長村　尚弘 (名古屋大)

LR模型におけるθ項への輻射補正のダイアグラムを用いた評価

JHEP 03 (2023) 150 [arXiv:2301.13405]

共同研究者 ：
久野純治, 北原哲平, 山田篤幸

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CP violation in the SM

$\mathcal{L}_{\rm SM}$ is a P & T violating theory.

$T\,(CP)$ **violaing parameters** ($\not{T} = \not{C}\not{P}$ under CPT theorem)

the complex phase $\delta_{\rm CKM}$ in the CKM matrix

- flavor off-diagonal
  - W bosons exchange
- observed $\delta_{\rm CKM} = 66^\circ$

the QCD θ angle

- the θ term $\left(\tilde{G}^{\hat{a}\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^{\hat{a}}_{\rho\sigma}\right)$

$$\mathcal{L}_{\rm SM} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$

  - renormalizable
  - purely gluonic, flavor diagonal
  - $\not{P}$ & $\not{T} (= \not{C}\not{P})$
  - 未発見

Fujikawa method

$T\,(CP)$-odd

1 flavor $\not{P}, \not{T}$ QCD : $\mathcal{L}_{\not{P},\not{T}} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} - m\bar{\psi}\psi - m_{\mathrm{CP}}\bar{\psi}i\gamma_5\psi$

mass diagonalize (chiral rotation)

—Fujikawa method—

K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

$$\mathcal{L}_{\not{P},\not{T}} \ni \bar{\theta} \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} - M\bar{\psi}_M\psi_M \quad \left(M = m + \frac{m_{\mathrm{CP}}^2}{m} \right)$$

the physical parameter

$$\bar{\theta} = \theta_G - \frac{m_{\mathrm{CP}}}{m}$$

Strong CP problem

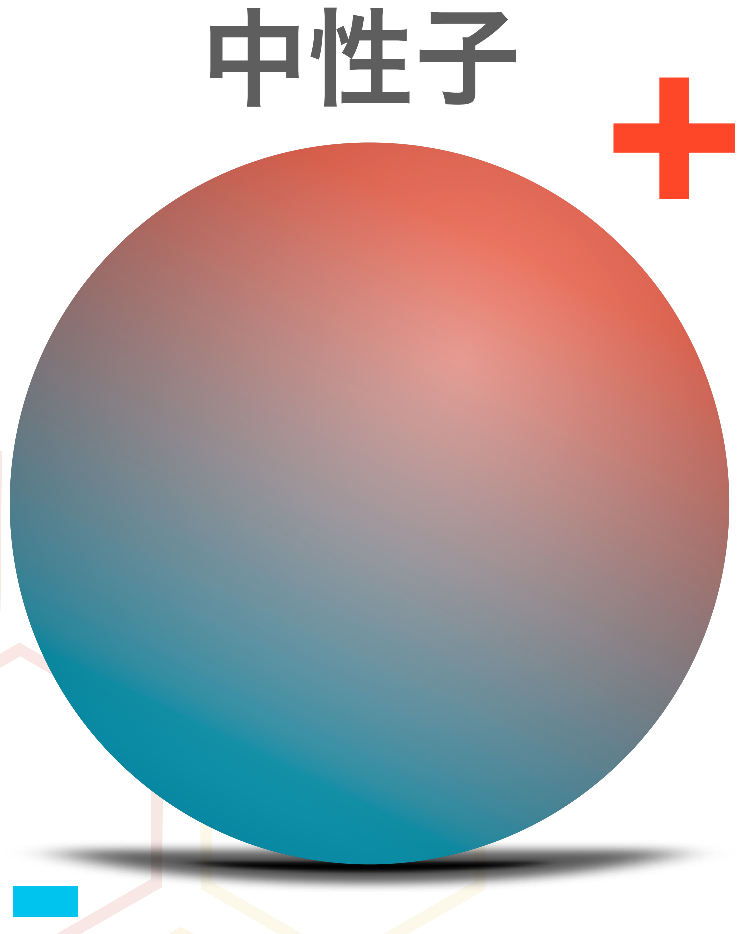
physical parameter $\bar{\theta}$ $(\bar{\theta} \neq \theta_G)$

$$\bar{\theta} = \theta_G - \arg\det\left[\mathcal{M}_u \mathcal{M}_d\right]$$

bare the chiral (ABJ) anomaly

γ_5-mass termからの寄与

S. L. Adler, Phys. Rev. **177** (1969) 2426-38
J. S. Bell, R. Jackiw, Nuovo. Cim. A **60** (1969) 47-61



neutron electric dipole moment (nEDM) d_n

- $\mathbf{E}$(electric field) - $\mathbf{S}$(spin) interaction $H_{\not P, \not T} = -d_n \mathbf{E} \cdot \frac{\mathbf{S}}{S}$
- the observable which is sensitive to flavor diagonal T violation

$$|d_n|_{\exp} < 1.8 \times 10^{-26}\ e\,\mathrm{cm}\ (90\%\ \mathrm{CL})$$

C. Abel, *et al.*, Phys. Rev. Lett. 124 (2020) 081803

$$|\bar{\theta}| \lesssim 1.2 \times 10^{-10} \quad (90\%\ \mathrm{CL})$$

Strong CP problem

strong CP problem

$$\delta_{\mathrm{CKM}} = \mathcal{O}(1) \quad \gg \quad |\bar{\theta}| \lesssim 1.2 \times 10^{-10} \quad (90\% \ \mathrm{CL})$$

Why is the QCD θ angle so small?

The SM cannot explain the small value of that angle.

solutions

- massless up quark
- axion
- extended P or CP

Solutions of the strong CP problem

promising solutions to the strong CP problem

axion

dynamical solution

predict a dark matter candidate

conflict with the quantum gravity

R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. **38** (1977)

the left-right model

parity(P) symmetric model

spontaneous breaking

$$\mathcal{L}_{\mathrm{SMEFT}} \ni \bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$ small!

free from the quantum gravity

M. A. B. Beg, H. S. Tsao, Phys. Rev. Lett. **41** (1978) 278

The parity symmetry forbids $G\tilde{G}$!!

$\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$: P- and $T(CP)$-odd operator

Solutions of the strong CP problem

promising solutions to the strong CP problem

どれくらい小さいの？

$\bar{\theta} < 10^{-10}$ を満たすの？

- ちゃんと計算されていない！
  - strong CP problemの解候補？
  - 実験検証精度？

the left-right model

parity(P) symmetric model

spontaneous breaking

$$\mathcal{L}_{\rm SMEFT} \ni \bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$ small!

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やりたいこと（研究課題）

従来の研究課題

最小なLR模型が誘起するQCD θ angle $\bar{\theta}$ がstrong CP problem, $\bar{\theta} < 10^{-10}$ を満足するかどうか明らかにしたい

$\bar{\theta}$ の構成要素である

bare θ_G tree level γ_5 mass 1-loop γ_5 mass 2-loop γ_5 mass …

パリティ対称性によって禁止

クォークのループ質量を求める

本研究の研究課題

$\bar{\theta}$ に対する寄与って質量だけ考慮すればいいの？

従来の計算法(γ_5 mass term + Fujikawa method)の修正・改良が必要なのではないか

結果

- 結果１：$\bar{\theta}$ に対する輻射補正を計算する**新しい手法の開発**

  diagrammatic method

- 結果２：新計算法を最小なLR模型へ応用し、誘起される$\bar{\theta}$ を評価した

Fujikawa method

$T\,(CP)$-odd

1 flavor $\not{P}, \not{T}$ QCD : $\mathcal{L}_{\not{P},\not{T}} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} - m\bar{\psi}\psi - m_{\rm CP}\bar{\psi}i\gamma_5\psi$

mass diagonalize (chiral rotation)

—Fujikawa method—

K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

$$\mathcal{L}_{\not{P},\not{T}} \ni \bar{\theta}\frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} - M\bar{\psi}_M\psi_M \qquad \left(M = m + \frac{m_{\rm CP}^2}{m}\right)$$

the physical parameter

$$\bar{\theta} = \theta_G - \frac{m_{\rm CP}}{m}$$

Conventional calculation of $\delta\bar{\theta}$

$T(CP)$-odd

$$\mathcal{L}_{\not P,\not T} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - \left(m + \delta m^{(1)}\right)\bar{\psi}\psi - \left(m_{\rm CP} + \delta m^{(1)}_{\rm CP}\right)\bar{\psi}i\gamma_5\psi$$

radiative corrections

mass diagonalize (chiral rotation)

—Fujikawa method—

+

radiative corrections to masses

$$\mathcal{L}_{\not P,\not T} \ni \bar{\theta}^{\rm loop} \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} - M^{(1)}\bar{\psi}_M\psi_M \quad \left(M^{(1)} = (m + \delta m^{(1)}) + \frac{(m_{\rm CP} + \delta m^{(1)}_{\rm CP})^2}{m + \delta m^{(1)}}\right)$$

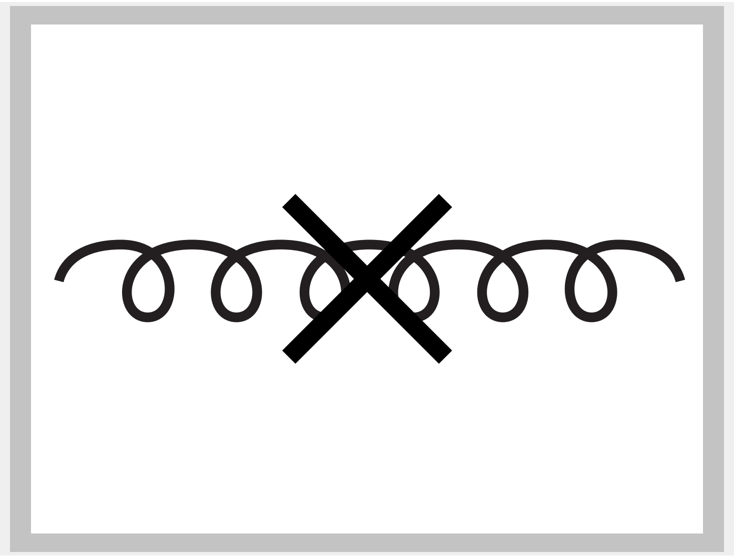
conventional radiative corrections to $\bar{\theta}$

$$\bar{\theta}^{\rm loop} \simeq \theta_G - \frac{m_{\rm CP}}{m} - \frac{\delta m^{(1)}_{\rm CP}}{m} + \frac{m_{\rm CP}}{m}\frac{\delta m^{(1)}}{m}$$

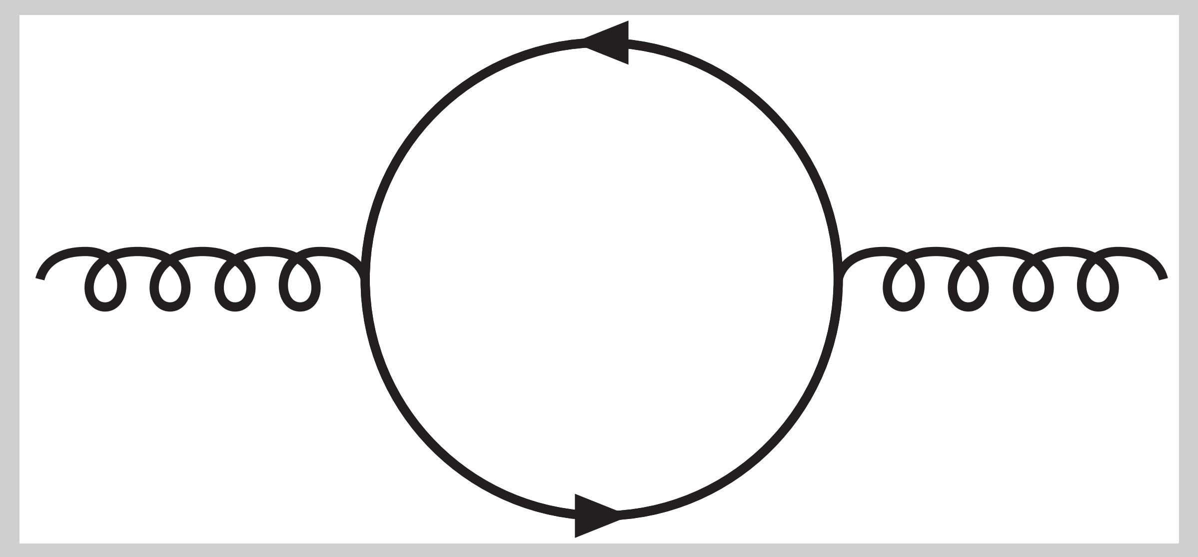
Diagrammatic approach to $\bar{\theta}$ corrections

$$\bar{\theta}\frac{\alpha_s}{8\pi}G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu}$$

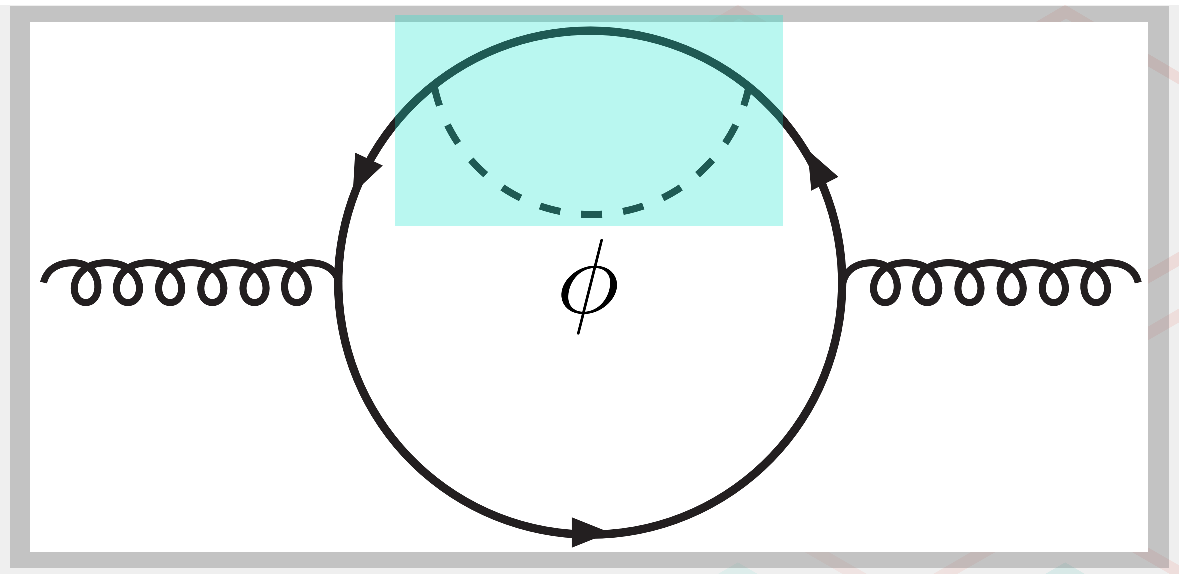
$\bar{\theta} = \theta_G$



$-\frac{m_{\mathrm{CP}}}{m}$

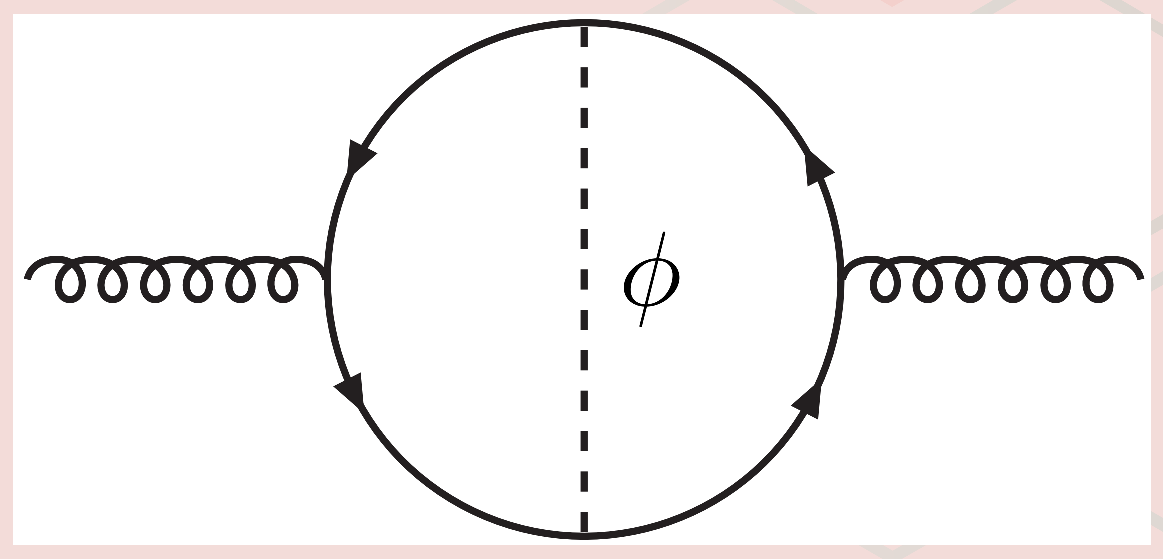


$-\frac{\delta m^{(1)}_{\mathrm{CP}}}{m} + \frac{m_{\mathrm{CP}}}{m}\frac{\delta m^{(1)}}{m}$



