量子計算の素粒子物理学への応用について

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PRESTO SAKIGAKE

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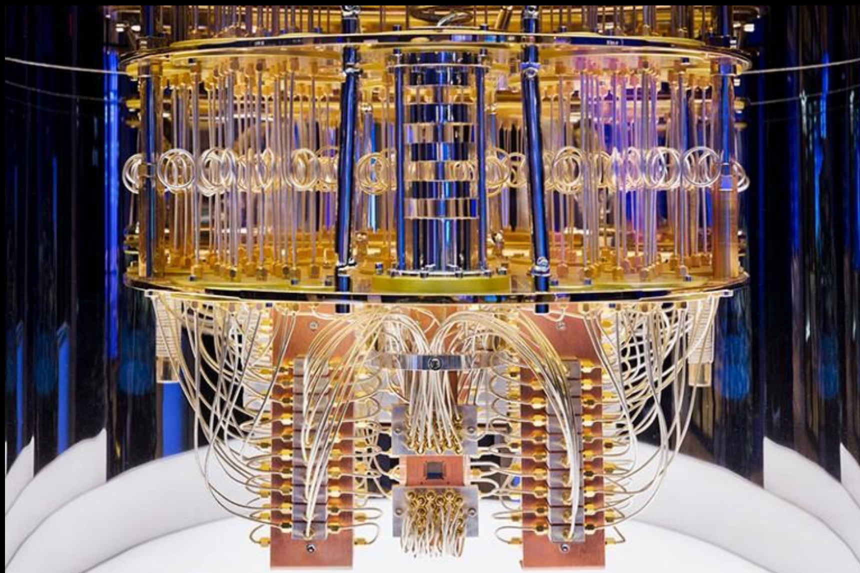
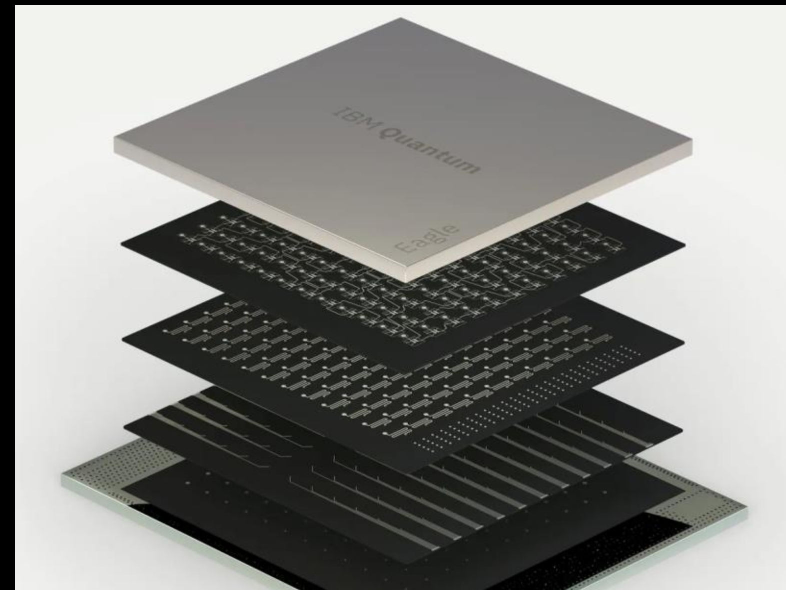
Quantum computer sounds growing well…





Article

Evidence for the utility of quantum computing before fault tolerance

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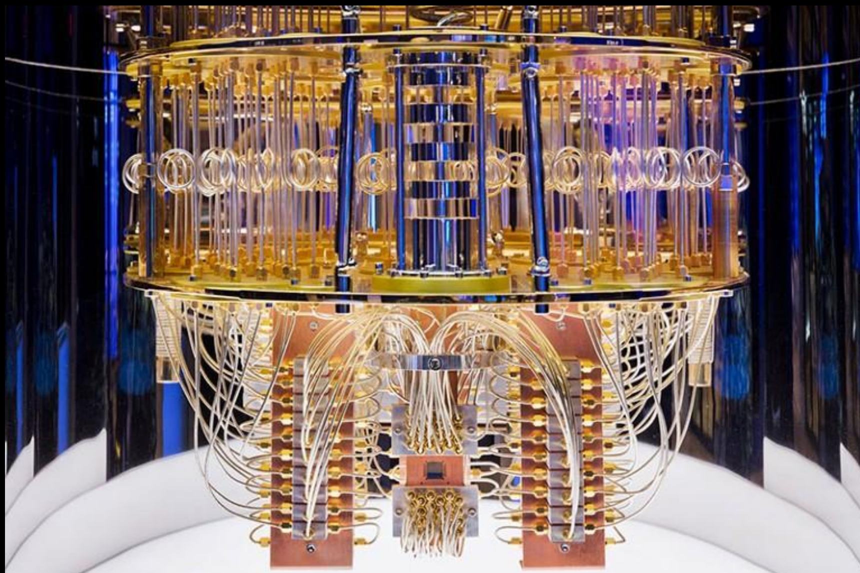
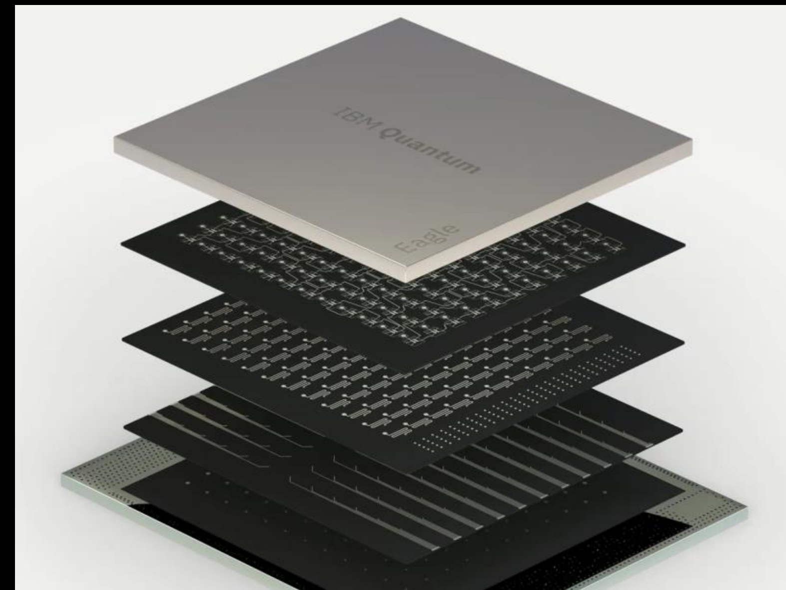
Quantum computing promises to offer substantial speed-ups over its classical

Quantum computer sounds growing well…



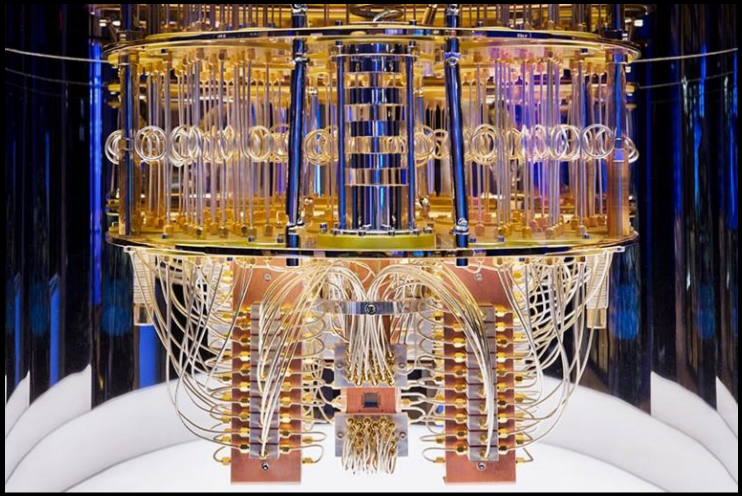


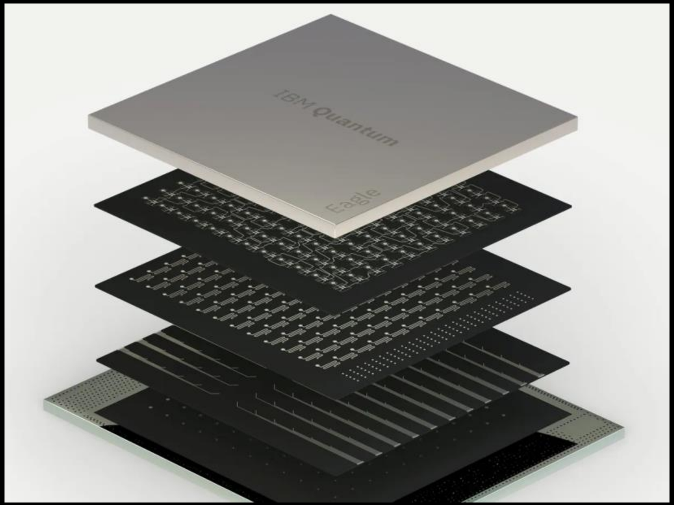
Article

Evidence for the utility of quantum computing before fault tolerance

How can we use it for us?

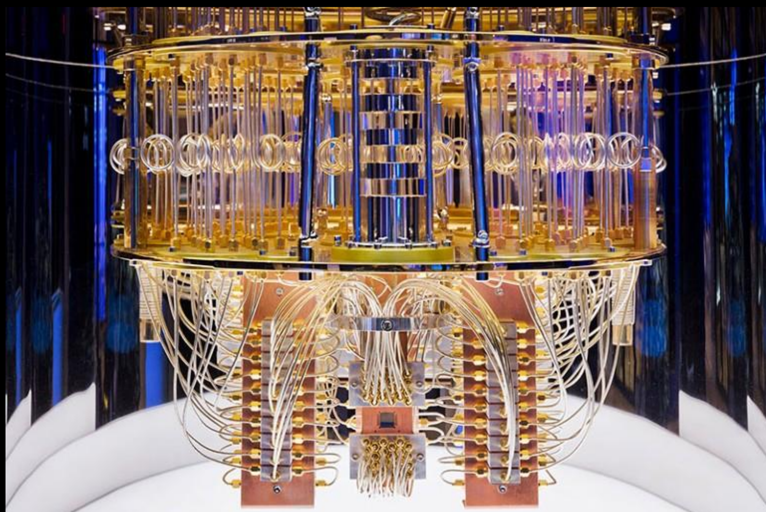
Applications mentioned in media ?

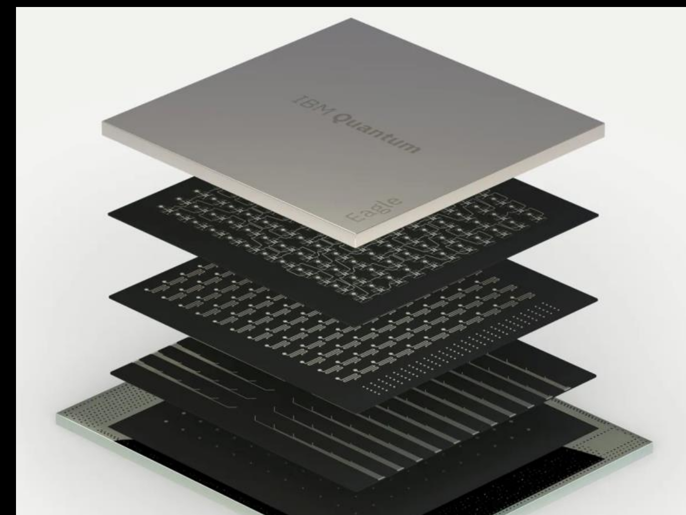


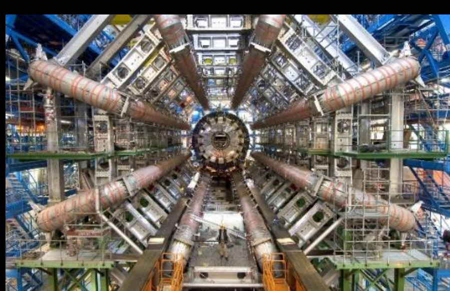


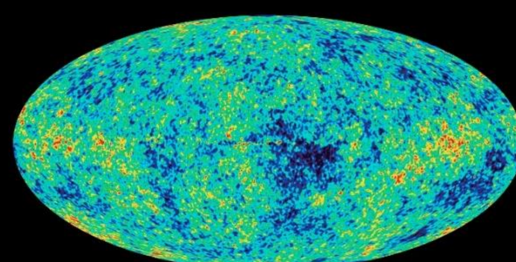
etc...

In my mind…









etc…

What is meant by

"Application of Quantum Computation to High Energy Physics" ??

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In general, it is

to replace (a part of) computations by quantum algorithm

Therefore,

physical meaning of qubits in quantum computer depends on contexts

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"Application of Quantum Computation to High Energy Physics" ??

In general, it is

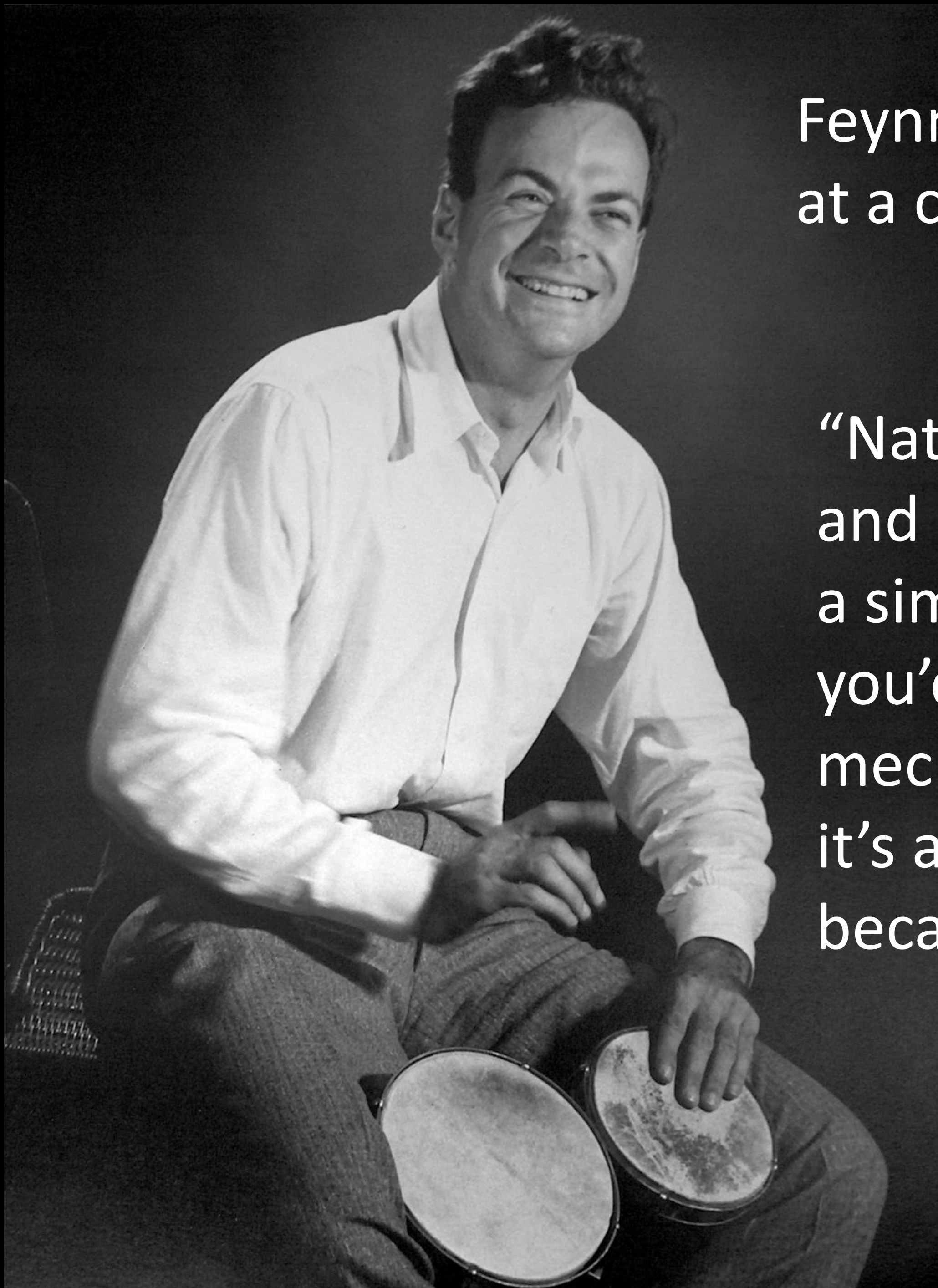
to replace (a part of) computations by quantum algorithm

Therefore,

physical meaning of qubits in quantum computer depends on contexts

Here,

qubits = states in quantum system



Feynman as a keynote speaker
at a conference in MIT (1981):

“Nature isn’t classical, dammit,
and if you want to make
a simulation of Nature,
you’d better make it quantum
mechanical, and by golly
it’s a wonderful problem
because it doesn’t look so easy.”

