QCD Axion:
A Unique Player in the Axiverse with Mixings

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Introduction

- QCD axion

$\begin{cases} \text{a solution to the strong CP problem} \\ \text{a (pseudo-)NG boson arising at SSB of Peccei-Quinn symmetry} \end{cases}$

QCD axion acquires a potential by QCD instanton effects:

$$V_{\rm QCD} = \chi(T)\left[1 - \cos\left(\frac{a}{f_a}\right)\right]$$

$$\chi(T) = \begin{cases} \chi_0 & (T < T_{\rm QCD}) \\ \chi_0 \left(\dfrac{T}{T_{\rm QCD}}\right)^n & (T \geq T_{\rm QCD}) \end{cases}$$

$\chi_0 = (75.6\,\text{MeV})^4$

$n = -8.16$

$T_{\rm QCD} = 153\,\text{MeV}$

QCD axion is a candidate of dark matter.

Introduction

- Misalignment of QCD Axion

Pre-inflationary PQ breaking → Misalignment, random a

(Post-inflationary PQ breaking → Random $a(x)$ & string-wall network)

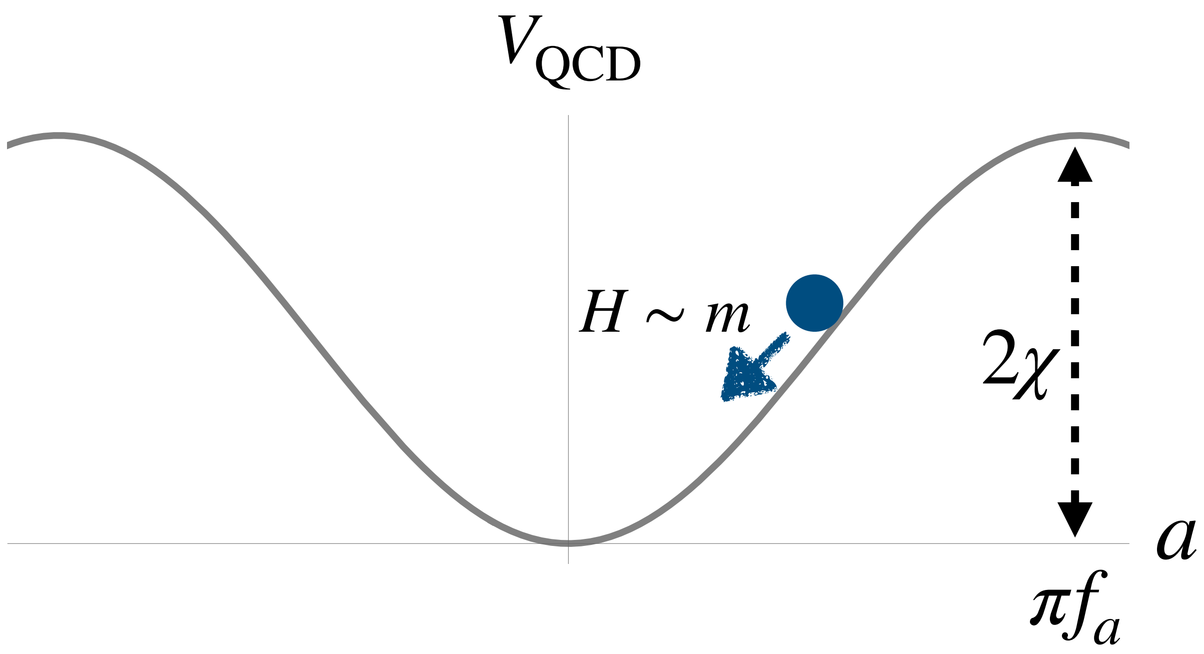
Energy scale : $\sim \chi$

Oscillation: $H \sim m = \sqrt{\chi}/f_a$

Larger abundance for larger f_a

If $a_{\rm ini}/f_a = \mathcal{O}(1)$,

$f_a = \mathcal{O}(10^{12})$ GeV for all DM.



$f_a > \mathcal{O}(10^{12})$ GeV requires a fine-tuning.

Introduction

- **Stochastic scenario** Graham, Scherlis [1805.07362], Takahashi, Yin, Guth [1805.08763]

  If the inflationary scale $H_{\rm inf} \lesssim T_{\rm QCD}$, $V_{\rm QCD}$ is present during inflation.
  Then, $a_{\rm ini}$ is no longer a random variable.

- Classical role

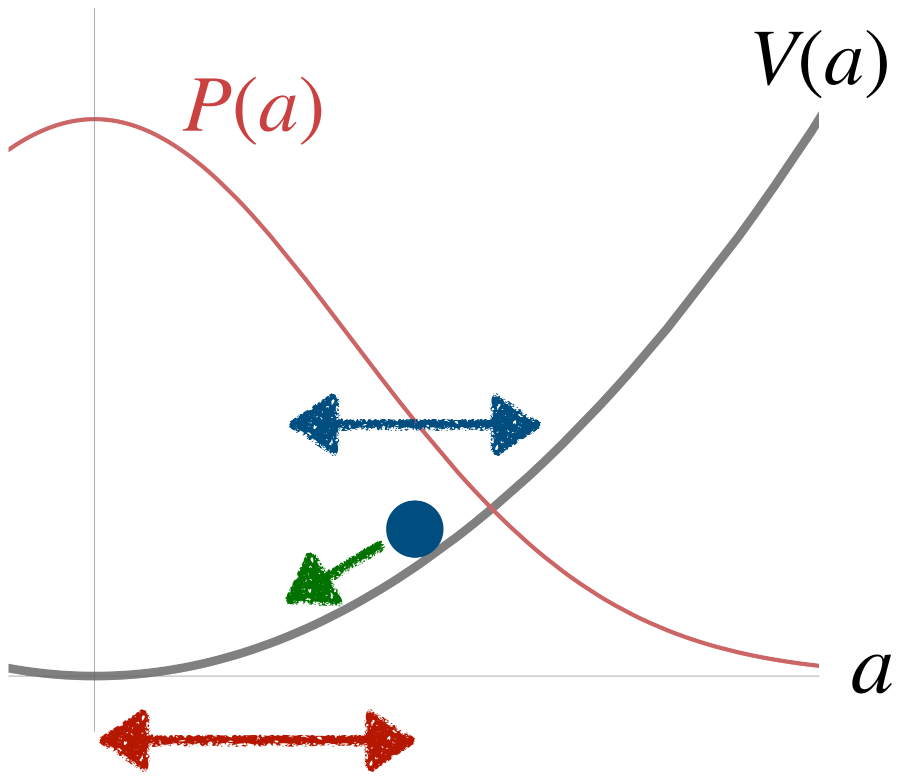
$$\Delta a = -\frac{m^2}{3H_{\rm inf}^2} a \quad \text{in each e-fold.}$$

$$\langle a^2 \rangle \to e^{-m^2/H_{\rm inf}^2} \langle a^2 \rangle$$

- Quantum fluctuations

$$\Delta a = \pm \frac{H_{\rm inf}}{2\pi} \quad \text{in each e-fold.}$$

$$\langle a^2 \rangle \to \langle a^2 \rangle + H_{\rm inf}^2$$



$$\langle a^2 \rangle \sim \frac{H_{\rm inf}^4}{m^2}$$

Introduction

- **Stochastic scenario** Graham, Scherlis [1805.07362], Takahashi, Yin, Guth [1805.08763]

Fokker-Planck equation:

$$\frac{\partial P(N,a)}{\partial N} = \frac{1}{3H_{\rm inf}^2}\frac{\partial}{\partial a}\left(\frac{\partial V(a)}{\partial a}P(N,a)\right) + \frac{H_{\rm inf}^2}{8\pi^2}\frac{\partial^2 P(N,a)}{\partial a^2}$$

$$P(N,a) \propto \exp\left[-\frac{8\pi^2}{3H_{\rm inf}^4}V(a)\right]$$

c.f.) $\langle \phi^2 \rangle \sim \frac{H_{\rm inf}^4}{m^2}$

Dynamically realize $a_{\rm ini}/f_a \ll 1$.

$f_a > 10^{12}\,\mathrm{GeV}$ is possible for low-scale inflation.

Note: This scenario requires very long inflation.
Free from isocurvature perturbations due to low $H_{\rm inf}$.

Introduction

- Axiverse

There can also be many axion(-like particle)s from string theory.

Their decay constants are typically the string scale ($\sim 10^{15\text{-}17}\,\mathrm{GeV}$).

Overabundance by the misalignment mechanism (moduli problem)

Again, this can be solved by low-scale inflation. Ho, Takahashi, Yin [1901.01240]

If they are coupled to gluons, they also acquire V_{QCD}.

Their mixing leads to non-trivial behavior in the stochastic scenario!

