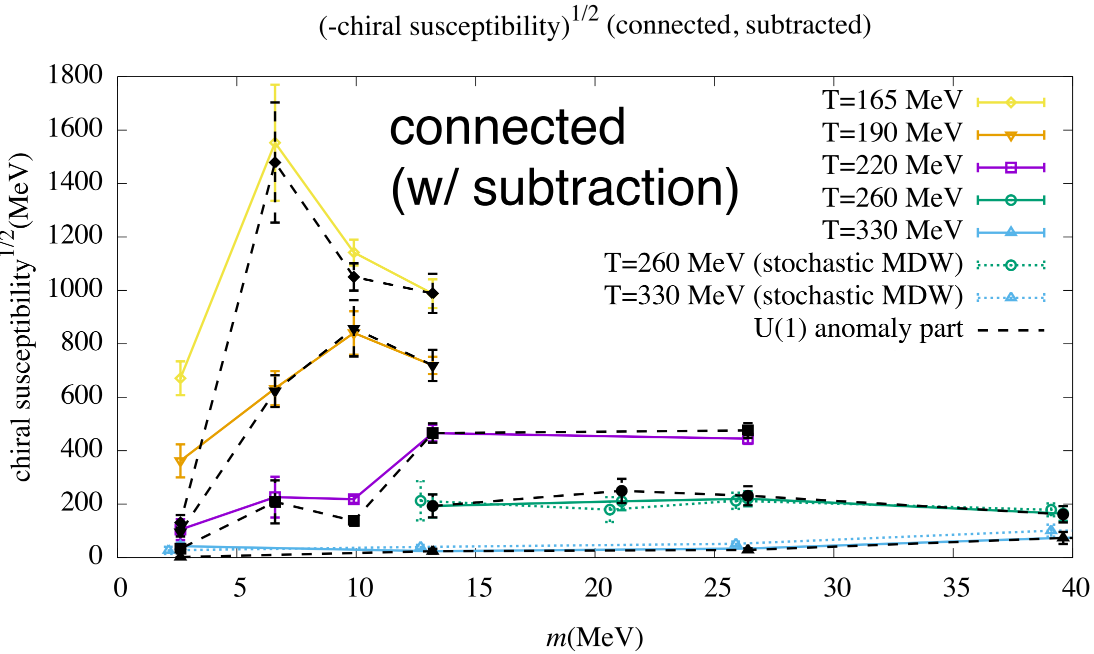
Low mode approximation

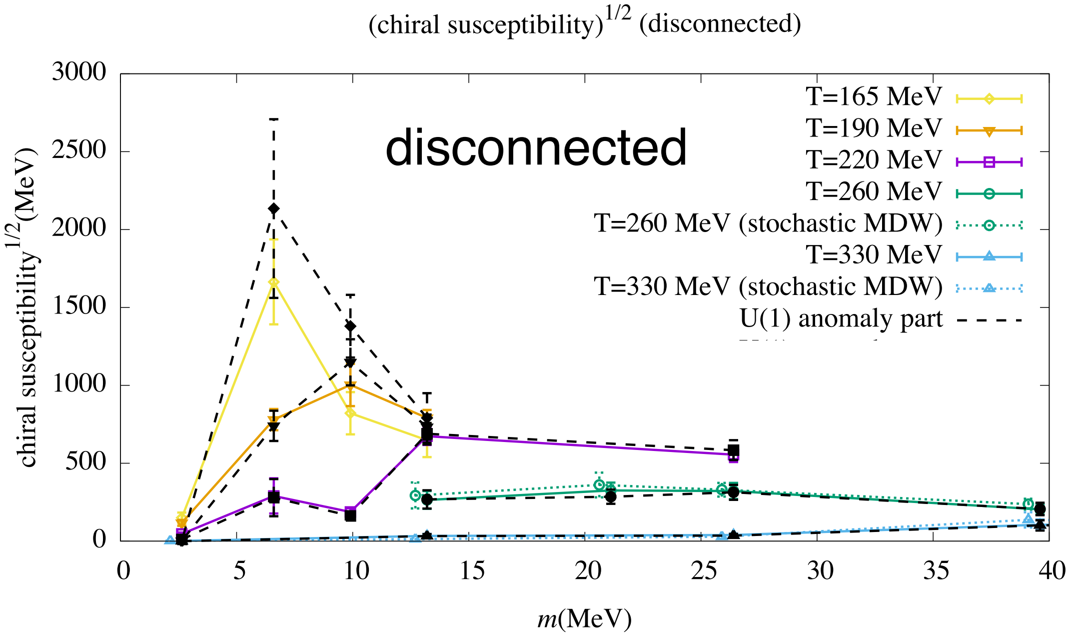


For T<= 260MeV, we find a good saturation and consistency with direct inversion of Mobius domain-wall Dirac operator (direct MDW) but T=330 MeV, it is not good; we use direct MDW.

Previous Nf=2 results at higher Ts

S.Aoki, Y. Aoki, HF, S. Hashimoto, C. Rohrhofer, K. Suzuki [JLQCD collaboration] PTEP2022 (2022) 2, 023B05 [arXiv:2103.05954]



