有限密度QCD型理論のコンフォーマルバウンドの破れ

Etsuko Itou (YITP, Kyoto U./ iTHEMS, RIKEN)

Based on K.Iida and EI, PTEP 2022 (2022) 11, 111B01

基研研究会　素粒子物理学の進展２０２３，YITP Kyoto University, 2023/08/29

Conformal bound (Holography bound)

conjecture (A.Cherman et al., 2009)

maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories

A bound on the speed of sound from holography

Aleksey Cherman[*] and Thomas D. Cohen[†]
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University of Maryland, College Park, MD 20742-4111

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Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

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for a broad class of 4-dim. theories

A bound on the speed of sound from holography

Aleksey Cherman* and Thomas D. Cohen†
Center for Fundamental Physics, Department of Physics

We found a strong evidence of $c_s^2/c^2 > 1/3$ in finite density

Nf=2 2color QCD using Lattice Monte Carlo

the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

Why QCD at finite density?

Introduction

May, 2023 @ U. of Minnesota

Finite density QCD

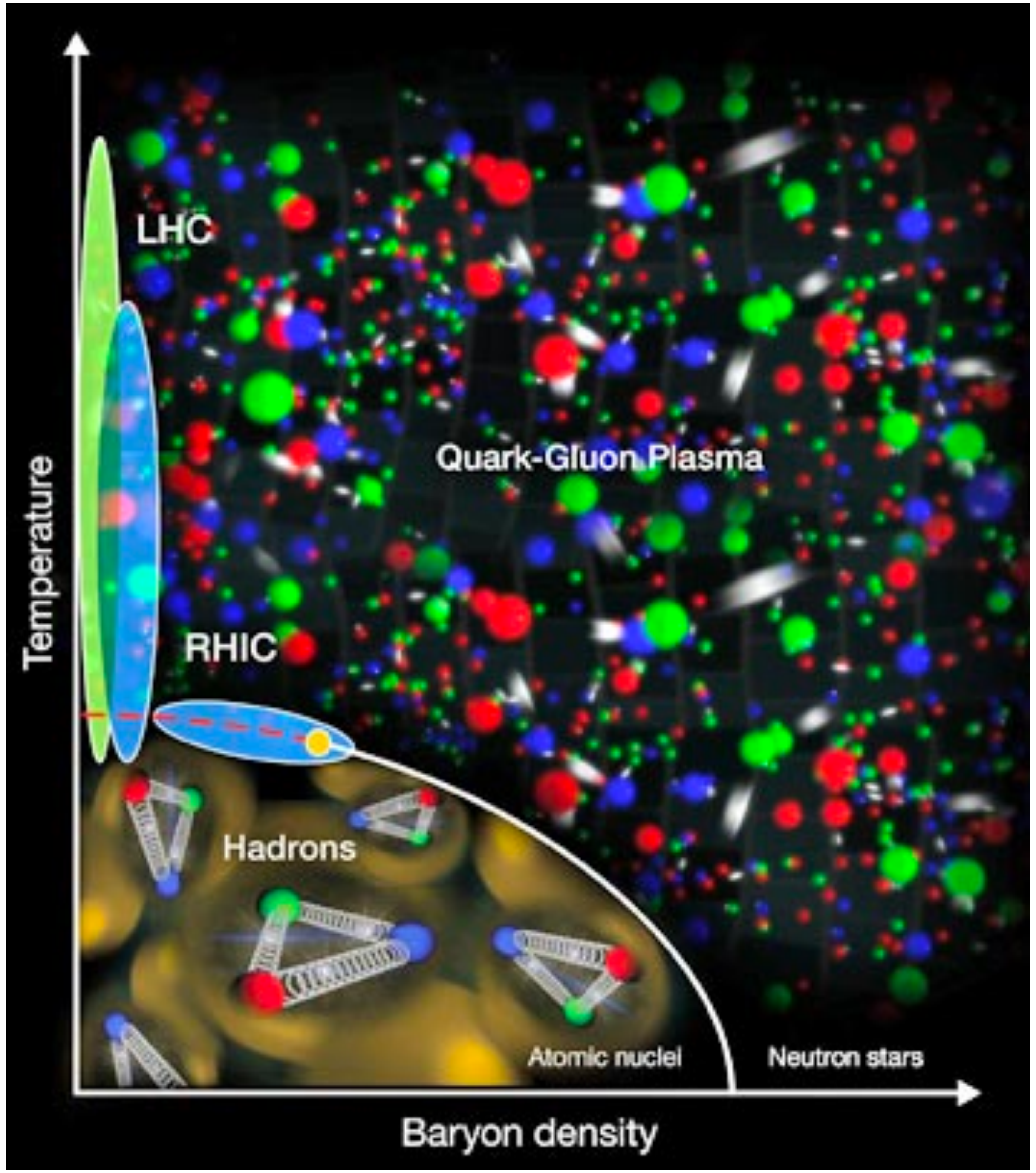
$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- At $\mu = 0$, QCD phenomena has been well understood for this 50 years
- We know almost nothing about QCD at finite density

Introduction

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

Finite density QCD

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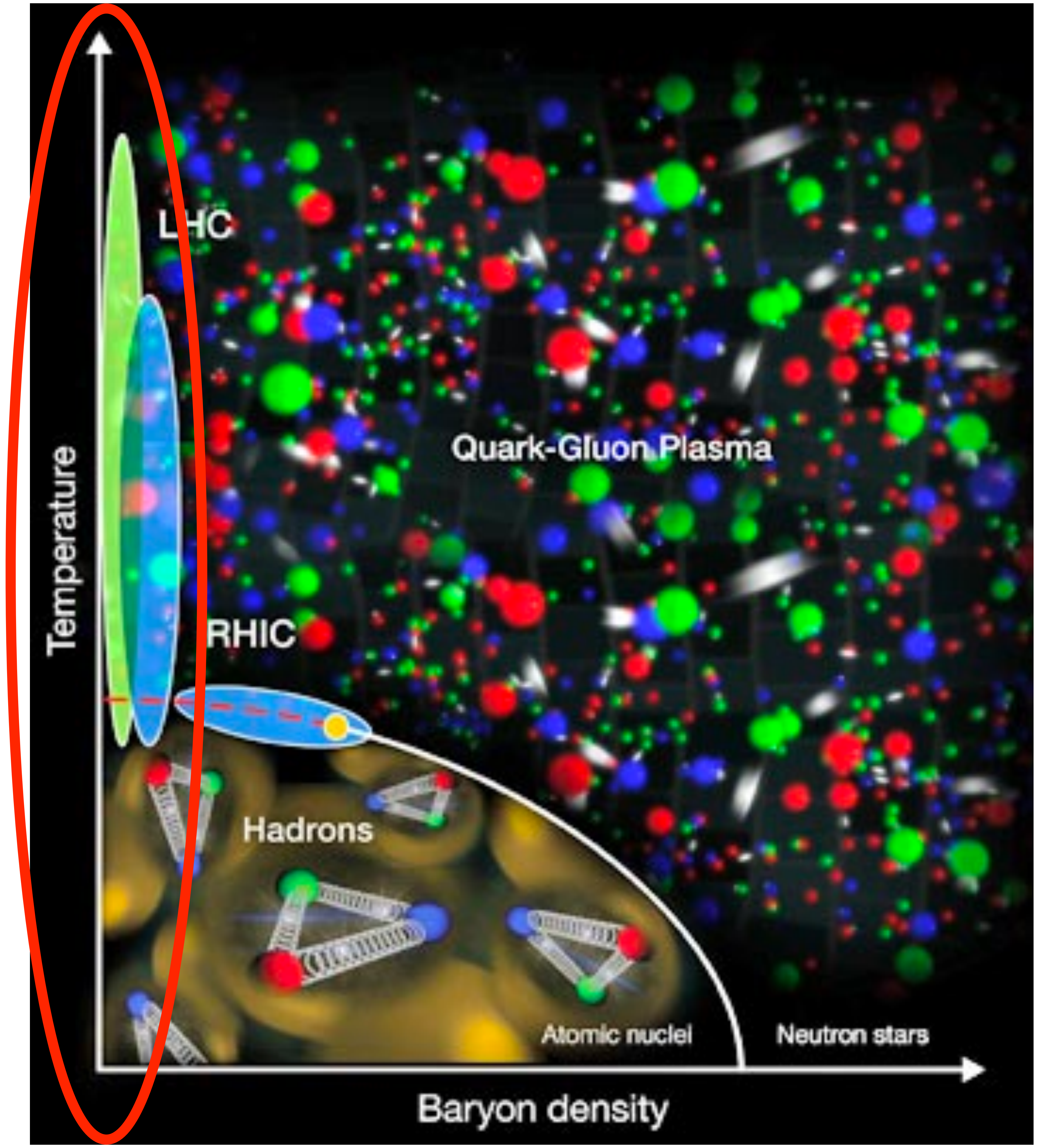
- Lattice gauge theory is only known nonperturbative and gauge invariant regularization method

Introduction

expected QCD phase diagram



T

$\propto \mu$ ©BNL/RHIC

Finite density QCD

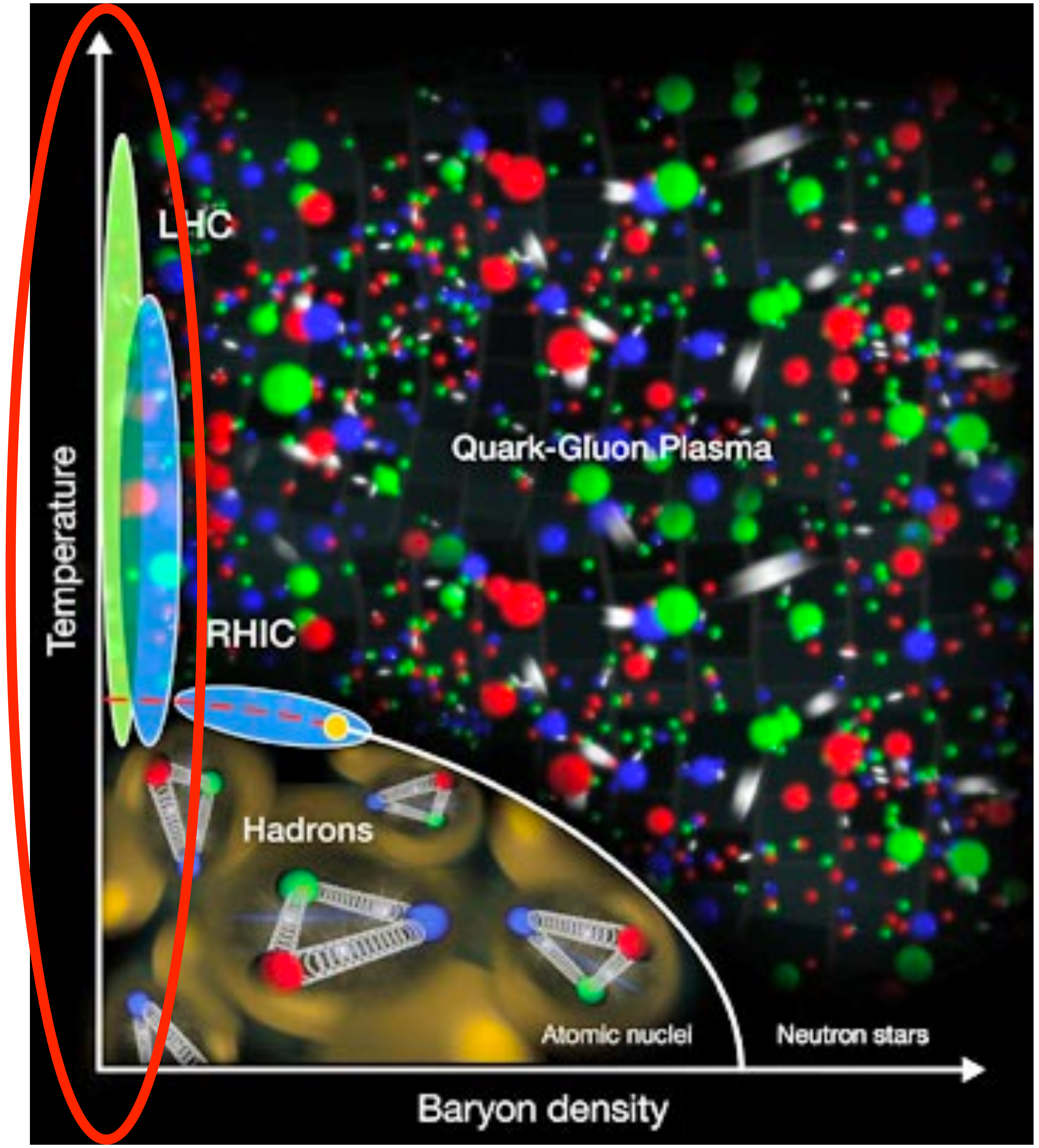
$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- Lattice gauge theory is only known nonperturbative and gauge invariant regularization method
- Finite-T QCD at $\mu = 0$ axis:

  studied by lattice MC and collider experiments

Sound velocity: finite-T transition

EoS and sound velocity at zero-μ



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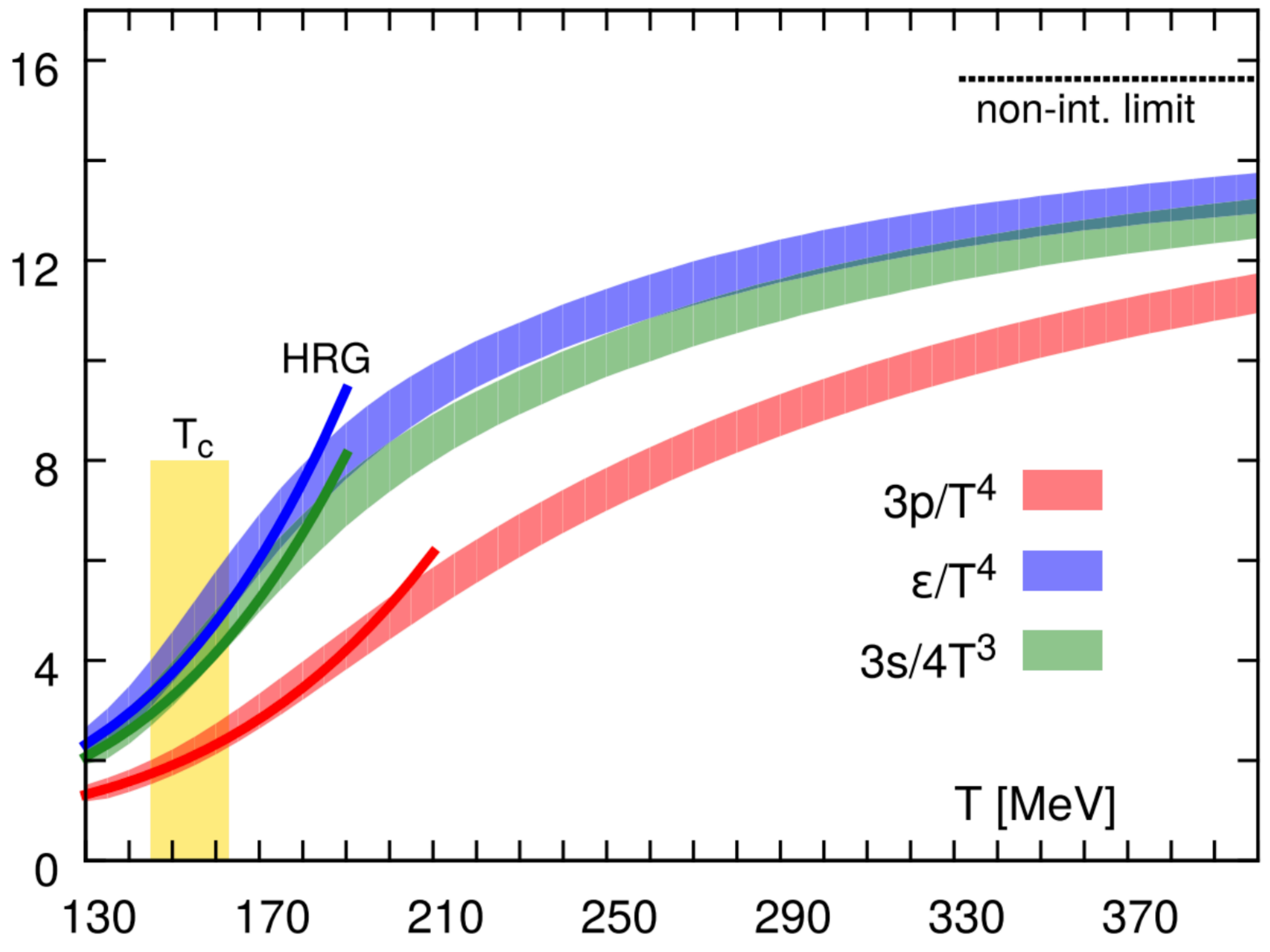
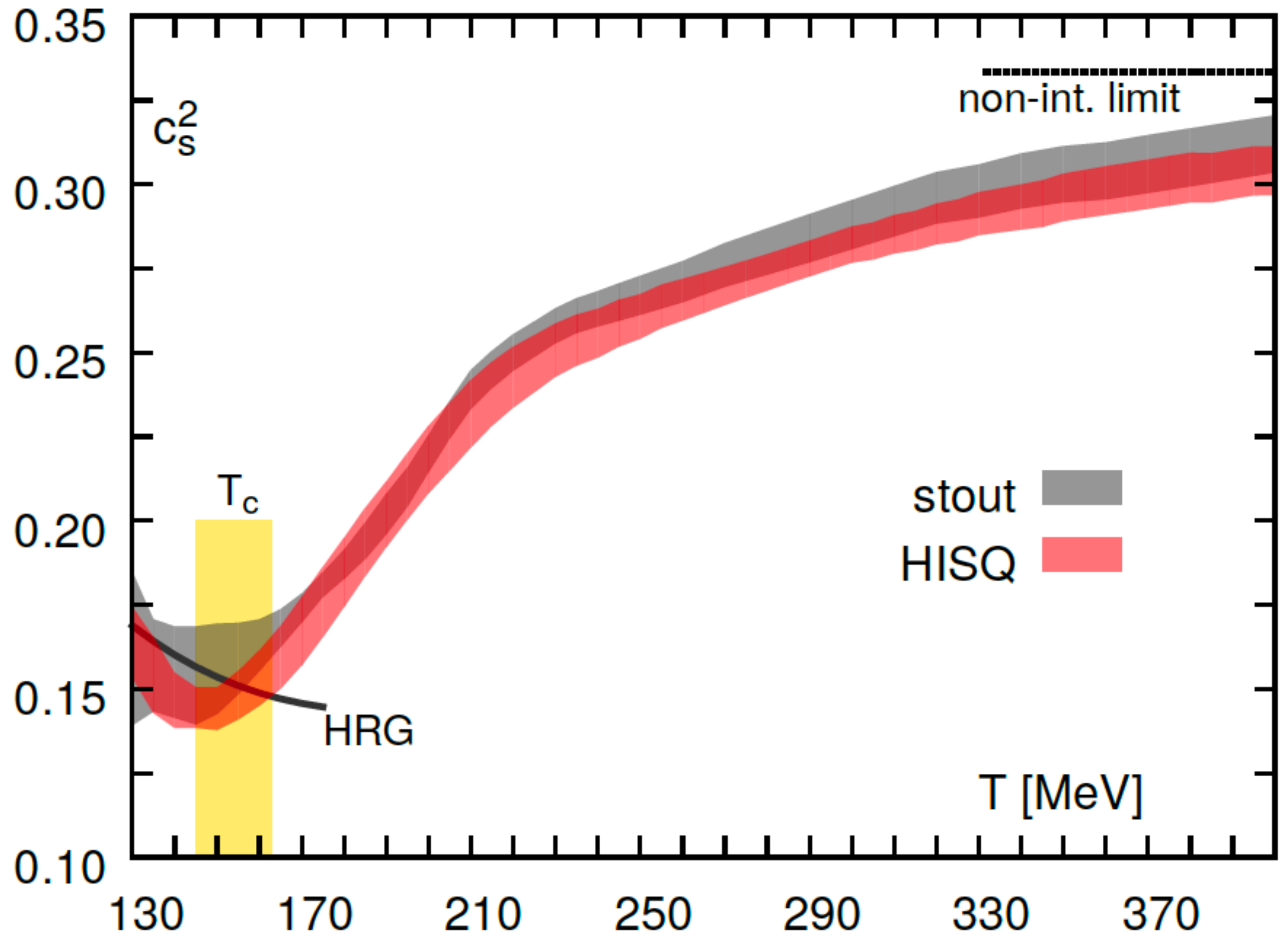
Finite Temperature transition
(Nf=2+1 QCD)

EoS
(p and ε)



Sound velocity
$c_s^2 = \partial p / \partial \epsilon$

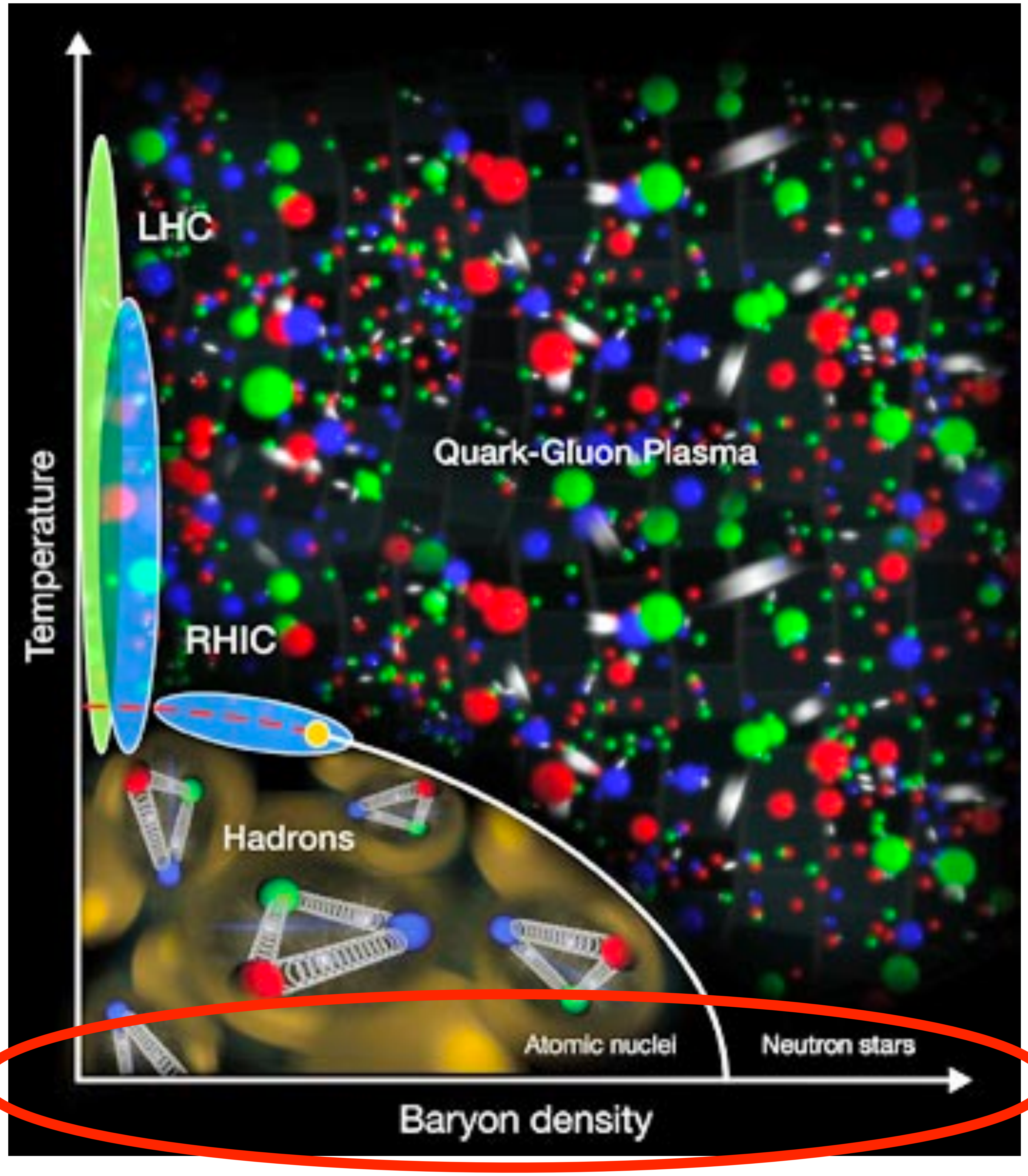


HotQCD (2014)

Introduction

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

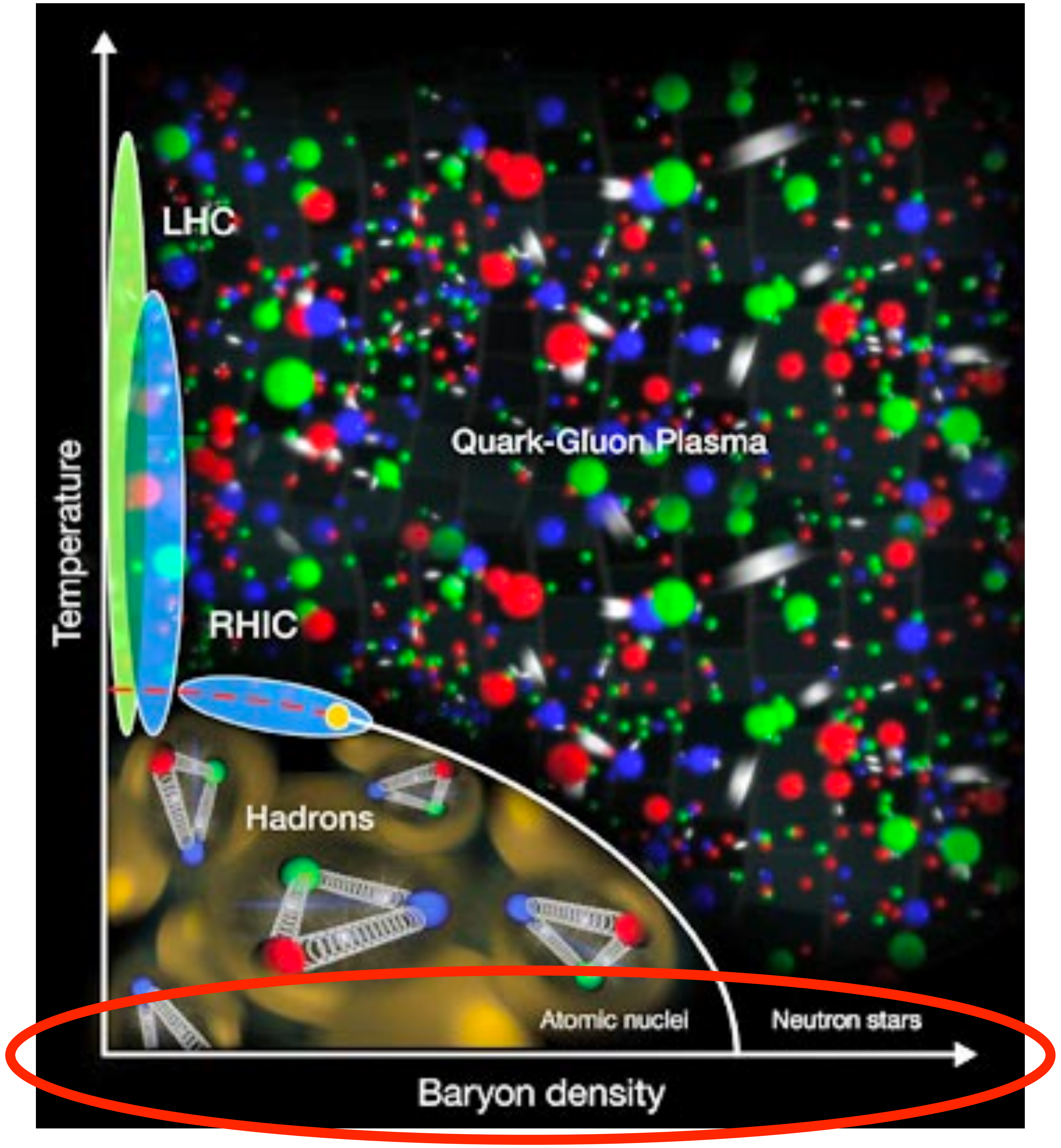
- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem
  (理論を変えるか,アルゴリズムを変えるか)

  永田桂太郎：「有限密度格子QCDと符号問題の現状と課題」
  素粒子論研究Vol.31(2020) No.1
  Prog.Part.Nucl.Phys. 127 (2022) 103991・e-Print: 2108.12423 [hep-lat]

- Experiments:
  Neutron star observations are (will be) ongoing
  Gravitational wave, LISA, NICER,...