Main topic

Model

- Model

Let us consider two axions, a and ϕ:

$$\mathscr{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(a, \phi)$$

$$V(a, \phi) = V_{\rm QCD}(a) + V_{a\phi}(a, \phi)$$

$$= \chi(T)\left[1 - \cos\left(\frac{a}{f_a}\right)\right] + m_\phi^2 f_\phi^2 \left[1 - \cos\left(\underbrace{N\frac{a}{f_a} + \frac{\phi}{f_\phi}}_{\text{mixing}}\right)\right]$$

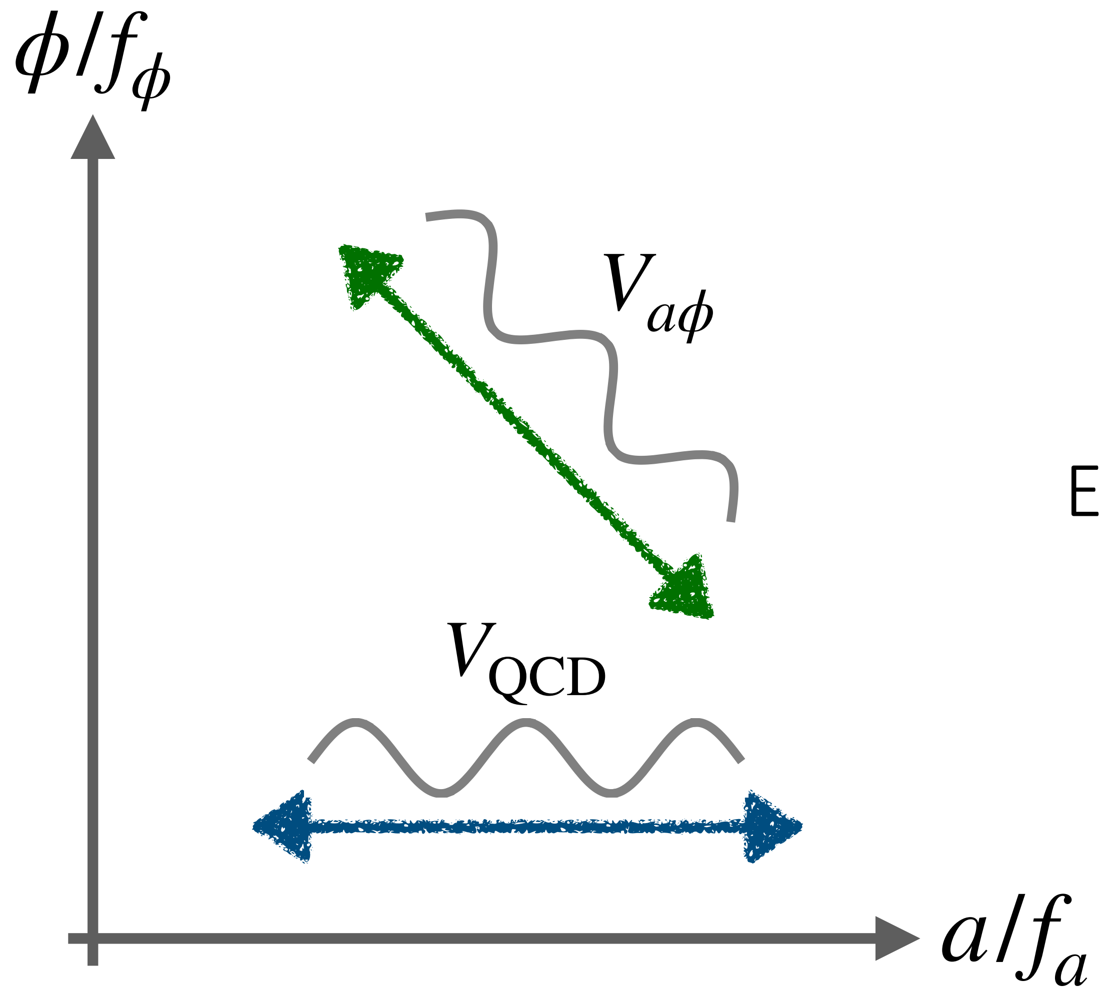
Here, we set $a = 0$ as the strong CP conserving point.

Typically, $f_a \sim f_\phi$ and we assume $f_a = f_\phi$ and $N = -1$ for simplicity.

Model

- Model

$$V(a,\phi) = \chi(T)\left[1-\cos\left(\frac{a}{f_a}\right)\right] + m_\phi^2 f_\phi^2\left[1-\cos\left(\underbrace{N\frac{a}{f_a}+\frac{\phi}{f_\phi}}_{\propto\,\Phi}\right)\right]$$



Each of $V_{\rm QCD}$ and $V_{a\phi}$ is flat in one direction.

Model

- Mass eigenstate

$$V = \chi(T)\left[1-\cos\left(\frac{a}{f_a}\right)\right] + m_\phi^2 f_\phi^2\left[1-\cos\left(N\frac{a}{f_a}+\frac{\phi}{f_\phi}\right)\right]$$

Around $a = \phi = 0$, the potential can be approximated as

$$V \simeq \frac{1}{2}\begin{pmatrix} a & \phi \end{pmatrix}\begin{pmatrix} \frac{\chi(T)+N^2 m_\phi^2 f_\phi^2}{f_a^2} & \frac{N m_\phi^2 f_\phi}{f_a} \\ \frac{N m_\phi^2 f_\phi}{f_a} & m_\phi^2 \end{pmatrix}\begin{pmatrix} a \\ \phi \end{pmatrix}$$

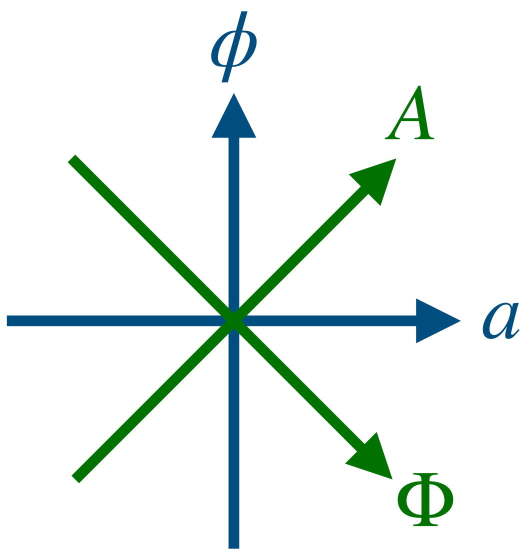
The mass matrix is orthogonalized as

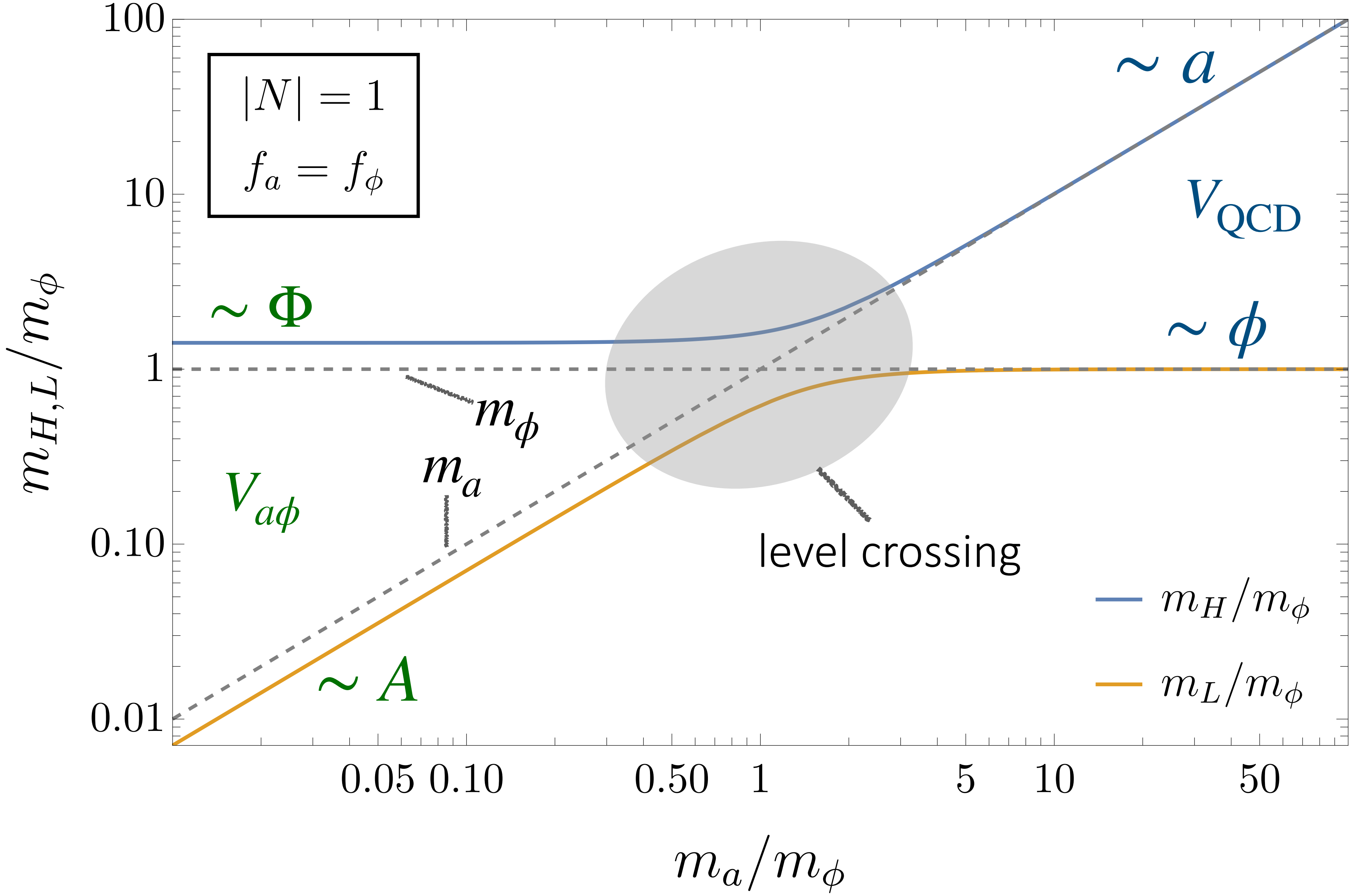
$$UMU^{\mathrm{T}} = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_L^2 \end{pmatrix}, \qquad U = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Mass eigenstates: $\begin{pmatrix} s_H \\ s_L \end{pmatrix} = U\begin{pmatrix} a \\ \phi \end{pmatrix}$

