Link soliton in models with B-L and Peccei-Quinn symmetries

Yu Hamada (KEK -> DESY)

arXiv: 2309.XXXXX

w/ M. Eto (Yamagata U.) and M. Nitta (Keio U.)

PPP2023, 30th August 2023 @ YITP

Knot soliton
II
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Introduction

$U(1)_{\mathrm{global}} \times U(1)_{\mathrm{gauge}}$ が自発的に破れる模型では、

変態的なソリトンが現れる(場合があります)

$U(1)_{PQ}$ $U(1)_{B-L}$

↑ ↑

$U(1)_{\rm global} \times U(1)_{\rm gauge}$ が自発的に破れる模型では、

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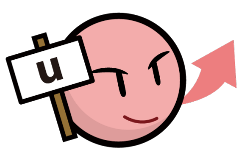
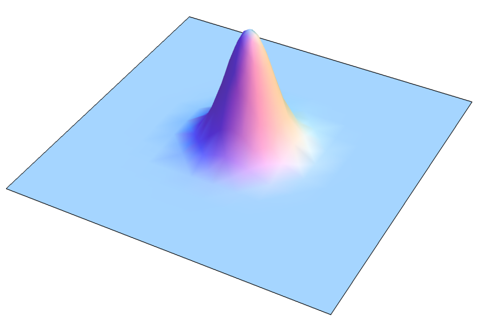
Soliton

- **Particle**：真空周りの摂動的ゆらぎ



- **Soliton：古典的でコヒーレントな励起** (``粒子のカタマリ'')



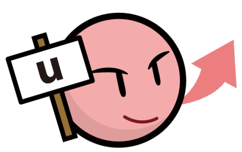
localized excitation

ex.) Tsunami

wikipedia
``神奈川沖浪裏''

Soliton

- **Particle**：真空周りの摂動的ゆらぎ



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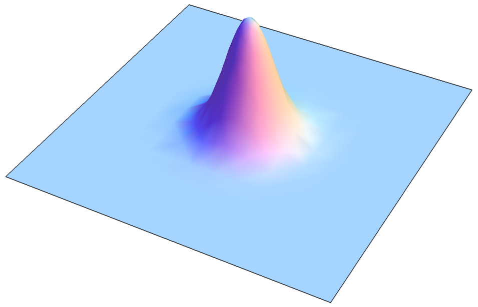

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special object? → not so special!

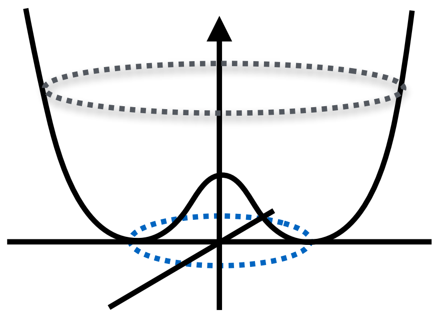
Eg.) Vortex string

[Abrikosov '58]

[Nielsen-Olesen '73]

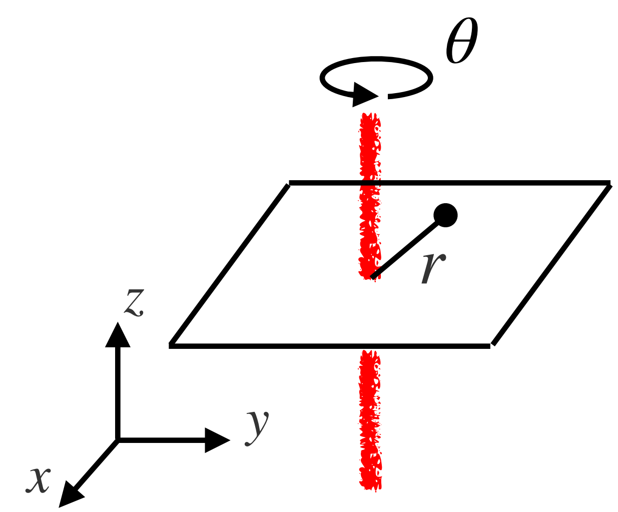
- 3+1 D Abelian-Higgs model

$\langle \phi \rangle = v \rightarrow \cancel{U(1)}$



$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4$$

- Introduce polar coordinate: $r = \sqrt{x^2 + y^2},\ \tan\theta = \frac{y}{x}$



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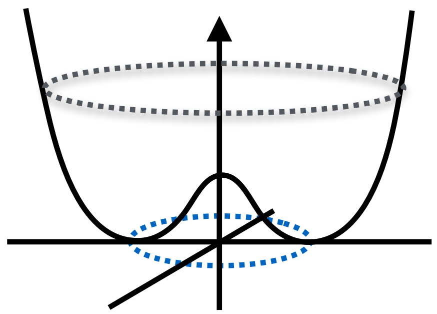

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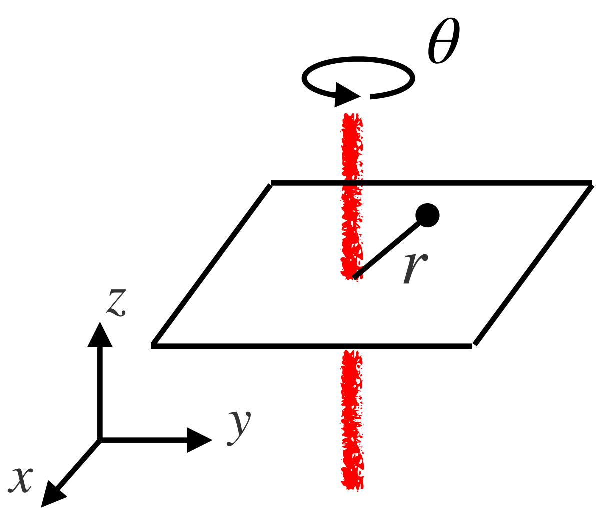

- **Field configuration:**

$$\phi(x) = v f(r) e^{i\theta} \qquad \vec{A}(x) = g^{-1} a(r) \vec{e_\theta}$$

ϕ's phase has winding # = 1

Eg.) Vortex string

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[Nielsen-Olesen '73]

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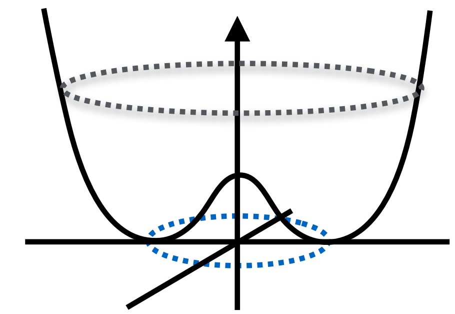

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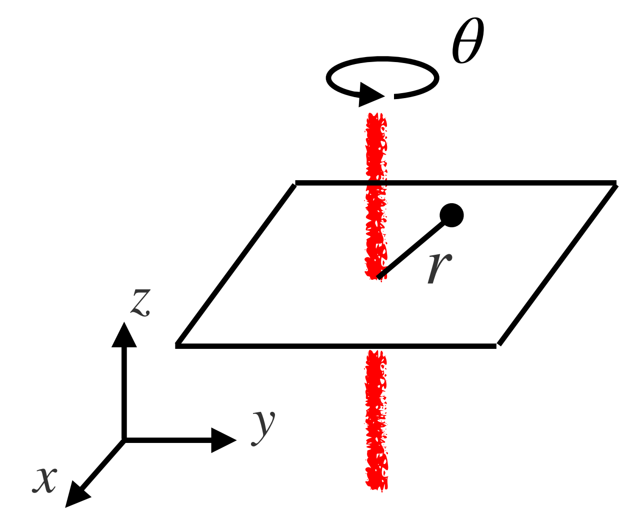

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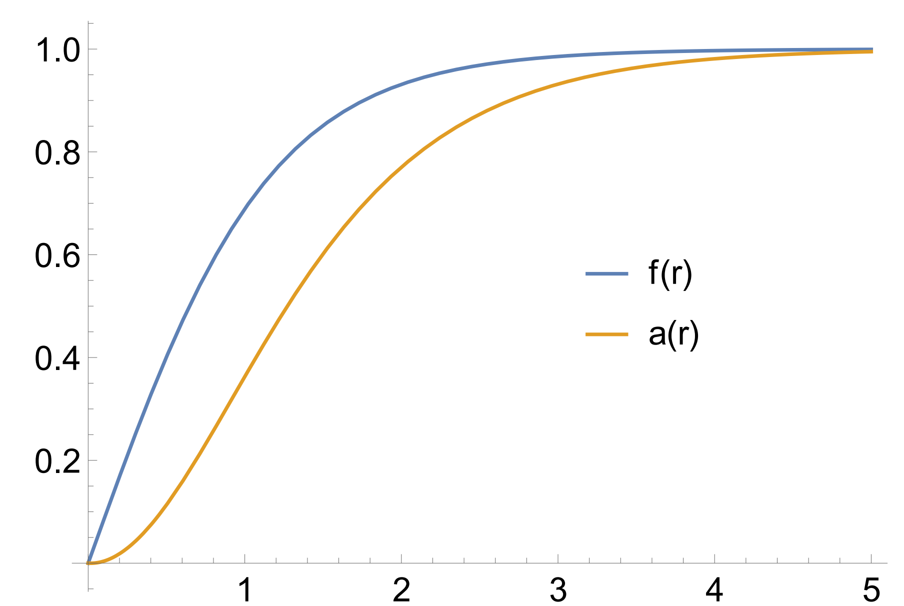
- **solving classical EOMs for $f(r)$ and $a(r)$:**



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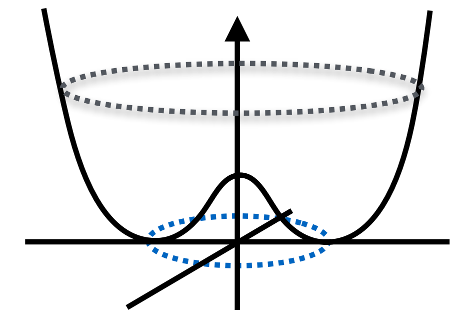

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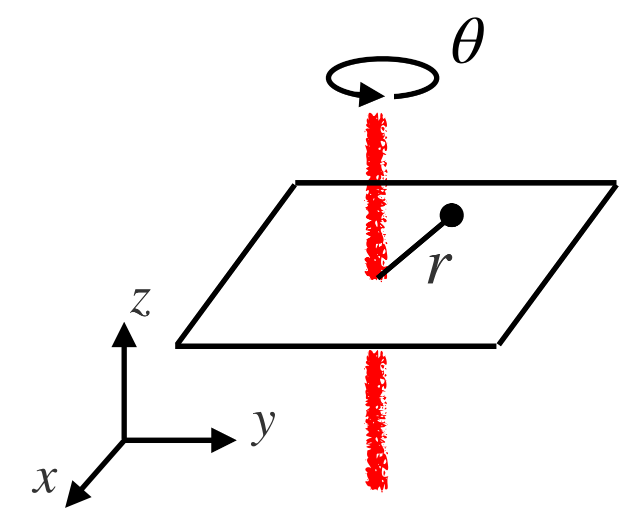

