Generalized Z string and its stability

Yukihiro Kanda (Nagoya University)

Based on PRD 107 (2023) 9, 096007 [arXiv: 2303.09517]

In collaboration with N. Maekawa (Nagoya University)

1. Introduction and motivation

Cosmic string … A classical solution having 1-dimensional excited region which is produced in symmetry breaking

Ex. In $U(1)$ Higgs model with a potential $V(\phi) = \lambda(|\phi|^2 - v^2)^2$,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi) \qquad (D_\mu\phi = (\partial_\mu - igA_\mu)\phi)$$

Nielsen-Olesen string [Nielsen, Olesen (1973)]

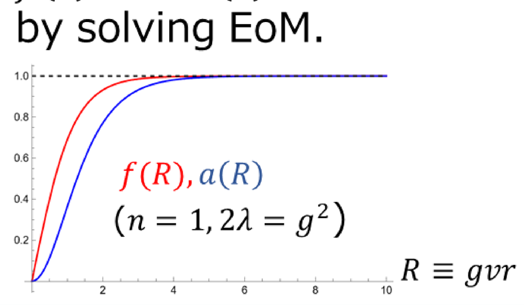
Scalar : $\phi_s(x) = f(r)ve^{in\theta}$ $\quad (n \in \mathbb{Z})$

Gauge boson : $\vec{A}_s(x) = \frac{na(r)}{gr}\vec{e}_\theta$, $A_s^0(x) = 0$

in the cylindrical coordinate (r, θ, z)

$(f(0) = a(0) = 0, f(\infty) = a(\infty) = 1)$

$f(r)$ and $a(r)$ are derived by solving EoM.



Recently, the NANOGrav 15-year data [arXiv:2306.16219] suggests not simple N-O strings but metastable strings or superstrings.

⬇

Z string in $SU(2)\times U(1)$ Higgs model with $V(H) = \lambda(H^\dagger H - v^2)^2$

$SU(2)\times U(1) \xrightarrow{H:\ (\mathbf{2}, 1/2)} U(1)$

$\cup \qquad \cup$

$U(1) \longrightarrow \times$

Related to the Z boson

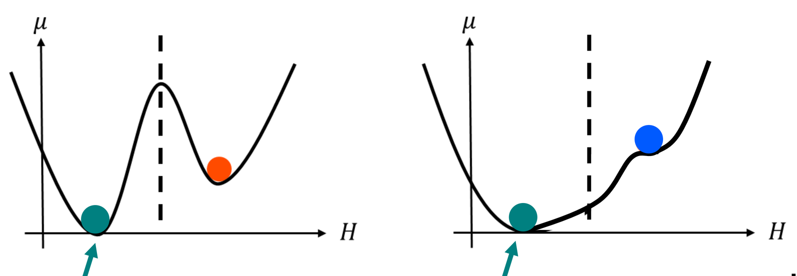
Consider N-O string

Z string [Vachaspati (1992)]

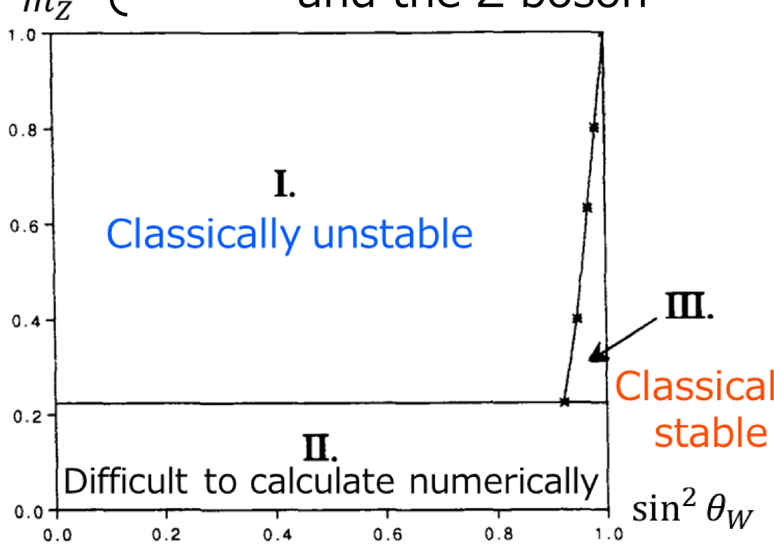
$$H = \begin{pmatrix} 0 \\ f(r)ve^{i\theta} \end{pmatrix}, \qquad \vec{Z} = -\frac{2z(r)}{\alpha r}\vec{e}_\theta,$$

(others) = 0 $\qquad (\alpha^2 = g_1^2 + g_2^2)$

Is Z string stable or unstable?