Sound velocity: finite density regime

EoS and sound velocity at low-T and high-μ



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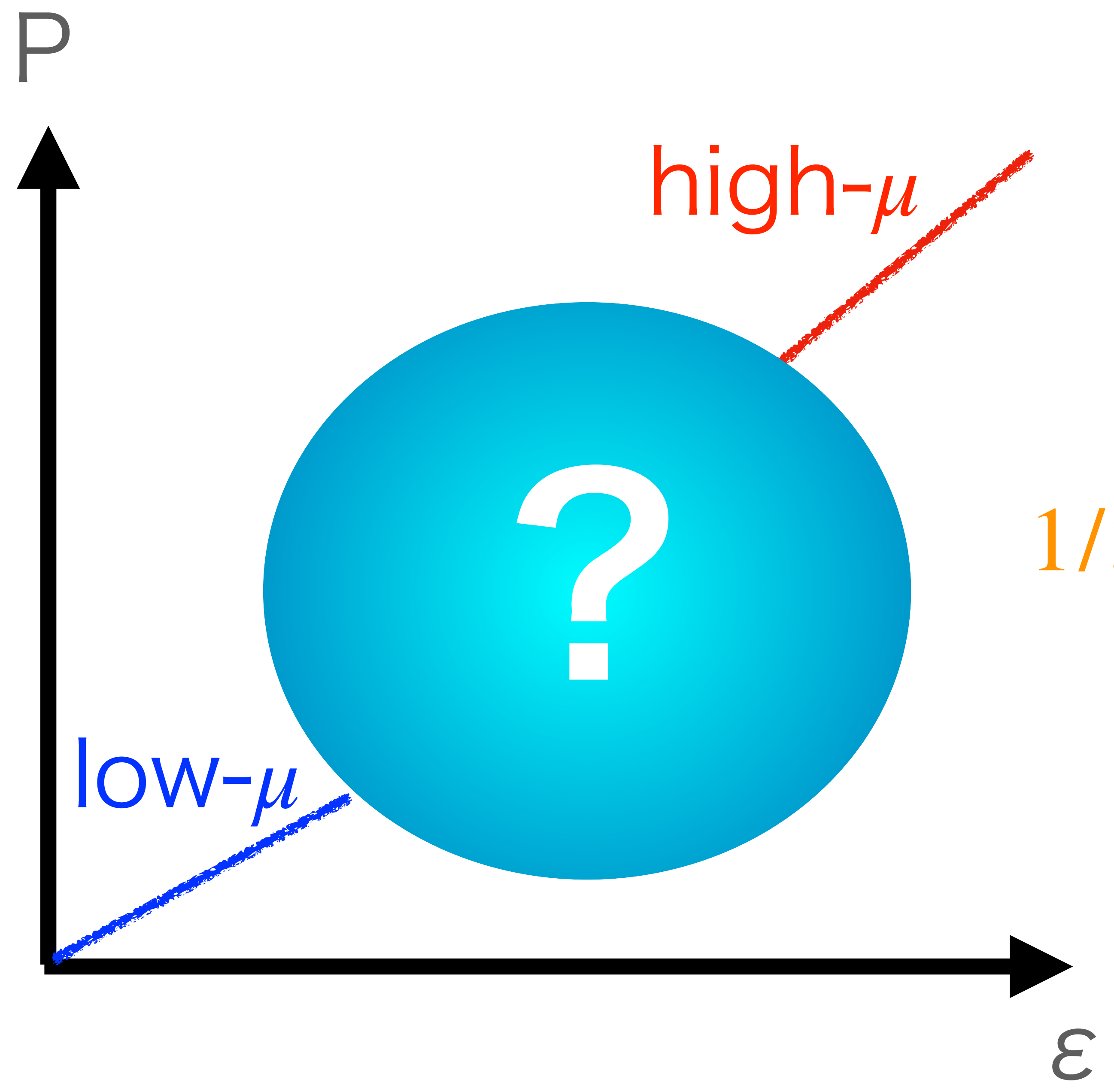
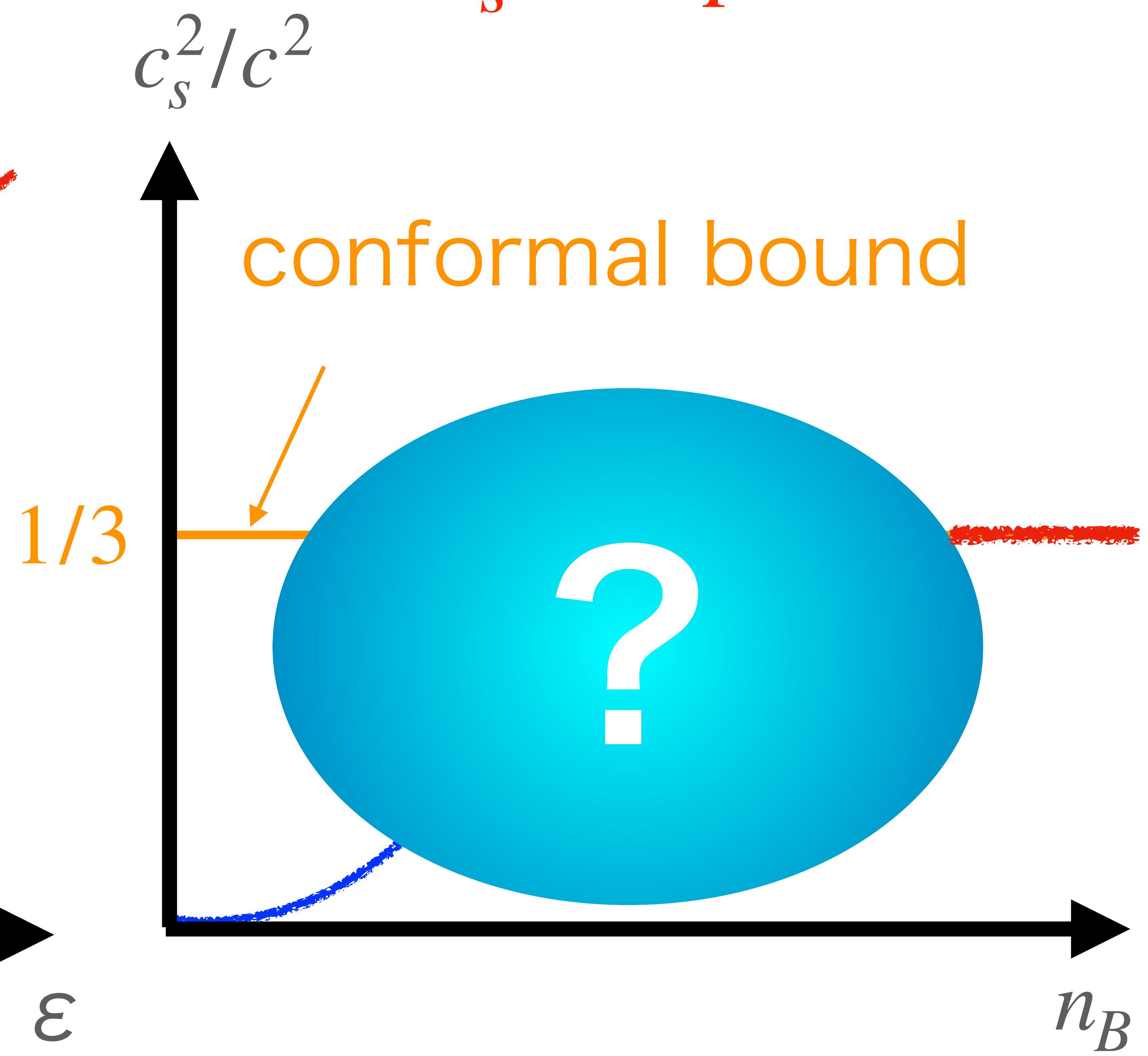
EoS

$p(\mu)$ vs $\varepsilon(\mu)$

Sound velocity

$c_s^2 = \partial p / \partial \epsilon$





low -μ ($n_B \lesssim 2n_0$): Hadronic matter

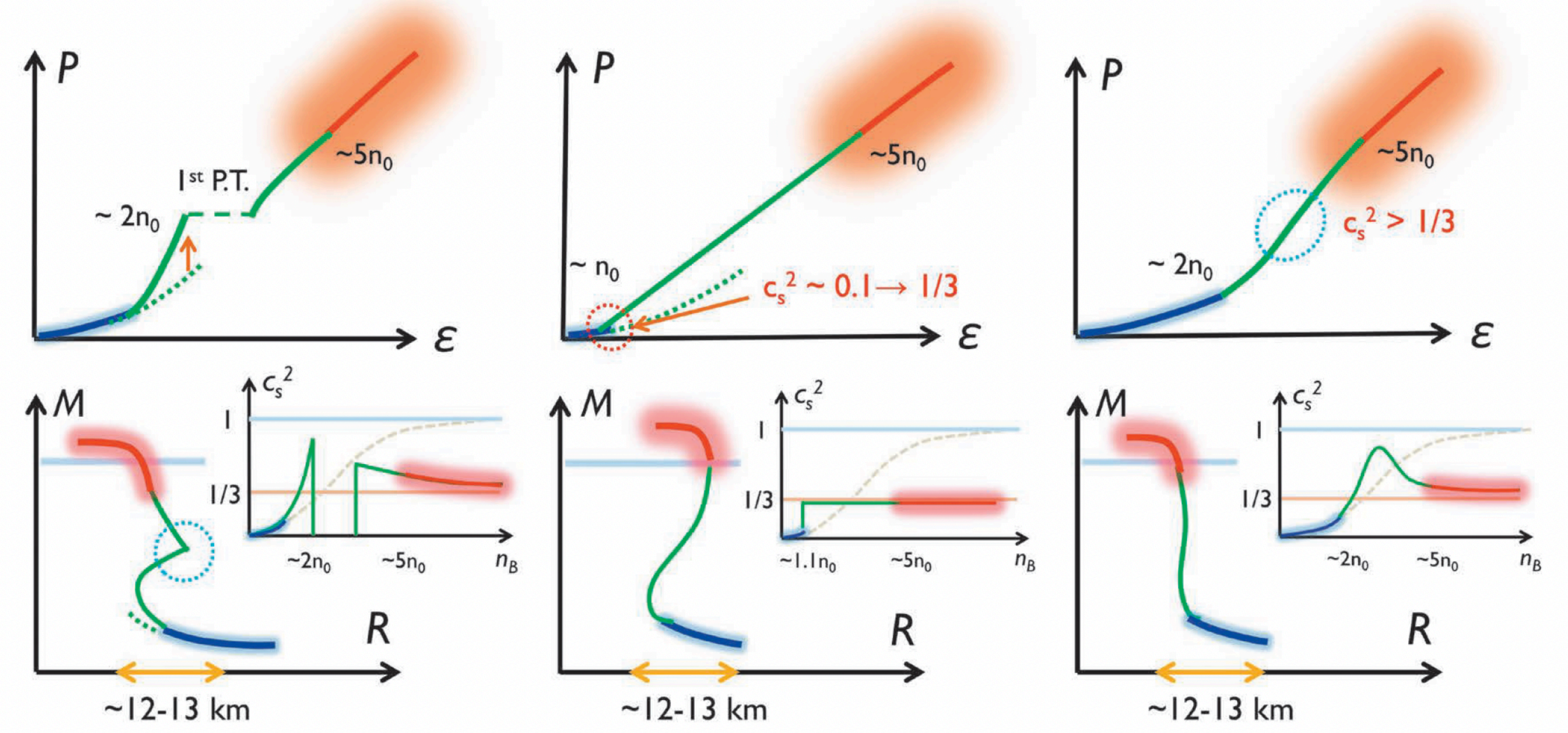
high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

EoS (ε vs. p), c_s and neutron star

Mass and radius of neutron star

T. Kojo, arXiv:2011.10940
物理学会誌2022年2月



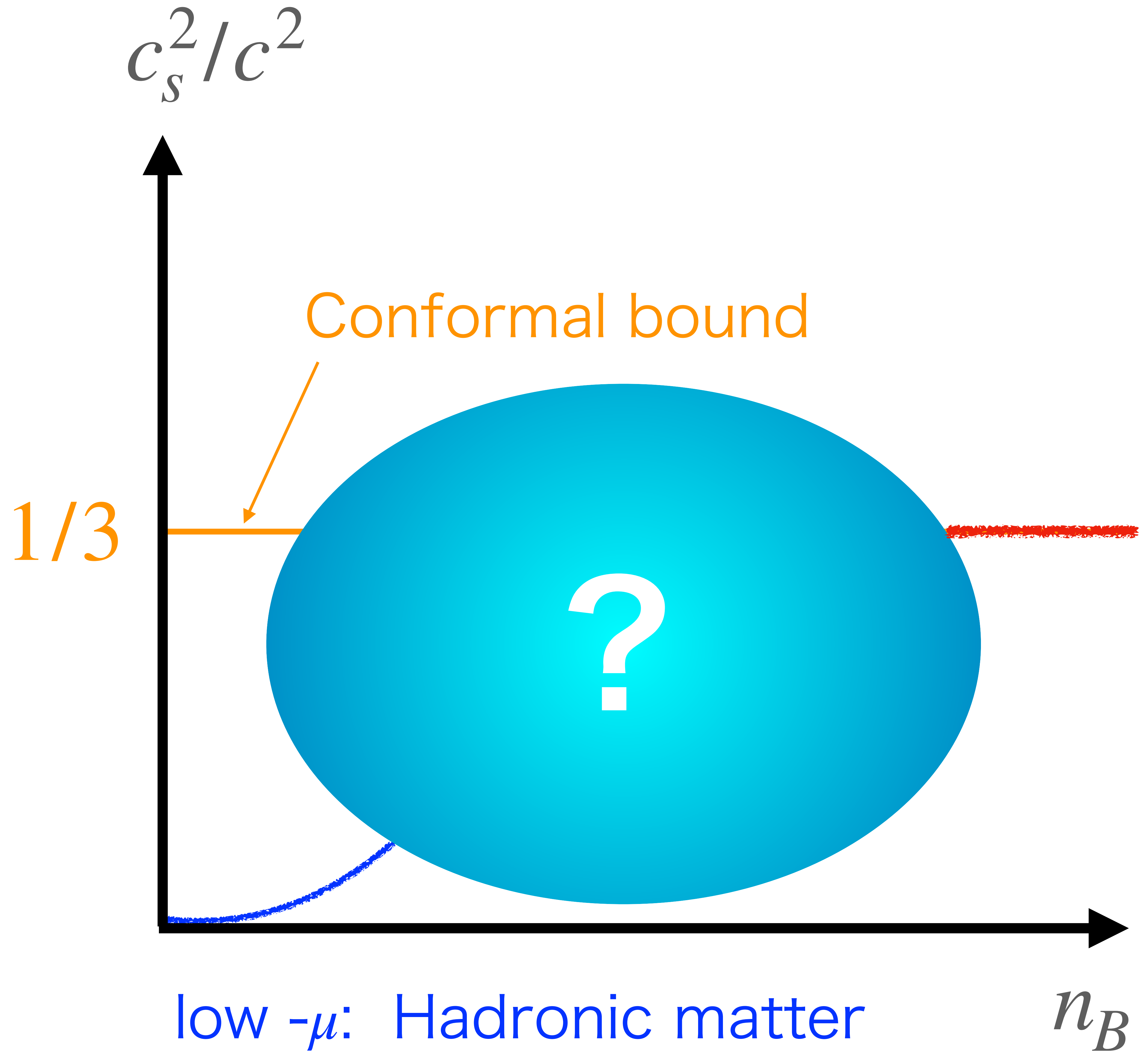
Sound velocity

$$c_s^2 = \partial p / \partial \epsilon$$

Mass-Radius of neutron star <=> EoS in dense QCD

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

  Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD the sign problem is absent!!

Two problems at low-T high-μ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)

➡ Reduce the color dof, 2color QCD
quarks becomes pseudo-real reps.
The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-μ (e.g. $\mu_q > m_\pi/2, \ \ m_N/3$),

It comes from the phase transition to superfluid phase(SSB of baryon sym.)

Kogut et al. NPB642 (2002)18

➡ Add an explicit breaking term of the sym., then take $j \to 0$ limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

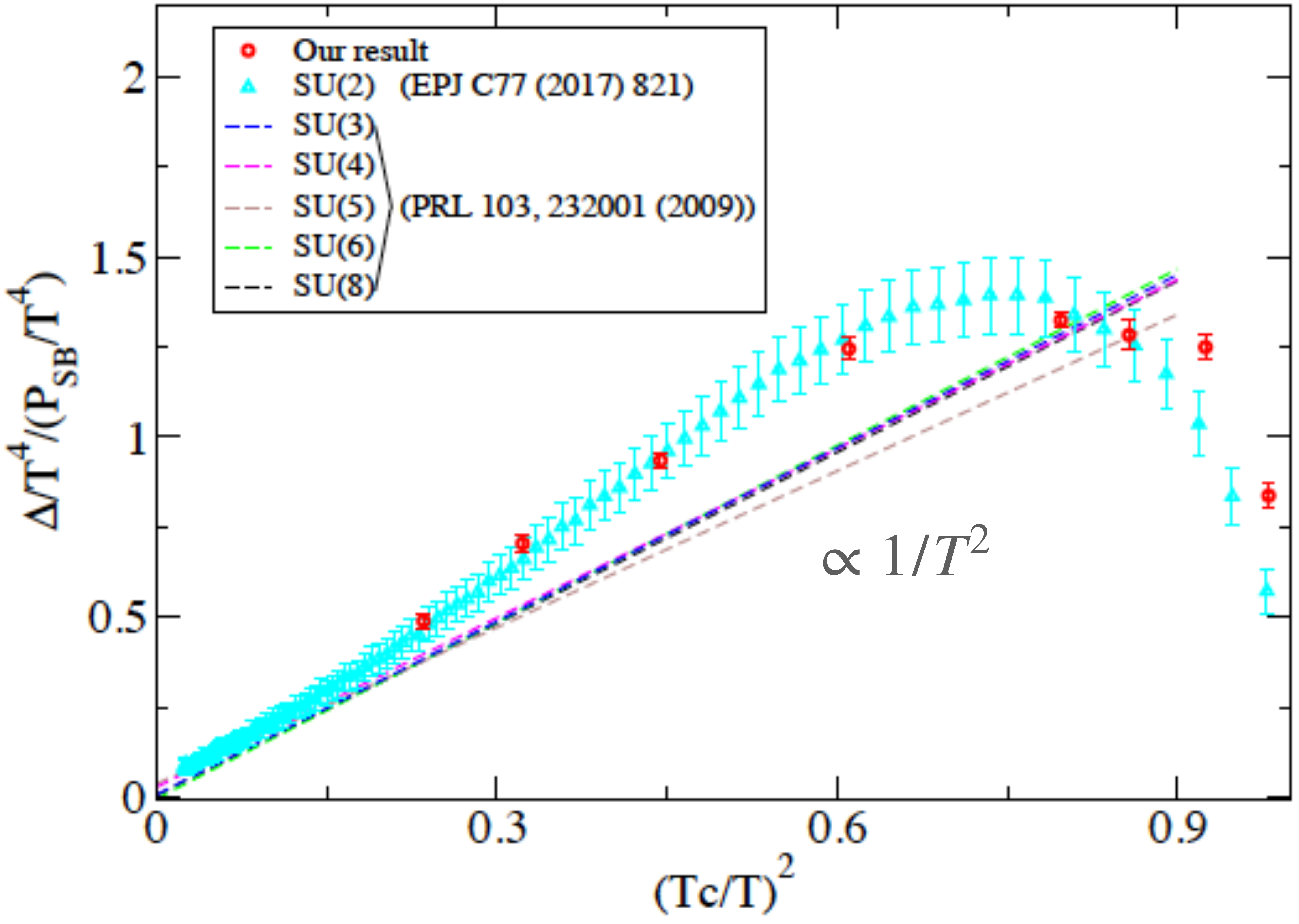
HMC simulations for whole T-μ regime are doable!