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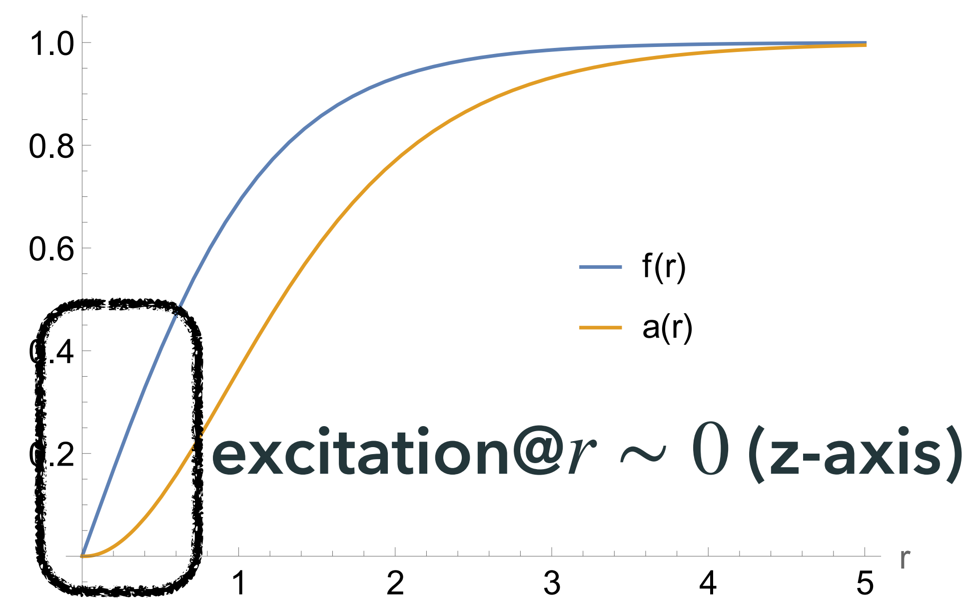


Vortex string in many systems

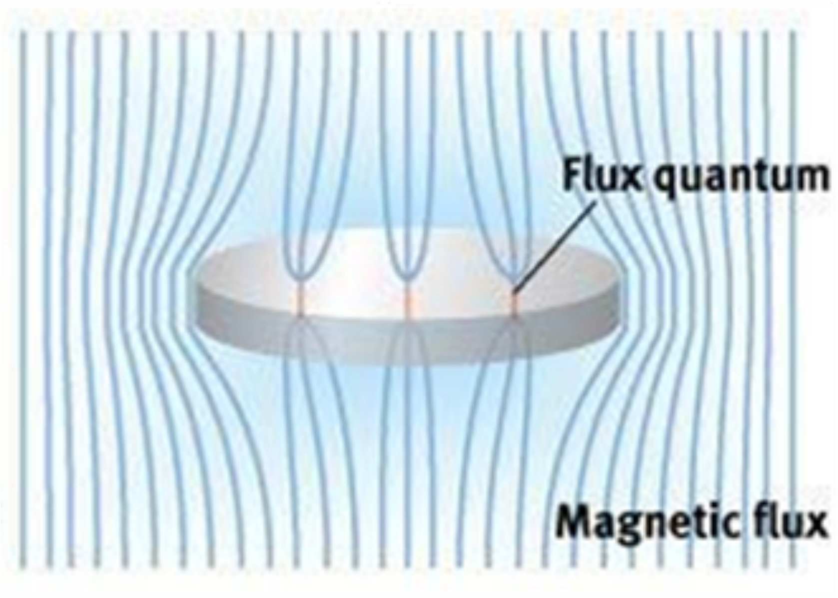
- **Magnetic flux tube in superconductor**
    - characterize phases of supercond.
- Vortex string in the universe: **Cosmic string**

  → cf. 北嶋さんトーク,

  神田さんポスター, 千歳さんポスター

    - Gravitational wave
    - **strong evidence of new physics**, but haven't yet been discovered.
- Superfluid vortex in neutron star

  → cf. 藤原さんトーク



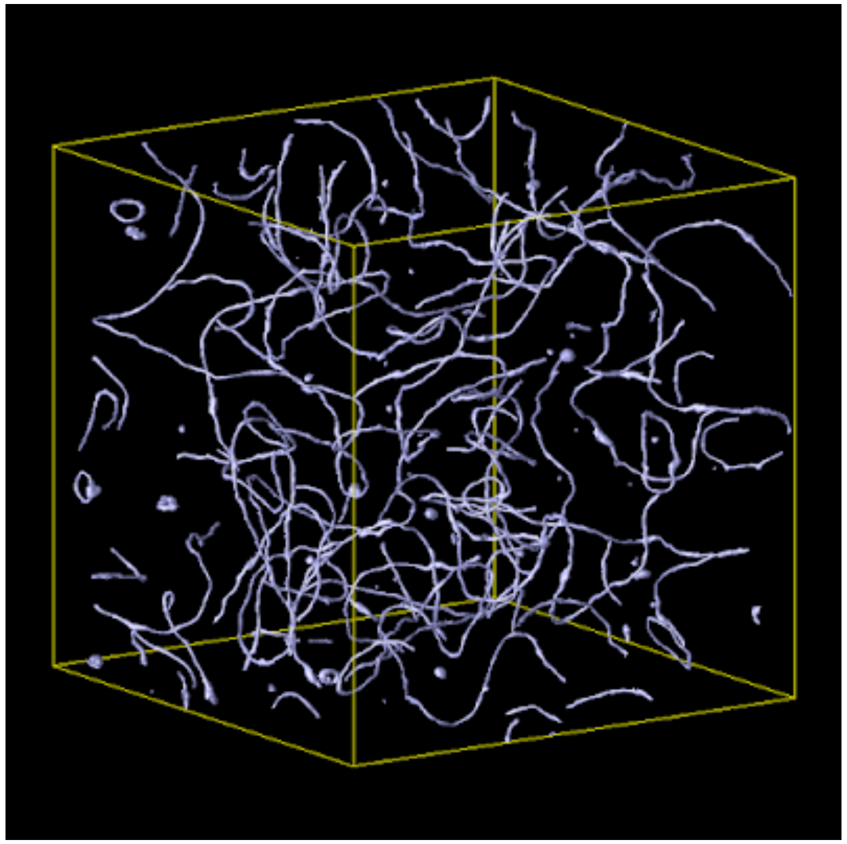


fig from slide by Hiramatsu

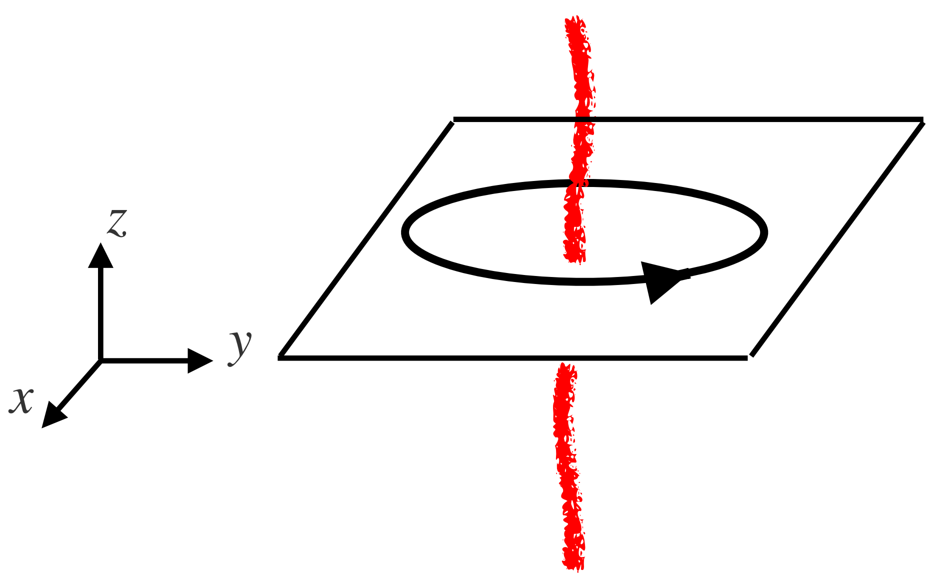
Global string vs Local string

- SSB of **gauged** $U(1)$ sym → **local** vortex string

→ **string中に"磁場フラックス"が存在:** $\int d^2x\, B = 2\pi/g$

- SSB of **global** $U(1)$ sym → **global** vortex string

→ 磁場無し



$$\phi(x) = v f(r) e^{i\theta}$$

stringの周りで NG場が 0 から 2π に変化

Knot soliton

Local string

(well-known)