Model

- Mass eigenstate





Axion dynamics

- Initial condition during inflation

During inflation, the BD distribution is applied to the eigenstates:

$$\sqrt{\langle s_{H0}^2 \rangle} = \sqrt{\frac{3}{8\pi^2}\frac{H_{\rm inf}^2}{m_{H0}}}\,, \quad \sqrt{\langle s_{L0}^2 \rangle} = \sqrt{\frac{3}{8\pi^2}\frac{H_{\rm inf}^2}{m_{L0}}}$$

"0" denotes quantities when $\chi = \chi_0$.

We parameterize the field value during inflation by

$$s_{H0,\rm init} = c_H\sqrt{\frac{3}{8\pi^2}\frac{H_{\rm inf}^2}{m_{H0}}}\,, \quad s_{L0,\rm init} = c_L\sqrt{\frac{3}{8\pi^2}\frac{H_{\rm inf}^2}{m_{L0}}}$$

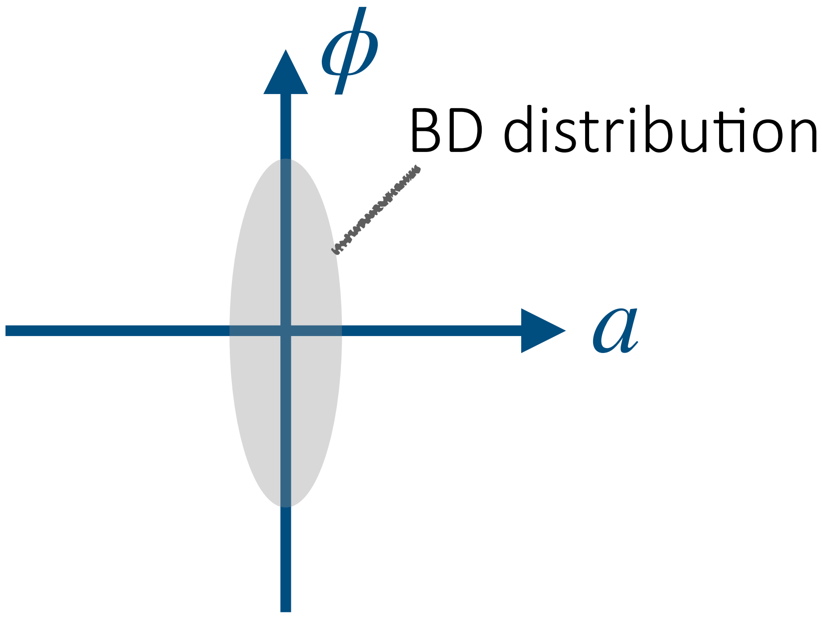
Then, we obtain the initial condition for the post-inflationary dynamics as

$$\begin{pmatrix} a_{\rm init} \\ \phi_{\rm init} \end{pmatrix} = U_0^{\rm T} \begin{pmatrix} s_{H0,\rm init} \\ s_{L0,\rm init} \end{pmatrix}$$

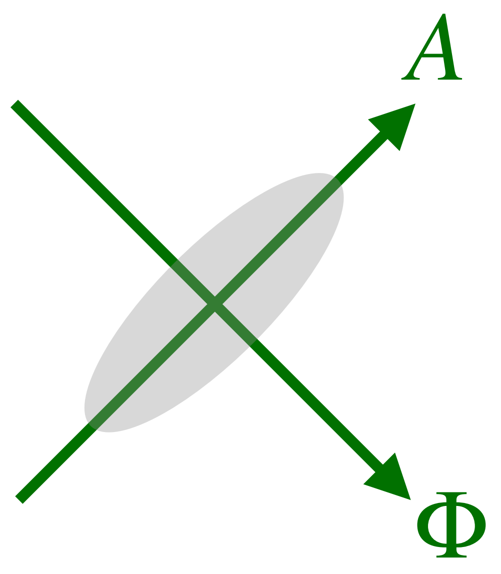
$$\sqrt{\langle s_{H0,L0}^2 \rangle} = \sqrt{\frac{3}{8\pi^2}\frac{H_{\rm inf}^2}{m_{H0,L0}}}$$

- BD distributions in two limits

If $m_{a0} \gg m_\phi$, $s_{H0} \simeq a$:



If $m_{a0} \ll m_\phi$, $s_{H0} \simeq \Phi$:



In the latter case, Φ is always a mass eigenstate. → Trivial dynamics

Axion dynamics

- After inflation

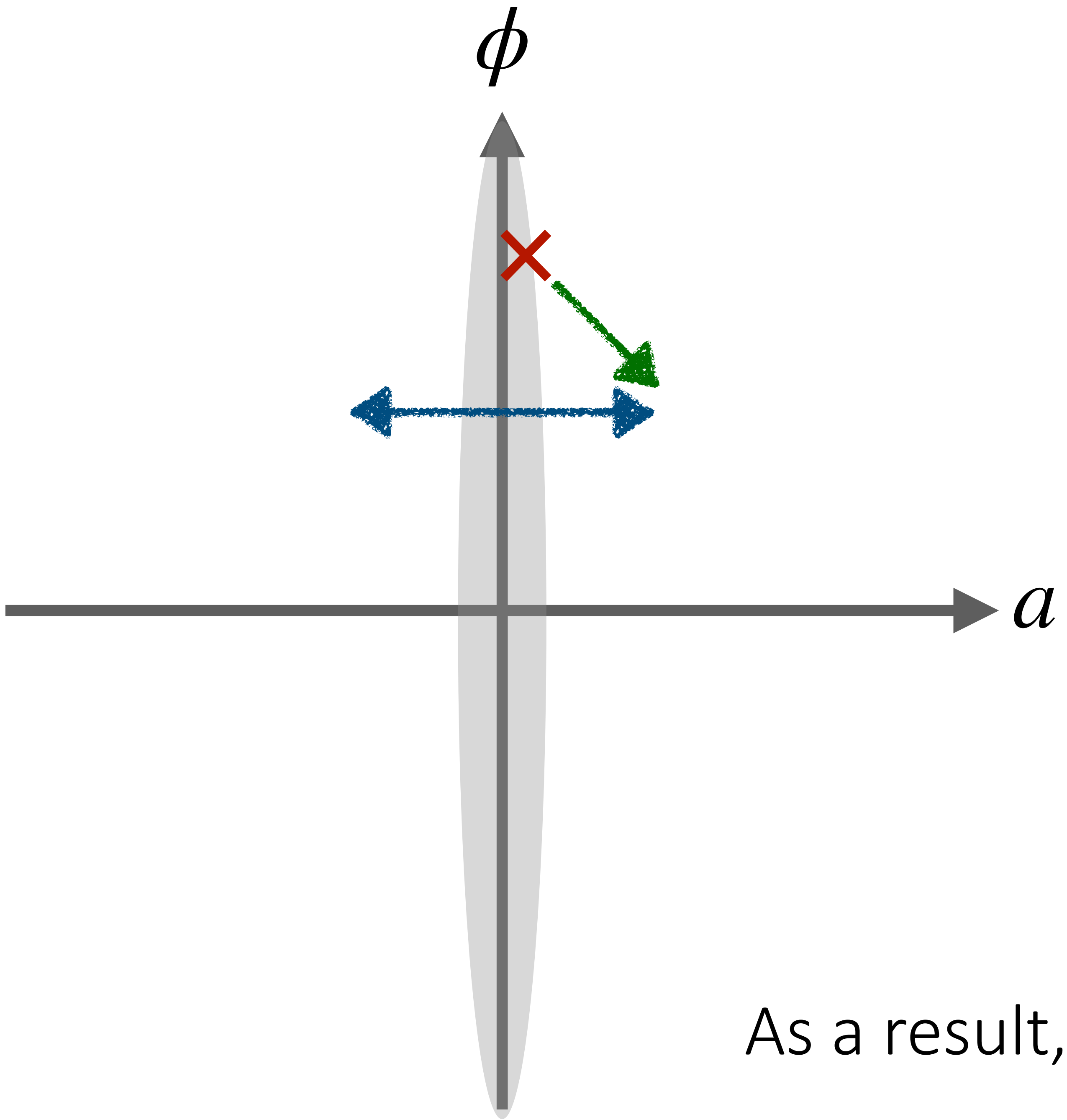
If the axions move only after $T < T_{\mathrm{QCD}}$,
a is a mass eigenstate when the potential works.

