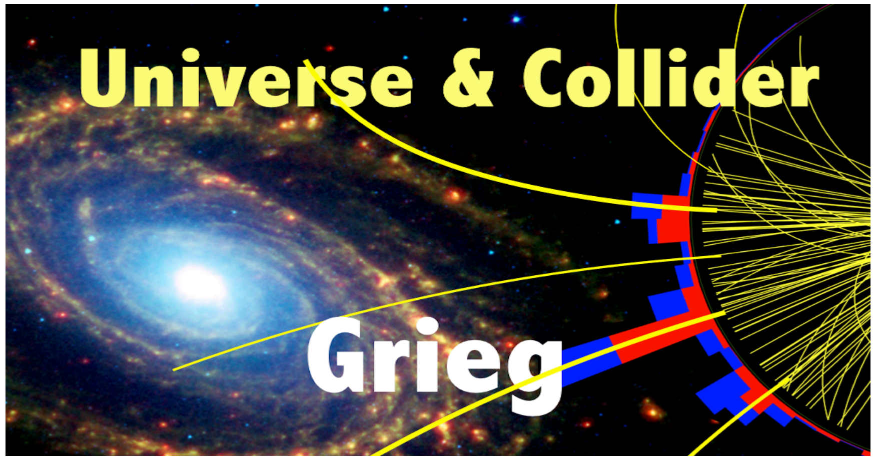
Current status of self-interacting dark matter

Ayuki Kamada (University of Warsaw)



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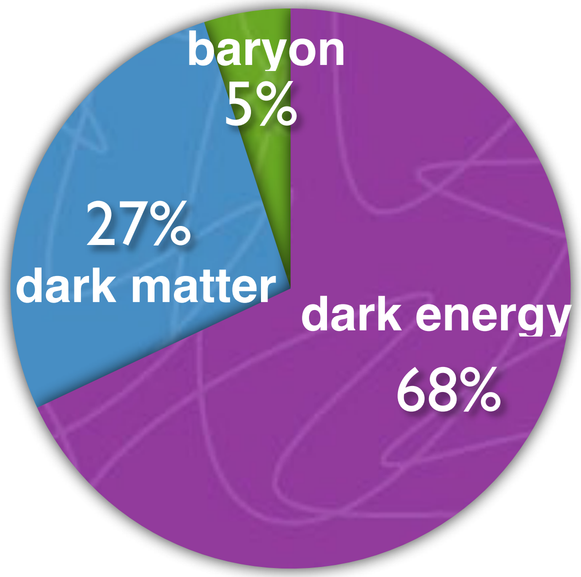
Aug. 28, 2023 @ PPP 2023

Dark matter

Dark matter

- evident from cosmological observations
  - cosmic microwave background (CMB)…
- essential to form galaxies in the Universe
- one of the biggest mysteries
  - astronomy, cosmology, particle physics…



cosmic energy budget

Gravitational probes

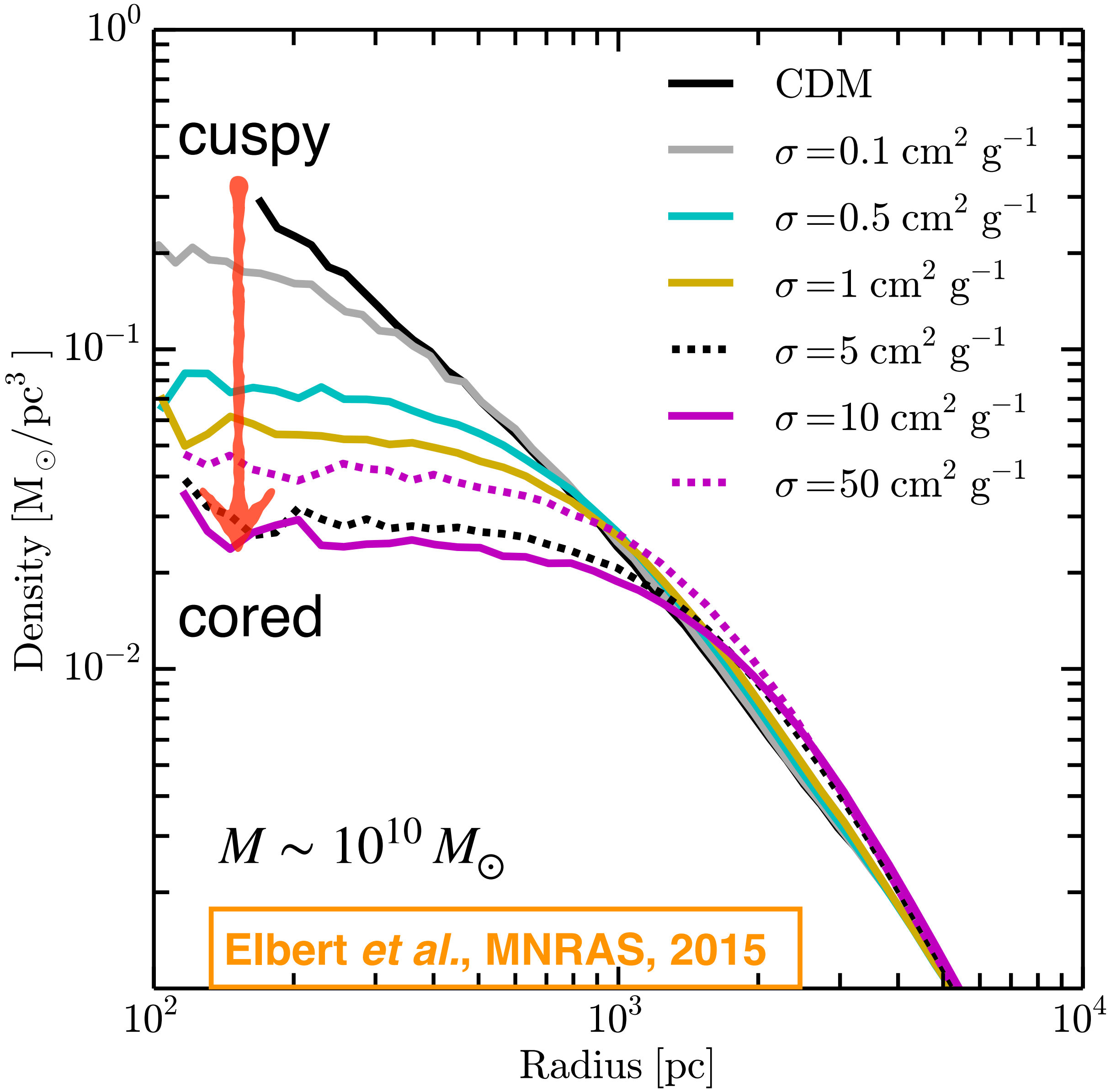
- complementary to direct, indirect and collider searches
- how the star distribution changes w/ properties of dark matter
  - all known properties of dark matter are derived in this way (including its existence; SM neutrinos are too hot to form galaxies)

Self-interacting dark matter

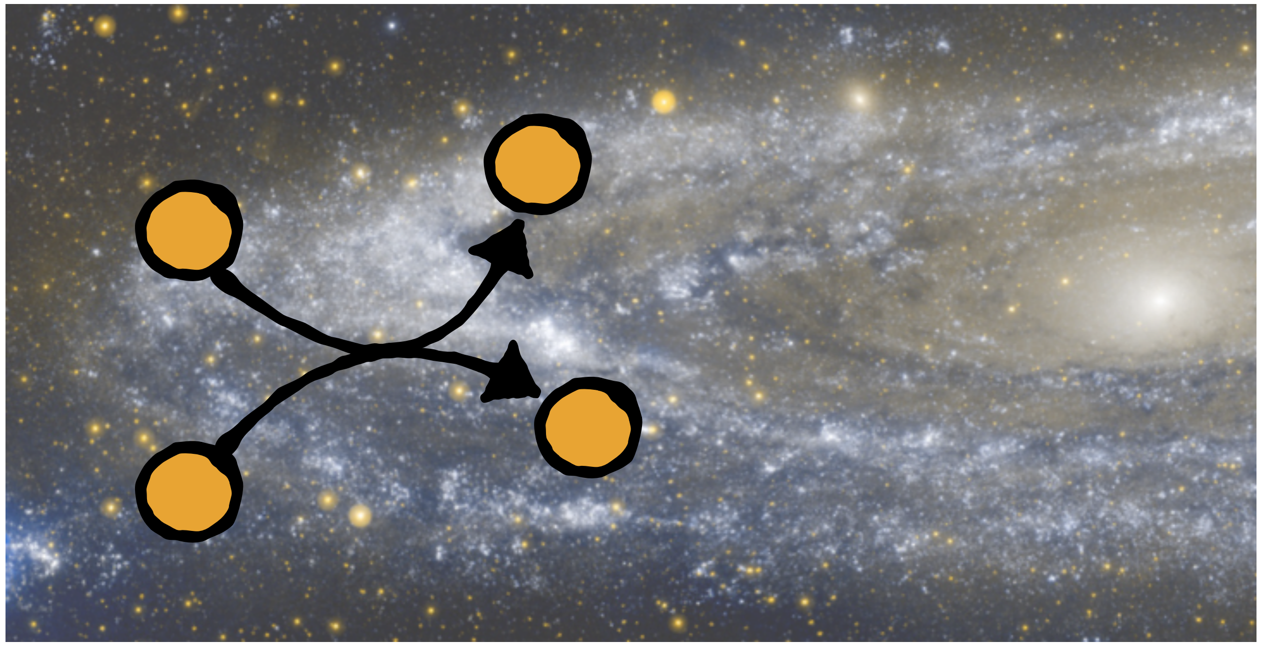
Self-interacting dark matter (SIDM)

- interactions among dark matter particles
  - hard to probe in other searches
  - dark matter density profile inside a halo turns from cuspy to cored
  - cored profile "appear to" provide better fit to astronomical data

$$\sigma/m \sim 1\,\mathrm{cm}^2/\mathrm{g} \sim 1\,\mathrm{barn/GeV}$$



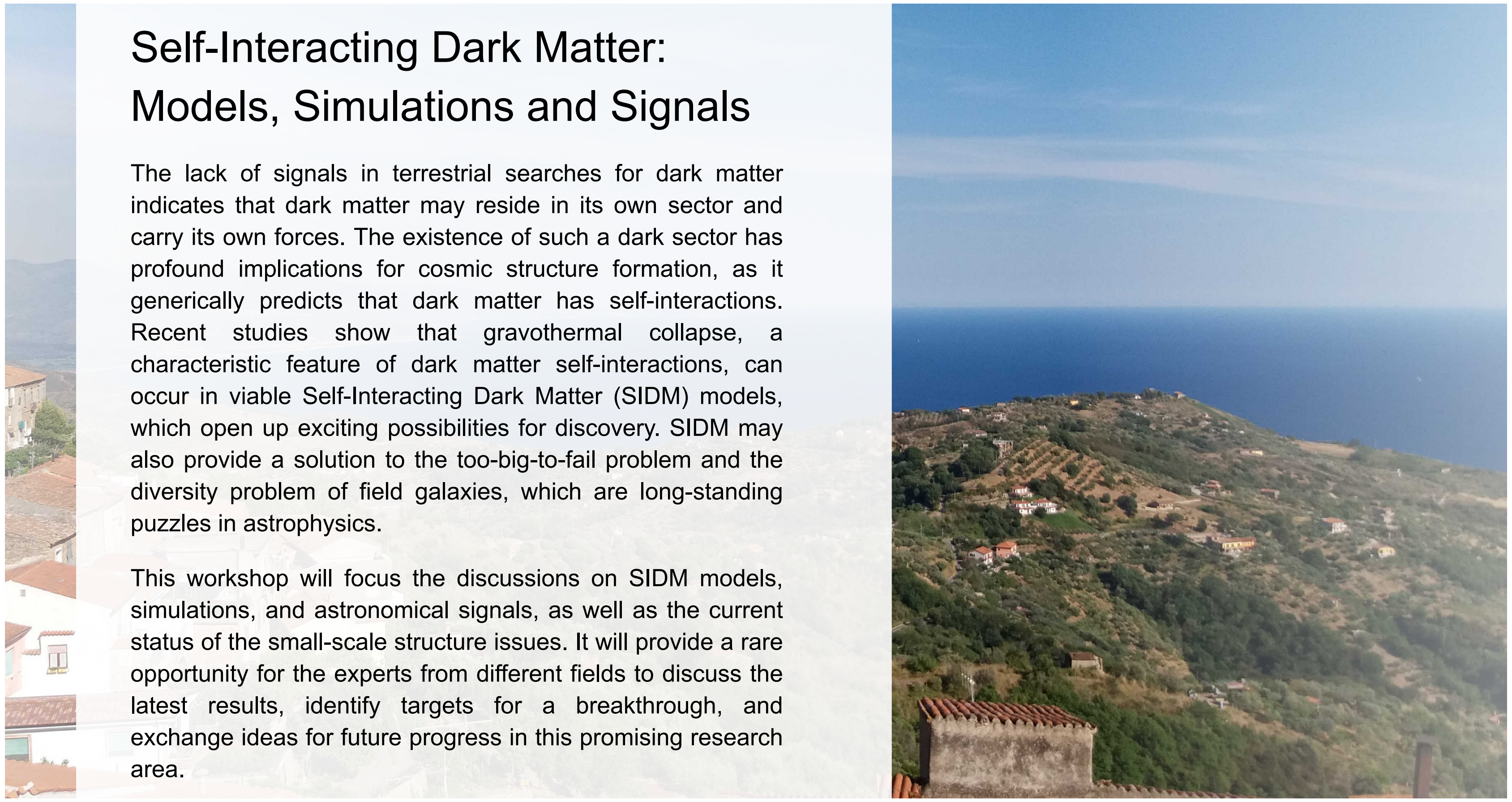
Elbert *et al.*, MNRAS, 2015



Growing interests

SIDM workshop during June 19-30 2023

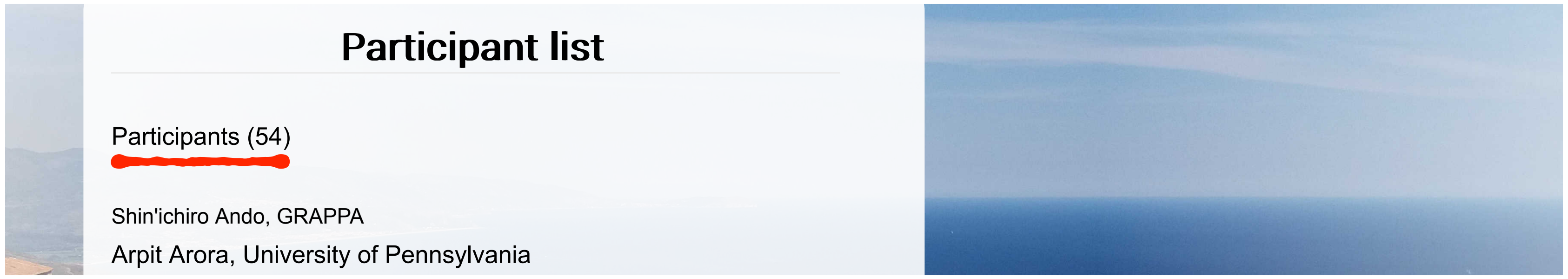
- focusing on SIDM but 50+ participants
- quickly developing and many things to do



Self-Interacting Dark Matter:
Models, Simulations and Signals

The lack of signals in terrestrial searches for dark matter indicates that dark matter may reside in its own sector and carry its own forces. The existence of such a dark sector has profound implications for cosmic structure formation, as it generically predicts that dark matter has self-interactions. Recent studies show that gravothermal collapse, a characteristic feature of dark matter self-interactions, can occur in viable Self-Interacting Dark Matter (SIDM) models, which open up exciting possibilities for discovery. SIDM may also provide a solution to the too-big-to-fail problem and the diversity problem of field galaxies, which are long-standing puzzles in astrophysics.

This workshop will focus the discussions on SIDM models, simulations, and astronomical signals, as well as the current status of the small-scale structure issues. It will provide a rare opportunity for the experts from different fields to discuss the latest results, identify targets for a breakthrough, and exchange ideas for future progress in this promising research area.



Participant list

Participants (54)

Shin'ichiro Ando, GRAPPA

Arpit Arora, University of Pennsylvania

Contents

Brief history of SIDM

- (1st stage) core vs cusp problem and constant cross section
- (2nd stage) cosmic-ray anomalies and velocity-dependent cross section

Frontier of SIDM (3rd stage)

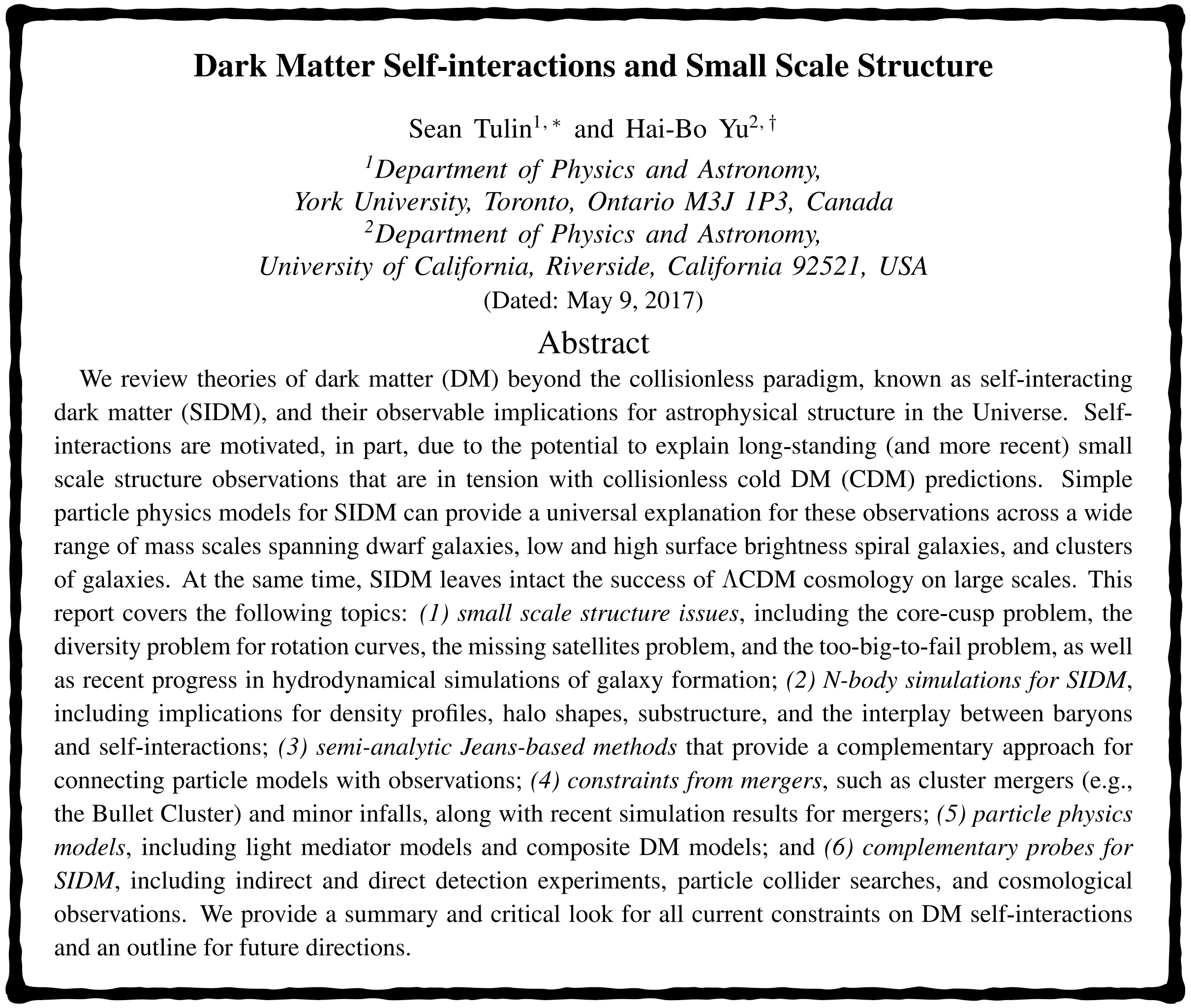
- diversity problems: dwarf spiral galaxies and satellite galaxies
- strong self-interaction and gravothermal collapse

Model-building aspects

- Sommerfeld enhancement and indirect detection

Contents

Brief history of SIDM



Dark Matter Self-interactions and Small Scale Structure

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(Dated: May 9, 2017)

Abstract

We review theories of dark matter (DM) beyond the collisionless paradigm, known as self-interacting dark matter (SIDM), and their observable implications for astrophysical structure in the Universe. Self-interactions are motivated, in part, due to the potential to explain long-standing (and more recent) small scale structure observations that are in tension with collisionless cold DM (CDM) predictions. Simple particle physics models for SIDM can provide a universal explanation for these observations across a wide range of mass scales spanning dwarf galaxies, low and high surface brightness spiral galaxies, and clusters of galaxies. At the same time, SIDM leaves intact the success of ΛCDM cosmology on large scales. This report covers the following topics: *(1) small scale structure issues*, including the core-cusp problem, the diversity problem for rotation curves, the missing satellites problem, and the too-big-to-fail problem, as well as recent progress in hydrodynamical simulations of galaxy formation; *(2) N-body simulations for SIDM*, including implications for density profiles, halo shapes, substructure, and the interplay between baryons and self-interactions; *(3) semi-analytic Jeans-based methods* that provide a complementary approach for connecting particle models with observations; *(4) constraints from mergers*, such as cluster mergers (e.g., the Bullet Cluster) and minor infalls, along with recent simulation results for mergers; *(5) particle physics models*, including light mediator models and composite DM models; and *(6) complementary probes for SIDM*, including indirect and direct detection experiments, particle collider searches, and cosmological observations. We provide a summary and critical look for all current constraints on DM self-interactions and an outline for future directions.

Astrophysical Tests of Dark Matter Self-Interactions

Susmita Adhikari[1,2*], Arka Banerjee[1], Kimberly K. Boddy[3], Francis-Yan Cyr-Racine[4],
Harry Desmond[5,6,7†], Cora Dvorkin[8], Bhuvnesh Jain[9], Felix Kahlhoefer[10],
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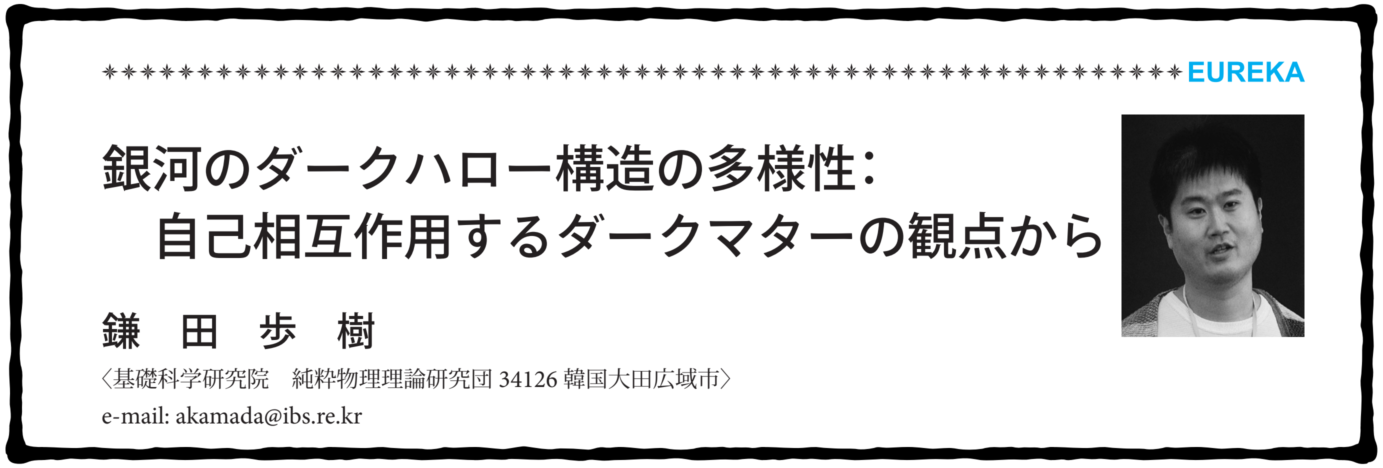

EUREKA

銀河のダークハロー構造の多様性：自己相互作用するダークマターの観点から

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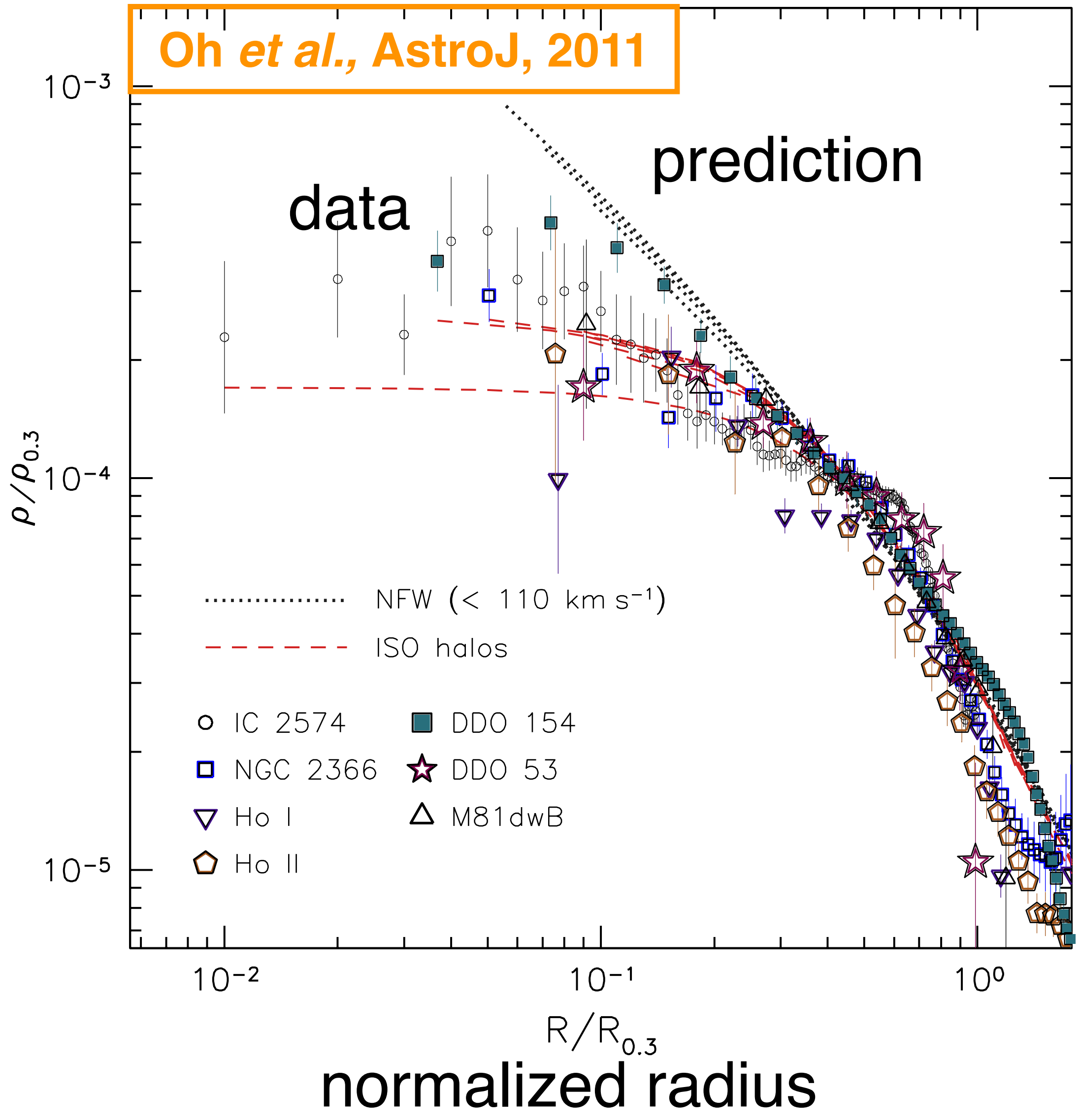
1st stage of SIDM