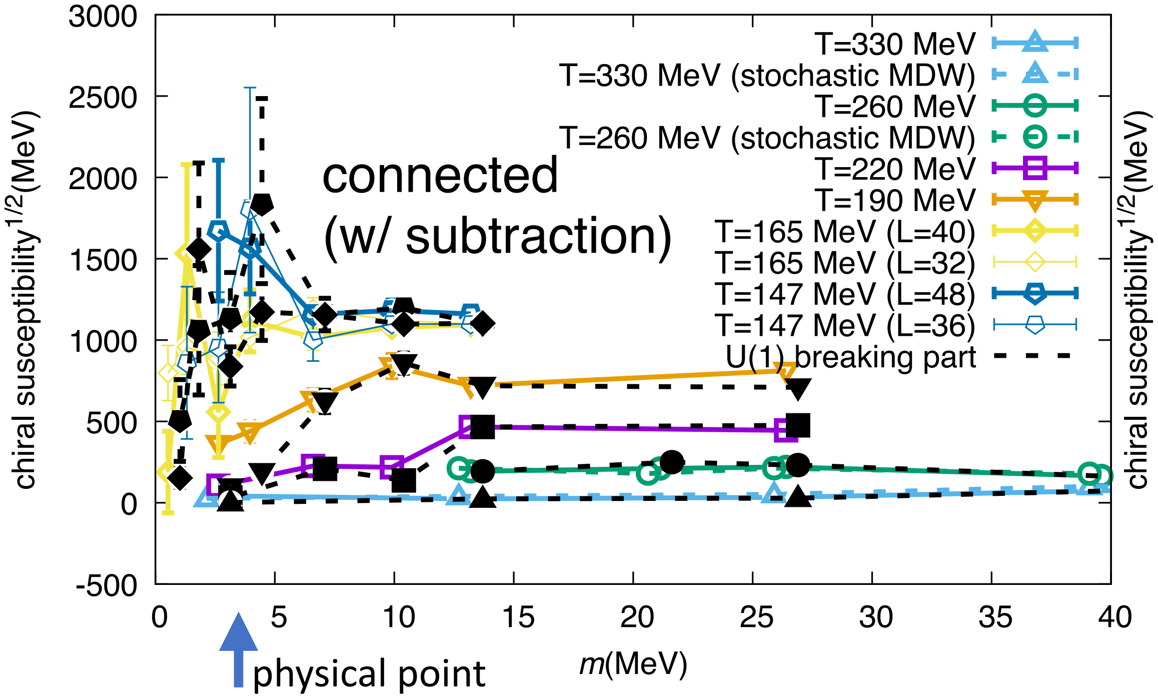


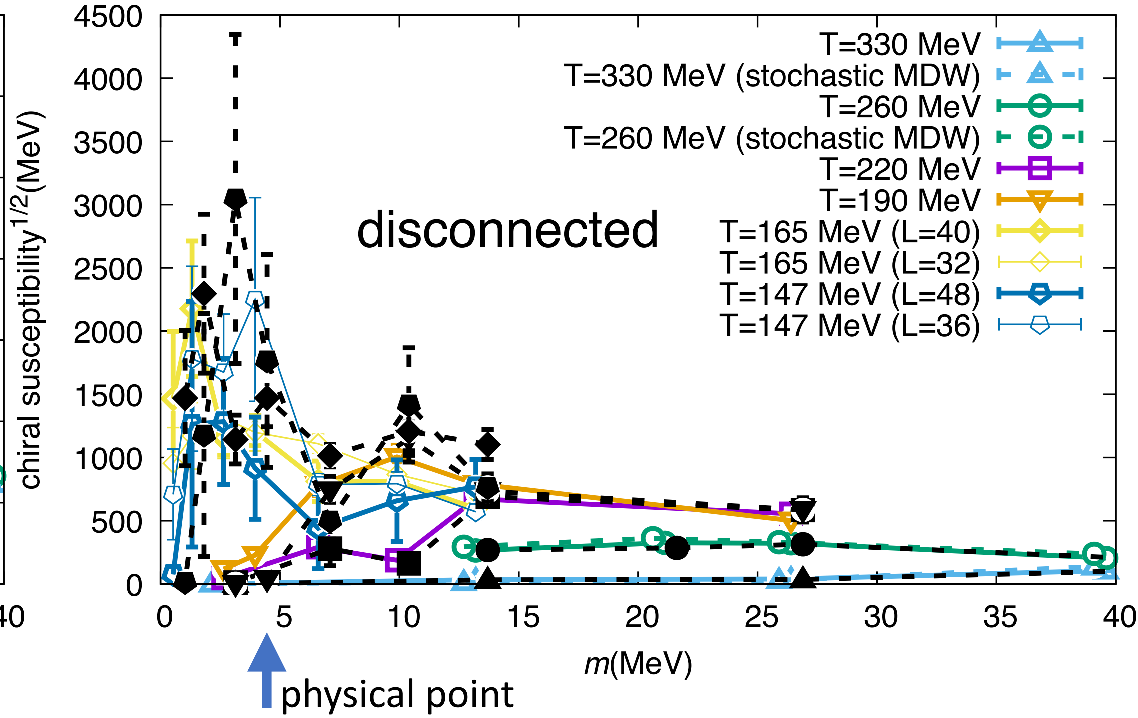
The dominance by axial U(1) anomaly is seen at 5 different Ts.

$$\chi^{\rm con.}(m) = \underbrace{-\Delta_{U(1)}(m) + \frac{\langle|Q(A)|\rangle}{m^2 V}}_{U(1)_A \text{ breaking}} \underbrace{-\frac{-\langle\bar{q}q\rangle_{\rm sub.}(m)}{m}}_{\text{mixed}}$$

$$\chi^{\rm dis.}(m) = \underbrace{\frac{N_f}{m^2}\chi_{\rm top.}(m)}_{U(1)_A \text{ breaking}} \underbrace{+\Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)}_{SU(2)xSU(2) \text{ breaking}}$$

Nf=2 QCD updates (w/ lower T and m and larger V)