(j->0 extrapolation is taken in all plots today)

2color QCD ≈ 3color QCD at $\mu = 0$

EoS shows very similar at least quenched QCD case

Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc) gauge theories with several Nc



➡

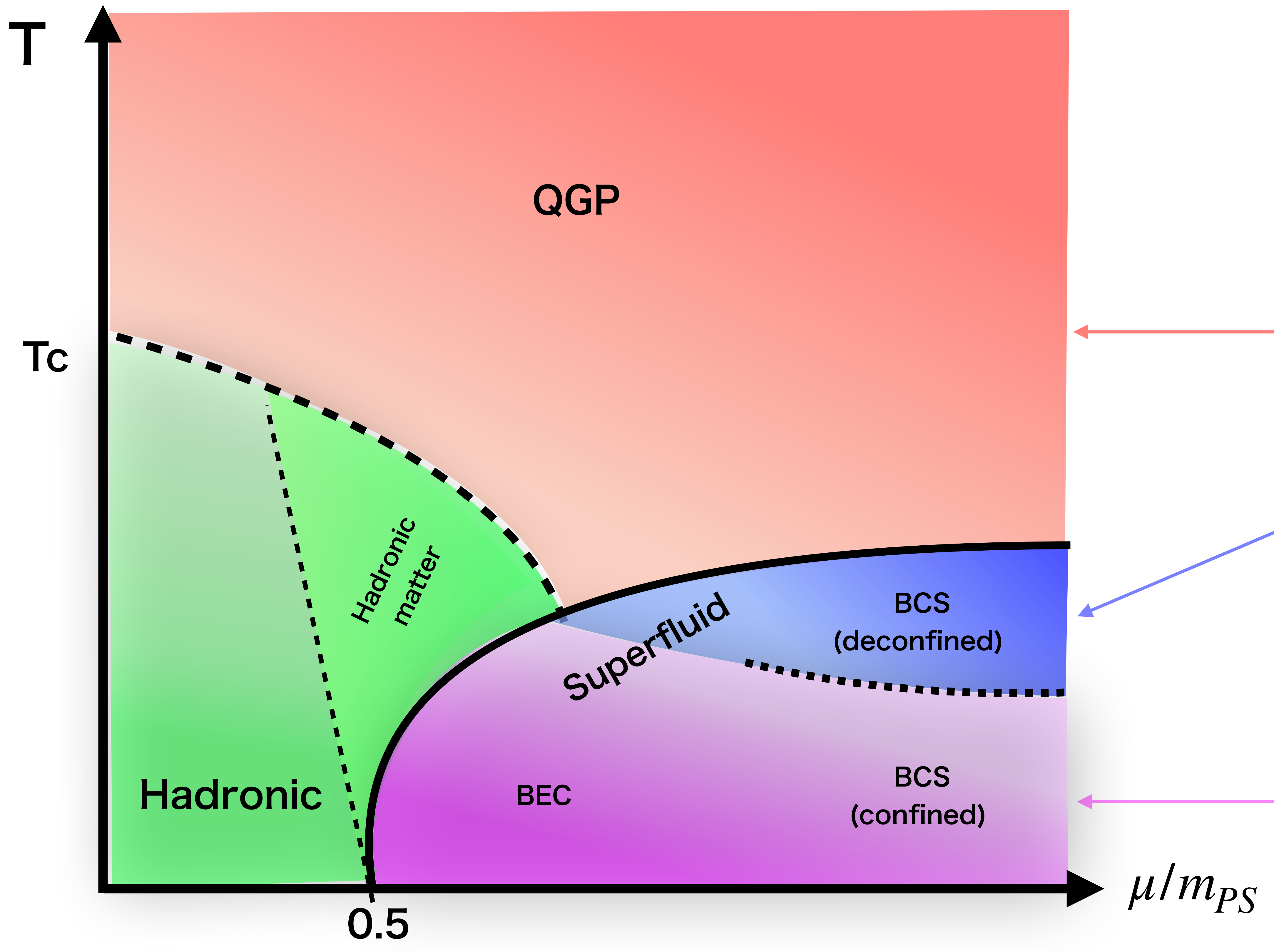
Find qualitative property of real dense 3color QCD from dense 2color QCD

T. Hirakida, EI, H. Kouno
PTEP 2019 (2019) 033B01

2color QCD phase diagram

(1) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(2) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(3) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(4) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Current status on 2color QCD phase diagram



At least three independent group studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

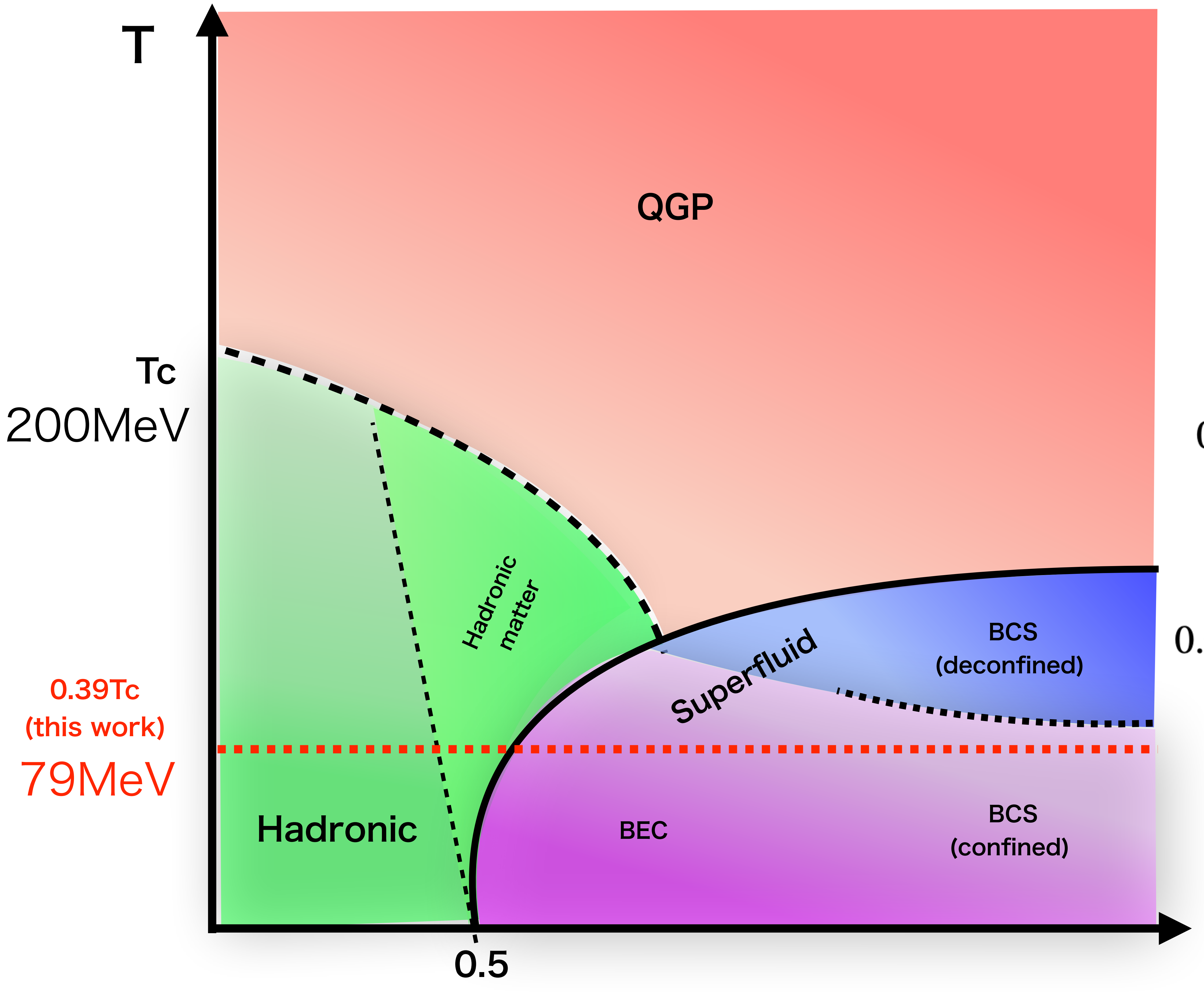
- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

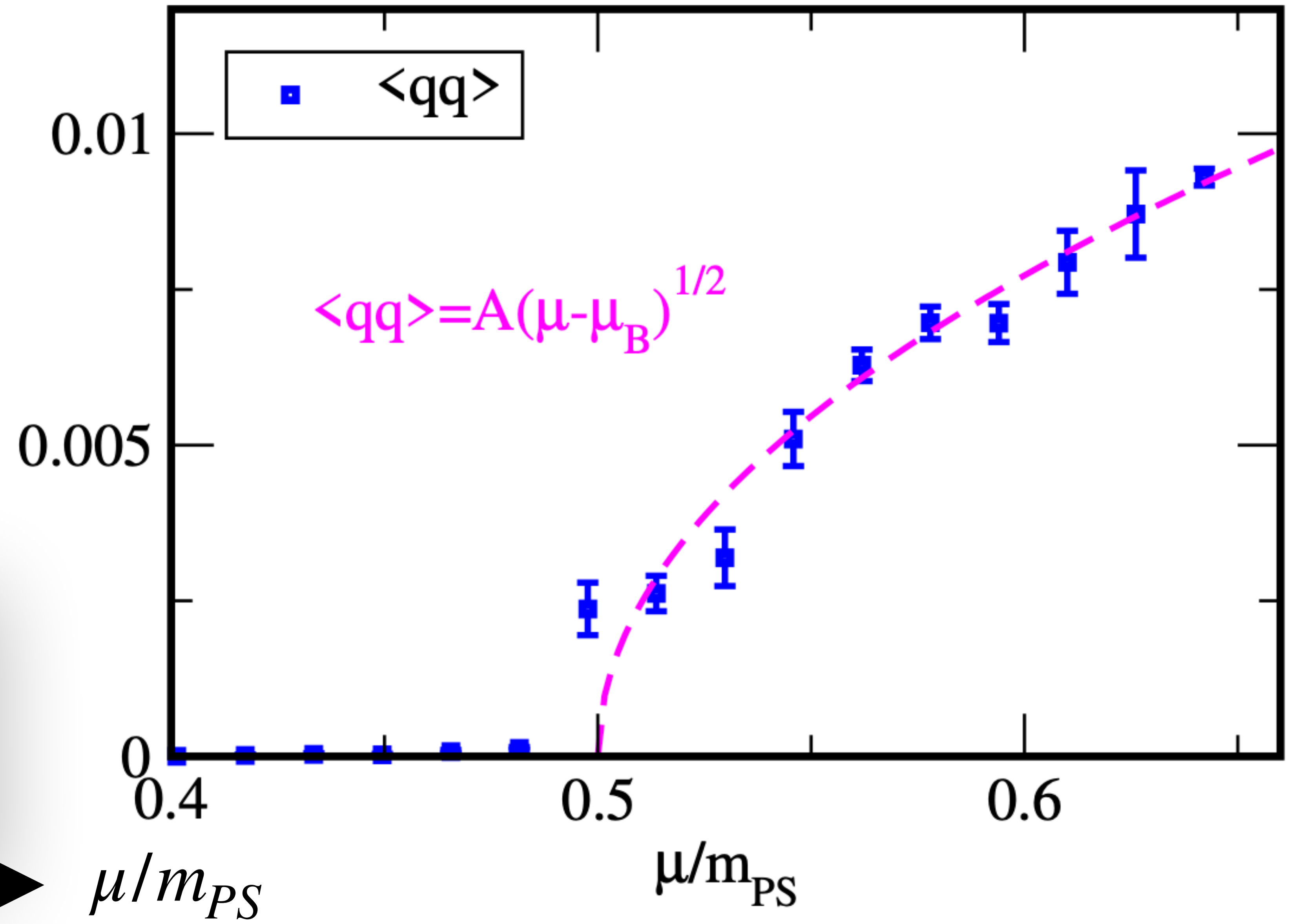
- 2color QCD phase diagram has been determined by independent works!

Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\text{tree}} \approx 1$

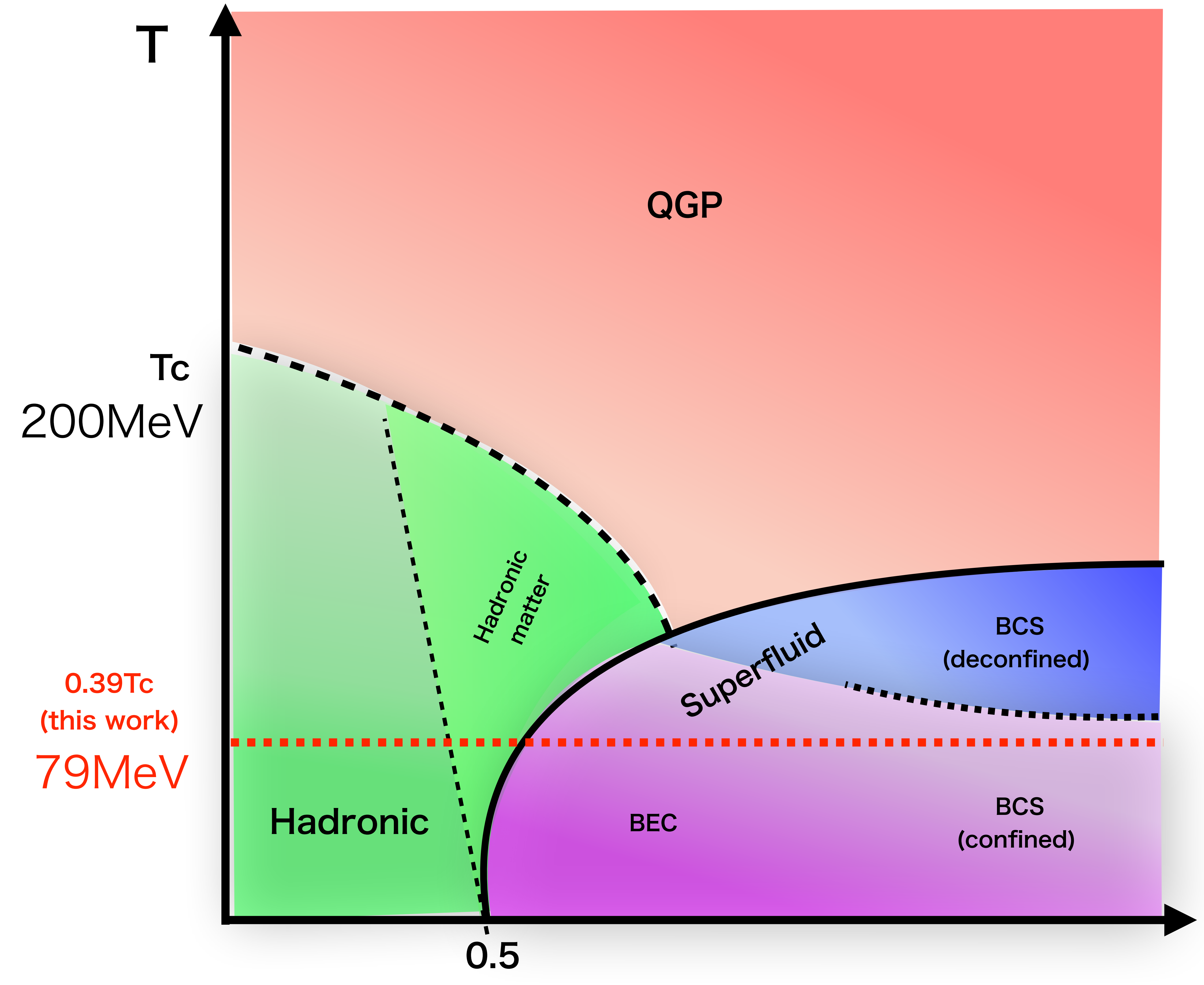


Scaling law of order param. is consistent with ChPT. (good analysis for $\mu \approx \mu_c$)

Kogut et al., NPB 582 (2000) 477

Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



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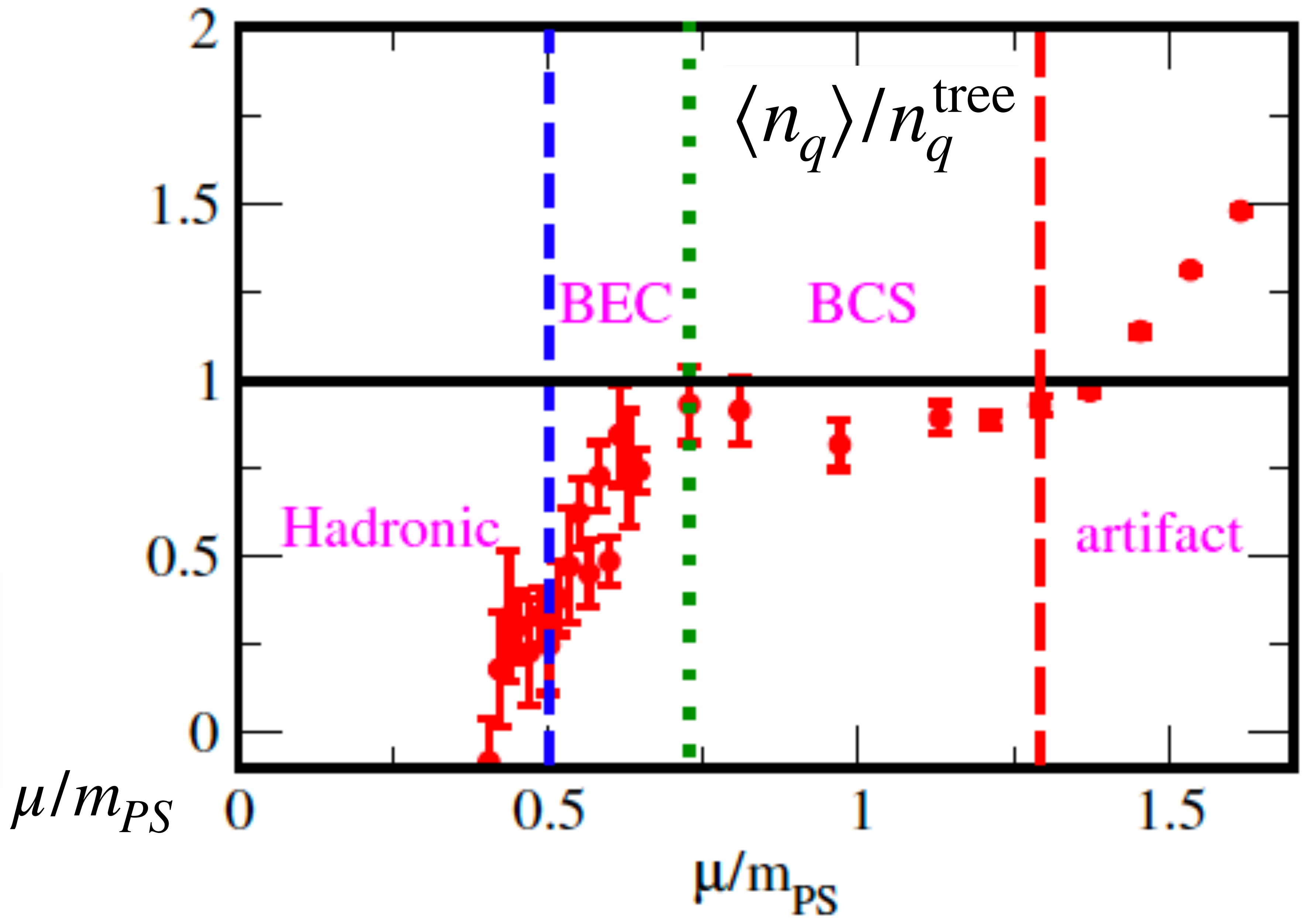

In high-μ, $\langle n_q \rangle \approx n_q^{\text{tree}}$

number density of free particle

BEC-BCS crossover

Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge)

  lattice size = 16^4

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle \right)$

  No renormalization for μ

  $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

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  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Technical steps

(1) Measure $\langle \cdot \rangle$ on the generated configuration

(2) **Nonperturbatively** calculate beta fn. at $\mu = 0$

(3) Numerical integration of n_q

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

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  Zero at $j \to 0$

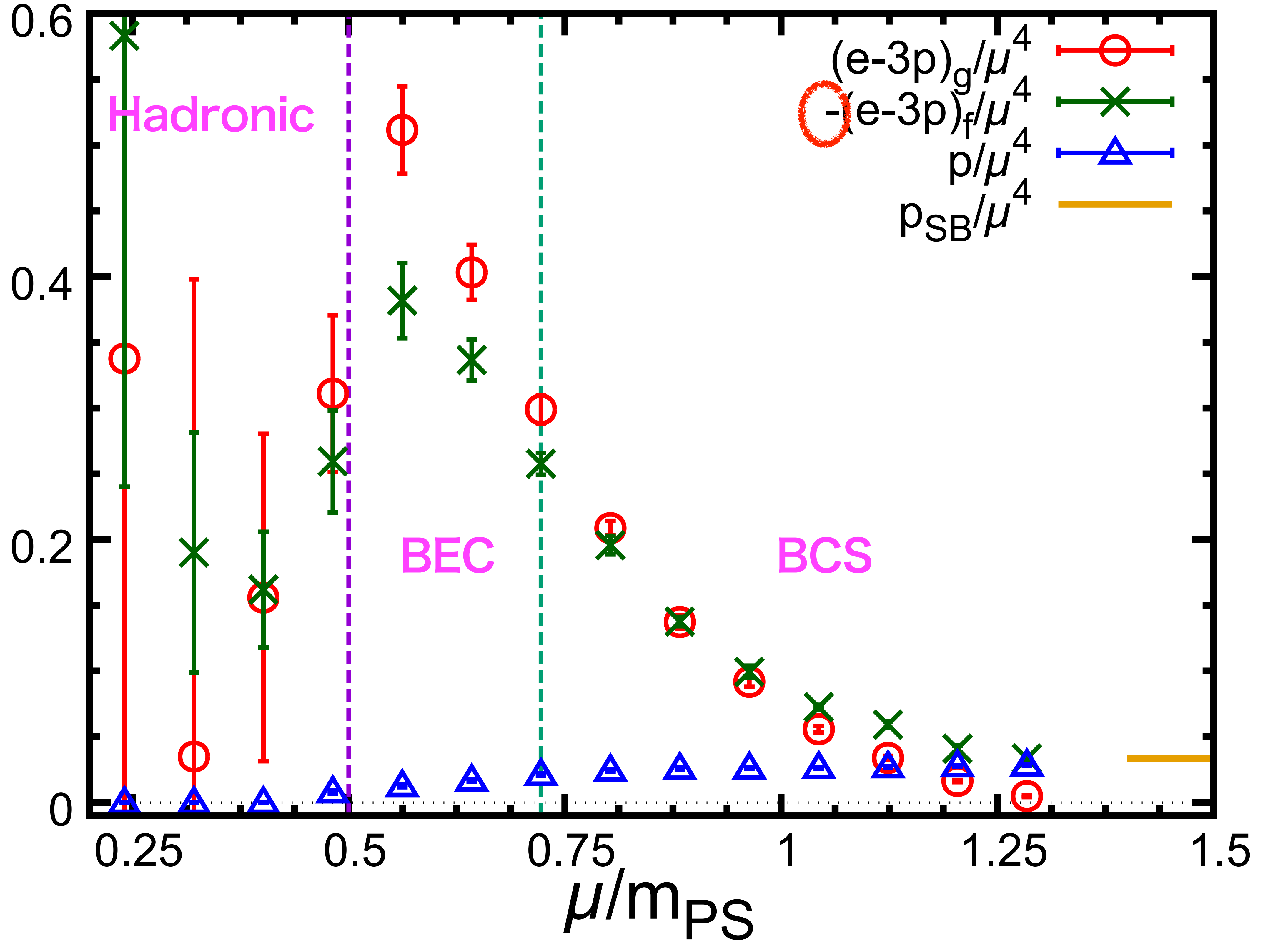
- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Nonperturbative beta-fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Trace anomaly and pressure



- Sum of trace anomaly, $(e-3p)_g + (e-3p)_f$

  zero in Hadronic phase

  positive in BEC phase

  positive -> negative in BCS phase

  Finally, fermions give the larger contribution

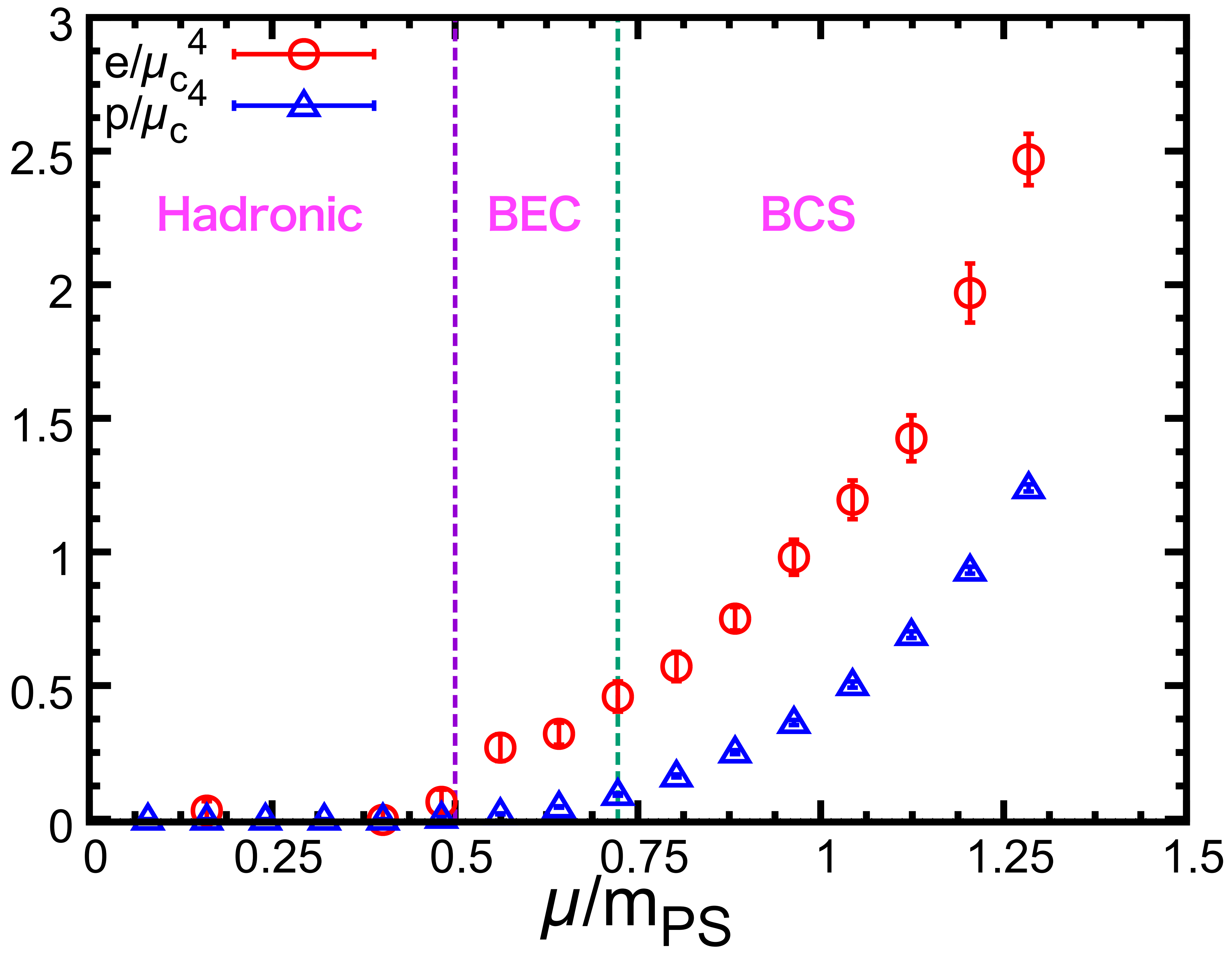
- Pressure increase monotonically

  In high density, it approaches

  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$

P and e as a function of μ

(Normalized by $1/\mu_c^4$ to be dim-less)

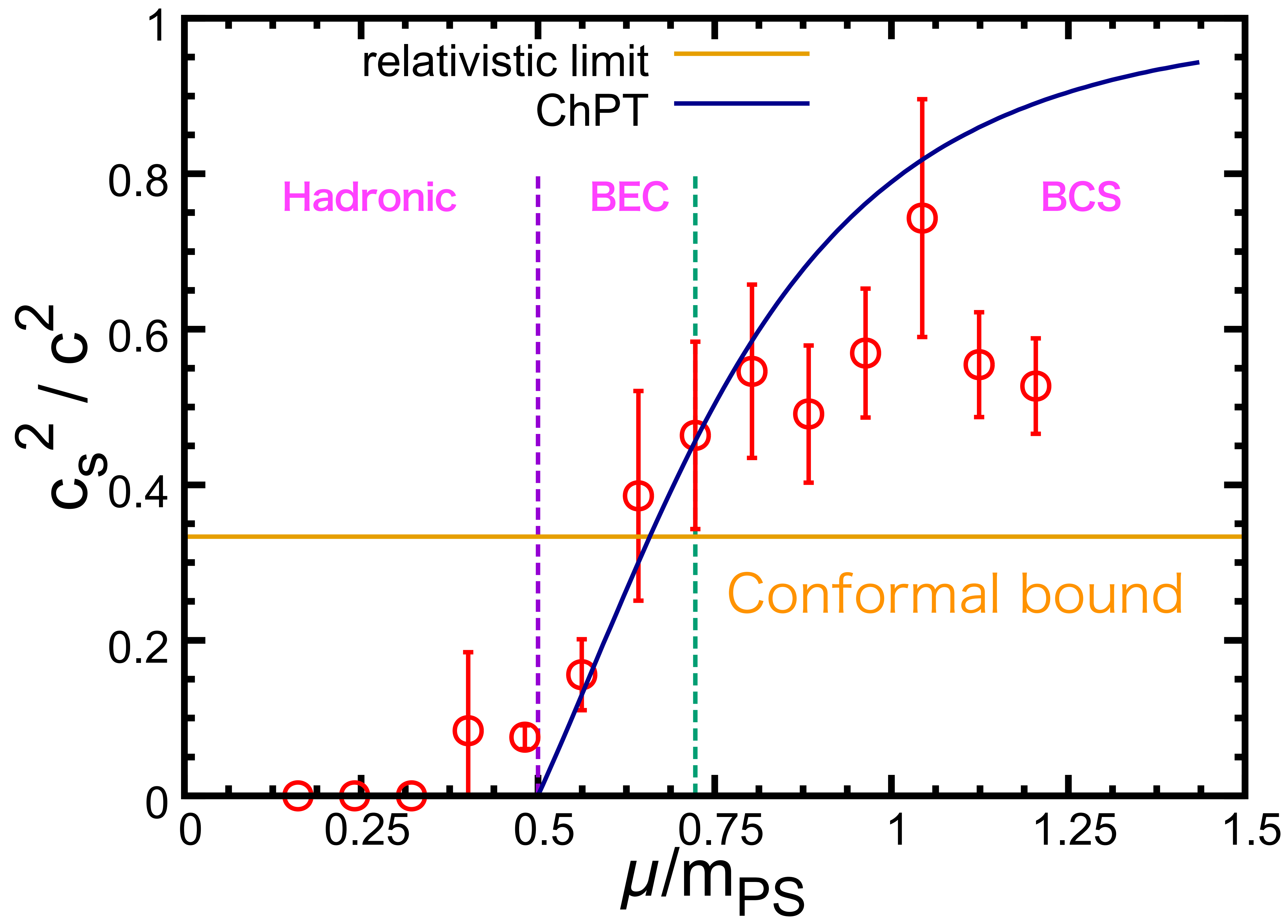


- P is zero in Hadronic phase since $n_q = 0$
- e is also zero in Hadronic phase by the cancelation between $(e-3p)_g$ and $(e-3p)_f$

From these data, the sound velocity is obtained

$$c_s^2/c^2 = \frac{\Delta p}{\Delta e} = \frac{p(\mu+\Delta\mu) - p(\mu-\Delta\mu)}{e(\mu+\Delta\mu) - e(\mu-\Delta\mu)}$$

Sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



Chiral Perturbation Theory (ChPT)

$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!!