Core vs cusp problem (1994)

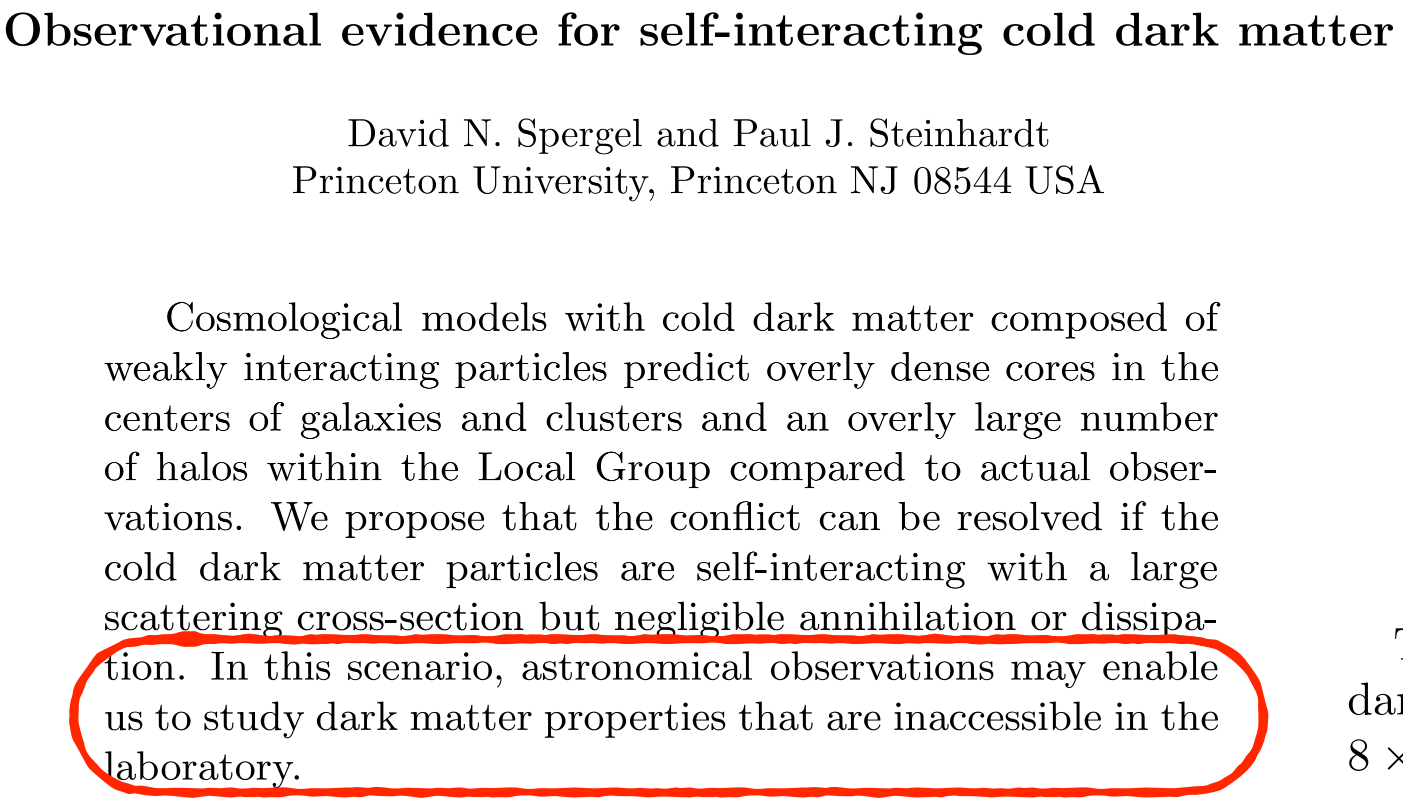
- dwarf galaxies appear to prefer a cored profile
- stellar feedback should be taken into account

SIDM as an explanation (1999)

normalized mass density



Oh *et al.*, AstroJ, 2011



Observational evidence for self-interacting cold dark matter

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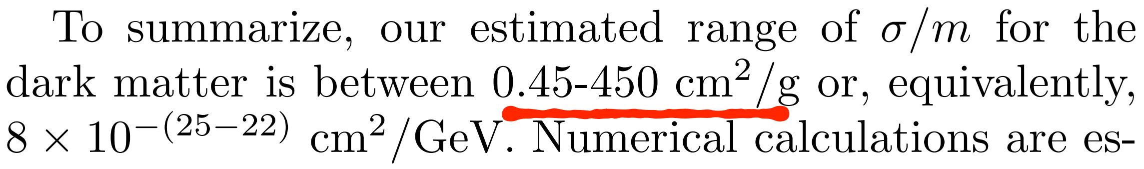
Cosmological models with cold dark matter composed of weakly interacting particles predict overly dense cores in the centers of galaxies and clusters and an overly large number of halos within the Local Group compared to actual observations. We propose that the conflict can be resolved if the cold dark matter particles are self-interacting with a large scattering cross-section but negligible annihilation or dissipation. In this scenario, astronomical observations may enable us to study dark matter properties that are inaccessible in the laboratory.

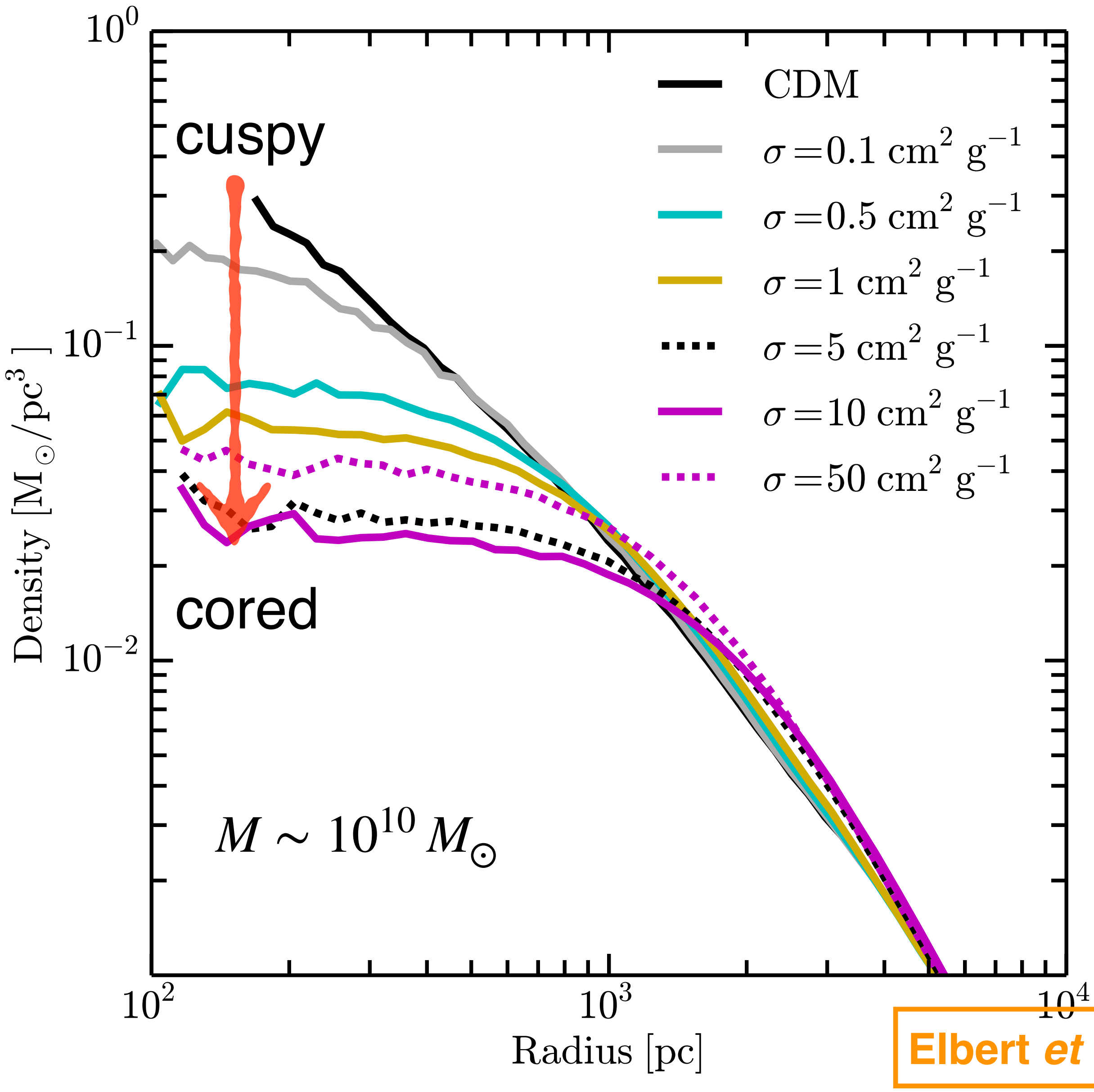


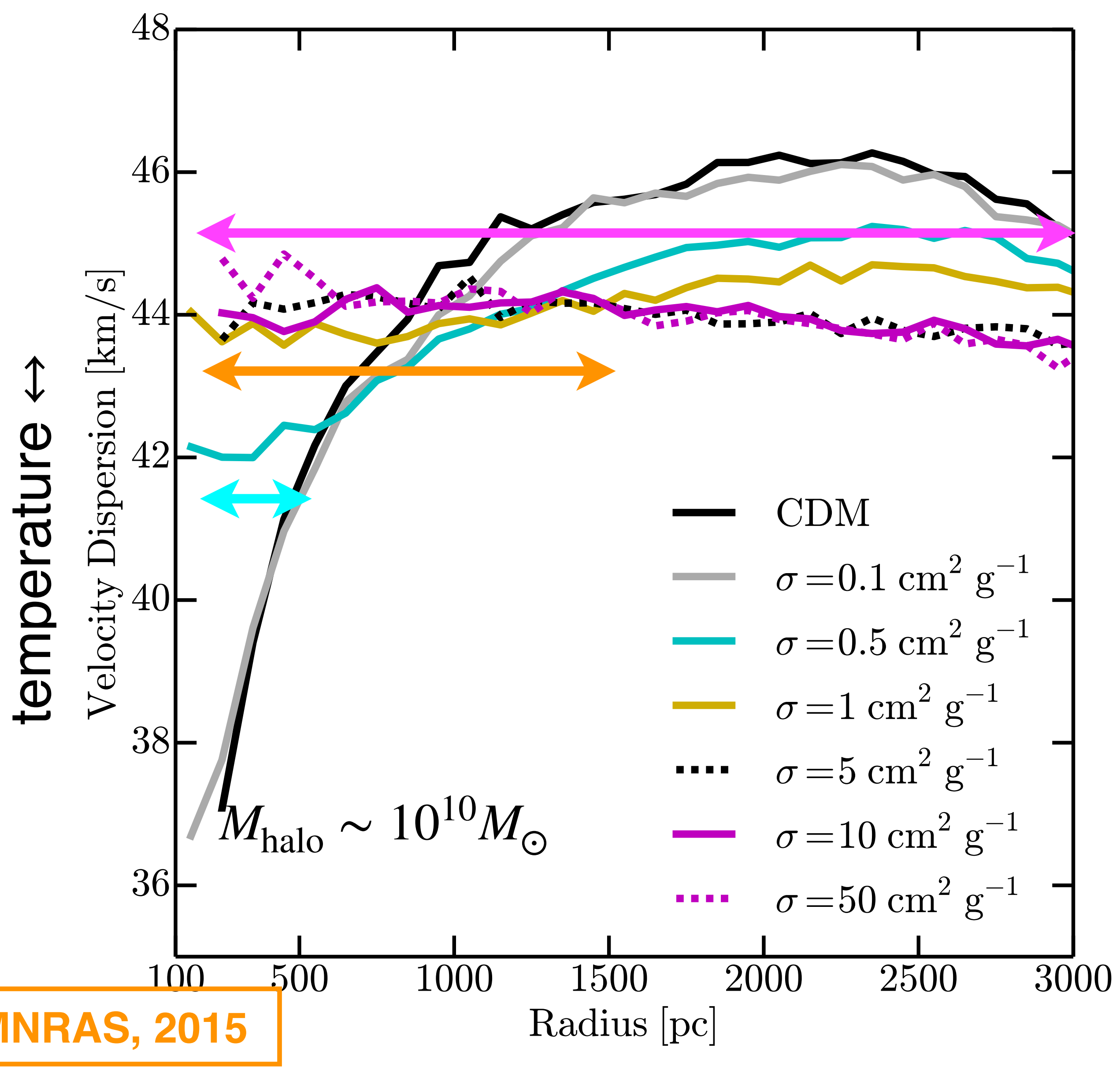
To summarize, our estimated range of σ/m for the dark matter is between 0.45-450 cm^2/g or, equivalently, $8 \times 10^{-(25-22)}$ cm^2/GeV. Numerical calculations are es-

Underlying physics of SIDM

How a core forms

- heat transfer inside a halo
- iso-thermal (equal temperature) region forms



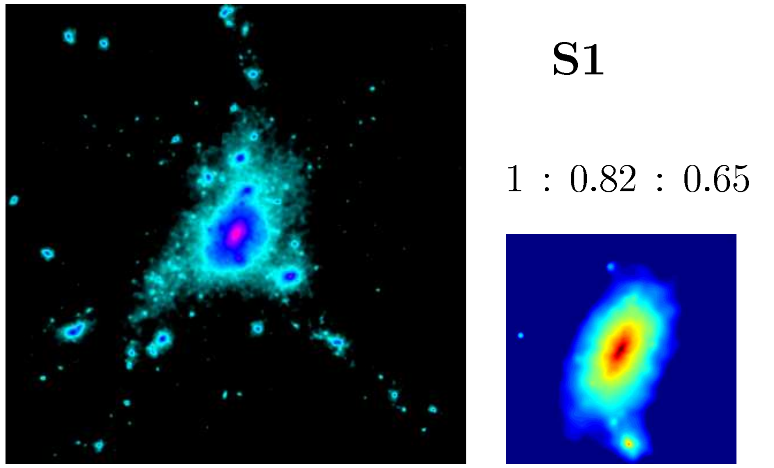


Elbert *et al.*, MNRAS, 2015

End of the 1st stage

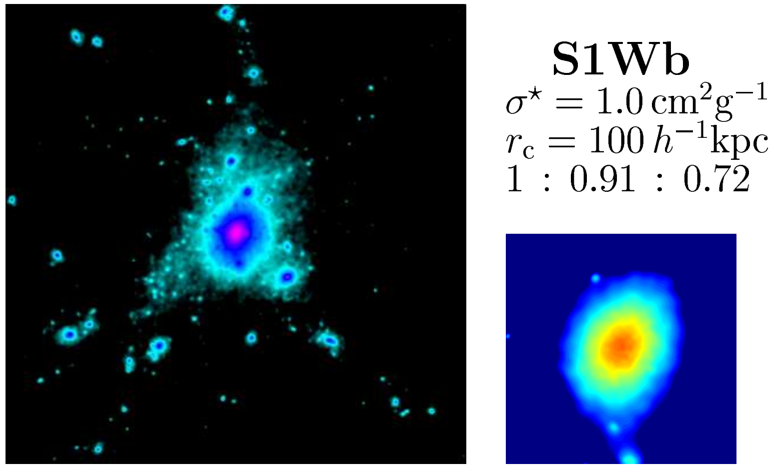
Spherical halo

- self-interaction turns a halo shape from elliptical into spherical in an iso-thermal region



Galaxy-cluster halo shape (2000)

- inferred from strong lensing



Yoshida *et al.*, ApJL, 2000

A TEST OF THE COLLISIONAL DARK MATTER HYPOTHESIS FROM CLUSTER LENSING

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Received 2000 February 8; accepted 2001 August 29

at $z_s = 1$. Assuming also a cluster velocity dispersion $\sigma = 1000$ km s^{-1} (roughly the minimum value required given the Einstein radius of the cluster), and a cluster age $t_c = 5 \times 10^9$ years, we obtain the upper limit

$$\frac{s_x}{m_x} < \frac{1}{\rho 2^{1/2} \sigma t_c} \simeq 10^{-25.5}\,\frac{\mathrm{cm}^2}{m_p} \simeq \underline{0.02\,\frac{\mathrm{cm}^2}{\mathrm{g}}}\,. \qquad (14)$$

- not consistent?

To summarize, our estimated range of σ/m for the dark matter is between 0.45-450 cm^2/g or, equivalently, $8 \times 10^{-(25-22)}$ cm^2/GeV. Numerical calculations are es-

Dormant decade

Velocity dependence (self-interaction)

- typical collision velocity in a galaxy cluster $\langle v_{\rm rel}\rangle \sim 10^3\,{\rm km/s}$
is 10 times higher than that in a dwarf galaxy $\langle v_{\rm rel}\rangle \sim 10^2\,{\rm km/s}$

- but SIDM was not considered for a decade after that

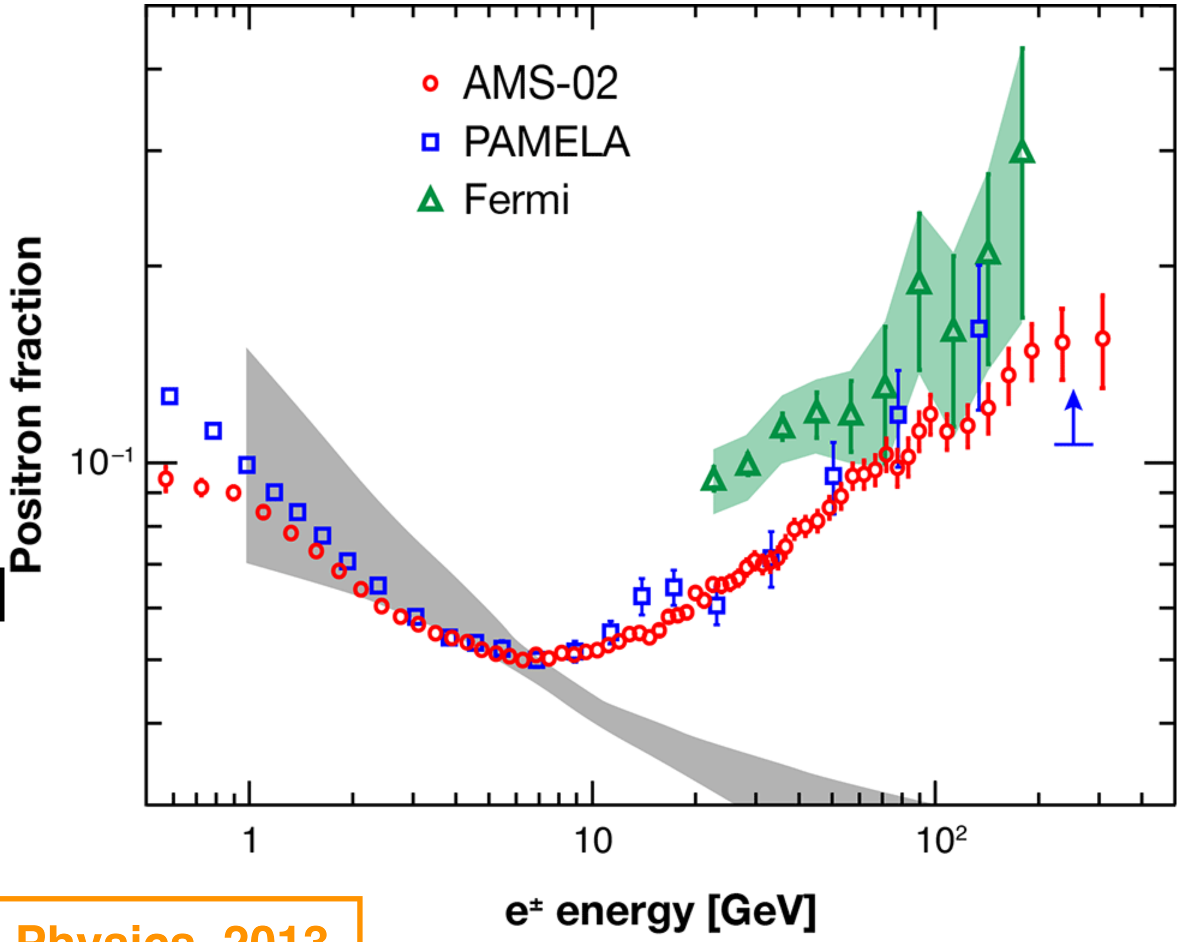
  - maybe not well motivated to consider a “complication” (velocity dependence)

Cosmic-ray anomalies (2008)

- Pamela, ATIC… reports an excess in electron/positron flux

- requires 100 times larger cross section (boost factor) than canonical value for correct relic abundance

$$\Omega h^2 = 0.1 \times \frac{3\times 10^{-26}\,{\rm cm^3/s}}{\langle \sigma_{\rm ann} v\rangle}$$



Coutu, Physics, 2013

Dark force and light mediator

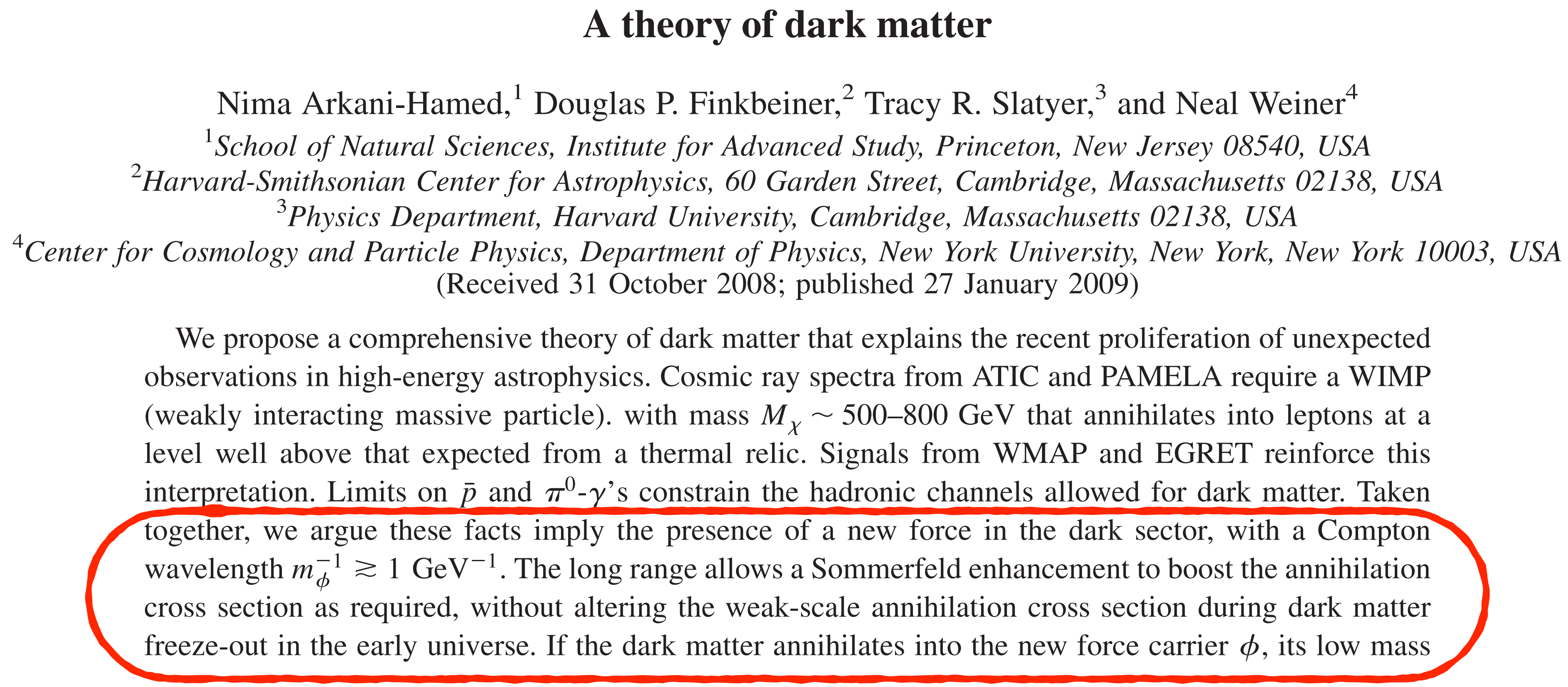
Velocity dependence (annihilation)

- typical collision velocity around the freeze-out $\langle v_{\rm rel} \rangle \simeq 1.5 \times 10^5\,{\rm km/s}$ is much higher than that in our MW galaxy $\langle v_{\rm rel} \rangle \sim 10^2\,{\rm km/s}$

Dark force and light mediator as an explanation (2009)

- dark matter may have its own “long-range” force $V = -\dfrac{\alpha_\chi}{r} e^{-m_\phi r}$
- non-minimal dark sector

- Yukawa potential



A theory of dark matter

Nima Arkani-Hamed,[1] Douglas P. Finkbeiner,[2] Tracy R. Slatyer,[3] and Neal Weiner[4]
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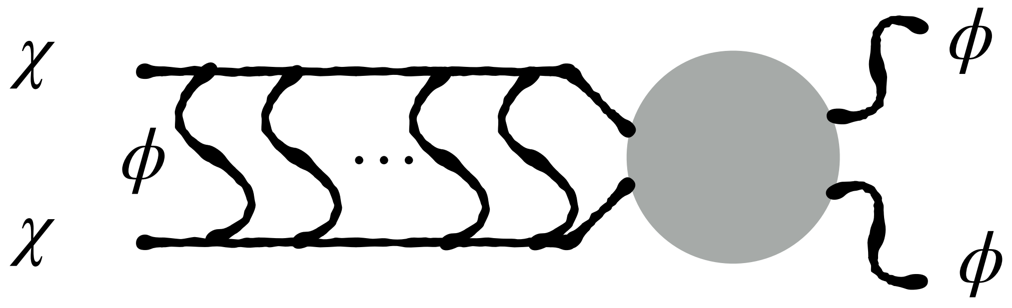
[2] *Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA*
[3] *Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA*
[4] *Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York 10003, USA*
(Received 31 October 2008; published 27 January 2009)