低温（0.9Tc）1/5 physical quark mass

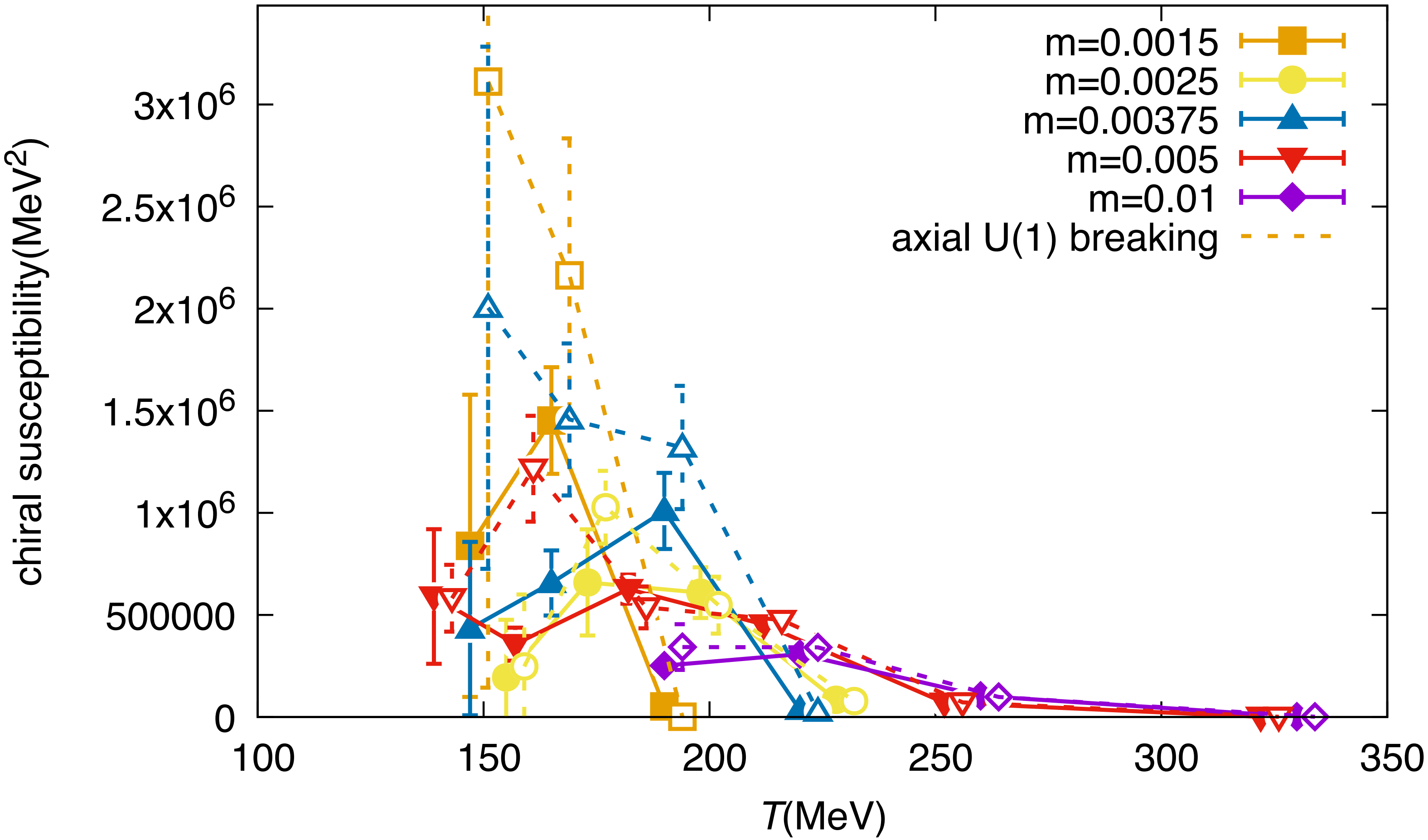
でもAxial U(1) の破れが支配的！

Colored open symbols: data for chiral susceptibility

Black filled symbols: axial U(1) anomaly part

Finite V effects look under control.