We consider the axion starts to move at $T > T_{\mathrm{QCD}}$:

$$3H(T_{\mathrm{QCD}}) \lesssim m_\phi \lesssim m_{a0}$$

At first, $V_{\mathrm{QCD}} \sim 0$ and the axions move in the Φ-direction.
After that, V_{QCD} arises and the axions move in 2D.

- Qualitative evolution



1. In the BD distribution,
   $a_{\rm ini}$ is typically small due to m_{a0}.

2. When $H \sim m_\phi > m_a$,
   (a, ϕ) rolls in the Φ-direction.

3. Then m_a grows and
   a has a large amplitude.
   (Typically $a \sim \phi$)

As a result, the abundance of a is enhanced.

Axion dynamics

- Numerical simulation

Assumption: $N = -1\,, \quad f_a = f_\phi \equiv f\,, \quad c_H = c_L = 1$

If $H_{\rm inf}$ is small enough, the potential is almost quadratic.
Then, $H_{\rm inf}$ does not change the qualitative behavior of the axions.

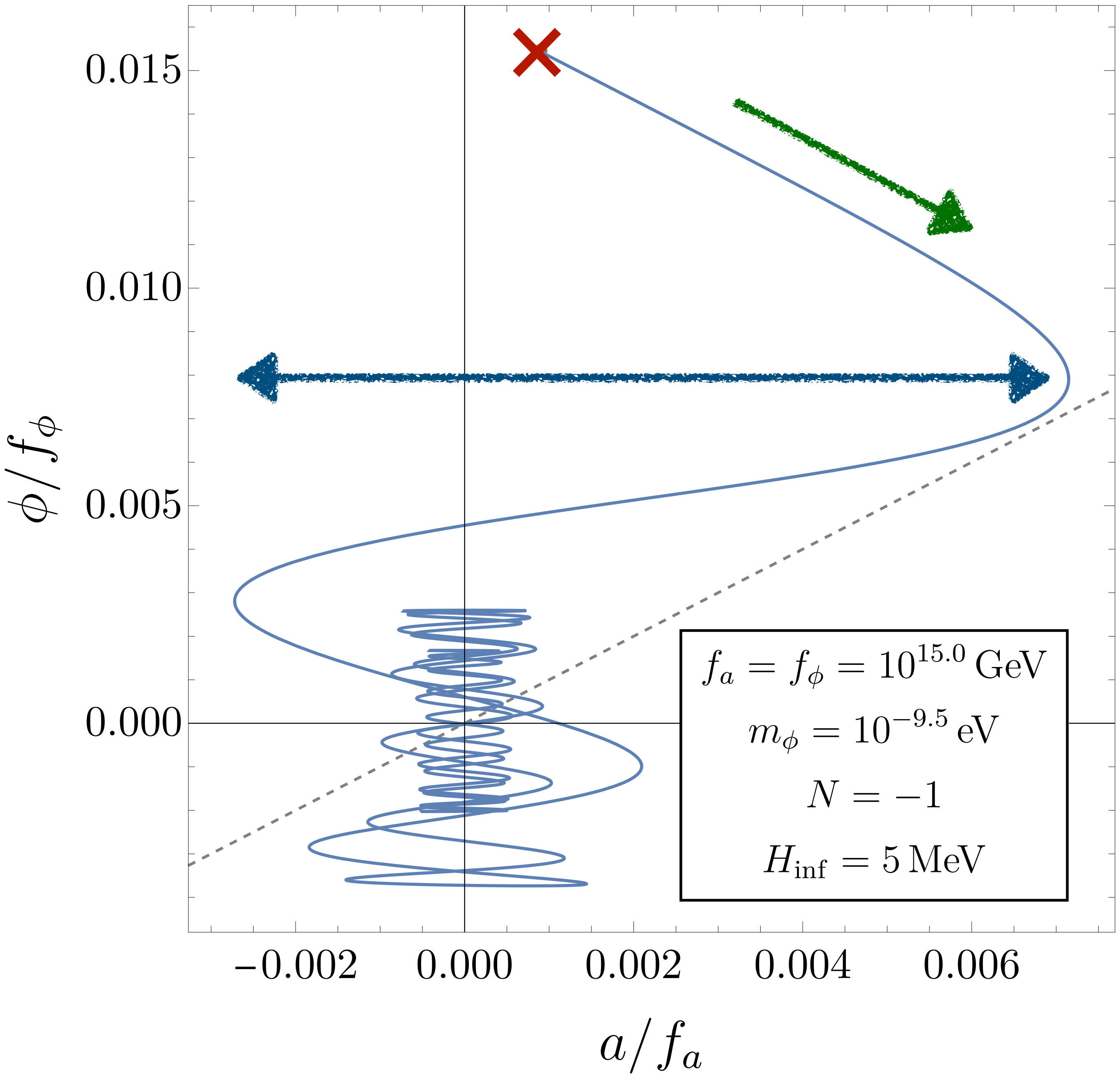
The remaining free parameters are f and m_ϕ.

For comparison, we also simulate two axions without mixing ($N = 0$).