Global string

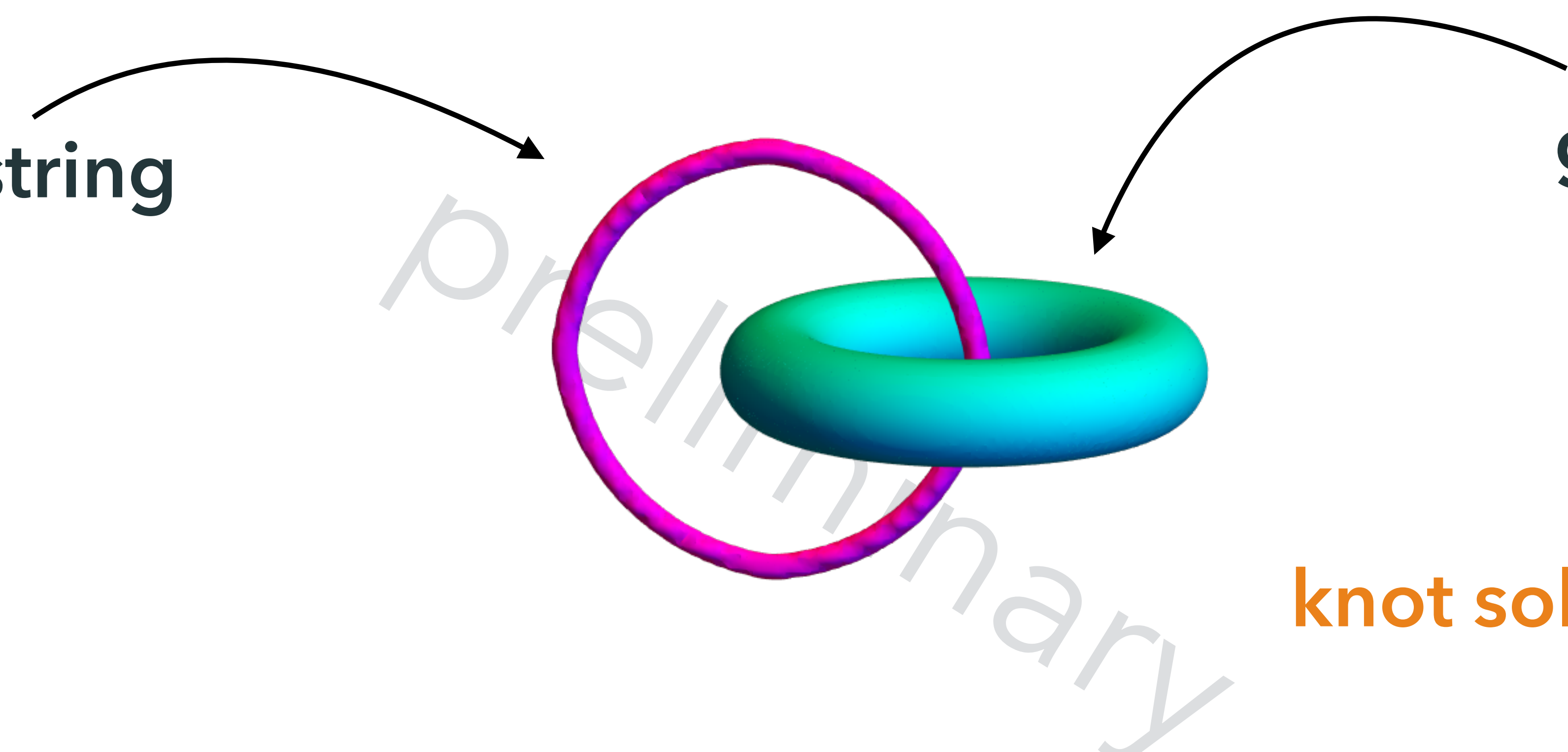
(well-known)

Knot soliton

Local string
(well-known)

combine

Global string
(well-known)



Plan of talk

- Introduction
- Knot soliton
- Application to phenomenology & cosmology
- Summary

Knot soliton

The model

Lagrangian:

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4} F_{\mu\nu}^2 - V(\phi_1, \phi_2)$$

$$V(\phi_1, \phi_2) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

- Symmetries:

	$U(1)_{\text{gauge}}$	$U(1)_{\text{global}}$
ϕ_1	1	0
ϕ_2	0	1

$D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$

- For $\kappa > 0$ & $\lambda > 0$, both symmetries are broken at the vacuum:

$$\langle \phi_1 \rangle = v_1, \ \langle \phi_2 \rangle = v_2$$

→ local string ($\phi_1 \sim v_1 e^{i\theta}$) & global string ($\phi_2 \sim v_2 e^{i\theta}$)

Chern-Simons coupling

3+1D theory:

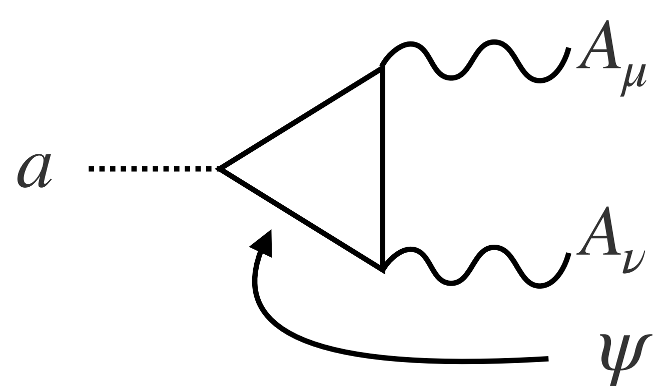
Chern-Simons coupling

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4}F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$V(\phi_1, \phi_2) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$a \equiv -i \arg(\phi_2) \qquad D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

- At the broken phase, CS coupling is induced by triangle anomaly.



dependent on matter sector

$$\Rightarrow c = \sum_f Q^f_{global}(Q^f_{gauge})^2$$ taken as free parameter in this talk

Chern-Simons coupling

3+1D theory:

Chern-Simons coupling

$$\mathscr{L} = |D_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 - \frac{1}{4g^2} F_{\mu\nu}^2 - V(\phi_1, \phi_2) + \frac{c}{16\pi^2} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

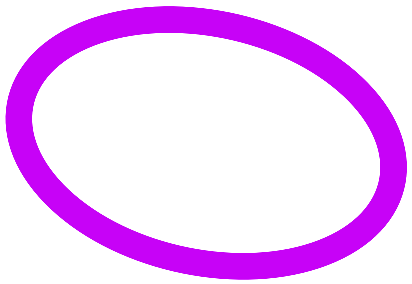
$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

$$a \equiv -i \arg(\phi_2) \qquad D_\mu \phi_1 = (\partial_\mu - igA_\mu)\phi_1$$

- CS couplingは単体のstring loopには効かない

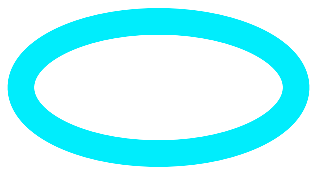
ϕ_1 string

(a is decoupled)



ϕ_2 string

(A_μ is decoupled)



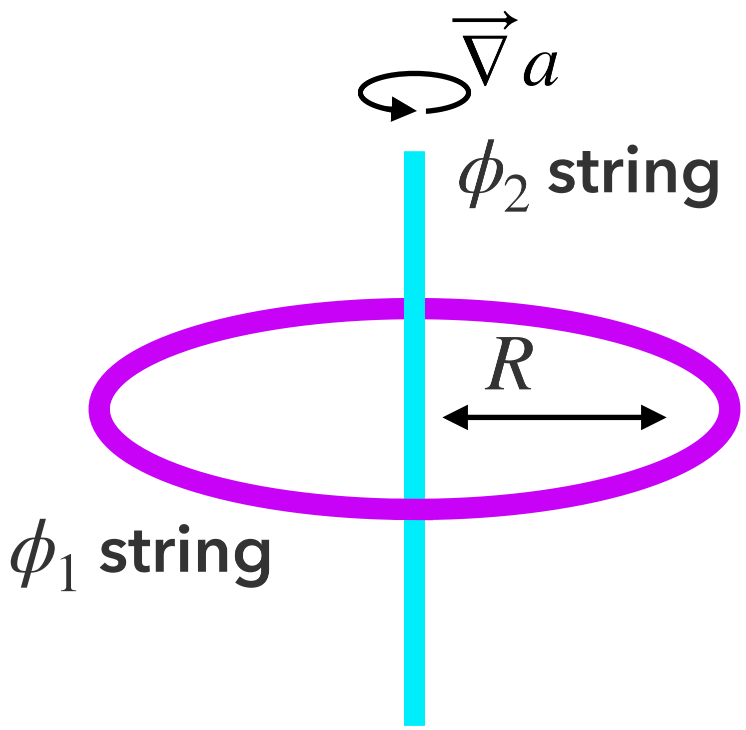
→ これらのloopは縮んで消える

Linking configuration

部分積分で書き直す:

$$\frac{c}{16\pi^2} a F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\frac{c}{16\pi^2}\left(\partial_i a\right) A_0 B^i$$

linkしてるとき、$\vec{\nabla} a$ は $\vec{B}$ と同じ向き $\Rightarrow \left(\partial_i a\right) A_0 B^i = \frac{1}{R} A_0 |\vec{B}|$



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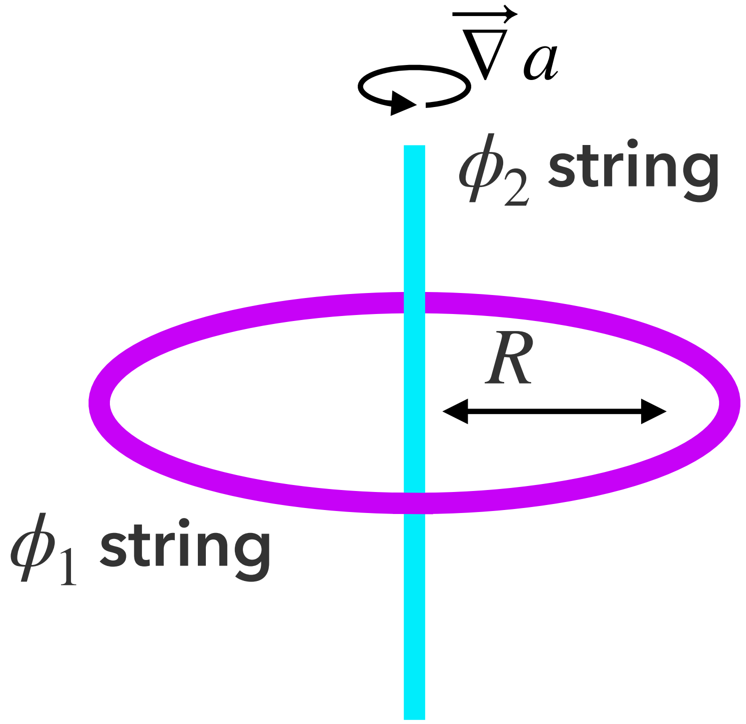

Gauss law:

$$\frac{\delta \mathscr{L}}{\delta A_0} = \partial_i E_i - g^2 J^0 + \frac{g^2 c}{16\pi^2 R} |\vec{B}| = 0$$