We propose a comprehensive theory of dark matter that explains the recent proliferation of unexpected observations in high-energy astrophysics. Cosmic ray spectra from ATIC and PAMELA require a WIMP (weakly interacting massive particle). with mass $M_\chi \sim 500$–800 GeV that annihilates into leptons at a level well above that expected from a thermal relic. Signals from WMAP and EGRET reinforce this interpretation. Limits on $\bar{p}$ and π^0-γ's constrain the hadronic channels allowed for dark matter. Taken together, we argue these facts imply the presence of a new force in the dark sector, with a Compton wavelength $m_\phi^{-1} \gtrsim 1\ {\rm GeV}^{-1}$. The long range allows a Sommerfeld enhancement to boost the annihilation cross section as required, without altering the weak-scale annihilation cross section during dark matter freeze-out in the early universe. If the dark matter annihilates into the new force carrier ϕ, its low mass

Sommerfeld enhancement

How dark force explains velocity dependence in annihilation

- attractive dark force increases the probability of finding two DM particles at the “zero” distance

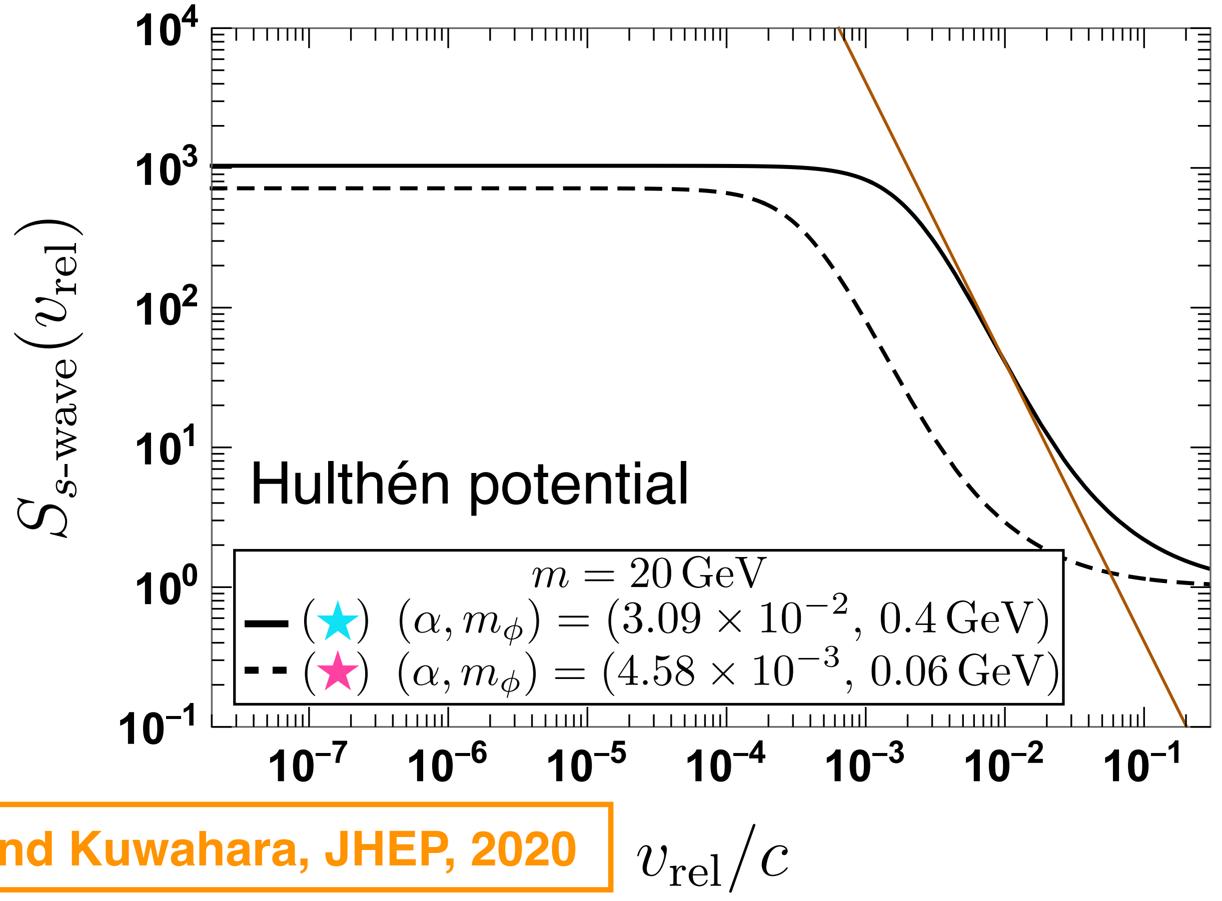


- annihilation cross section is enhanced at low velocity (Sommerfeld enhancement)

  - high velocity particle does not care the potential

$$(\sigma_{\rm ann} v_{\rm rel}) = S(\sigma^{(0)}_{\rm ann} v_{\rm rel})$$

- without potential



AK, Kim and Kuwahara, JHEP, 2020

Scattering in quantum mechanics

Weinberg, “Lectures on Quantum Mechanics”

Schrödinger equation

$$\left[-\frac{1}{2\mu}\nabla^2 + V(r)\right]\psi_k(\vec{x}) = E\psi_k(\vec{x}) \qquad E = \frac{k^2}{2\mu} \qquad k = \mu v_{\mathrm{rel}}$$

- potential from long-range force

- reduced mass ($\mu = m/2$ for identical particle)

- scattering state (energy-eigenstate of Schrödinger equation)

$$\psi_k(\vec{x}) \to e^{ikz} + f(k,\theta)\frac{e^{ikr}}{r} \qquad r \to \infty$$

- (in-coming) plane wave

- scattering amplitude

- out-going spherical wave

Partial-wave decomposition

$$e^{ikz} = \sum_{\ell=0}^{\infty} \frac{1}{2ikr}(2\ell+1)R_{k,\ell}(r)P_\ell(\cos\theta)\left(e^{ikr} - e^{-i(kr-\ell\pi)}\right)$$

$$\psi_k(\vec{x}) = \sum_{\ell=0}^{\infty} \frac{1}{k} e^{i\left(\frac{1}{2}\ell\pi + \delta_\ell\right)}(2\ell+1)R_{k,\ell}(r)P_\ell(\cos\theta)$$

- radial Schrödinger equation

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + k^2 - \frac{\ell(\ell+1)}{r^2} - 2\mu V(r)\right]R_{k,\ell}(r) = 0$$

Scattering in quantum mechanics

Scattering phase

- radial wave function at infinity

$$R_{k,\ell}(r) \to \frac{\sin(kr - \frac{1}{2}\ell\pi + \delta_\ell)}{r} \quad r \to \infty$$

$$f(k,\theta) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(k) P_\ell(\cos\theta) \quad f_\ell(k) = \frac{e^{2i\delta_\ell} - 1}{2ik}$$

$$\sigma = \sum_{\ell=0}^{\infty} \sigma_\ell \quad \sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell(k)$$

- diagonalized S-matrix $S_\ell = e^{2i\delta_\ell}$

Sommerfeld enhancement

Iengo, JHEP, 2009

Cassel, J.Phys.G, 2010

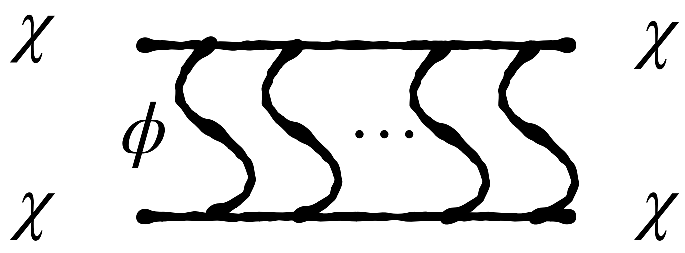
- radial wave function around the origin
  - annihilation through the contact interaction (delta function potential)

$$S_{k,\ell} = \left| \frac{R_{k,\ell}(r)}{R^{(0)}_{k,\ell}(r)} \right|^2 \quad r \to 0$$

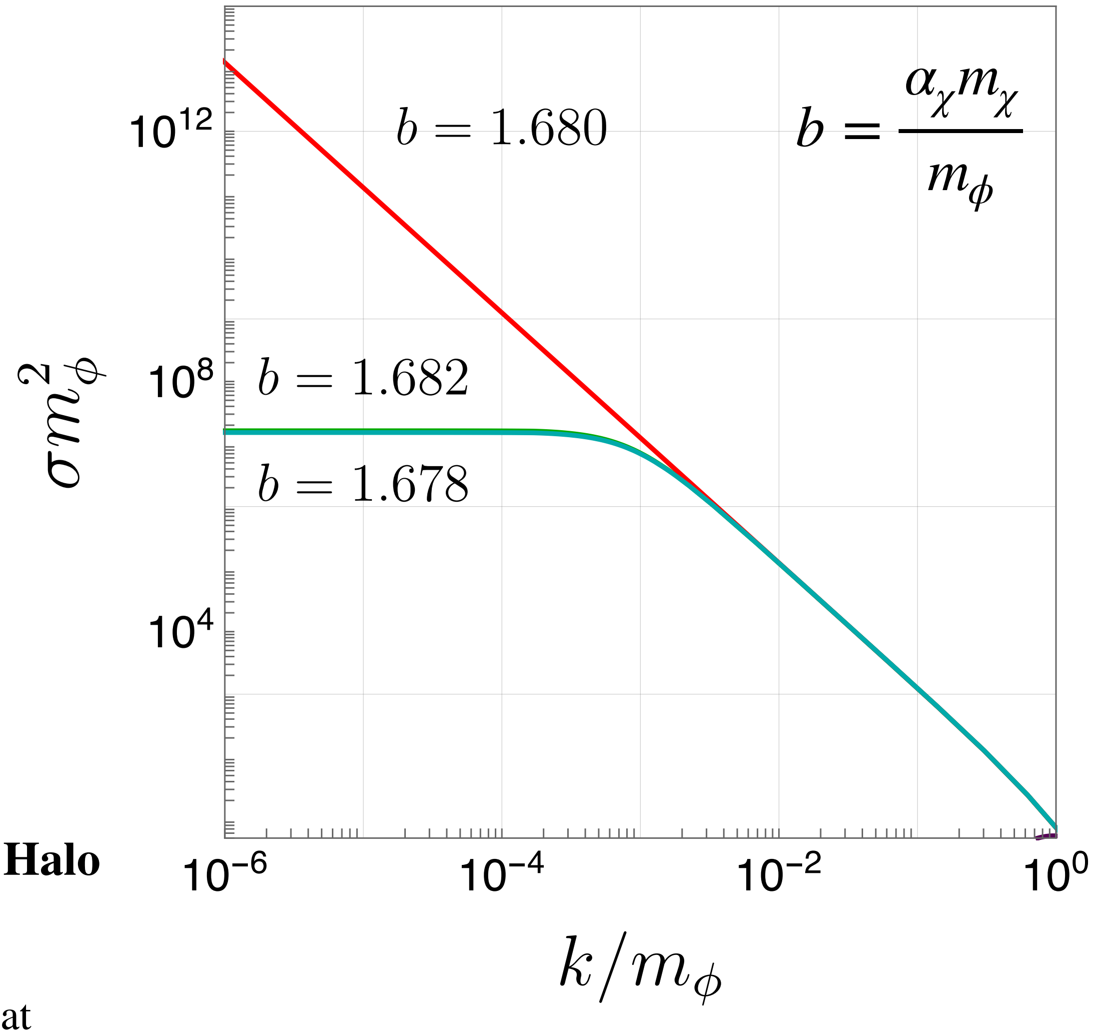
- without potential

2nd stage of SIDM

Dark force also introduces velocity dependence in self-interaction

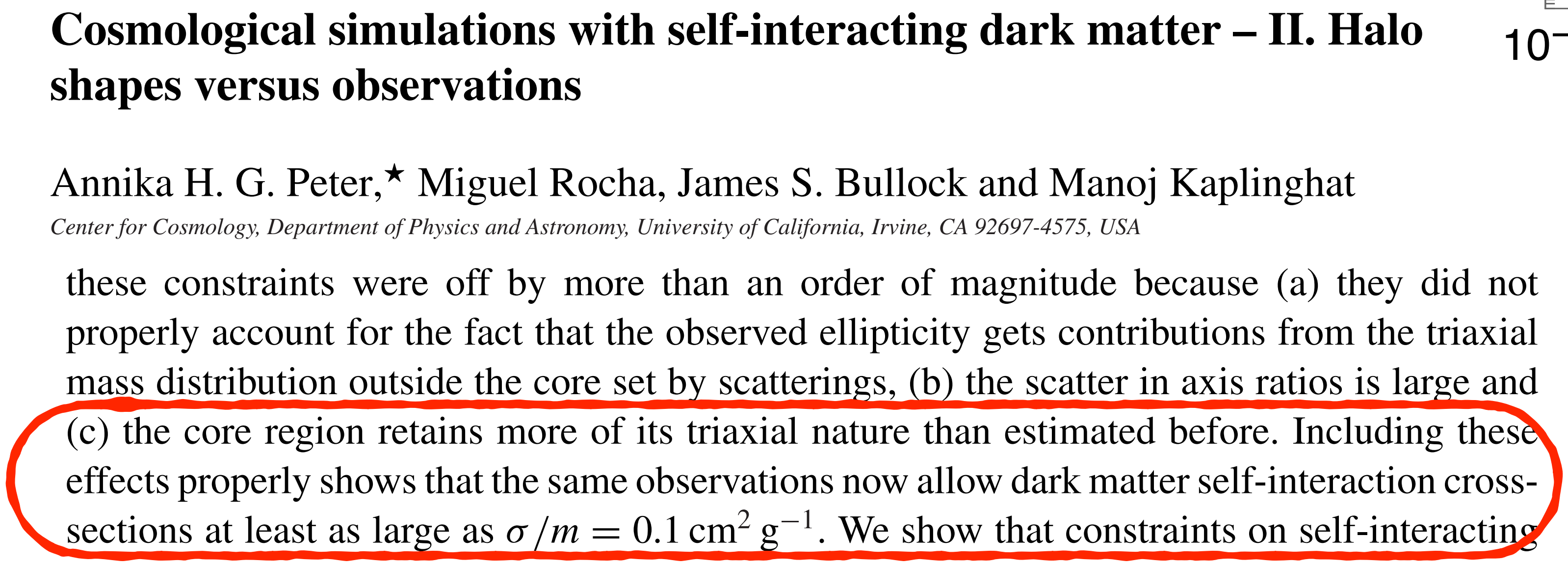


- people “re”-started to study SIDM again



AK, Kuwahara and Patel, arXiv:2303.17961

Re-assessment of galaxy-cluster halo shape (2014)



Cosmological simulations with self-interacting dark matter – II. Halo shapes versus observations

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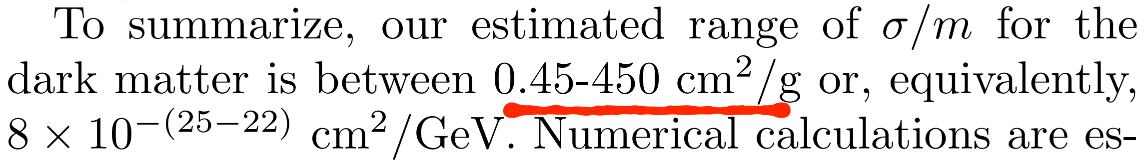
Center for Cosmology, Department of Physics and Astronomy, University of California, Irvine, CA 92697-4575, USA

these constraints were off by more than an order of magnitude because (a) they did not properly account for the fact that the observed ellipticity gets contributions from the triaxial mass distribution outside the core set by scatterings, (b) the scatter in axis ratios is large and (c) the core region retains more of its triaxial nature than estimated before. Including these effects properly shows that the same observations now allow dark matter self-interaction cross-sections at least as large as $\sigma/m = 0.1\ \mathrm{cm}^2\ \mathrm{g}^{-1}$. We show that constraints on self-interacting



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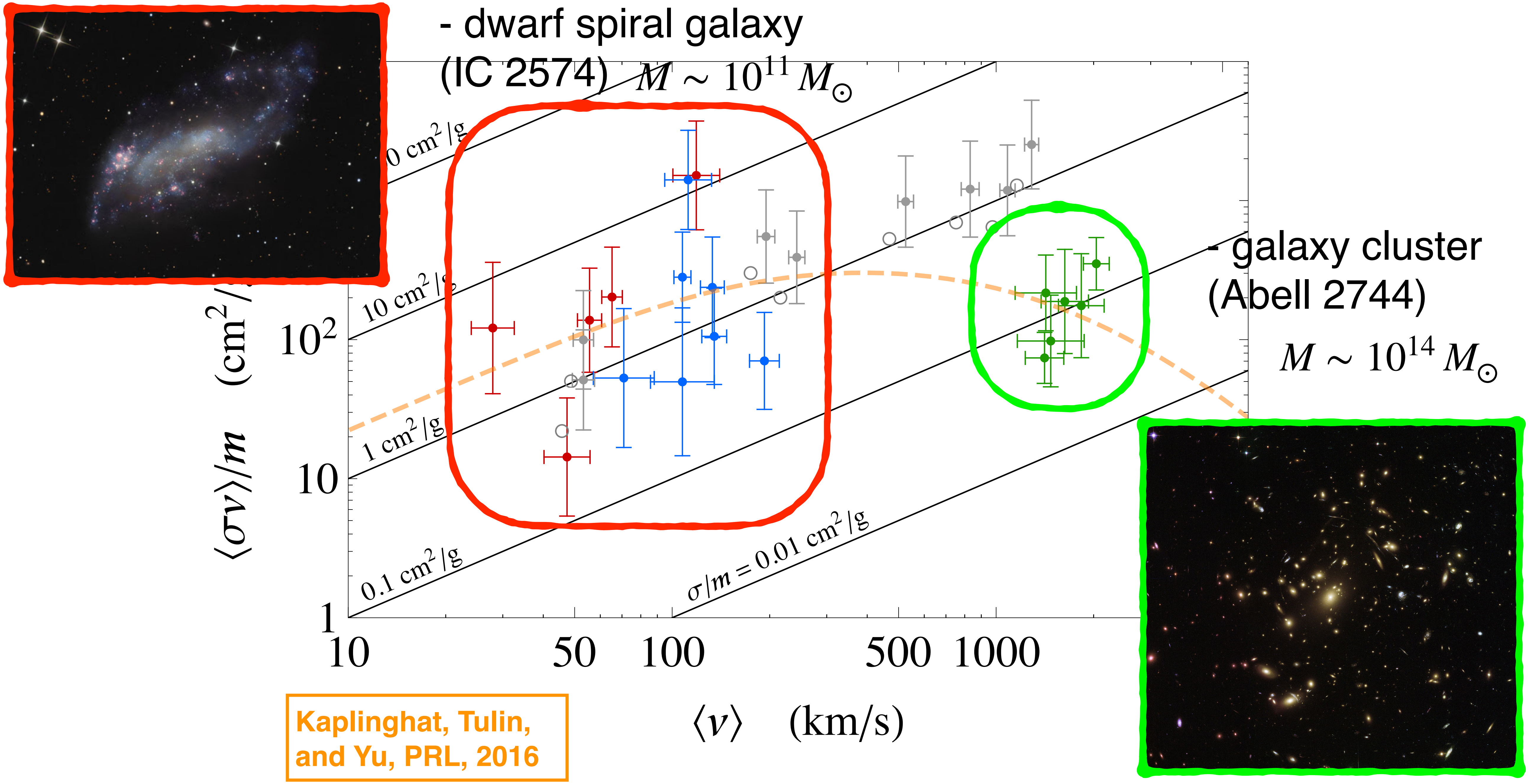
“Data” points

Overview

- cores in various-size halos may prefer velocity dependence of self-scattering cross section



- dwarf spiral galaxy (IC 2574)

- galaxy cluster (Abell 2744)

Kaplinghat, Tulin, and Yu, PRL, 2016

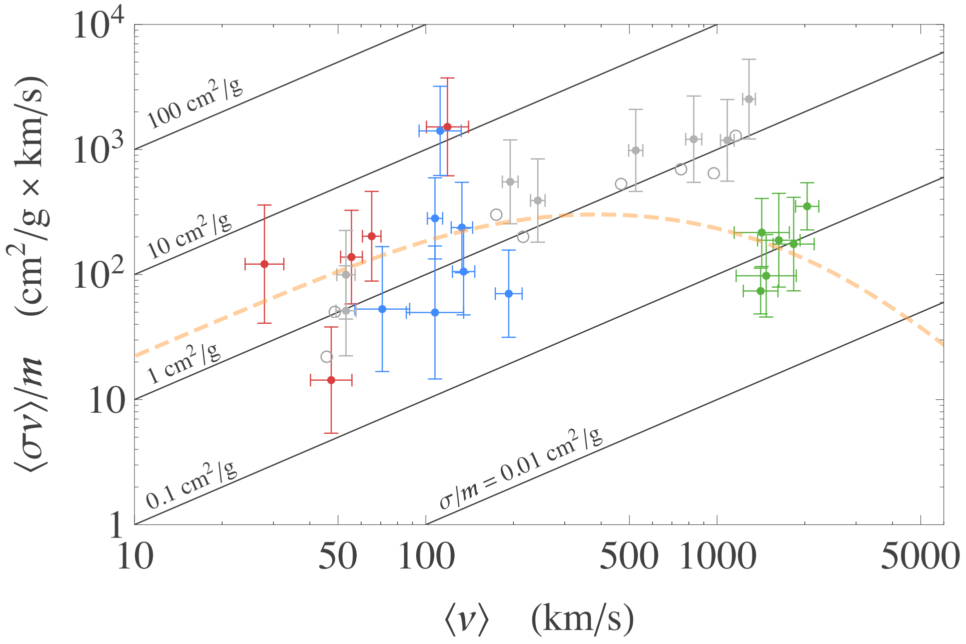
Data points

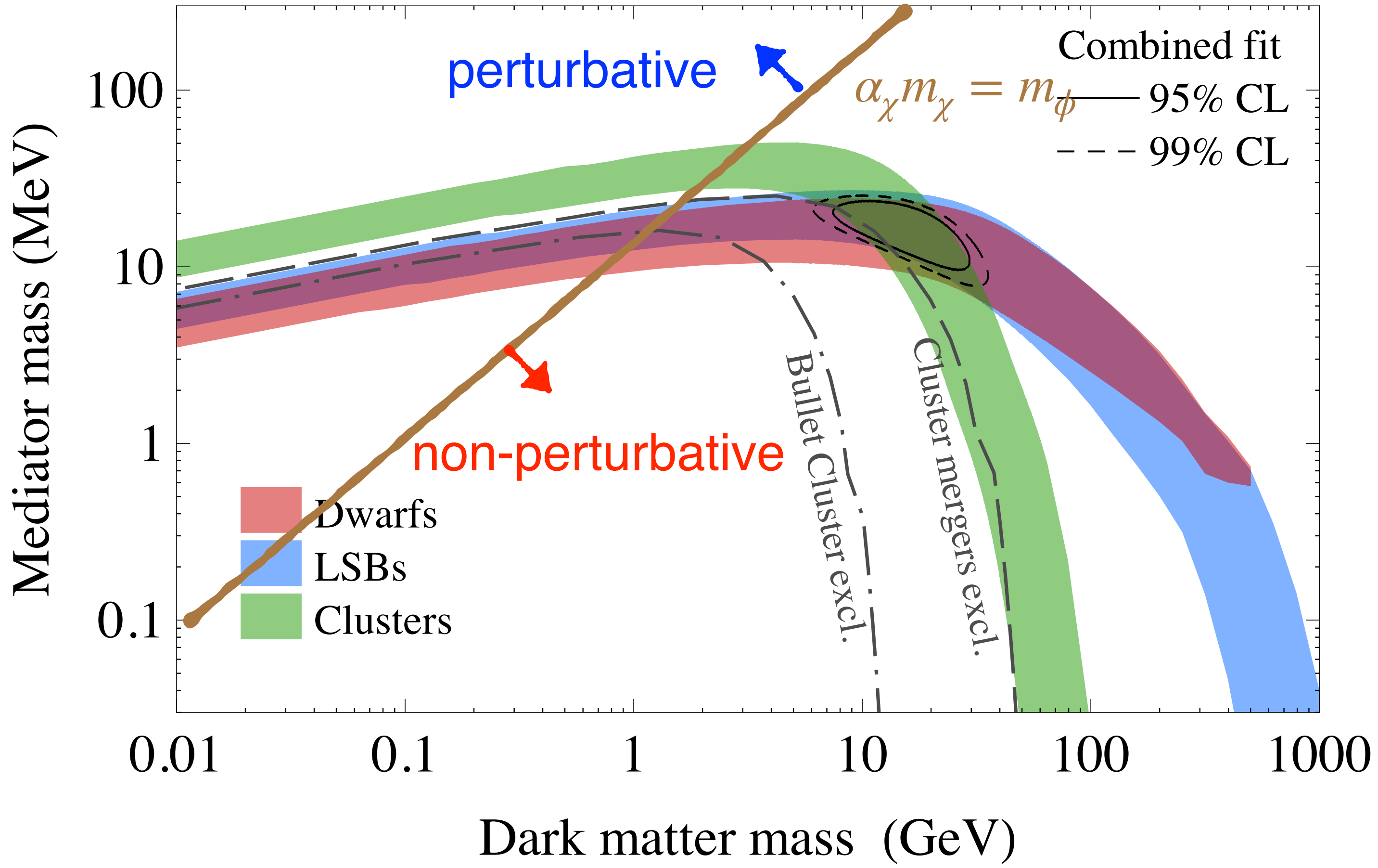
Light mediator fit to data

- DM and mediator masses are pinned down $m_\chi \sim 10\,\mathrm{GeV}$ $m_\phi \sim 10\,\mathrm{MeV}$

  - repulsive Yukawa (for simplicity)

$$\alpha_\chi = \alpha_{\mathrm{em}}$$





- perturbative: Born (tree-level) approximation is good

- non-perturbative: need to solve Schrödinger equation (resummation of ladder diagrams)

Tulin, Yu, and Zurek, PRD, 2013

Summary

Self-interacting dark matter

- turns a density profile from core to cusp

Velocity dependence

- inferred by combining observations of different size halos
- larger cross section for smaller velocity is preferred
- non-minimal dark sector
    - light mediator

Cross section computation

- non-perturbative effect is important
    - solve Schrodinger equation

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Frontier of SIDM (3rd stage)

- diversity problems: dwarf spiral galaxies and satellite galaxies
- strong self-interaction and gravothermal collapse