Son and Stephanov (2001) : 3color QCD with isospin μ

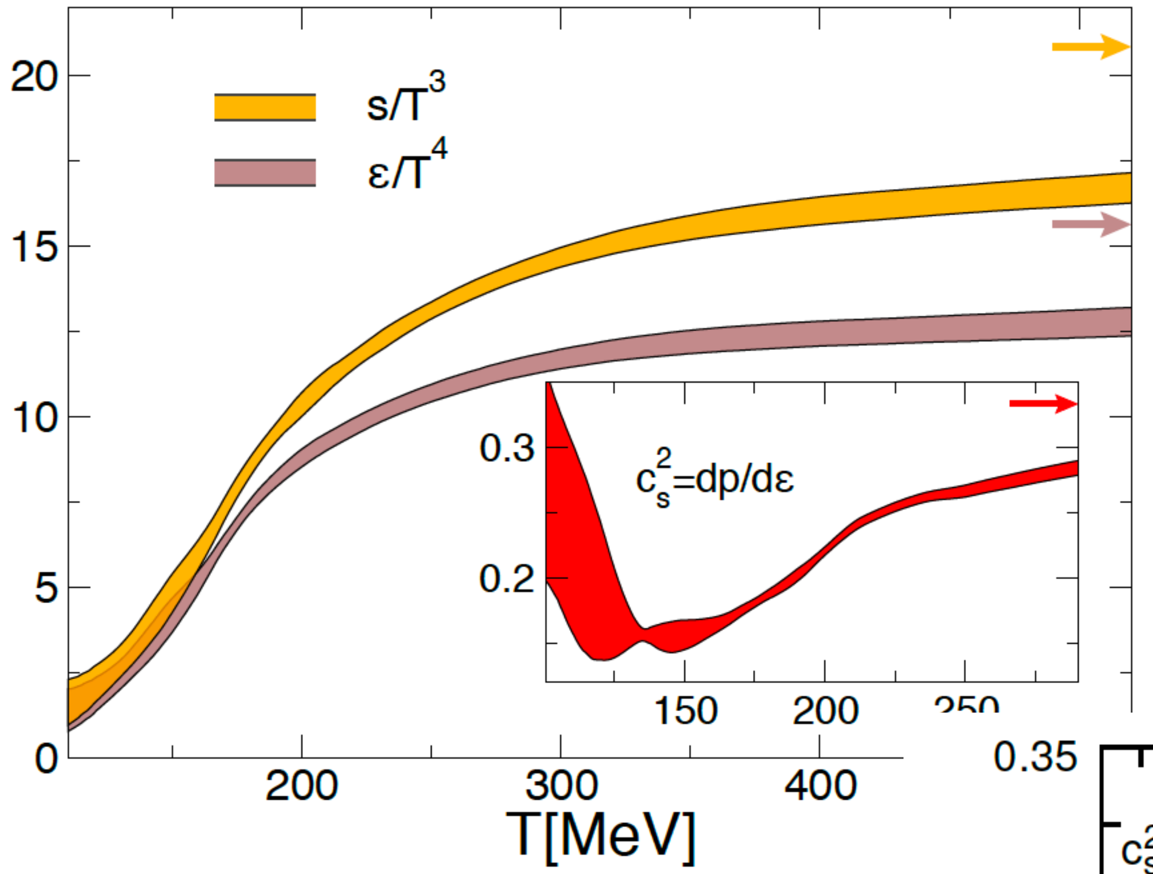
Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- In BEC phase, our result is consistent with ChPT.
- c_s^2/c^2 exceeds the relativistic limit
- In high-density, it peaks around $\mu \approx m_{PS}$.

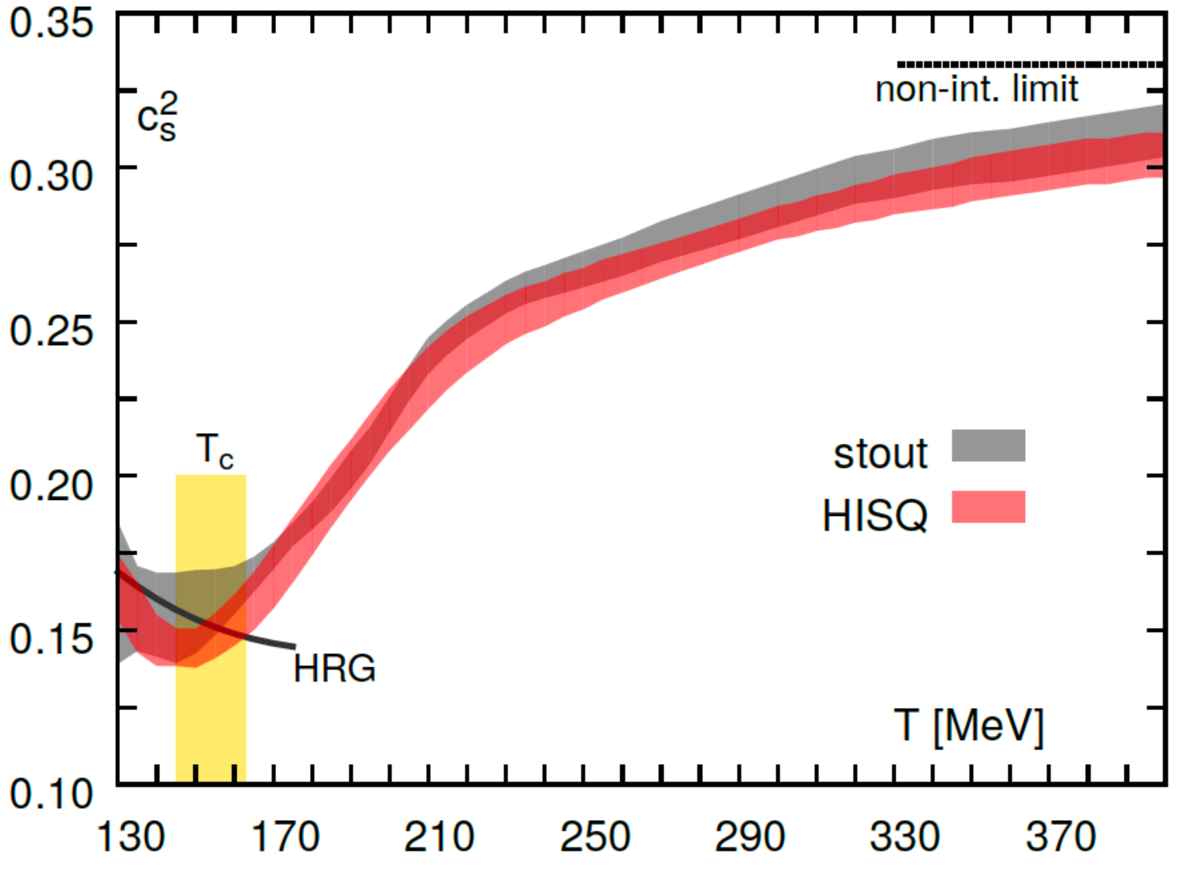
"Stiffen" and then "soften" picture as density increases

Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)



Borsanyi et al. (2013)

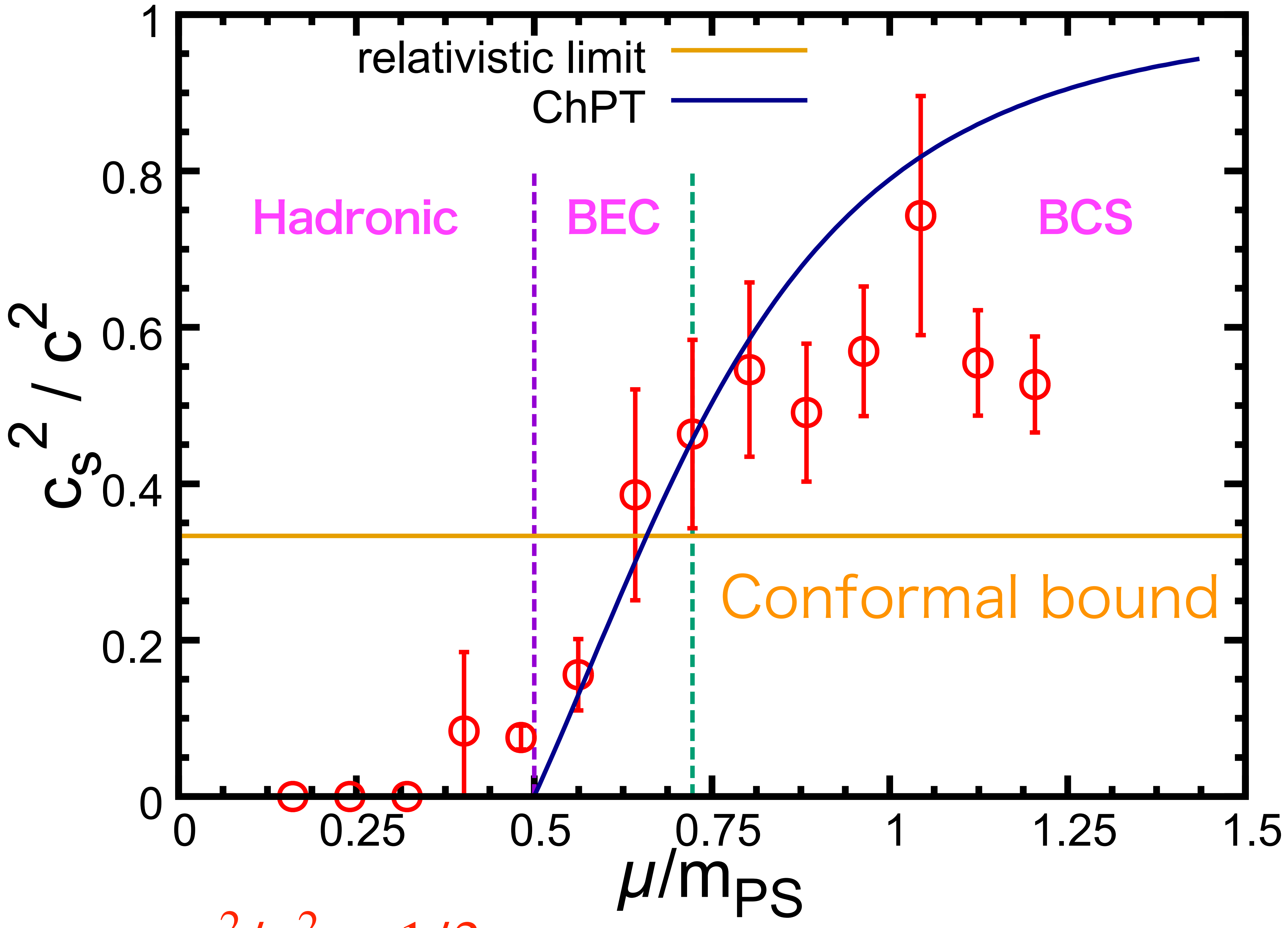


HotQCD (2014)

- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

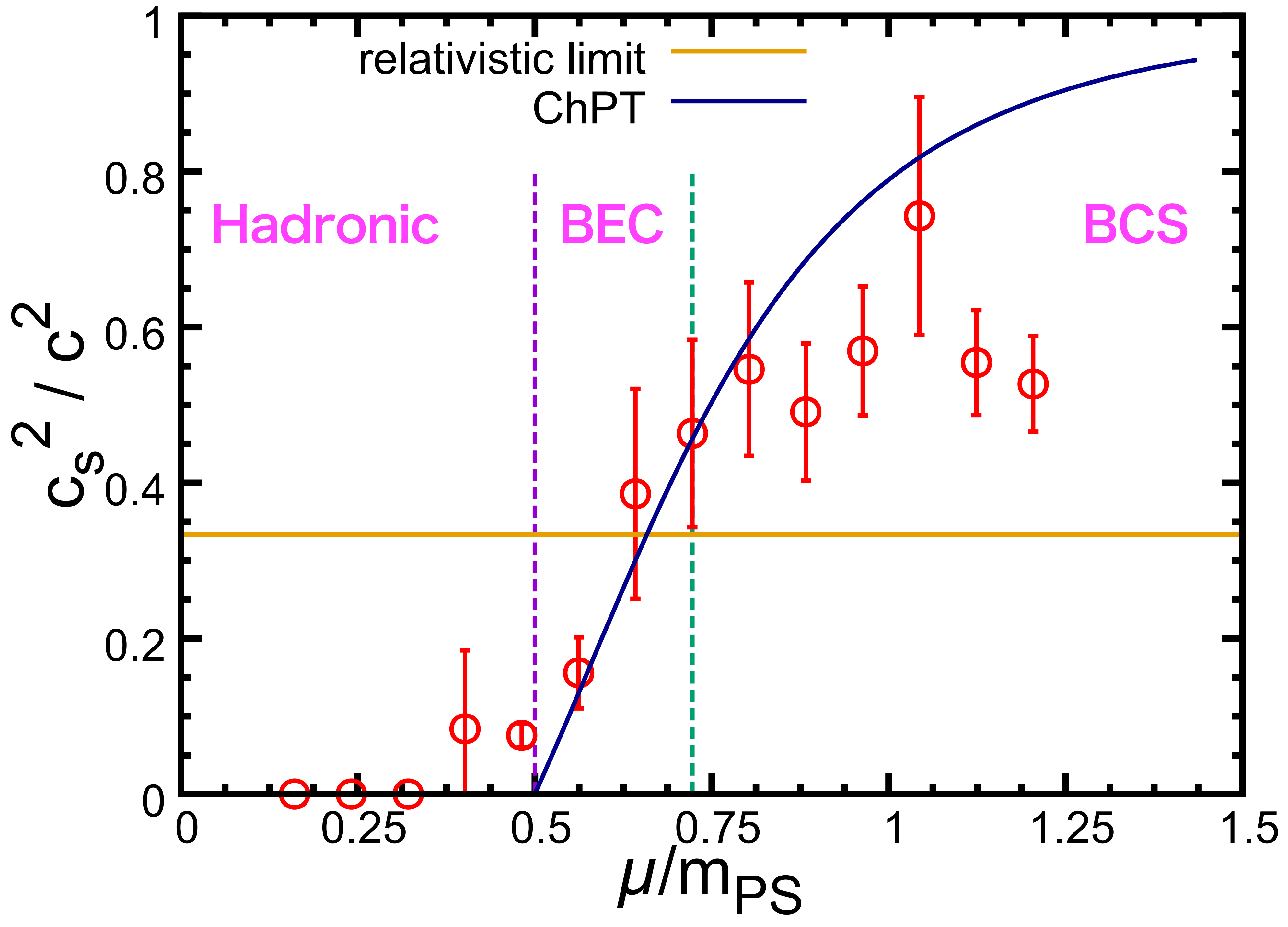
Iida and El arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

Further high density?

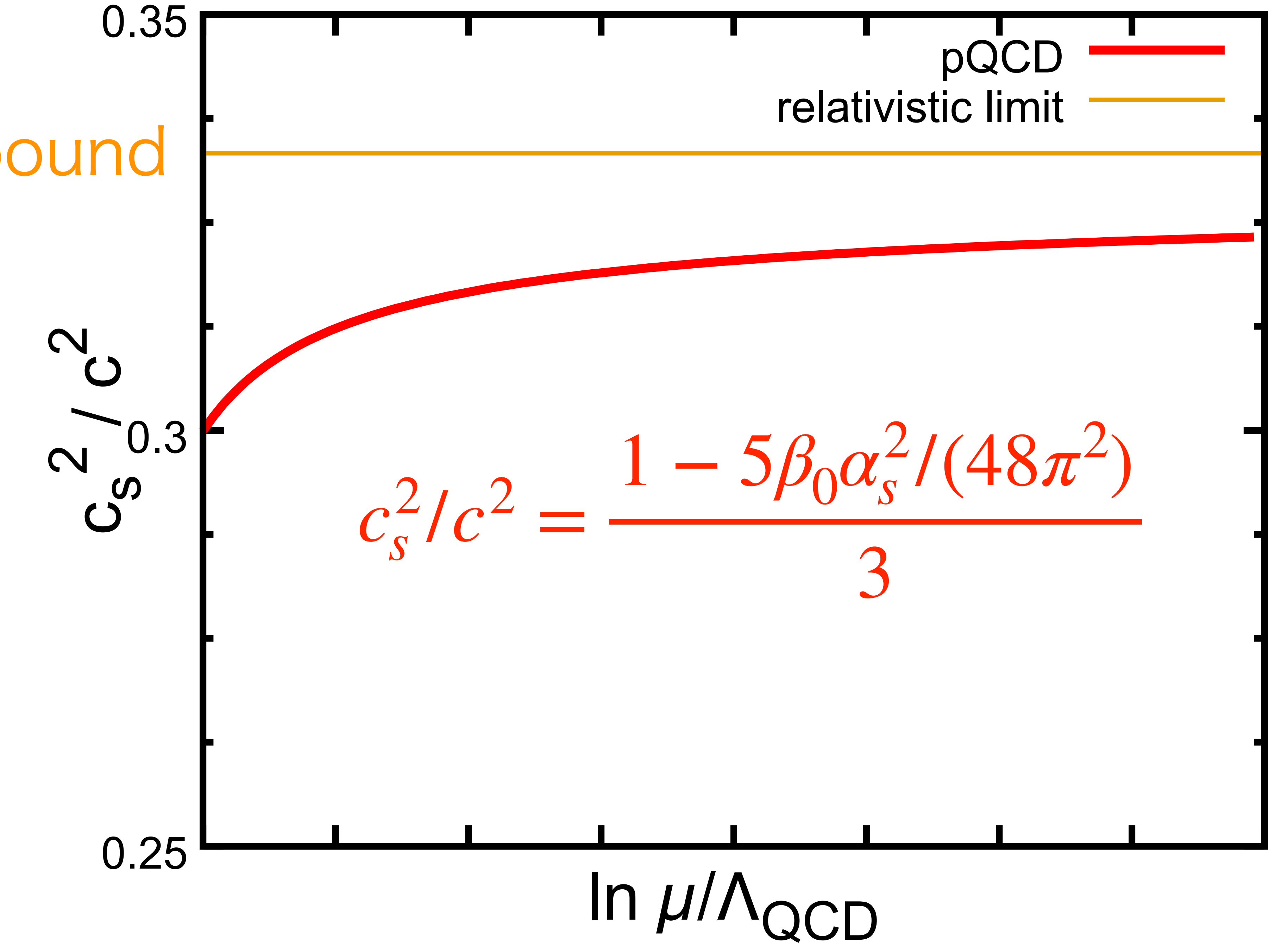
Kojo, Baym, Hatsuda (2021)



pQCD prediction

(Ultra high-density regime)