This talk:

Application of Quantum Computation to Quantum Field Theory (QFT)

▪ <u>Generic motivation:</u>

simply would like to use powerful computers?

▪ <u>Specific motivation:</u>

Quantum computation is suitable for operator formalism

$\longrightarrow$ Liberation from infamous sign problem in Monte Carlo?

(next slide)

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

(this point will be elaborated tomorrow)

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \quad \Rightarrow \quad \int d\phi \ \mathcal{O}(\phi) e^{-S(\phi)}$$

②

Sign problem in Monte Carlo simulation

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& make path integral finite dimensional:

$$\int D\phi\ \mathcal{O}(\phi)e^{-S[\phi]} \Rightarrow \int d\phi\ \mathcal{O}(\phi)e^{-S(\phi)}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle\mathcal{O}(\phi)\rangle \simeq \frac{1}{\sharp(\text{samples})}\sum_{i\in\text{samples}}\mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi\ \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

Sign problem in Monte Carlo simulation (Cont'd)

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Examples w/ sign problem:

- topological term —— complex action
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- real time —— " $e^{iS(\phi)}$ " *much worse*

In operator formalism,

sign problem is absent from the beginning

(∃ various approaches within framework of path integral formalism but I'll skip it)

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Cost of operator formalism

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Quantum computers do this job?

Contents

0. Introduction

1. Quantum computation

2. Ising model

3. Schwinger model [Chakraborty-MH-Kikuchi-Izubuchi-Tomiya ’20] [MH-Itou-Kikuchi-Nagano-Okuda ’21] [MH-Itou-Kikuchi-Tanizaki ’21]

4. Future prospects

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"computational basis"

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \quad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \qquad |1\rangle = |\downarrow\rangle$$

(We don't need to mind how it is realized as "users")

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij}|ij\rangle, \qquad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1,\cdots i_N=0,1} c_{i_1\cdots i_N}|i_1 \cdots i_N\rangle,$$

$$|i_1 i_2 \cdots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

Rule of the game

Do something interesting by a combination of

1. action of Unitary operators:

$$|\psi\rangle \quad \text{—}\boxed{U}\text{—} \quad U|\psi\rangle$$

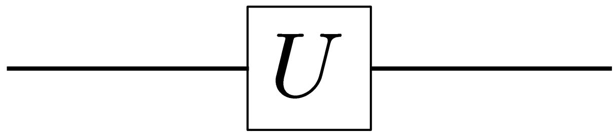
&

2.

Rule of the game

Do something interesting by a combination of

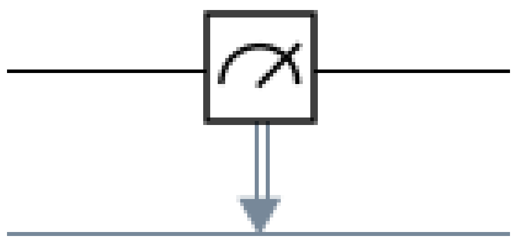
1. action of Unitary operators:

$|\psi\rangle$  $U|\psi\rangle$

&

2. measurements:

$|\psi\rangle$
$= \alpha|0\rangle + \beta|1\rangle$



c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Unitary gates used here

X, Y, Z gates: (just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X is "NOT": $X|0\rangle = |1\rangle, \; X|1\rangle = |0\rangle$

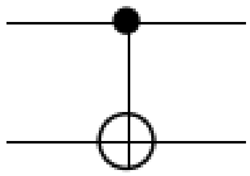
R_X, R_Y, R_Z gates:

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Controlled X (NOT) gate:

$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



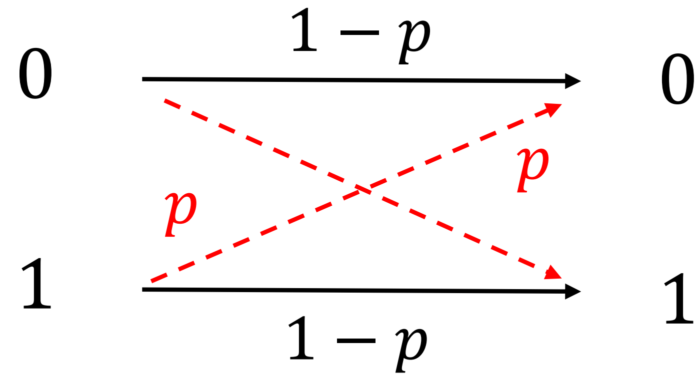
Errors in classical computers

Computer interacts w/ environment ⇒ error/noise

Errors in classical computers

Computer interacts w/ environment → error/noise

one bit →



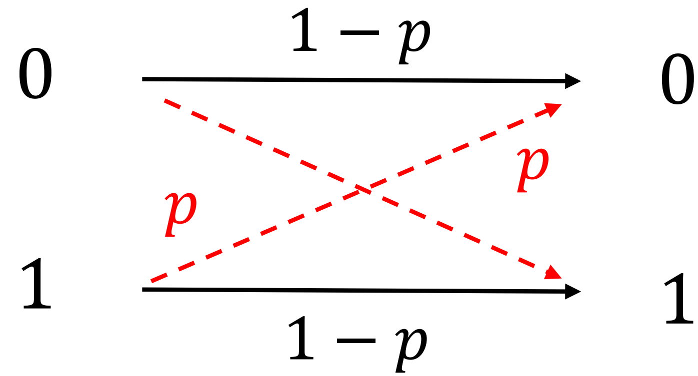
Suppose we send a bit but have "error" in probability p

A simple way to correct errors:

Errors in classical computers

Computer interacts w/ environment ➡ error/noise

one bit ➡



Suppose we send a bit but have “error” in probability p

A simple way to correct errors:

① Duplicate the bit (encoding): $0 \to 000, \quad 1 \to 111$

② Error detection & correction by “majority voting”:

$$001 \to 000, \quad 011 \to 111, \quad \text{etc...}$$

➡ $P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

(we'll come back to this point tomorrow)

Computer interacts w/ environment $\Longrightarrow$ error/noise

Unknown unitary operators are multiplied:

(in addition to decoherence & measurement errors)

$$|\psi\rangle \xrightarrow{\text{error!}} U|\psi\rangle$$

not only bit flip!

We need to include "quantum error corrections"
but it seems to require a huge number of qubits

~ major obstruction of the development

(Classical) simulator for Quantum computer

Quantum computation $\subset$ Linear algebra

The same algorithm can be implemented in classical computer
but w/o speed-up (1 quantum step = many classical steps)

Simulator = Tool to simulate quantum computer by classical computer

- Doesn't have errors → ideal answers

  (More precisely, classical computer also has errors but its error correction is established)

- The same code can be run in quantum computer w/ speed-up

Useful to test algorithm & estimate computational resources

(~# of qubits, gates)

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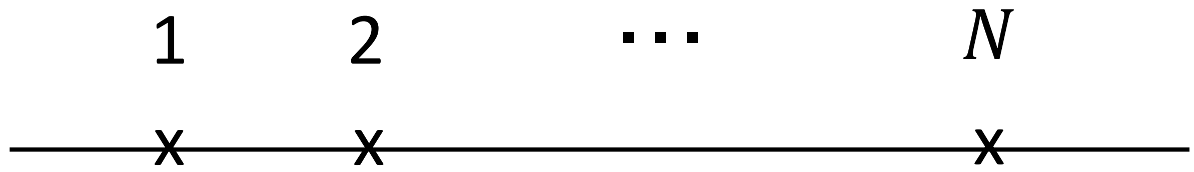
4. Future prospects

The (1+1)d transverse Ising model



Hamiltonian (w/ open b.c.):

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

$$\hat{H} = -J\sum_{n=1}^{N-1} Z_n Z_{n+1} - h\sum_{n=1}^{N} X_n$$

Let’s construct the time evolution op. $e^{-i\hat{H}t}$

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1 Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

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How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Step 1: Suzuki-Trotter decomposition:

($\exists$ higher order improvements)

$$e^{-i\hat{H}t} = \left(e^{-i\hat{H}\frac{t}{M}}\right)^M$$

(M: large positive integer)

$$\simeq \left(e^{-iH_X\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

The 1st one is trivial:

acting on qubit 2 acting on qubit 1

$$e^{-iH_X\frac{t}{M}} = e^{-i\frac{ht}{M}X_2} e^{-i\frac{ht}{M}X_1} = R_X^{(2)}\left(\frac{2ht}{M}\right) R_X^{(1)}\left(\frac{2ht}{M}\right)$$

Time evolution operator (Cont'd)

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The 2nd one is nontrivial:

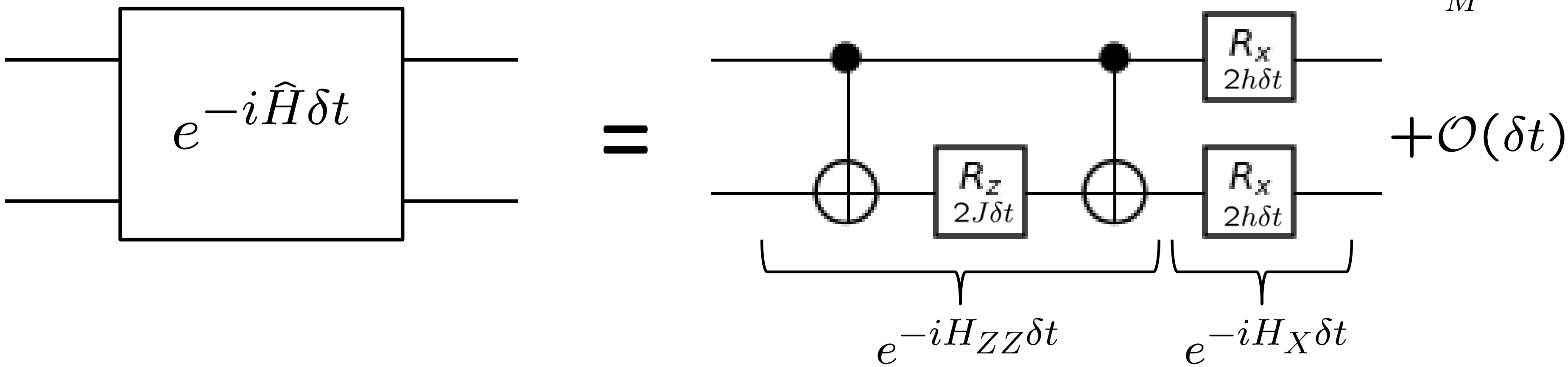
$$e^{-iH_{ZZ}\frac{t}{M}} = e^{-i\frac{Jt}{M}Z_1Z_2} = \cos\frac{Jt}{M} - iZ_1Z_2\sin\frac{Jt}{M}$$

One can show

$$e^{-i\frac{Jt}{M}Z_1Z_2} = CX R_Z^{(2)}\left(\frac{2Jt}{M}\right)CX$$

“Computational cost” for large size system

$\delta t = \frac{t}{M} \ll 1$



Classical computer

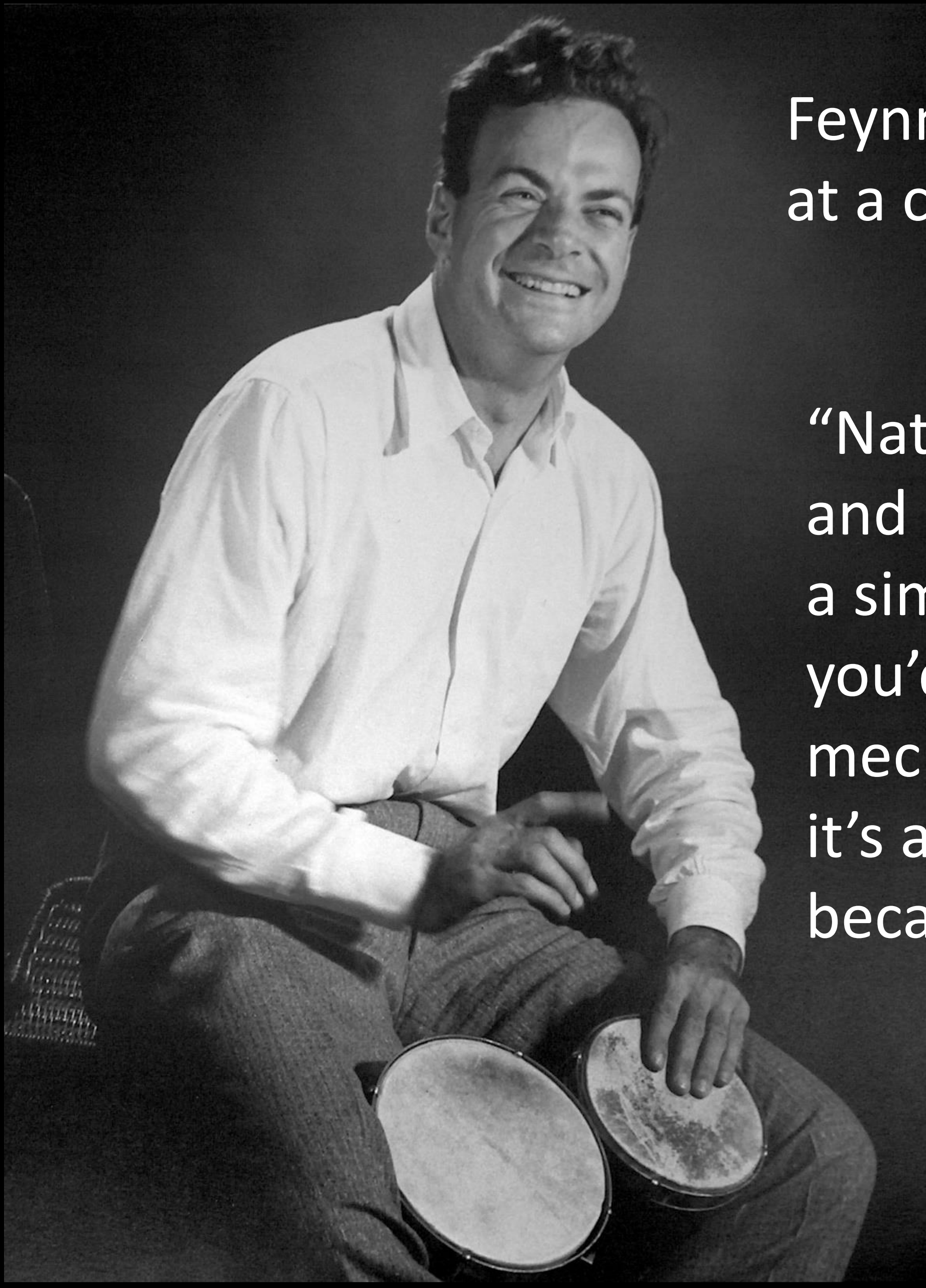
multiplications of matrices to vectors w/ sizes $= 2^N$

exponentially large steps

Quantum computer

- time evolution $= \mathcal{O}(NM)$ experimental operations

polynomial steps



Feynman as a keynote speaker at a conference in MIT (1981):

"Nature isn't classical, dammit,
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4. Future prospects

“Regularization” of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

⟶ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  — Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  — Hilbert sp. at each site is ∞ dimensional
  (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  — ∞ dimensional Hilbert sp. in higher dimensions

Let's consider charge-q Schwinger model:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i} q A_\mu)\psi - m\, \overline{\psi}\psi$$

Field content:

- $U(1)$ gauge field
- charge-q Dirac fermion

Let's explore

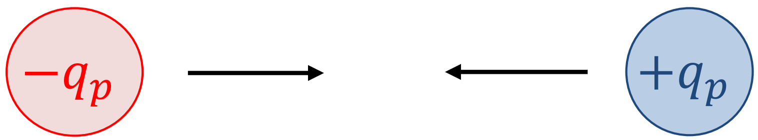
screening vs confinement problem

(next slide)

Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

Coulomb law in 1+1d

||

confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}(1 - e^{-q\mu x})$$

screening

- massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$ *screening*

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g\ )$$

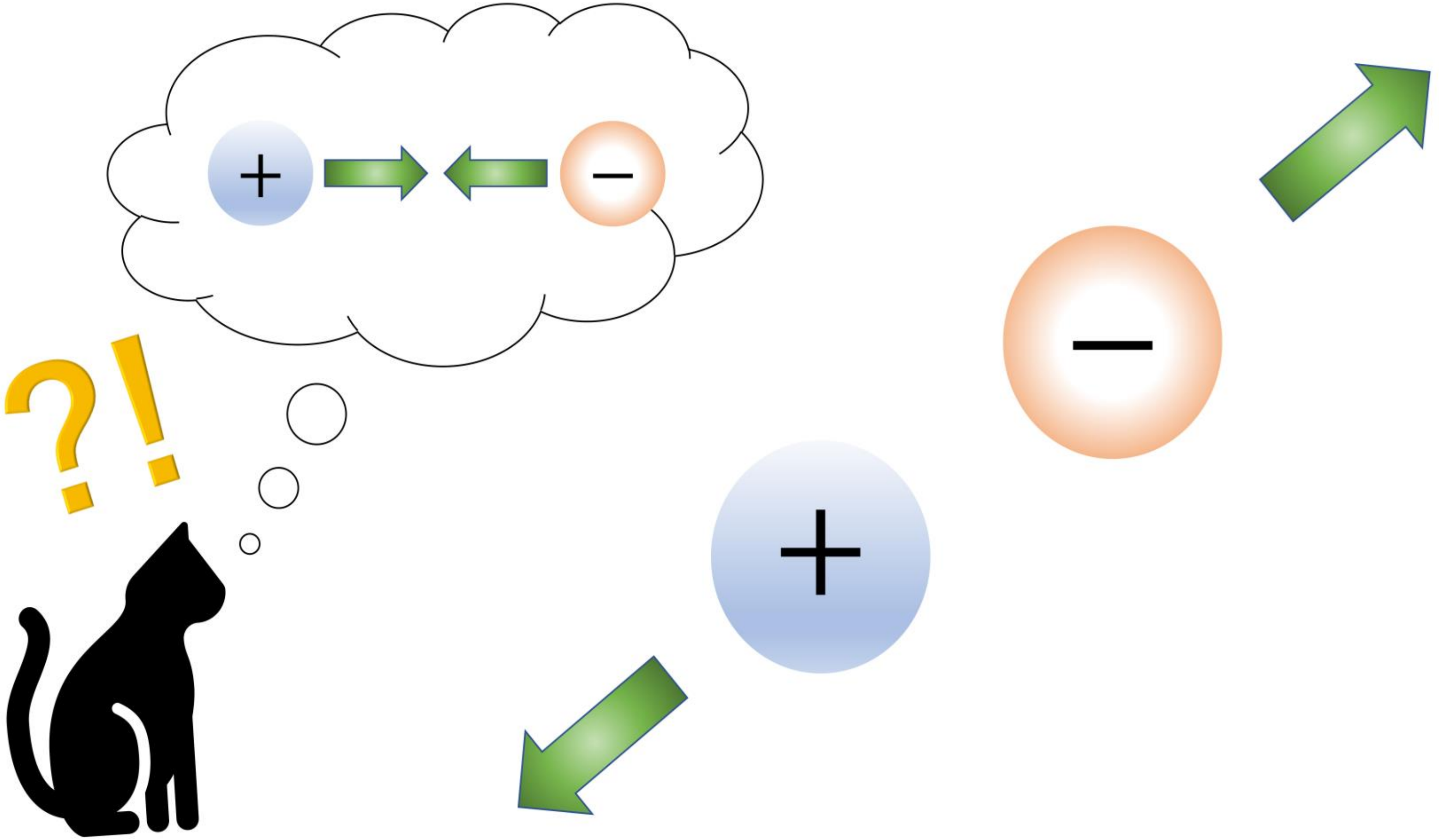
$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \end{cases}$$

= Const. for $\mathrm{q_p/q} = \mathbf{Z}$ *screening*

$\propto x$ for $\mathrm{q_p/q} \neq \mathbf{Z}$ *confinement?*

*but sometimes **negative** slope!*

That is, as changing the parameters…



Let’s explore this aspect by quantum simulation!

UT me! MARKET

UTme!トップ | 新規登録/ログイン | よくある質問 | カート

おすすめ | ランキング | スペシャル | 新着 | 検索

DETAIL アイテムの詳細

DESIGNED BY

masazumi318

アイテム

¥1,990

- ベーシックTシャツ
- ベーシックTシャツ（2021年モデル）
- Uクルーネック T(半袖)
- ベーシック T シャツ（長袖）
- Uクルーネック T（ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- KIDS カラークルーネック T(半袖)
- KIDS Uクルーネック T（ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- トートバッグ

Charge-q Schwinger model

Continuum:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

Taking temporal gauge $A_0 = 0$, (Π: conjugate momentum of A_1)

$$H(x) = \frac{g^2}{2} \left(\Pi - \frac{\theta_0}{2\pi} \right)^2 - \bar{\psi}\, \mathrm{i}\, \gamma^1 (\partial_1 + \mathrm{i}\, q A_1)\psi + m\bar{\psi}\psi,$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - q g \bar{\psi} \gamma^0 \psi$$

Put the theory on lattice

▪ **Fermion** (on site):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

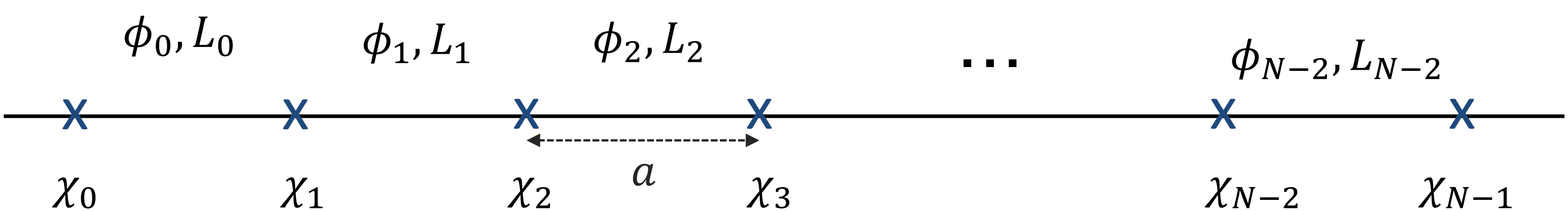
ψ_u → odd site

ψ_d → even site

a: *lattice spacing*

▪ **Gauge field** (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



Lattice theory w/ staggered fermion

Hamiltonian:

$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\theta_0}{2\pi}\right)^2 - \mathrm{i}w\sum_{n=0}^{N-2}\left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n$$

$$\left(w = \frac{1}{2a}, J = \frac{g^2 a}{2}\right)$$

Commutation relation:

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Gauss law:

$$L_n - L_{n-1} = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q \sum_{j=1}^{n} \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

Then,

$$H = -\mathrm{i}w \sum_{n=1}^{N-1} \left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.} \right] + m \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n$$
$$+ J \sum_{n=1}^{N} \left[\frac{\theta_0}{2\pi} + q \sum_{j=1}^{n} \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \right]^2 .$$

This acts on finite dimensional Hilbert space

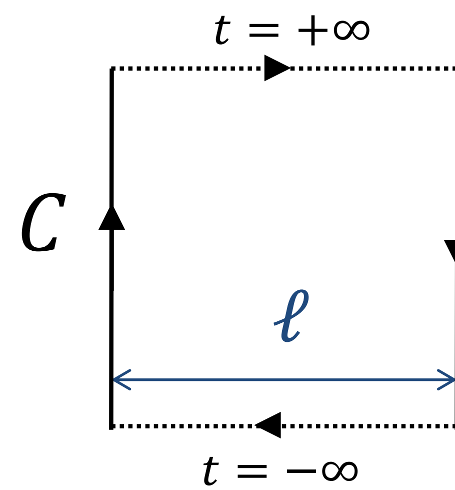
Insertion of the probe charges

① Introduce the probe charges $\pm q_p$:

$$e^{iq_p \int_C A}$$

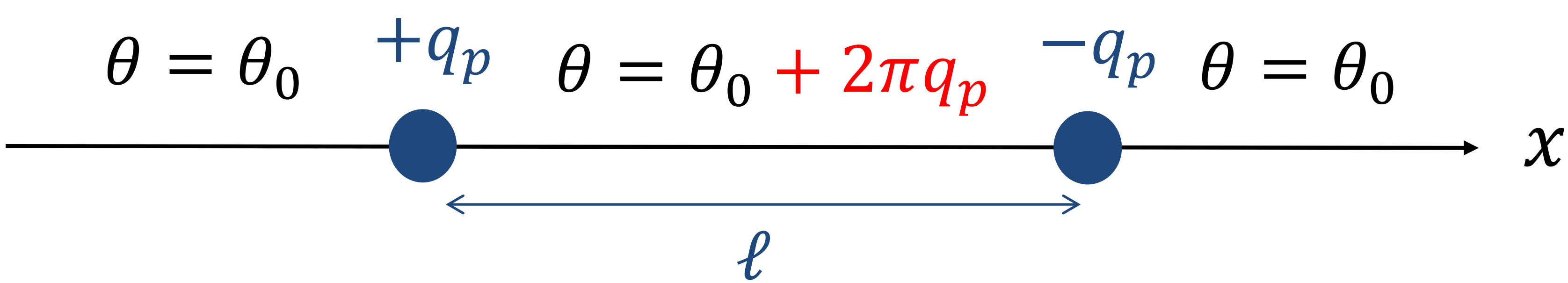
$$||$$

$$e^{iq_p \int_{S,\partial S=C} F}$$



local θ-term w/ $\theta = 2\pi q_p$!!

② Include it to the action & switch to Hamilton formalism



③ Compute the ground state energy (in the presence of the probes)

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

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(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Now the system is purely a spin system:

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger\chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n \ + J\sum_{n=1}^{N}\left[\frac{\vartheta_n}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger\chi_j - \frac{1-(-1)^j}{2}\right)\right]^2$$

⇓

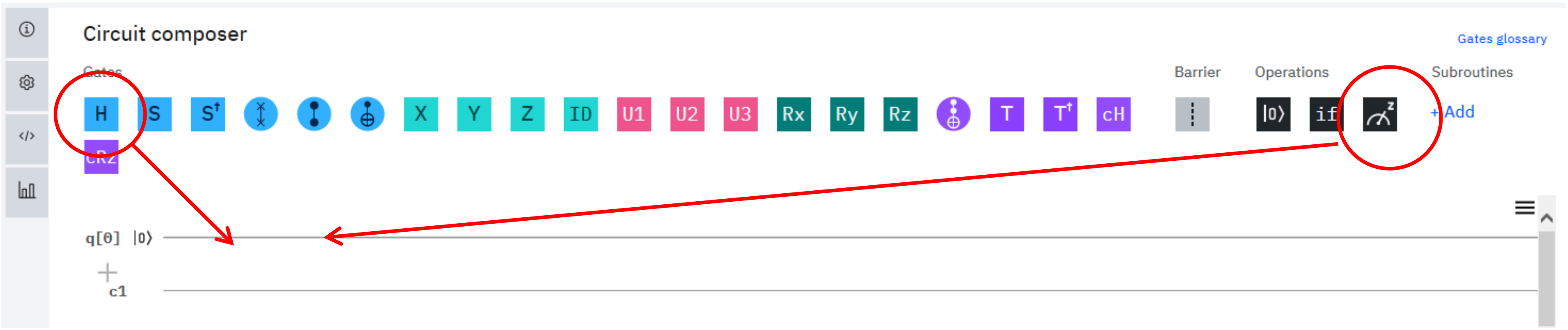
$$H = J\sum_{n=0}^{N-2}\left[q\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

Qubit description of the Schwinger model !!

Atmosphere (?) of using quantum computer…

Suppose we'd like to measure the state: $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Screenshot of IBM Quantum Experience:



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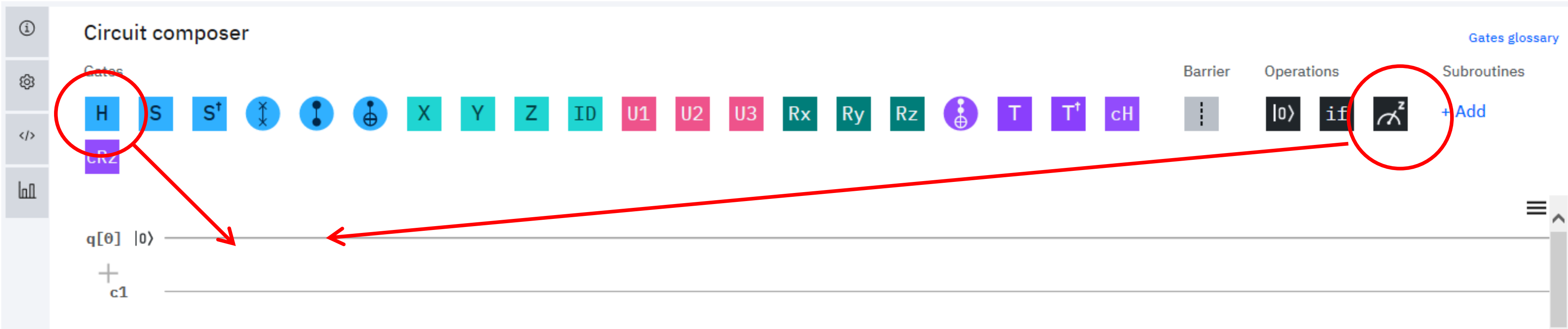
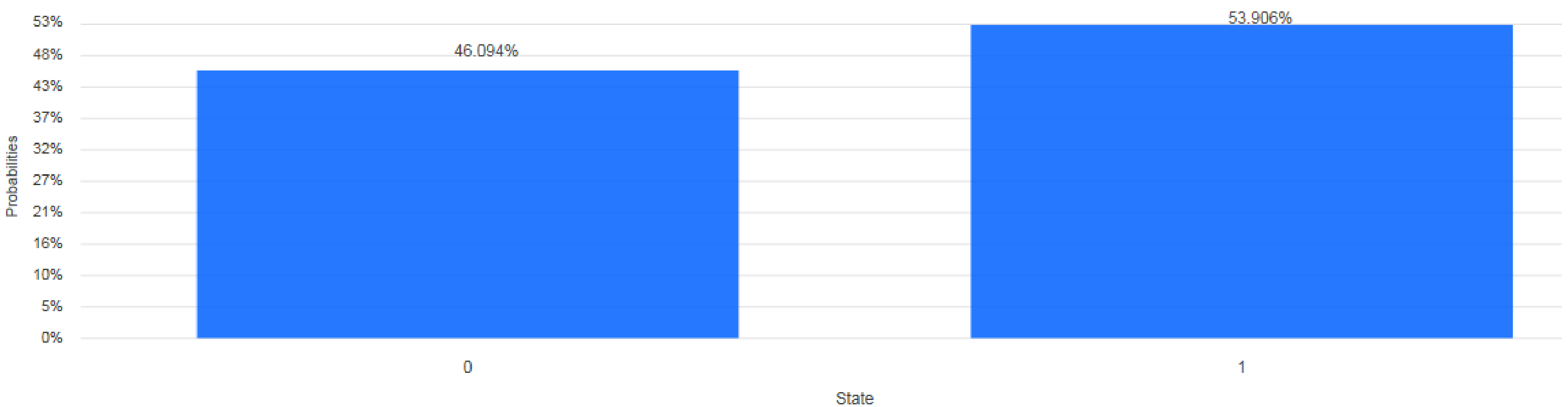


Output of 1024 times measurements ("shots") :



Idea: express physical quantities in terms of "probabilities" & measure the "probabilities"

Constructing vacuum (ground state)

$^{\exists}$various quantum algorithms to construct vacuum:

- adiabatic state preparation
- algorithms based on variational method
- imaginary time evolution

etc...