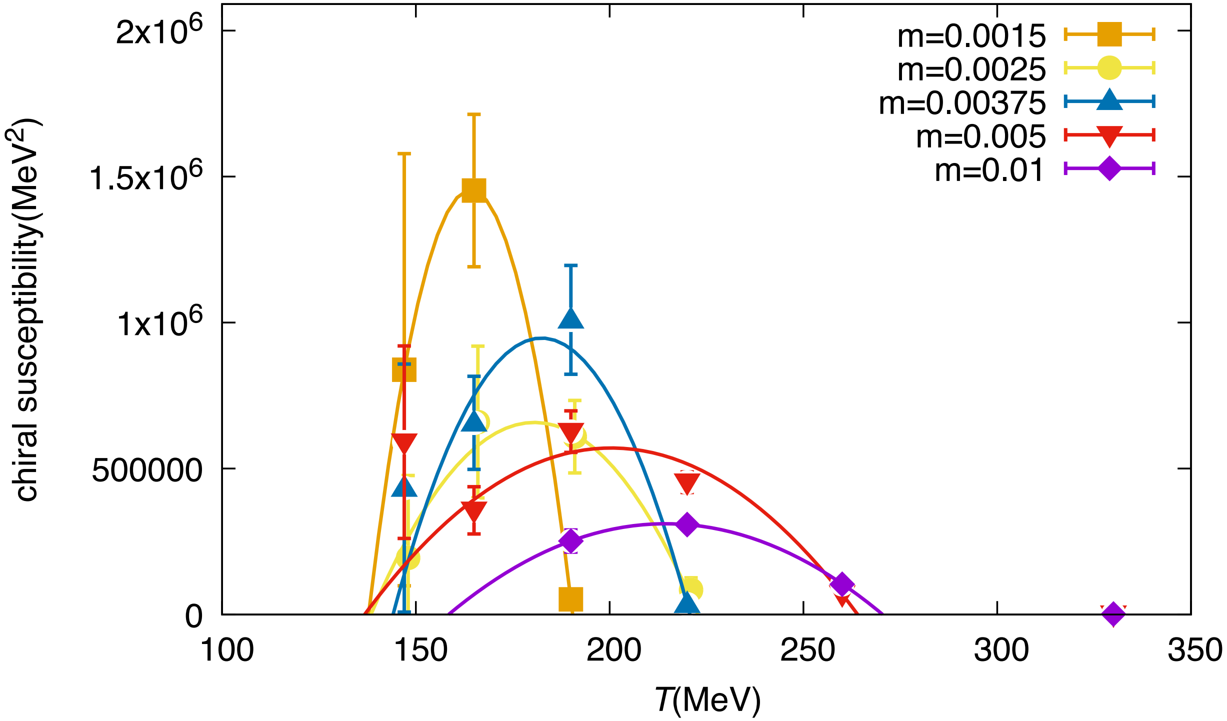
T dependence of disconnected part

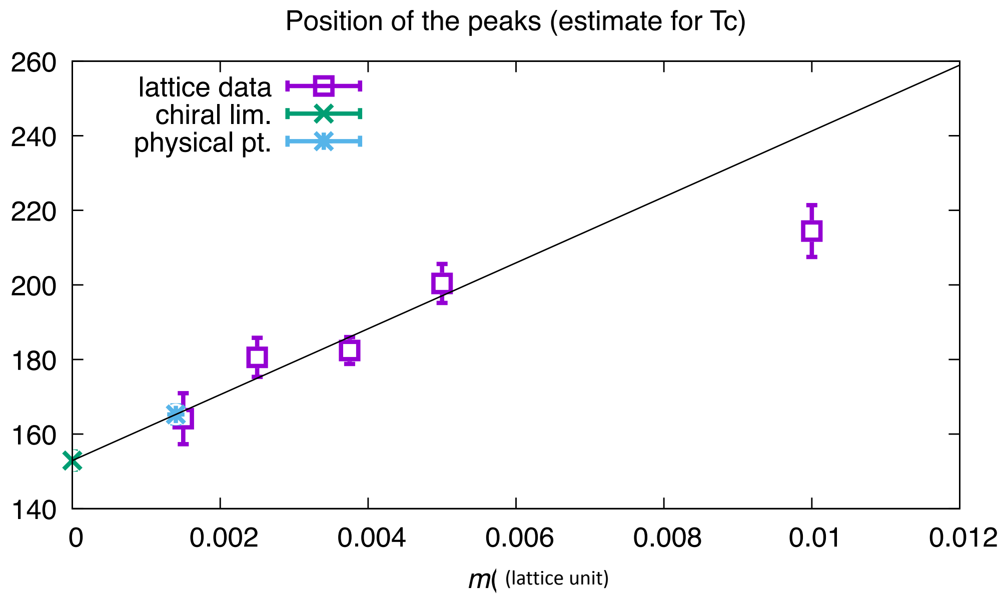


At higer T than the peaks, axial U(1) breaking is dominant.