Conformal bound

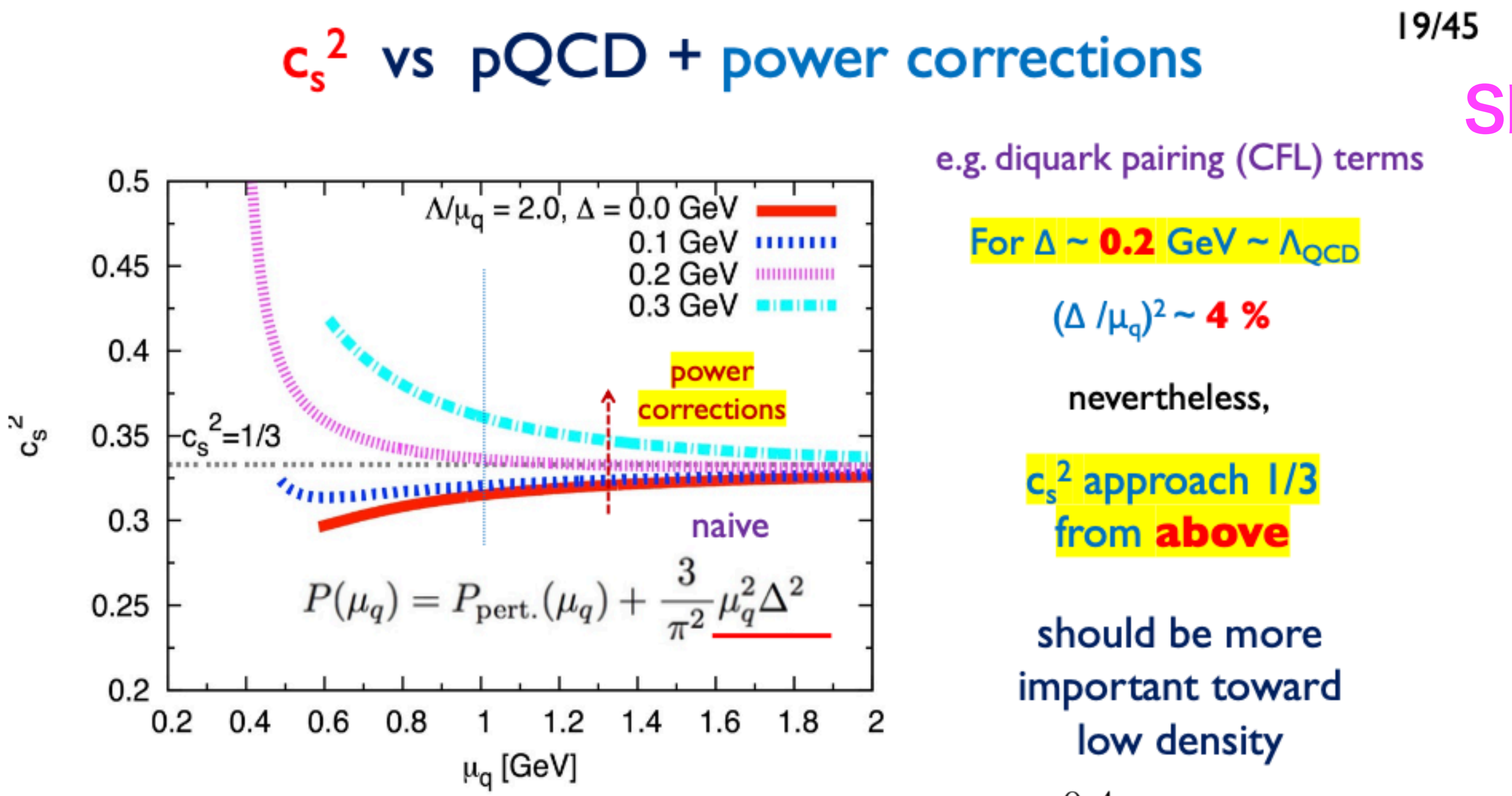


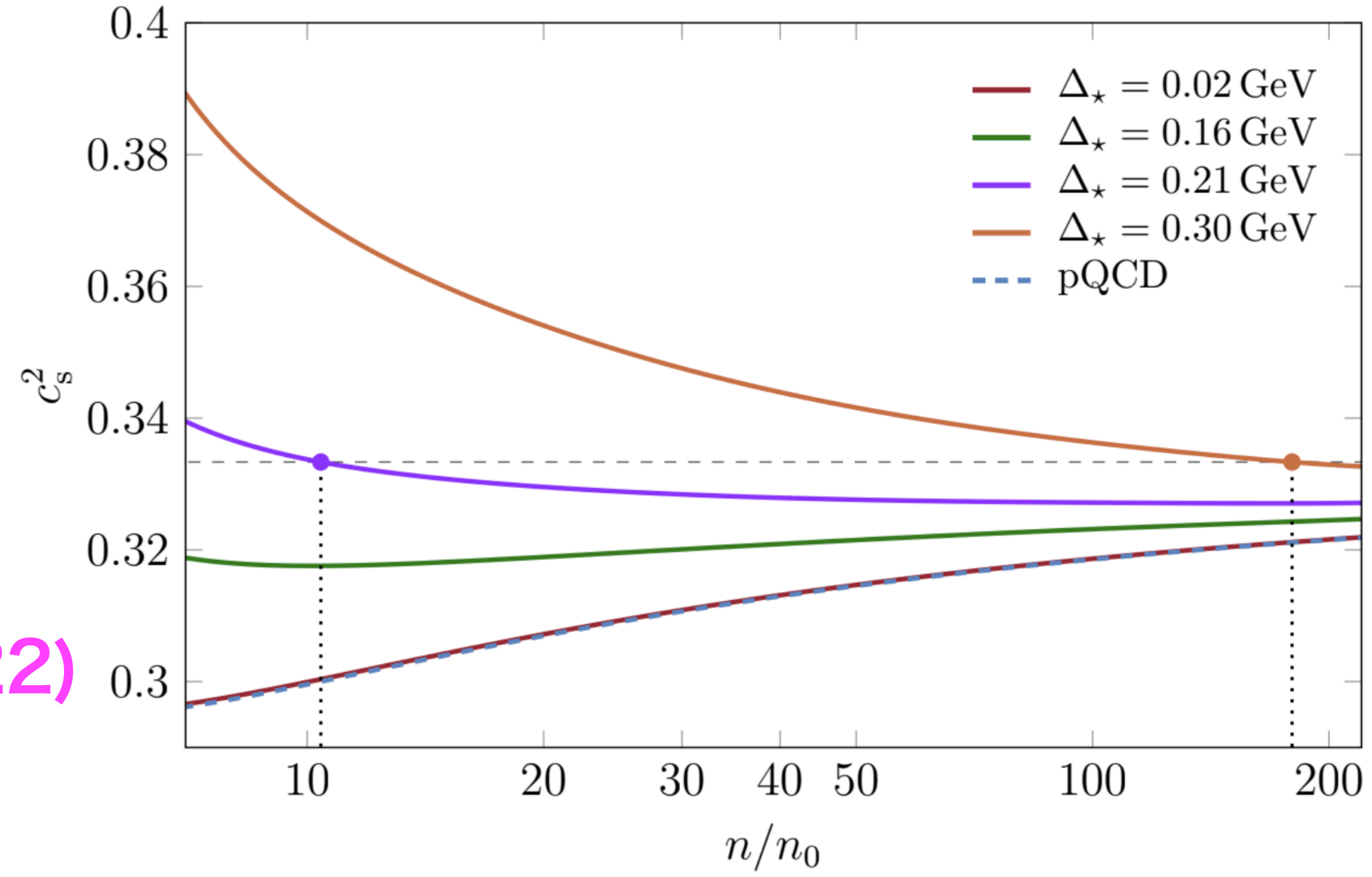
- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$
  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

Further high density?

pQCD + power correction due to diquark gap

Slide by Kojo (2019)

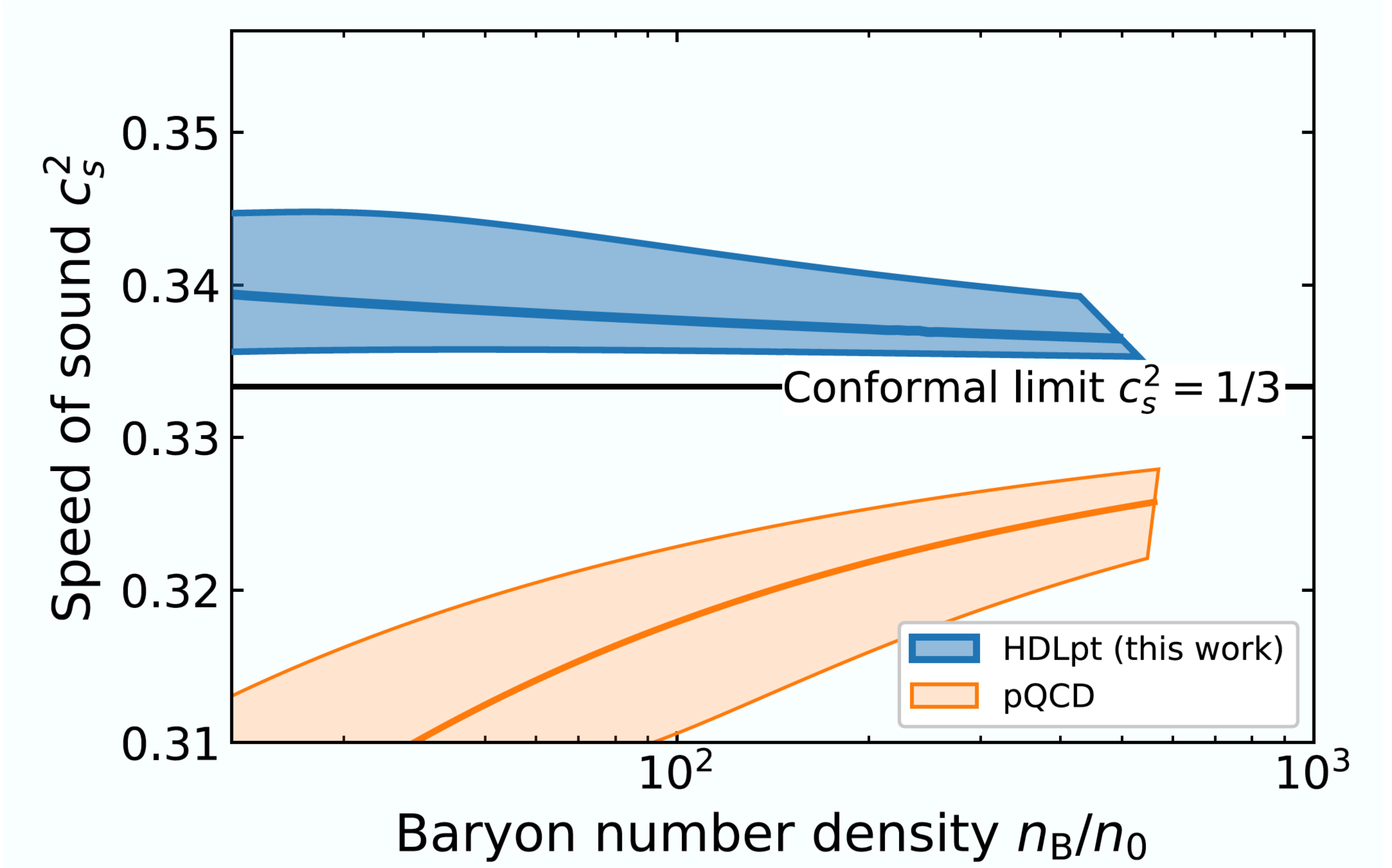




fRG analysis
Braun, Geissel, Schallmo(2022)

Hard thermal loop resummation

Fujimoto and Fukushima(2021)



- Open question: How c_s^2/c^2 approaches 1/3; from below or from above?

Summary and future work

- Sound velocity exceeds the conformal bound in finite-μ QCD-like theory

  First counterexample of conformal bound conjecture using lattice MC

  It seems to have a peak after BEC-BCS crossover

  cf.) cond-mat model study also find a peak after BEC-BCS

  Tajima and Liang (2022)

- Find a mechanism of a peak structure

  - quark saturation?(Kojo,Suenaga), strong coupling with trace anomaly? (McLerran,Fukushima et al.), others?

  - attractive or repulsive force between hadrons?

    => extended HAL QCD method in finite density

    work in progress with K.Murakami

    => mass spectrum in superfluid phase

    Suenaga, Murakami, EI, Iida (PRD,2023)
    K.Murakami's Lattice proceedings

  - independent of the color dof?

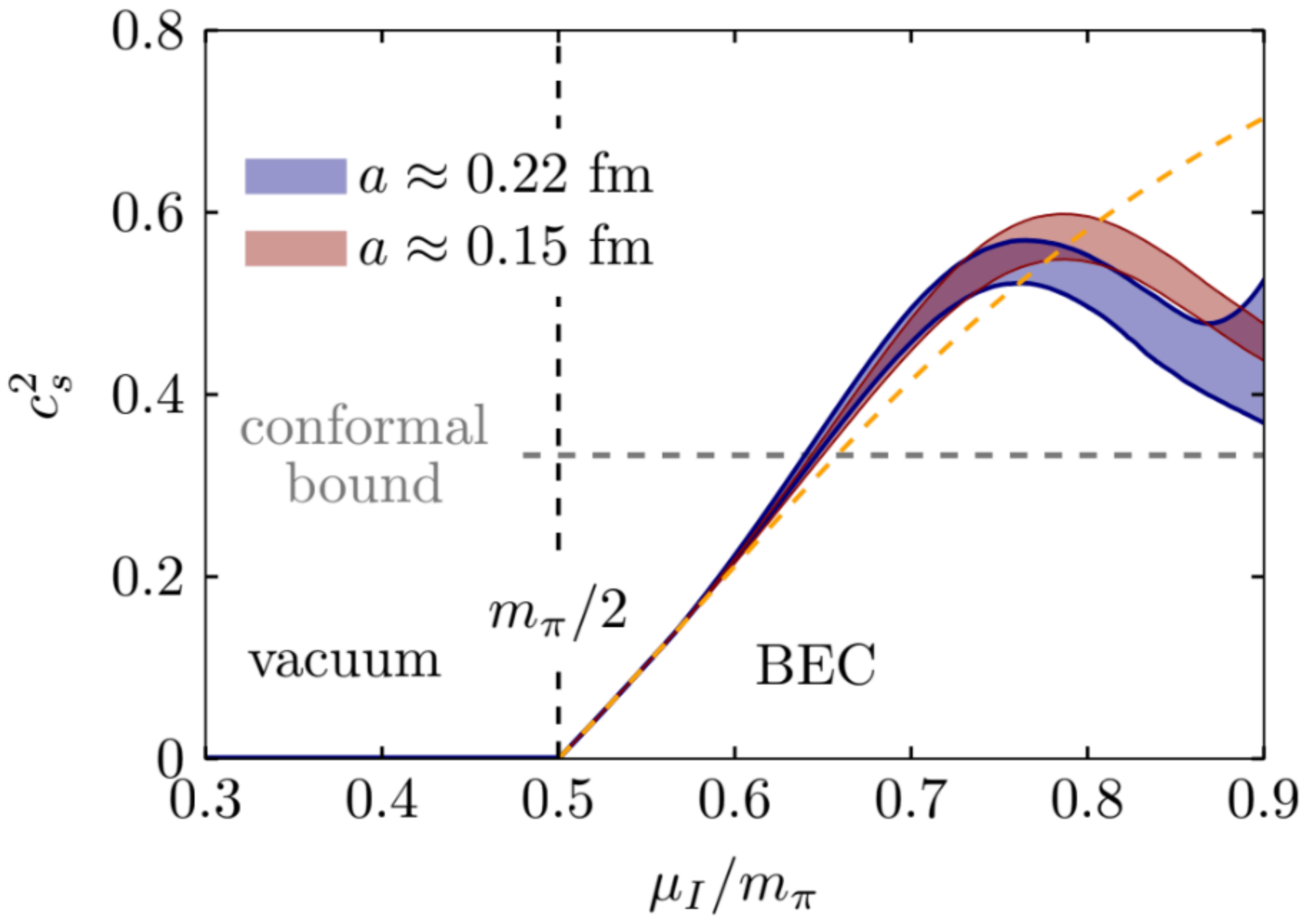
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

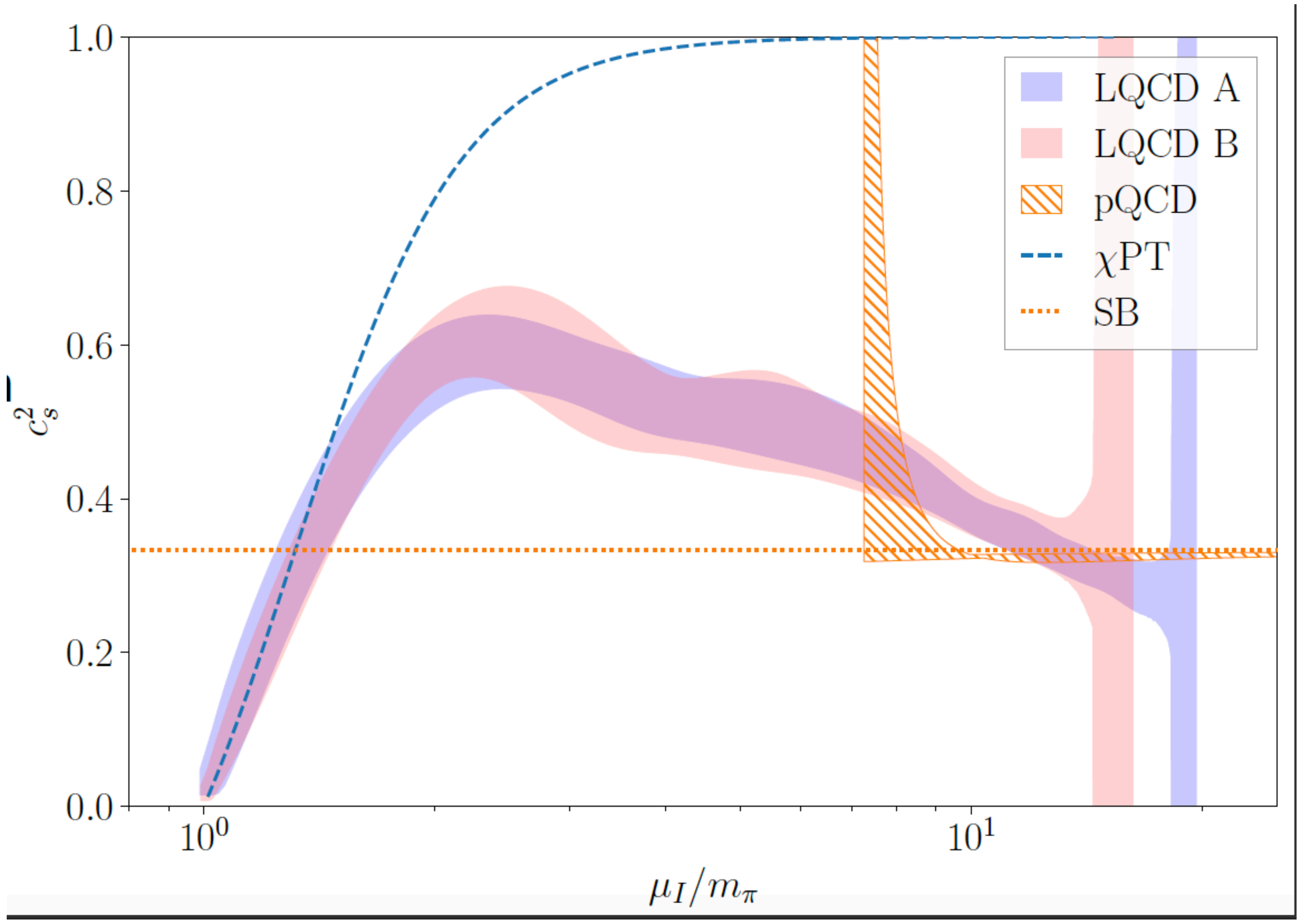
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014
(W.Detmold's talk Monday)

Result with spline interpolation



New algorithm for n-point fn. calc.



Counterexamples of conformal bound

N=4 SYM at finite density

PHYSICAL REVIEW D **94**, 106008 (2016)

Breaking the sound barrier in holography

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(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

Evidence against a first-order phase transition in neutron star cores: impact of new data

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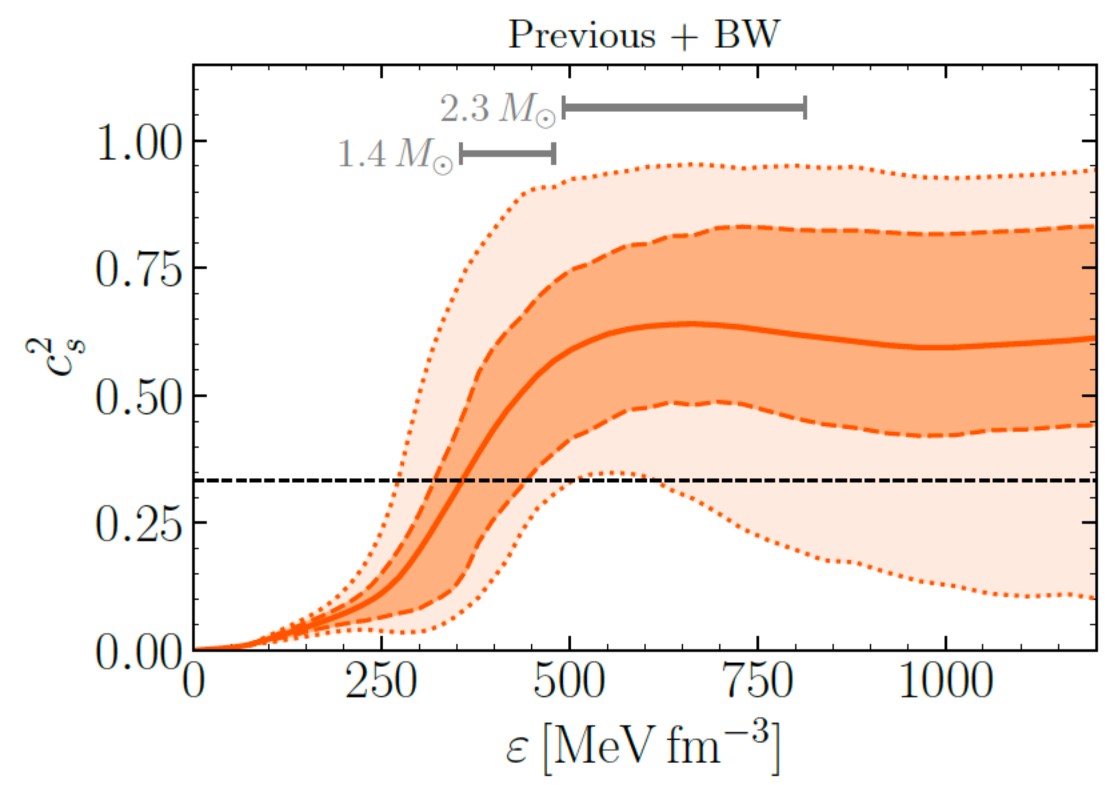
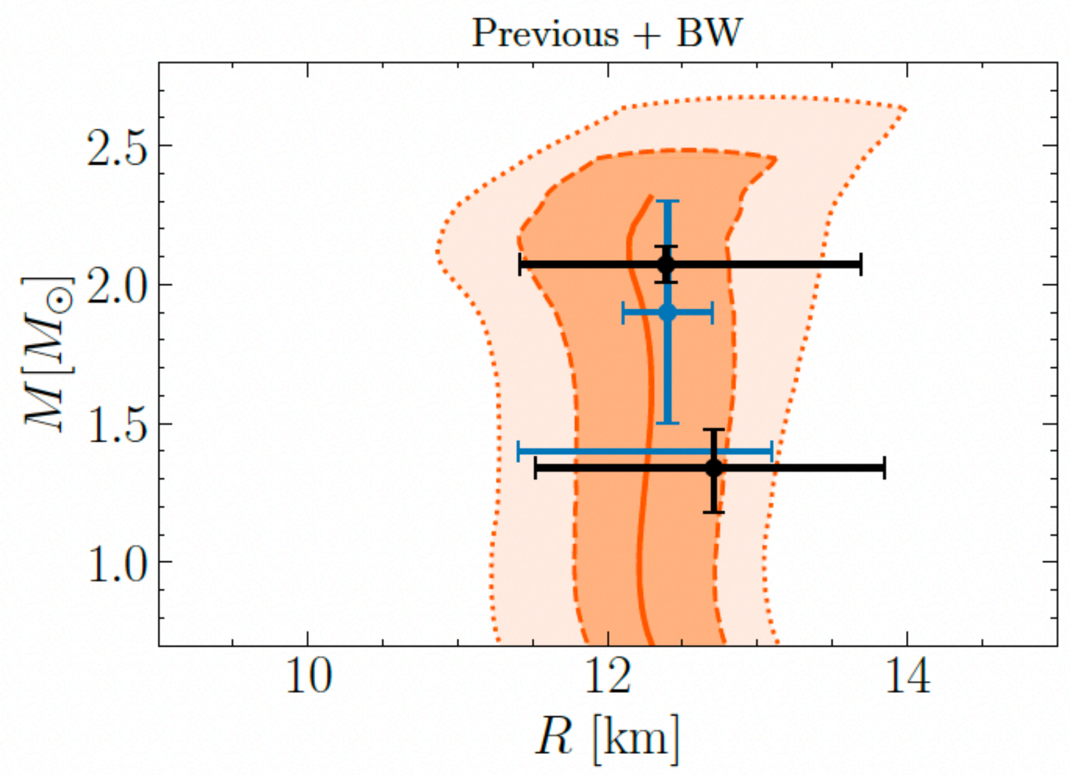
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

arXiv:2306.06218

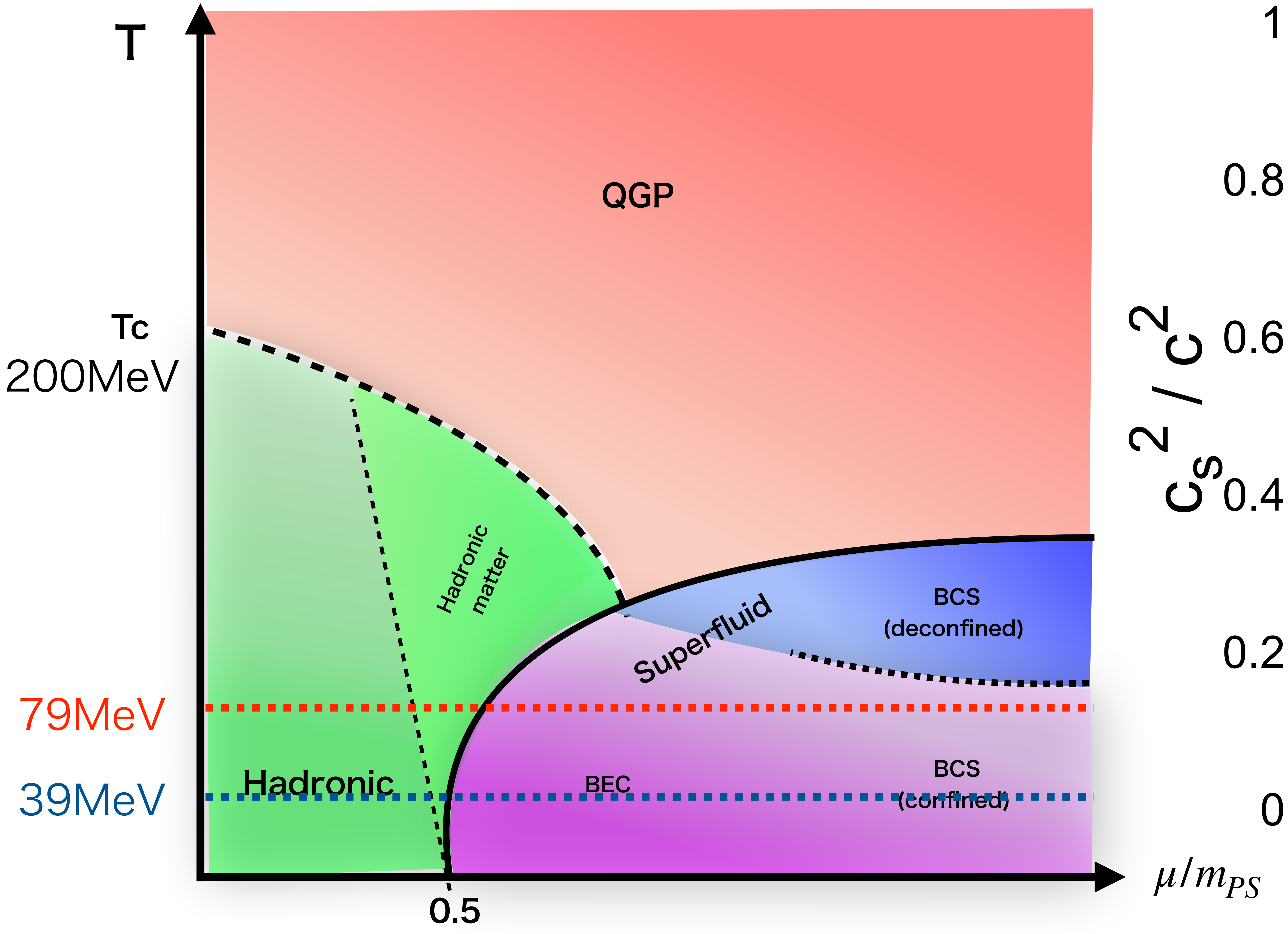
Bayian analyses of recent observation data of neutron star