Results

Axion dynamics

- Trajectory

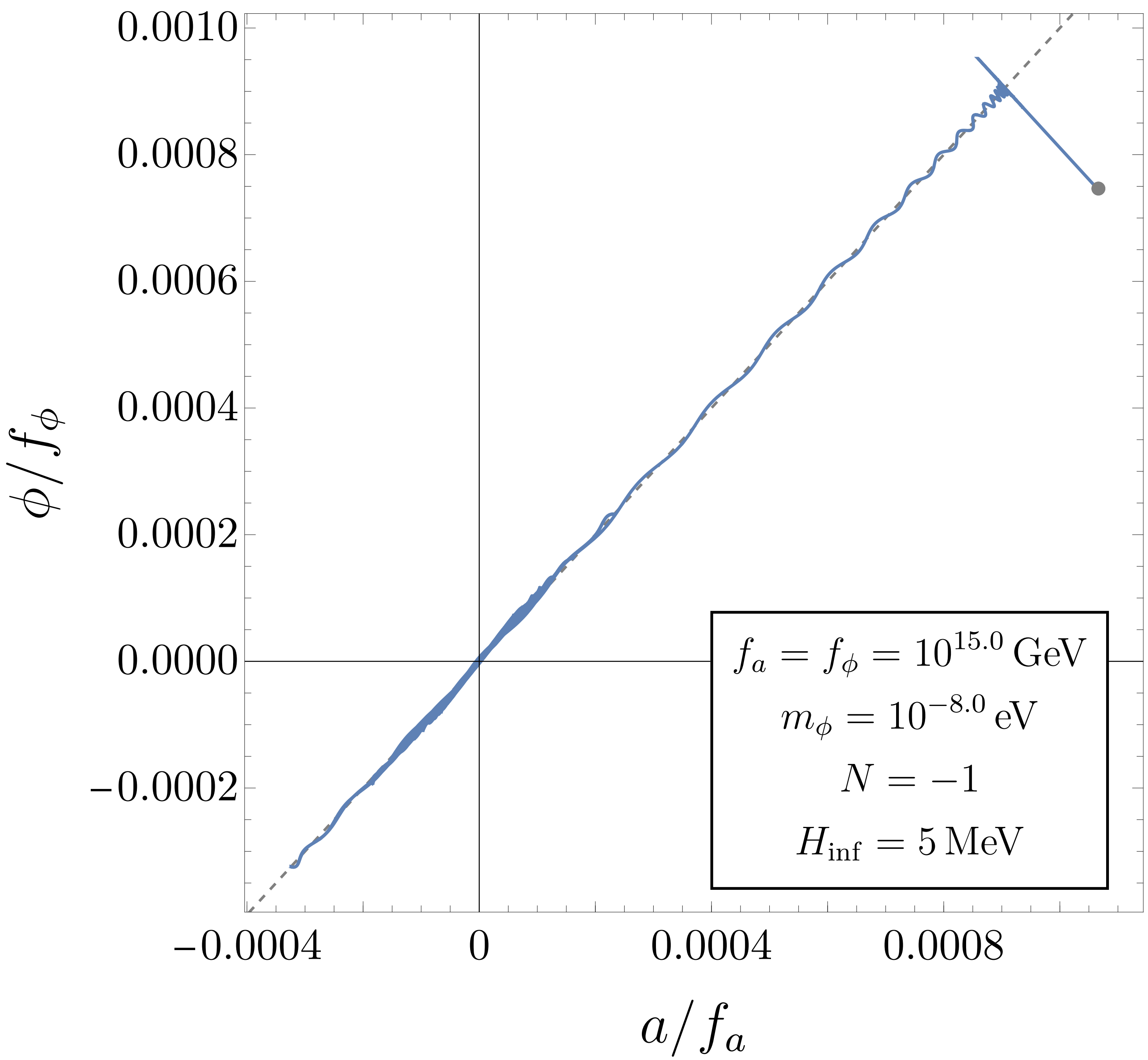


$m_\phi \sim H(T)$ when $m_a(T) \sim H(T)$

→ Large enhancement

Axion dynamics

- Trajectory

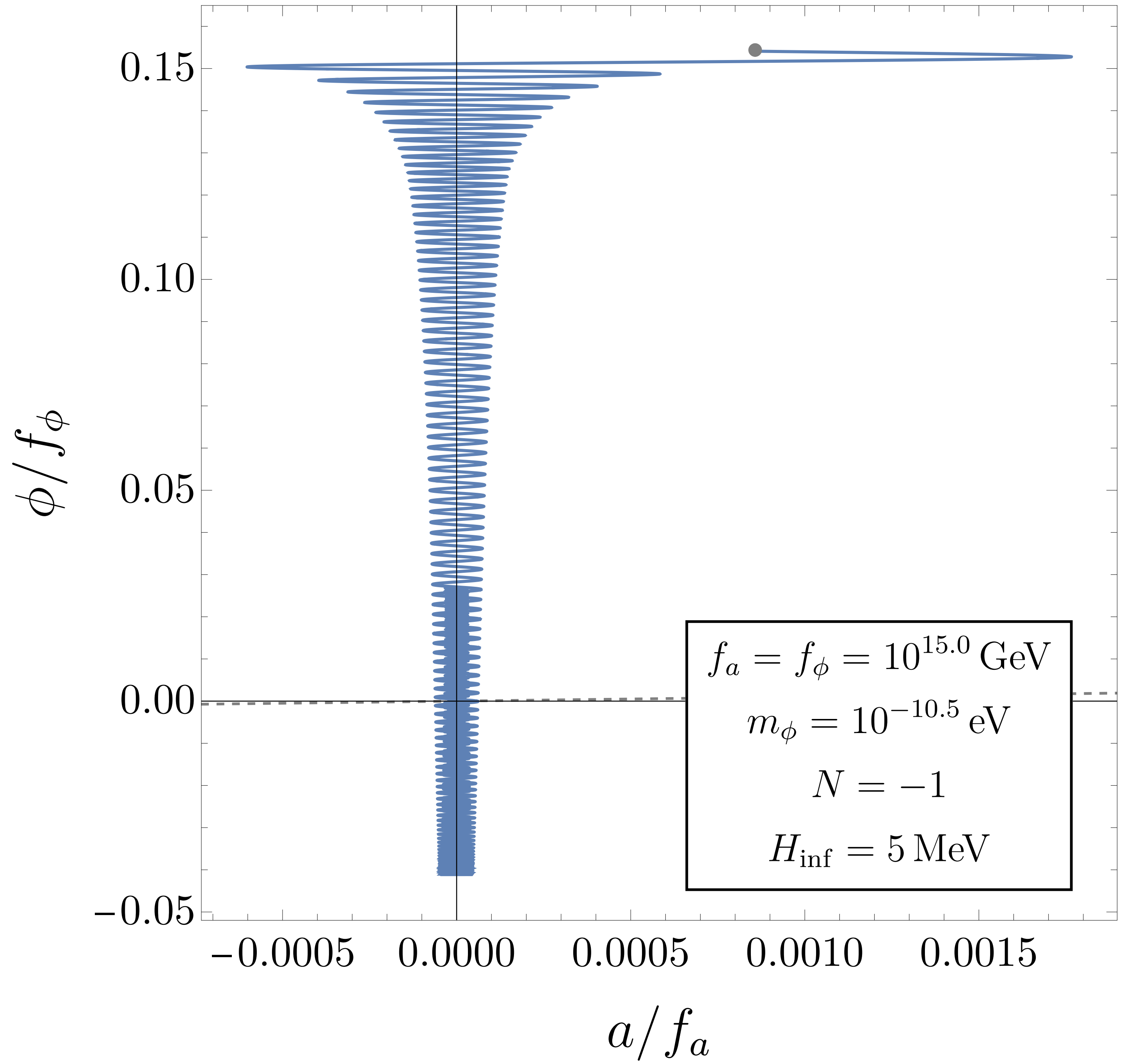


$m_\phi > m_{a0}$

→ Φ & A are independent.