But it deviates at lower T.

Determination of Tc (very preliminary)

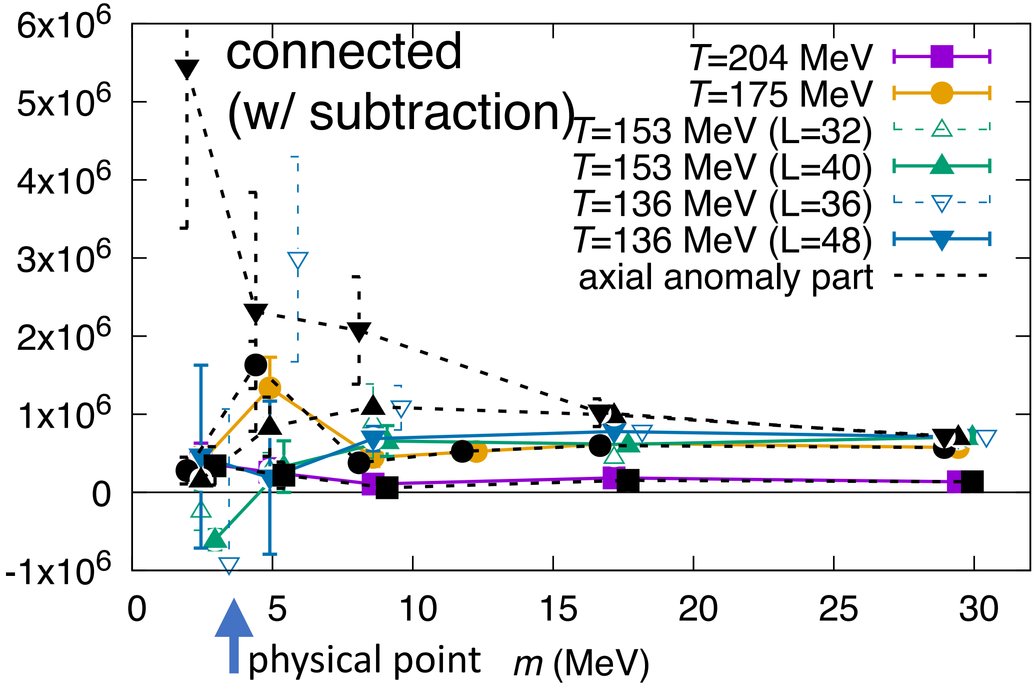


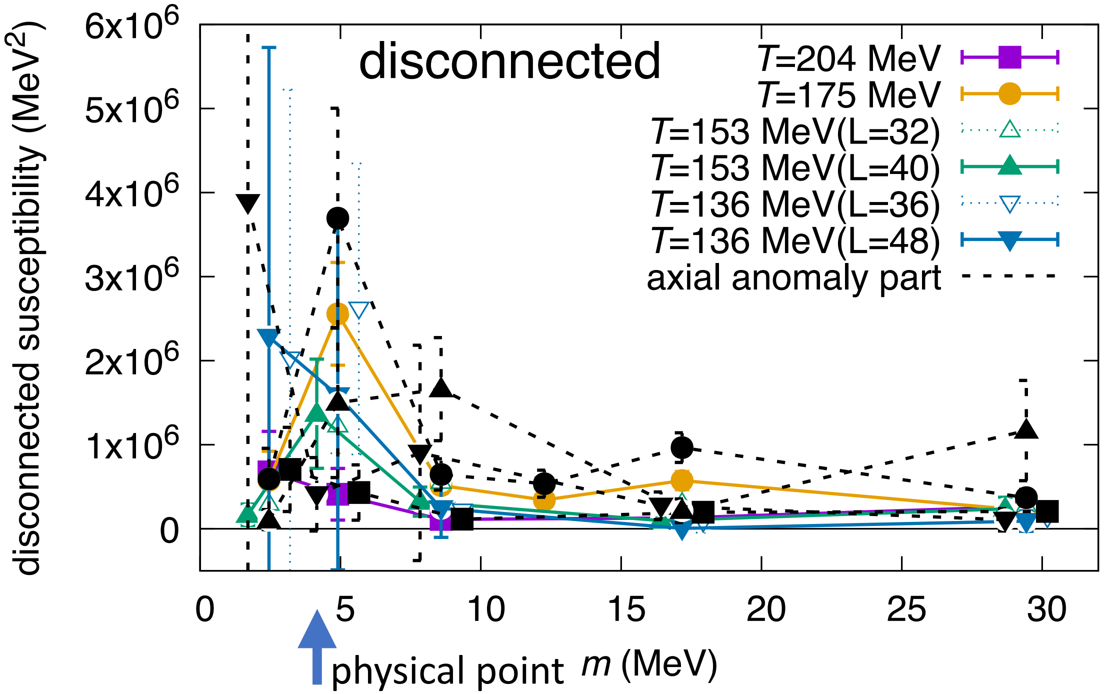


From a quadratic estimate for the position of the peak, we obtain Tc (physical pt.) = 165(3)MeV, Tc(chiral limit)=153(3) MeV.

Nf=2+1results

Colored open symbols: data for chiral susceptibility
Black filled symbols: axial U(1) anomaly part





Axial U(1) dominance is seen.
However, statistically noisy.
Different Vs are consistent.
At the physical point m~4MeV, pseuco-critical T is estimated to be 140-150MeV.

Subtlety in the total contribution

$$\chi(m) = \chi^{\mathrm{con.}}(m) + \chi^{\mathrm{dis.}}(m)$$

$$\chi^{\mathrm{con.}}(m) = -\Delta_{U(1)}(m) + \frac{\langle |Q(A)| \rangle}{m^2 V} - \frac{-\langle \bar{q}q \rangle_{\mathrm{sub.}}(m)}{m}$$

U(1)$_A$ breaking contribution

mixed

$$\chi^{\mathrm{dis.}}(m) = \frac{N_f}{m^2}\chi_{\mathrm{top.}}(m) + \Delta^{(1)}_{SU(2)}(m) - \Delta^{(2)}_{SU(2)}(m)$$

SU(2)xSU(2) breaking

a large cancellation

O(1/V$^{1/2}$) effect

It is difficult to see what survives in the total contribution.

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✔ 2. U(1)$_A$ contribution to chiral susceptibility

カイラル感受率における axial U(1) の破れの寄与を非摂動的に分離が可能。