(assumption: a scalar ϕ which interacts with ψ)

$+$ **New!**



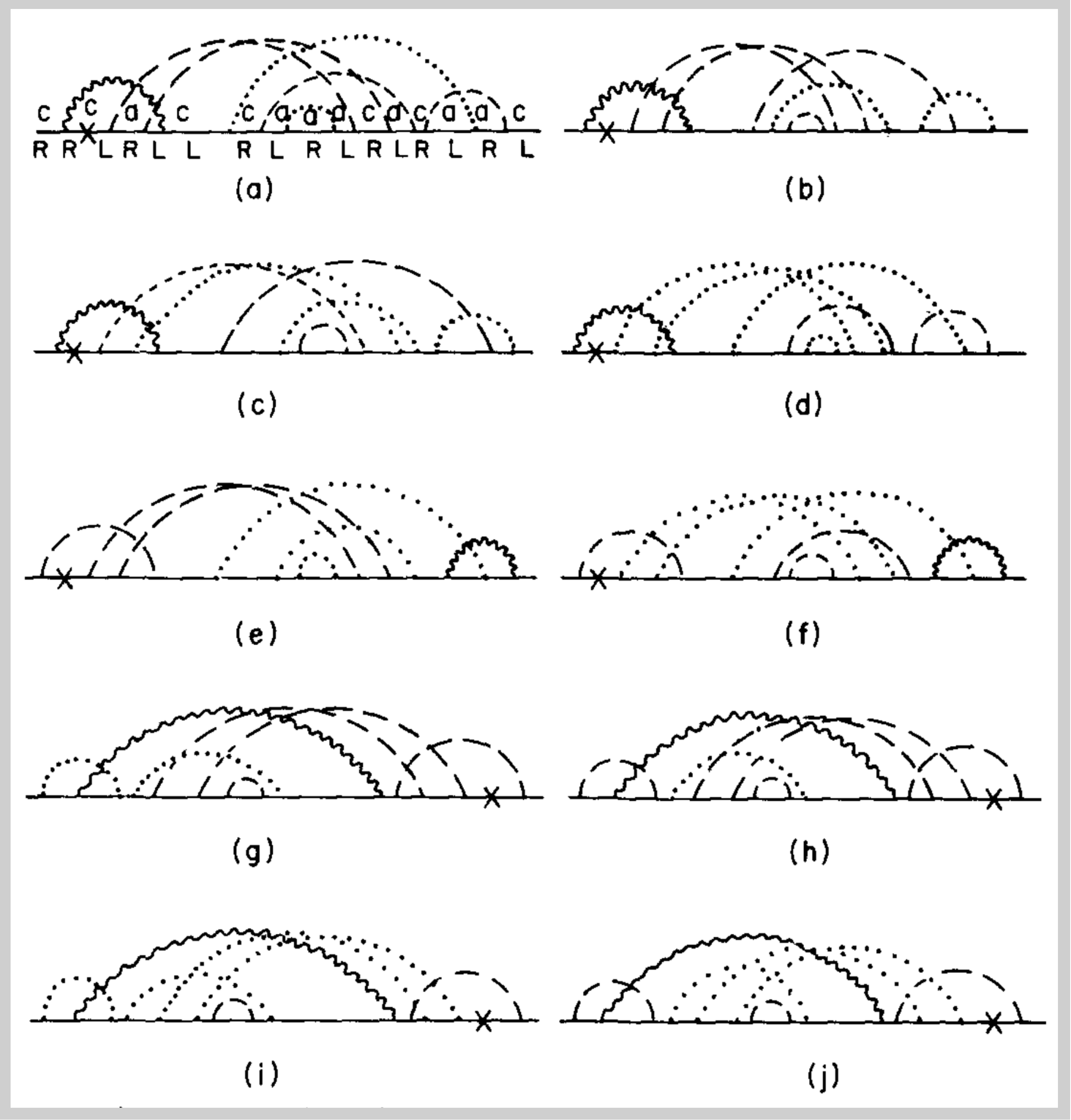
$+\cdots$

CKM contribution to $\bar{\theta}$ in SM

$$\bar{\theta} = \theta_G - \arg\det\left[\mathcal{M}_u \mathcal{M}_d\right]$$

CKM phase ⟶ quark mass phase ⟶ $\bar{\theta}$

- 7-loop

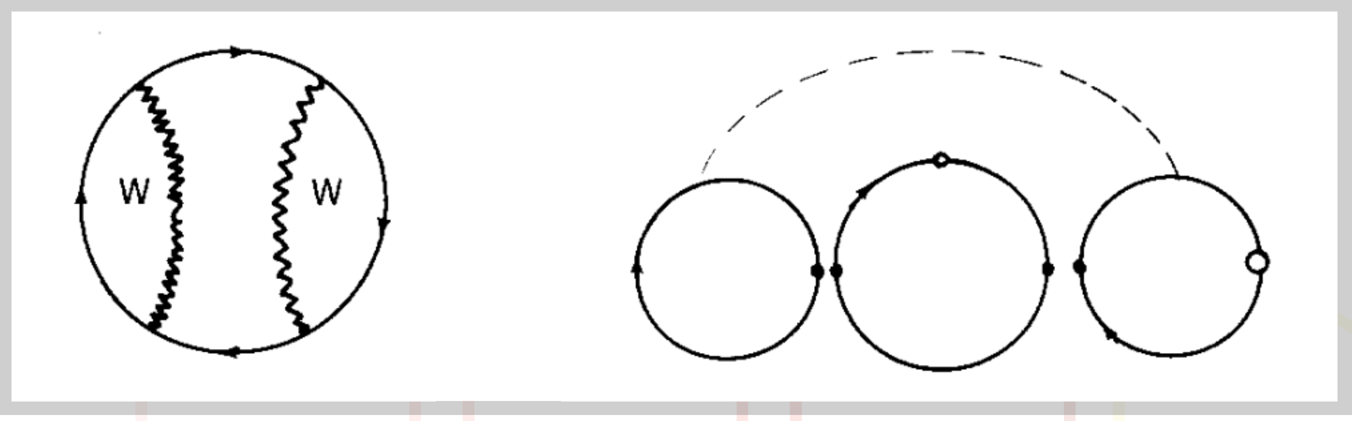


J. R. Ellis, M. K. Gaillard, Nucl. Phys. B **150** (1979) 141-162

diagrammatic approach

CKM phase ⟶ $\bar{\theta}$

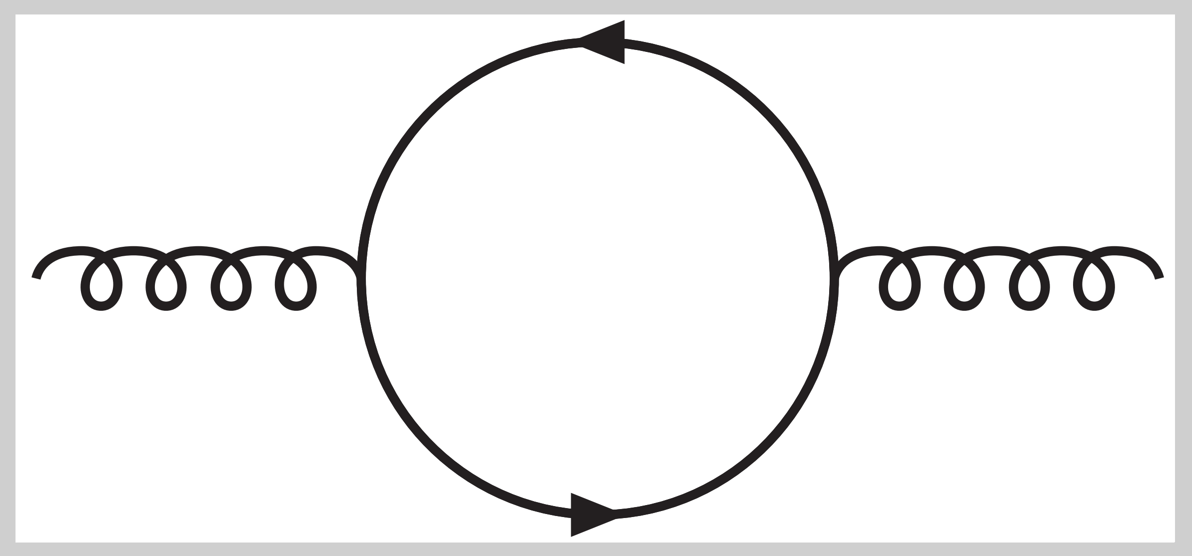
- 4-loop

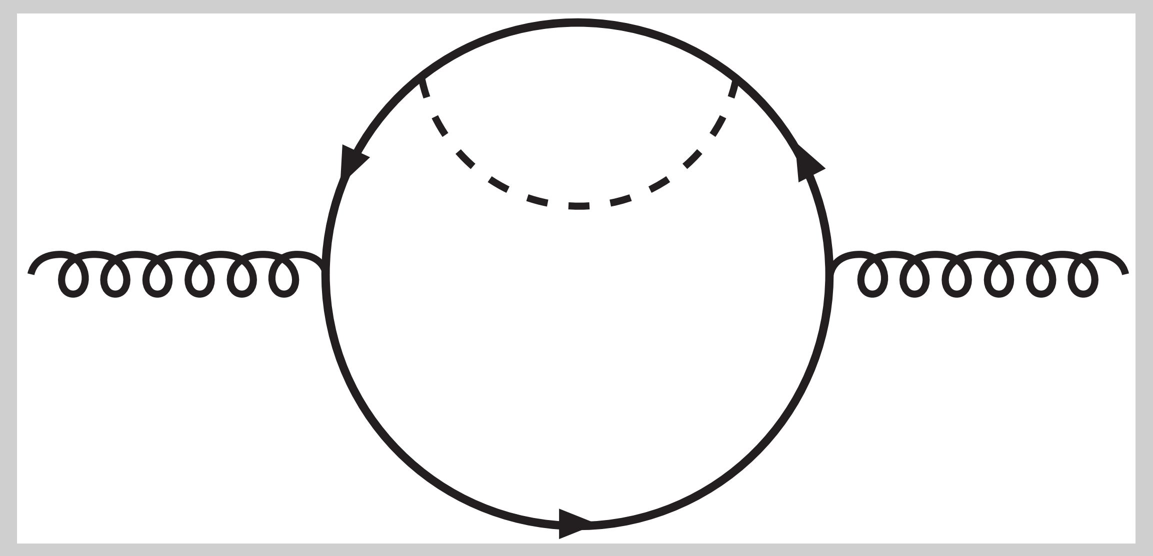


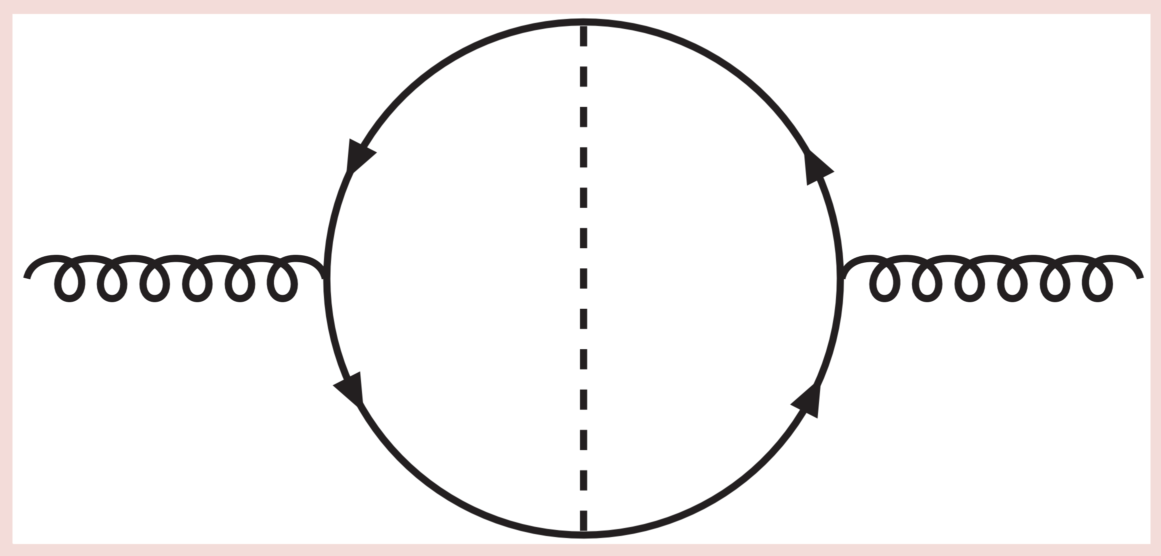
- This is consistent with the quark (chromo-) EDMs generating at 3-loop from the CKM phase.
- no detailed calculation

I. B. Khriplobich, Phys. Lett. B **174** (1986) 193-196

Difficulty in calculating the loop diagrams







difficulty: the theta term cannot be derived perturbatively.