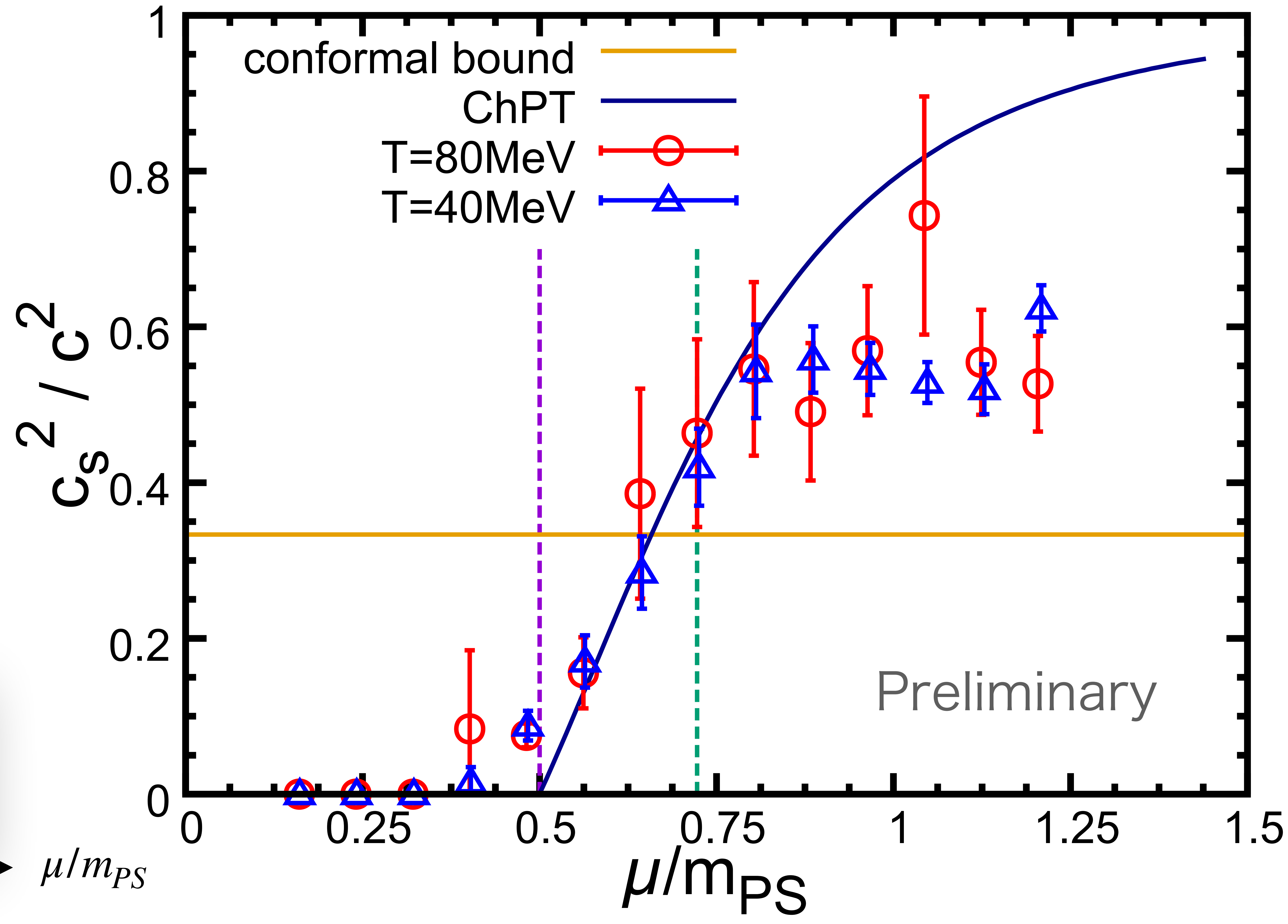




Further low temperature

T~40MeV data: Phase diagram (μ_c value, BEC-BCS crossover) is not changed





backup

Our projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

  Phase diagram by Lattice simulation
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0

  Scale setting of Lattice simulation
- K.Iida, K.Ishiguro, EI, arXiv: 2111.13067 (PoS, Lattice 2021)

  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01

  Velocity of sound by Lattice simulation
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023)

  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472 (PoS, Lattice 2022)

  Mass spectrum by Lattice simulation

Conformal bound (Holography bound)?

conjecture : $c_s^2/c^2 \leq 1/3$ is valid for a broad class of 4-dim. theories

A bound on the speed of sound from holography

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Abhinav Nellore[‡]
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

We found a strong evidence of $c_s^2/c^2 > 1/3$ in finite density QCD-like theory

using Lattice Monte Carlo

Implementation QC2D with diquark source term

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

construct a single bilinear form of fermion fields

$$S_F = (\bar{\psi}_1 \ \ \bar{\varphi}) \begin{pmatrix} \Delta(\mu) & J\gamma_5 \\ -J\gamma_5 & \Delta(-\mu) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi$$

Here, $\Psi = \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix}$

$$\bar{\varphi} = -\bar{\psi}_2^T C \tau_2, \quad \varphi = C^{-1}\tau_2 \bar{\psi}_2^T$$

$\mathscr{M}$ has non-diagonal components, calculations of det[M] and inverse of M are hard…

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} \Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2 & 0 \\ 0 & \Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2 \end{pmatrix}$$

J (=$j\kappa$) term lifts the eigenvalue of Dirac op.

Note that Ψ denotes 2-flavor, $\det \mathcal{M}$ gives Nf=2 action

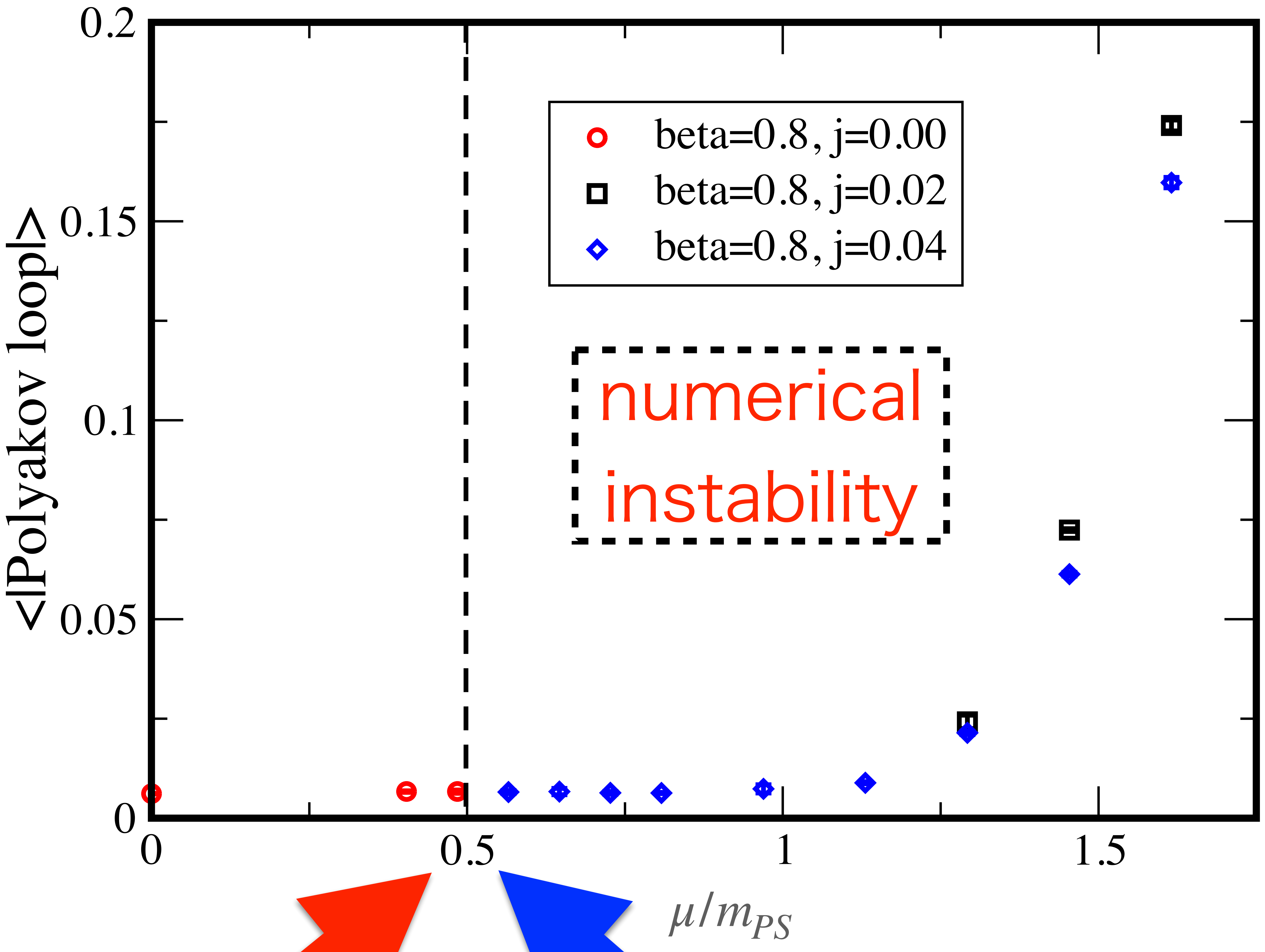
$\det \mathcal{M}^\dagger \mathcal{M}$ is 4-flavor theory

RHMC algorithm

HMC calculation w or w/o diquark source term

According to chiral perturbation theory,

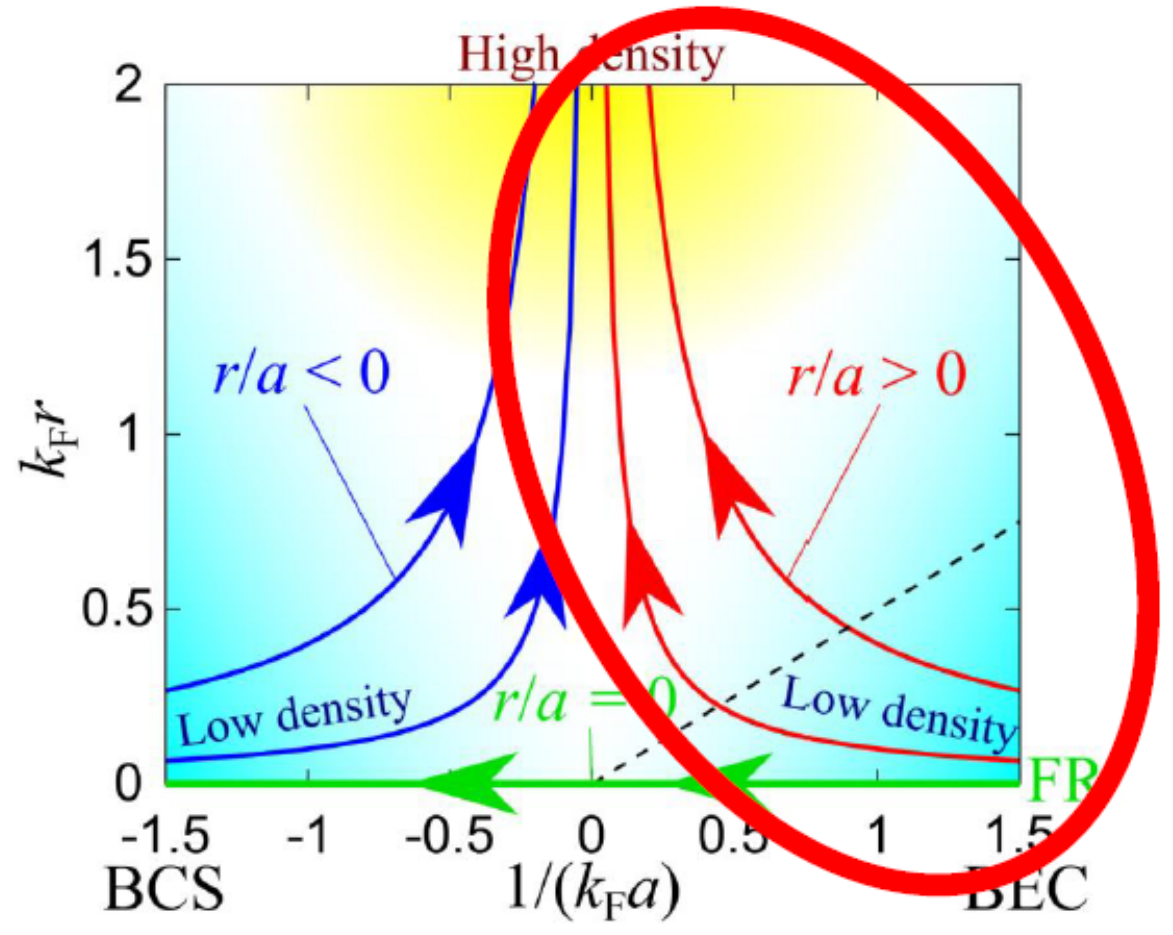
the hadronic-superfluid phase transition occurs at $\mu/m_{PS} \sim 0.5$

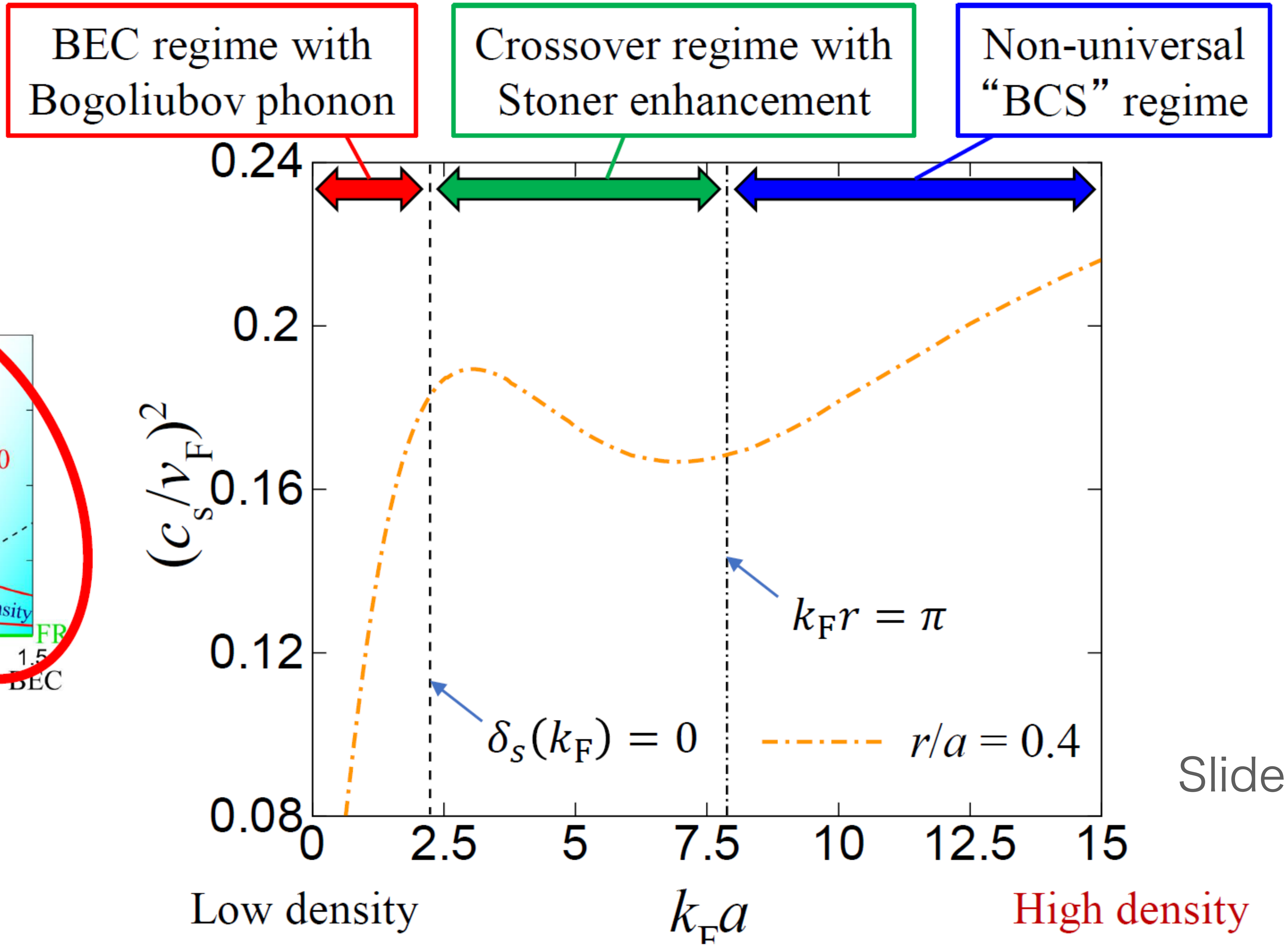


HMC without j is doable
(minimum MC step ~ 1/800)

HMC without j cannot run even with
a tiny MC step(~1/1000)

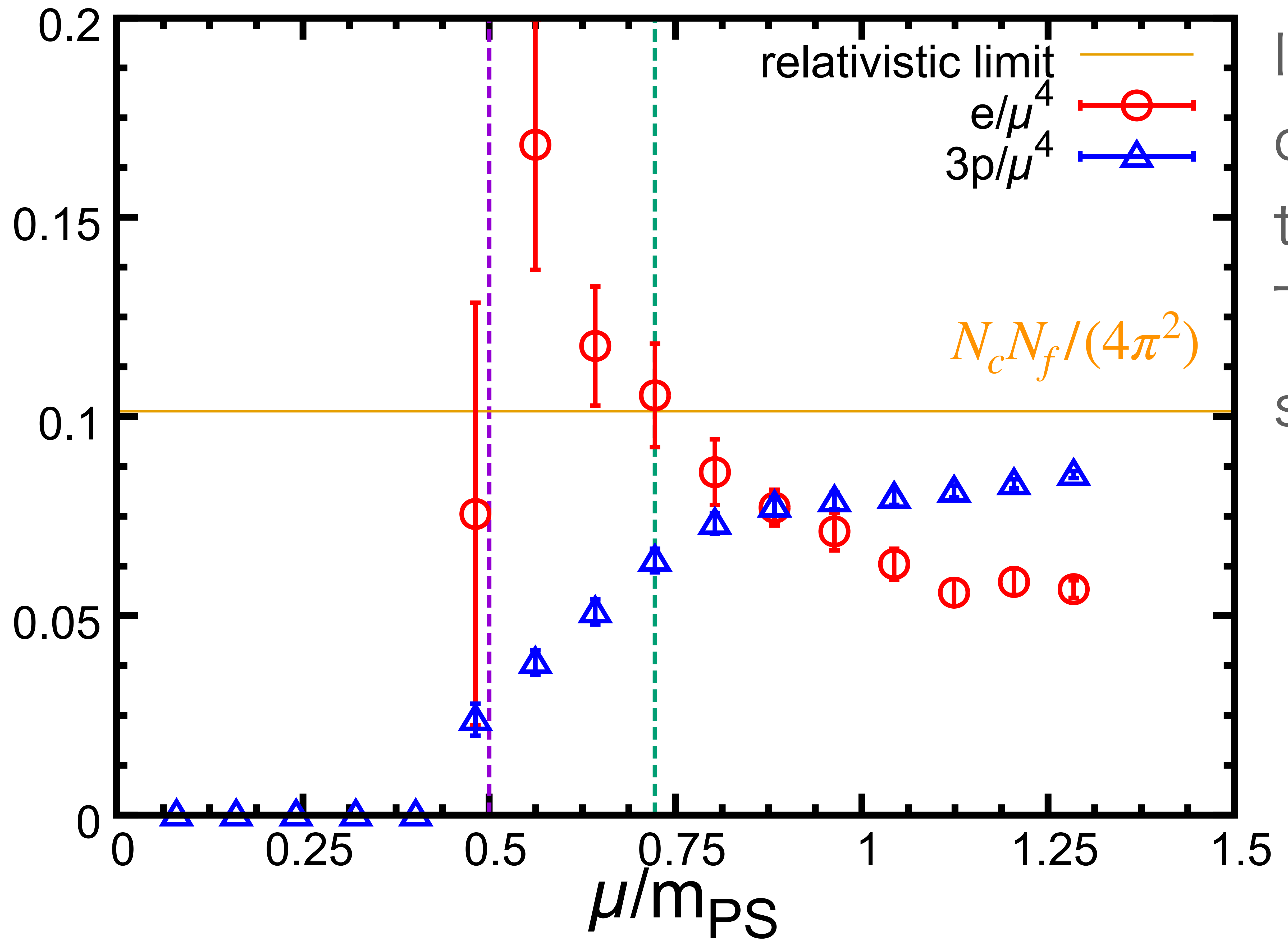
Example of cond.mat. model





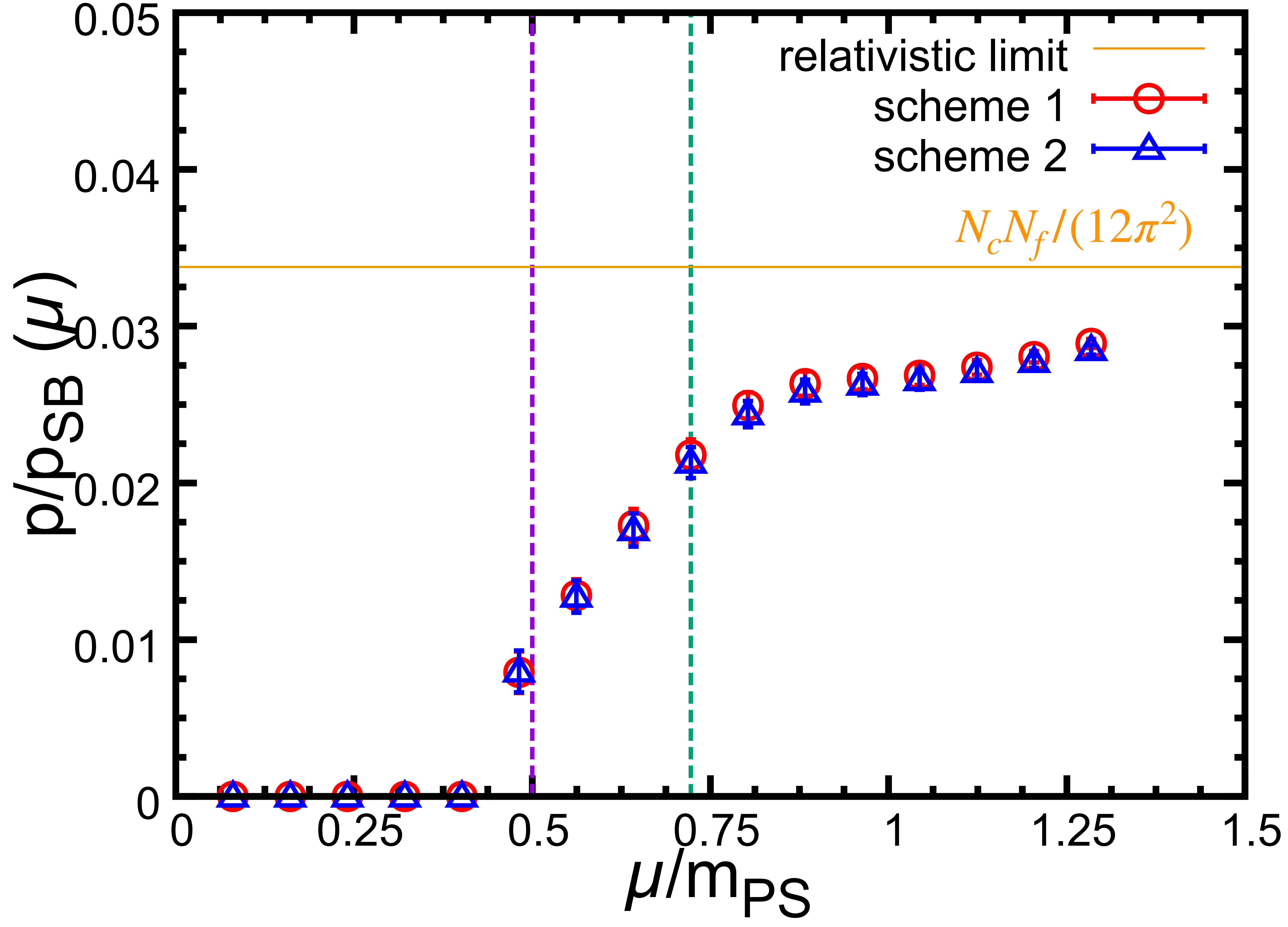
Slide by H.Tajima

scaling of p and e in high density



In massive fermion theory, the trace anomaly does not vanish because the mass term breaks the scale invariance.
The mass term will give a negative contribution, so that we expect $e/\mu^4 < e_{SB}/\mu^4 = N_cN_f/(4\pi^2)$