Here, let's apply

adiabatic state preparation

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2:

Step 3:

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

- $H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}}$
- $\left|\frac{dH_A}{dt}\right| \ll 1$ for $T \gg 1$

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- $H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}}$
- $\left|\frac{dH_A}{dt}\right| \ll 1$ for $T \gg 1$

Step 3: Use the adiabatic theorem

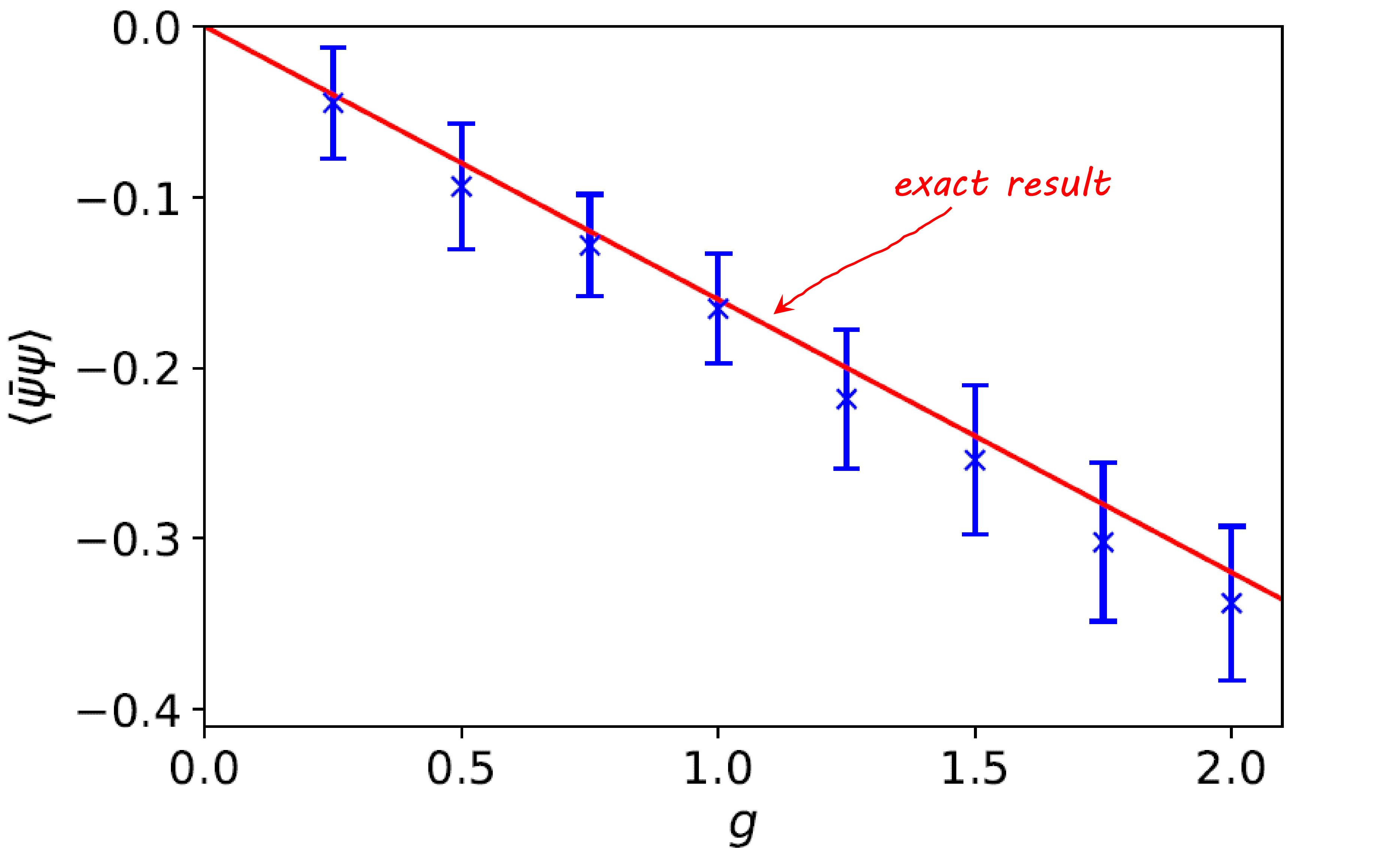
If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of H_{target} is obtained by

$$|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\, H_A(t)\right) |\mathrm{vac}_0\rangle$$

Matching exact result ($q = 1$ & $m = 0$) (after continuum limit)

$T = 100, \delta t = 0.1, N_{\max} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



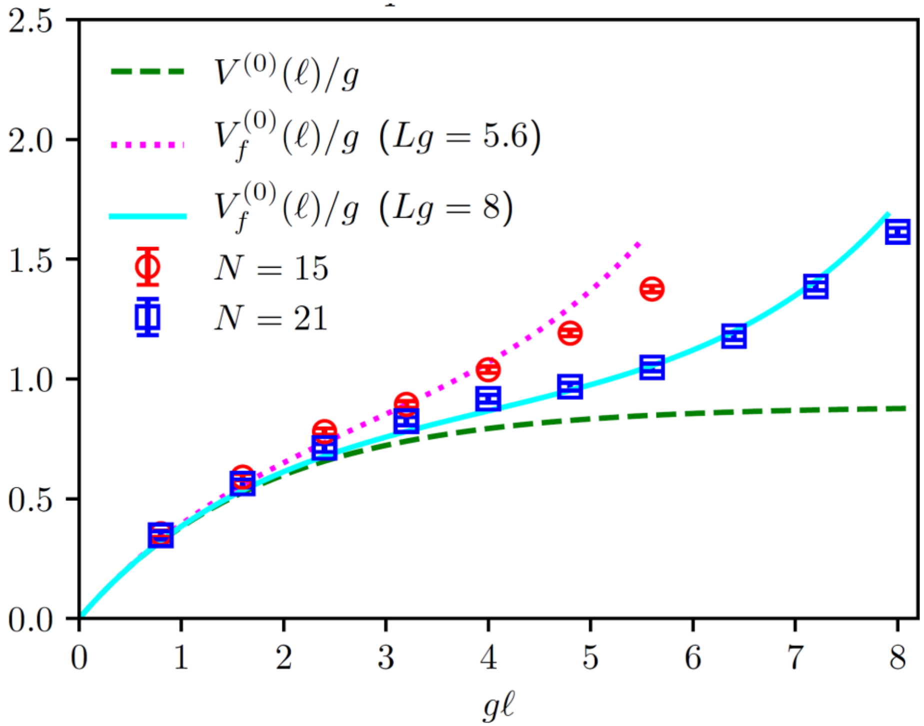
Massless vs massive for $\theta_0 = 0$ & $q_p/q \in \mathbb{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

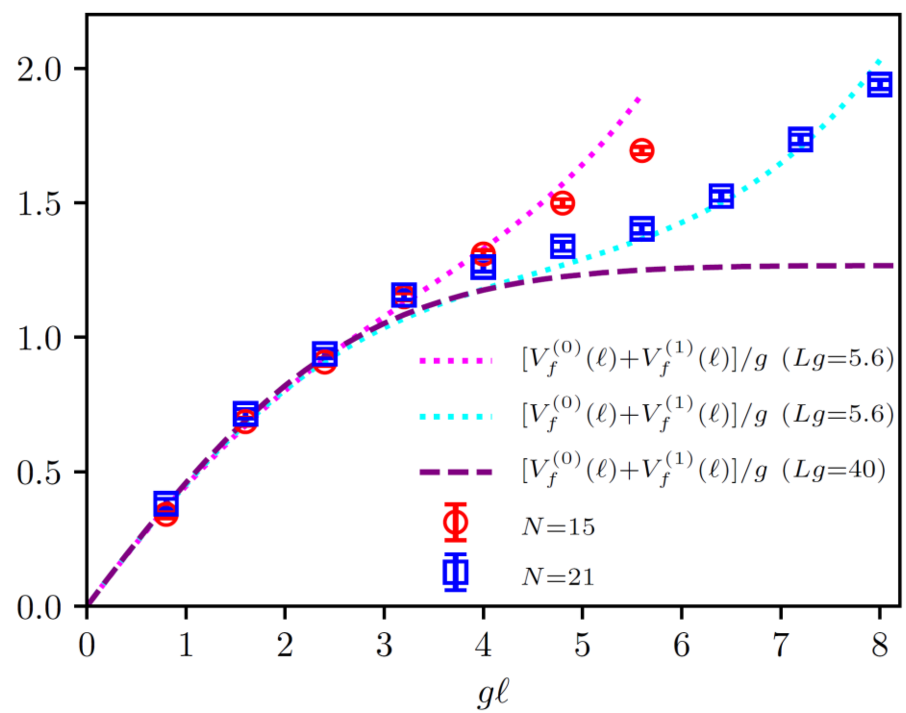
Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1$

Lines: analytical results in the continuum limit (finite & ∞ vols.)

$q_p = 1, m = 0$



$q_p = 1, m/g = 0.2$



Consistent w/ expected screening behavior

Results for $\theta_0 = 0$ & $q_p/q \notin \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1/4, m = 0$ & 0.2

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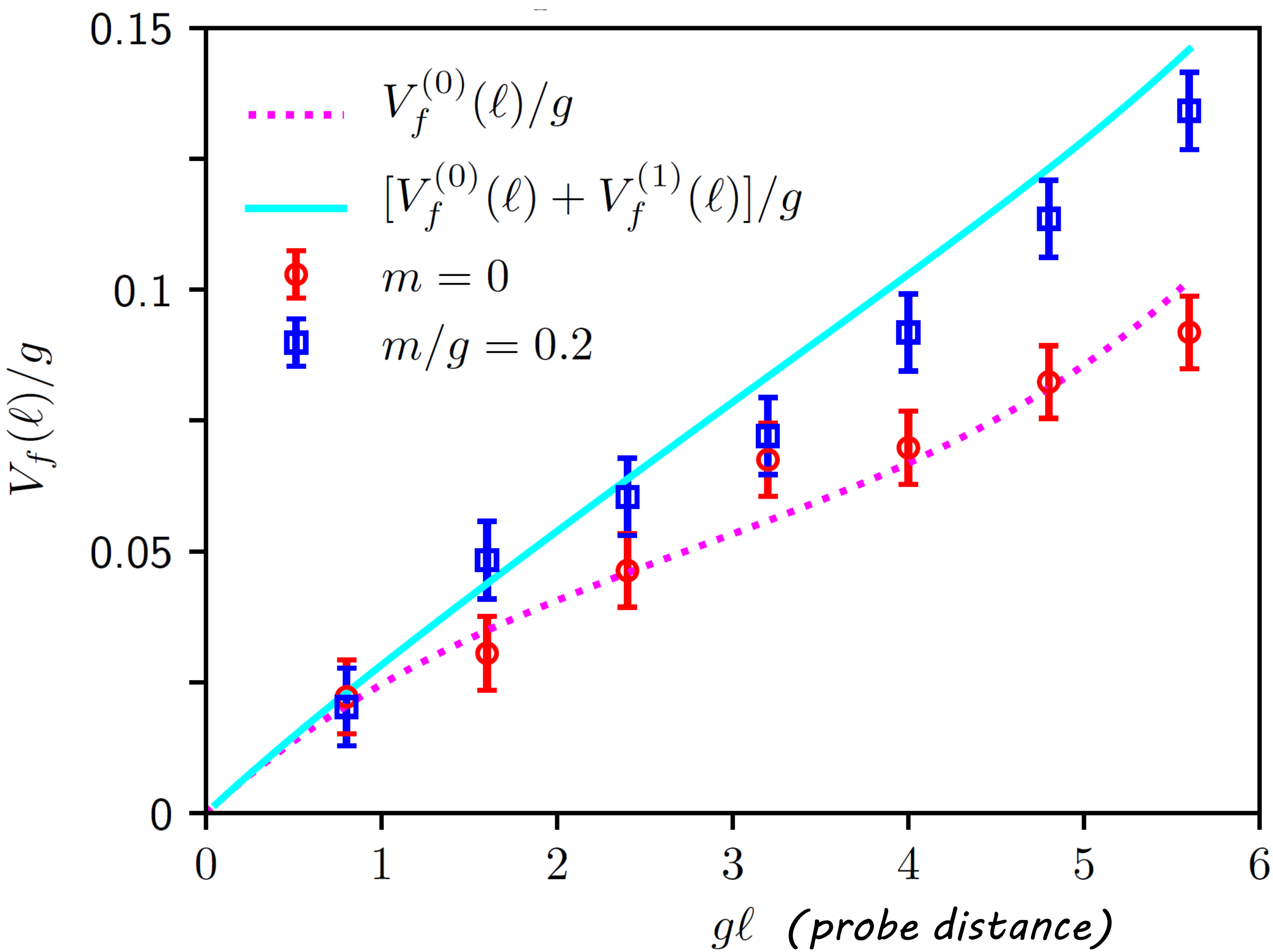

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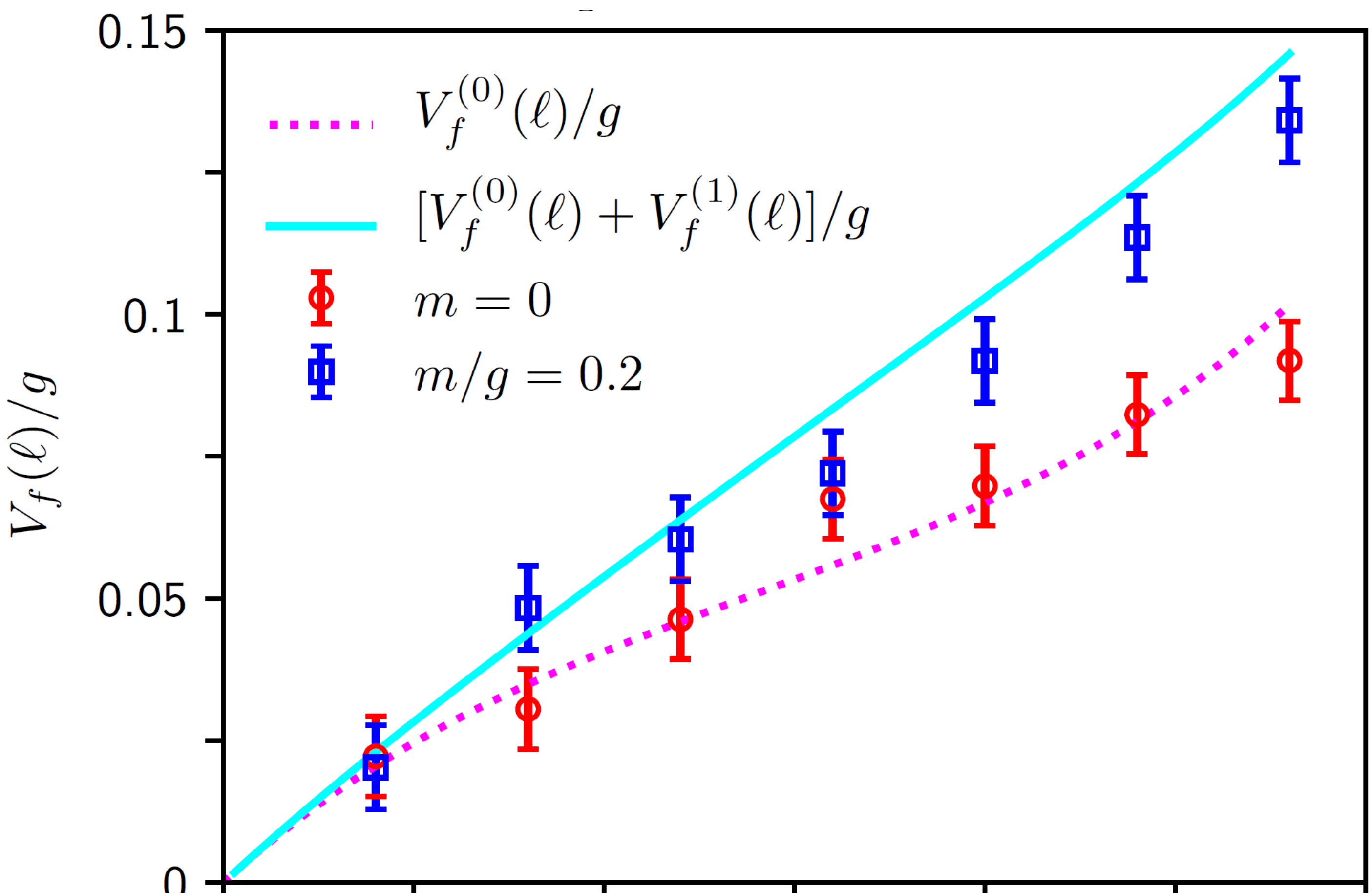
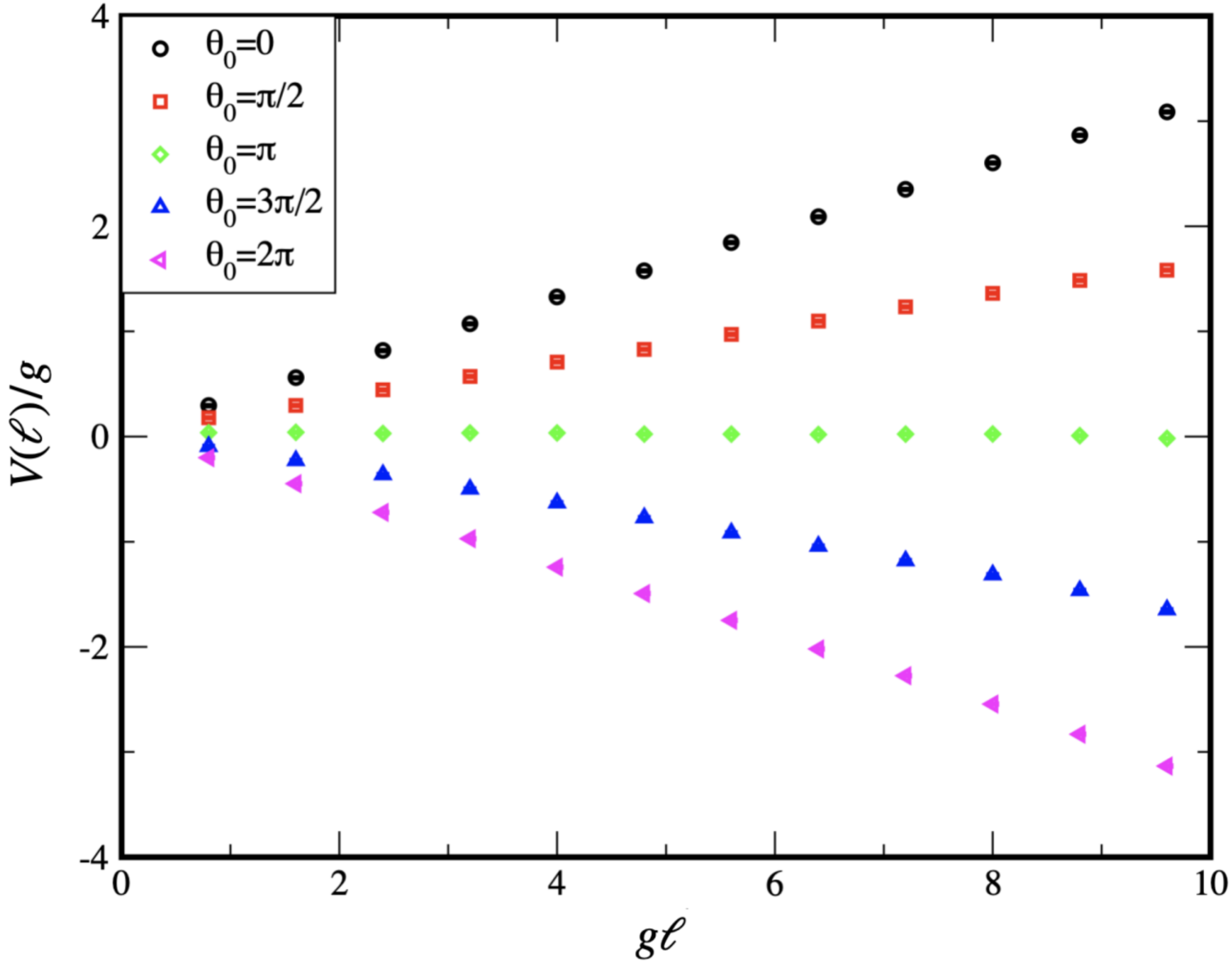


Consistent w/ expected confinement behavior

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$



Sign(tension) changes as changing θ-angle!!