$\because$ total derivative $G^{\hat{a}}_{\mu\nu}\tilde{G}^{\hat{a}\mu\nu} = \partial^\mu \epsilon_{\mu\nu\rho\sigma}\left(A^{\hat{a}}_\nu G^{\hat{a}}_{\rho\sigma} - \frac{g_s}{3} f^{\hat{a}\hat{b}\hat{c}} A^{\hat{a}}_\nu A^{\hat{b}}_\rho A^{\hat{a}}_\sigma\right)$ $\sum p^\mu = 0$

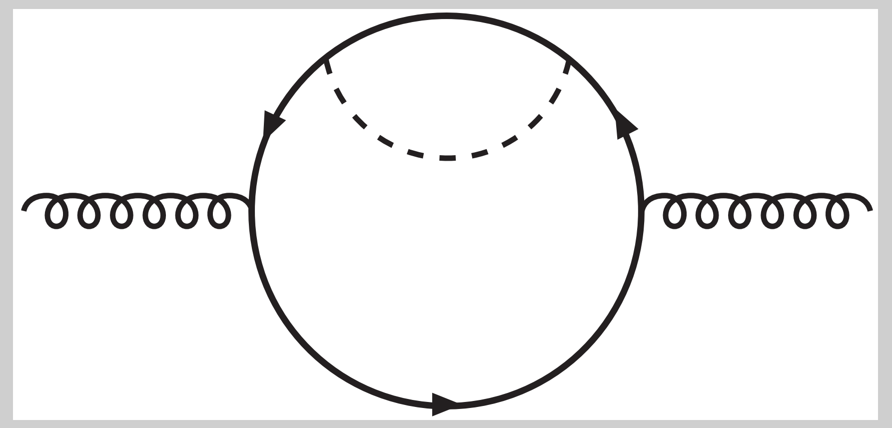
strategy

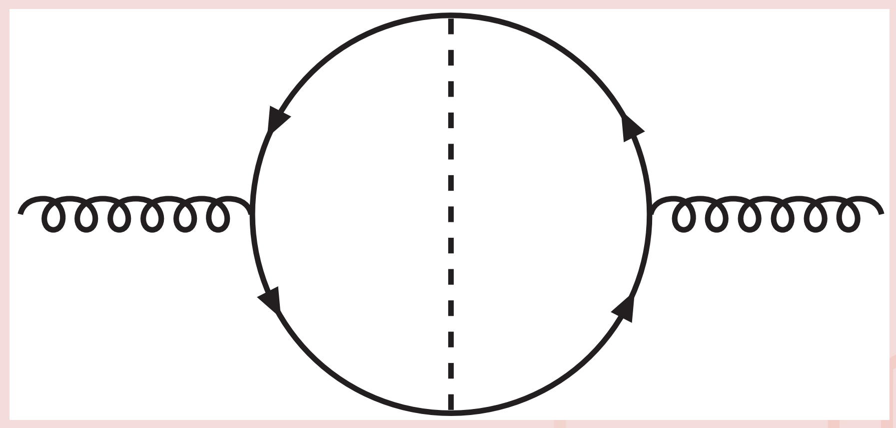
- building a gluon effective theory described by not gauge field $A^{\hat{a}}_\mu$ but the field strength $G^{\hat{a}}_{\mu\nu}$
- temporarily breaking the translation symmetry
- fixing a background field strength $G^a_{\mu\nu}$

Fock-Schwinger gauge method

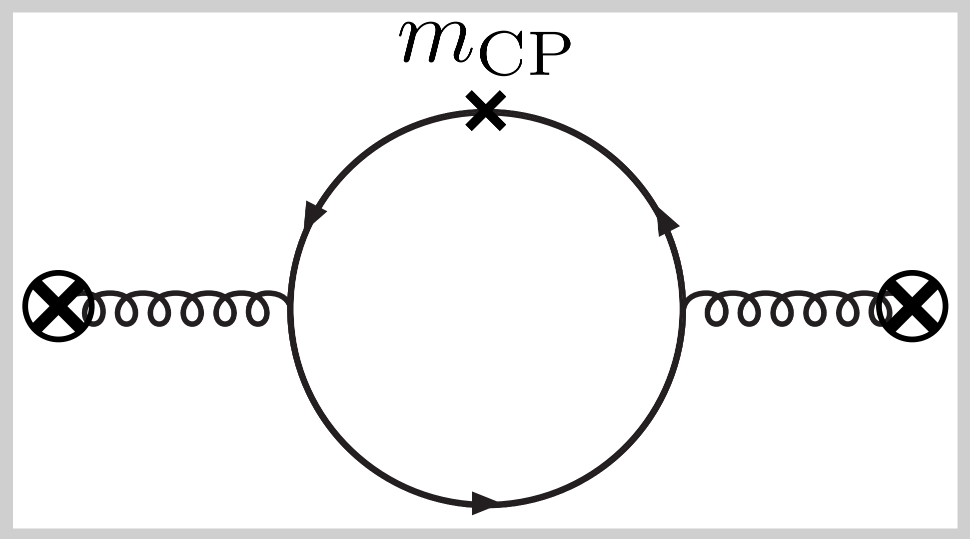
Fock-Schwinger gauge: $(x^\mu - x_0^\mu) A_\mu^{\hat{a}}(x) = 0$ V. A. Novikov, *et al.*, Fortsch. Phys. **32** (1984) 585

- fixing a background $A_\mu^{\hat{a}}(x) = \frac{1}{2}(x^\nu - x_0^\nu) G_{\nu\mu}^{\hat{a}}(x_0) + \cdots$
- breaks the translation symmetry, but it revives in the result of gauge invariant quantities
  S. N. Nikolaev, *et al.*, Nucl. Phys. **B** 213 (1983) 285-304
- calculable





$$\mathcal{L} = \bar{\psi} i \not{D} \psi - m \bar{\psi}\psi - m_{\mathrm{CP}} \bar{\psi} i \gamma_5 \psi - \frac{1}{4} G_{\mu\nu}^{\hat{a}} G^{\hat{a}\mu\nu} + \theta_G \frac{\alpha_s}{8\pi} G_{\mu\nu}^{\hat{a}} \tilde{G}^{\hat{a}\mu\nu}$$

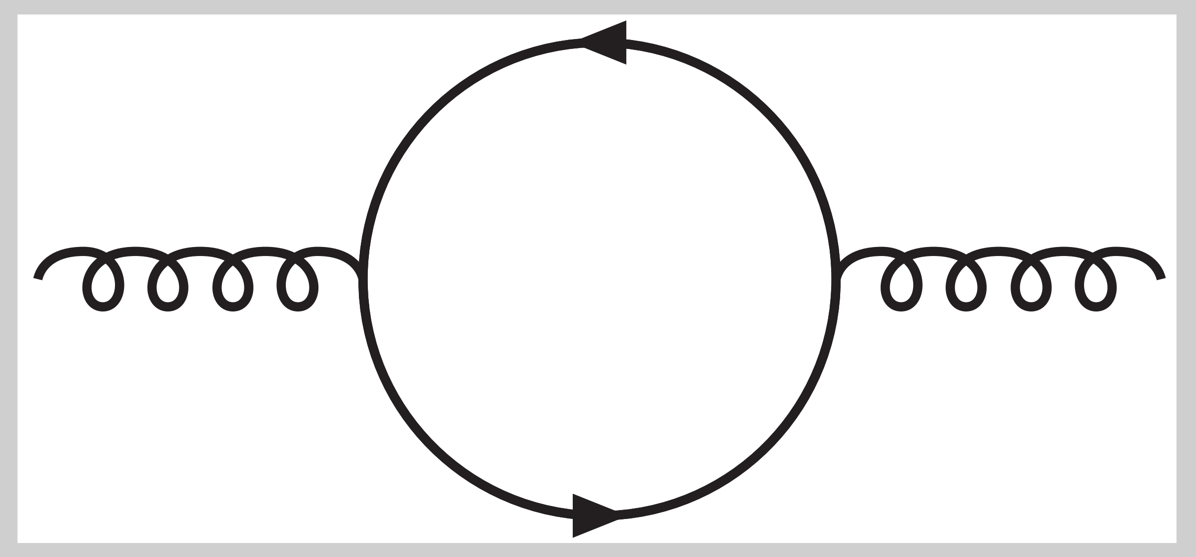
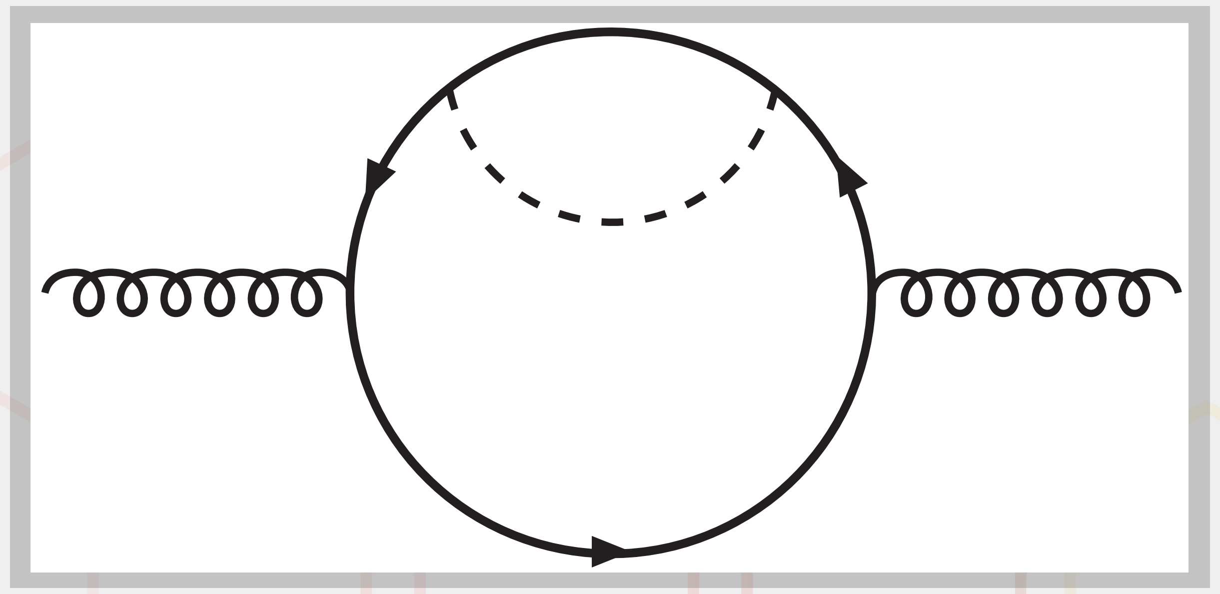
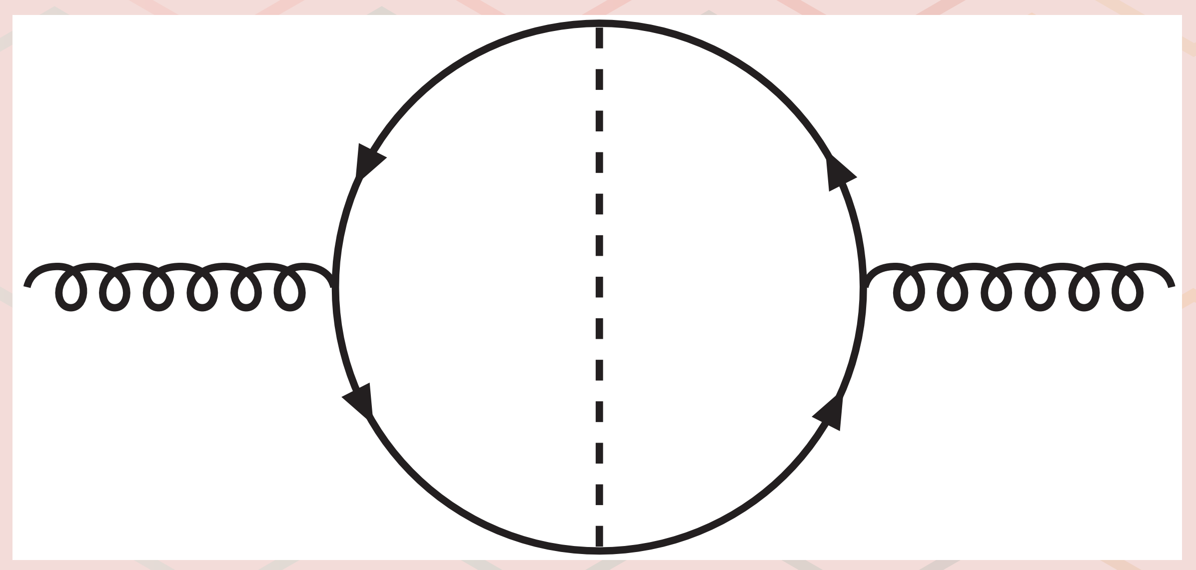


$$-\frac{m_{\mathrm{CP}}}{m}$$

(Fujikawa method)

consistent!

1st result

corrections to $\bar{\theta}$	conventional method		**the diagrammatic method (Fock-Schwinger gauge)**
tree	tree γ_5-mass $m_{\rm CP}$ + Fujikawa method	consistent!	
1-loop corrections	loop mass $\delta m_{\rm CP}$ + Fujikawa method		
1-loop corrections (not mass)	✕		 **New!!**

Minimal LR model

SM: $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

LR model: $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory

$P_{\rm gen}$: generalized parity symmetry

$SU(2)_L$ singlet $\longrightarrow$ $SU(2)_R$ doublet

LR model $\xrightarrow[\langle H' \rangle = (0, v')]{}$ SM (low energy EFT)

$SU(2)_{L/R}$ singlet, vector-like(VL) quarks

(SM quark masses by seesaw mechanism)

	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	$U(1)_Y$
$Q_L^i \equiv (u_L^i, d_L^i)^T$	$\square$	$\square$	$\mathbf{1}$	$1/6$	$(1/6, 1/6)$
$Q_R^i \equiv (u_R^i, d_R^i)^T$	$\square$	$\mathbf{1}$	$\square$	$1/6$	$(2/3, -1/3)$
H	$\mathbf{1}$	$\square$	$\mathbf{1}$	$1/2$	$(1/2, 1/2)$
H'	$\mathbf{1}$	$\mathbf{1}$	$\square$	$1/2$	$(1, 0)$
U_L^a	$\square$	$\mathbf{1}$	$\mathbf{1}$	$2/3$	$2/3$
U_R^a	$\square$	$\mathbf{1}$	$\mathbf{1}$	$2/3$	$2/3$
D_L^a	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-1/3$	$-1/3$
D_R^a	$\square$	$\mathbf{1}$	$\mathbf{1}$	$-1/3$	$-1/3$

$\gtrsim 1\mathrm{TeV}$

Seesaw mechanism

mass matrix (light flavor: $Q^i_{L/R}$, heavy flavor: $U^a_{L/R}$, $D^a_{L/R}$)

$$-\mathcal{L}_Y = \bar{Q}^i_L x_u^{ia} U^a_R \tilde{H} + \bar{Q}^i_R x_u^{ia} U^a_L \tilde{H}' + M_u^a \bar{U}^a_L U^a_R + \bar{Q}^i_L x_d^{ia} D^a_R H + \bar{Q}^i_R x_d^{ia} D^a_L H' + M_d^a \bar{D}^a_L D^a_R + \text{h.c.}$$

Yukawa (light × heavy × Higgs) Dirac masses

seesaw mechanism

$$(\bar{u}^i_L, \bar{U}^a_L) \begin{pmatrix} 0 & x_u^{ib} v \\ x_u^{\dagger aj} v' & M_u^a \delta^{ab} \end{pmatrix} \begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix}$$

$$(\bar{d}^i_L, \bar{D}^a_L) \begin{pmatrix} 0 & x_d^{ib} v \\ x_d^{\dagger aj} v' & M_d^a \delta^{ab} \end{pmatrix} \begin{pmatrix} d^j_R \\ D^b_R \end{pmatrix}$$

CP phases

$$x_u = V^\dagger_{\text{CKM}} \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}(\theta_{u3}, \theta_{u8}) V_U \frac{\sqrt{M_u}}{\sqrt{v'}}, \qquad 1(V_{\text{CKM}}) + 2(\theta_{u3}, \theta_{u8}) + 1(V_U) = 4$$

$$x_d = \frac{\sqrt{m_d}}{\sqrt{v}} \bar{\Phi}(\theta_{d3}, \theta_{d8}) V_D \frac{\sqrt{M_d}}{\sqrt{v'}} \qquad 2(\theta_{d3}, \theta_{d8}) + 1(V_D) = 3$$

($V_{U/D}$: CKM-like matrix, $\bar{\Phi}(\theta_{q3}, \theta_{q8})$: two $U(1)$ phases)