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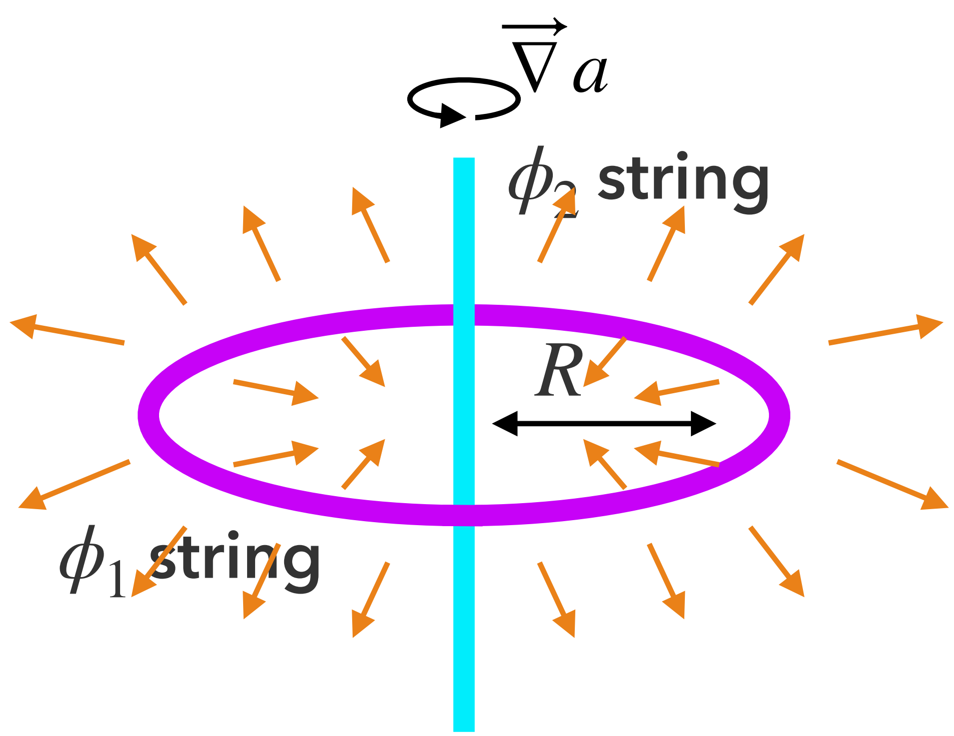

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$\vec{B}$ induces the electric field

→ ϕ_1 stringのloop は電場による反発で縮まない

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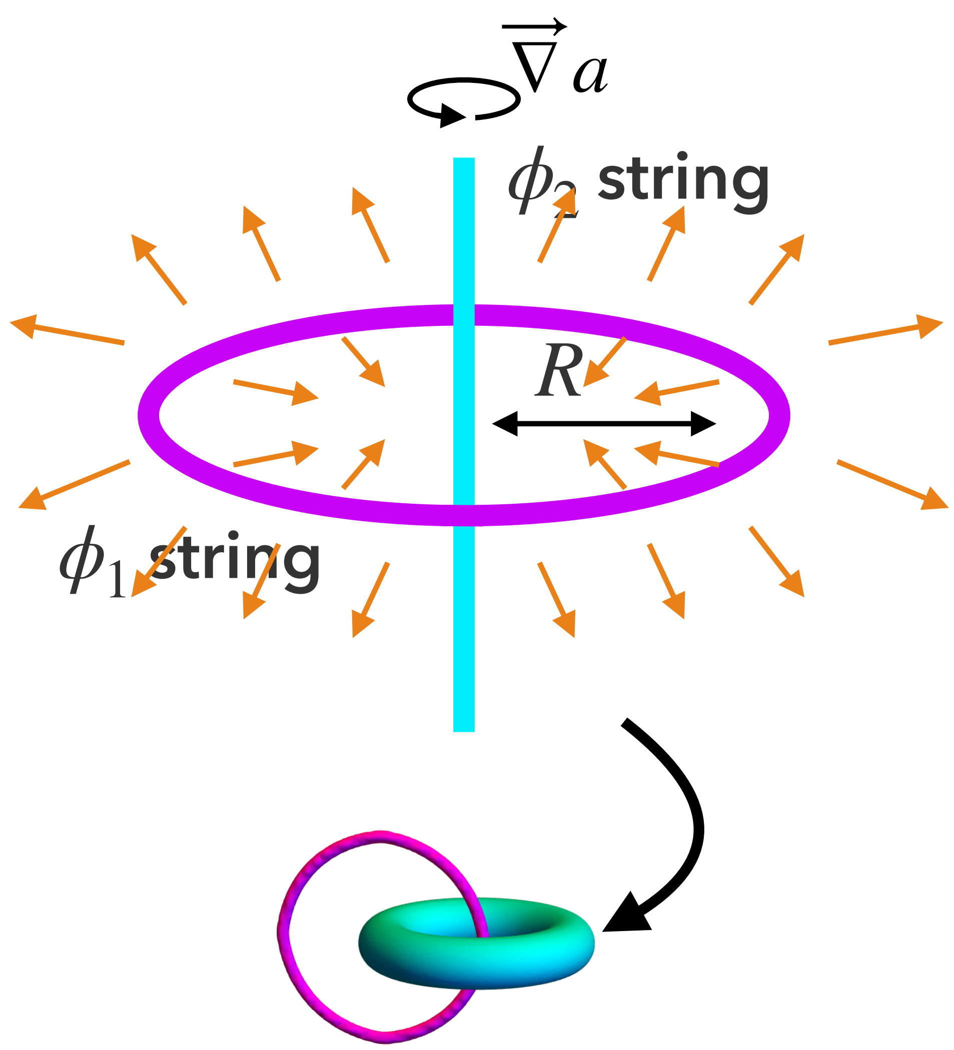

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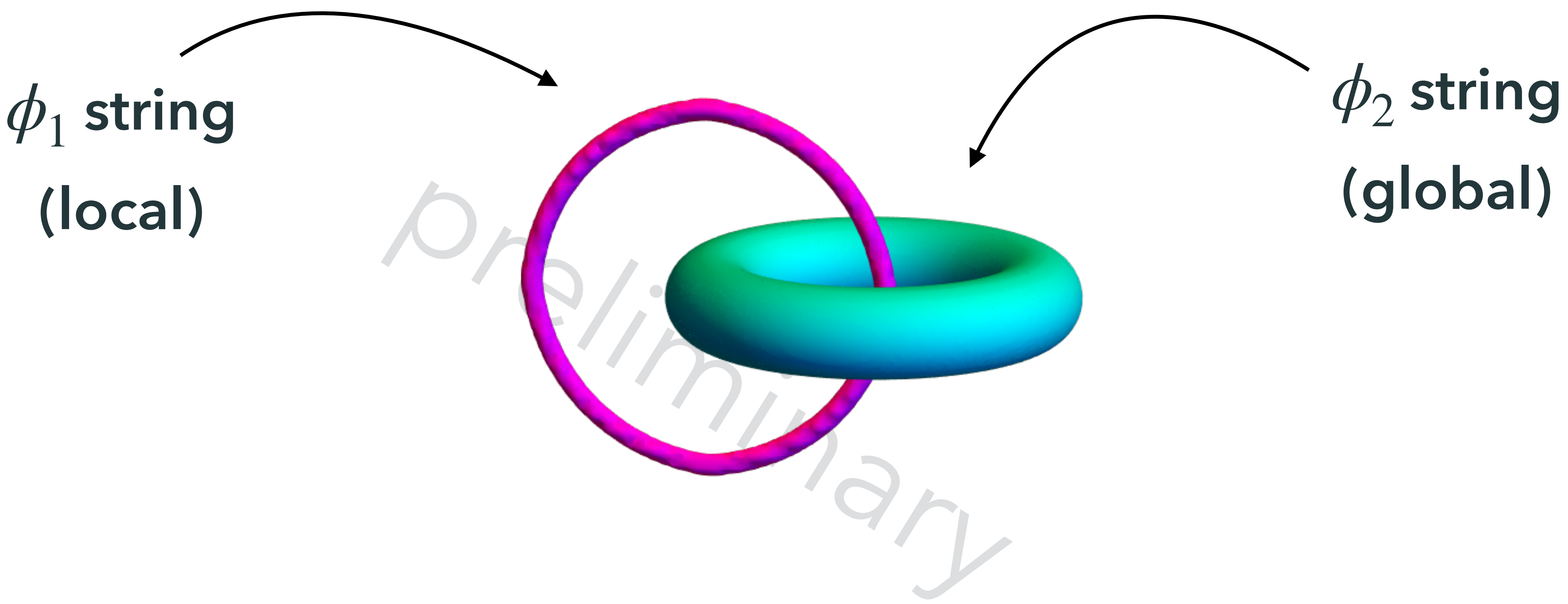
Numerical solution



$$\lambda/g^2 = 1600,\ \chi/g^2 = 312,\ \kappa/g^2 = 1.6,\ v_2/v_1 = 0.1,\ gv_1 = 2.5,\ g^2c/(16\pi^2) = 20$$

Knot stability

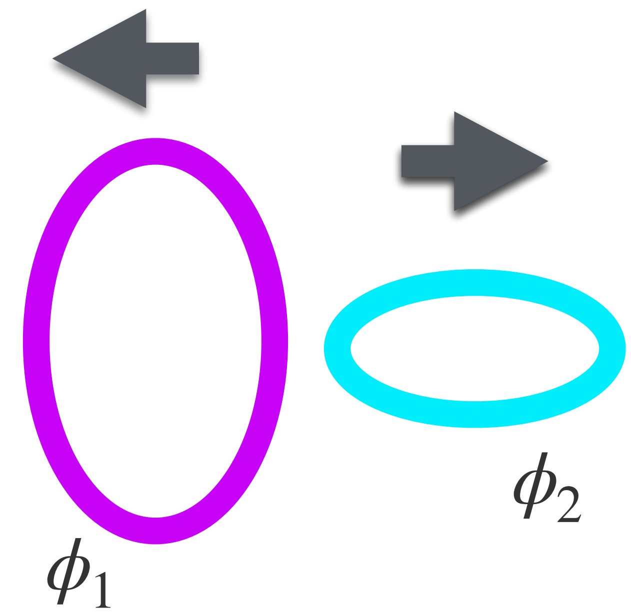
- can decay by delinking?

  → $\lambda \gg g^2, \kappa, \chi$ prevents delink

$$V(\phi) = \lambda \left(|\phi_1|^2 + |\phi_2|^2 - \mu^2 \right)^2 - \kappa |\phi_1|^2 |\phi_2|^2 + \chi |\phi_2|^4$$

→ non-linear σ-model w/ $O(4)$ sym. → $O(3)$ sym.

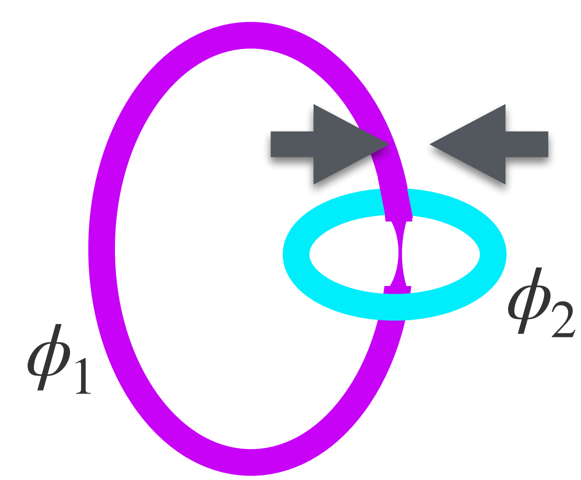
linking # = skyrmion # [Gudnason-Nitta '20]



- Loop of ϕ_2 string can shrink infinitely?