✔ 3. Numerical results

Tc以上の高温でカイラル感受率のシグナルのほとんどを axial U(1) の破れが占めている。

4. Summary

まとめ

1. Nf=2 and 2+1 lattice QCD をカイラル対称性を精密にたもつ domain-wall/overlap クォーク作用で実行→カイラル感受率におけるU(1)量子異常の寄与を正確に抽出.
2. 今回はNf=2低温側 with mass reweighting (~1/5 physical point)、Nf=2+1の新しい結果について発表。
3. T>=Tcにおいて、Connected/disconnected カイラル感受率は U(1) の破れが支配的(相転移のトリガーはaxial U(1)なのではないか)

   Connected part ~ axial U(1) susceptibility.
   Disconnected part ~ top. susceptibility x $2/m^2$

What if axial U(1) “restored”?

Not only $SU(2)_L \times SU(2)_R$ but also $U(1)_A$
may be restored at Tc.

Then, the effective action = SU(2)xSU(2)[or O(4)] linear sigma model needs additional degrees of freedom.

-> effective potential becomes complicated

-> 1st-order transition is favored [Pisarski & Wilczek]

(the same suggestion as Yonekura-san’s PPP2020 talk but from different point of view.)

What if chiral phase transition is 1st order?

* 1st order region may be spanned to finite quark mass.
* If physical point is not a crossover but 1st order, QCD may explain dark matter (Witten) [Yonekura-san's PPP2020 talk]
* Gravitational waves due to QCD bubbles.
* Axion dark matter scenario may be difficult (abundance is likely to be too big).

Interesting! But we have not detected its sign.