Assumptions: $\tilde{M} \equiv M_d^a = M_u^1 \gg M_u^2 \gg M_u^3$

$\bar{\theta}$ parameter in the minimal LR model

bare θ

$$\mathcal{L}_{\rm Left-Right} \ni \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu}$$

prohibited by $P_{\rm gen}$

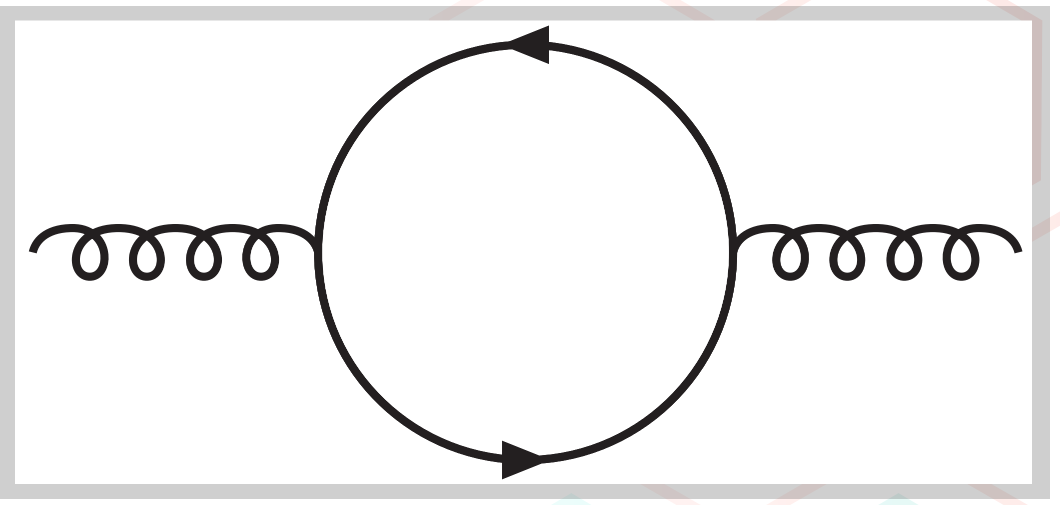
tree (Fujikawa method+tree mass)

the mass matrix

$x_{u/d}$ はUVのYukawa

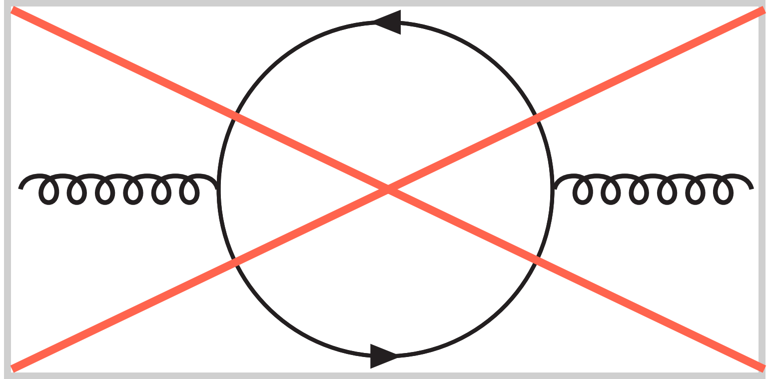
$$\left(\overline{u}^i_L, \overline{U}^a_L\right) \begin{pmatrix} 0 & x_u^{ib} v \\ x_u^{\dagger aj} v' & M_u^a \delta^{ab} \end{pmatrix} \begin{pmatrix} u^j_R \\ U^b_R \end{pmatrix} \equiv \overline{\mathcal{U}}^p_L \mathcal{M}_u^{(0)pq} \mathcal{U}^q_R$$

$$\arg\det\left[\mathcal{M}_u \mathcal{M}_d\right] = 0$$

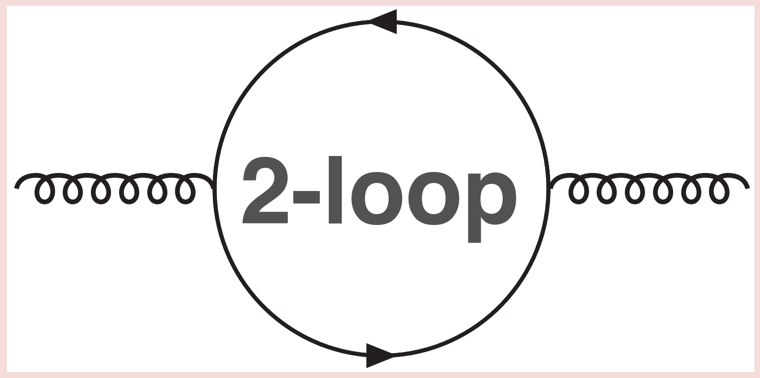


$= 0$

$\bar{\theta} = \theta_G +$



$+$



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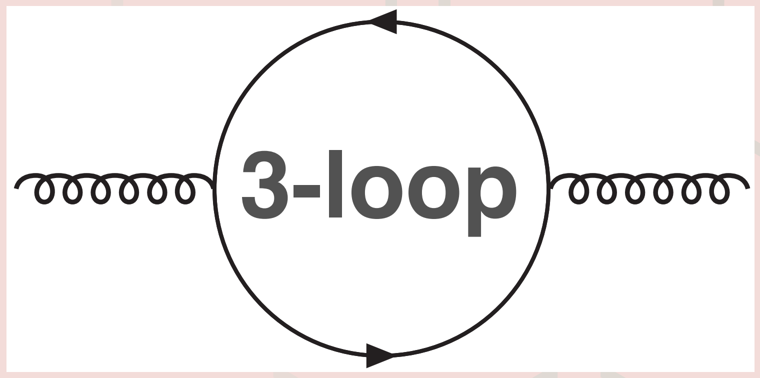


$+ \cdots$

Non-vanishing radiative $\bar{\theta}$ corrections

$$\mathcal{O}(x^4)\cdots \ \mathrm{Im}\ \mathrm{Tr}\left(A^a_q A^b_{q'}\right) f(M^a_q, M^b_{q'}) = \frac{1}{2}\mathrm{Im}\ \mathrm{Tr}\left(A^a_q A^b_{q'}\right) f(M^a_q, M^b_{q'}) - \frac{1}{2}\mathrm{Im}\ \mathrm{Tr}\left(A^b_{q'} A^a_q\right) f(M^a_q, M^b_{q'})$$

$$= \frac{1}{2}\mathrm{Im}\ \mathrm{Tr}\left(A^a_q A^b_{q'} - A^a_q A^b_{q'}\right) f(M^a_q, M^b_{q'})$$

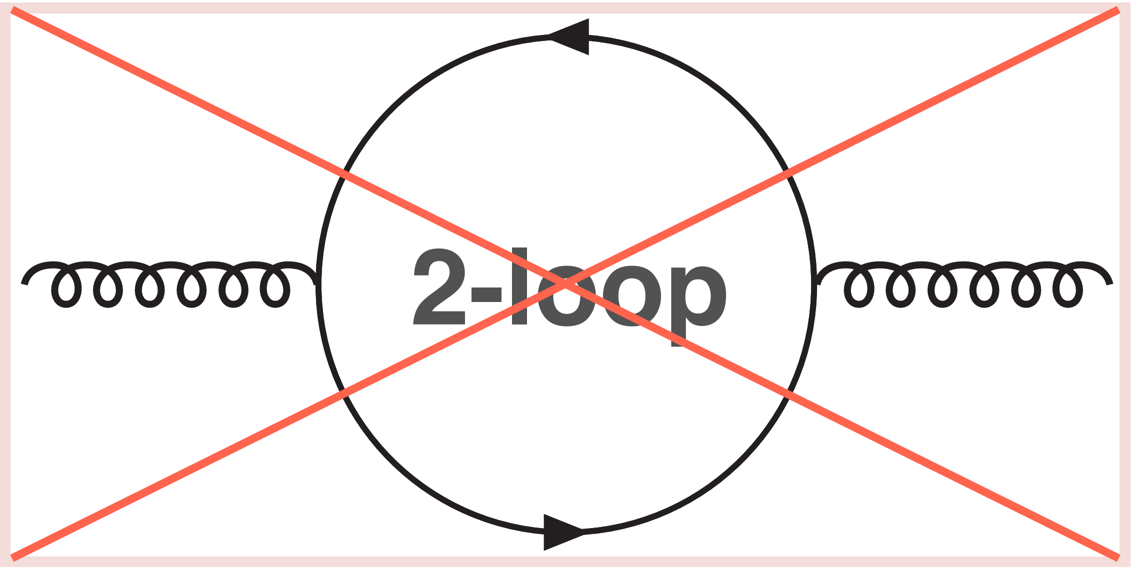
$$= 0$$

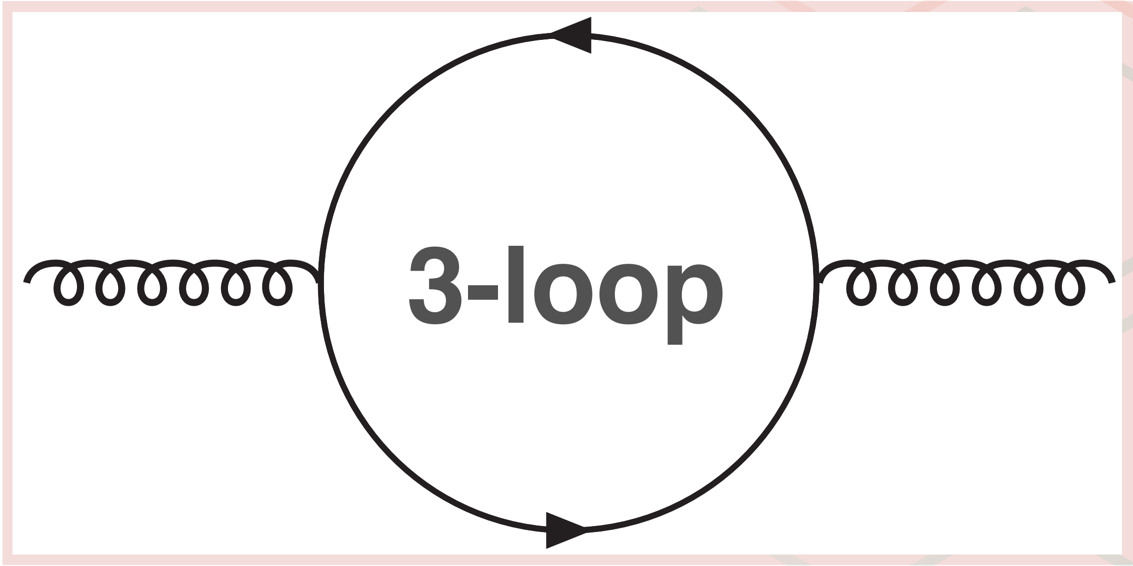
$$(A^a_q)^{ij} = x^{ia}_q x^{\dagger aj}_q$$

- f is a real loop function
- cyclicity in Tr

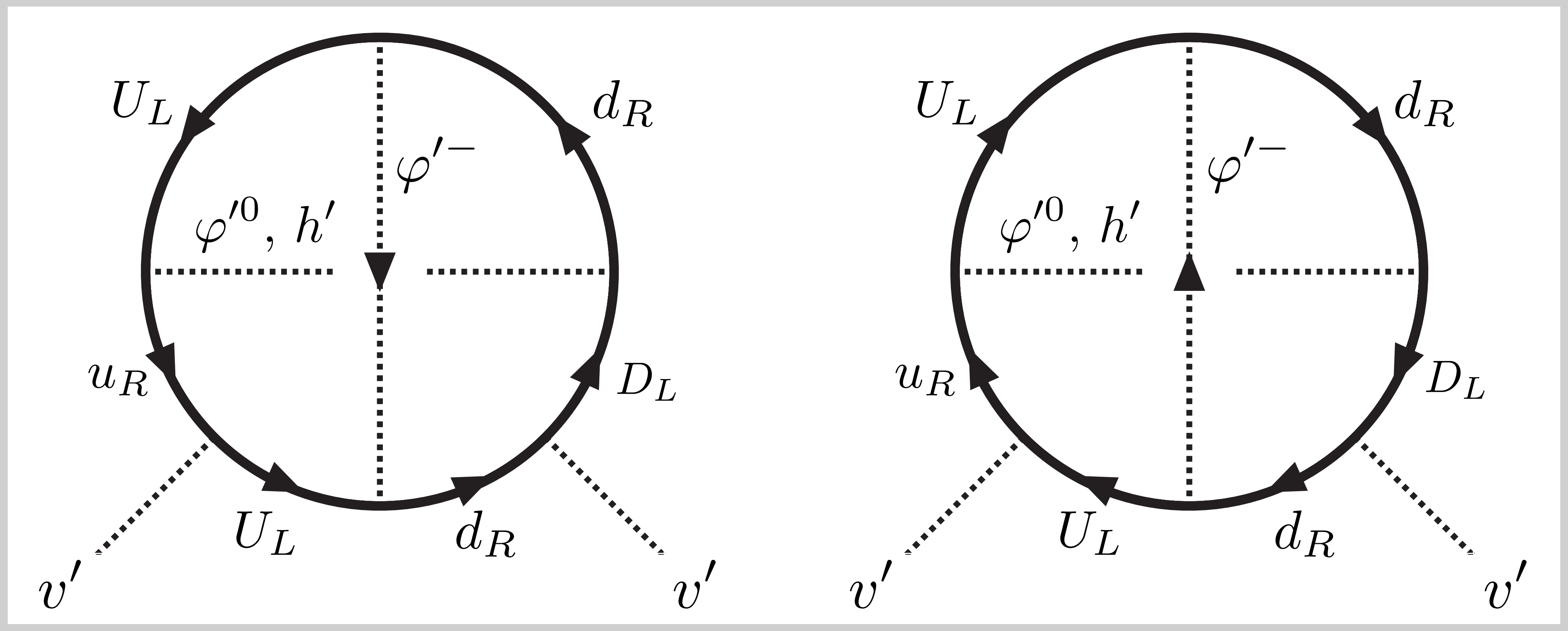
totally anti-symmetric

non-vanishing: $\mathrm{Im}\ \mathrm{Tr}\left(A^a_q\left[A^b_{q'}, A^c_{q''}\right]\right) f(M^a_q, M^b_{q'}, M^c_{q''})$