  → $v_2/v_1 \ll 1$ prevents shrinking

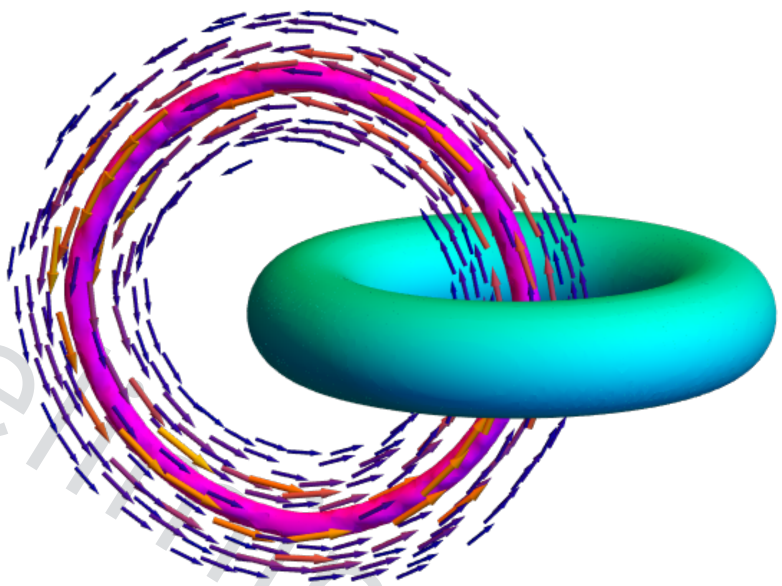
  typically $v_2 \lesssim 0.1 v_1$



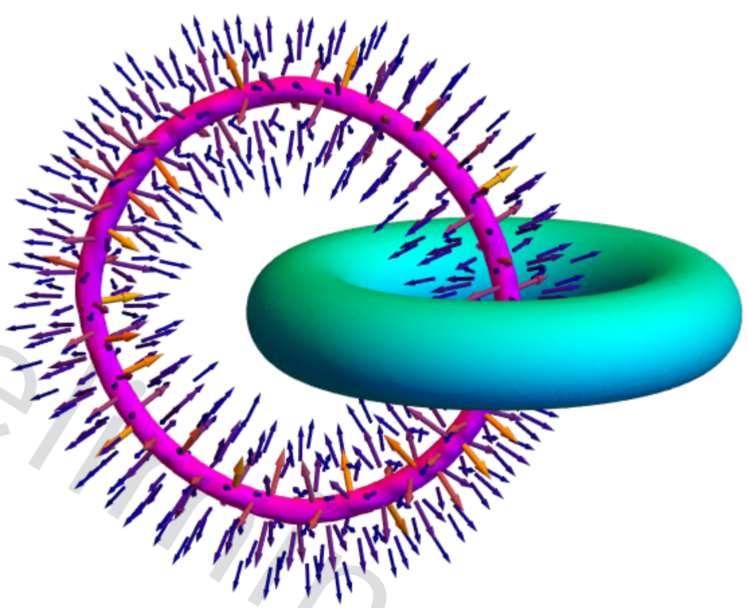
classically stable under these two conditions

Magnetic field & Electric field

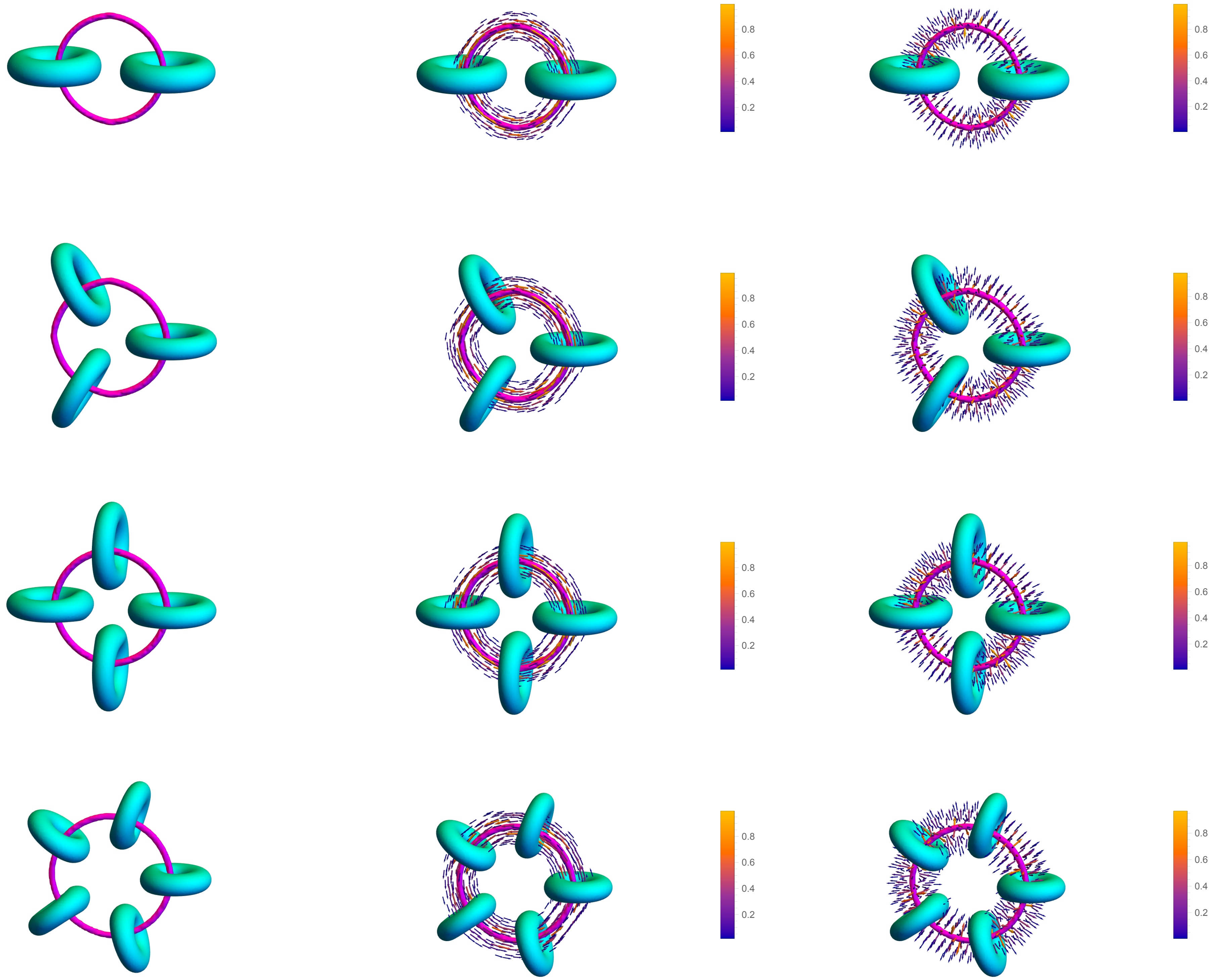
$\overrightarrow{B}$



$\overrightarrow{E} = \overrightarrow{\nabla} A_0$



Higher linking number



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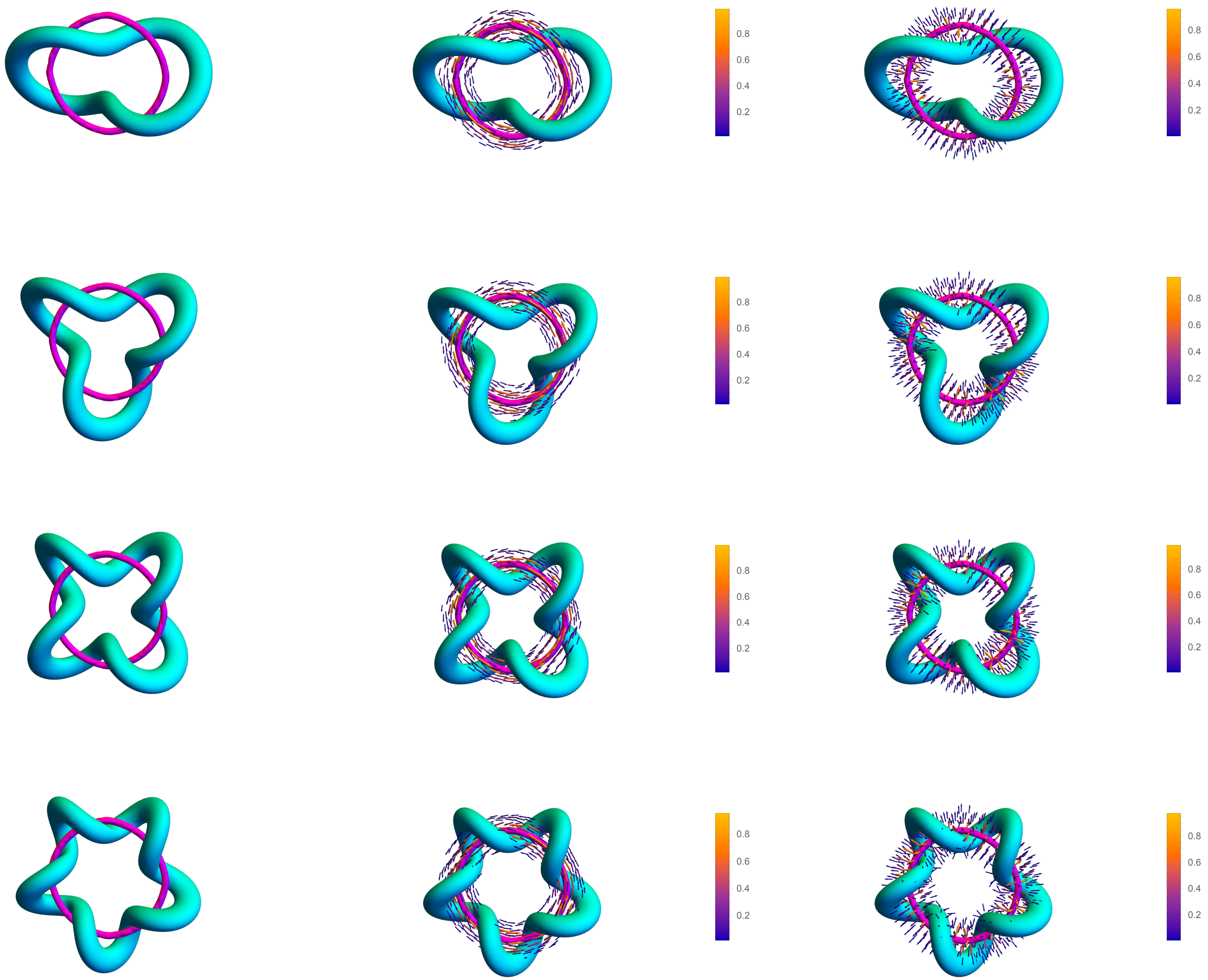