Model-building aspects

- Sommerfeld enhancement and indirect detection

Diversity in dwarf spiral galaxies

Rotation curves

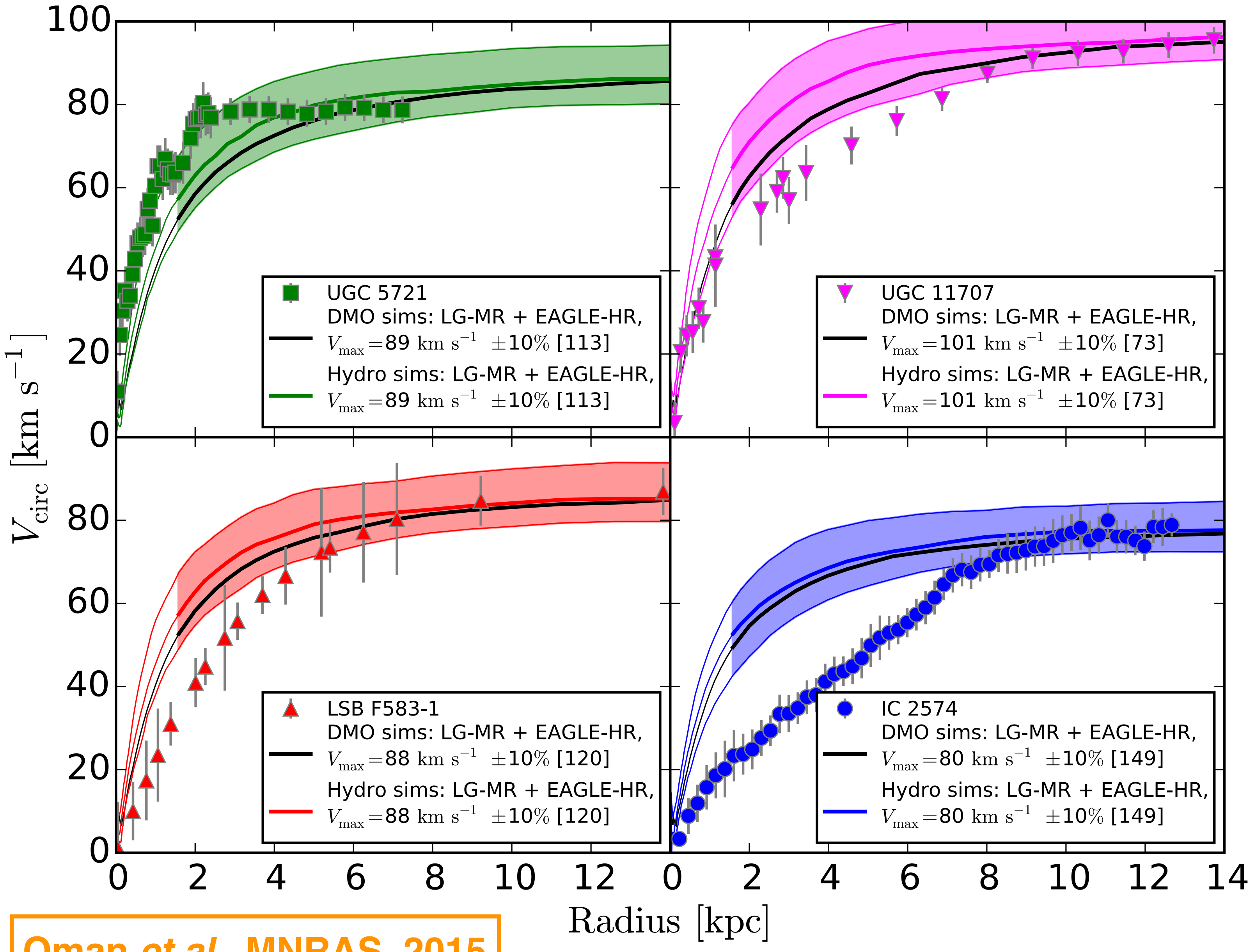
- simulation: inner circular velocity is almost uniquely determined by outer circular velocity

$$\rho_{\rm NFW} = \frac{\rho_s}{r/r_s(1+r/r_s)^2}$$

- ρ_s and r_s are not independent (concentration-mass relation)

↕

- observation: diverse inner circular velocity

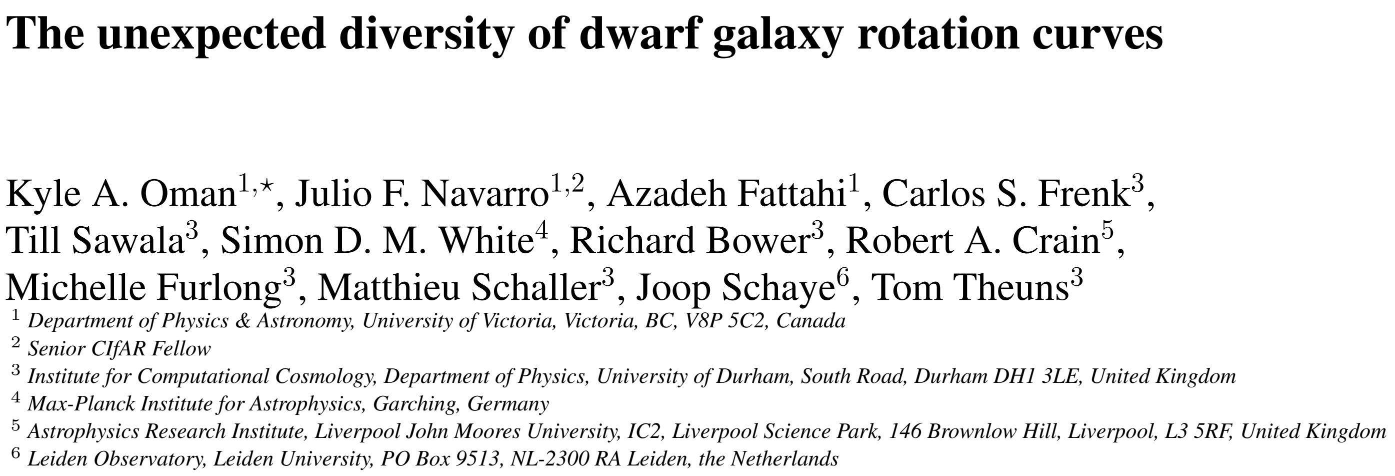


Oman *et al.*, MNRAS, 2015

Can SIDM explain it?

Naively, no

- SIDM has a universal impact



The unexpected diversity of dwarf galaxy rotation curves

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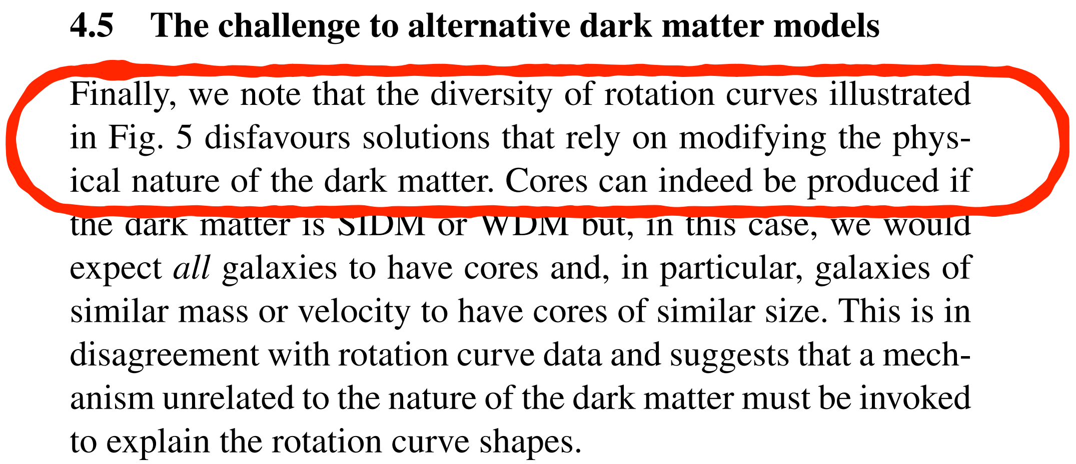
[6] *Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, the Netherlands*



4.5 The challenge to alternative dark matter models

Finally, we note that the diversity of rotation curves illustrated in Fig. 5 disfavours solutions that rely on modifying the physical nature of the dark matter. Cores can indeed be produced if the dark matter is SIDM or WDM but, in this case, we would expect *all* galaxies to have cores and, in particular, galaxies of similar mass or velocity to have cores of similar size. This is in disagreement with rotation curve data and suggests that a mechanism unrelated to the nature of the dark matter must be invoked to explain the rotation curve shapes.

Really? But galactic disks show diversity

- different disk sizes in different halos
- SIDM profile is exponentially sensitive to baryon distribution

$$\rho_{\rm DM}(\vec{x}) = \rho^0_{\rm DM}\exp(-\phi(\vec{x})/\sigma^2)$$

$$\Delta\phi = 4\pi G(\rho_{\rm DM} + \boxed{\rho_{\rm baryon}})$$

- iso-thermal region forms through self-interaction

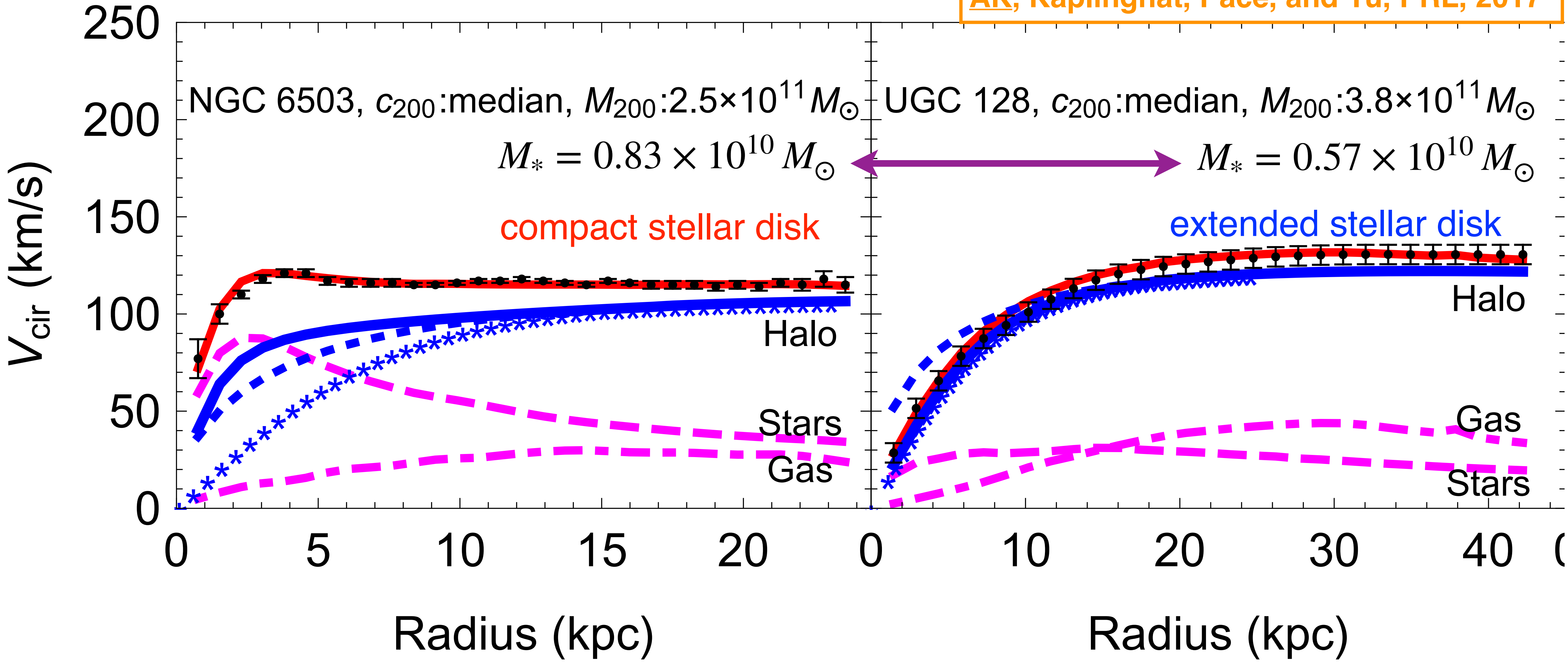
SIDM explanation

SIDM reproduces diversity (unlike a naive expectation)

- compact disk→ redistribute SIDM significantly
- extended disk→ unchange SIDM distribution

$\sigma/m = 3\,\mathrm{cm}^2/\mathrm{g}$

AK, Kaplinghat, Pace, and Yu, PRL, 2017



Diversity in MW satellites

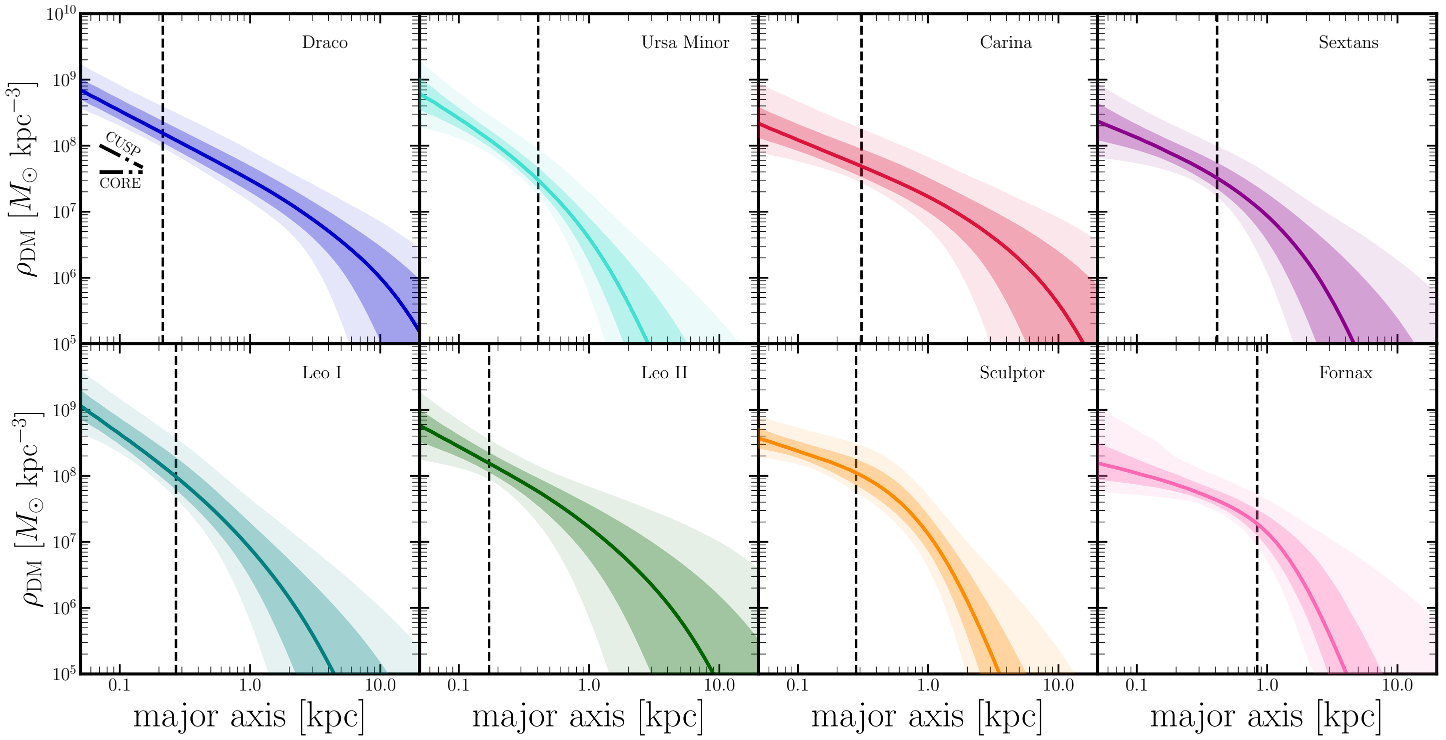
MW satellites (classical)

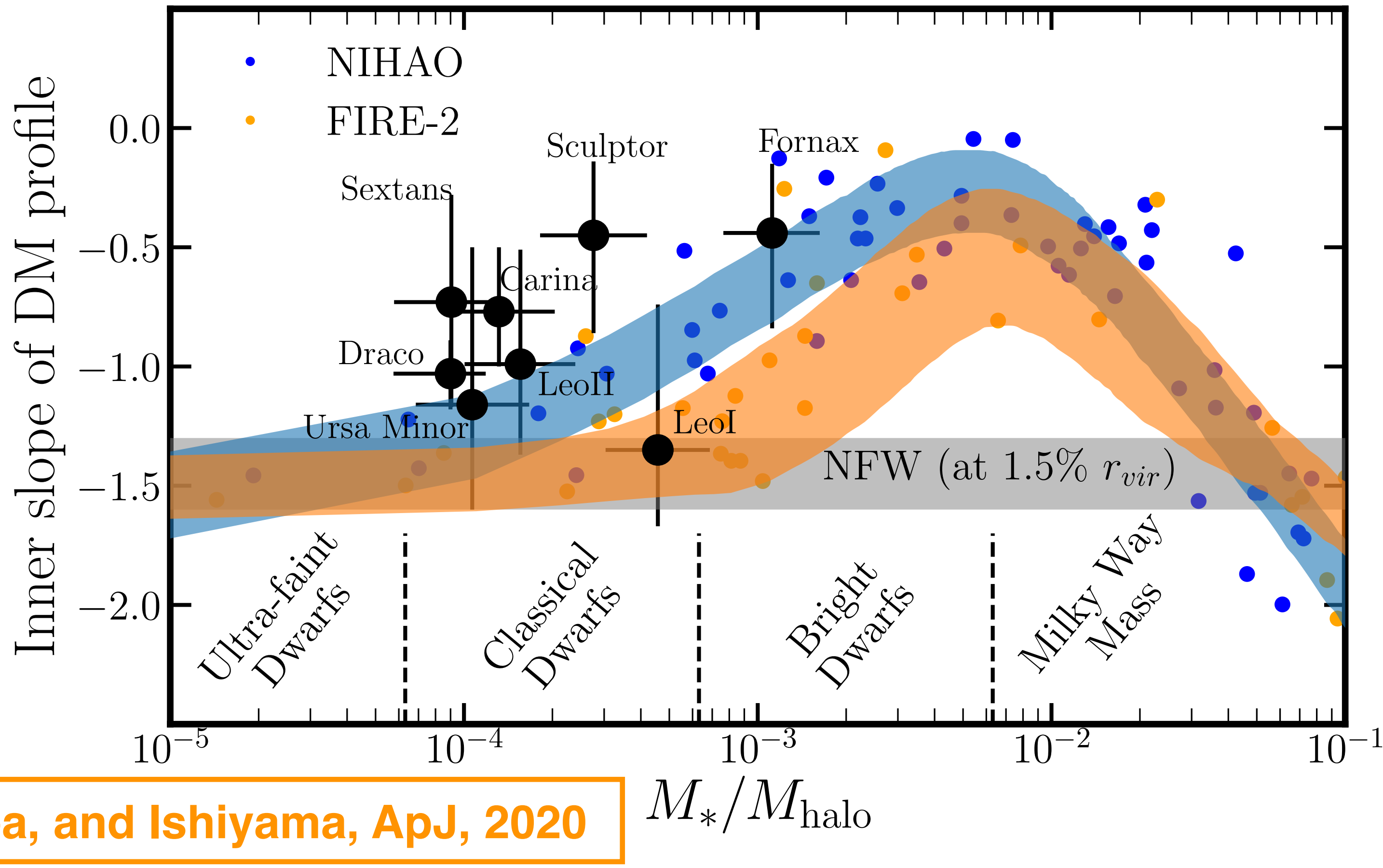
- mass distribution is determined by line-of-sight velocity dispersion (LOSVD) profile

- shows diversity in inner slope and density, though uncertainty is still large

SIDM again? Naively, no

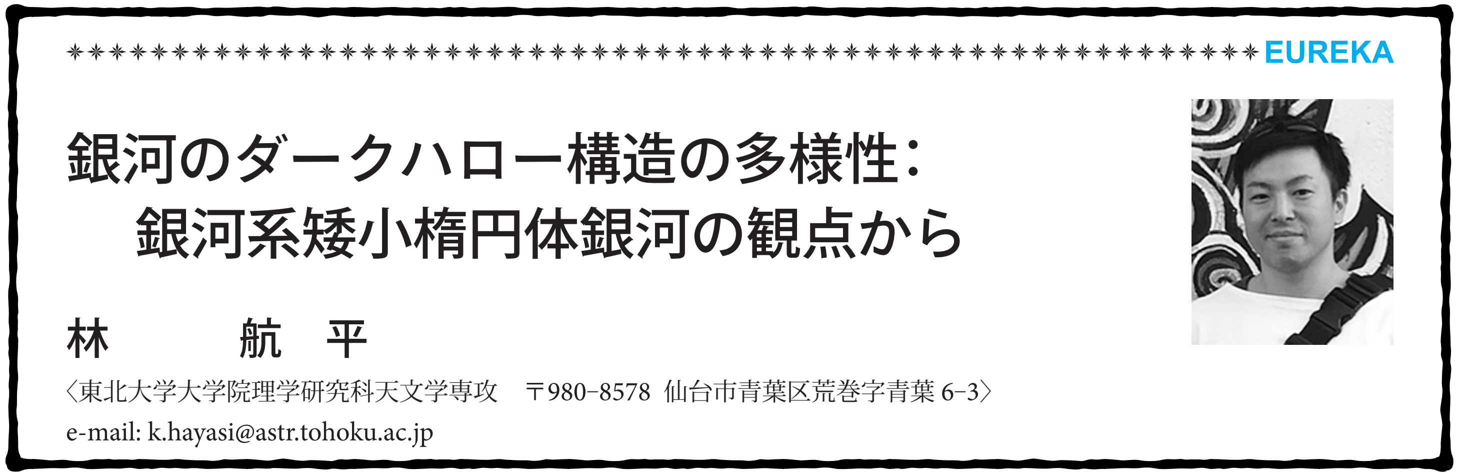
- satellite galaxies have only negligible amount of baryons





Hayashi, Chiba, and Ishiyama, ApJ, 2020

Diversity in MW satellites



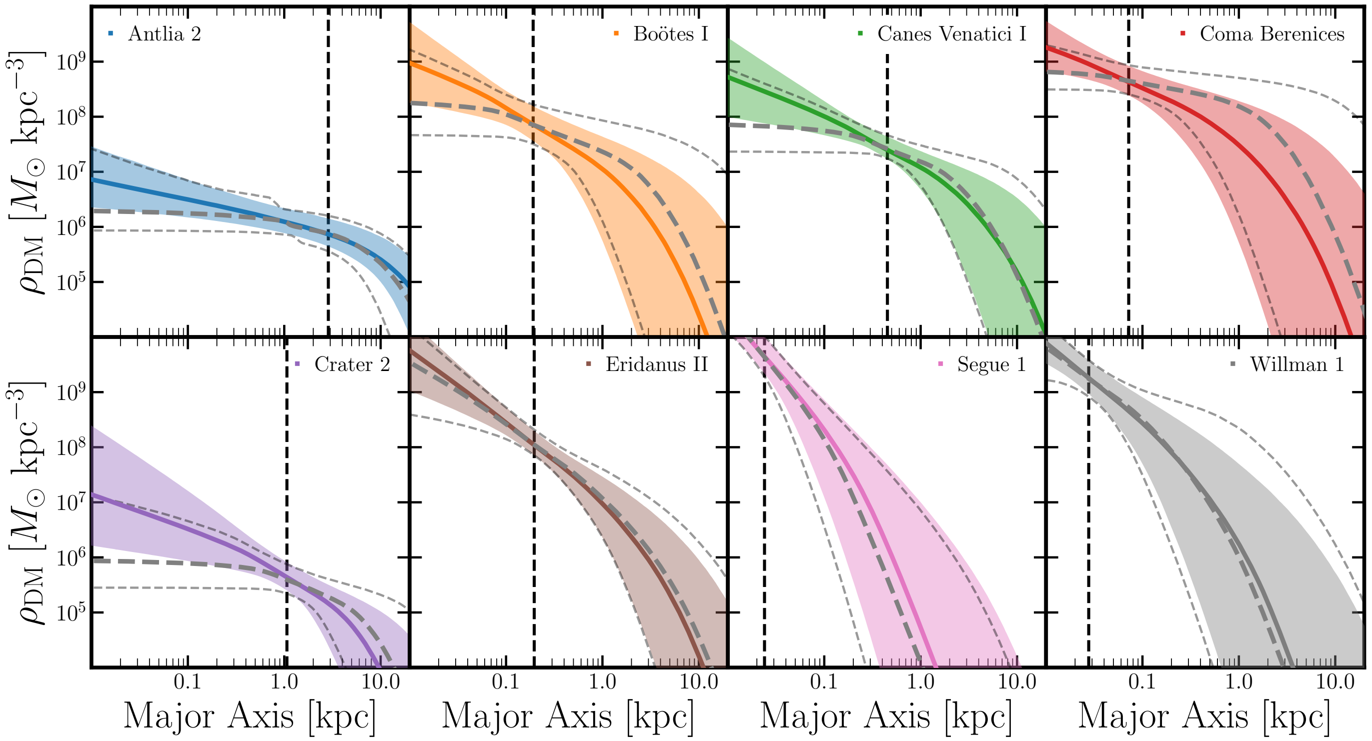
＊＊＊EUREKA

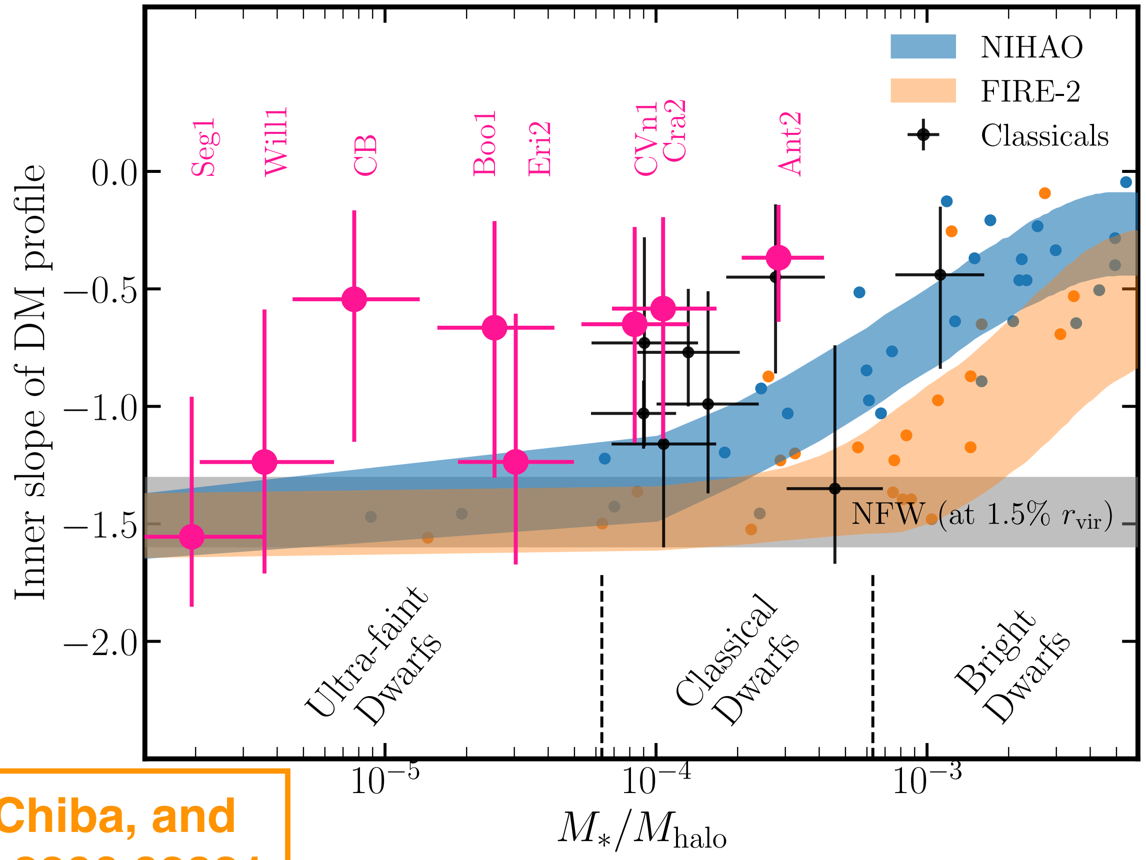
銀河のダークハロー構造の多様性：
銀河系矮小楕円体銀河の観点から

林　　航　平

〈東北大学大学院理学研究科天文学専攻　〒980–8578 仙台市青葉区荒巻字青葉 6–3〉
e-mail: k.hayasi@astr.tohoku.ac.jp

MW satellites (ultra-faint)