3-loop contributions up to loop function



$$(A_q^a)^{ij} \equiv x_q^{ia} x_q^{\dagger aj}$$

$$x_u = V_{\rm CKM}^\dagger \frac{\sqrt{m_u}}{\sqrt{v}} \bar{\Phi}\left(\theta_{u3}, \theta_{u8}\right) V_U \frac{\sqrt{M_u}}{\sqrt{v'}}$$

$$x_d = \frac{\sqrt{m_d}}{\sqrt{v}} \frac{\sqrt{M_d}}{\sqrt{v'}}$$

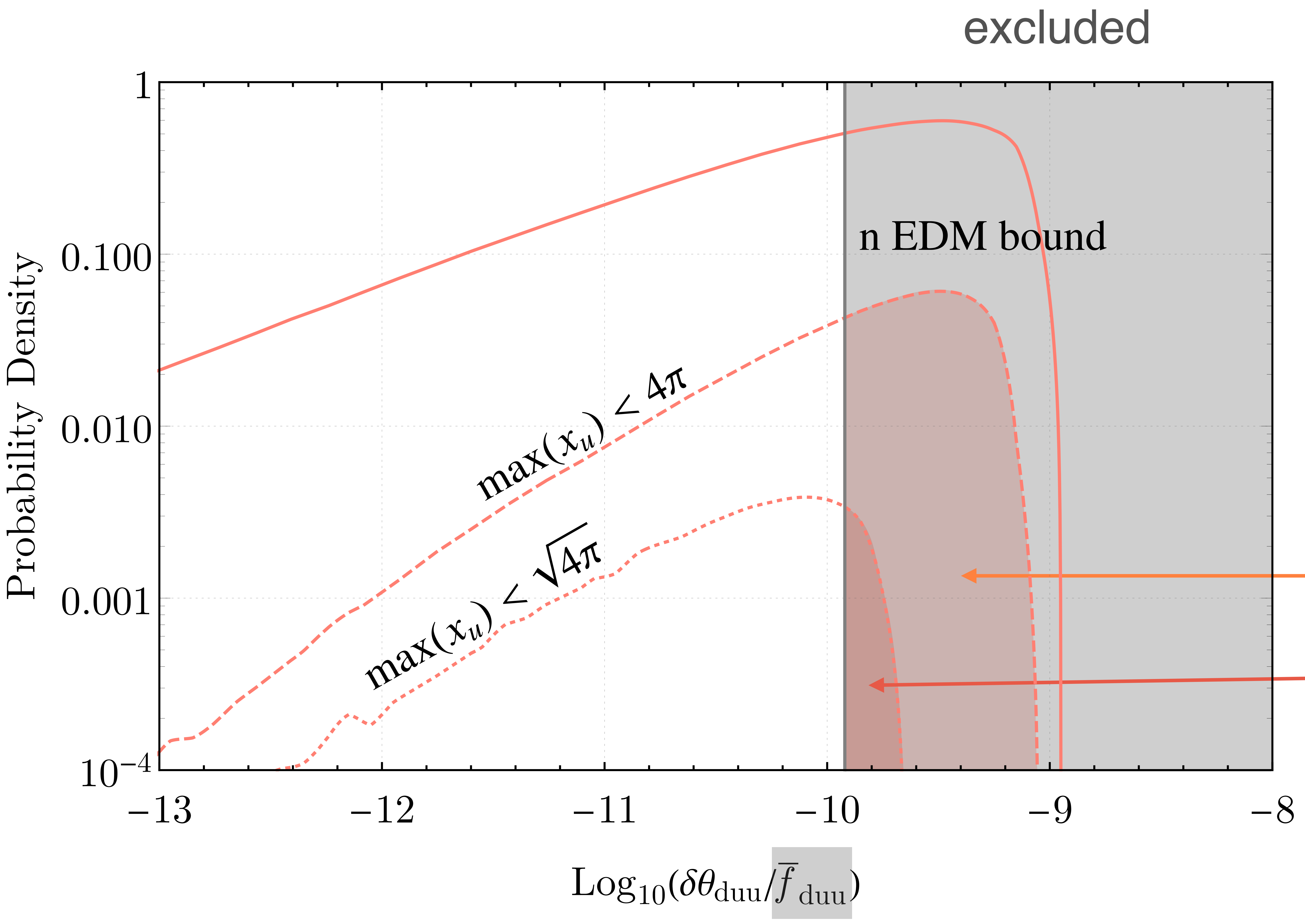
$$\delta\theta_{duu} \approx \frac{1}{(16\pi^2)^2} \frac{v'^2}{\widetilde{M}^2} \operatorname{Im}\operatorname{tr}\left(A_d^a \left[A_u^b, A_u^c\right]\right) f_{duu}^{abc}$$

$$\approx \frac{4}{(16\pi^2)^2} \frac{v'^2}{\widetilde{M}^2} \frac{\widetilde{M}_d M_u^b M_u^c}{v'^3} \frac{m_b m_t^{\frac{3}{2}} \sqrt{m_c}}{v^3} \operatorname{Im}\left(V_{\rm CKM}^{\dagger 33} V_U'^{3b} V_U'^{\dagger b3} V_U'^{3c} V_U'^{\dagger c2} V_{\rm CKM}^{23}\right) \tilde{f}_{duu}^{bc}$$

free parameters

- the hierarchy in the VL quark masses $\tilde{M} \equiv M_d^a = M_u^1 \gg M_u^2 \gg M_u^3 = v'$
- angles: 3 mixing angles in V_U + (2+1) CP phases in $\bar{\Phi}$ & V_U

Induced $\bar{\theta}$ in the minimal LR model



including uncertainty of a loop function

parameter set

- mass hierarchy in VL quarks

$$\tilde{M} \equiv M_d^a = M_u^1 = M_u^2 = 10^3 M_u^3$$

- 6 angles are taken at random $[0, 2\pi]$

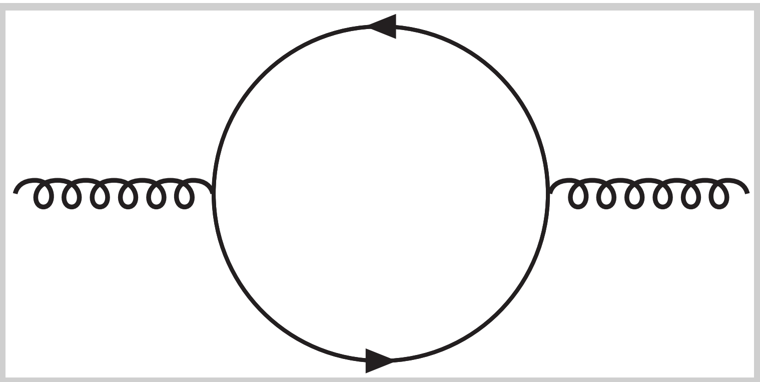
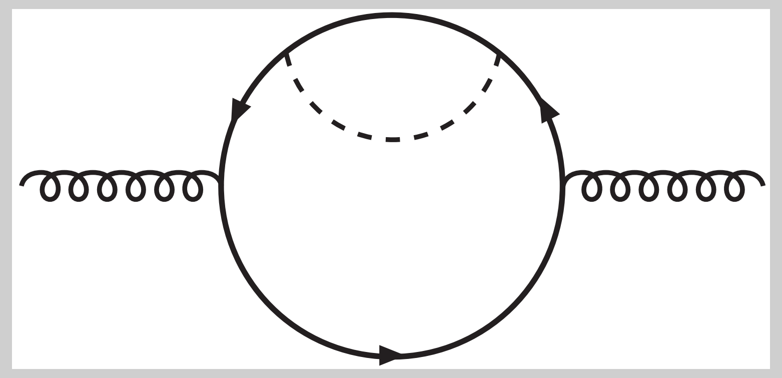
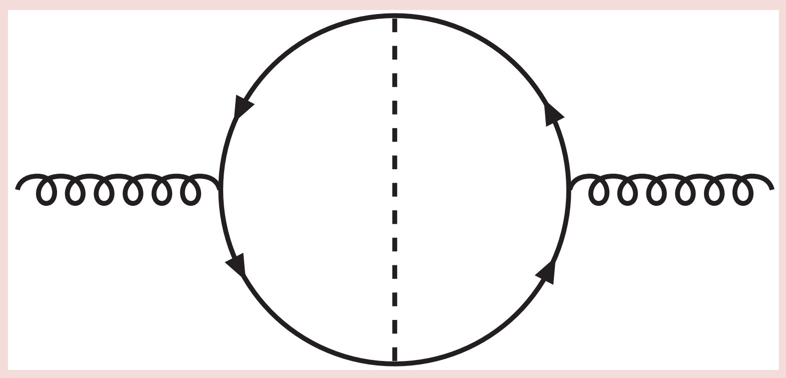
excluded region ($\bar{f}_{\mathrm{duu}} = 1$)

$\max(x_u) < 4\pi$	58.9%
$\max(x_u) < \sqrt{4\pi}$	10.6%

This model can be probed by a little experimental improvement!

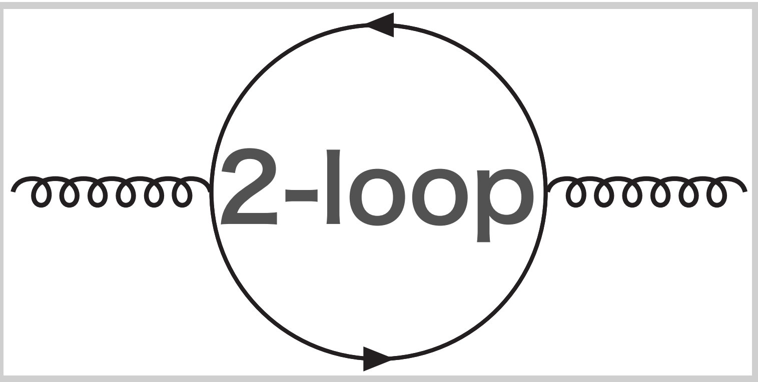
Summary

- **1st result: novel method to calculate radiative corrections to $\bar{\theta}$**

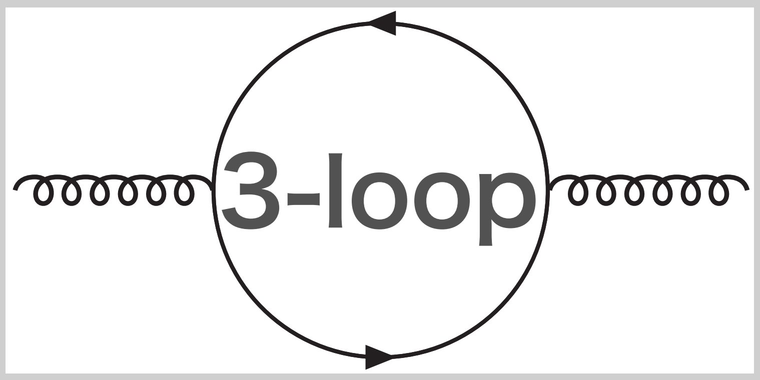
$\bar{\theta} = \theta_G +$  $+$  $+$  $+ \cdots$

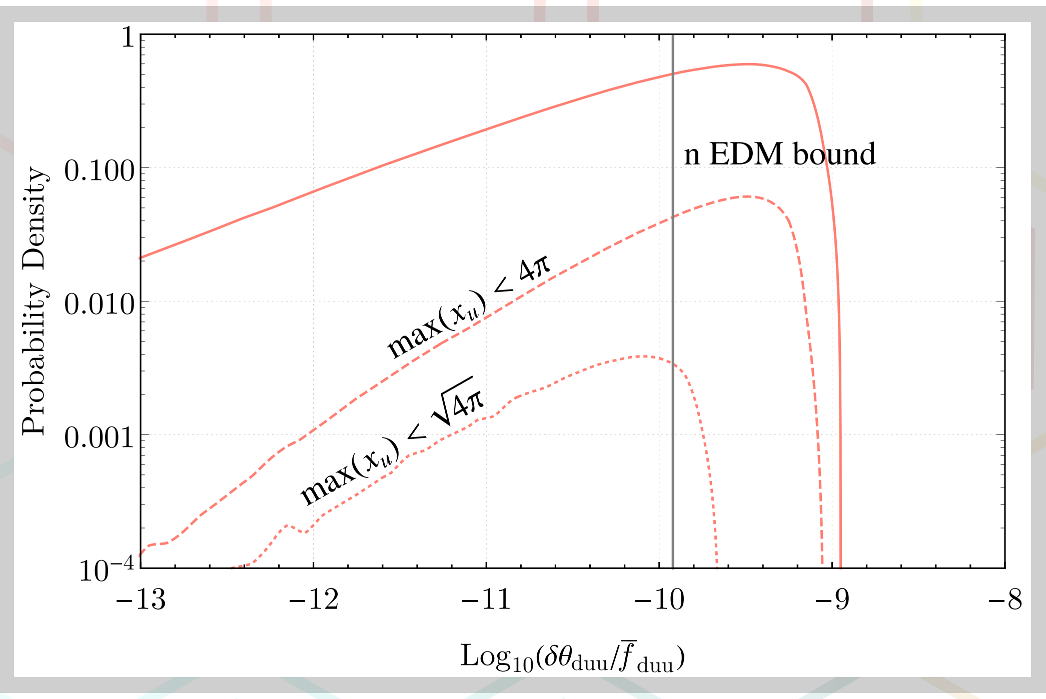
New!

- **2nd result: in the minimal LR model,**



$= 0$





high sensitivity!

Backup

axion and quantum gravity

Peccei-Quinn mechanism : SM × global symmetry $U(1)_{\rm PQ}$

$$\mathcal{L} \ni \left(\bar{\theta} - \frac{a}{f_a}\right) \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} \xrightarrow[\langle a \rangle = f_a \bar{\theta}]{} 0$$

a :axion field

The $U(1)_{\rm PQ}$ symmetry has to be exact.

R. D. Peccei, H. R. Quinn, Phys. Rev. **38** (1977) 1440-1443

⇕

The quantum gravity imposes a global symmetry does not exist.

Y. Zeldovich, Phys. Lett. A **59** (1976) 254

D. Harlow, H. Ooguri Phys. Rev. Lett. **122** (2019) 191601

Fujikawa method

$$S_{\mathrm{QCD}} = \int d^4x \left\{ \sum_{f=u,d,s,c,b,t} \bar{f} i \not{D} f - \left(\mathcal{M}_u^{ij} \bar{u}_L^i u_R^j + \mathcal{M}_u^{\dagger ij} \bar{u}_R^i u_L^j + \mathcal{M}_d^{ij} \bar{d}_L^i d_R^j + \mathcal{M}_d^{\dagger ij} \bar{d}_R^i d_L^j \right) \right.$$
$$\left. - \frac{1}{4} G^{\hat{a}}_{\mu\nu} G^{\hat{a}\mu\nu} + \theta_G \frac{\alpha_s}{8\pi} G^{\hat{a}}_{\mu\nu} \tilde{G}^{\hat{a}\mu\nu} \right\}$$

mass diagonalize (chiral rotation)

—Fujikawa method—

K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

$$\bar{\theta} = \theta_G - \arg\det\left[\mathcal{M}_u \mathcal{M}_d\right]$$

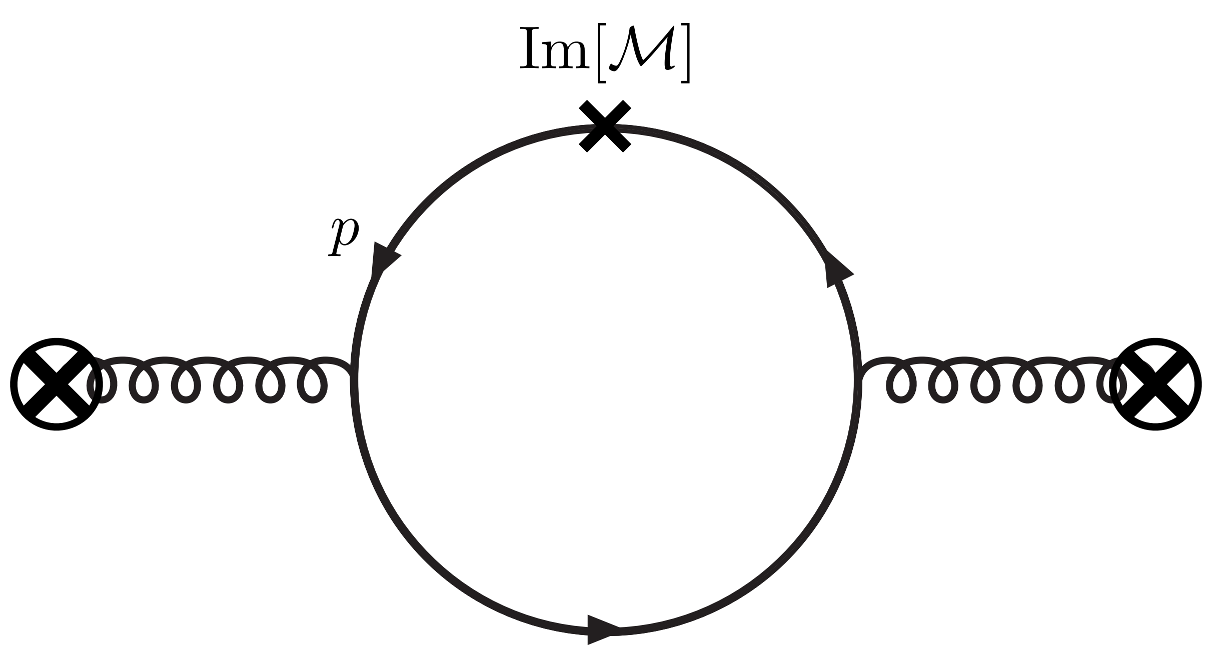
~~**integrating quarks out**~~

Effective aspect of the novel calculation method

$\bar{\theta} = \theta_G \boxed{- \arg\det\left[\mathcal{M}_u \mathcal{M}_d\right]}$

~~integrating quarks out~~

Consistency is checked.