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Application to phenomenology & cosmology

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$U(1)_{\mathrm{global}} \times U(1)_{\mathrm{gauge}}$ が自発的に破れる模型では、

変態的なソリトンが現れる(場合があります)

The model

	$U(1)_{B-L}$	$U(1)_{\rm PQ}$
ϕ_1	2	0
ϕ_2	0	1
KSVZ-like Q	Q^f_{B-L}	$Q^f_{\rm PQ}$
ν_R	-1	0
SM	q: $1/3$ l: -1	0

$v_1 \sim v_2 \sim 10^{9-12}\,\mathrm{GeV}$

←特定しない

$$\Rightarrow c = \sum_f Q^f_{global}(Q^f_{gauge})^2$$

- $\mathscr{L} \supset y_R\,\phi_1^*\,\bar{\nu}_R\nu_R^c \to \langle\phi_1\rangle$ gives Majorana mass → type-I seesaw

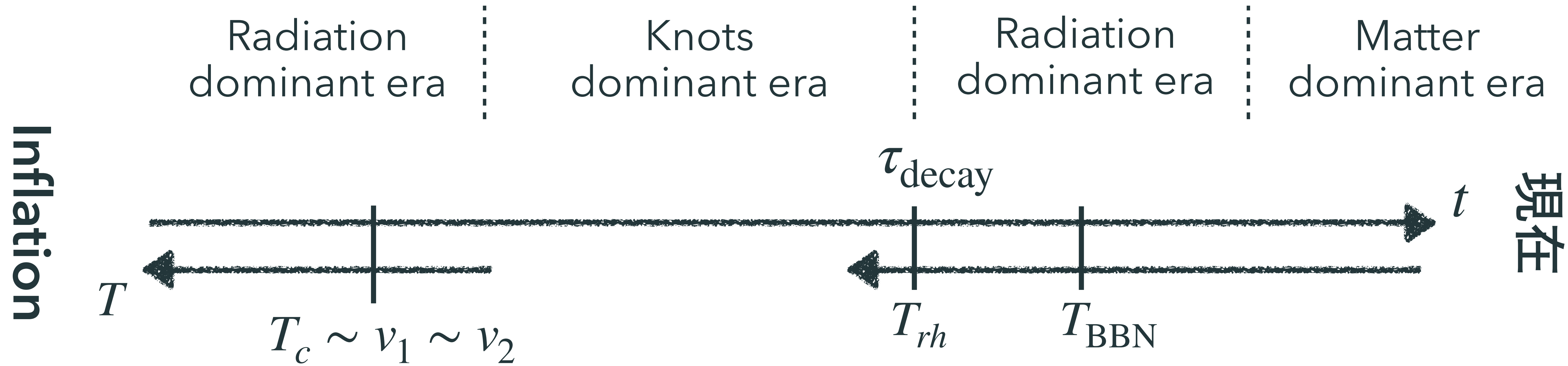
[Minkowski '77] [Yanagida '79] [Gell-Mann+ '79] [Mohapatra-Senjanovic+ '80]

- phase of ϕ_2 (a) is identified as QCD axion

[Peccei-Quinn '77] [Weinberg '78] [Wilczek '78]

→ solution of strong CP problem & Dark matter

Fate of knot soliton



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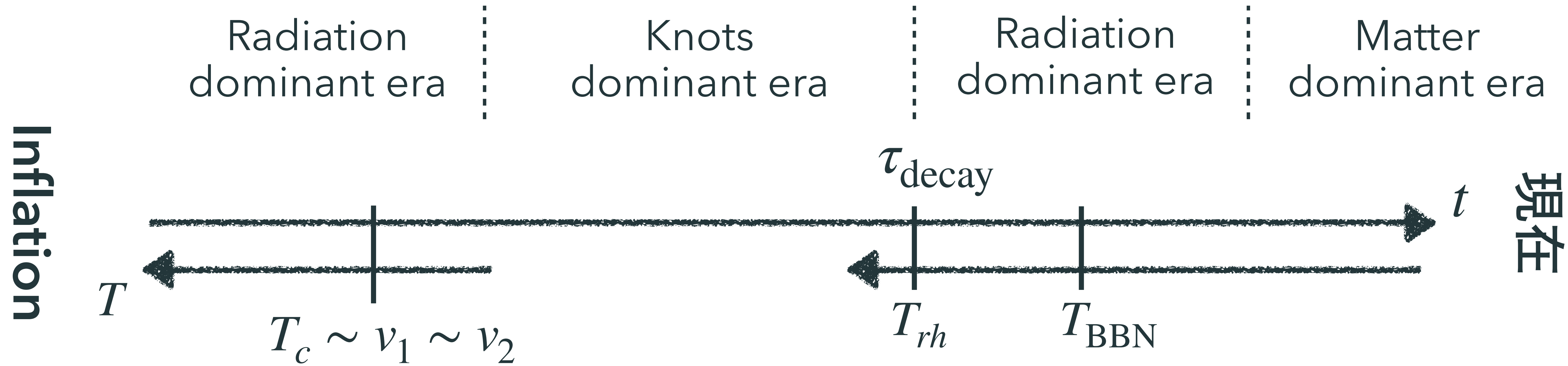
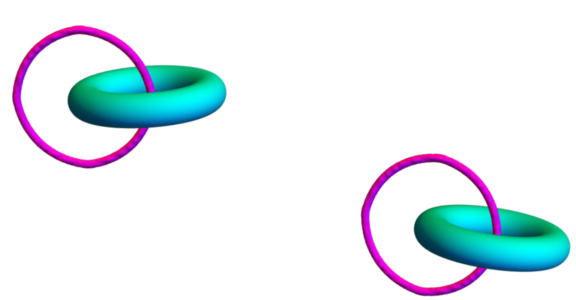


SSB: ~~$U(1)_{PQ} \times U(1)_{B-L}$~~

→knotとϕ_1, ϕ_2のcosmic stringが生成



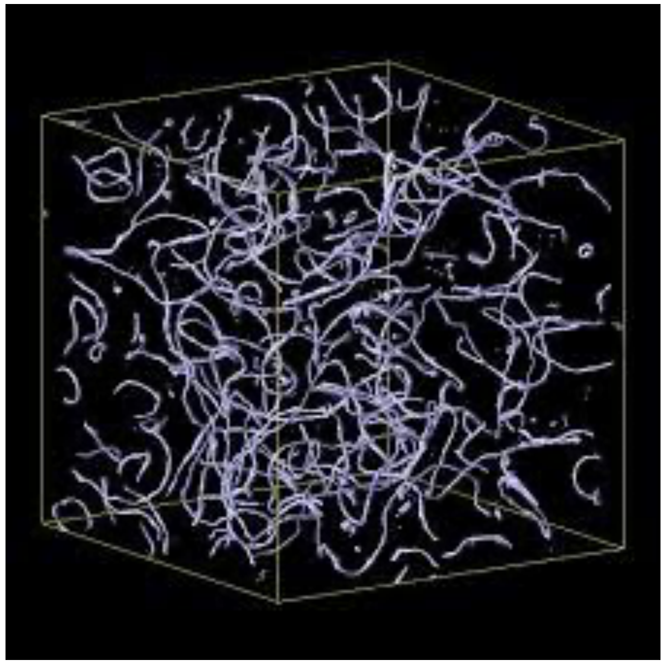


fig from slide by Hiramatsu

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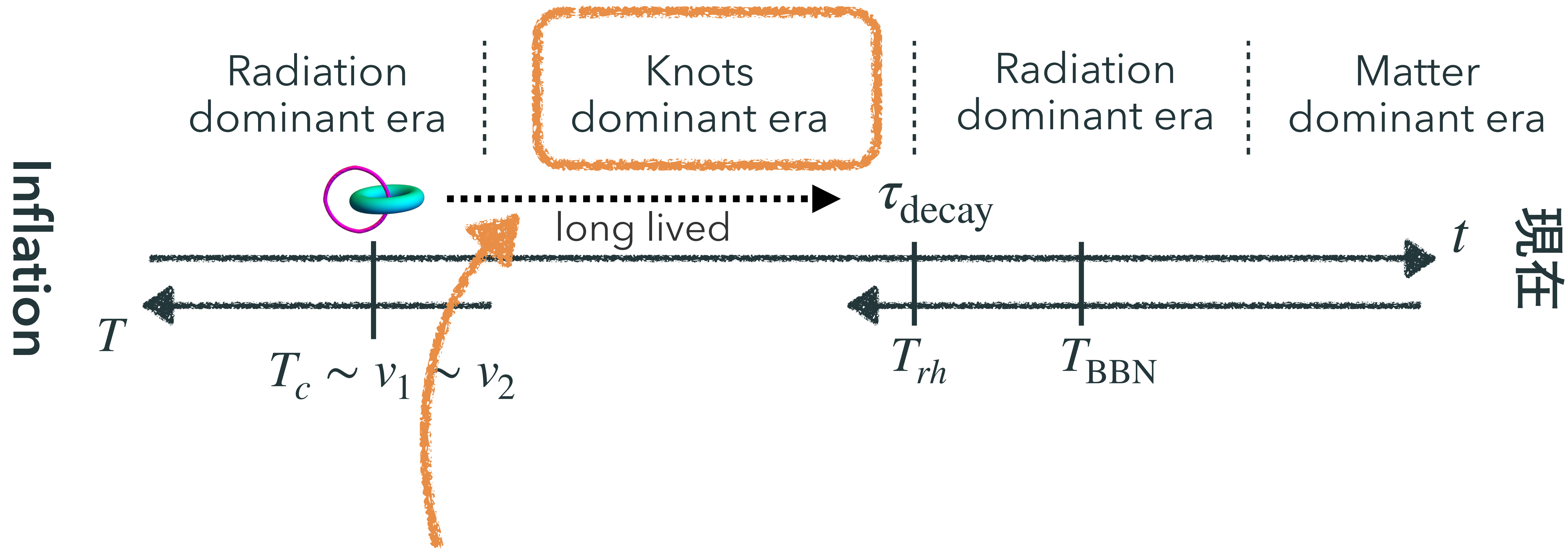
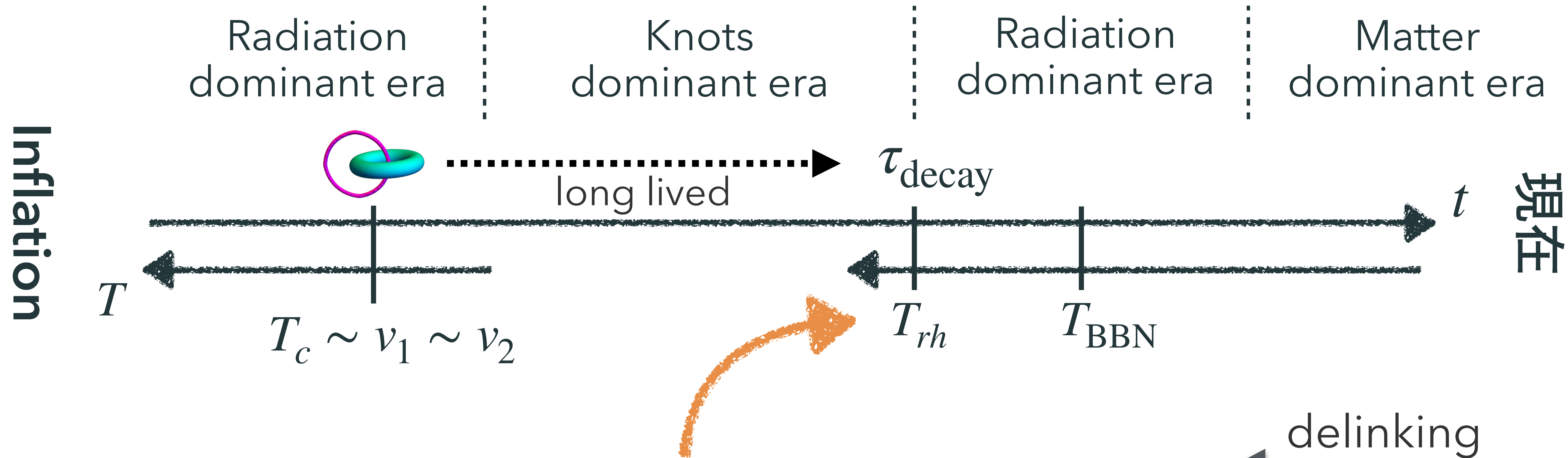