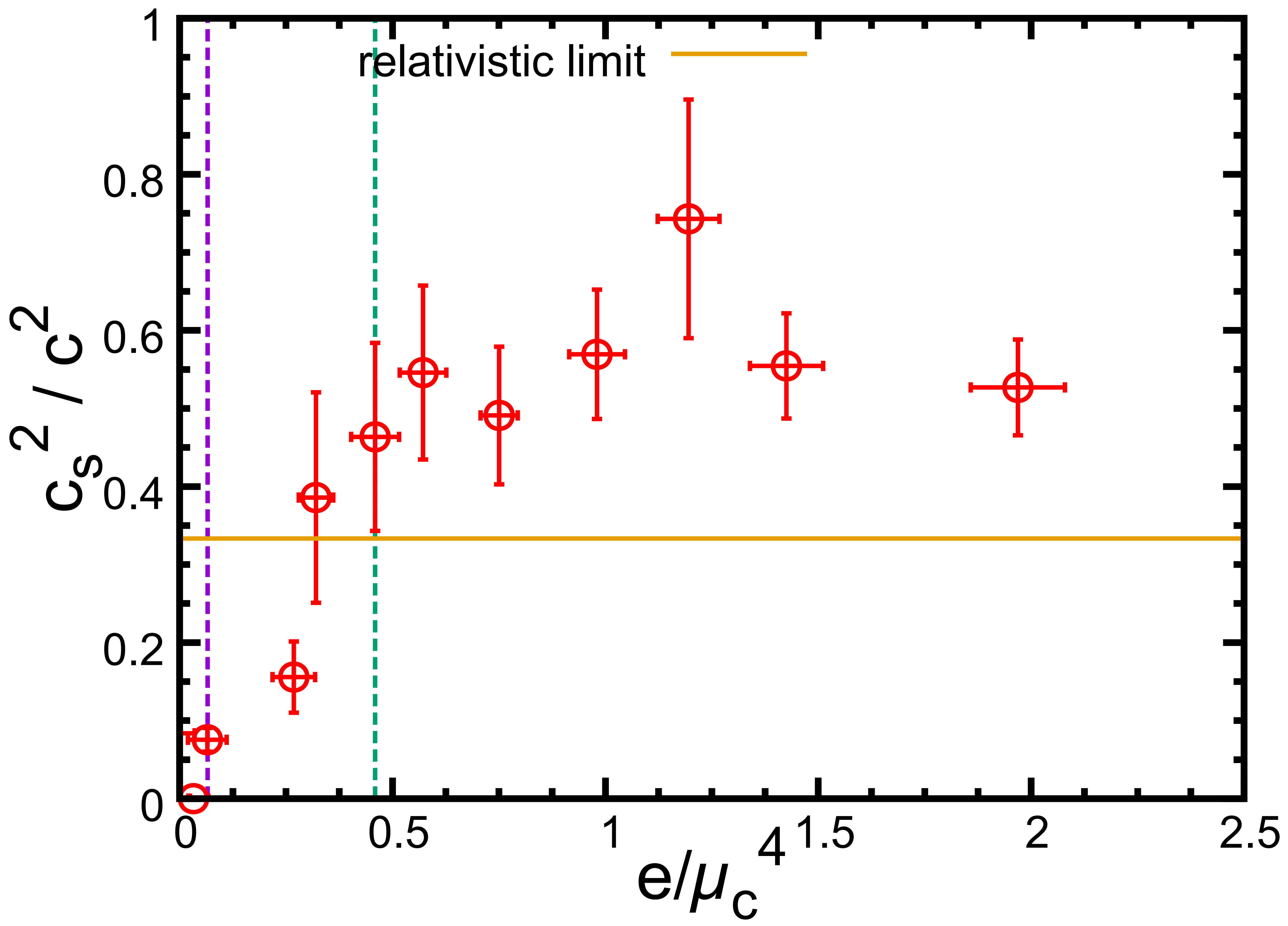
Scheme dependence of pressure

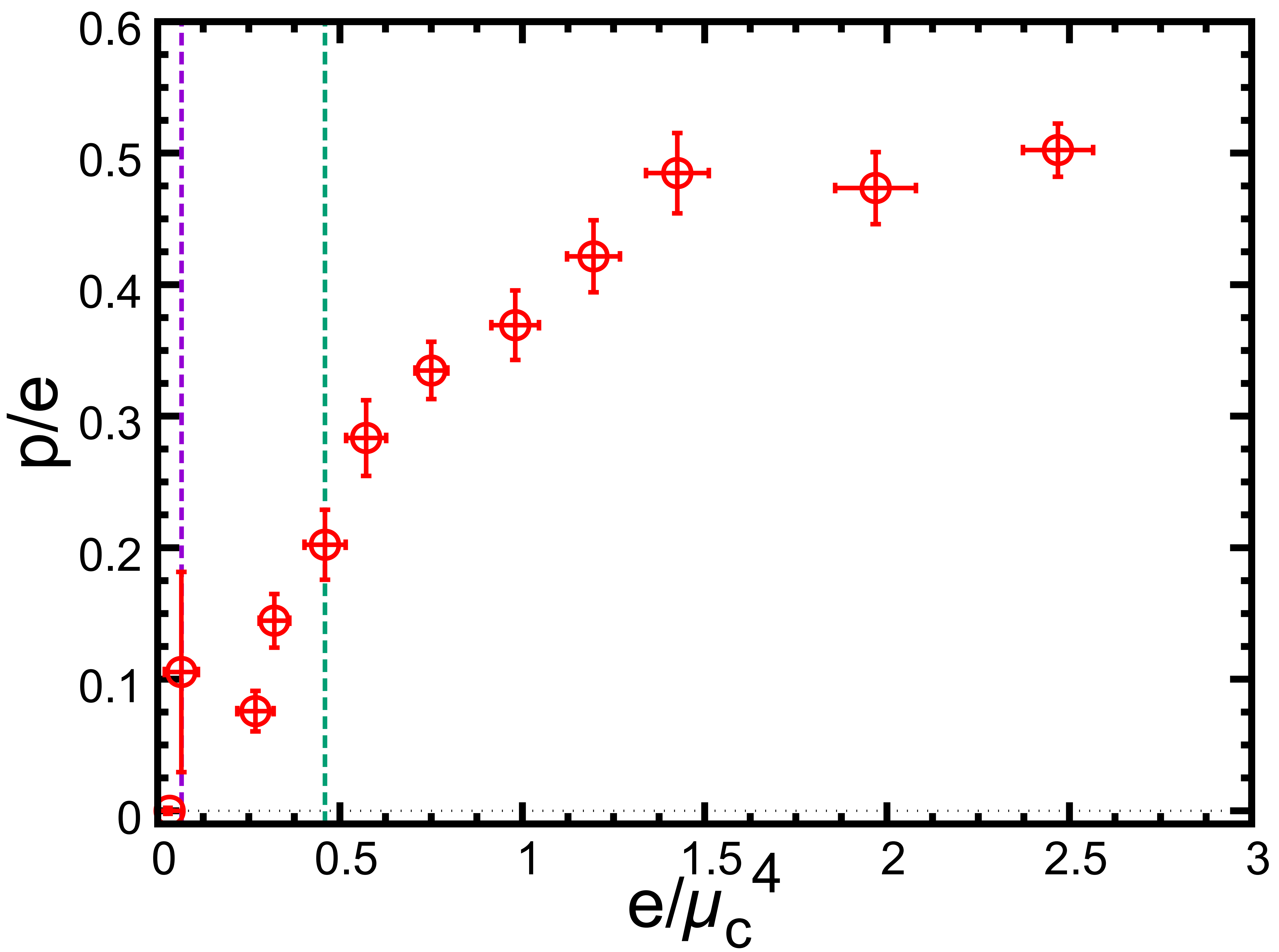


$$\mathrm{I}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\rm lat}(\mu')d\mu'}\,; \qquad (28)$$

$$\mathrm{II}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} \frac{n_{SB}^{\rm cont}}{n_{SB}^{\rm lat}}(\mu')\, n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\rm cont}(\mu')d\mu'}\,, \qquad (29)$$

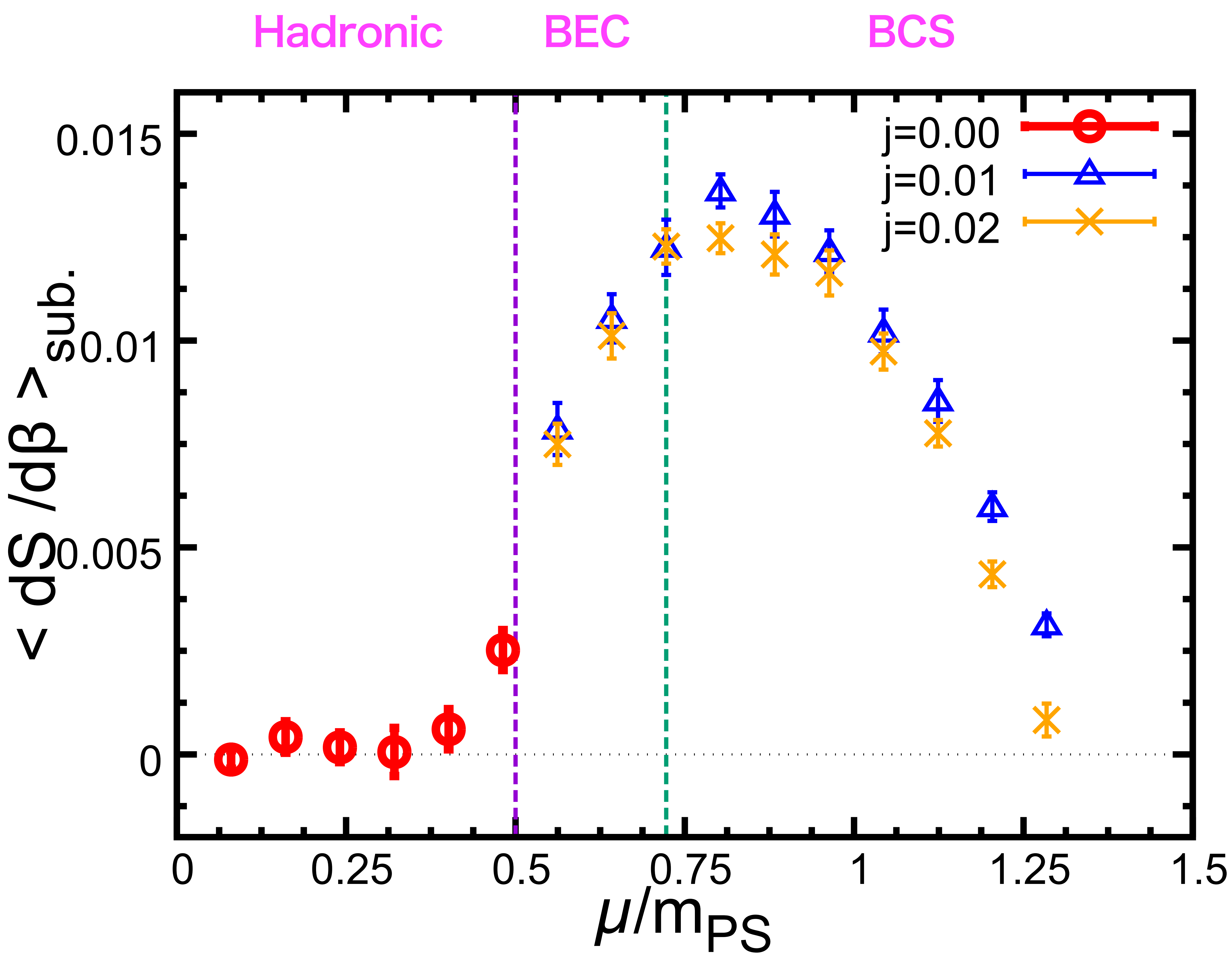
Sound velocity (ratio $\Delta p/\Delta e$) vs energy





μ-dependence of gauge action

value of Iwasaki gauge action knows the phase structure!



Our definition of each phase

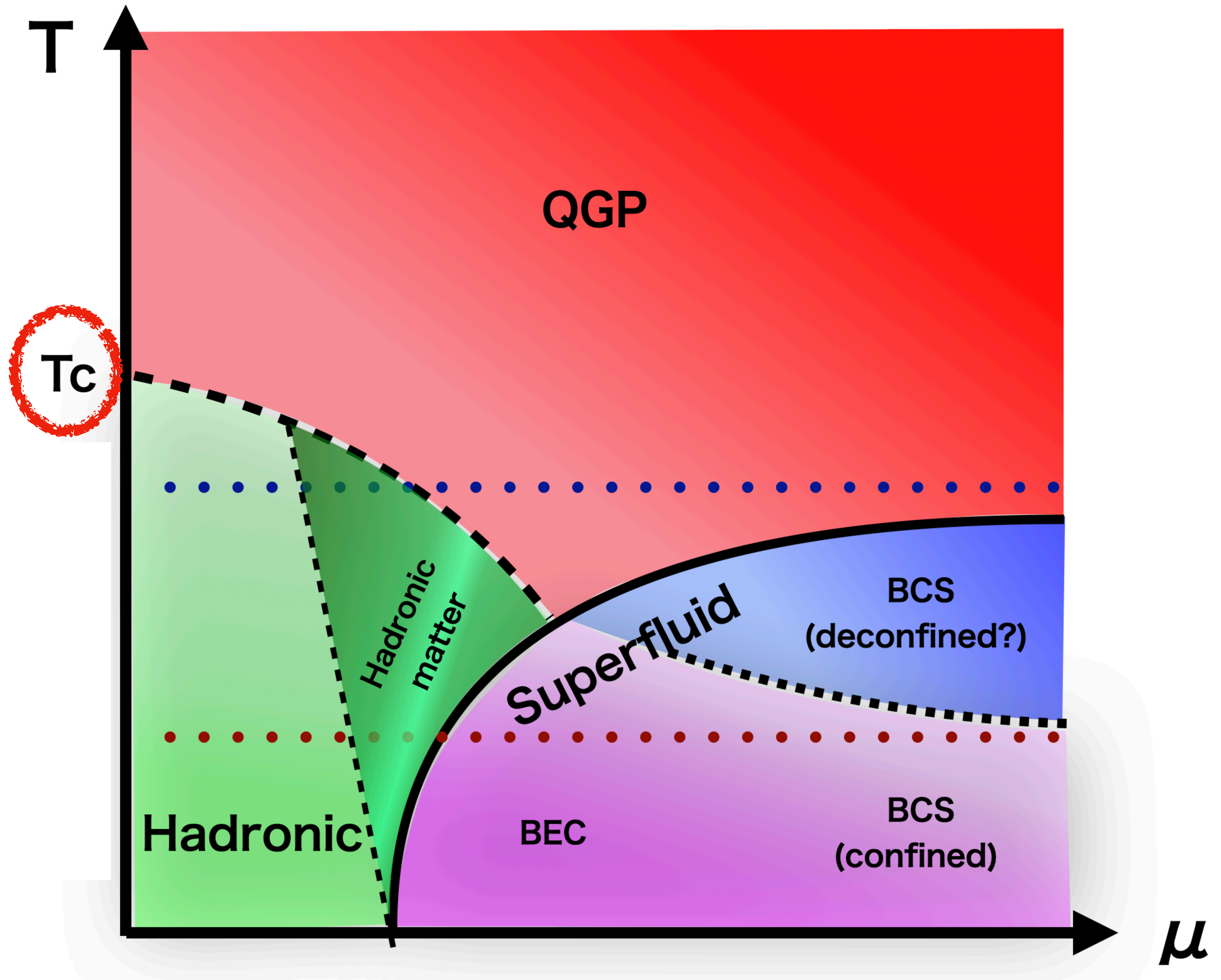
	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\rm tree} \approx 1$

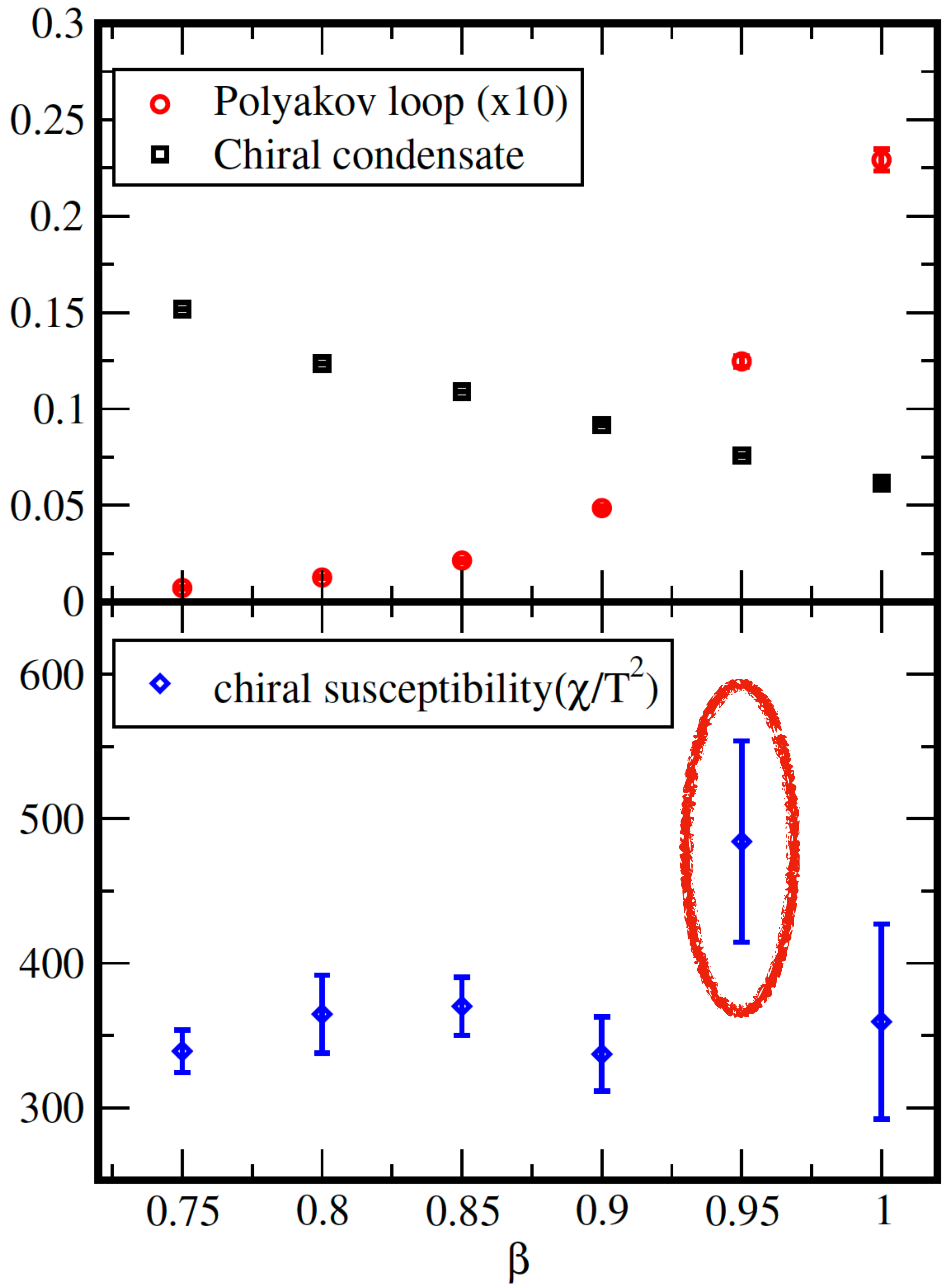
Phase diagram

Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

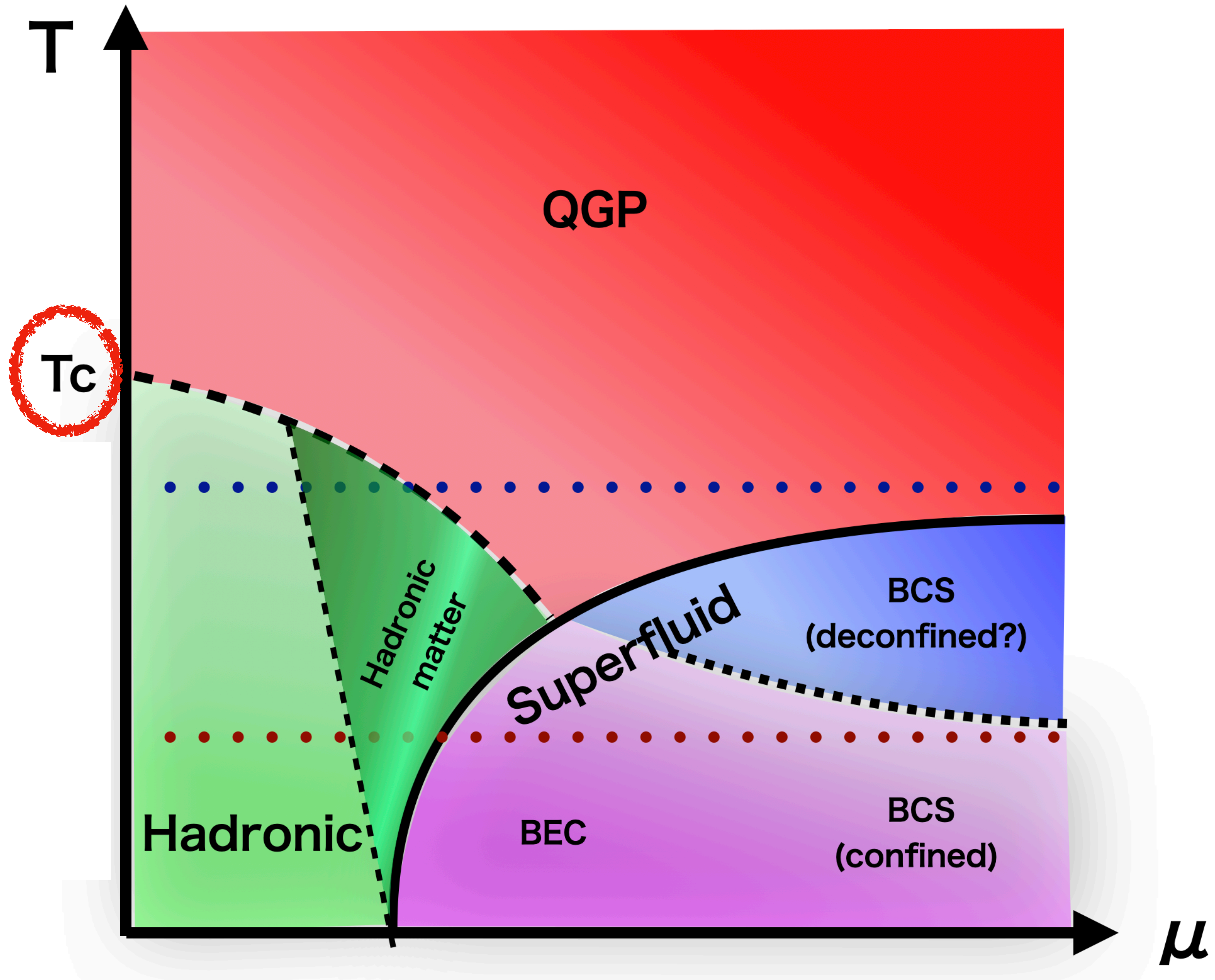
- Tc at $\mu = 0$ from chiral susceptibility





Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

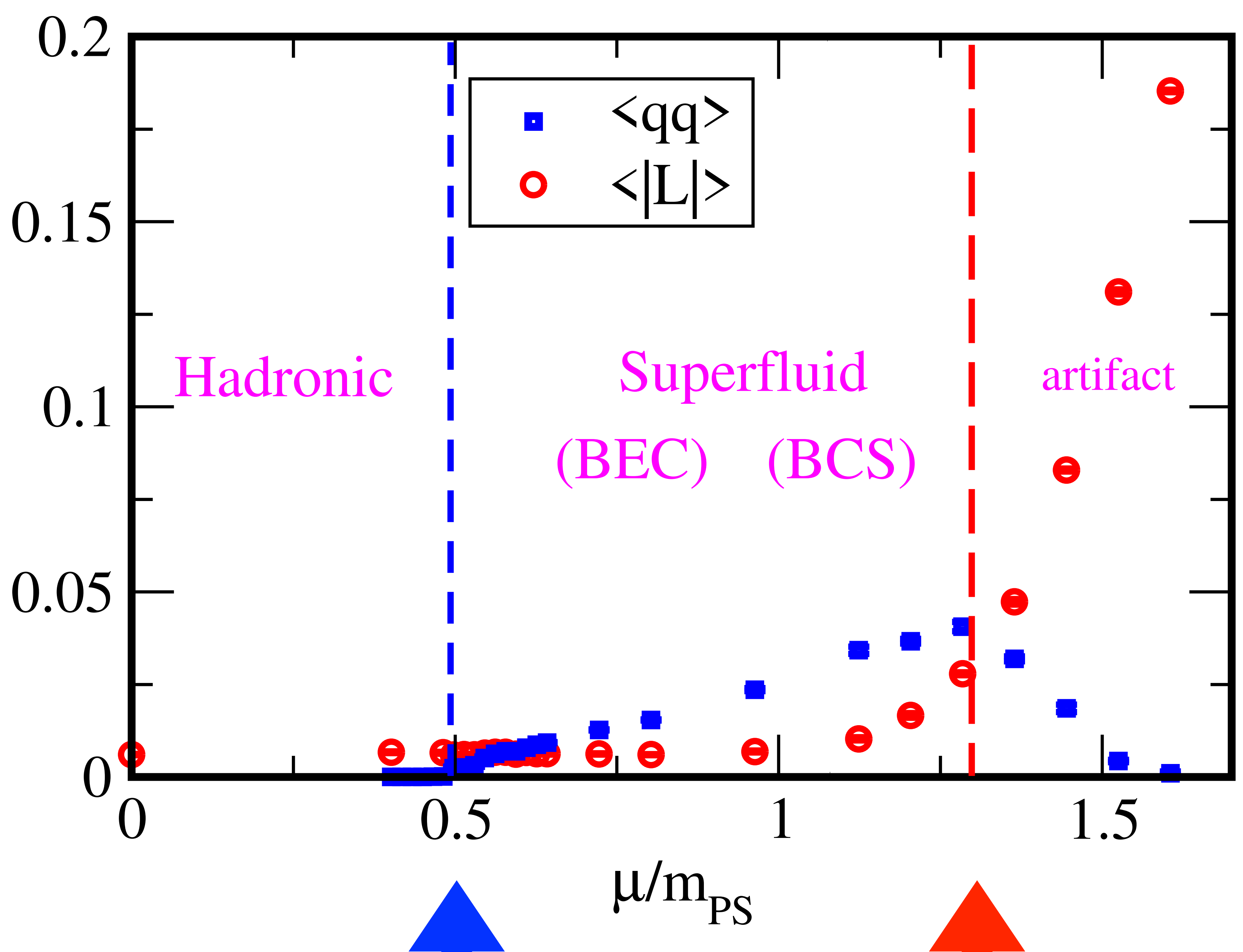


- Tc at $\mu = 0$ from chiral susceptibility
- Assume Tc=200MeV

  Tc is realize Nt=10, $\beta = 0.95$ (a=0.1[fm])
- Find relationship between β (lattice bare coupling) and a (lattice spacing)