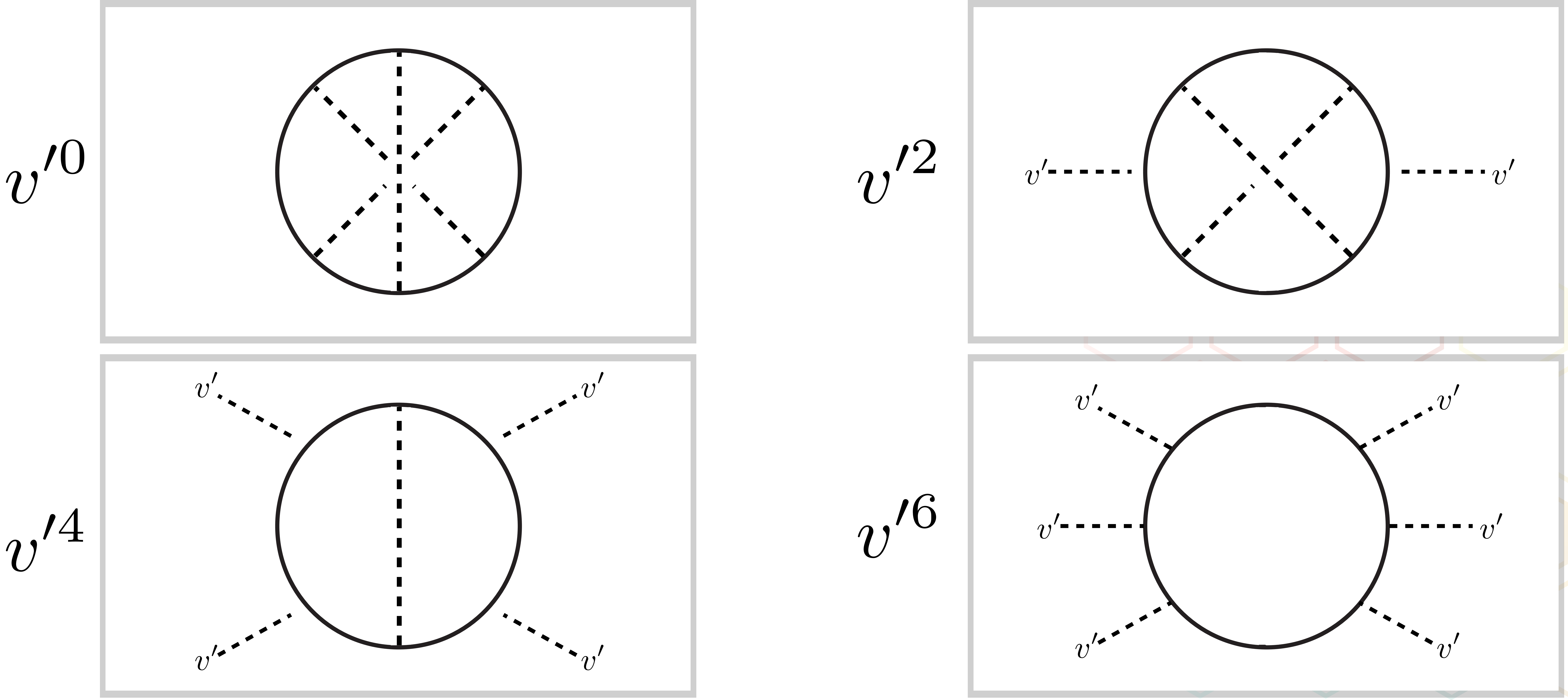
integrating quarks out

- the perturbative bound: $\mu = \Lambda_{\rm QCD}$
- $m_u \, , m_d \, , m_s < \Lambda_{\rm QCD}$

Non-vanishing contribution of $\bar{\theta}$

$$\mathcal{O}(x^6):\ \mathrm{Im}\ \mathrm{Tr}\left(A_q^a\left[A_{q'}^b,\ A_{q''}^c\right]\right) f\left(M_q^a, M_{q'}^b, M_{q''}^c\right)$$

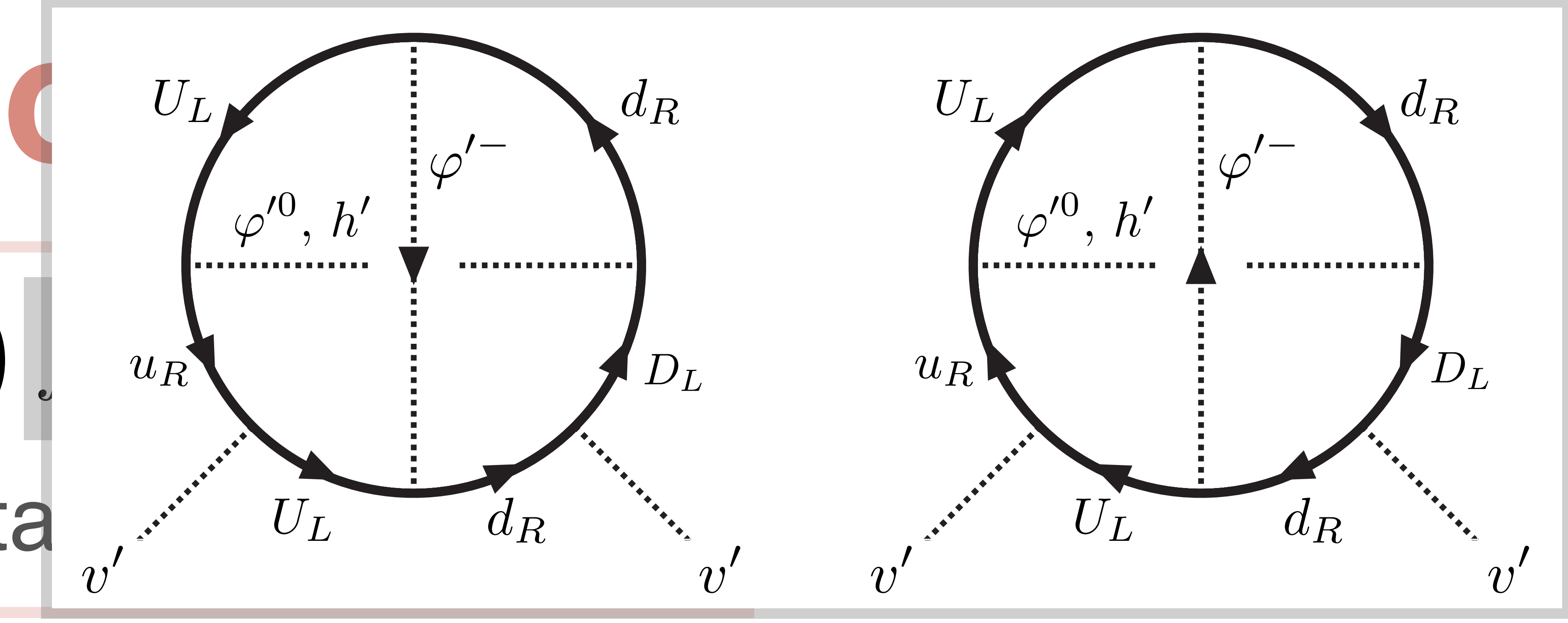
totally antisymmetric



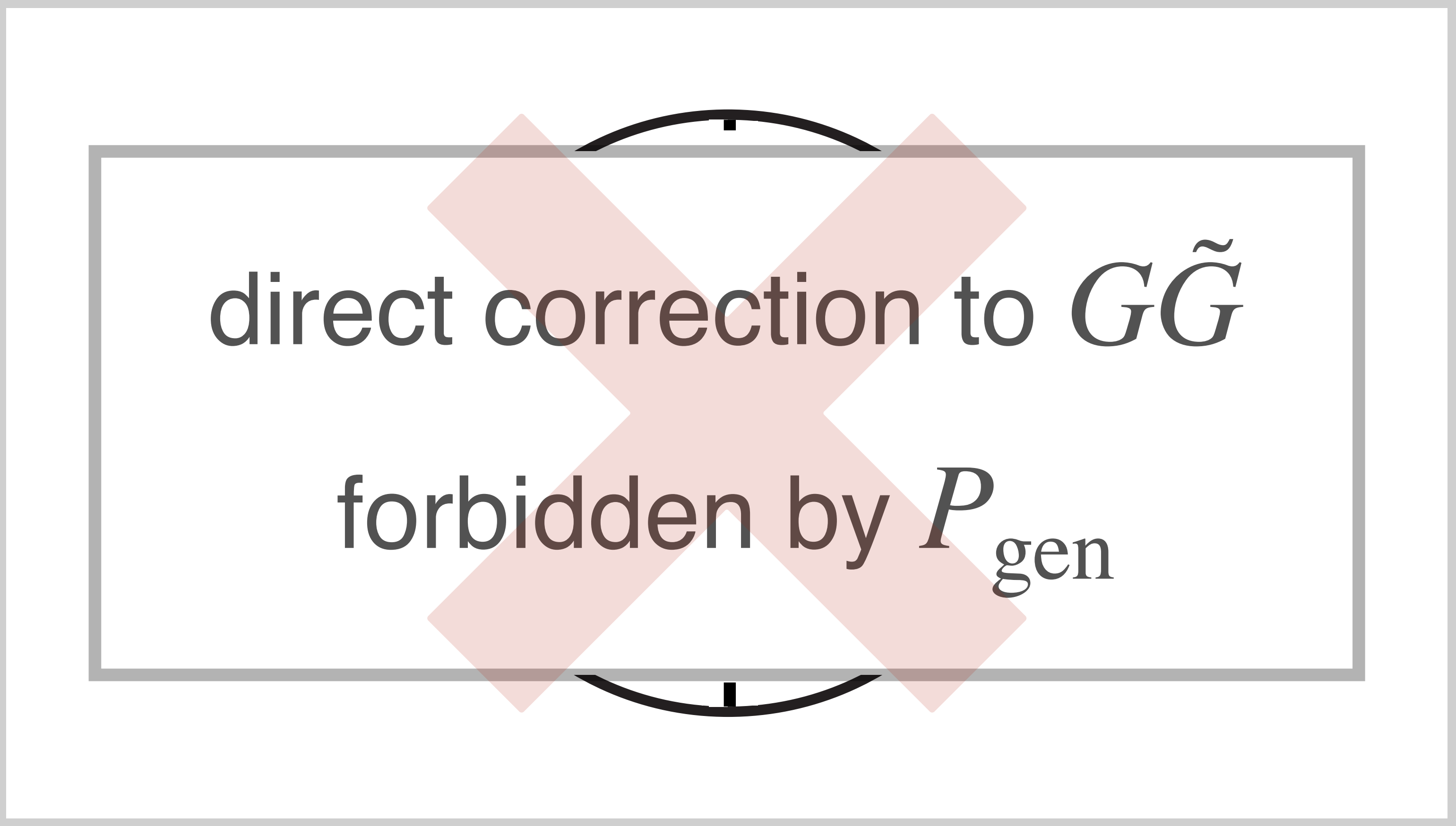
Non-vanishing contributi

$\mathcal{O}(x^6)$: Im Tr $\left(A_q^a \left[A_{q'}^b,\, A_{q''}^c\right]\right)$