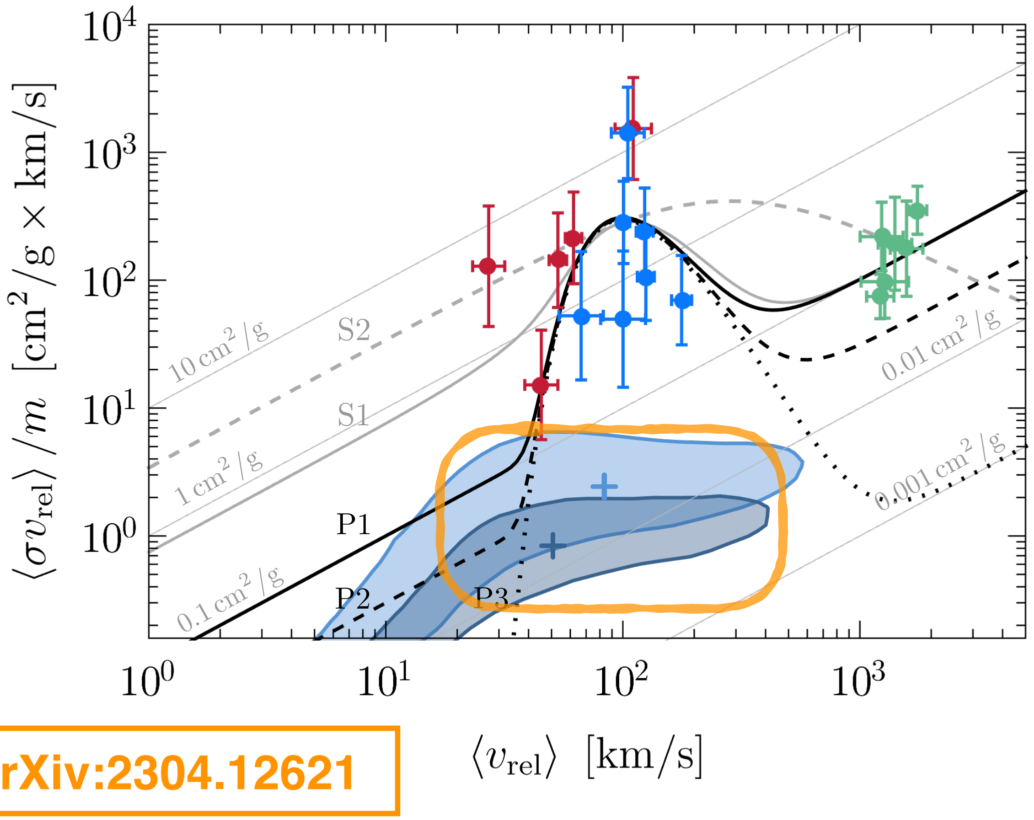


Hayashi, Hirai, Chiba, and Ishiyama, arXiv:2206.02821

Two possibilities on the table

Resonant SIDM

Chu, Garcia-Cely, and Murayama, PRL, 2019



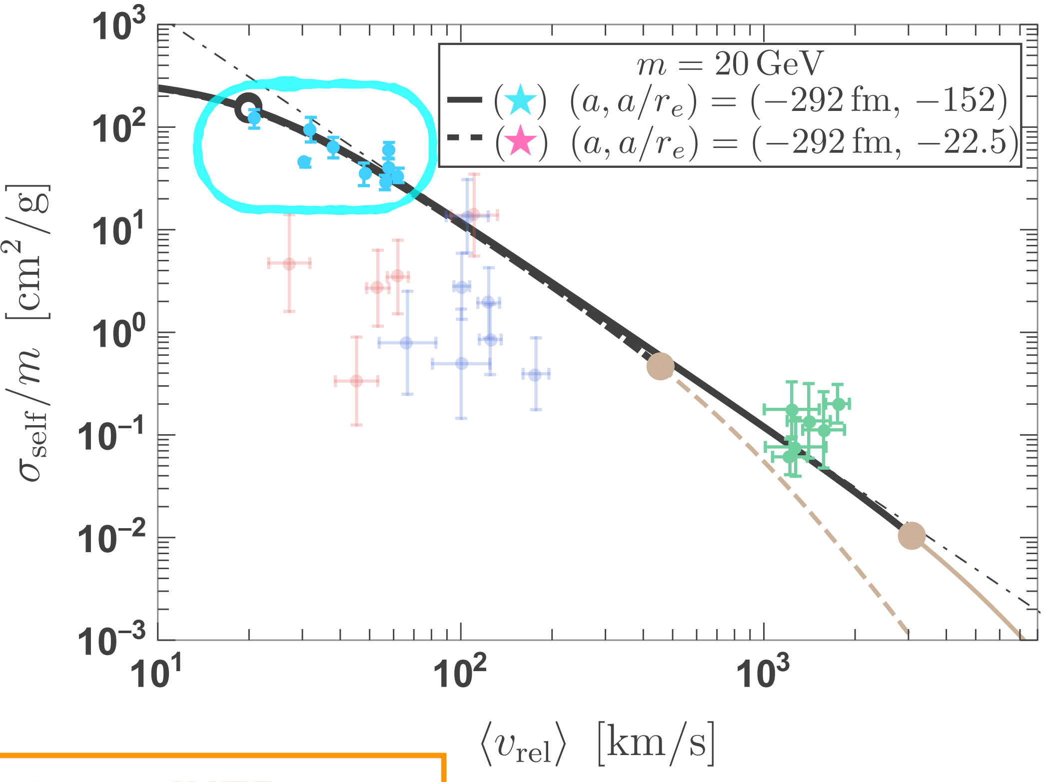
- explain only cuspy profile by taking a small cross section at low velocity

$$\sigma/m < 0.1\,{\rm cm^2/g} \quad \langle v_{\rm rel}\rangle \sim 30\,{\rm km/s}$$

- leave cored profile for stellar feedbacks

Hayashi *et al.*, PRD, 2021 **AK and Kim, arXiv:2304.12621**

Strong SIDM



- explain diversity by taking a further large cross section at low velocity

$$\sigma/m \sim 40\,{\rm cm^2/g}$$

- gravothermal collapse is sensitive to initial profiles and orbits in MW

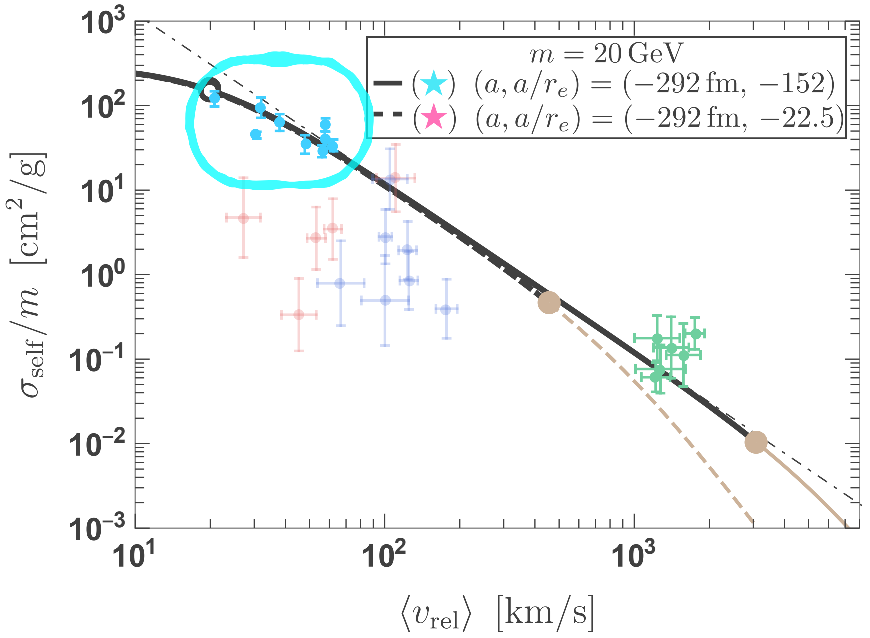
Correa, MNRAS, 2021 **AK, Kim and Kuwahara, JHEP, 2020**

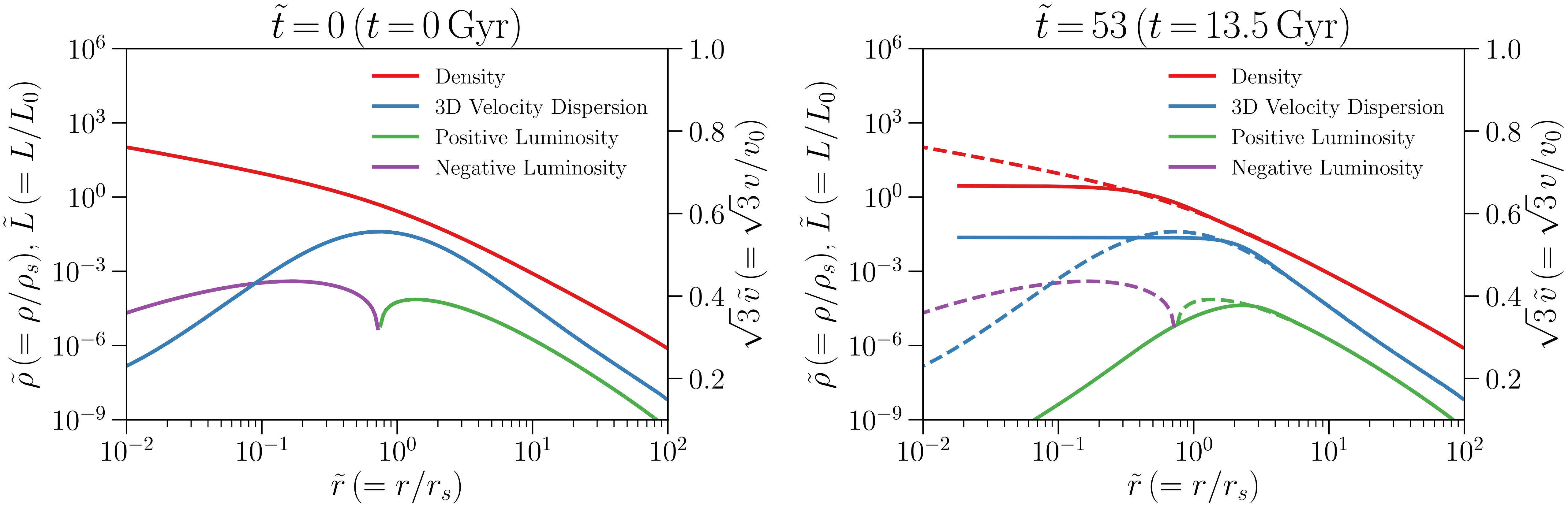
Strong SIDM

Gravothermal collapse

- SIDM halo evolution: core expansion
  → core collapse

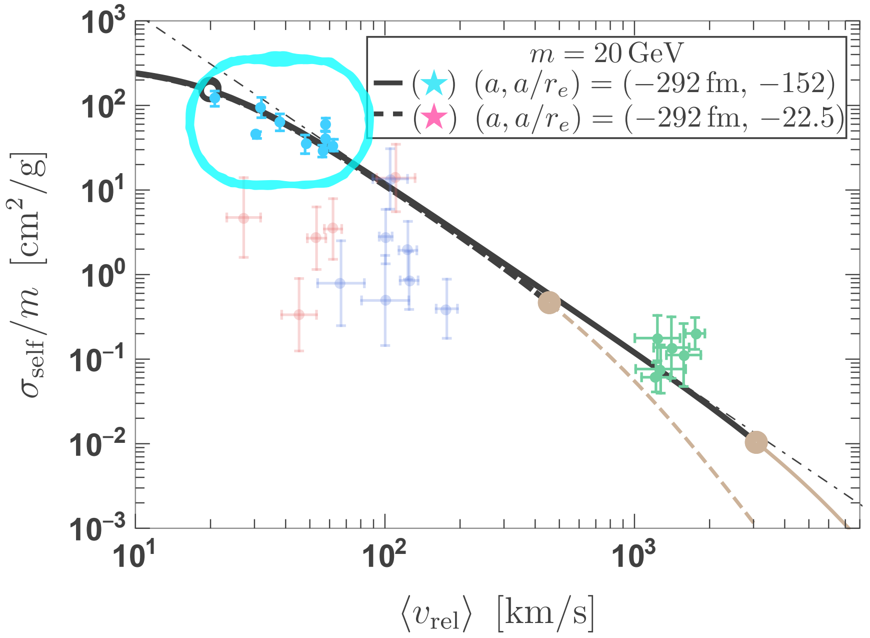
- core expansion lasts till the temperature profile gets flat (thermalization)





Nishikawa, Boddy, and Kaplinghat, PRD, 2020

Strong SIDM

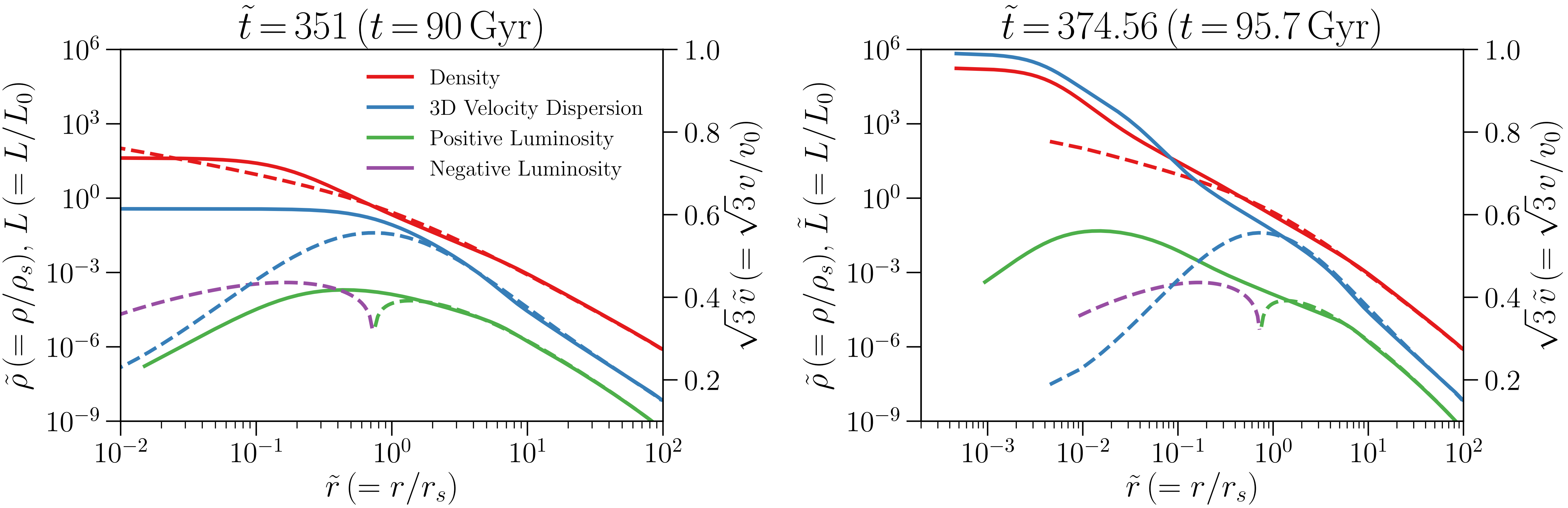


Gravothermal collapse

- SIDM halo evolution: core expansion → core collapse

- core contraction proceeds by depositing heat to the outer region

    - heat deposit → lower energy but higher temperature (negative heat capacity)

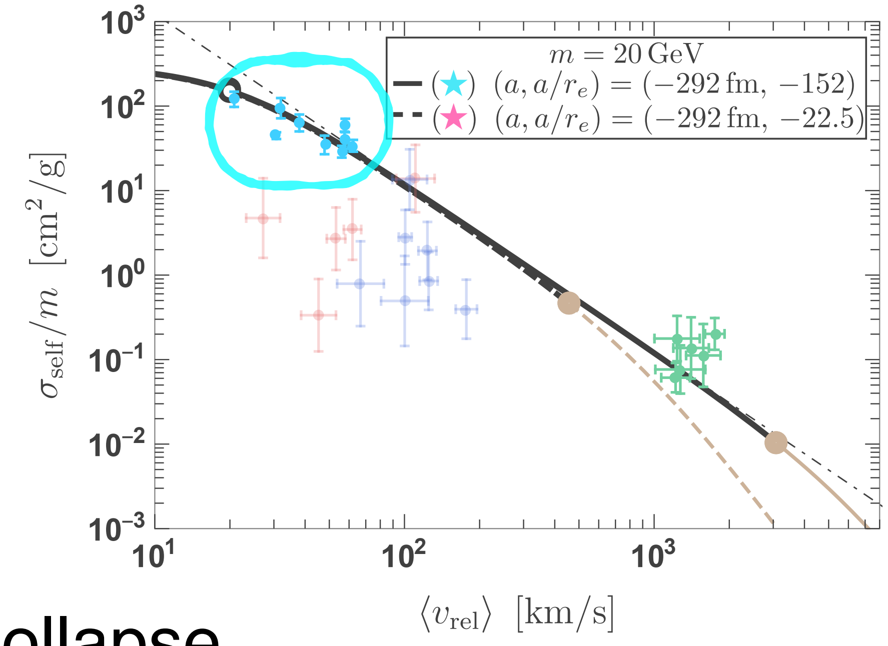


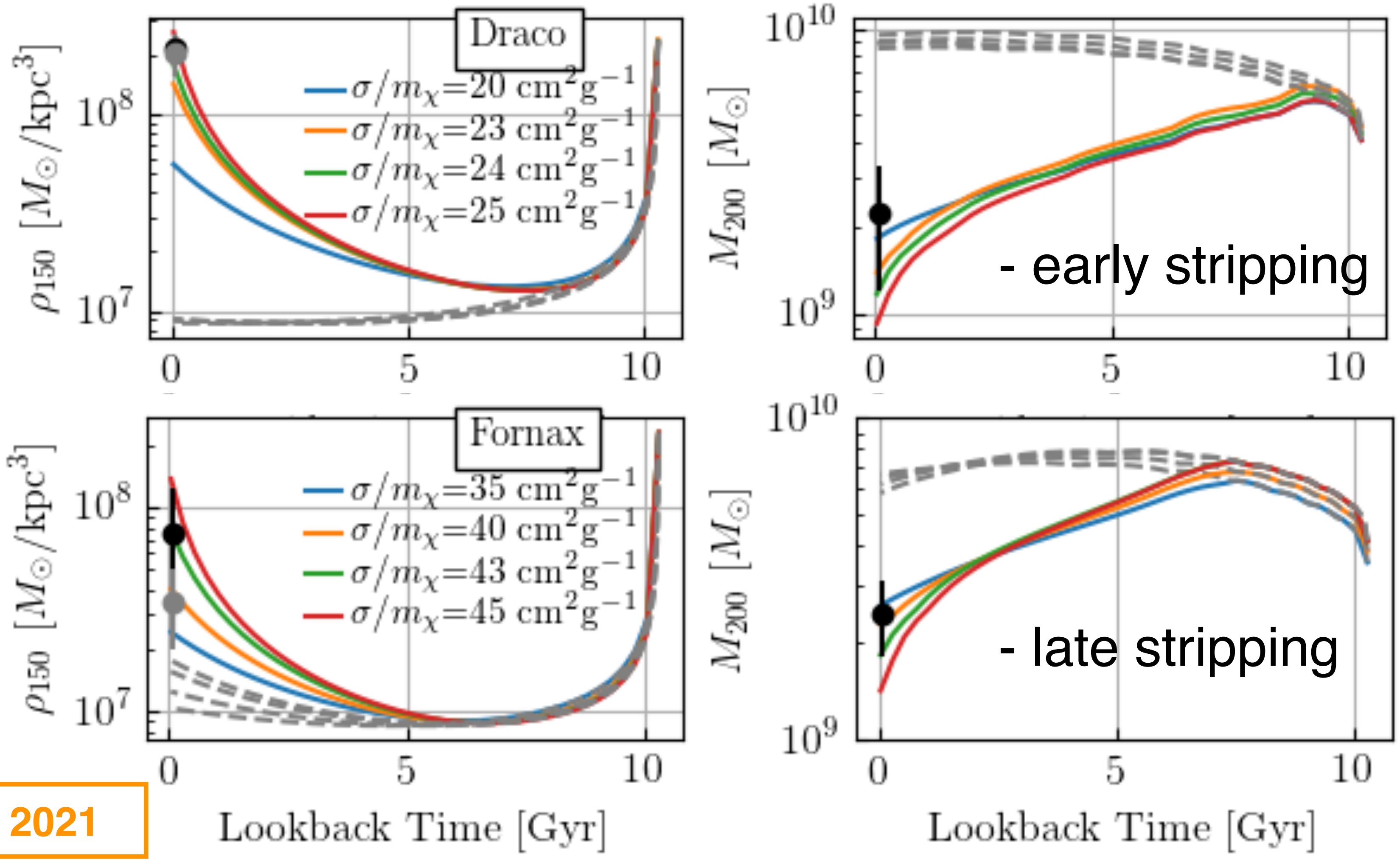
Nishikawa, Boddy, and Kaplinghat, PRD, 2020

Strong SIDM

Gravothermal collapse

- very sensitive to initial profiles and orbits in MW
  - tidal stripping: different orbits in MW → different "initial" profiles
  - tidal stripping accelerates gravothermal collapse





- early stripping

- late stripping

Correa, MNRAS, 2021

On-going efforts

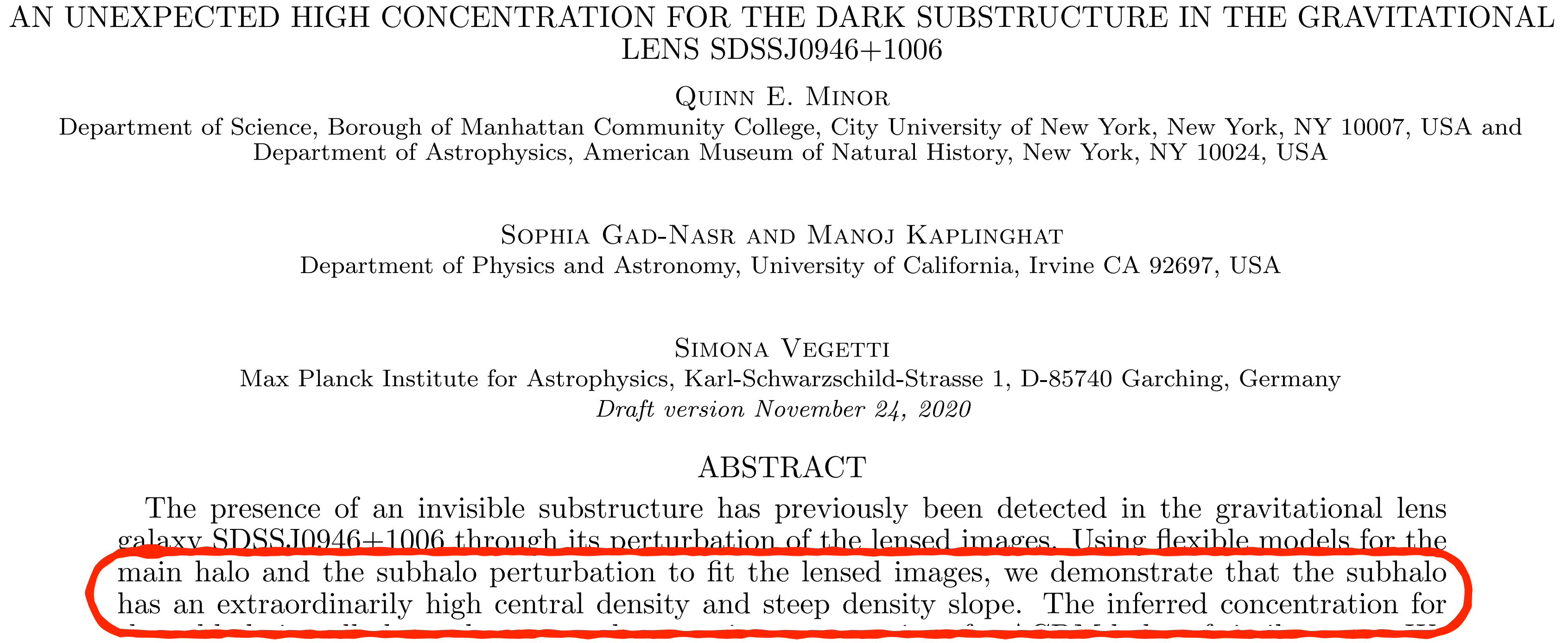
More precise prediction

- isolated halo + tidal stripping through gravothermal-fluid modeling
- cosmological simulations (time-consuming) are limited so far

How to examine?