重いmatterと同様に振る舞うので、その

うちknot solitonのエネルギー密度が宇宙

のエネルギーを占める

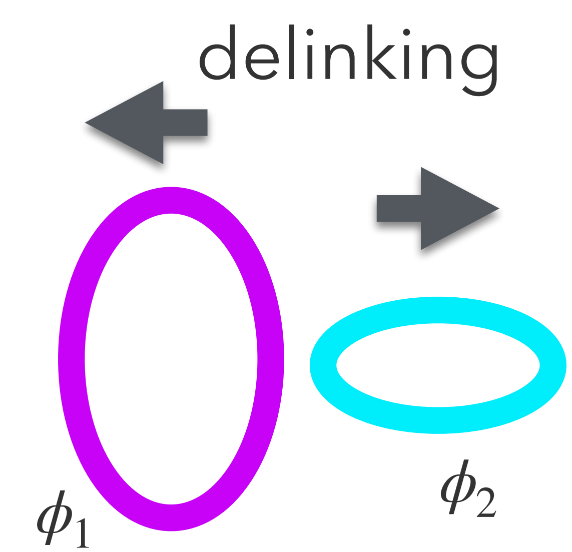
Fate of knot soliton



量子効果でknotが軽い粒子に崩壊

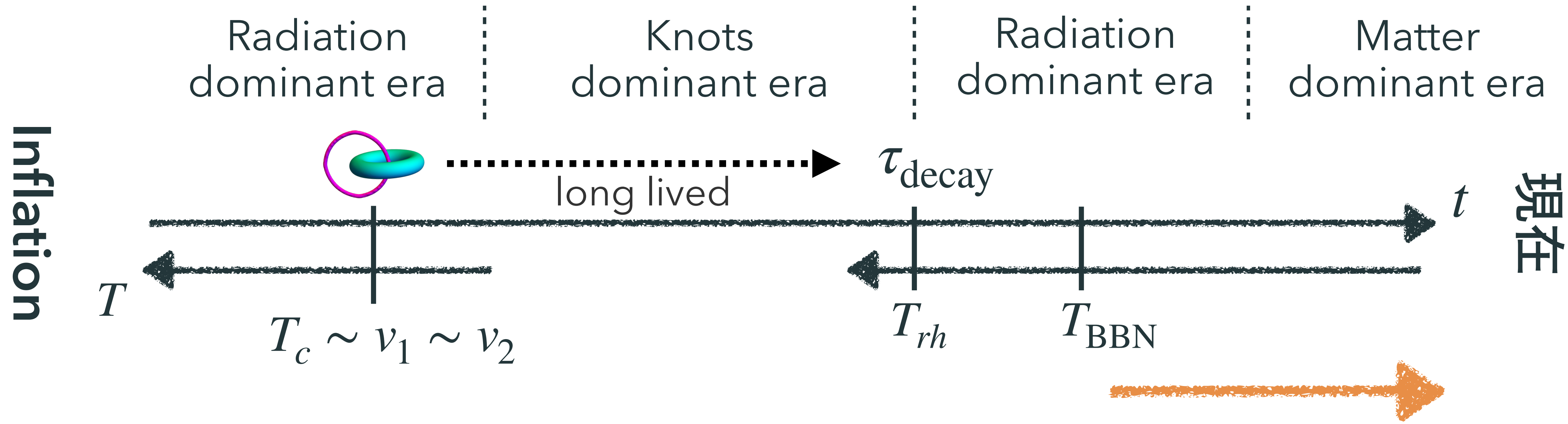
→ 熱浴の温度が上がる

(secondary reheating)



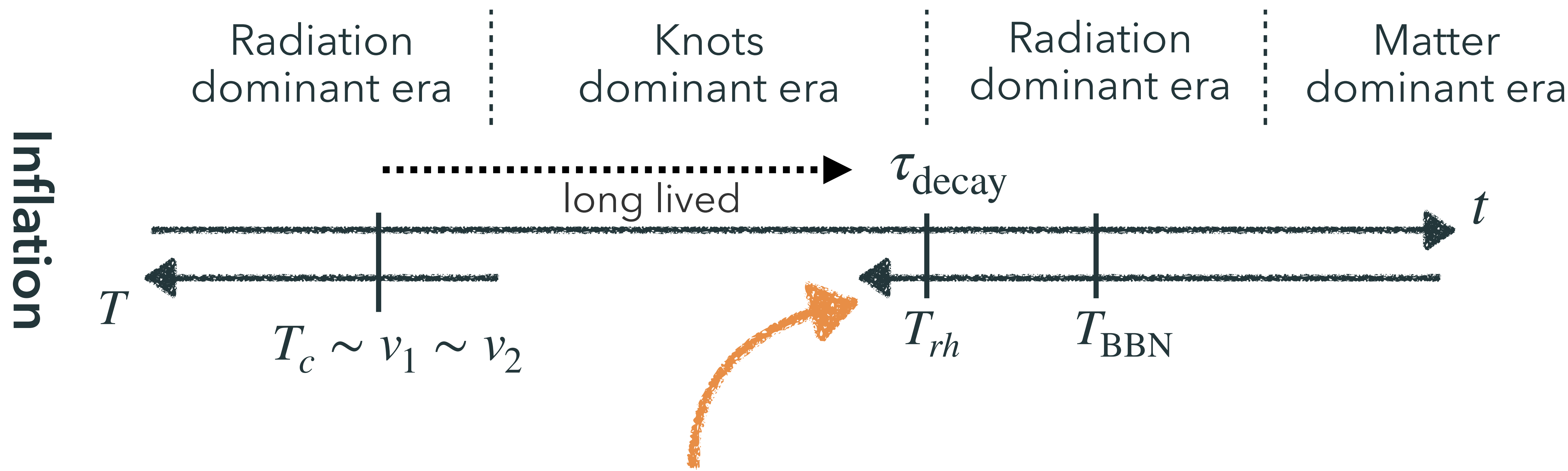
$$\tau_{\rm decay}^{-1} \sim \Gamma \sim g v_1 \exp\left[-\frac{4}{3}\sqrt{\frac{\lambda v_1}{g v_2}}\right] \qquad T_{rh} \sim \sqrt{g_X v_1 M_{\rm pl}} \exp\left[-\frac{2}{3}\sqrt{\frac{\kappa}{\tan\beta}}\right]$$