Future prospects

Near future prospect

In near future, available device is so-called [Preskill '18]

Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

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On such device,

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⇨ nice if $^{\exists}$a way to reduce errors w/o increasing qubits

⇨ "quantum error mitigation"

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Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

On such device,

- quantum error correction can't be enough

⇨ nice if $^{\exists}$a way to reduce errors w/o increasing qubits

⇨ "quantum error mitigation"

- algorithms w/ less gates are preferred

⇨ Hybrid quantum-classical algorithm

(Popular one for finding vacuum: "variational method")

Quantum Error mitigation

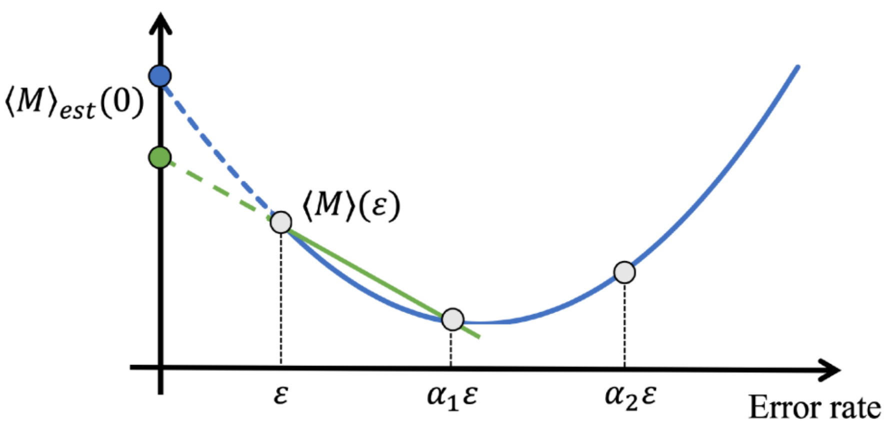
[Figs. are from Endo-Cai-Benjamin-Yuan '20]

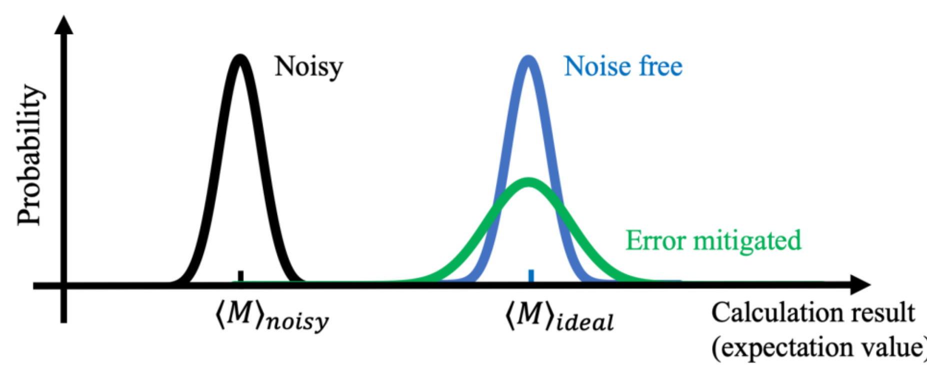
the simplest way = extrapolation

In general,

difficult to decrease errors but possible to increase them

⇨ error-free result by fitting as a function of error rate





This doesn't need to increase qubits but needs more shots

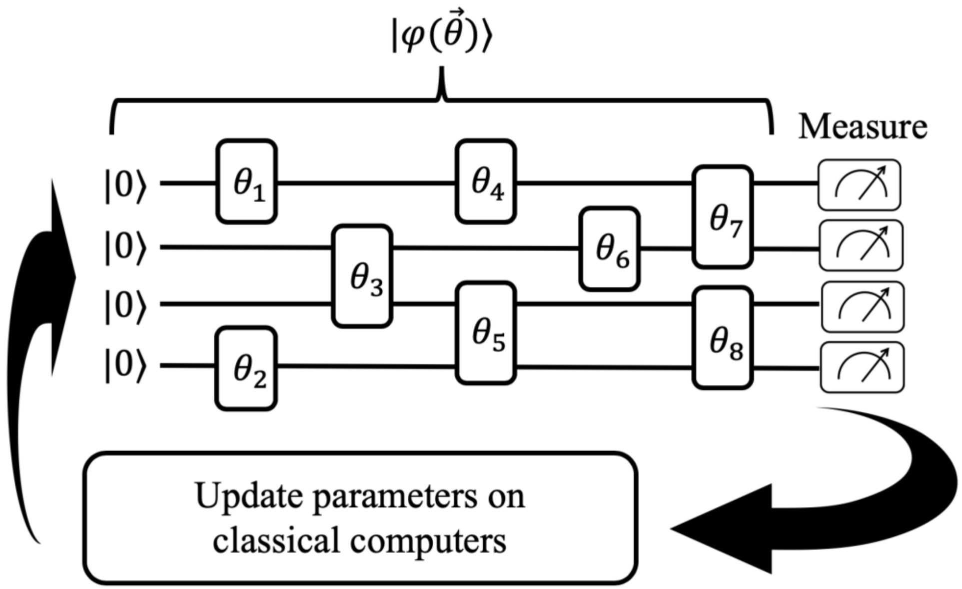
Variational quantum algorithm

Idea:

[Fig. is from Endo-Cai-Benjamin-Yuan '20]

Acting gates & measurements ⇨ Quantum computer

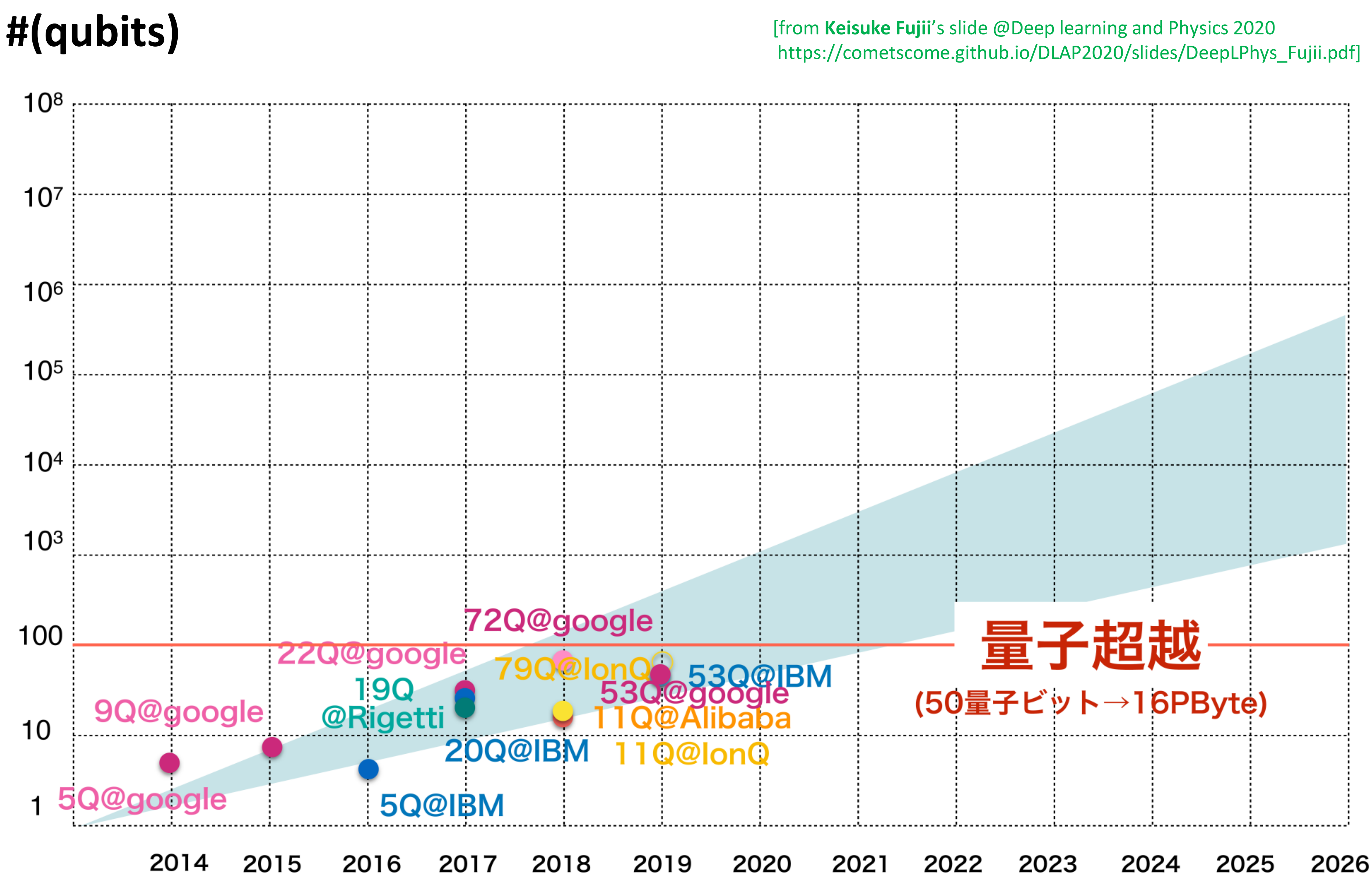
Parameter optimization ⇨ Classical computer



This method needs much less gates than adiabatic state preparation but it's not guaranteed to get true ground state

“Quantum” Moore’s law?

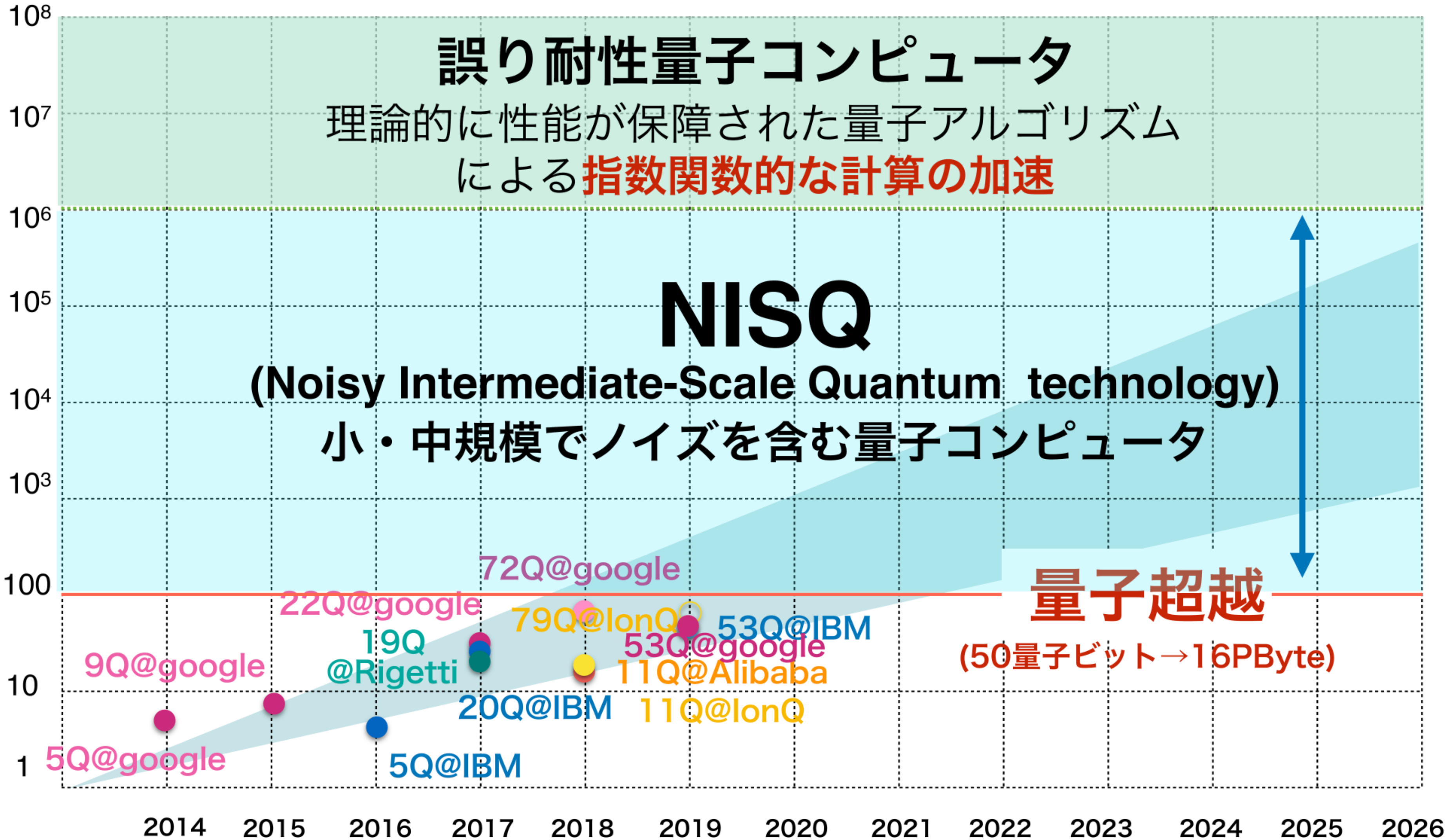
[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]



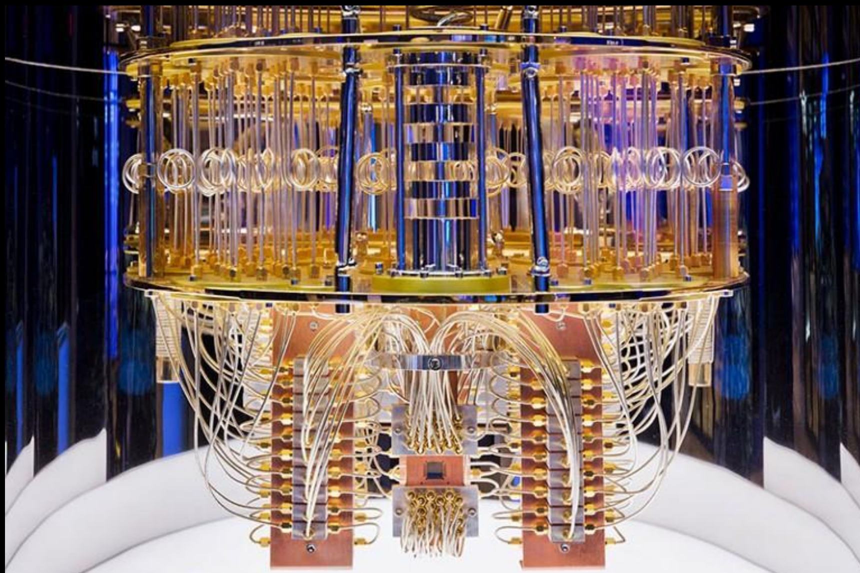
“Quantum” Moore’s law?