- Resonant SIDM: density breaks develop (different from constant SIDM)
- Strong SIDM: collapsed halos
  - through perturbations of strong-lensed systems

AK and Kim, arXiv:2304.12621



AN UNEXPECTED HIGH CONCENTRATION FOR THE DARK SUBSTRUCTURE IN THE GRAVITATIONAL LENS SDSSJ0946+1006

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Draft version November 24, 2020

ABSTRACT

The presence of an invisible substructure has previously been detected in the gravitational lens galaxy SDSSJ0946+1006 through its perturbation of the lensed images. Using flexible models for the main halo and the subhalo perturbation to fit the lensed images, we demonstrate that the subhalo has an extraordinarily high central density and steep density slope. The inferred concentration for

Summary

Diversity problem

- more complicated than just core vs cusp
- rotation curves of dwarf galaxies $\langle v_{\rm rel} \rangle \sim 100\,{\rm km/s}$
- inner density of MW satellites (large uncertainty) $\langle v_{\rm rel} \rangle \sim 30\,{\rm km/s}$

SIDM

- explain diversity of rotation curves through diversity in galactic disks

  $\sigma/m \sim 1\,{\rm cm^2/g}$

- Strong SIDM: explain diversity of inner density through gravothermal collapse and diversity in initial profiles and orbits in MW

  $\sigma/m \sim 40\,{\rm cm^2/g}$

- Resonant SIDM: explain only cuspy satellites, while leaving cored satellites for stellar feedback

  $\sigma/m < 0.1\,{\rm cm^2/g}$

Contents

Brief history of SIDM

- (1st stage) core vs cusp problem and constant cross section
- (2nd stage) cosmic-ray anomalies and velocity-dependent cross section

Frontier of SIDM (3rd stage)

- diversity problems: galaxies and satellite galaxies
- strong self-interaction and gravothermal collapse

Model-building aspects

- Sommerfeld enhancement and indirect detection

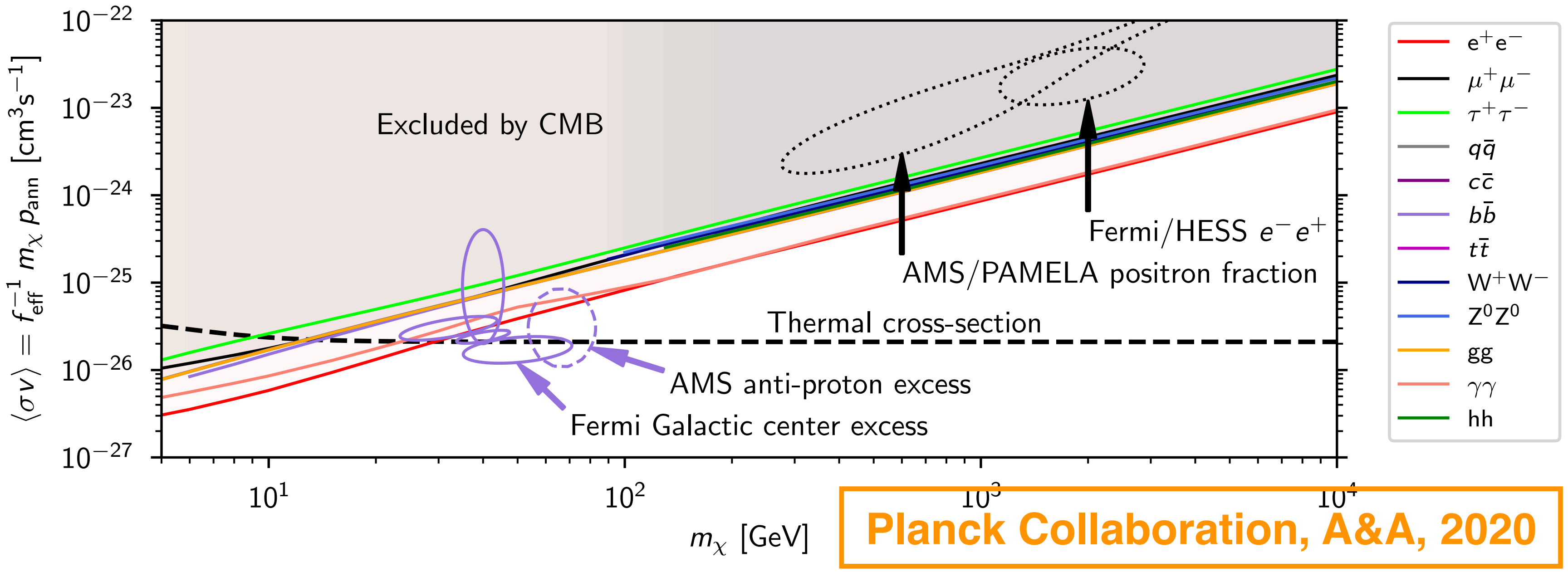
What should be concerned?

Light mediator

- naively overcloses the Universe
- decay or efficient annihilation

Enhanced annihilation

- large Sommerfeld enhancement by long-range force
- CMB constraints are relevant
    - energy deposit around the last scattering

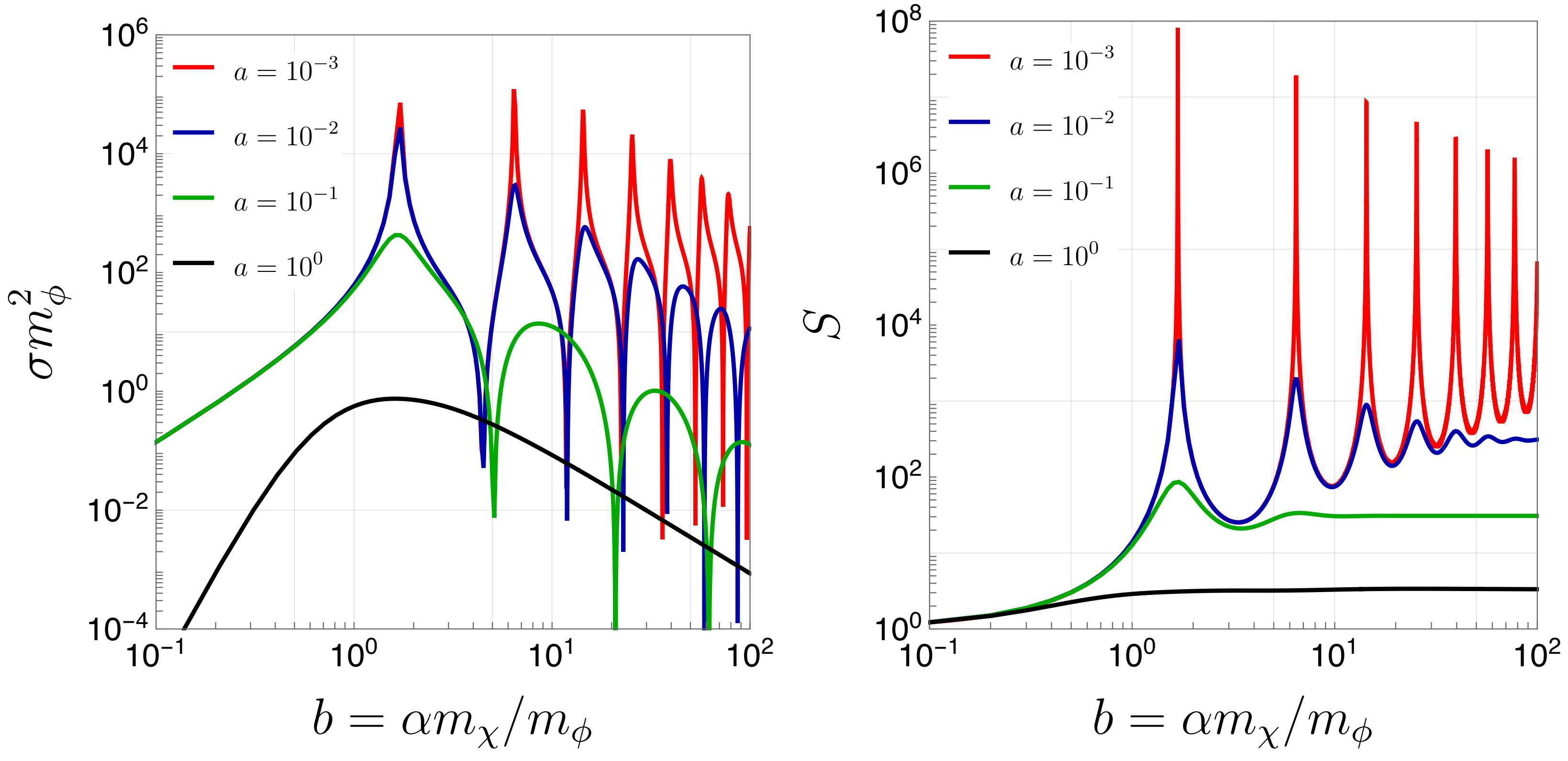


Planck Collaboration, A&A, 2020

Enhanced annihilation

Sommerfeld enhancement and self-scattering

- tightly correlated
  - resonant enhancement occurs at the same parameter point



- annihilation amplitude and scattering phase are related by Watson theorem
  - model-independent

$$\Gamma_\ell(k^2 + i\epsilon) = e^{2i\delta_\ell}\Gamma_\ell(k^2 + i\epsilon)^*$$

AK, Kuwahara and Patel, arXiv:2303.17961

Resonant enhancement

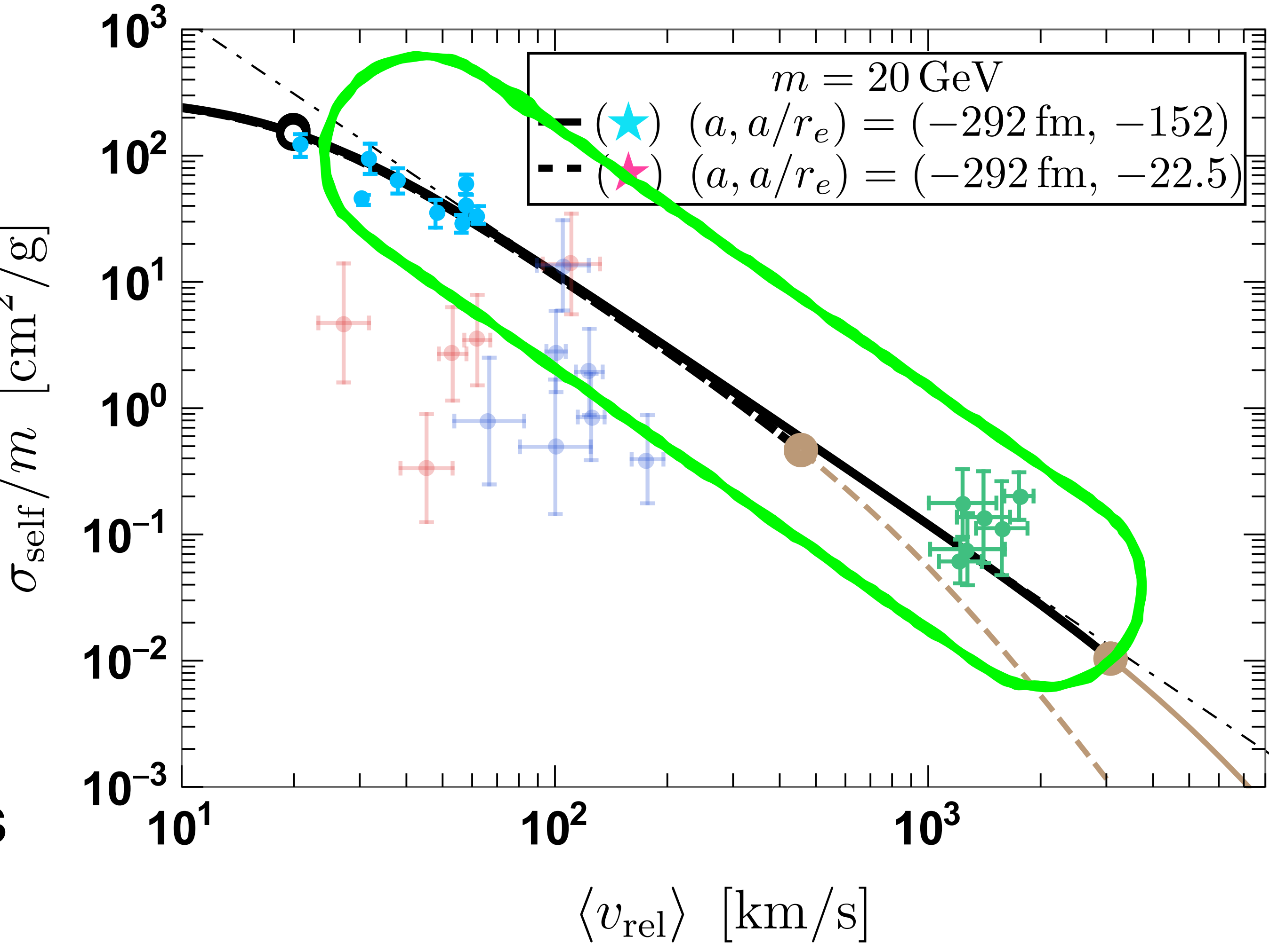
Maximally self-interacting dark matter

- s -wave Unitarity

$$\sigma^{\rm el} = \sigma^{\rm el,max}_{\ell=0} = \frac{16\pi}{m^2 v_{\rm rel}^2}$$

- comparison with “data”
  - $\sigma^{\rm el,max}_{\ell} \propto 1/v_{\rm rel}^2$ is in good agreement with data
  - depends solely on DM mass

    $m \simeq 20\,{\rm GeV}$
  - large $|a/r_{\rm e}|$ in effective range theory



AK, Kim, and Kuwahara, JHEP, 2020

Possible ways to go

Evade constraints

- s-wave annihilation into electromagnetic energy is disfavored

- p-wave annihilation, annihilation into neutrinos

  - $L_\mu - L_\tau$ model (muon g-2)

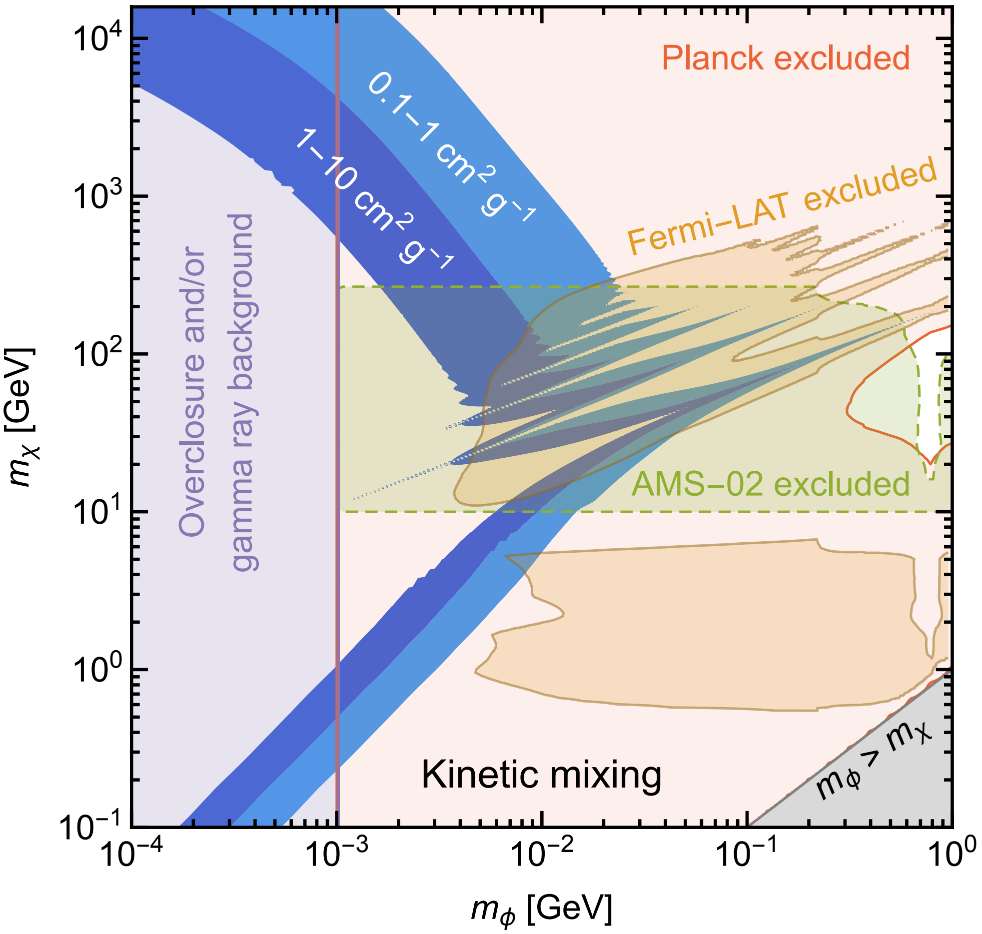
AK, Kaneta, Yanagi, and Yu, JHEP, 2018

- asymmetric dark matter

  - almost no late-time annihilation

  - dark baryon dark matter (naturally explain large cross section)

Ibe, AK, Kobayashi, and Nakano, JHEP, 2018



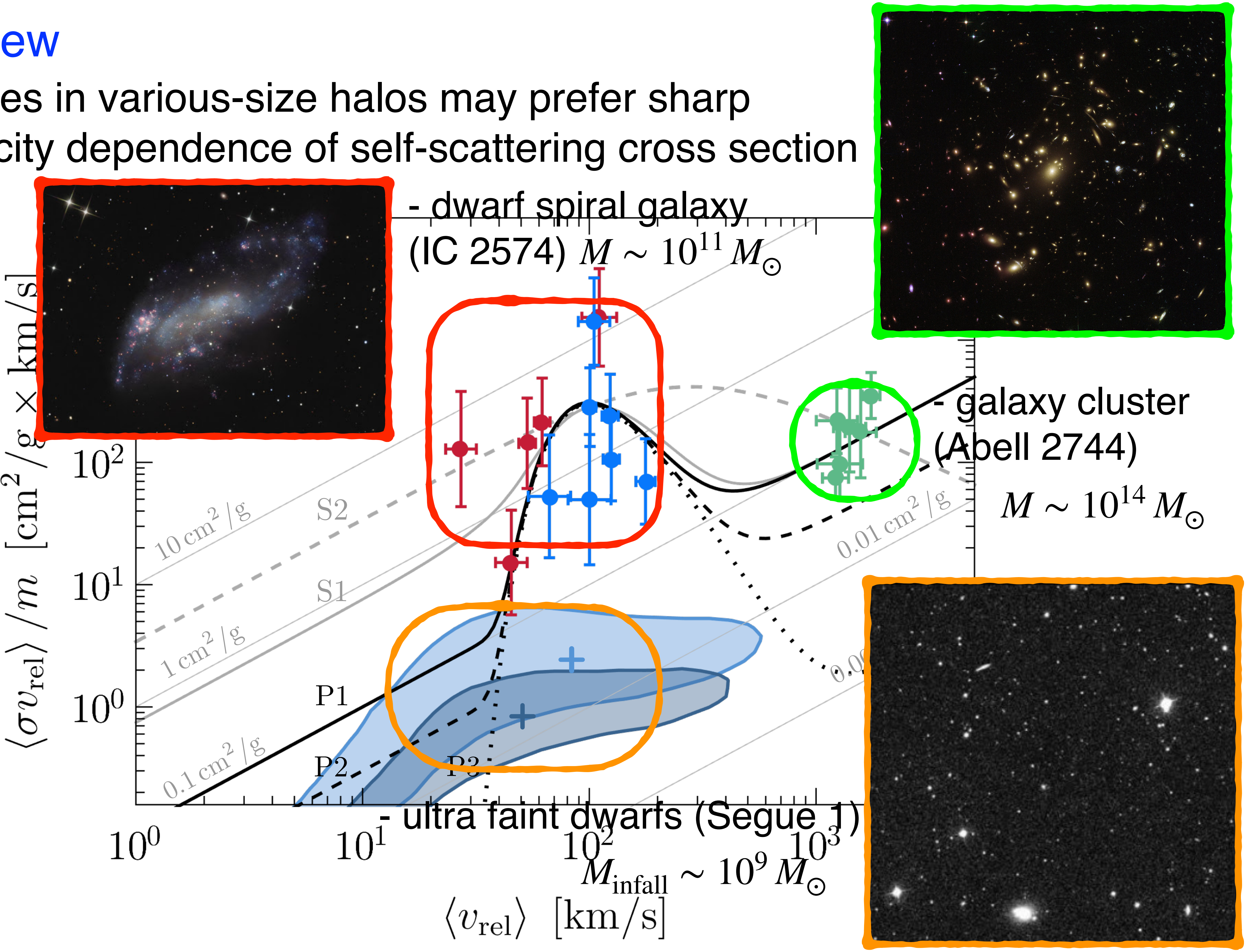
Bringmann, Kahlhoefer, Schmidt-Hoberg and Walia, JHEP, 2020

Thank you

“Data” points

Overview

- cores in various-size halos may prefer sharp velocity dependence of self-scattering cross section



- dwarf spiral galaxy (IC 2574) $M \sim 10^{11} M_\odot$

- galaxy cluster (Abell 2744) $M \sim 10^{14} M_\odot$

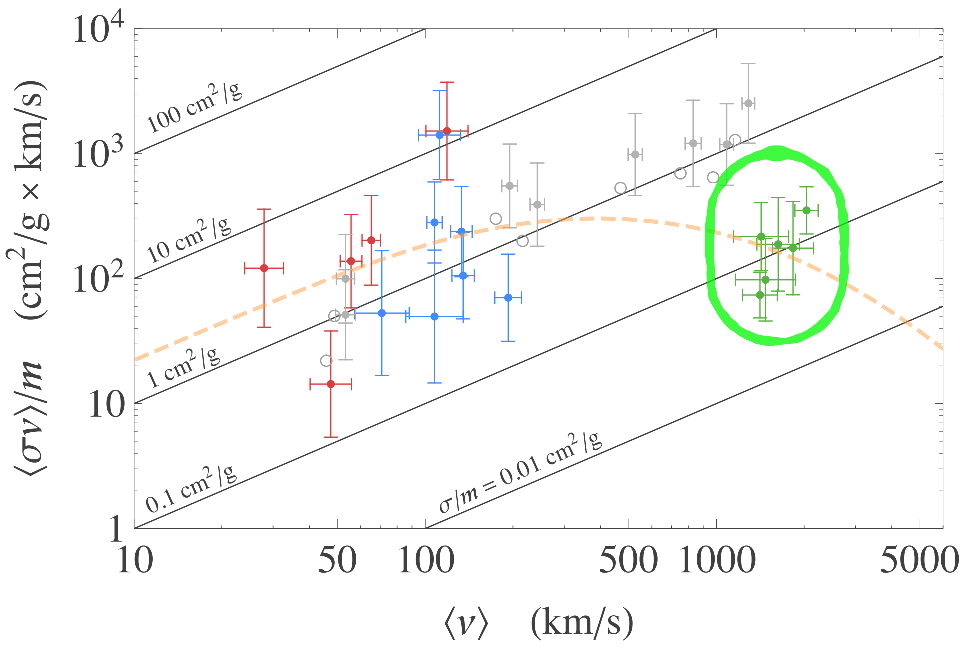
- ultra faint dwarfs (Segue 1) $M_{\rm infall} \sim 10^{9} M_\odot$

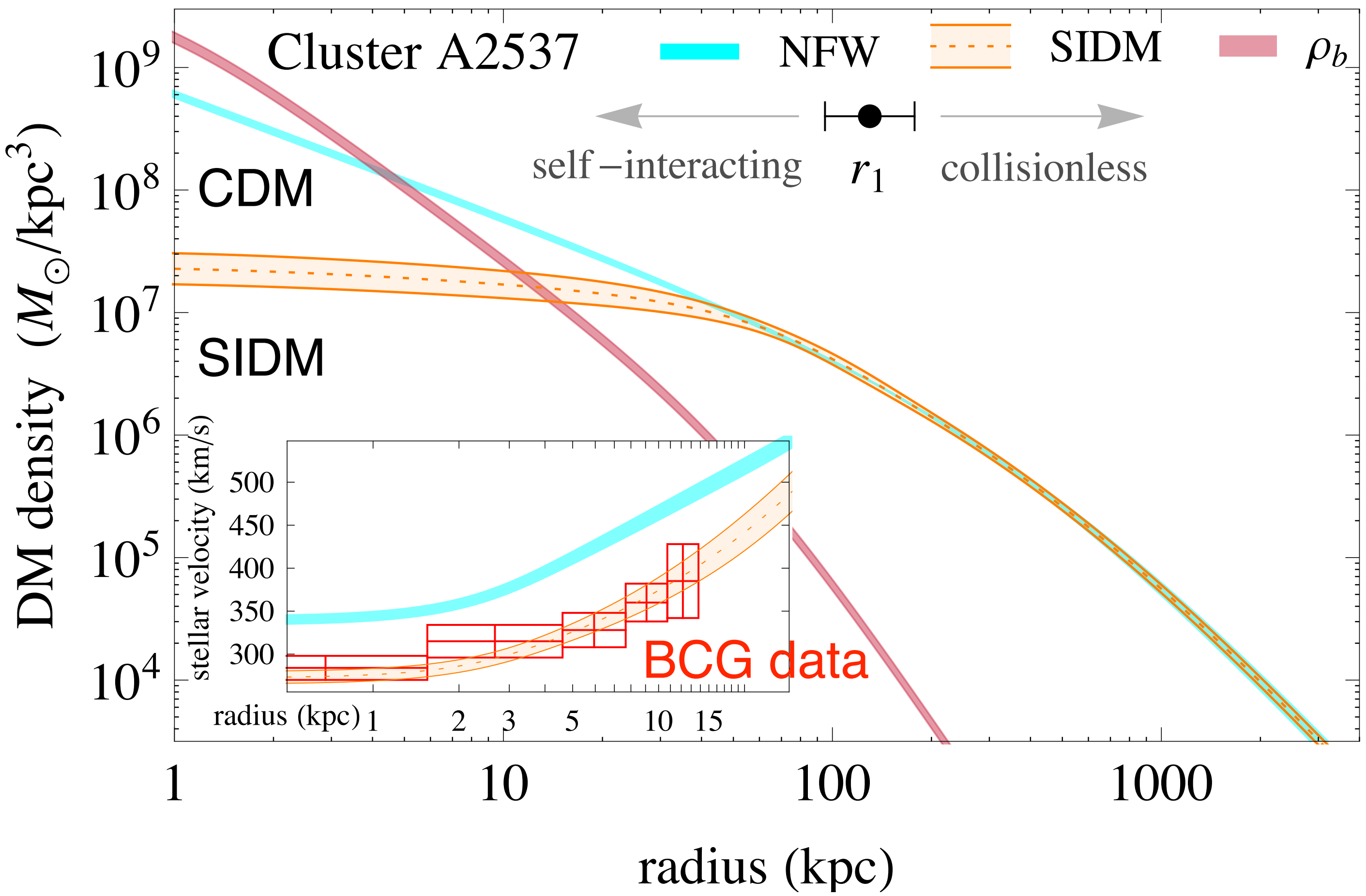
Data points

Galaxy clusters (GCs)

- mass distribution in the outer region is determined by strong/weak gravitational lensing