$\frac{m_H}{m_Z}$ (m_H, m_Z: the masses of the Higgs and the Z boson)



➡ Z string is unstable in the SM.
[James, Perivolaropoulos, Vachaspati (1992)]

In more general, we can consider embedded strings for sub-$U(1)$ breaking, but their stability has not been studied except for the Z string. [Vachaspati, Barriola, Bucher (1994)]

To study the embedded strings which are produced in physics beyond the SM, we should answer

① When are they produced? ➡ Our work! (This presentation)

② How are they observed? ➡ Our future work…

2. Generalized Z string

$SU(N)\times U(1)$ Higgs model

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_\mu\phi|^2 - \lambda(|\phi|^2 - v^2)^2 \qquad \phi: \left(\mathbf{N}, \tfrac{1}{2}\right)$$

We write neutral massive gauge boson as

$$\tilde{Z}_\mu \equiv \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}G_\mu^{N^2-1} - \frac{g_1}{\alpha_N}B_\mu$$

$$\alpha_N^2 \equiv \frac{2(N-1)}{N}g_N^2 + g_1^2 \qquad T^{N^2-1} = \frac{1}{\sqrt{2N(N-1)}}\mathrm{diag}(1, \dots, 1, 1-N)$$

Generalized Z-string

$$\phi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f(r)ve^{i\theta} \end{pmatrix}, \quad \vec{\tilde{Z}} = -\frac{2z(r)}{\alpha_N r}\vec{e}_\theta, \quad \text{(others)}=0$$

$f(0) = z(0) = 0, f(\infty) = z(\infty) = 1$

When $N = 2$, it is the same as the Z string

To study the classical stability, we consider all the perturbation.

Higgs: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_{N-1}(x) \\ f(r)e^{in\theta} + \delta\phi(x) \end{pmatrix}$, Gauge boson: $\begin{pmatrix} \vec{G}^a(x) & \vec{G}^+(x) \\ \vec{G}^-(x) & \end{pmatrix}$,

$\vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\vec{e}_\theta + \delta\vec{\tilde{Z}}(x)$

$\vec{\tilde{A}}(x) \equiv \frac{g_1}{\alpha_N}\vec{G}^{N^2-1} + \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}\vec{B}$ Diagonal part

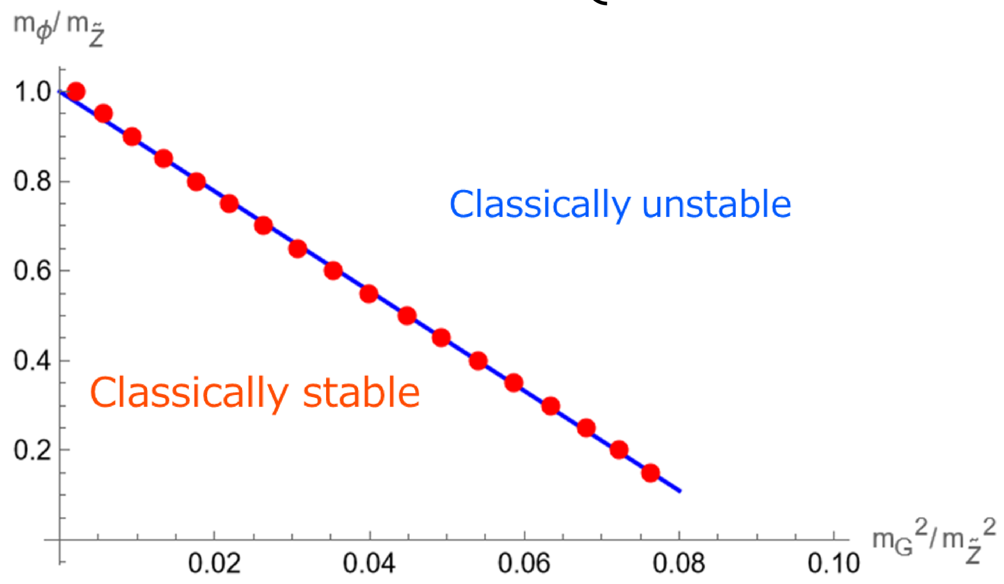
:$SU(N-1)$ adjoint
:$SU(N-1)$ fundamental
:$SU(N-1)$ singlet

➡ Variation of the energy density $\delta\mu = \delta\mu_{ad} + \delta\mu_s + \delta\mu_f$ ⬅ Check it!
$\qquad (>0) \quad (>0)$

We have found $\delta\mu_f = \sum_{k=1}^{N-1}\delta\mu_k\left(r; \frac{m_\phi}{m_{\tilde{Z}}}, \frac{m_G}{m_{\tilde{Z}}}\right)$ ⬅ The stability depends on only the two mass ratios.