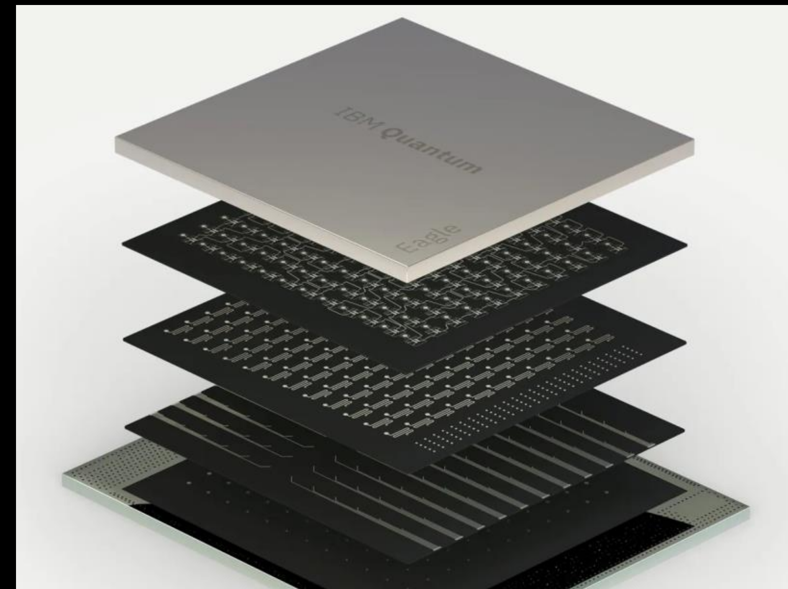
#(qubits)

[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]



The challenge by IBM’s 127-qubit device





Article

Evidence for the utility of quantum computing before fault tolerance

https://doi.org/10.1038/s41586-023-06096-3

Received: 24 February 2023

Accepted: 18 April 2023

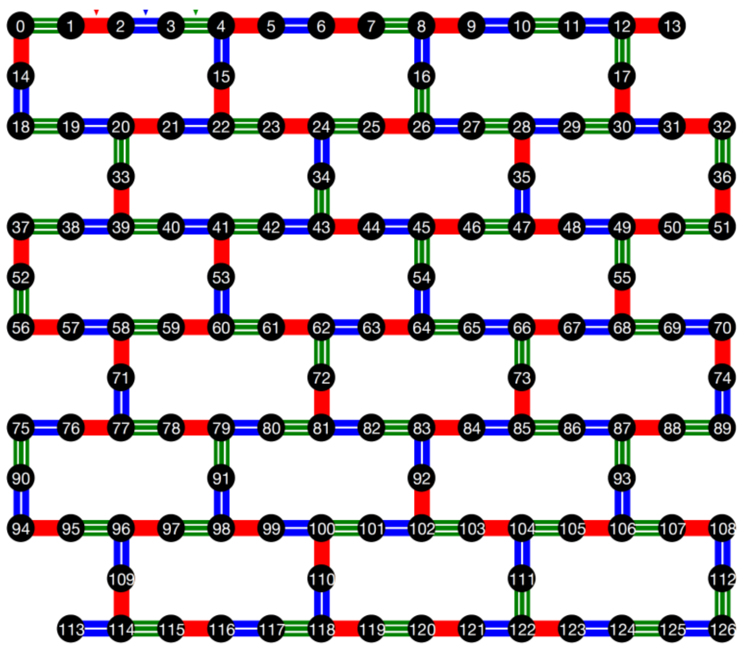
Published online: 14 June 2023

Youngseok Kim[1,6] ✉, Andrew Eddins[2,6] ✉, Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1] ✉

Quantum computing promises to offer substantial speed-ups over its classical

The challenge by IBM's 127-qubit device (cont'd)

Task: time evolution of Ising model on a lattice
w/ shape = the qubit config. of the device



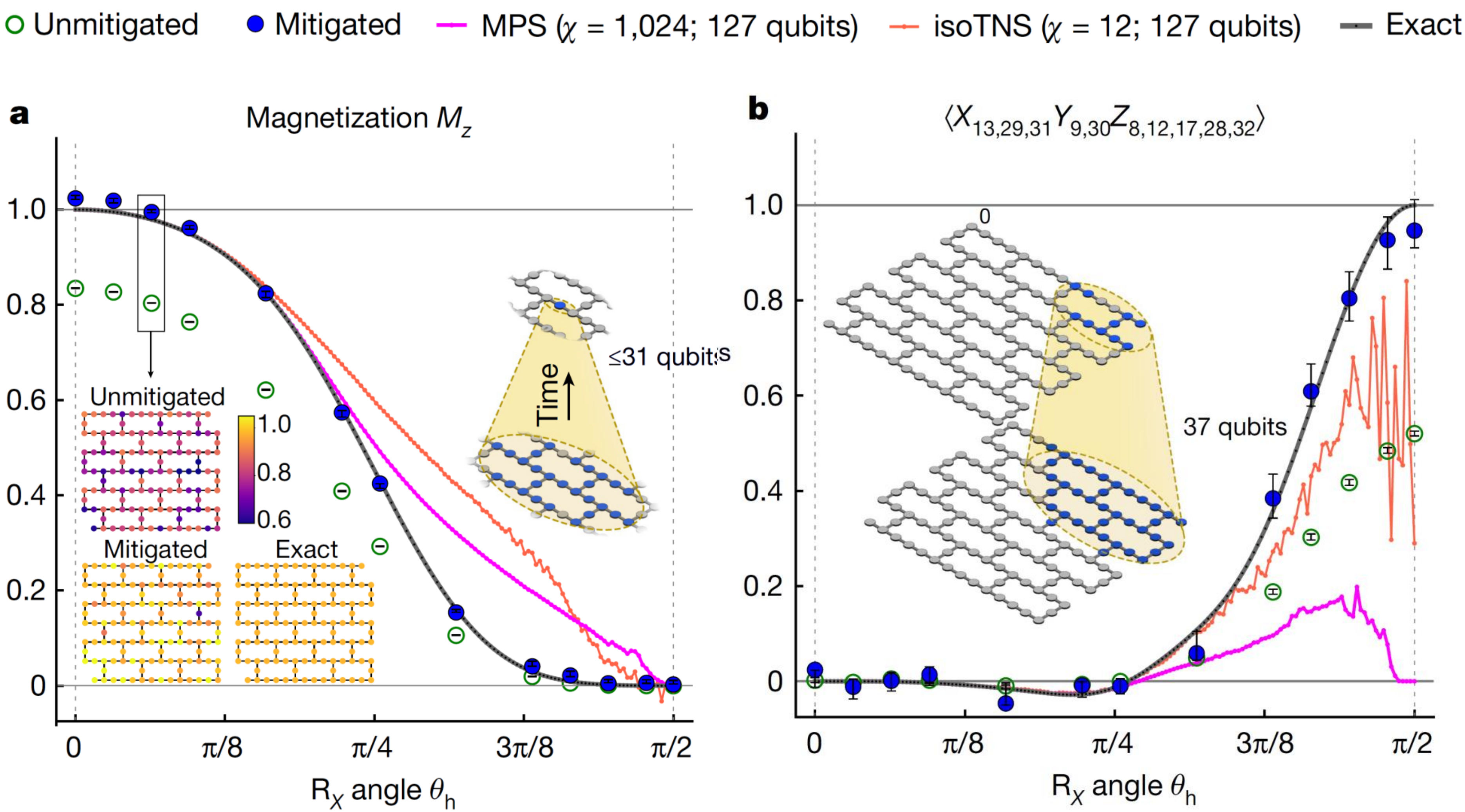
$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j + h \sum_i X_i,$$

$$|\psi(t)\rangle := e^{-iHt}|00\cdots 0\rangle$$

$$\langle\psi(t)|\mathcal{O}|\psi(t)\rangle$$

Strategy: Suzuki-Trotter approximation
+ error mitigation by extrapolation

The challenge by IBM's 127-qubit device (cont'd)



"Quantum supremacy"?

But…

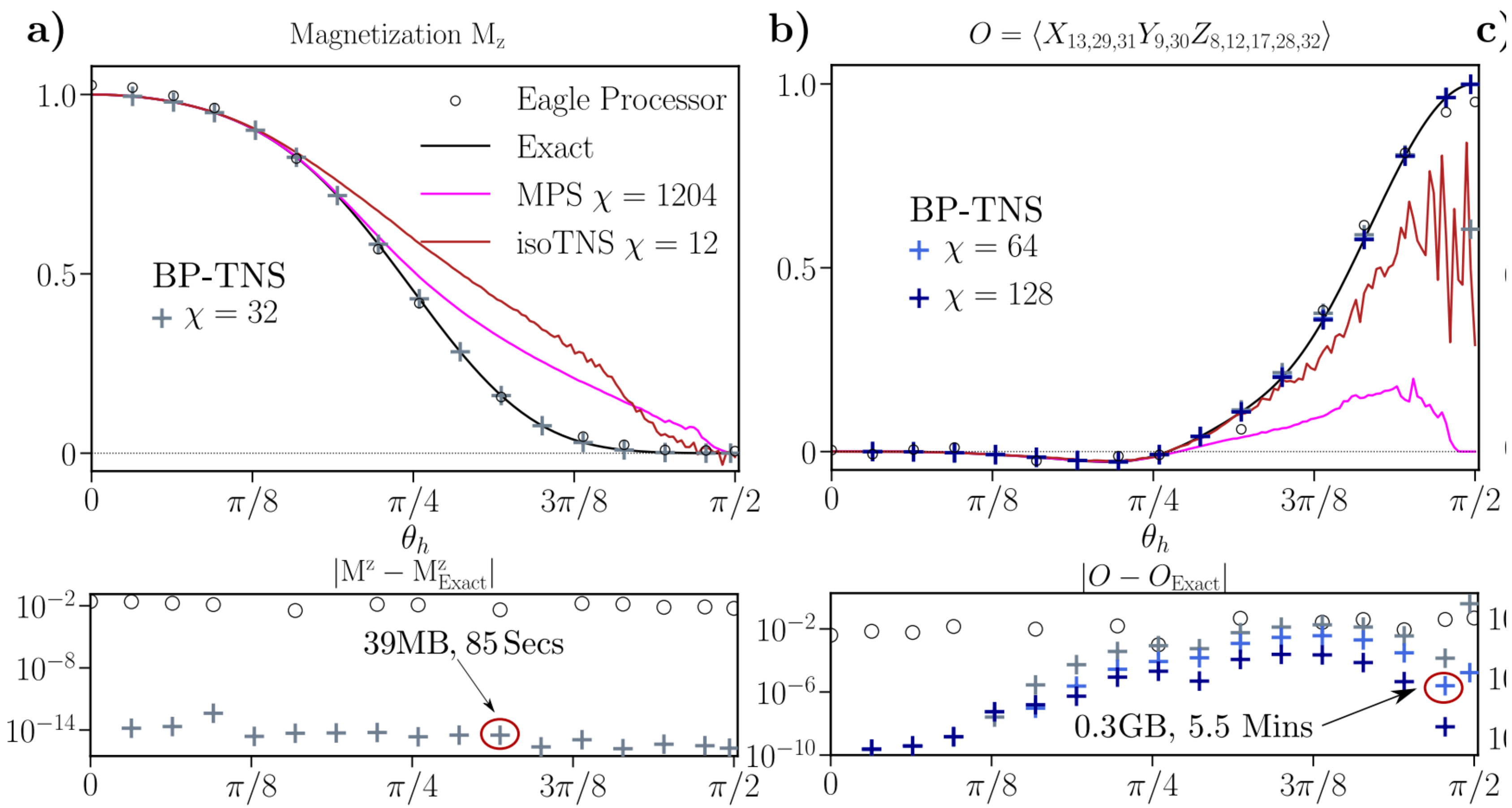
arXiv > quant-ph > arXiv:2306.14887

Quantum Physics

[Submitted on 26 Jun 2023]

Efficient tensor network simulation of IBM's kicked Ising experiment

Joseph Tindall, Matt Fishman, Miles Stoudenmire, Dries Sels



Applications PPP People may be interested

- 100 qubit simulation of Schwinger model [Farrell-Illa-Ciavarella-Savage ’23]
- Scattering [Jordan-Lee-Preskiill ’17]
- Inflation (scalar in curved spacetime) [Liu-Li ’20]
- Boltzmann eq. [Yamazaki-Uchida-Fujisawa-Yoshida ’23, Higuchi-Pedersen-Yoshikawa ’23]
- Dark sector showers [Chigusa-Yamazaki ’22]
- Schwinger model in open quantum system [De Jong-Metcalf-Mulligan-Ploskon-Ringer-Yao ’20, de Jong-Lee-Mulligan-Ploskon-Ringer-Yao ’21, Lee-Mulligan-Ringer-Yao ’23]
- Quantum many body scars in 2+1d SU(2) YM [Hayata-Hidaka ’23]
- Imaging stars w/ error correction [Huang-Brennen-Ouyang ’22]

Patterns to write papers

1. Find a bottle neck of (classical) numerical computation in your problem
2. Is there a corresponding quantum algorithm?

Patterns to write papers

1. Find a bottle neck of (classical) numerical computation in your problem
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⇩ No

Make the algorithm!

Patterns to write papers

1. Find a bottle neck of (classical) numerical computation in your problem

2. Is there a corresponding quantum algorithm?

⇩ Yes

Is there an application to your problem?

⇩ No

Make the algorithm!

Patterns to write papers

1. Find a bottle neck of (classical) numerical computation in your problem

2. Is there a corresponding quantum algorithm?

⇩ Yes

Is there an application to your problem?

⇩ Yes

Improve methods or get physically new results!

⇩ No

Make the algorithm!

Patterns to write papers

1. Find a bottle neck of (classical) numerical computation in your problem

2. Is there a corresponding quantum algorithm?

⇩ Yes

Is there an application to your problem?

⇩ Yes

Improve methods or get physically new results!

⇘ No

Propose the application & estimate complexity!

⇩ No

Make the algorithm!

Thanks!

Appendix

FTQC vs NISQ

Fault Tolerant Quantum Computer (FTQC)

- large quantum computer w/ sufficient error correction
- our dream
- expected to show "quantum supremacy" if it is realized
- not sure if it is realized in future

Noisy Intermediate-Scale Quantum computer (NISQ)

[cf. Preskill '18]

- intermediate quantum computer w/ non-negligible errors
- current/near future device
- not sure if $^\exists$ problems to give "quantum supremacy"

Symmetries in charge-q Schwinger model

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

- $\boldsymbol{Z_q}$ chiral symmetry for $m = 0$

  —— ABJ anomaly: $U(1)_A \to \boldsymbol{Z_q}$

  —— known to be spontaneously broken

- $\boldsymbol{Z_q}$ 1-form symmetry

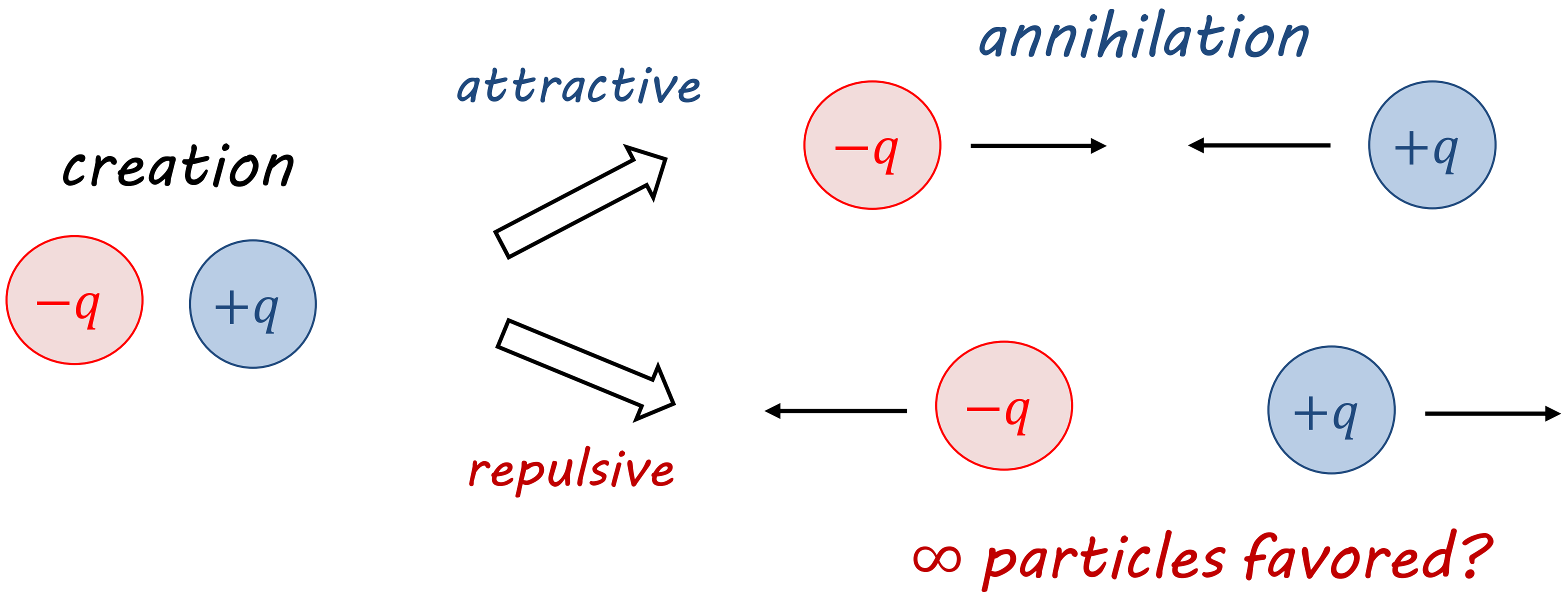
  —— remnant of $U(1)$ 1-form sym. in pure Maxwell

  —— Hilbert sp. is decomposed into q-sectors *"universe"*

  (cf. common for $(d-1)$-form sym. in d dimensions)

FAQs on negative tension behavior

Q1. It sounds that many pair creations are favored.
Is the theory unstable?