- stellar kinematics in the central region (brightest cluster galaxies) prefer cored SIDM profile





$$\sigma/m \sim 0.1\,\mathrm{cm}^2/\mathrm{g}$$

$$\langle v_{\mathrm{rel}}\rangle \sim 10^3\,\mathrm{km/s}$$

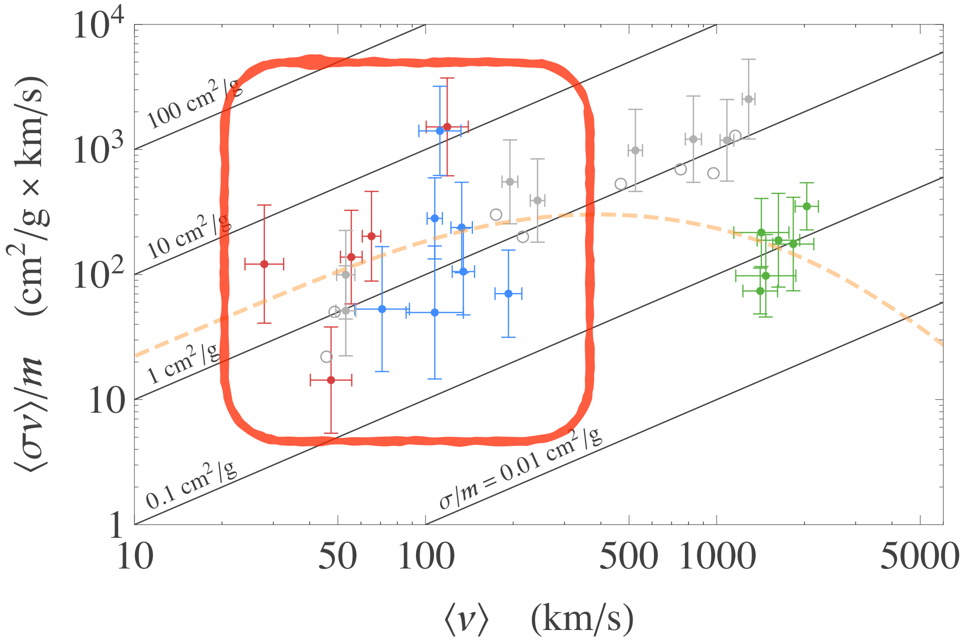
Kaplinghat, Tulin, and Yu, PRL, 2016

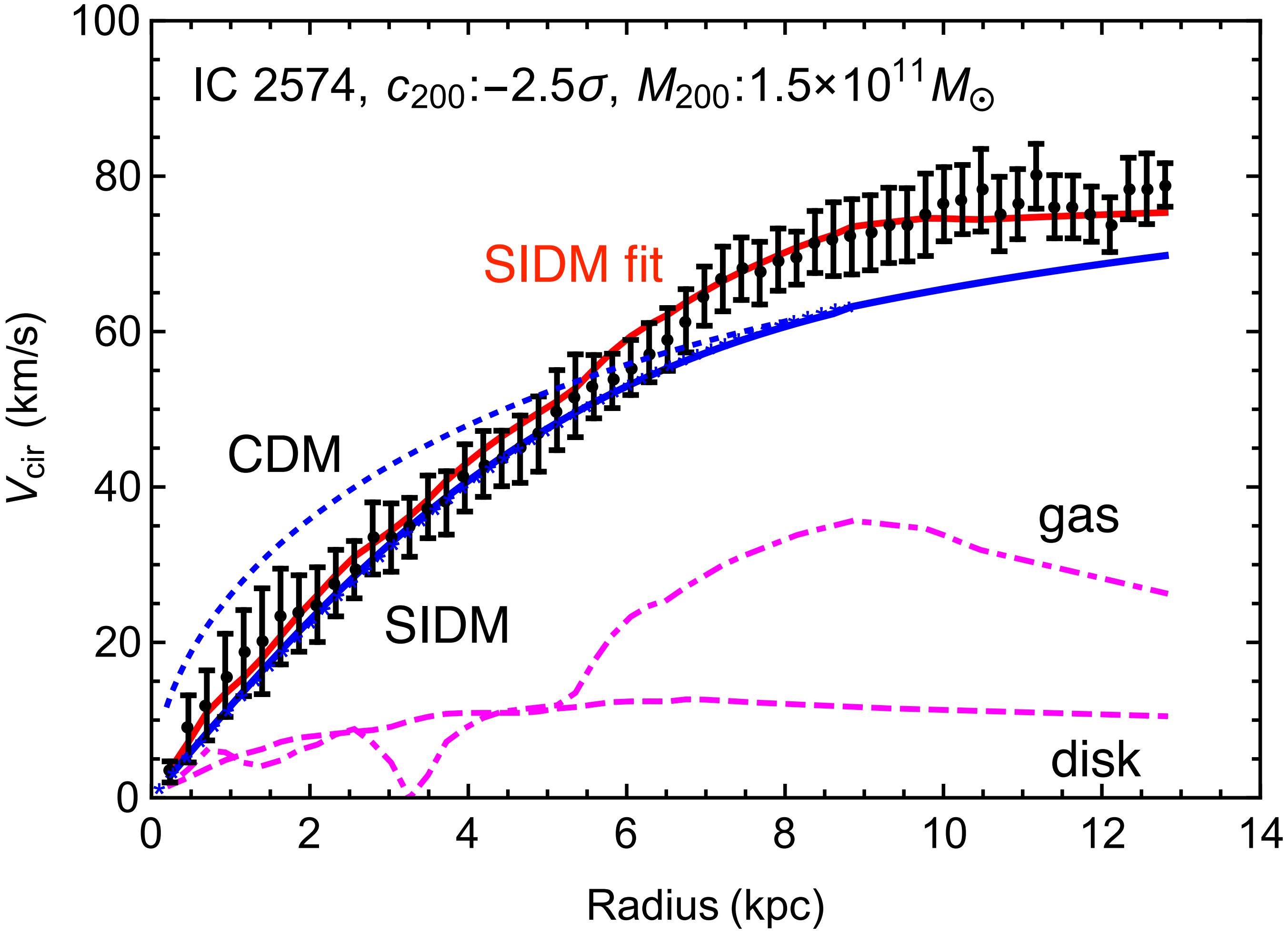
Data points

Dwarf spiral galaxies

- mass distribution is broadly determined by rotation curves

- rotation velocity in central region (of some galaxies) prefer cored SIDM profile





$$\sigma/m \sim 1\,\mathrm{cm^2/g}$$

$$\langle v_{\rm rel} \rangle \sim 10^2\,\mathrm{km/s}$$

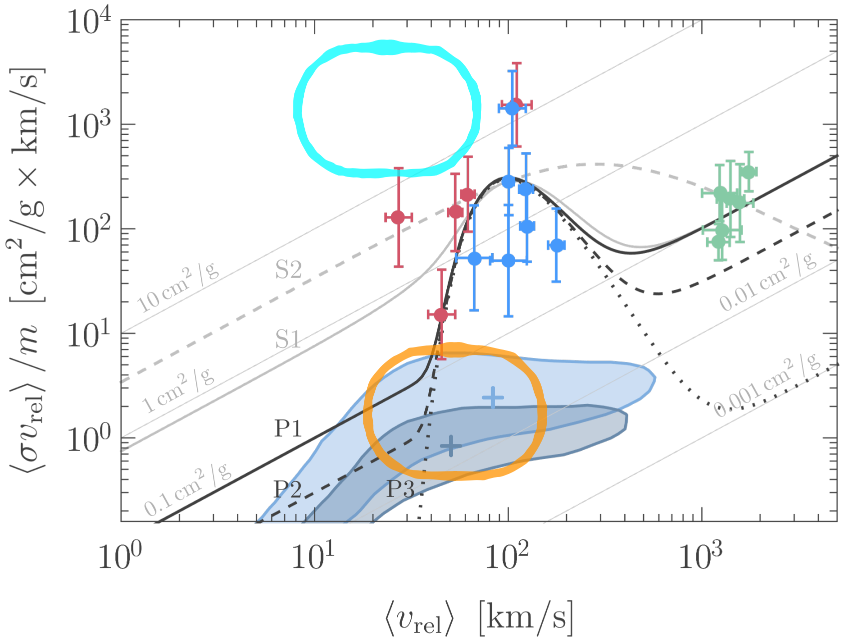
AK, Kaplinghat, Pace, and Yu, PRL, 2017

Data points

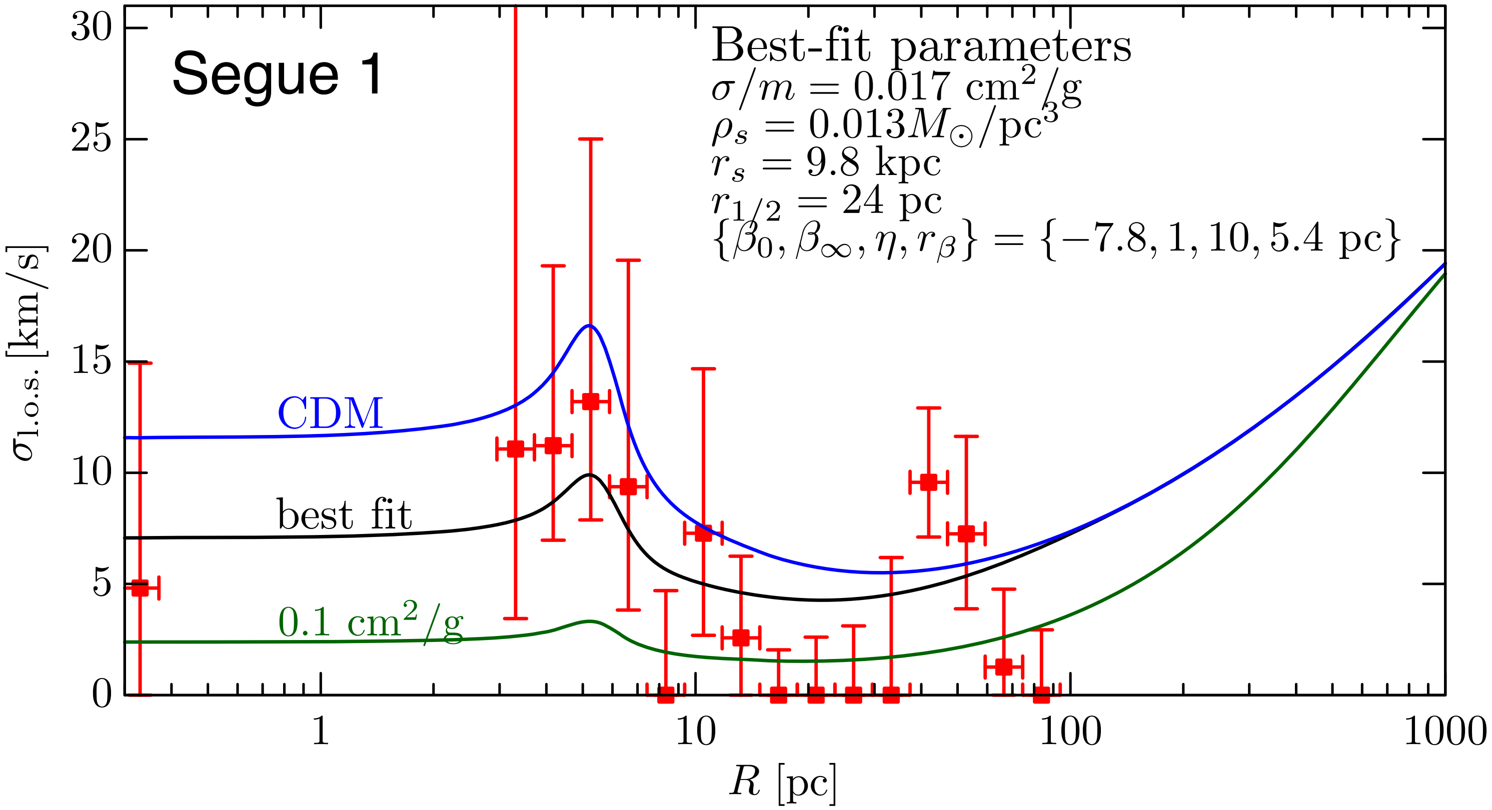
Ultra faint dwarf (UFD) galaxies

- mass distribution is determined by line-of-sight velocity dispersion (LOSVD) profile

- LOSVD in the central region (of some UFDs) prefer cuspy CDM profile



Hayashi *et al.*, PRD, 2021



$$\sigma/m < 0.1\,\mathrm{cm}^2/\mathrm{g}$$

$$\langle v_{\rm rel}\rangle \sim 30\,\mathrm{km/s}$$

- gravothermal collapse?

Correa, MNRAS, 2021

Evolution of resonant SIDM halos

Gravothermal modeling of isolated halo

- assuming hydrostatic equilibrium in the course of evolution

$$\frac{\partial}{\partial r}(\rho v^2) = -\rho\frac{GM}{r^2} \qquad \frac{\partial}{\partial r}M = 4\pi r^2\rho$$

- self-scattering leads to heat conduction

$$\frac{D}{Dt}\ln\left(\frac{\nu^3}{\rho}\right) = -\frac{1}{4\pi r^2\rho\nu^2}\frac{\partial L}{\partial r} = \frac{1}{3t_{\mathrm{cond.}}} \qquad \frac{L}{4\pi r^2} = -\kappa\frac{\partial T}{\partial r}$$

- heat conduction timescale

- naive interpolation between LMFP and SMFP regimes

$$\kappa^{-1} = \kappa_{\mathrm{LMFP}}^{-1} + \kappa_{\mathrm{SMFP}}^{-1}$$

- SMFP $\kappa_{\mathrm{SMFP}} = \frac{3}{2}b\frac{\nu}{\sigma_0 K_5(\nu)}$

$$b = \frac{25\sqrt{\pi}}{32} \simeq 1.38 \qquad K_p(\nu) = \frac{\langle\sigma v_{\mathrm{rel}}^p\rangle}{\sigma_0\langle v_{\mathrm{rel}}^p\rangle}$$

Outmezguine *et al.*, MNRAS, 2023

- LMFP $\kappa_{\mathrm{LMFP}} = \frac{3C}{2\pi^{3/2}}\frac{\rho\nu^3\sigma_0 K_1(\nu)}{Gm^2}$

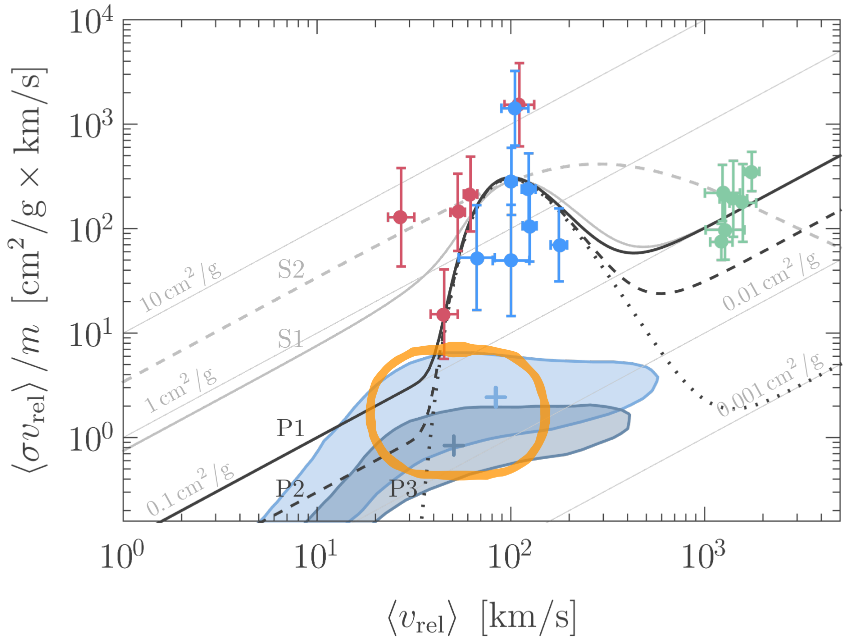
$p = 3?$ Outmezguine *et al.*, MNRAS, 2023

$p = 5?$ Yang *et al.*, ApJ, 2023

$C \simeq 0.75$ Koda and Shapiro, MNRAS, 2011

- start with NFW profile
  - mean c-M relation

Resonant SIDM

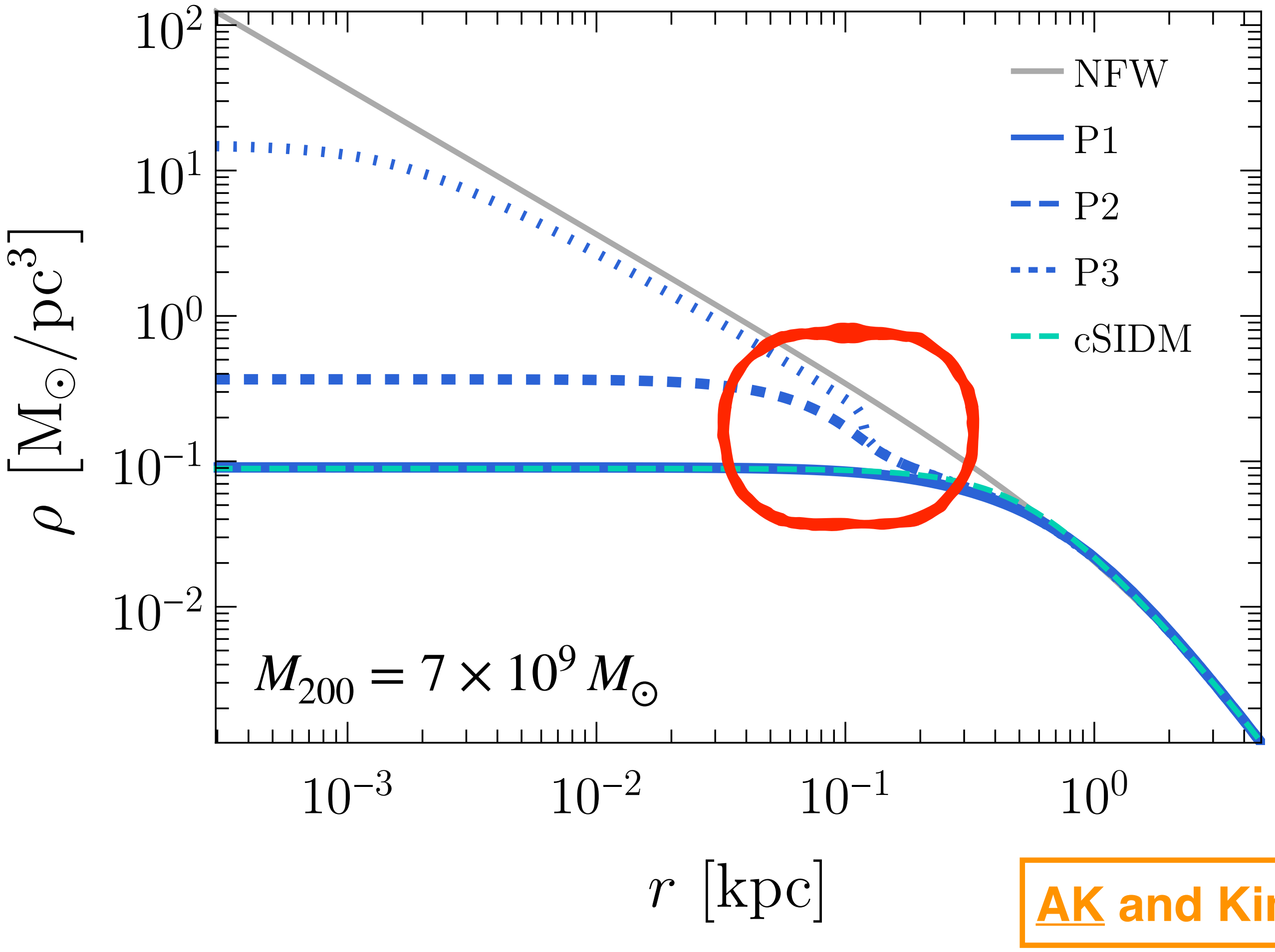


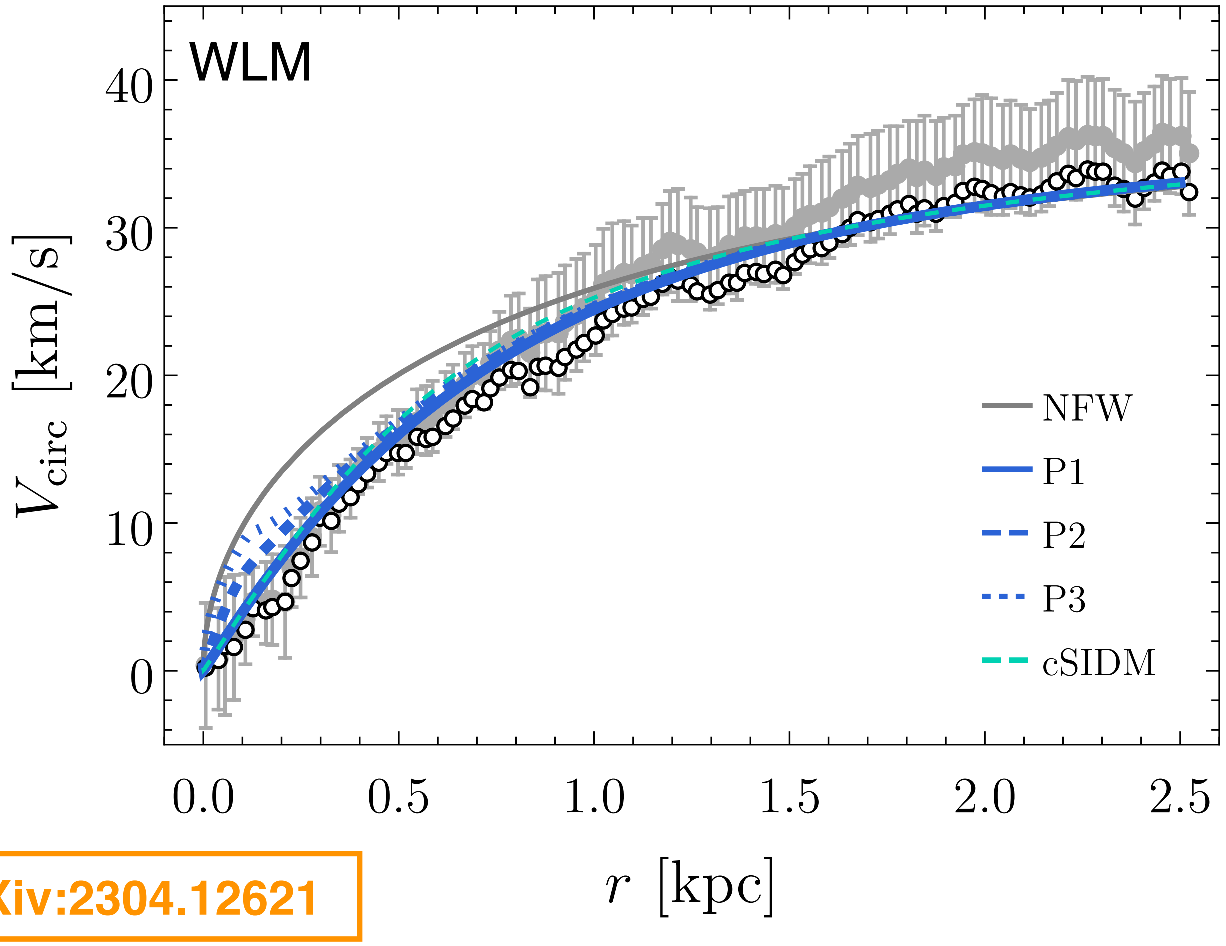
Density break

- formation, development and thermalization of density break in Resonant SIDM halo

- P3 benchmark halo has a circular velocity profile transiting from constant SIDM to NFW around 0.1 kpc