Fate of knot soliton

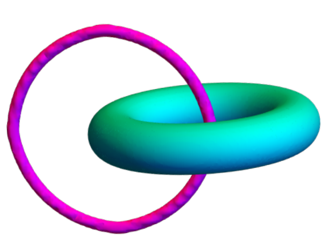


それ以降はビッグバン元素合成を経て
標準宇宙論と同じ

Leptogenesis via soliton



- knot solitonの崩壊でRH neutrino ができる



$\to N_i \to H, L$

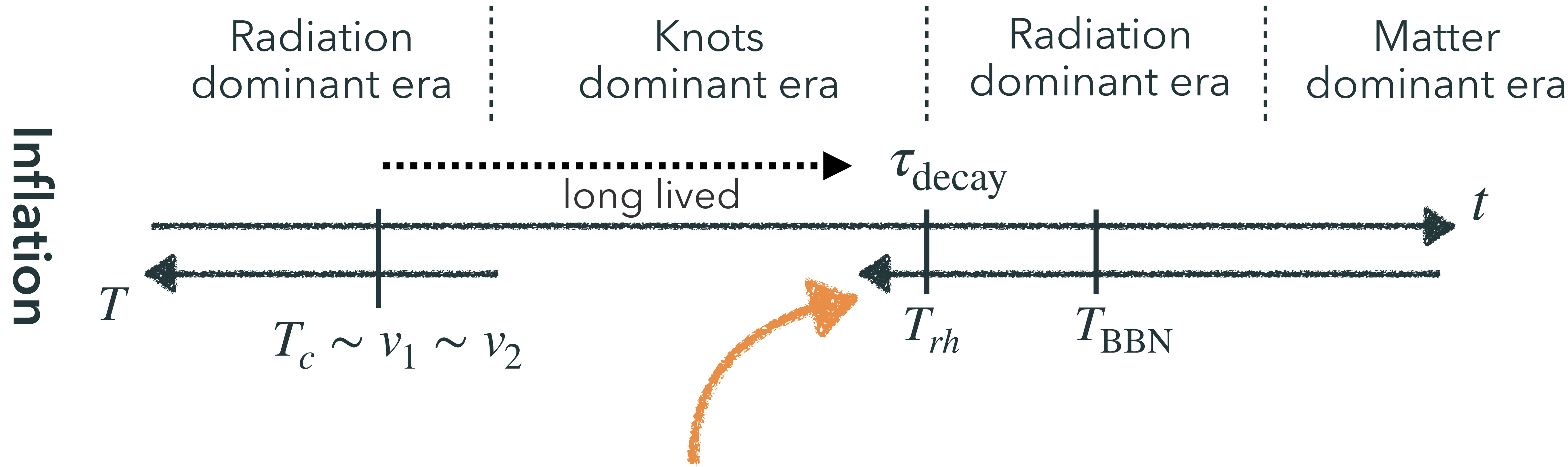
$\to X_\mu$

$M_{N_1} \gg T_{rh}$

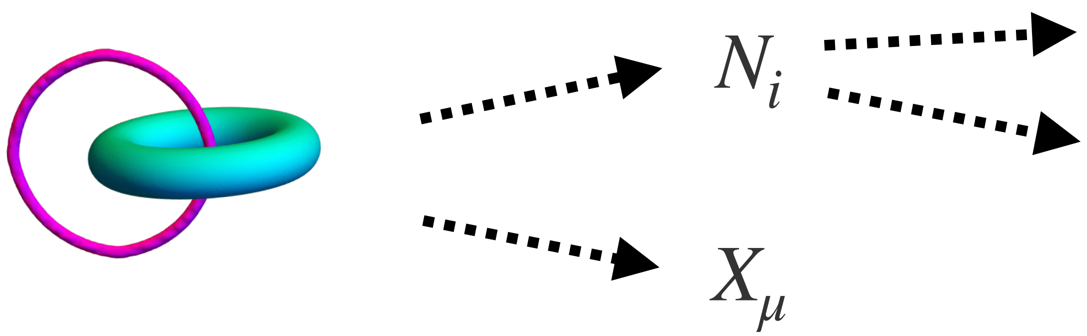
$f_{N_1} \in [0,1]$: energy fraction of N_1

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq 8.2 \times 10^{-11} f_{N_1} \left(\frac{T_{rh}}{10^6\,\mathrm{GeV}} \right) \left(\frac{m_3}{0.05\,\mathrm{eV}} \right) \delta_{\rm eff}$$

Leptogenesis via soliton



- knot solitonの崩壊でRH neutrino ができる



$M_{N_1} \gg T_{rh}$

$f_{N_1} \in [0,1]$: energy fraction of N_1

resonant case: $Y_B \lesssim 8.0 \times 10^{-11} f_{N_1} \left(\frac{T_{rh}}{10^2\,\text{GeV}} \right) \left(\frac{10^{12}\,\text{GeV}}{M_{R1}} \right)$

Testability by gravitational wave

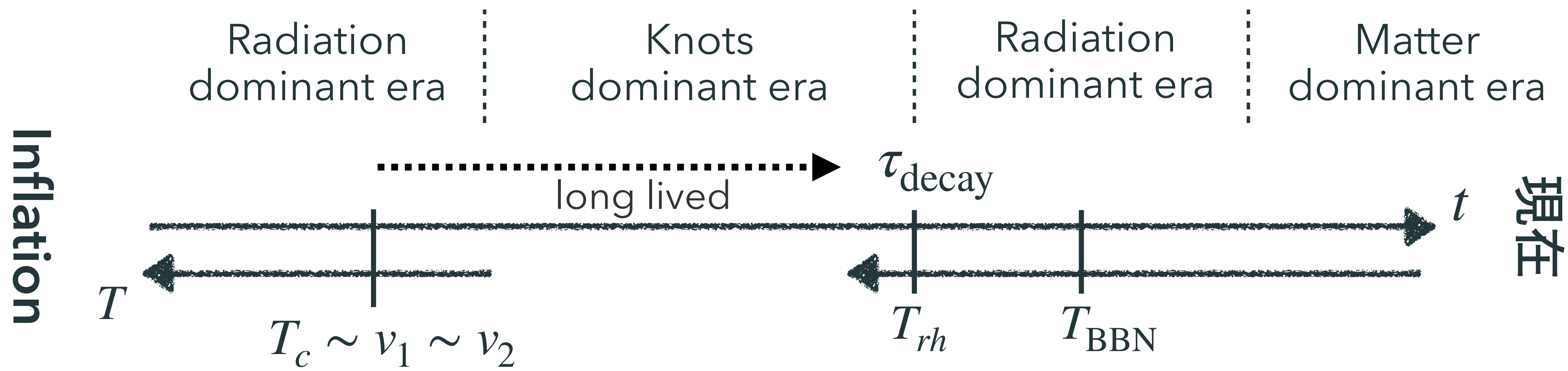
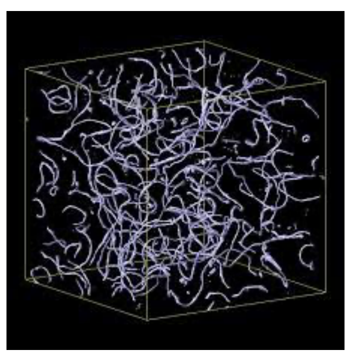


fig from slide by Hiramatsu

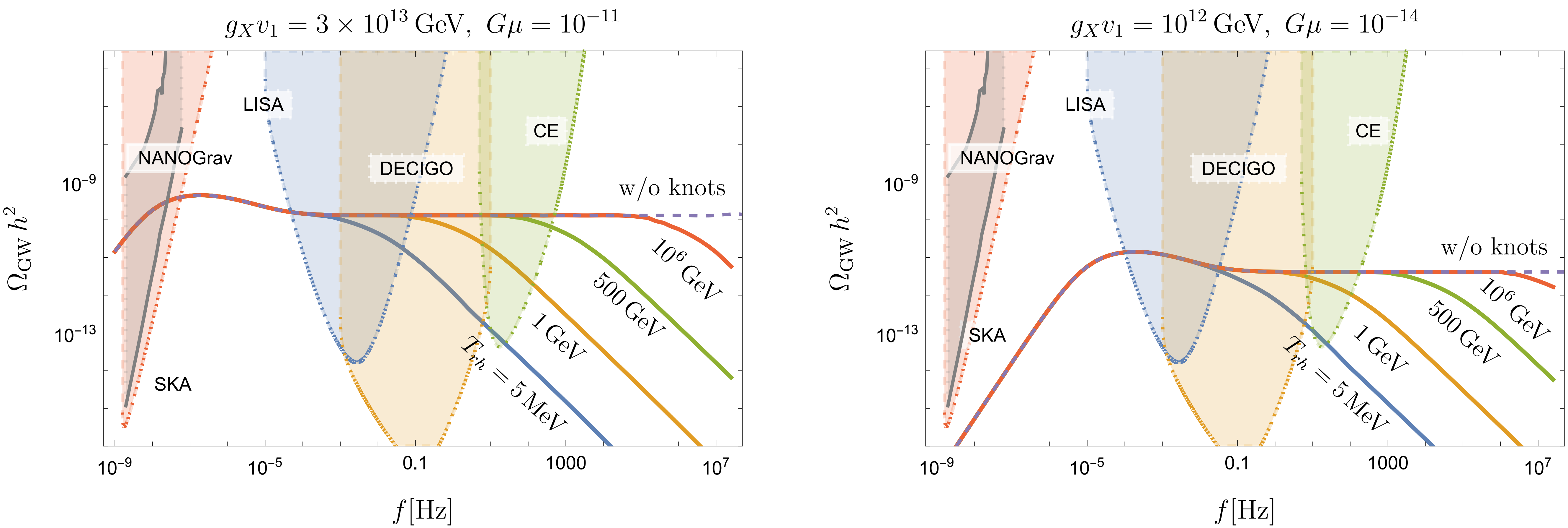


- ϕ_1, ϕ_2 のcosmic stringから現在まで背景重力波が出続ける
- knot dominantでは宇宙膨張の速さがradiation dominantと異なる

→cosmic string由来の背景重力波の波形に影響を与える [Cui+, 1711.03104]

→重力波観測でこのシナリオがテストできる

Testability by gravitational wave



- knotが無い通常の重力波スペクトラムは右側でフラット
- knot dominant eraのせいで右側が落ちる

→将来観測により、フラットな場合と区別できる

Summary

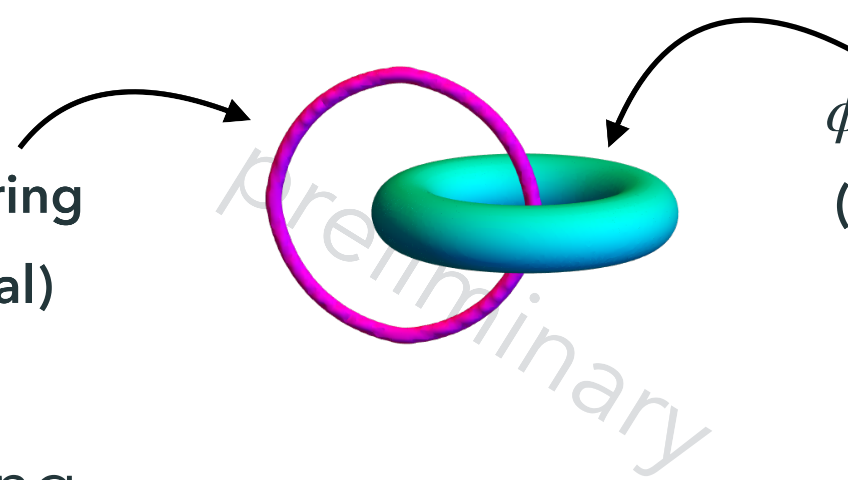
- Massage of this talk:

SSB of $U(1)_{\rm global} \times U(1)_{\rm gauge}$ symmetries admit existence of knot soliton!

- Key: Chern-Simons coupling $\dfrac{c}{16\pi^2}\displaystyle\int d^4x\, aF\tilde{F}$

- motivative setup:

$$\begin{cases} U(1)_{\rm global} = U(1)_{PQ} \\ U(1)_{\rm gauge} = U(1)_{B-L} \end{cases}$$



→ knot made of axion string & B-L string

- can be tested by gravitational wave

- can produce baryon asymmetry