Axion dynamics

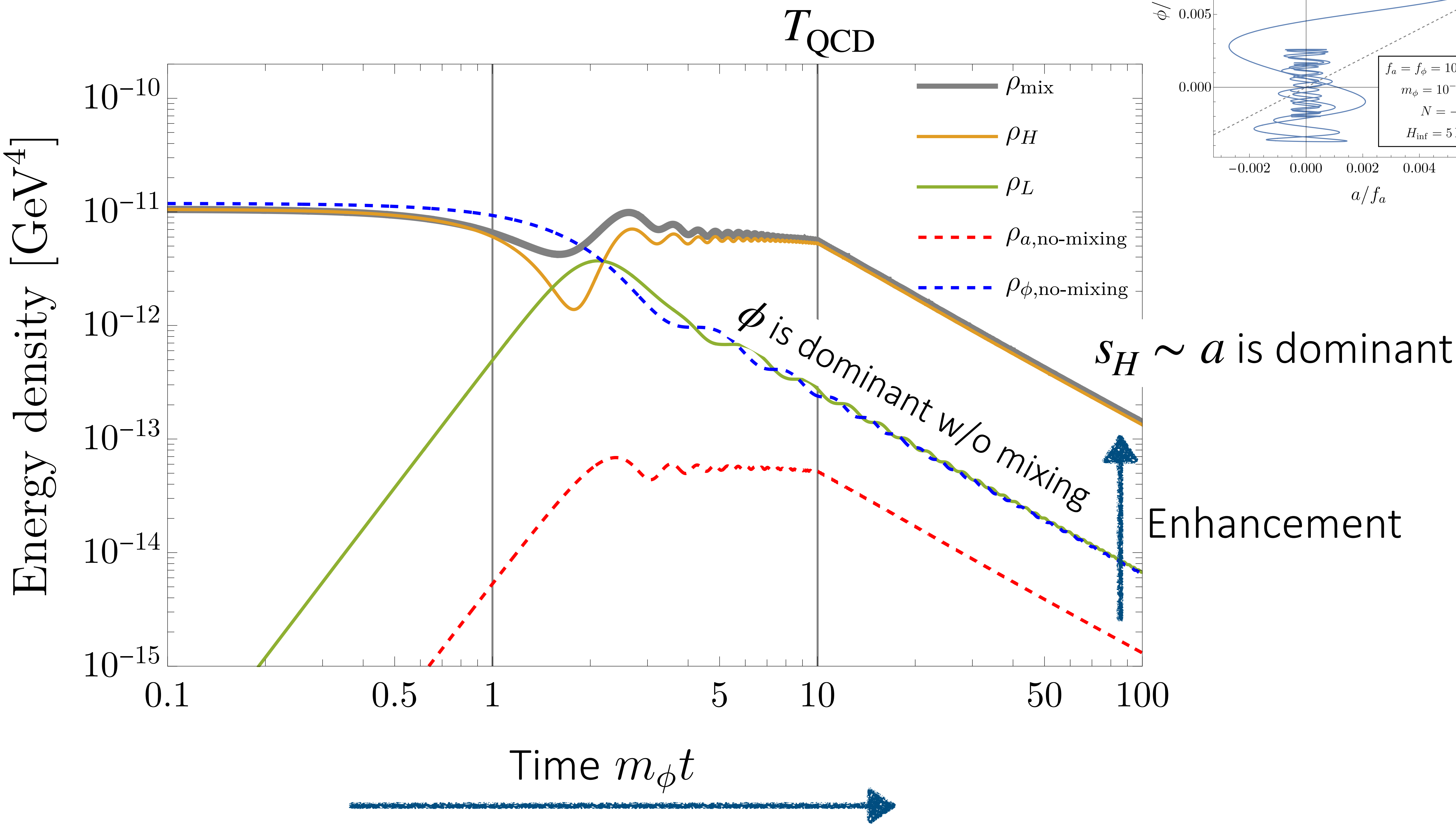
- Trajectory

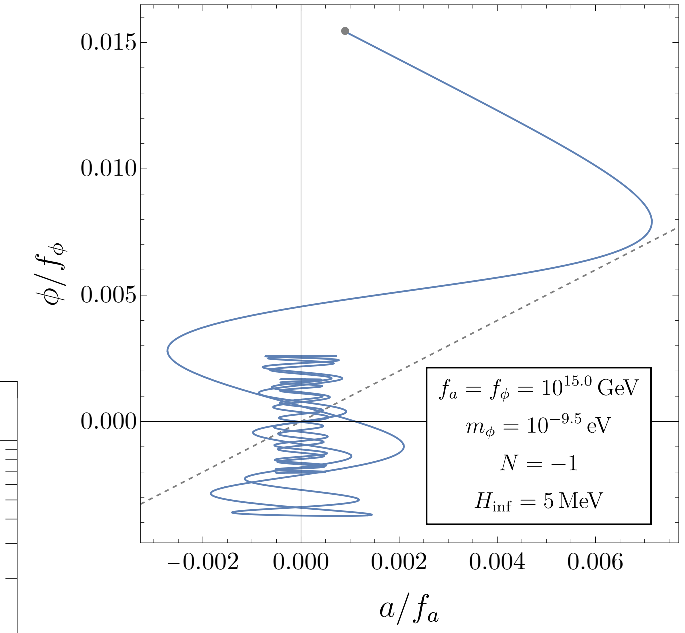


$m_\phi < H(T_{\rm QCD})$

→ a & ϕ are independent.