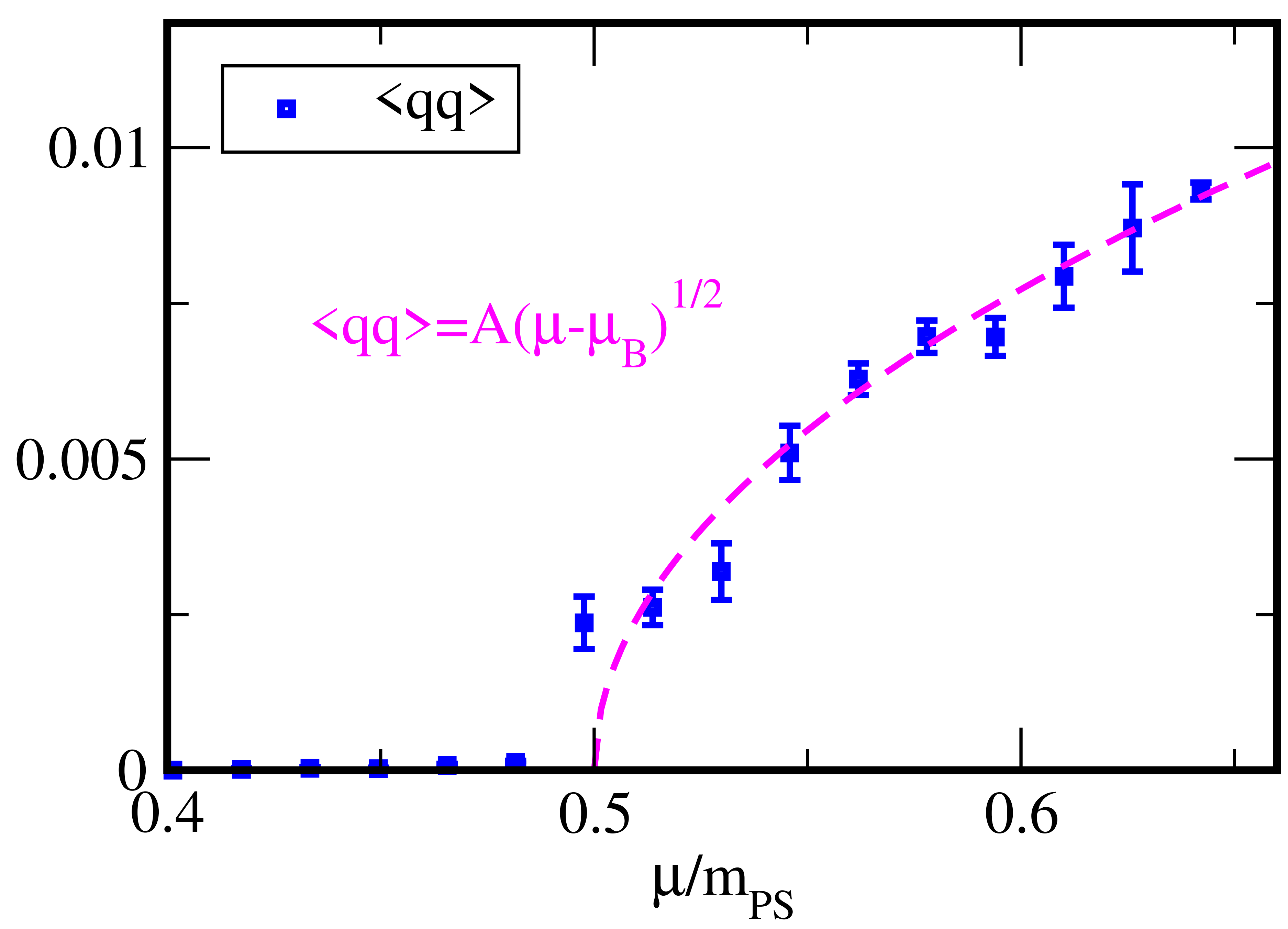
  In finite density simulation, a=0.1658[fm]

Order parameters in j=0 limit



$\mu_B/m_{PS} \simeq 0.50$

$\mu/m_{PS} \simeq 1.28$
$(\mu_D/m_{PS} \simeq 1.44)$

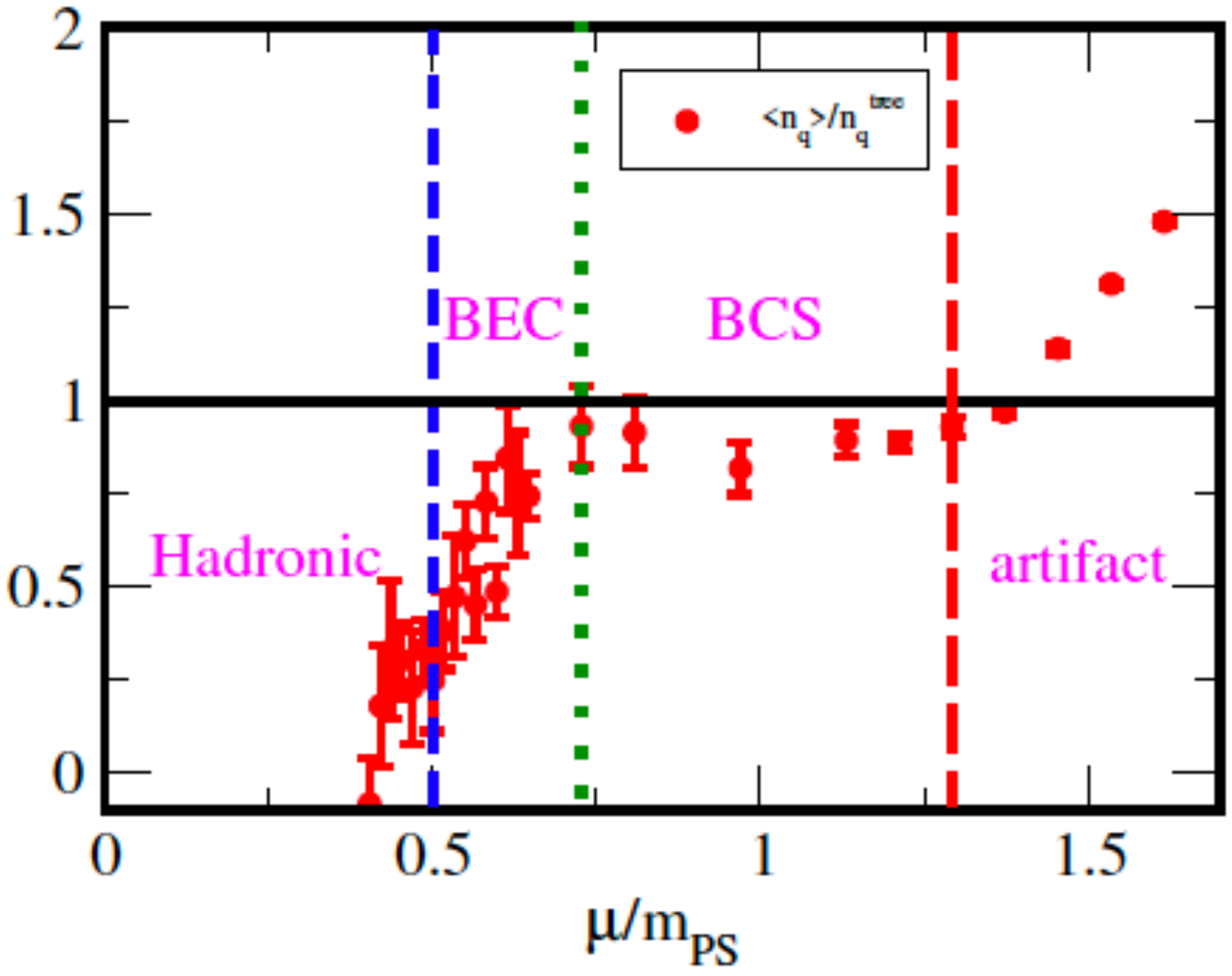


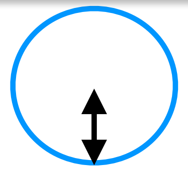
Scaling law of order param.
is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly
NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

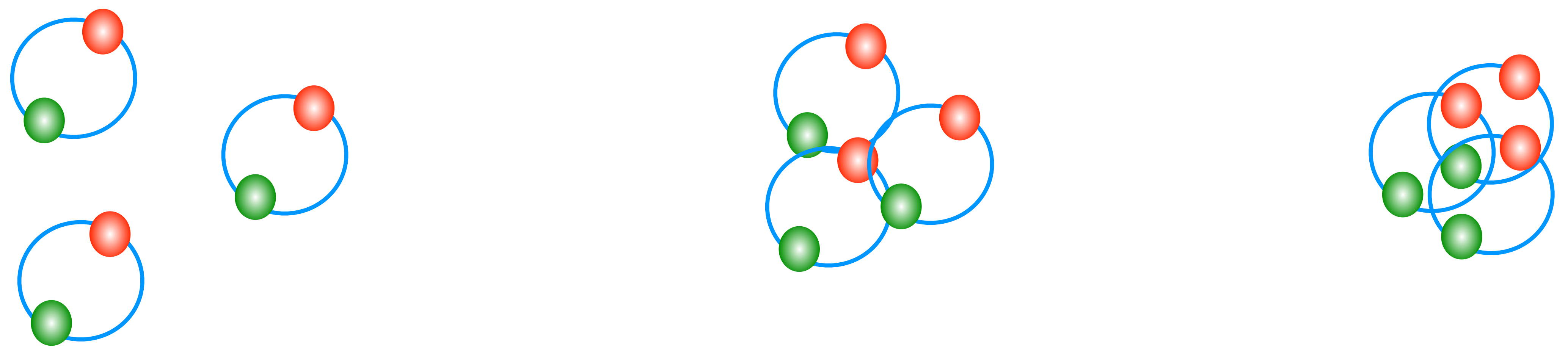
BEC/BCS crossover



 Δ^{-1} Δ: gap energy

BEC phase

BCS phase



Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$

Quarks behave free particles

density

Number density of free particle

$$n_q^{\rm tree}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$$

J->0 extrapolation

Diquark condensate has a strong j dependence

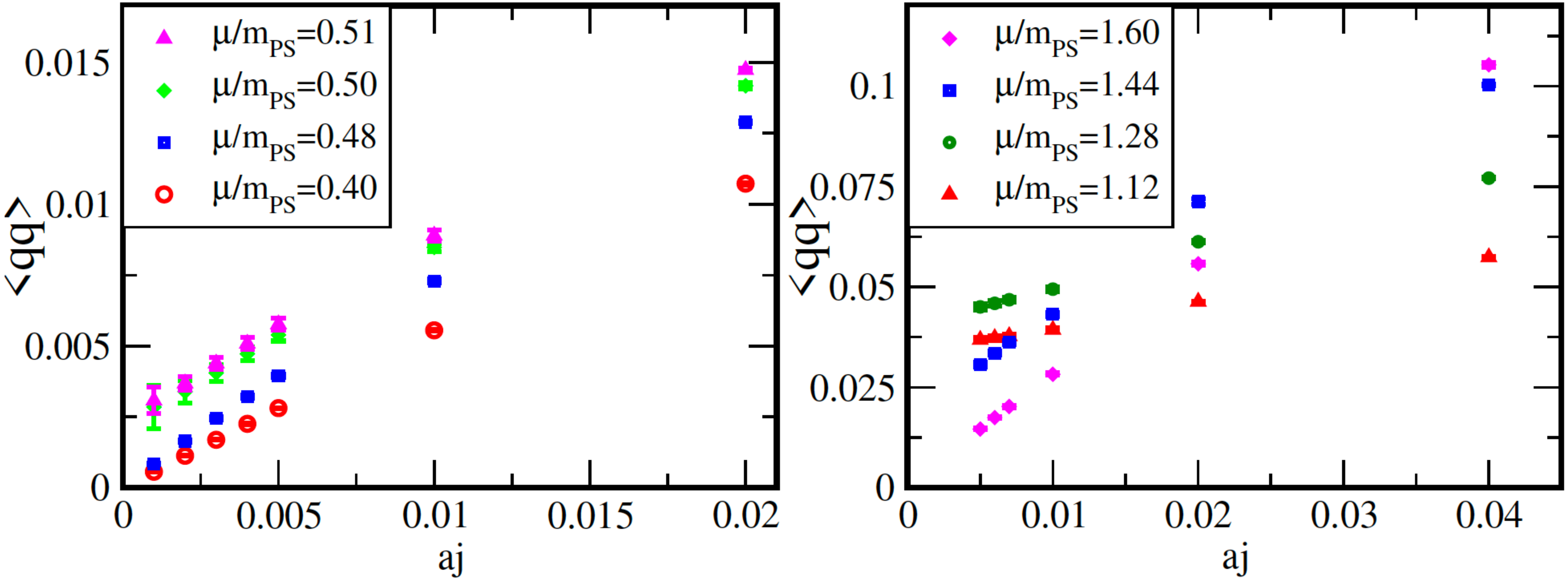
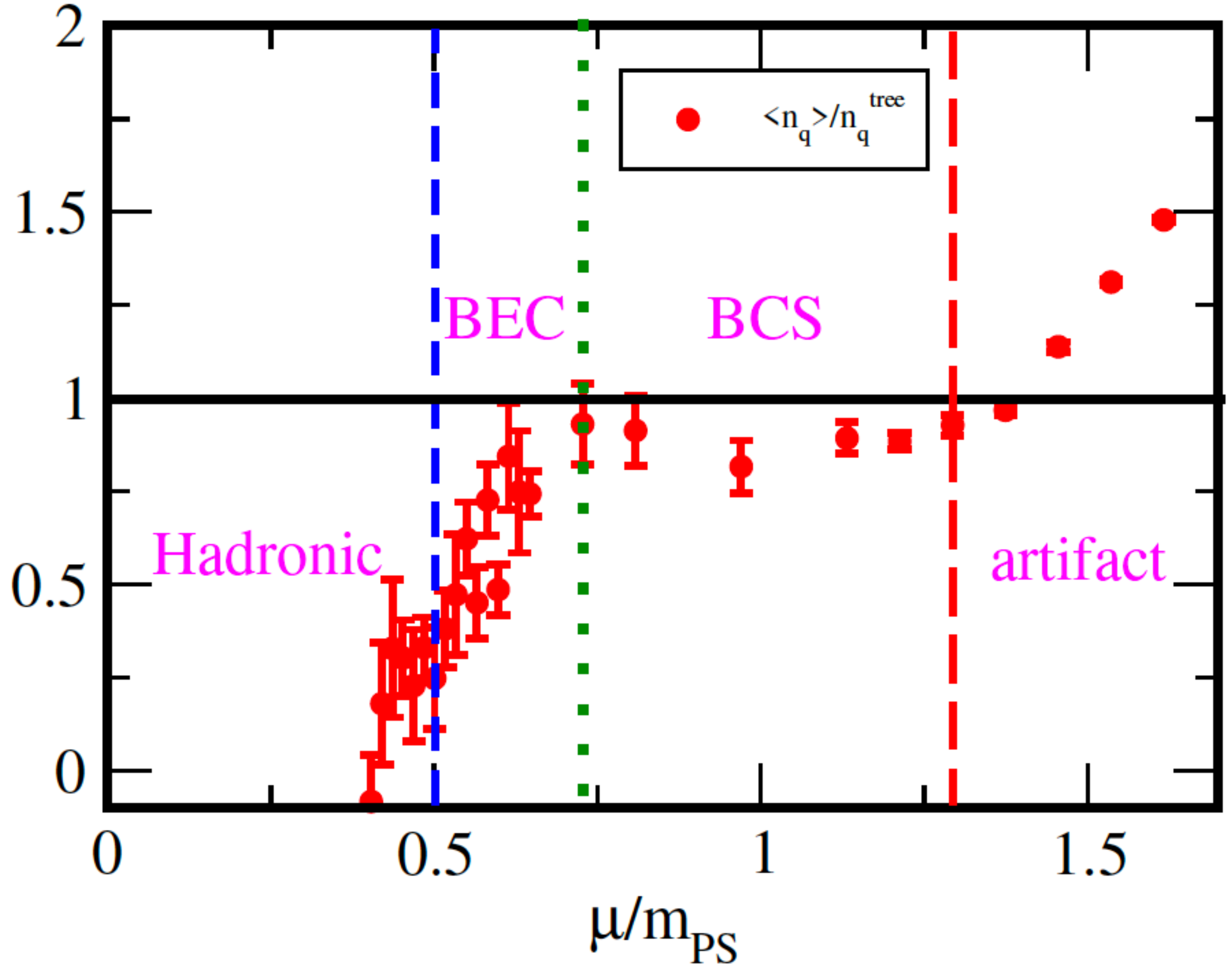
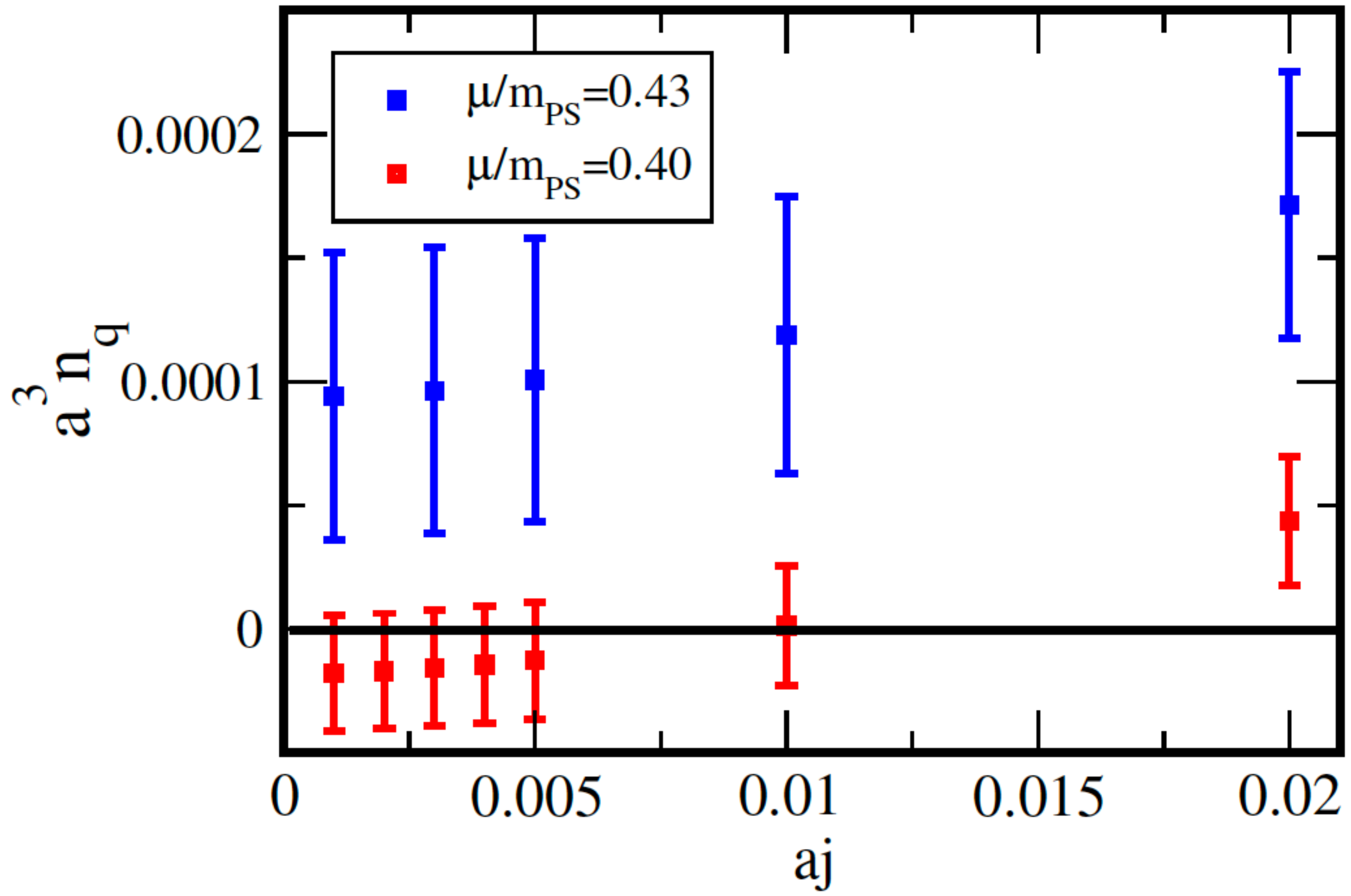


Figure 5. The j-dependence of the diquark condensate for several μ/m_{PS}.

J->0 extrapolation

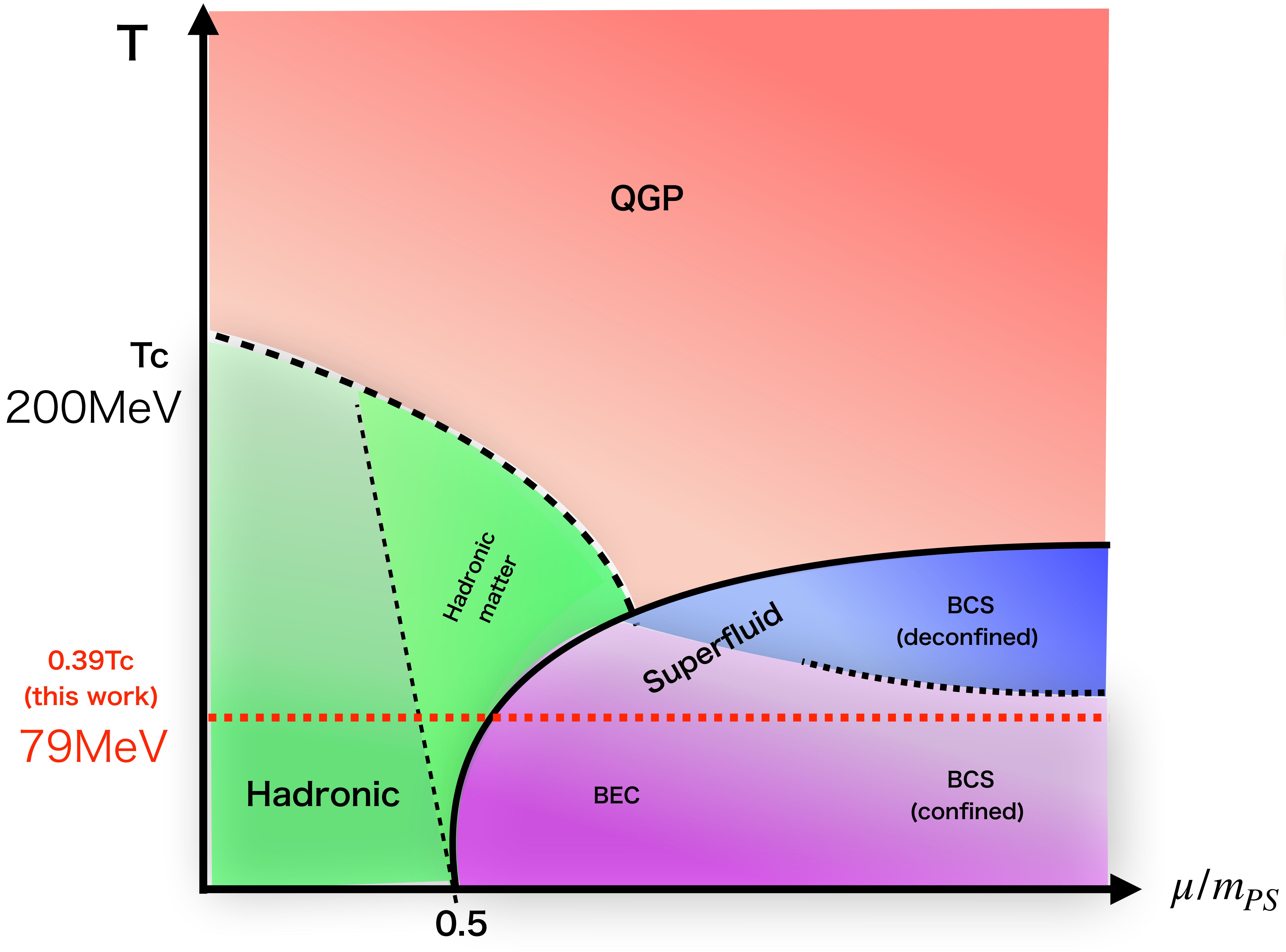
Chiral condensate and n_q have a mild j-dependence





Phase diagram of 2color QCD

Comparison with 3color QCD



Fukushima-Hatsuda (2010)