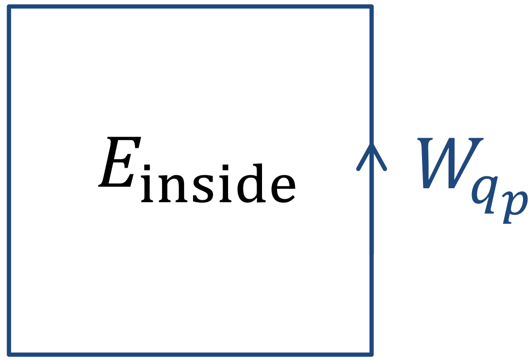


—— No. Negative tension appears only for $q_p \neq q\boldsymbol{Z}$.

So, such unstable pair creations do not occur.

FAQs on negative tension behavior (cont'd)

[cf. MH-Itou-Kikuchi-Tanizaki '21]



E_{outside} $(= E_0?)$

Q2. It sounds $E_{\text{inside}} < E_{\text{outside}}$. Strange?

—— Inside & outside are in different sectors decomposed by Z_q 1-form sym.

$$\mathcal{H} = \bigoplus_{\ell=0}^{q-1} \mathcal{H}_\ell$$ *"universe"*

E_{inside} & E_{outside} are lowest in each universe:

$$E_{\text{inside}} = \min_{\mathcal{H}_{\ell+q_p}} (E), \quad E_{\text{outside}} = \min_{\mathcal{H}_\ell} (E)$$

Comment on adiabatic state preparation

$$(\text{"systematic error"}) \sim \frac{1}{T\,(\text{gap})^2}$$

😄 Advantage:

- guaranteed to be correct for $T \gg 1$ & $\delta t \ll 1$
  if $H_A(t)$ has a unique gapped vacuum
- can directly get excited states under some conditions

☹️ Disadvantage:

- doesn’t work for degenerate vacua
- costly — likely requires many gates

➡ more appropriate for FTQC than NISQ

Without probes

VEV of mass operator (chiral condensation)

$$\langle \bar{\psi}(x)\psi(x)\rangle = \langle \mathrm{vac}|\bar{\psi}(x)\psi(x)|\mathrm{vac}\rangle$$

Instead of the local op., we analyze the average over the space:

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle$$

Once we get the vacuum, we can compute the VEV as

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle = \frac{1}{2Na}\sum_{n=1}^{N}(-1)^n \sum_{i_1\cdots i_N=0,1}\langle \mathrm{vac}|Z_n|i_1\cdots i_N\rangle\langle i_1\cdots i_N|\mathrm{vac}\rangle$$

$$= \frac{1}{2Na}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{n+i_n}\,|\langle i_1\cdots i_N|\mathrm{vac}\rangle|^2$$

How can we obtain the vacuum?

Massless case

For massless case,

θ is absorbed by chiral rotation ⇨ $\theta = 0$ w/o loss of generality

No sign problem

Nevertheless,

it's difficult in conventional approach because computation of fermion determinant becomes very heavy

$^\exists$ Exact result:

[Hetrick-Hosotani '88]

$$\langle \bar{\psi}(x)\psi(x) \rangle = -\frac{e^{\gamma}}{2\pi^{3/2}} g \simeq -0.160g$$

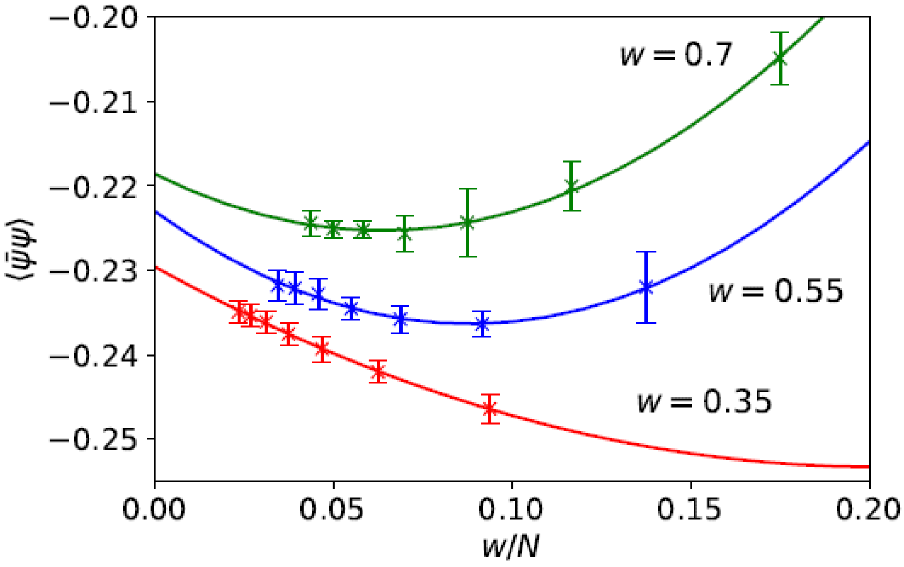
Can we reproduce it?

Thermodynamic & Continuum limit

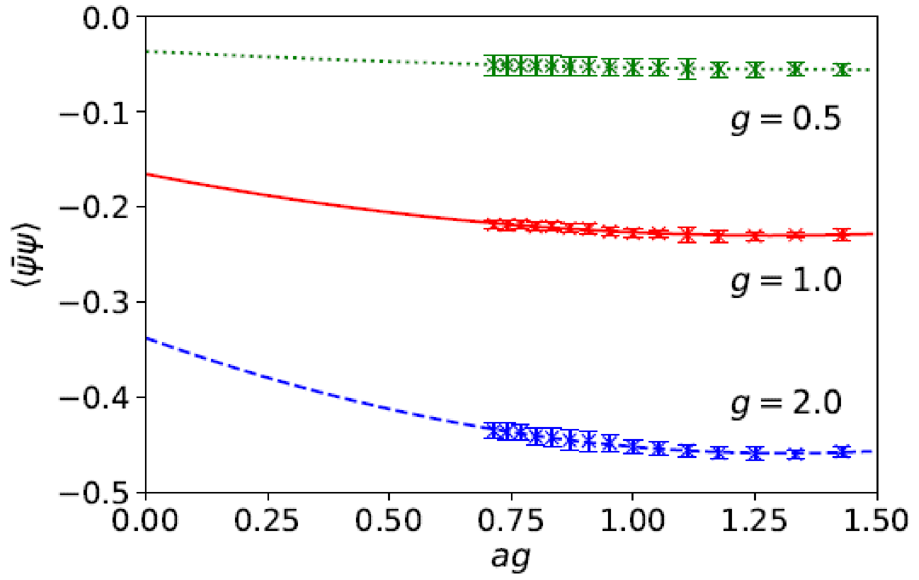
$g = 1, m = 0, N_{\max} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)

Thermodynamic limit (w/ fixed a)



Continuum limit (after $V \to \infty$)



Estimation of systematic errors

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Approximation of vacuum:

$$|\mathrm{vac}> \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0> \equiv |\mathrm{vac}_A\rangle$$

Approximation of VEV:

$$\langle \mathcal{O} \rangle \equiv \langle \mathrm{vac}|\mathcal{O}|\mathrm{vac}\rangle \simeq \langle \mathrm{vac}_A|\mathcal{O}|\mathrm{vac}_A\rangle$$

Introduce the quantity

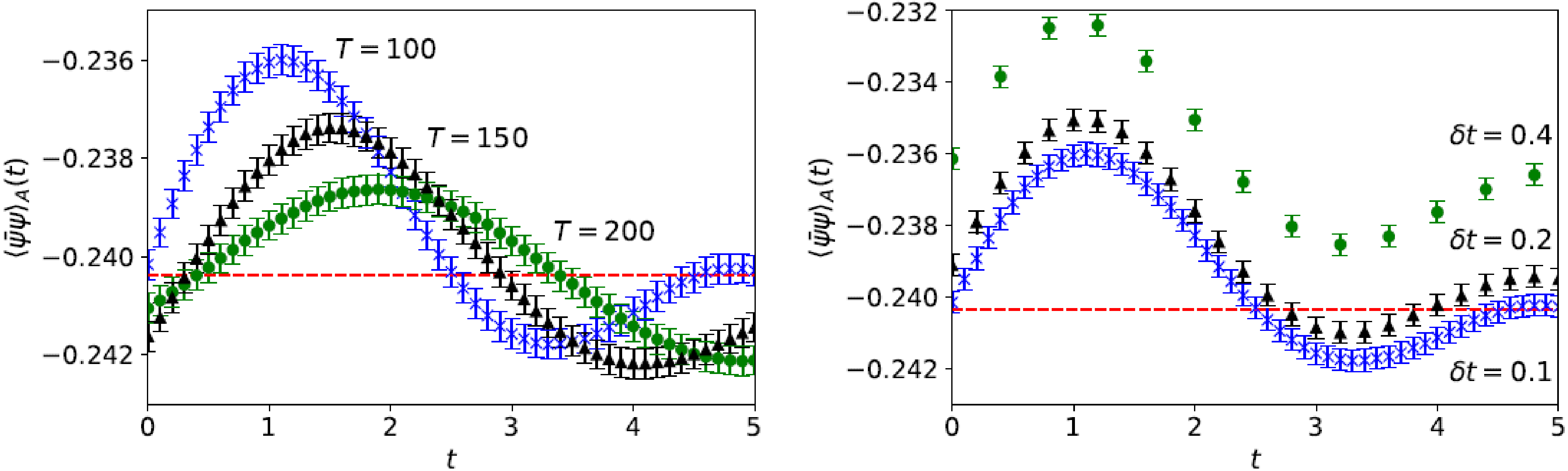
$$\langle \mathcal{O} \rangle_A(t) \equiv \langle \mathrm{vac}_A| e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\mathrm{vac}_A\rangle$$

- independent of t if $|\mathrm{vac}_A\rangle = |\mathrm{vac}\rangle$
- dependent on t if $|\mathrm{vac}_A\rangle \neq |\mathrm{vac}\rangle$

This quantity describes intrinsic ambiguities in prediction

⇒ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)



Oscillating around the correct value

➡ Define central value & error as

$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)+\min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)-\min\langle\mathcal{O}\rangle_A(t)\right)$$

Massive case

Result of mass perturbation theory: [Adam '98]

$$\langle \bar{\psi}(x)\psi(x) \rangle \simeq -0.160g + 0.322m \cos\theta + \mathcal{O}(m^2)$$

However,

$\exists$ subtlety in comparison: this quantity is UV divergent

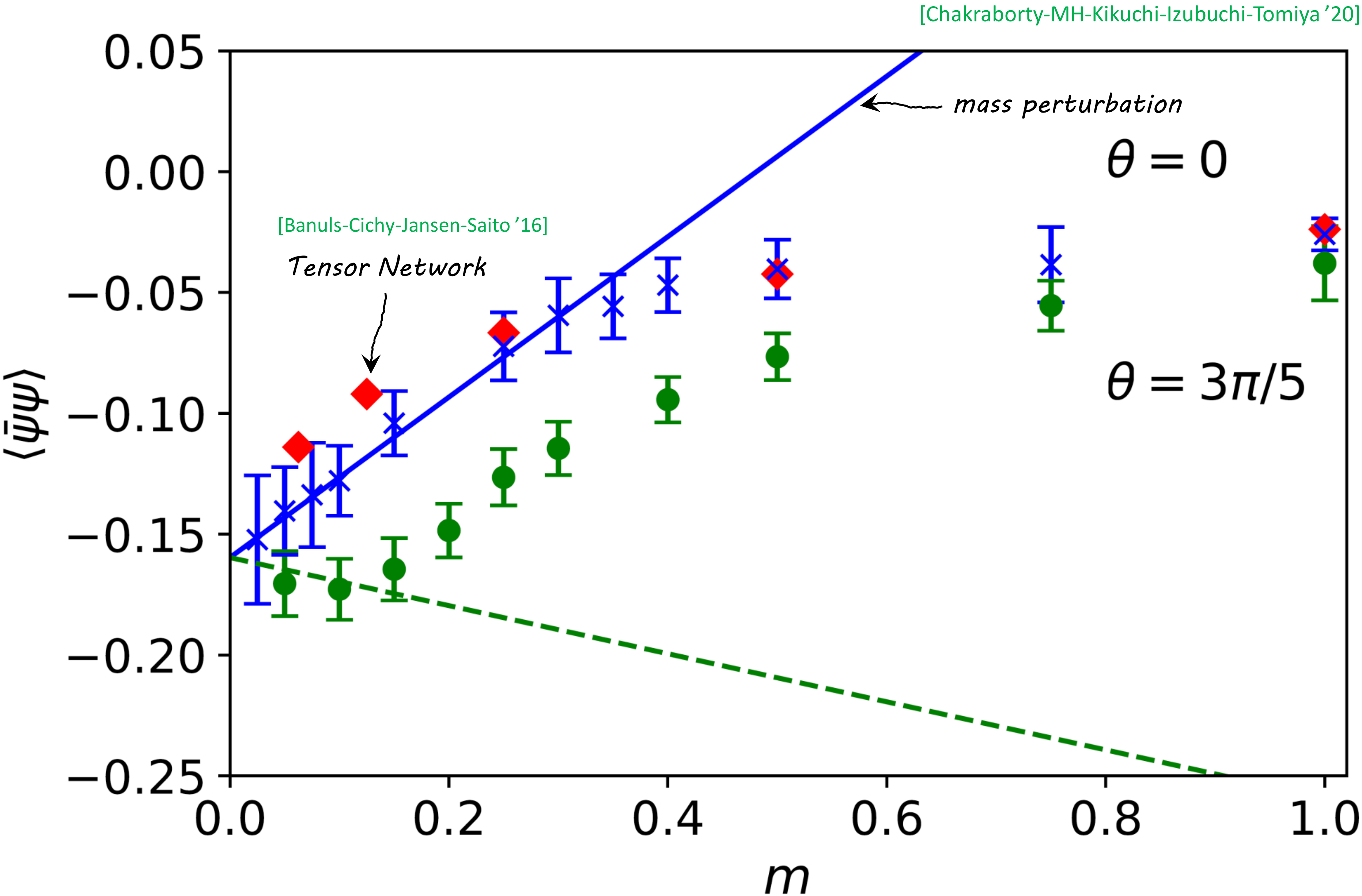
$(\sim m \log \Lambda)$

➡ Use a regularization scheme to have the same finite part

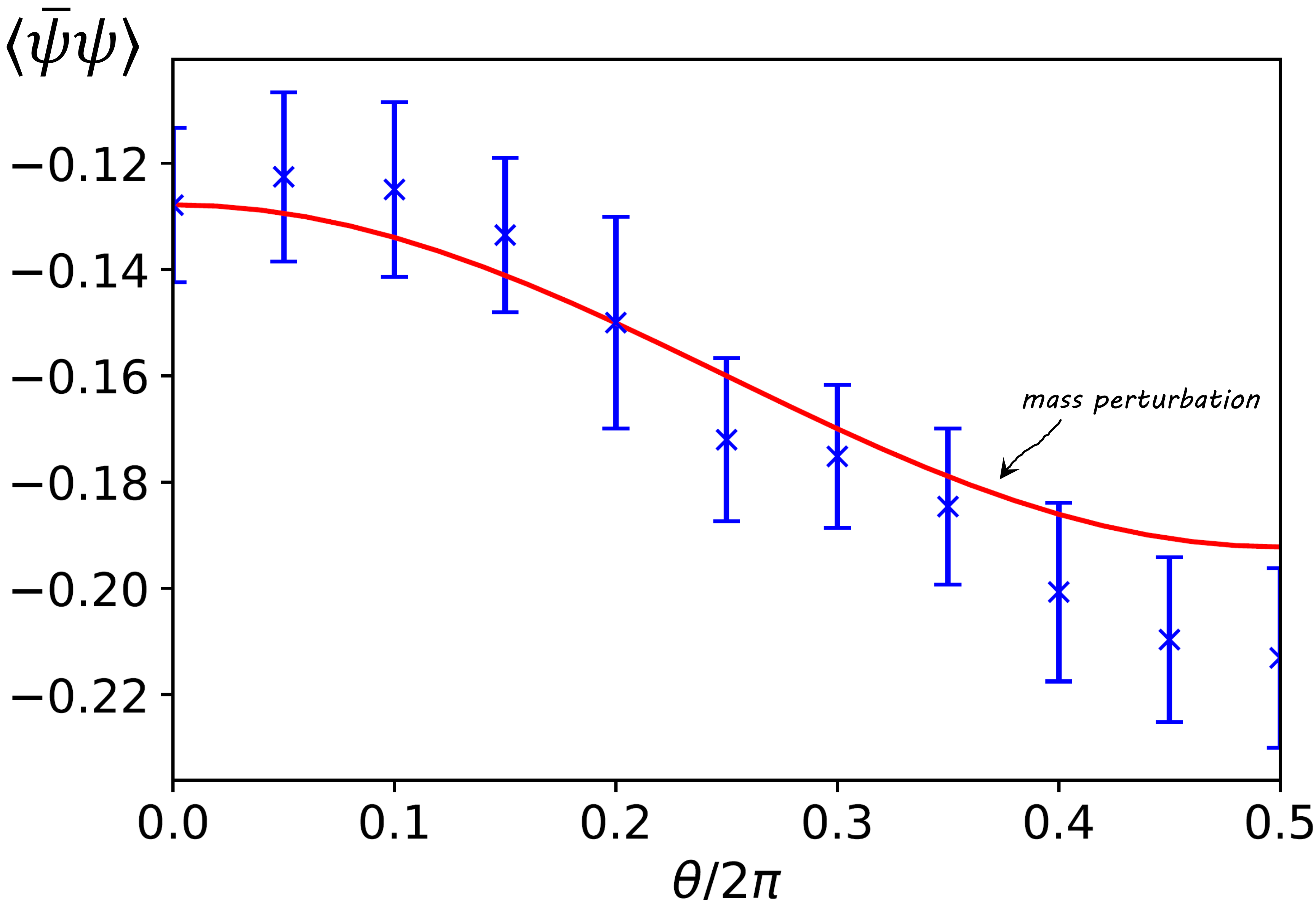
Here we subtract free theory result before taking continuum limit:

$$\lim_{a\to 0}\left[\langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{\text{free}}\right]$$