- Energy density

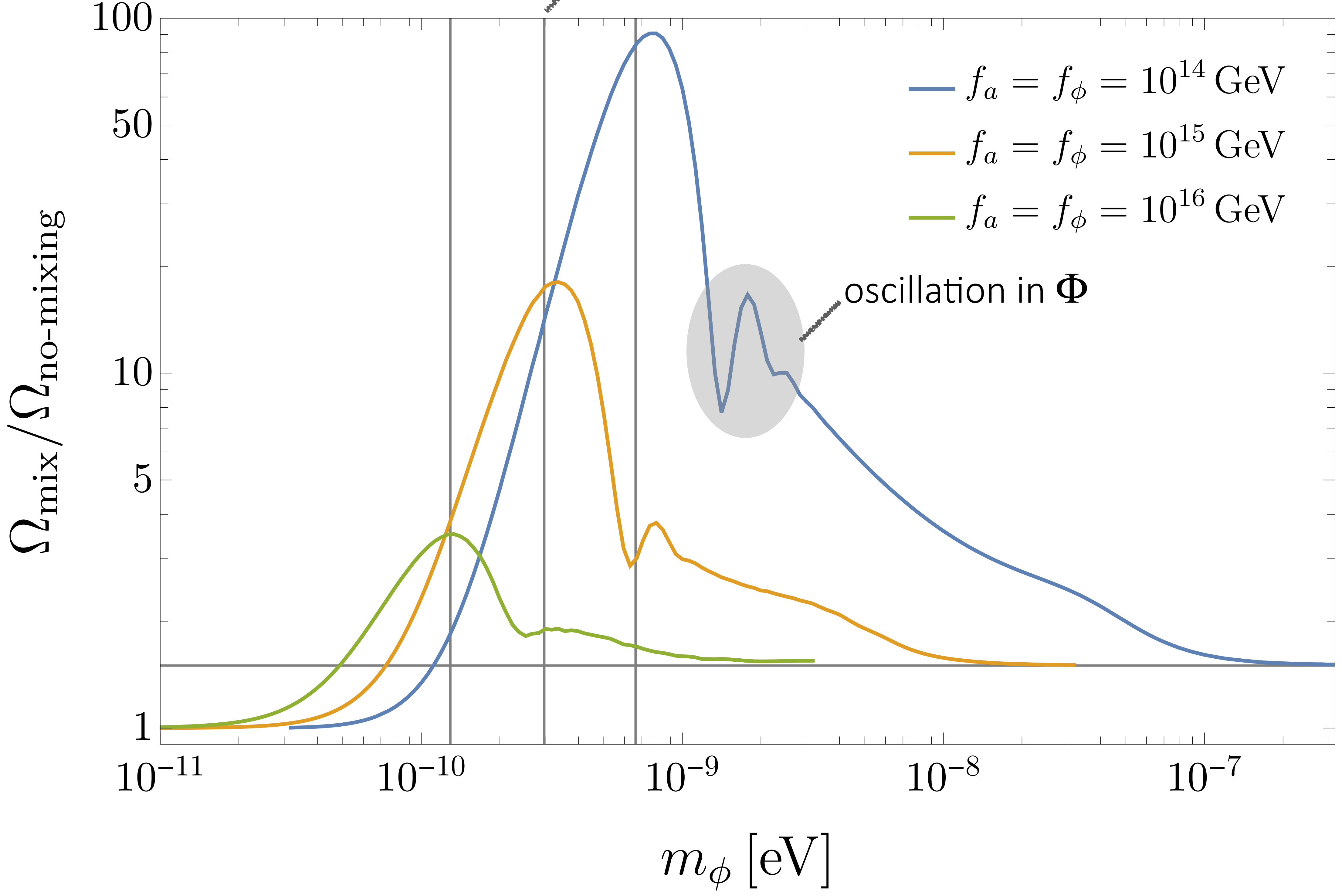




Axion dynamics

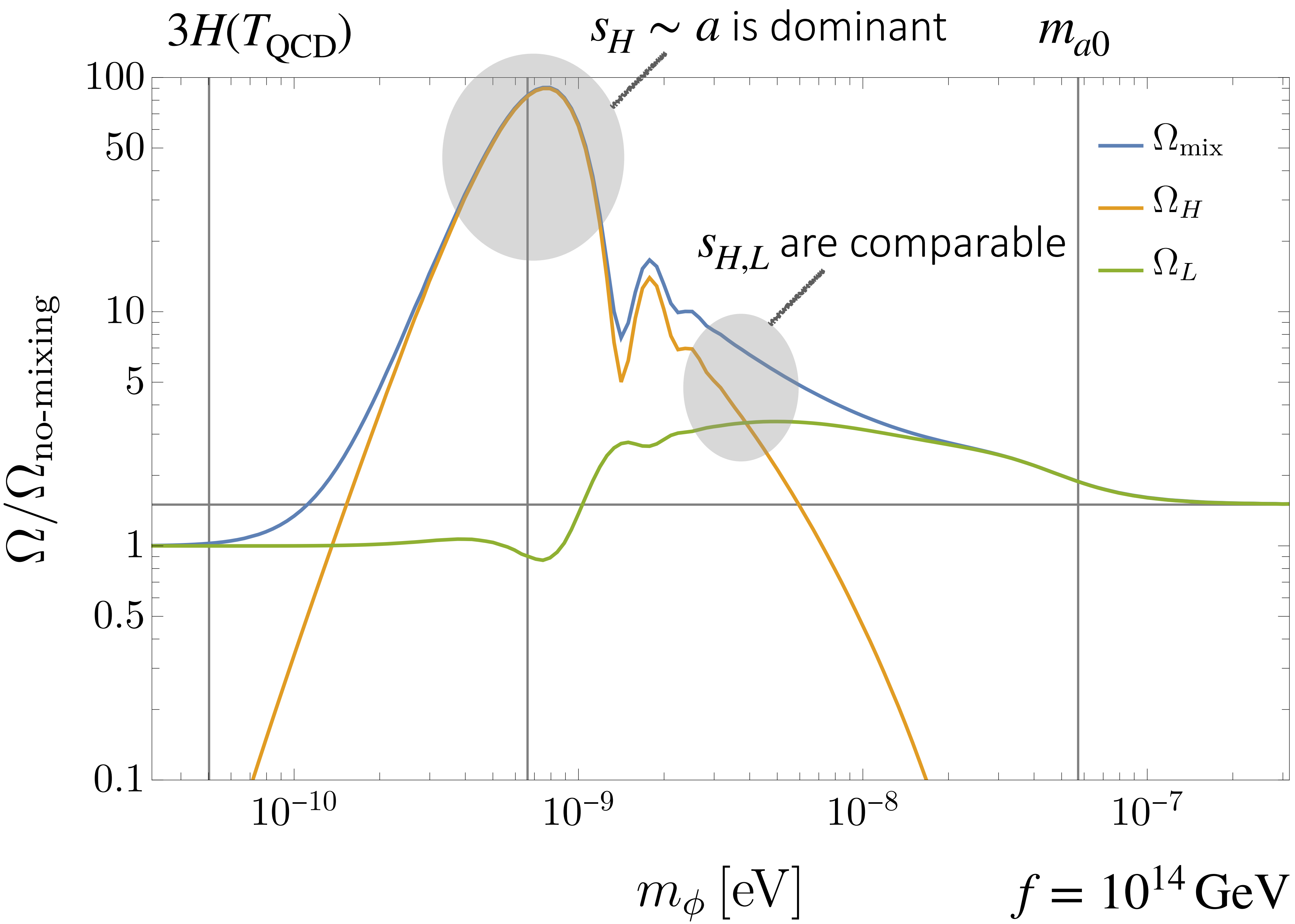
- Enhancement

$m_\phi \sim H(T)$ when $m_a(T) \sim H(T)$



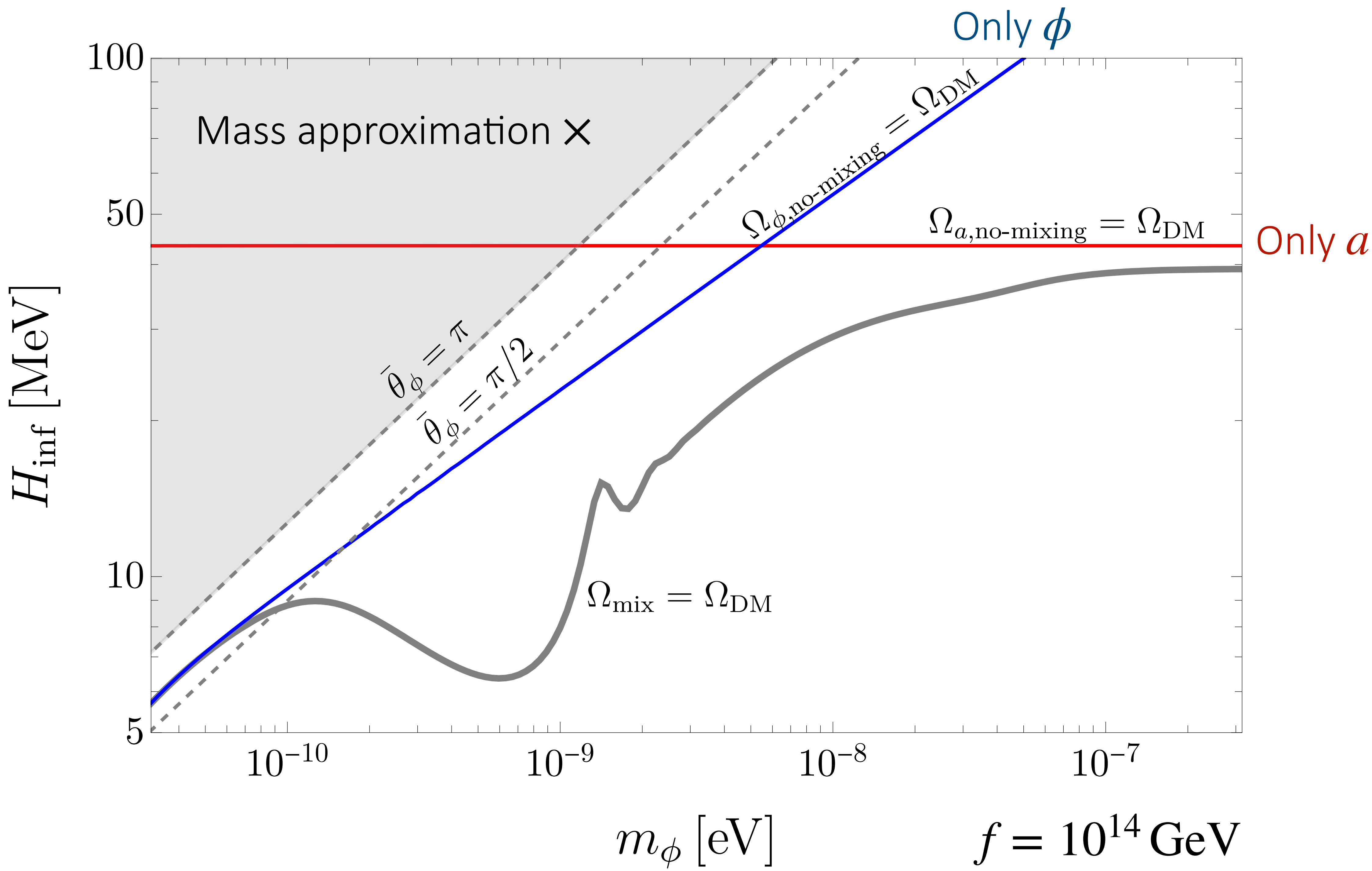
Axion dynamics

- Energy fraction



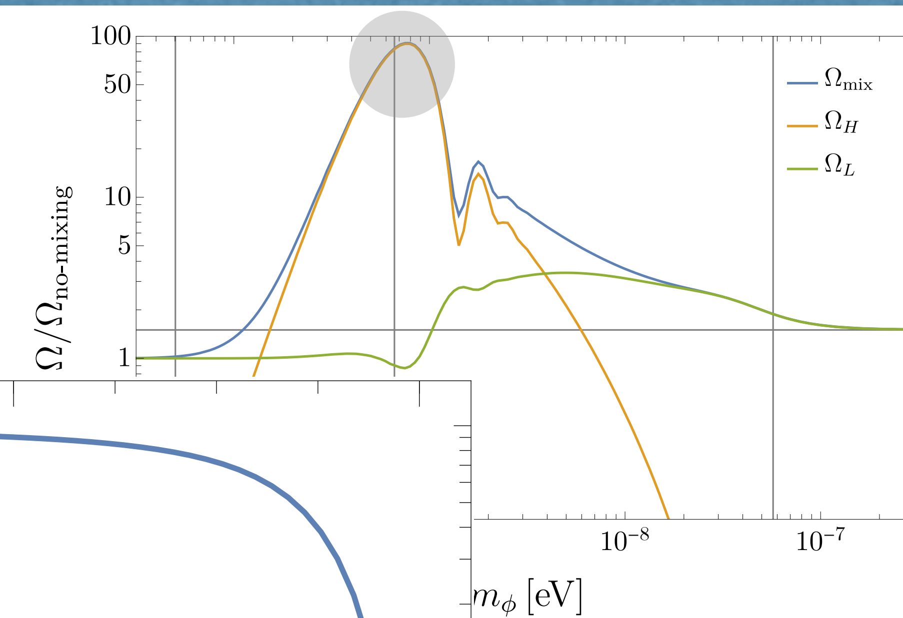
Axion dynamics

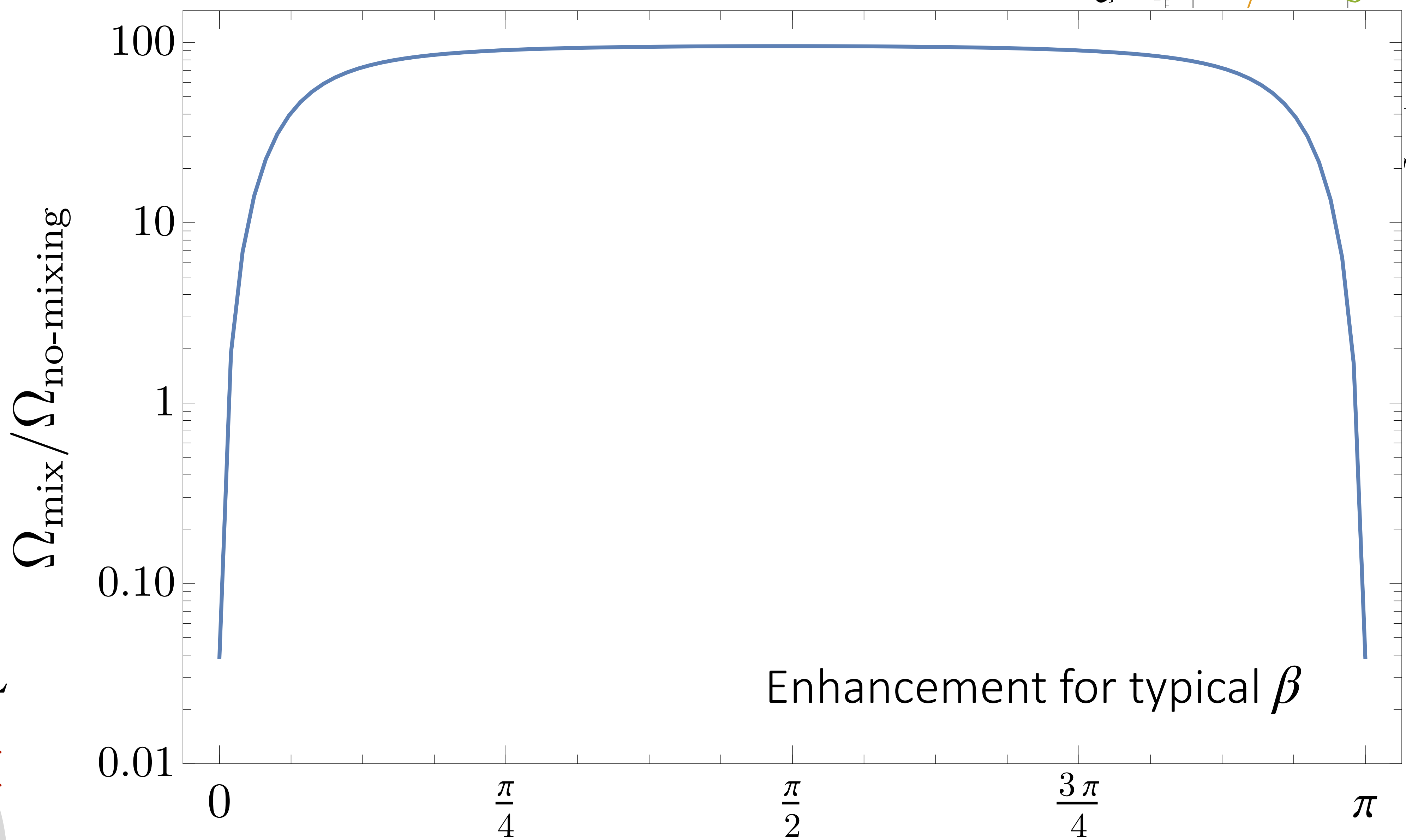
- To explain dark matter

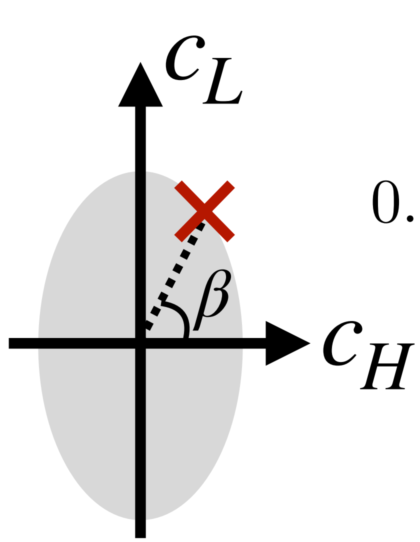


Axion dynamics

- Initial condition and enhancement



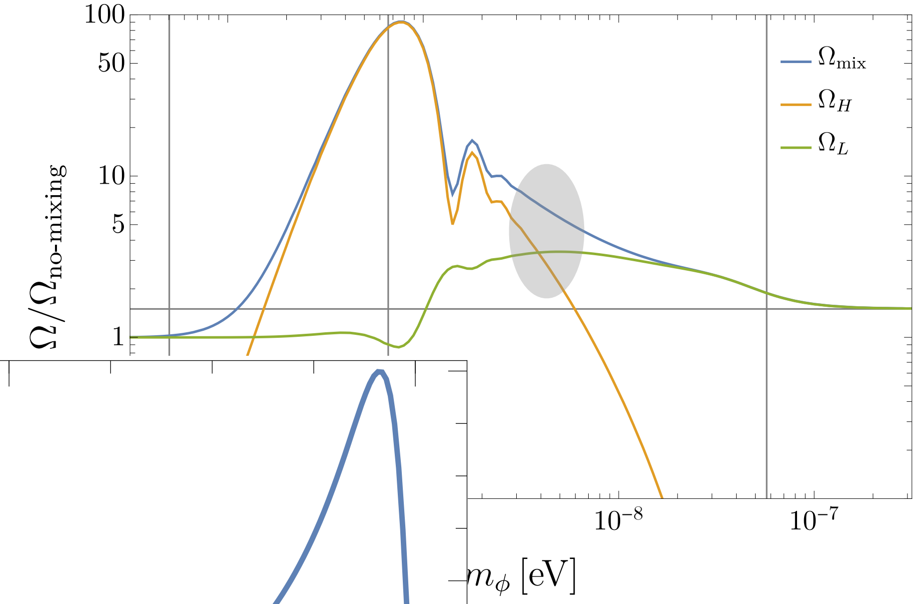


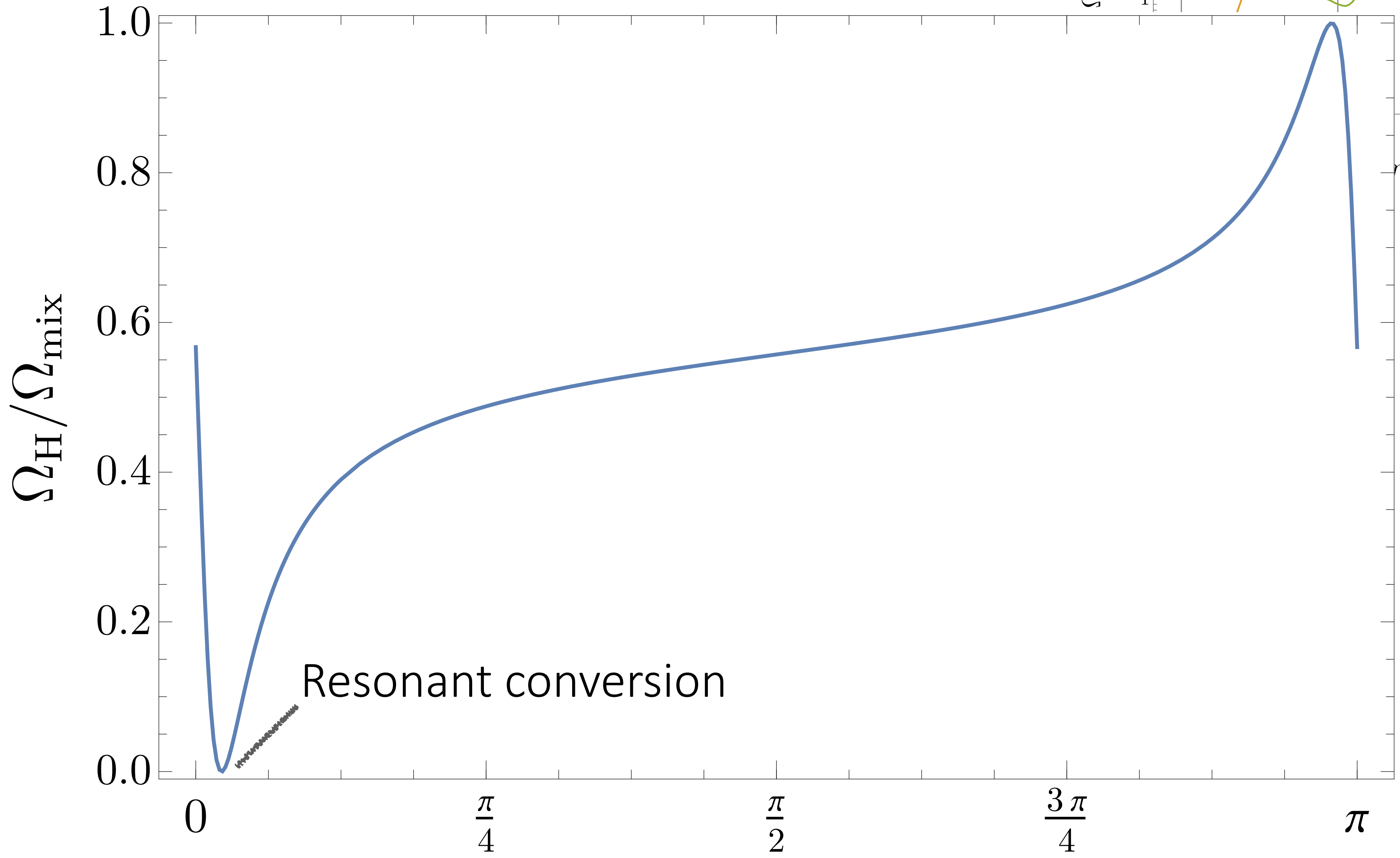