tota



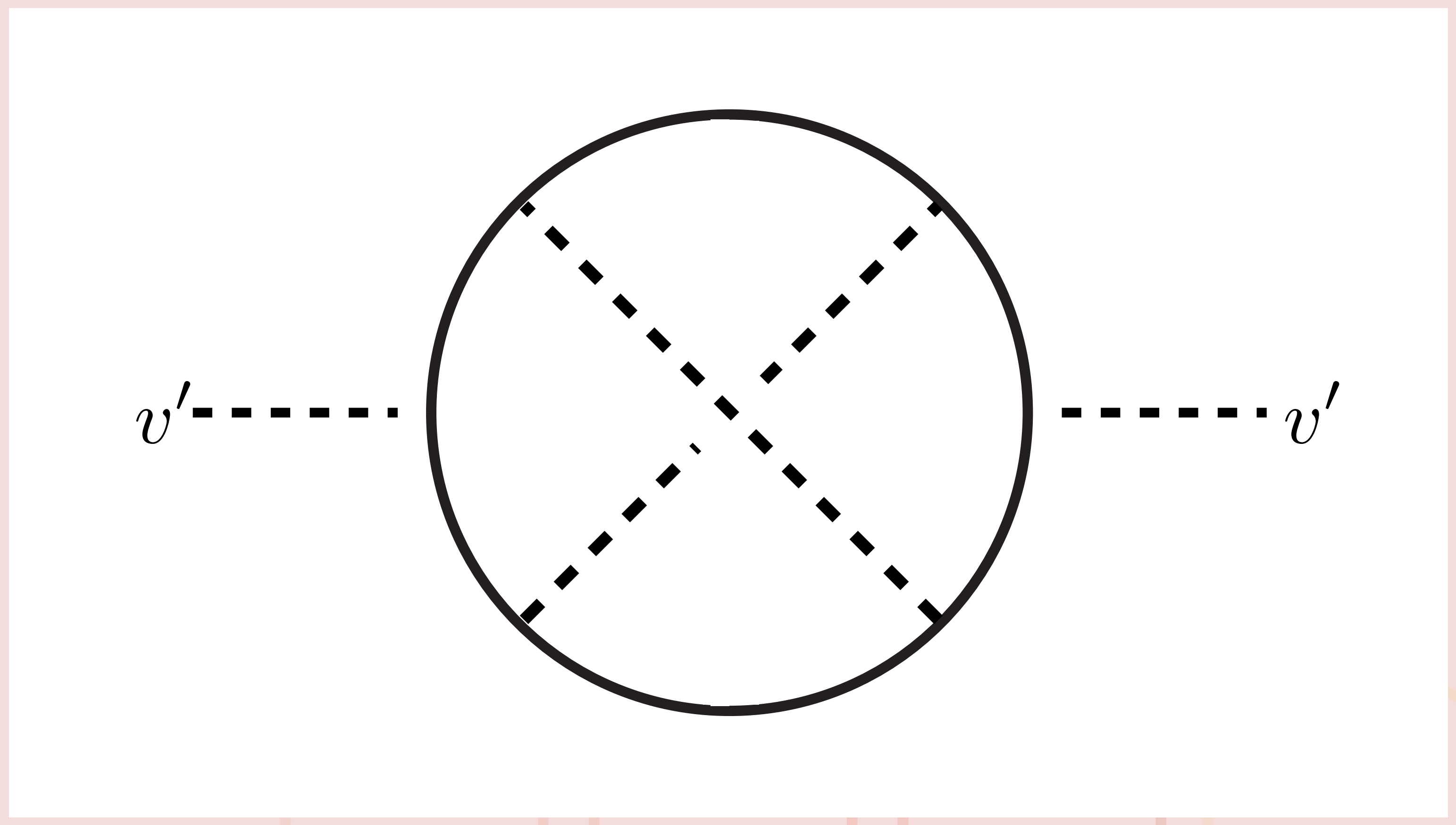
v'^0



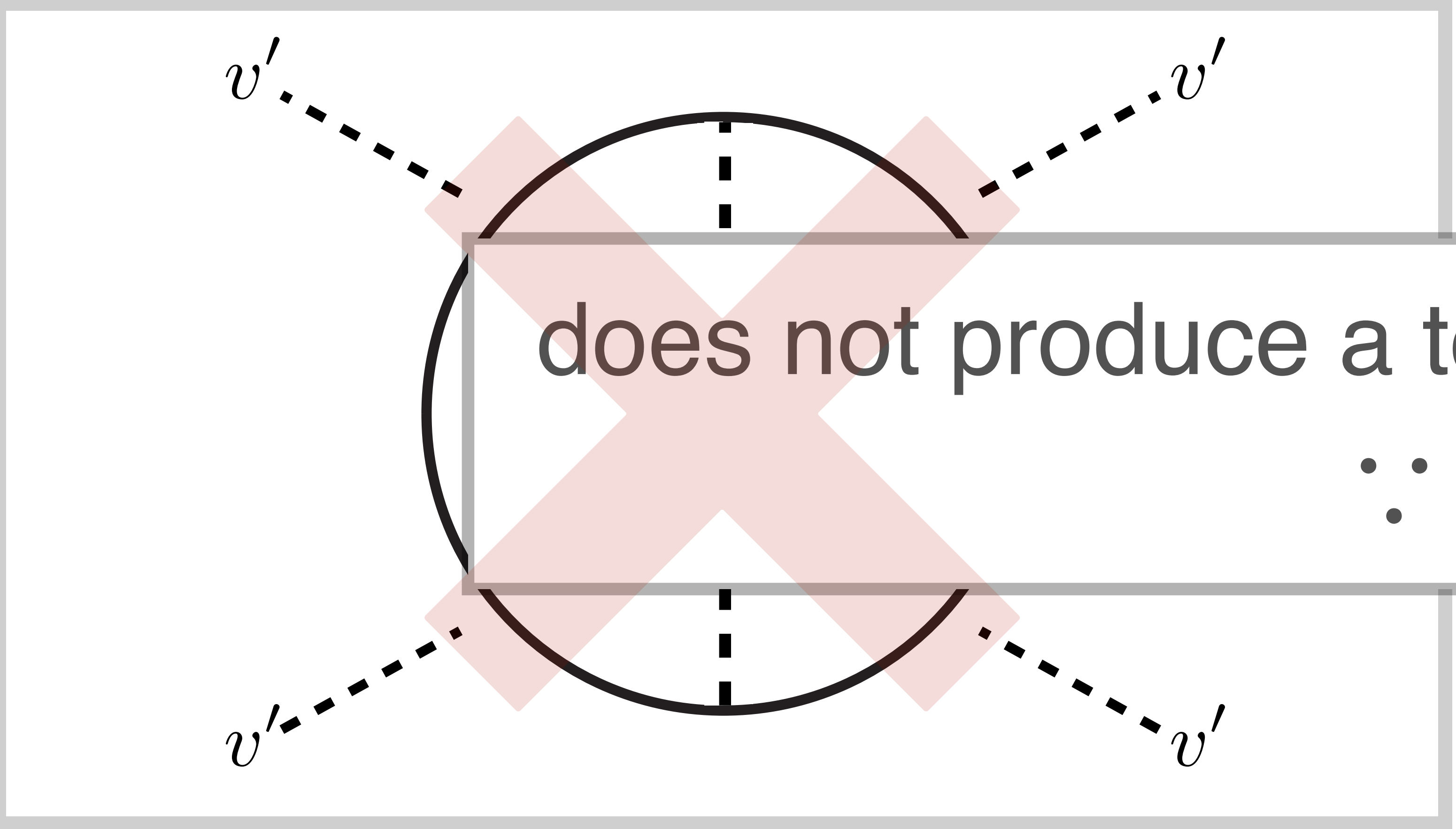
direct correction to $G\tilde{G}$

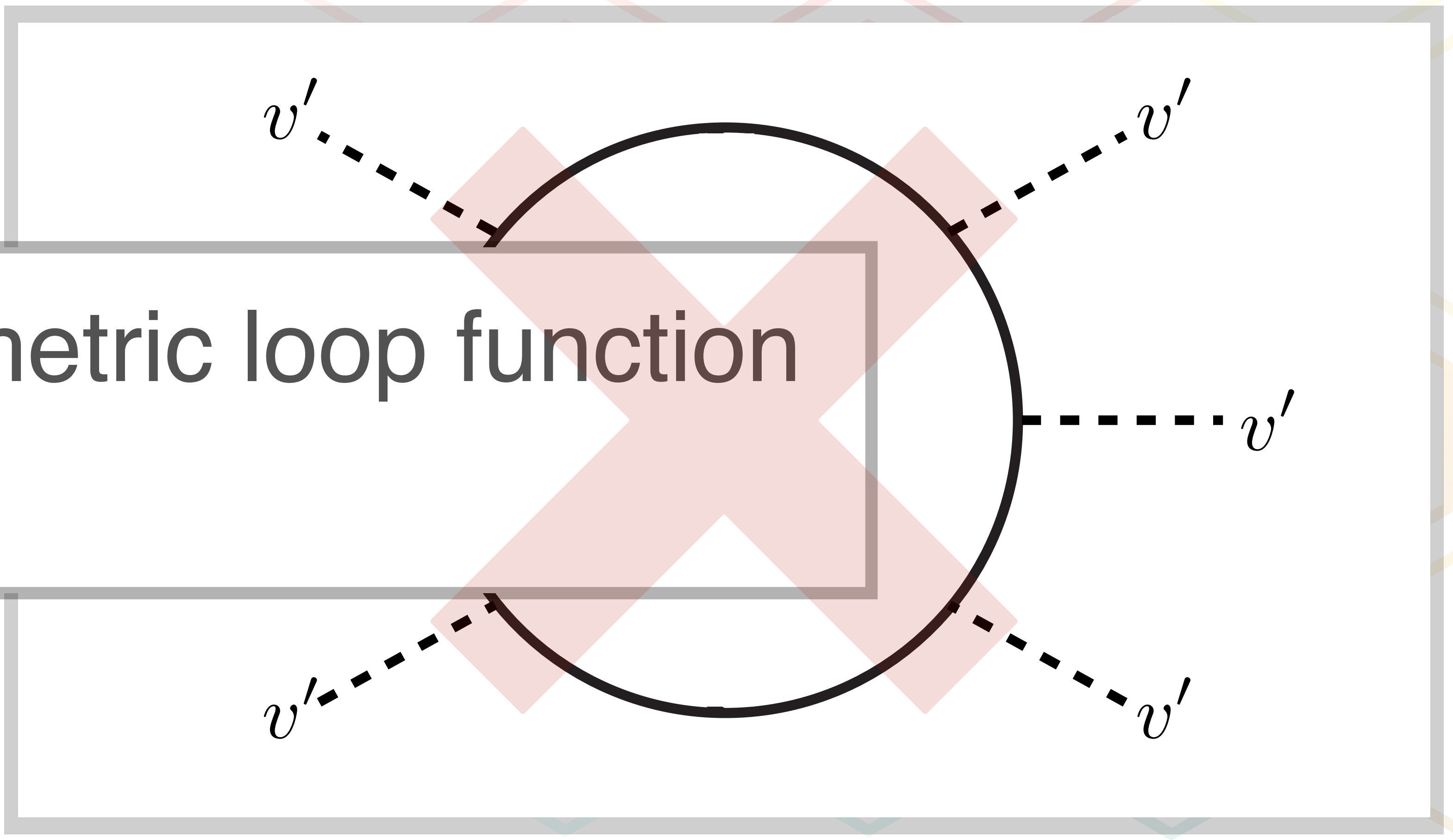
forbidden by $P_{\rm gen}$

v'^2



v'^4





does not produce a totally antisymmetric loop function

∵ mass insertion

Interactions

gauge
$$\mathcal{L}_{W+W'} = -\frac{g}{\sqrt{2}}\left(\bar{u}^i_L\gamma^\mu d^i_L W^+_\mu + \bar{d}^i_L\gamma^\mu u^i_L W^-_\mu\right) - \frac{g}{\sqrt{2}}\left(\bar{u}^i_R\gamma^\mu d^i_R W'^+_\mu + \bar{d}^i_R\gamma^\mu u^i_R W'^-_\mu\right)$$
$$= -\frac{g}{\sqrt{2}}\left[\left(V^{Pi}_{uL}V^{\dagger iQ}_{uL}\right)\bar{\mathcal{U}}^P_{ML}\gamma^\mu\mathcal{U}^Q_{ML}W^+_\mu + \left(V^{Pi}_{dL}V^{\dagger iQ}_{dL}\right)\bar{\mathcal{D}}^P_{ML}\gamma^\mu\mathcal{D}^Q_{ML}W^-_\mu\right]$$
$$-\frac{g}{\sqrt{2}}\left[\left(V^{Pi}_{uR}V^{\dagger iQ}_{uR}\right)\bar{\mathcal{U}}^P_{MR}\gamma^\mu\mathcal{U}^Q_{MR}W'^+_\mu + \left(V^{Pi}_{dR}V^{\dagger iQ}_{dR}\right)\bar{\mathcal{D}}^P_{MR}\gamma^\mu\mathcal{D}^Q_{MR}W'^-_\mu\right]$$

charged NG
$$-\mathcal{L}_{\varphi^\pm,\varphi'^\pm} = \left(V^{Pi}_{uL}x^{ia}_d V^{\dagger aQ}_{dR}\right)\bar{\mathcal{U}}^P_{ML}\mathcal{D}^Q_{MR}\varphi^+ - \left(V^{Pi}_{dL}x^{ia}_u V^{\dagger aQ}_{uR}\right)\bar{\mathcal{D}}^P_{ML}\mathcal{U}^Q_{MR}\varphi^-$$
$$+\left(V^{Pi}_{uR}x^{ia}_d V^{\dagger aQ}_{dL}\right)\bar{\mathcal{U}}^P_{MR}\mathcal{D}^Q_{ML}\varphi'^+ - \left(V^{Pi}_{dR}x^{ia}_u V^{\dagger aQ}_{uL}\right)\bar{\mathcal{D}}^P_{MR}\mathcal{U}^Q_{ML}\varphi'^-$$
$$+\,\mathrm{h.c.}$$

neutral NG
$$-\mathcal{L}_{\varphi^0,\varphi'^0} = \frac{i}{\sqrt{2}}\left(V^{Pi}_{uL}x^{ia}_u V^{\dagger aQ}_{uR}\right)\bar{\mathcal{U}}^P_{ML}\mathcal{U}^Q_{MR}\varphi^0 - \frac{i}{\sqrt{2}}\left(V^{Pi}_{dL}x^{ia}_d V^{\dagger aQ}_{dR}\right)\bar{\mathcal{D}}^P_{ML}\mathcal{D}^Q_{MR}\varphi^0$$
$$+\frac{i}{\sqrt{2}}\left(V^{Pi}_{uR}x^{ia}_u V^{\dagger aQ}_{uL}\right)\bar{\mathcal{U}}^P_{MR}\mathcal{U}^Q_{ML}\varphi'^0 - \frac{i}{\sqrt{2}}\left(V^{Pi}_{dR}x^{ia}_d V^{\dagger aQ}_{dL}\right)\bar{\mathcal{D}}^P_{MR}\mathcal{D}^Q_{ML}\varphi'^0$$
$$+\,\mathrm{h.c.}$$

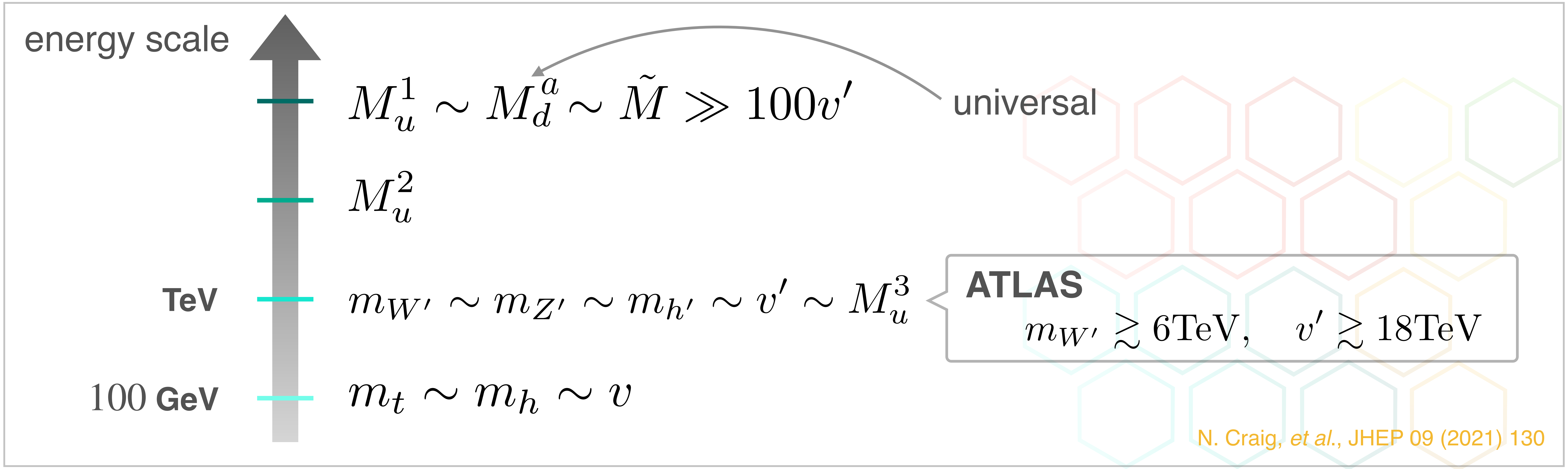
Mass spectrum

- up-type VL quark mass hierarchy ⟶ up-type quark mass hierarchy

$$M_u^1 \gg M_u^2 \gg M_u^3$$

- down-type Yukawa (x_d) components ⟶ down-type quark mass hierarchy (∵ mild)

$$M_d^1 = M_d^2 = M_d^3$$

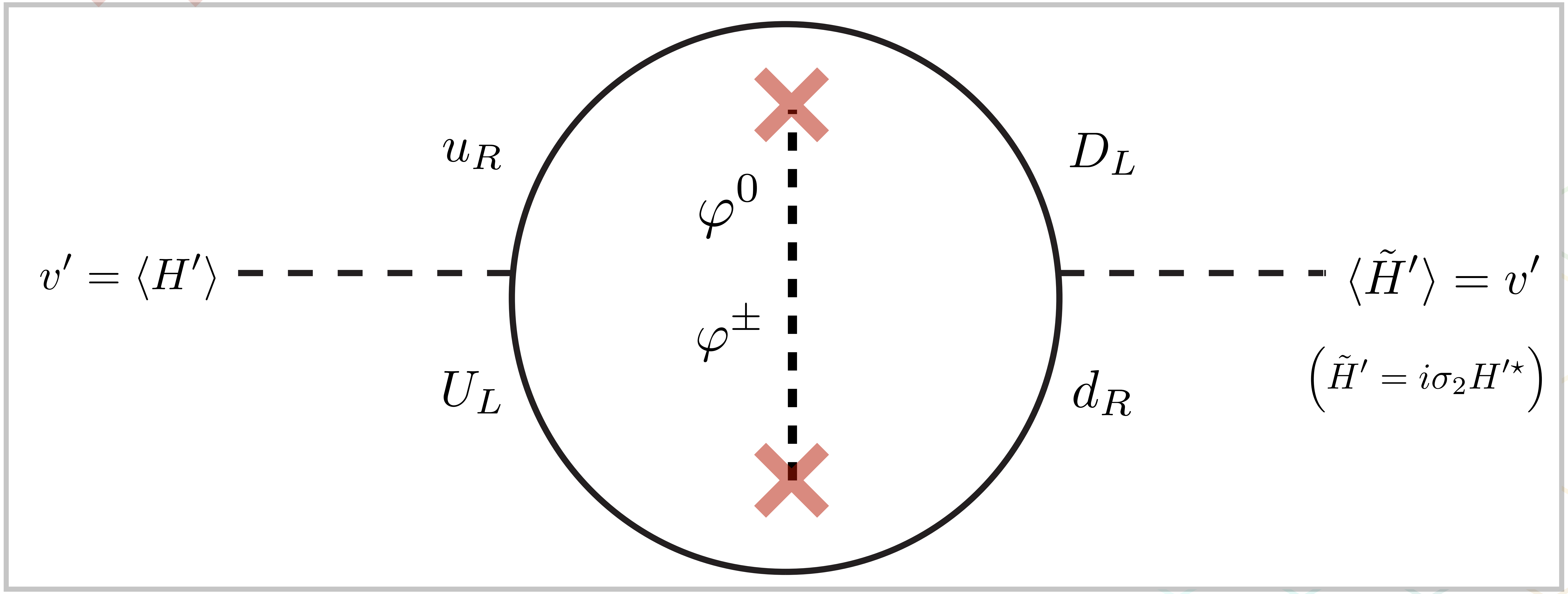


Loops of $SU(2)_L$ Higgs

~~2-loop~~

~~3-loop~~

$$\frac{|H'|^2 - \cancel{|H|^2}}{\Lambda^2} G\tilde{G} \to \frac{v'^2 - \cancel{v^2}}{\Lambda^2} G\tilde{G}$$