Chiral condens. for massive case at g=1



θ dependence at $m = 0.1$ & $g = 1$

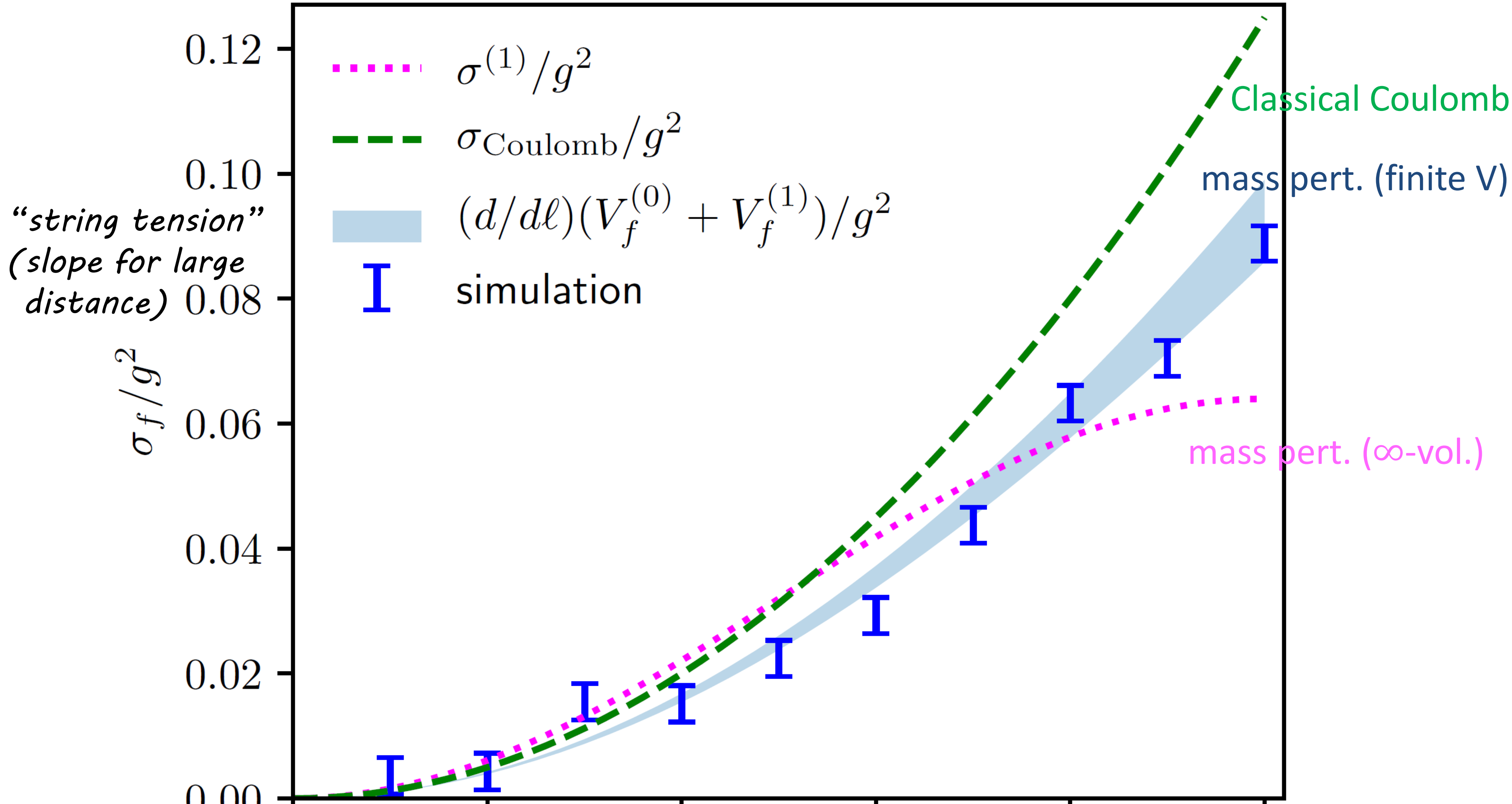


With probes

“String tension” for $\theta_0 = 0$

[MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, m/g = 0.2$

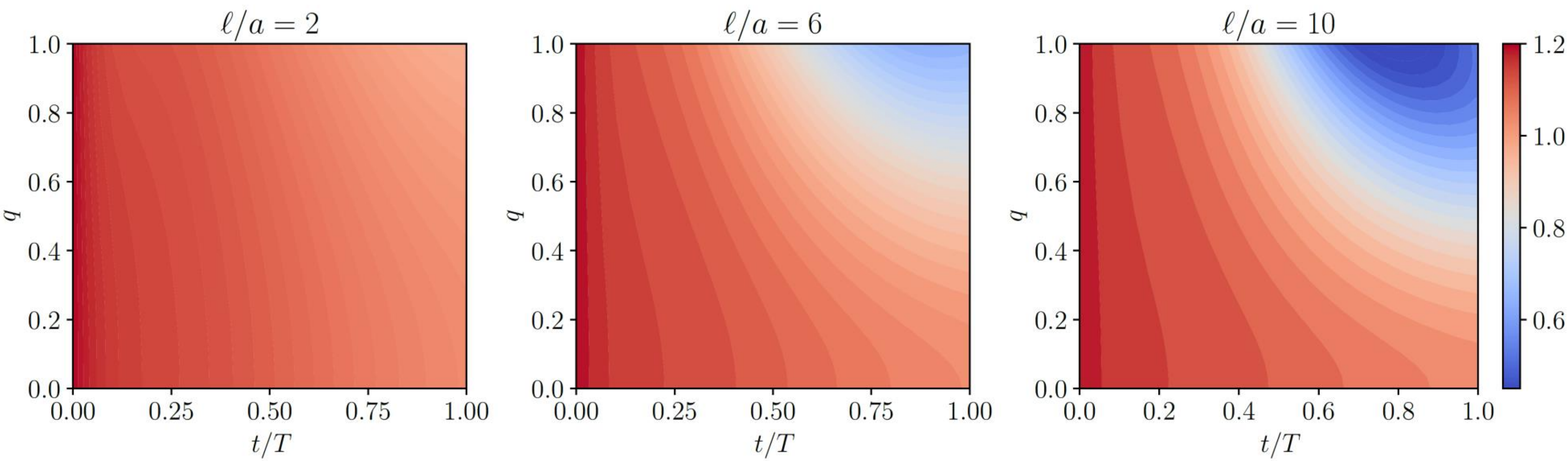


confinement by nontrivial dynamics!

Comment: density plots of energy gap

(known as "Tuna slice plot" inside the collaboration) [MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, q_p/q = 1, m/g = 0.15$



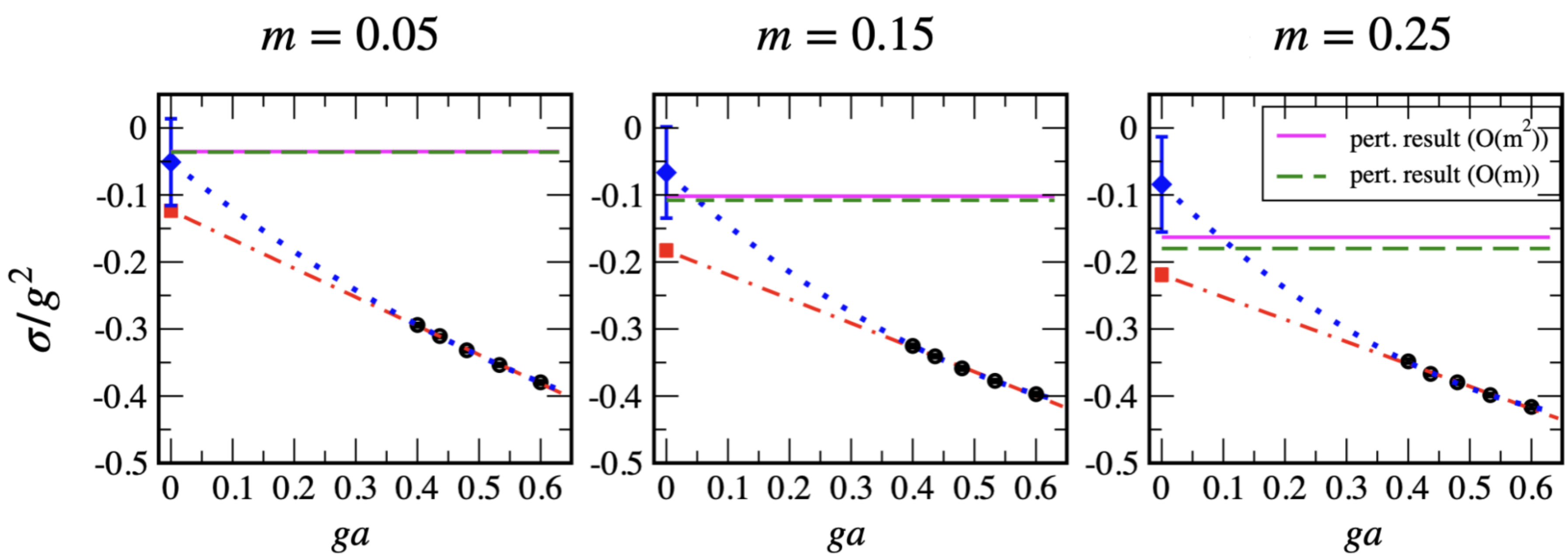
smaller gap for larger ℓ

→ larger systematic error for larger ℓ

Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

$g = 1,\ (\mathrm{Vol.}) = 9.6/g, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$

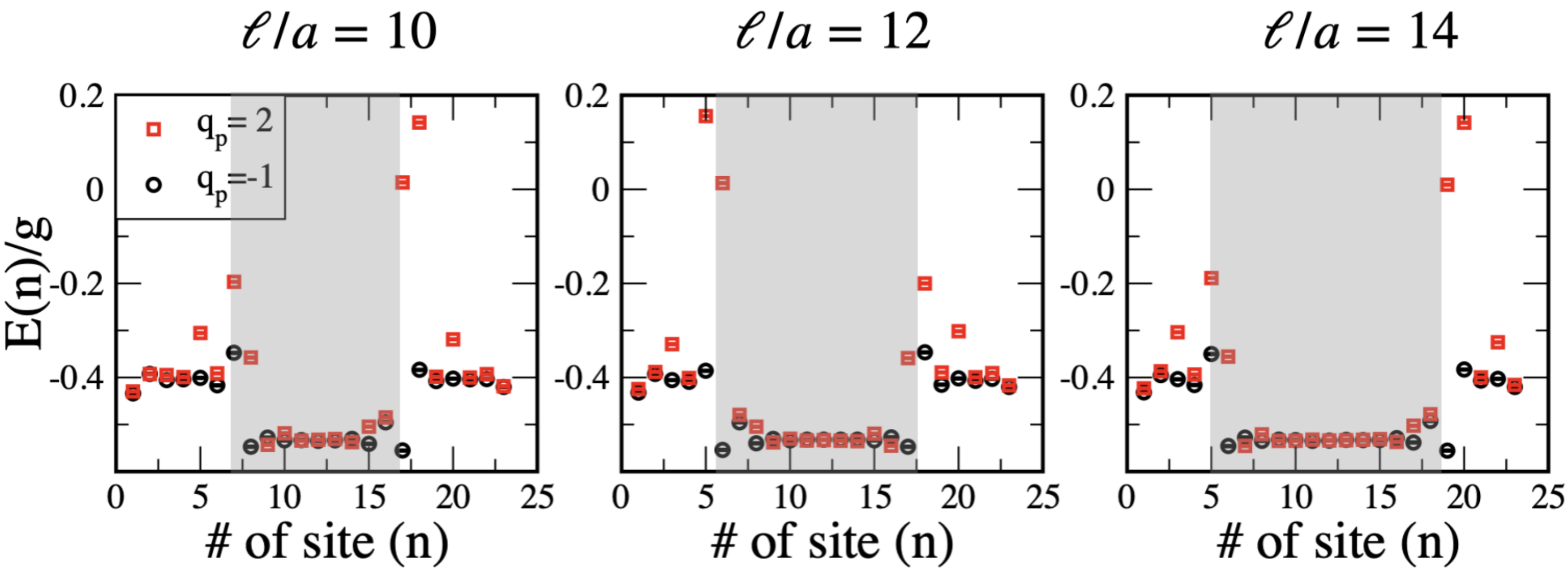


basically agrees with mass perturbation theory

Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki ’21]

$$g = 1, a = 0.4, N = 25, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$

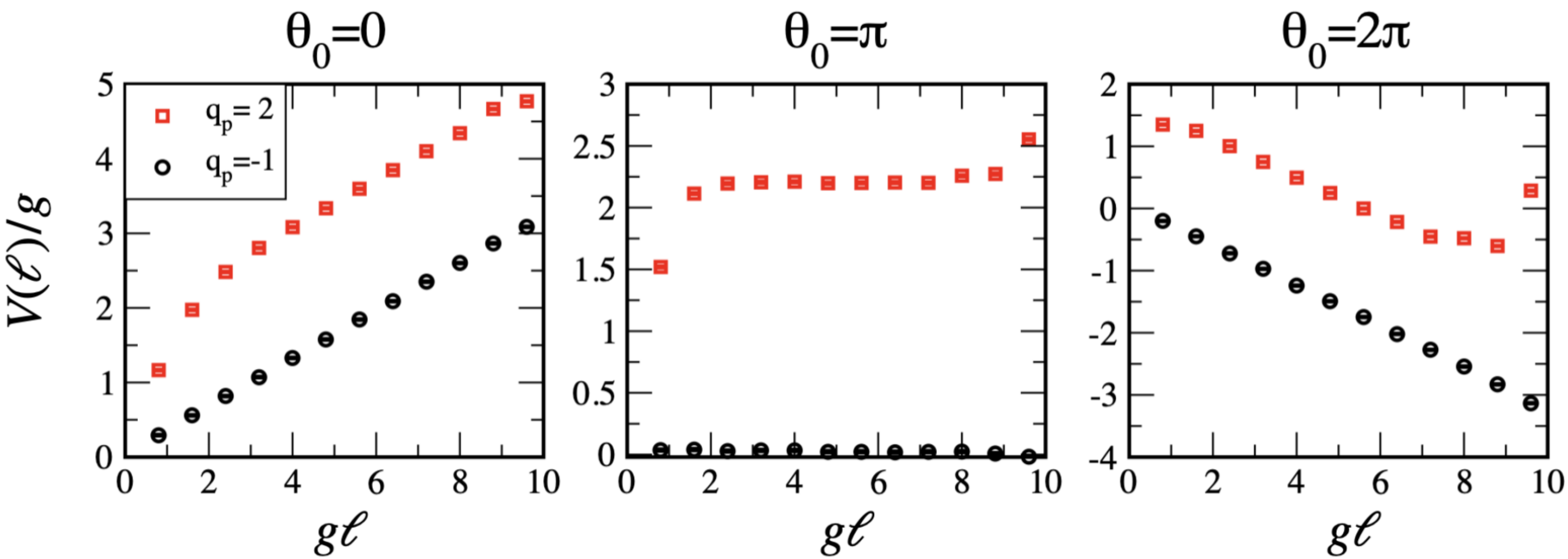


Lower energy inside the probes!!

Comparison of $q_p/q = -1/3$ & $q_p/q = 2/3$

[MH-Itou-Kikuchi-Tanizaki '21]

Parameters: $\mathrm{q} = 3, g = 1, a = 0.4, N = 25, T = 99, m = 0.15$



Similar slopes → (approximate) Z_3 symmetry

Adiabatic scheduling

[MH-Itou-Kikuchi-Tanizaki ’21]

$$N = 17, ga = 0.40, m = 0.20, q_p = 2, \theta_0 = 2\pi,$$