Our result

($m_\phi, m_{\tilde{Z}}, m_G$: the masses of the scalar, the neutral gauge boson and the charged gauge boson)



Approximate stable region

$$\frac{m_\phi}{m_{\tilde{Z}}} \leq 1 - 11\frac{m_G^2}{m_{\tilde{Z}}^2}$$

or

$$g_1 \geq \sqrt{\frac{11}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N$$

g_1 should be at least 3 times bigger than g_N for string formation.

3. Applications for the gauge group unification

We consider the case that $SU(N)$ and $U(1)$ are unified into the same simple group G.

$\phi = (\mathbf{N}, q, \mathbf{1})\big|_{g_1'} = (\mathbf{N}, 1/2, \mathbf{1})\big|_{g_1}$

$G \to \cdots \to SU(N)\times U(1)\times H \to SU(N-1)\times U(1)\times H$

$g_U = g_N = g_1'$ —RG running→ $g_N = \alpha_{RG}g_1' = \frac{\alpha_{RG}}{2q}g_1$

Substituting it for our result ⬇

Constraint for q
$$q^2 \geq \alpha_{RG}^2\left[\frac{2.75}{1 - m_\phi/m_{\tilde{Z}}} - \frac{N-1}{2N}\right]$$

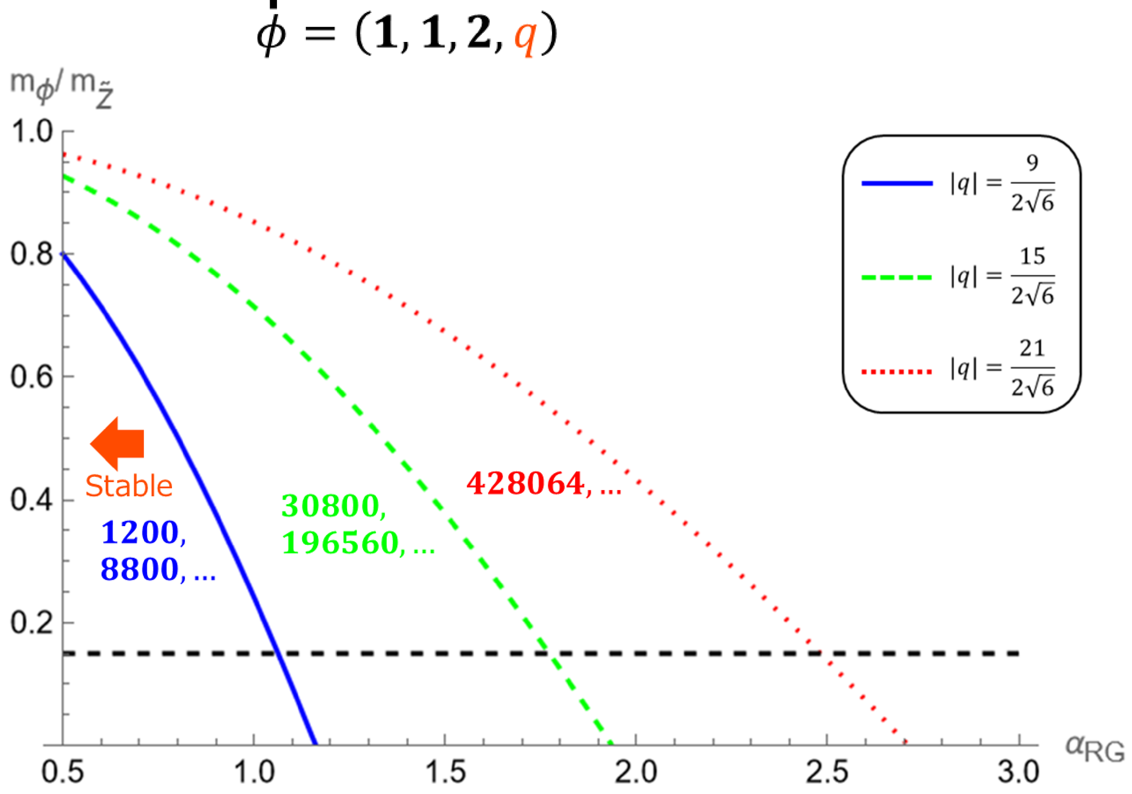
⇕

Constraint for the representation of ϕ in G

We apply it for two breaking patterns from $SO(10)$ to $SU(3)\times SU(2)\times U(1)$.

