Quark and lepton hierarchies from S_4' modular flavor symmetry

Junichiro Kawamura, Institute for Basic Science CTPU

with Y.Abe (U. of Wisconsin), T.Higaki (Keio U.), T.Kobayashi (Hokkaido U.).

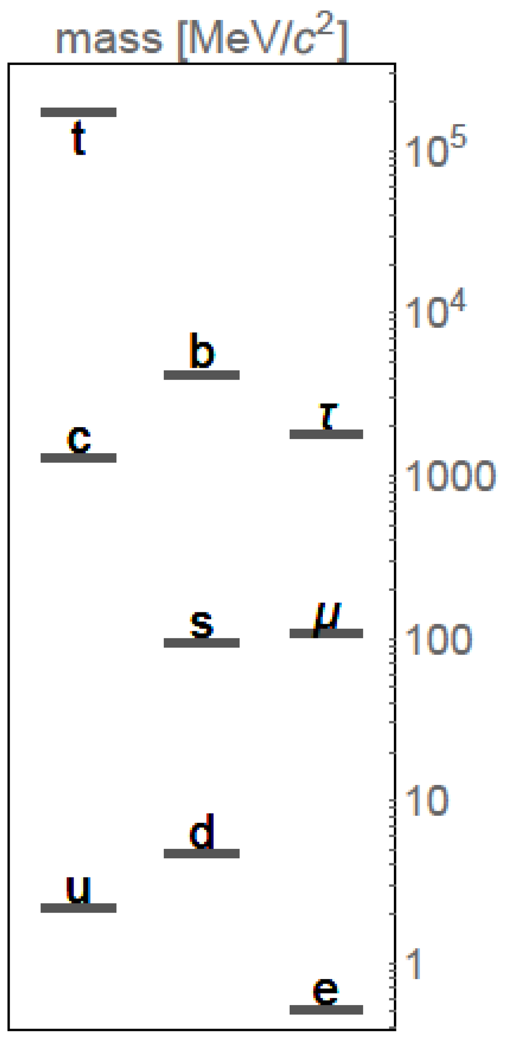
based on 2301.07439, **2302.11183 [PLB]**, 2307.01419 [JHEP]

1. Quark and lepton hierarchies

- there are 3 gens. of quarks and leptons
- masses are hierarchical
- CKM angles : $\theta_{13} \ll \theta_{23} \ll \theta_{12}$
- PMNS angles: $\theta_{13} < \theta_{23} \sim \theta_{12}$

How to explain the hierarchies ?

$\longrightarrow$ modular symmetry !



2. Modular flavor symmetry

➢ finite modular symmetry $\Gamma_N' := \Gamma(1)/\Gamma(N)$

$$\Gamma(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv 1 \mod N \in \mathbb{N} \,\middle|\, a,b,c,d \in \mathbb{Z}\,, ad - bc = 1 \right\}$$

generators: $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\longrightarrow$ $S^2 = R,\quad (ST)^3 = R^2 = 1,\quad TR = RT,\quad T^N = 1$

$\longrightarrow$ same as discrete flavor symmetries, e.g. $\Gamma_4' \simeq S_4'$

➢ modular forms

under modular sym., Yukawa couplings are "modular forms"

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \to (c\tau + d)^k \rho(r) Y_r^{(k)}(\tau) \qquad \text{for } \tau \overset{\Gamma}{\to} \frac{a\tau+b}{c\tau+d}$$

- τ is a complex scalar, called modulus (field)
- $k \in \mathbb{N}$ is a modular weight
- r labels representation whose matrix is $\rho(r)$

➢ residual symmetry

ex) if $\mathrm{Im}\,\tau \gg 1$, $T : \tau \to \tau + 1$ is unbroken

$\longrightarrow$ "residual" $\mathbb{Z}_N^T$ symmetry is approximately unbroken

ex) modular form for $k = 1, r = \hat{3}$ under $\Gamma_4' \simeq S_4'$

$\mathbb{Z}_4^T$-charge

$$Y_{\hat{3}}^{(1)}(\tau) \sim \begin{pmatrix} \sqrt{2}\epsilon(\tau) \\ \epsilon(\tau)^2 \\ -1 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 0 \end{matrix} \qquad \epsilon(\tau) \sim 2\exp\left(\frac{2\pi i \tau}{4}\right) \ll 1$$

$\longleftrightarrow$ Froggatt-Nielsen [FN] mechanism $\left(\frac{\langle \phi \rangle}{\Lambda}\right)^n \Leftrightarrow \epsilon(\tau)^n$

natural and predictive realization of FN mech.

3. S_4' model ...first model for both Q and L

➢ Representations

LH doublet quark $Q = 3$, $\mathbb{Z}_4^T$-charge (2,3,1)

RH up quark $u^c = 1 \oplus 1 \oplus \hat{1}'$, 0 0 1

RH down quark $d^c = 1 \oplus 1 \oplus 1$, 0 0 0

LH doublet lepton $L = 1 \oplus 1 \oplus 1$, 0 0 0

RH charged lepton $e^c = 3$, (2,3,1)

➢ Modular weights

$$k_Q = 4\,,\quad (k_{u_1}, k_{u_2}, k_{u_3}) = (0,4,3),\quad (k_{d_1}, k_{d_2}, k_{d_3}) = (0,2,4)$$

$$(k_{L_1}, k_{L_2}, k_{L_3}) = (0,2,4),\qquad k_e = 4$$

➢ Textures of masses and CKM /PMNS matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t \qquad \cot\beta := \langle H_d \rangle / \langle H_u \rangle$$

$$(m_d, m_s, m_b) \sim (m_e, m_\mu, m_\tau) \sim (\epsilon^3, \epsilon^2, \epsilon) m_t \cot\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \qquad V^{PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- $N = 4$ is minimal for different hierarchies in up and down Qs
- additional factors from $(2\mathrm{Im}\tau)^k$ by canonical normalization

➢ Numerical results

$\tan\beta = 3.7, \mathrm{Im}\,\tau = 2.8$ with coefficients $\in [0.62, 5.4]$

masses at GUT scale

obs.	value	center	error
$y_u/10^{-6}$	4.44	2.85	0.88
$y_c/10^{-3}$	1.481	1.479	0.052
y_t	0.5322	0.5320	0.0053
$y_d/10^{-5}$	1.94	1.93	0.21
$y_s/10^{-4}$	3.88	3.82	0.21
$y_b/10^{-2}$	2.097	2.100	0.021
$y_e/10^{-6}$	7.816	7.816	0.047
$y_\mu/10^{-3}$	1.6496	1.6500	0.0099
$y_\tau/10^{-2}$	2.808	2.805	0.028

CKM/PMNS angles

s_{12}	0.22520	0.22541	0.00072
$s_{23}/10^{-2}$	4.007	3.998	0.064
$s_{13}/10^{-3}$	3.43	3.48	0.13
δ_{CKM}	1.2395	1.2080	0.0540
$R_{32}^{21}/10^{-2}$	3.053	3.070	0.084
s_{12}^2	0.302	0.307	0.013
s_{23}^2	0.547	0.546	0.021
$s_{13}^2/10^{-2}$	2.203	2.200	0.070
δ_{PMNS}	-0.85	-2.01	0.63

well-fit ($< 1.8\sigma$) to the data

4. Summary

- modular sym. realizes non-Abelian discrete sym. wo/ flavon
- FN-like mechanism by a residual symmetry
- constructed the first model for both Q and L
- successfully explains the experimental data
- $SU(5)$ GUT model is possible for $\Gamma_6^{(\prime)}$ ...2307.01419