$$\beta = \arctan \frac{c_L}{c_H}$$

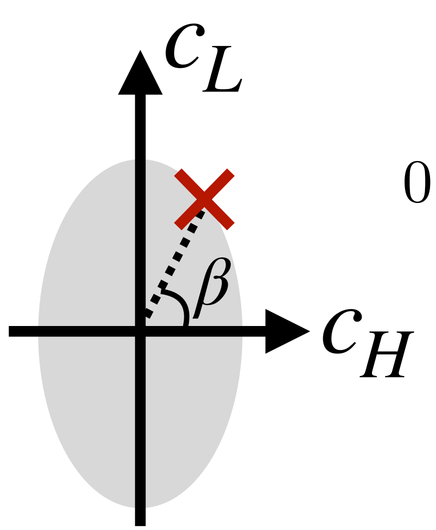
Axion dynamics

- Initial condition and energy fraction





$$\beta = \arctan \frac{c_L}{c_H}$$



Summary

- We studied the axion dynamics with mixing in the stochastic scenario.

- Due to the T-dependence of $V_{\rm QCD}$,
  the mass eigenstates also change in time.

- Depending on the axion masses,
  - the QCD axion is enhanced and dominant. → "unique player"
  - two axions compose mixed dark matter.

- For example, QCD axion with $f \sim 10^{14}\,\mathrm{GeV}$
  can be probed by DM-radio experiments.