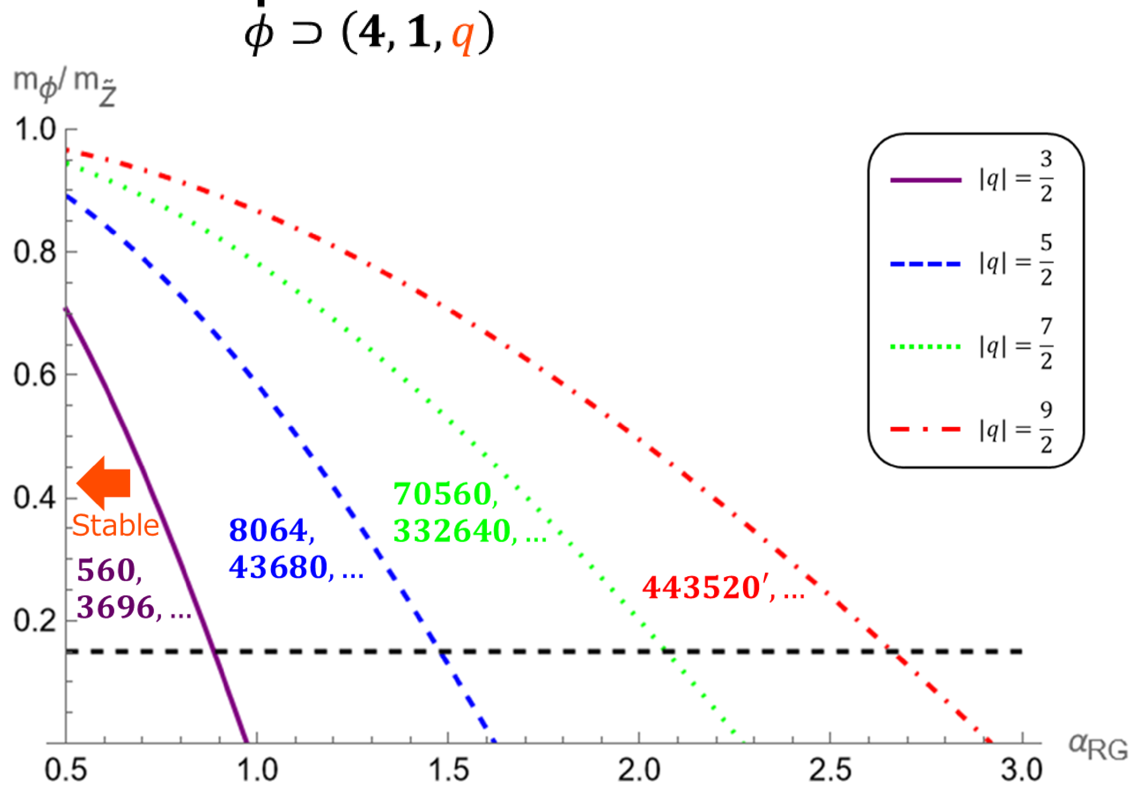
① $SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \to SU(3)_C\times SU(2)_L\times U(1)_Y$

↑ $\phi = (\mathbf{1}, \mathbf{1}, \mathbf{2}, q)$



② $SO(10) \to SU(4)_C \times SU(2)_L \times U(1)_X \to SU(3)_C\times SU(2)_L\times U(1)_Y$

↑ $\phi \supset (\mathbf{4}, \mathbf{1}, q)$



To produce the generalized Z string in some GUT models,

- Very high representation Higgs are needed.
- The SM fermions are not simply unified.

Summary

- Embedded strings are not topological defects, but the classical solutions having 1-dimensional excited region (= cosmic string)
- We have generalized the Z string for the $SU(N)\times U(1)$ Higgs model and found that its stability can be determined the ratios of the masses $(m_\phi/m_{\tilde{Z}}, m_G/m_{\tilde{Z}})$. It is consistent with the results for the Z string studied in 1993.
- We have applied the formation condition to the case that $SU(N)$ and $U(1)$ have the same origin. We have found that a higher dimensional scalar is needed for the generalized Z string formation.