  - may be a distinctive signature if observed by any chance





AK and Kim, arXiv:2304.12621

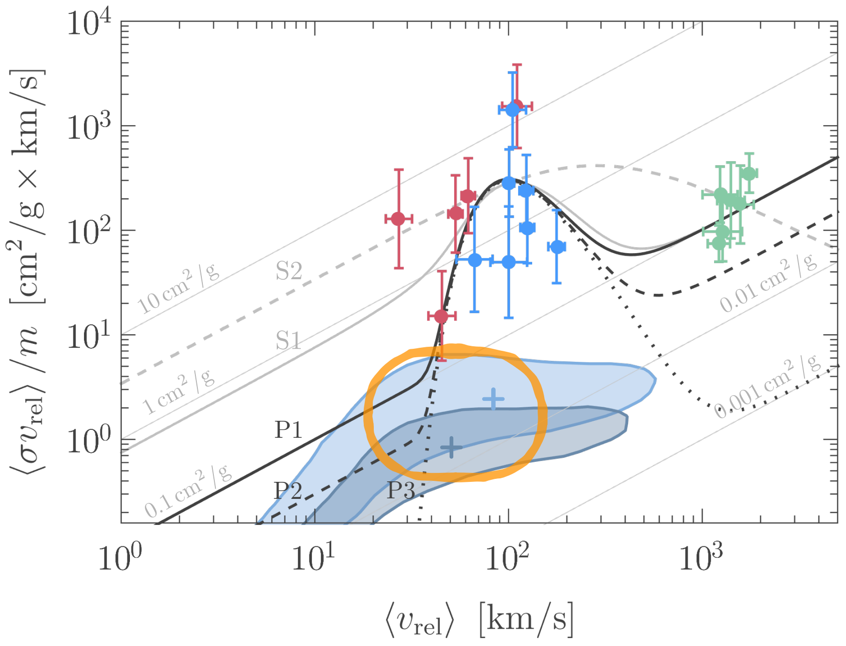
Resonant SIDM

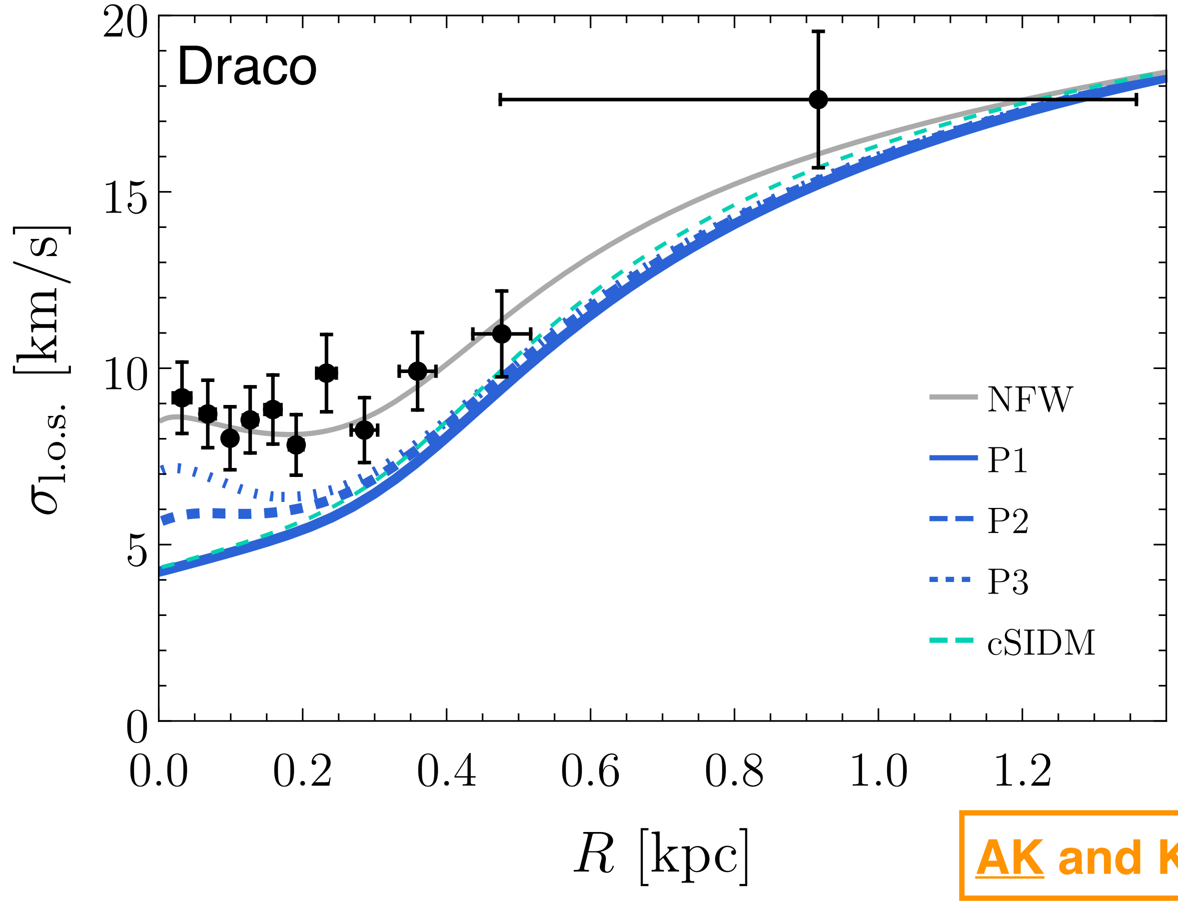
LOSVD profile of MW satellites

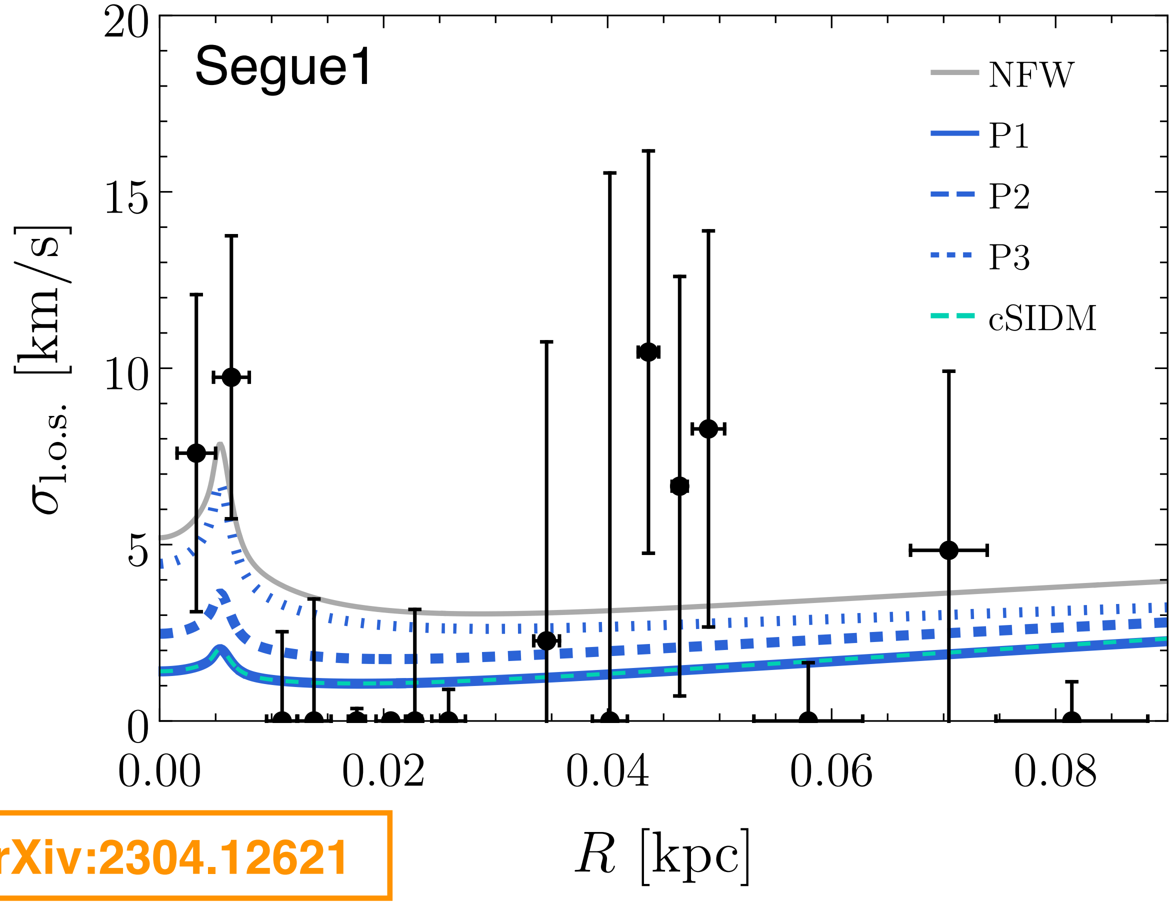
- stellar kinematic parameters are fixed to best fit values for NFW profile

Hayashi *et al.*, PRD, 2021

- P3 benchmark halo shows a transition from constant SIDM to NFW around 0.1 kpc
    - may fit the data better than constant SIDM





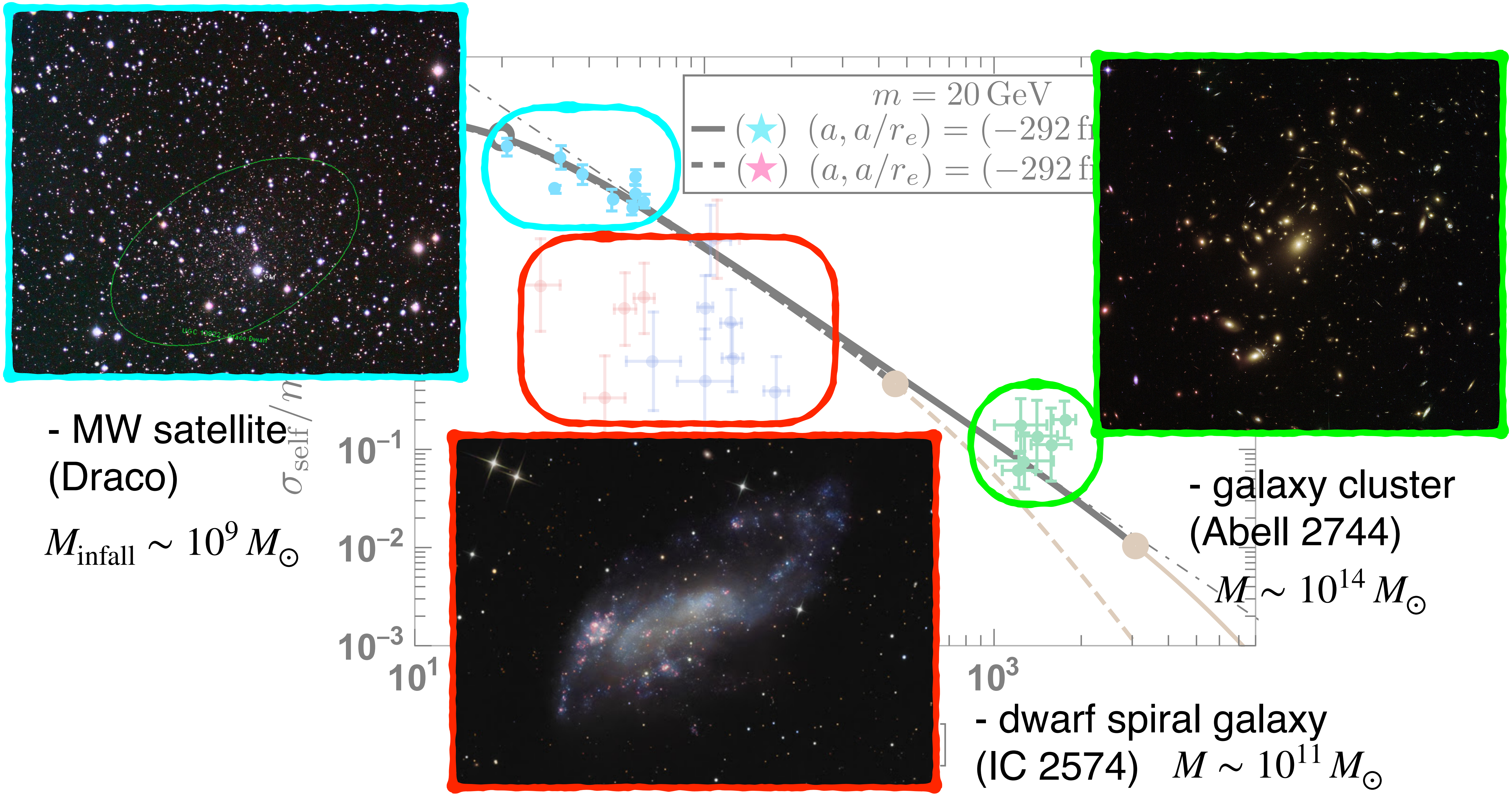


AK and Kim, arXiv:2304.12621

Data points

Overview

- cores in various-size halos



- MW satellite (Draco)

$M_{\rm infall} \sim 10^9\, M_\odot$

- galaxy cluster (Abell 2744)

$M \sim 10^{14}\, M_\odot$

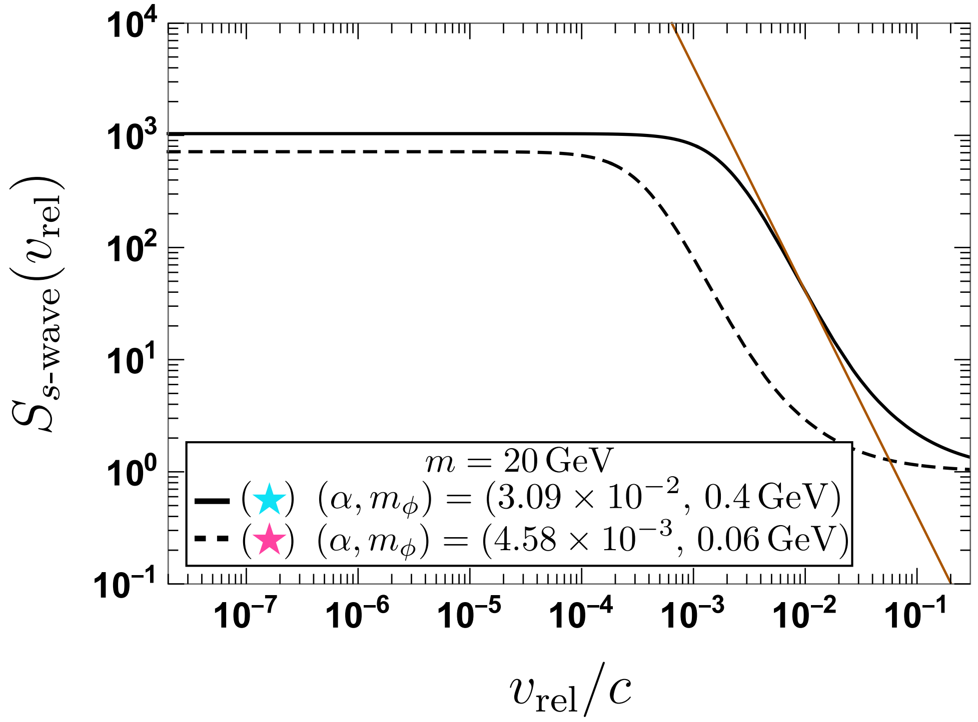
- dwarf spiral galaxy (IC 2574) $M \sim 10^{11}\, M_\odot$

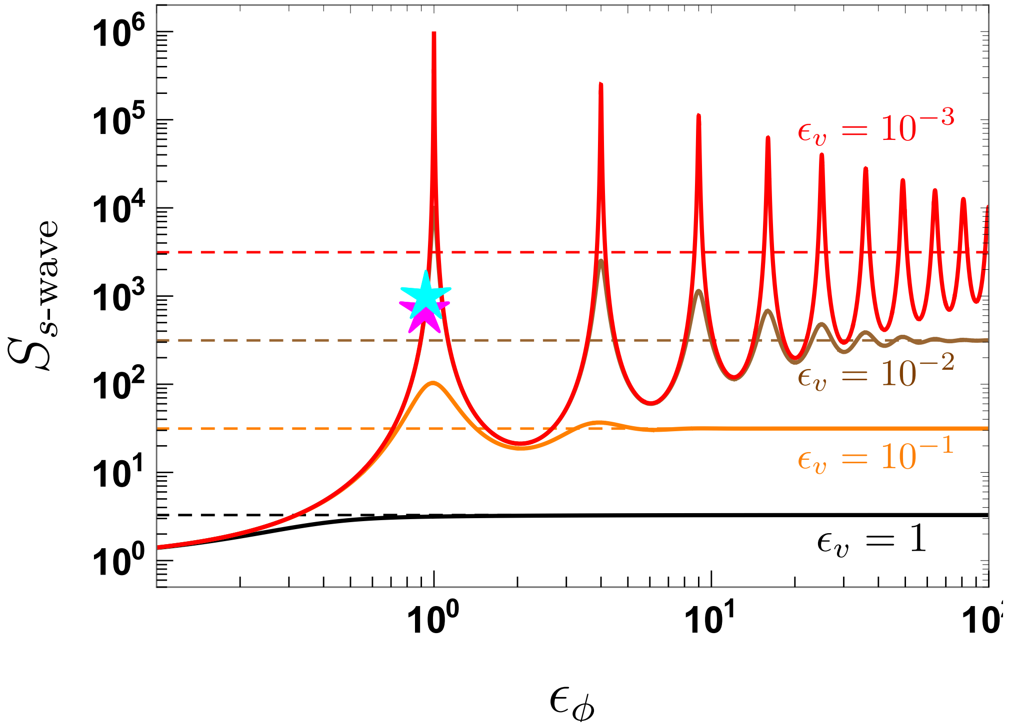
Maximally SIDM

Annihilation (Sommerfeld enhancement)

- almost zero-energy virtual level/bound state also enhances annihilation

$$(\sigma_{\rm ann} v_{\rm rel})_{\rm w/potential} = S(\sigma_{\rm ann} v_{\rm rel})_{\rm w/o}$$



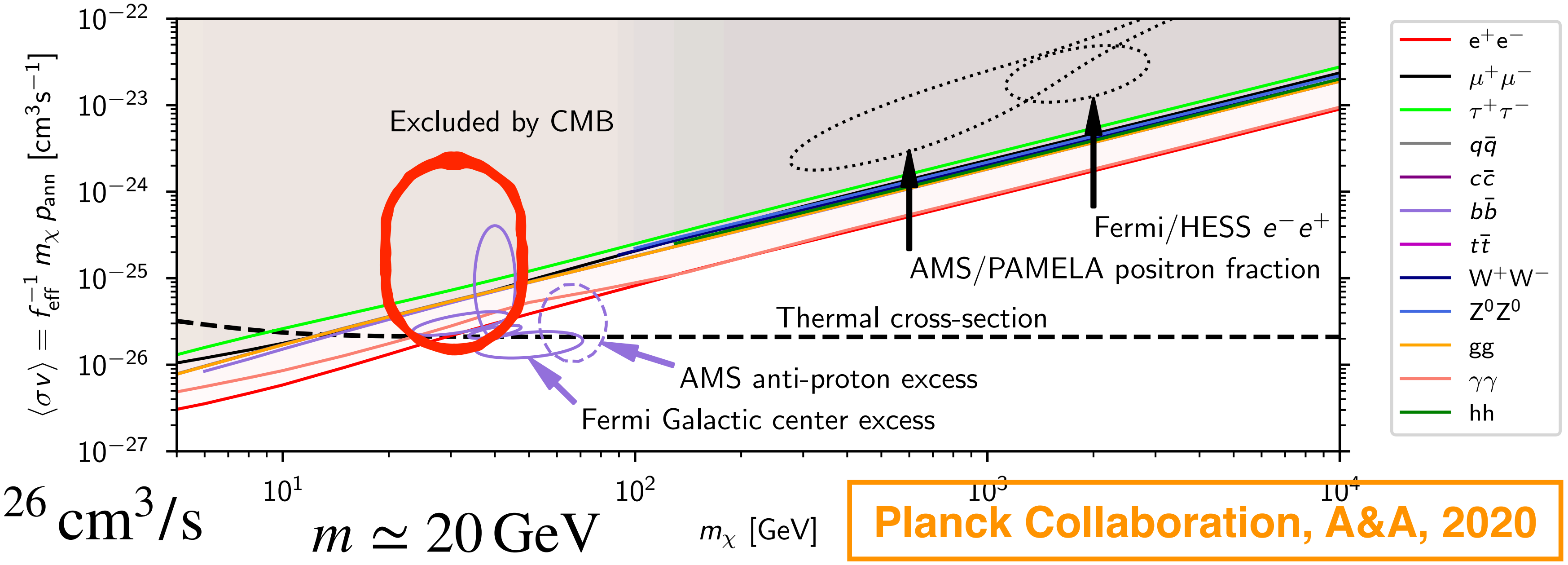


$$\epsilon_v = \frac{v_{\rm rel}}{2\alpha}$$

$$\epsilon_\phi = \frac{\alpha m}{\delta}$$

$$\delta = \zeta(2) m_\phi$$

- cosmological constraint is crucial for s-wave freeze-out DM



$$(\sigma_{\rm ann} v_{\rm rel})_{\rm w/o} \simeq 3\times10^{-26}\,\mathrm{cm^3/s}$$

$$m \simeq 20\,\mathrm{GeV}$$

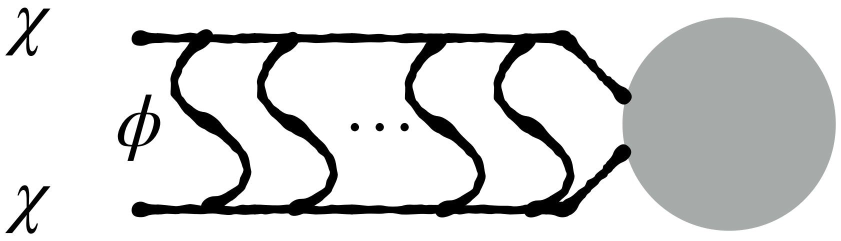
Planck Collaboration, A&A, 2020

Watson theorem

Oller, “A Brief Introduction to Dispersion Relations”

Annihilation matrix element

$$\Gamma_\alpha(k^2+i\epsilon)=\langle 0|\Theta_\chi|\psi^+_{\alpha,k}\rangle$$



- in-state as a whole

- inserting out states

$$\Gamma_\alpha(k^2+i\epsilon)=\sum_\beta S_{\beta\alpha}(k)\langle 0|\Theta_\chi|\psi^-_{\beta,k}\rangle=\sum_\beta S_{\beta\alpha}(k)\Gamma_\beta(k^2-i\epsilon)$$

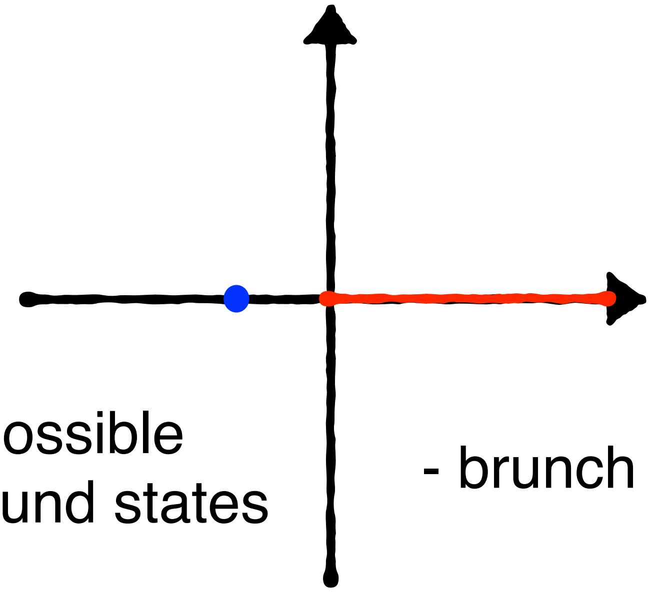
- assuming the real matrix element (T-invariance)

$$\Gamma_\alpha(k^2+i\epsilon)=\sum_\beta S_{\beta\alpha}(k)\Gamma_\beta(k^2+i\epsilon)^*$$

- for a partial wave

$$\Gamma_\ell(k^2+i\epsilon)=e^{2i\delta_\ell}\Gamma_\ell(k^2+i\epsilon)^*$$

- complex k^2 plane



- possible bound states

- brunch cut

Omnès solution

Omnès function

$$\Omega_\ell(k^2) = \exp[\omega_\ell(k^2)] \qquad \omega(k^2) = \frac{1}{\pi} \fint_0^\infty dq^2 \frac{\delta_\ell(q)}{q^2 - k^2}$$

- principal value

- computed by phase shift and reproduce the brunch cut

$$F_\ell(k^2) = \prod_{b_\ell} \frac{k^2}{k^2 + \kappa_{b,\ell}^2}$$

- rational function reproducing bound-state poles (Levinson theorem)

$$\boxed{\Gamma_\ell(k^2) = \Omega_\ell(k^2) F_\ell(k^2)}$$

- from Liouville theorem

  - we normalize $\delta_\ell(k) \to 0 \quad \Gamma_\ell(k^2) \to 1 \; k^2 \to \infty$

  - scattering phase and Sommerfeld enhancement are negligible at high velocity

Sommerfeld enhancement

$$S_\ell = |\Gamma_\ell(k^2)|^2$$

Around resonances

Levison theorem

Weinberg, “Lectures on Quantum Mechanics”

- # of bound states is given by phase shift

$$\delta_\ell(k \to 0) - \delta_\ell(k \to \infty) = \left[\#b_\ell \left(+\frac{1}{2}\right)\right]\pi$$

- excluding virtual states

- zero in our normalization

- only for s-wave resonances

- underlying idea
  - consider the system confined in a large sphere

$$R_{k\ell}(r) \to \frac{\sin(kr - \frac{1}{2}\ell\pi + \delta_\ell)}{r}$$

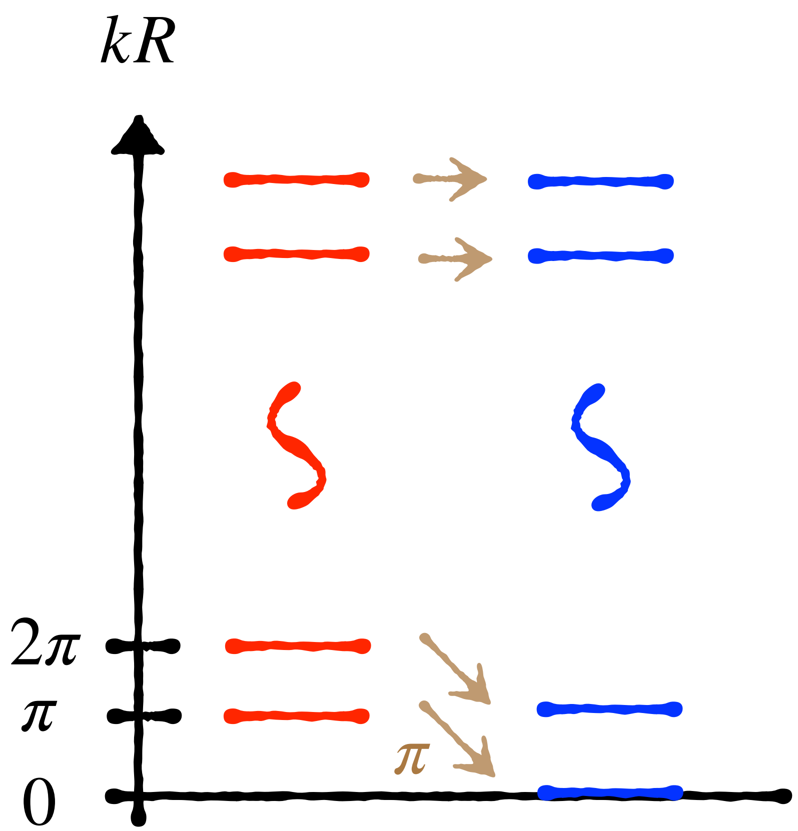
$$kR - \frac{1}{2}\ell\pi + \delta_\ell = n\pi$$

$$n = 0, \pm 1, \pm 2 \ldots$$

$r \to \infty$

$k > 0$

  - scattering states are discretized (countable infinity)
  - decrease in # of scattering states = # of bound states
    - total number does not change



- w/o potential

- w/ potential

Around resonances

AK, Kuwahara and Patel, arXiv:2303.17961

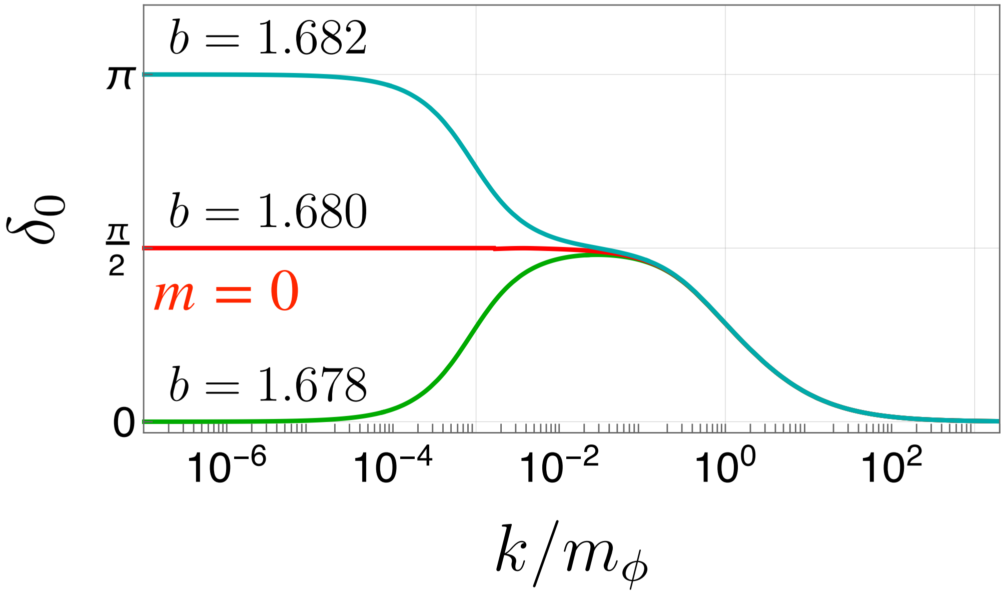
Effective range theory

- s-wave resonances

$$k \to 0 \quad k\cot\delta_0 \to -\frac{1}{a_0} + \frac{r_{e0}}{2}k^2$$

$$a_0 \to \infty$$

$$k \to 0 \quad \delta_0 \to \left(\frac{1}{2}+m\right)\pi \quad m = 0, 1, 2\ldots$$



- Omnès function

for m=0 (later)

$$\omega_0(k^2) = \frac{1}{\pi}\int_0^\infty dq^2 \frac{\delta_0(q)}{q^2-k^2}$$

$$F_0(k^2) = 1$$

$$k \to 0 \quad \to -\left(\frac{1}{2}+m\right)\ln(r_{e,0}^2 k^2)$$

$$\Gamma_0(k^2) = \exp[\omega_\ell(k^2)]F_0(k^2) \quad S_0 = |\Gamma_0(k^2)|^2$$

$$k \to 0 \quad \to \frac{F_0(k^2)}{k^{1+2m}}$$