Collider & flavor experimental bound

- **ATLAS (charged lepton + missing)** ⟶ **charged boson mass $m_{W'}$**

$$m_{W'} \gtrsim 6\text{TeV}, \quad v' \gtrsim 18\text{TeV}$$

- **Future Circular Collider (FCC), 100TeV pp collider**

$$m_{W'}, m_{Z'} \sim 40\text{TeV}, \quad v' \gtrsim 120\text{TeV}$$

: fine-tuning problem
in the scalar potential

- **one-loop FCNCs, kaon mixing**

$$(\Delta m_K)_{u,c} \approx -6 \cdot 10^{-16}\text{GeV}\left(\frac{6\text{TeV}}{m_{W'}}\right)^2, \quad |\epsilon_K|_{u,c} \approx 7 \cdot 10^{-5}\left(\frac{6\text{TeV}}{m_{W'}}\right)^2$$

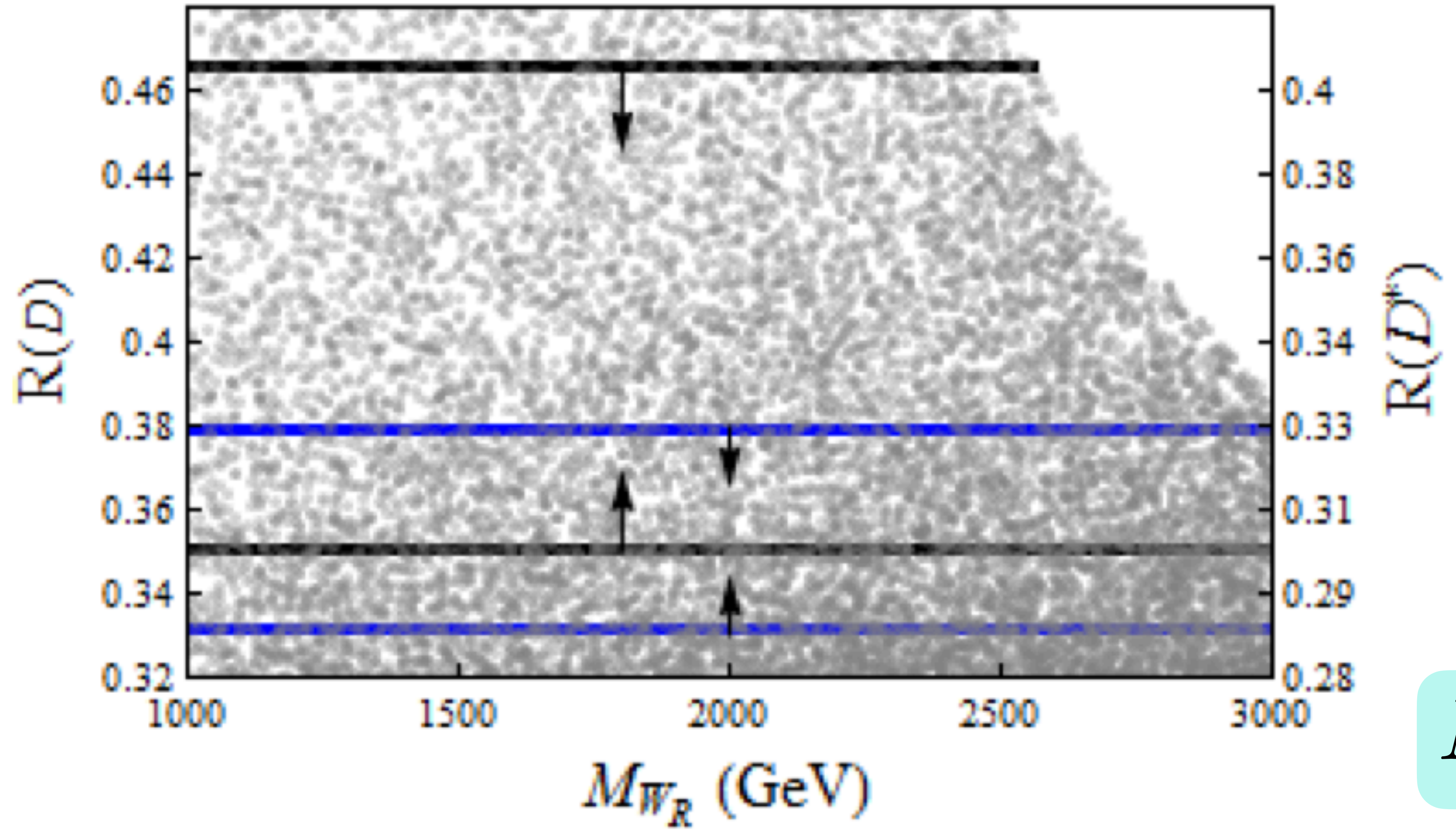
an order of magnitude below the theoretical error in the SM prediction

N. Craig, I. Garcia Garcia, G. Koszegi, and A. McCune, JHEP **09** (2021) 130

B anomaly in the LR model

$R(D), R(D^*)$ **anomaly**

$$R(D) = \frac{\Gamma(B \to D\tau\nu)}{\Gamma(B \to D\ell\nu)}, \quad R(D^*) = \frac{\Gamma(B \to D^*\tau\nu)}{\Gamma(B \to D^*\ell\nu)}$$



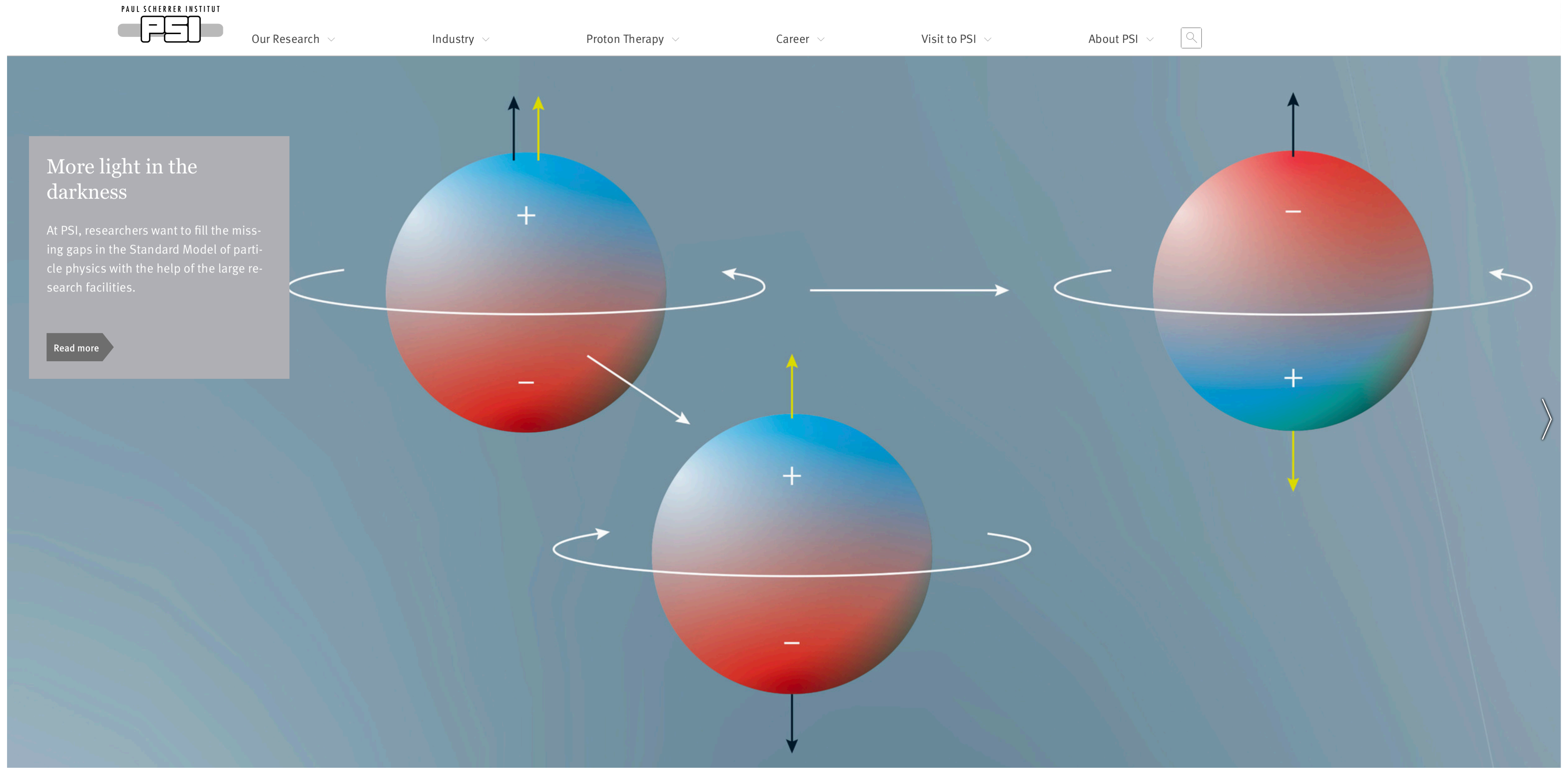
excluded

$M_{W_R} < 3\,\text{TeV}$

Figure 4: $R(D, D^*)$ scatter-plot is shown by varying g_R and M_{W_R}. The boundaries of $R(D)$ and $R(D^*)$ anomalies are shown by black and blue lines respectively. We show 1 σ allowed regions.

K. S. Babu, B. Dutta and R. N. Mohapatra, JHEP 01 (2019), 168

Neutron EDM experiment



https://www.psi.ch/en

neutron EDM (nEDM) experiment

- Paul-Scherrer Institute (PSI)
- ultracold neutron
  +
  Ramsey method
- result in 2020（measured in 2015 ~ 2016）

$$|d_n| < 1.8 \times 10^{-26} e\,\mathrm{cm} \quad (90\%\mathrm{C.L.})$$

C. Abel, Phys. Rev. Lett. 124 (2020) 081803

future experiment: TUCAN(TRIUMF Ultra-Cold Advanced Neutron)

Canada & Japan

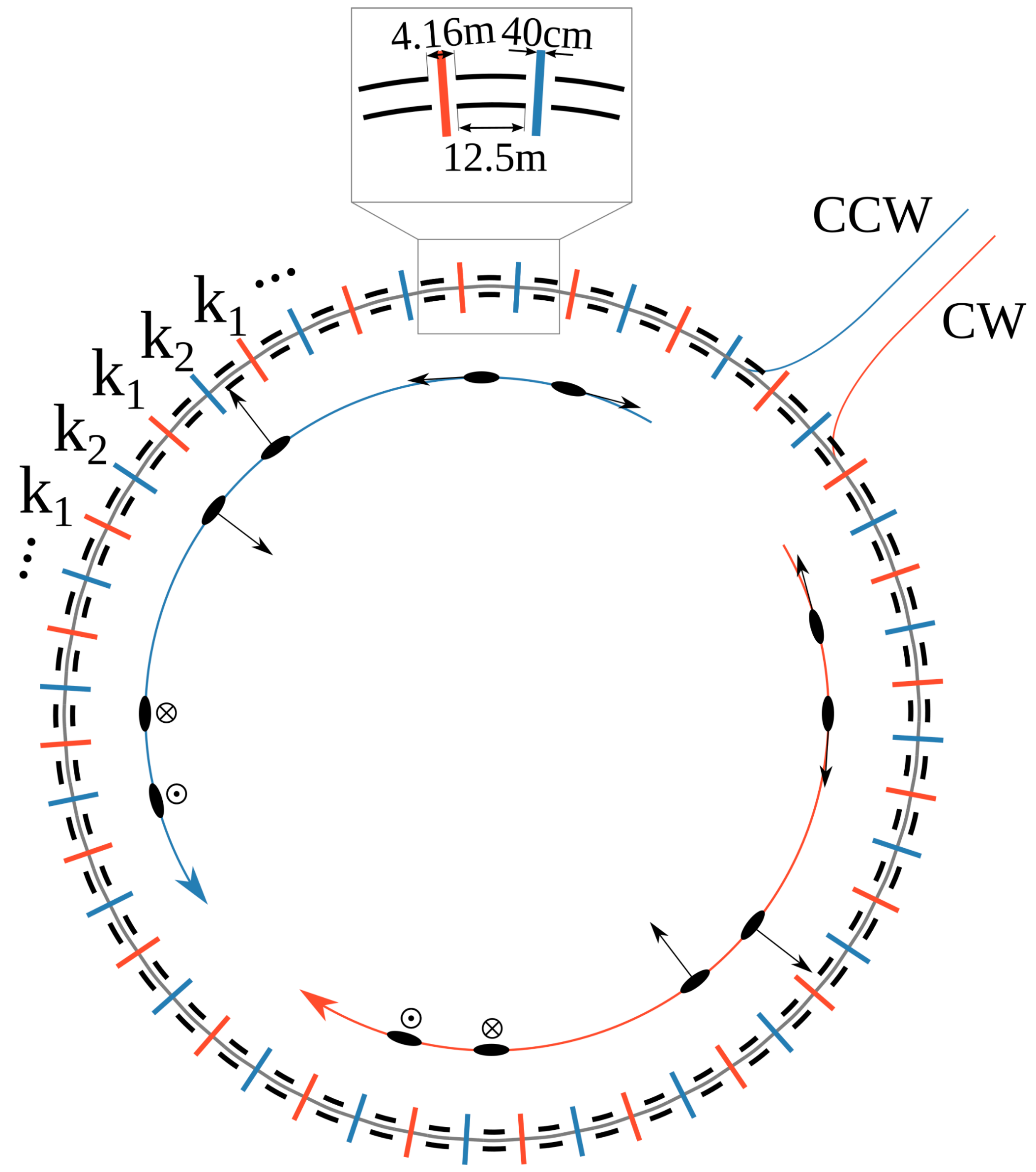
Aim: $|d_n| \lesssim 1 \times 10^{-27} e\,\mathrm{cm}$

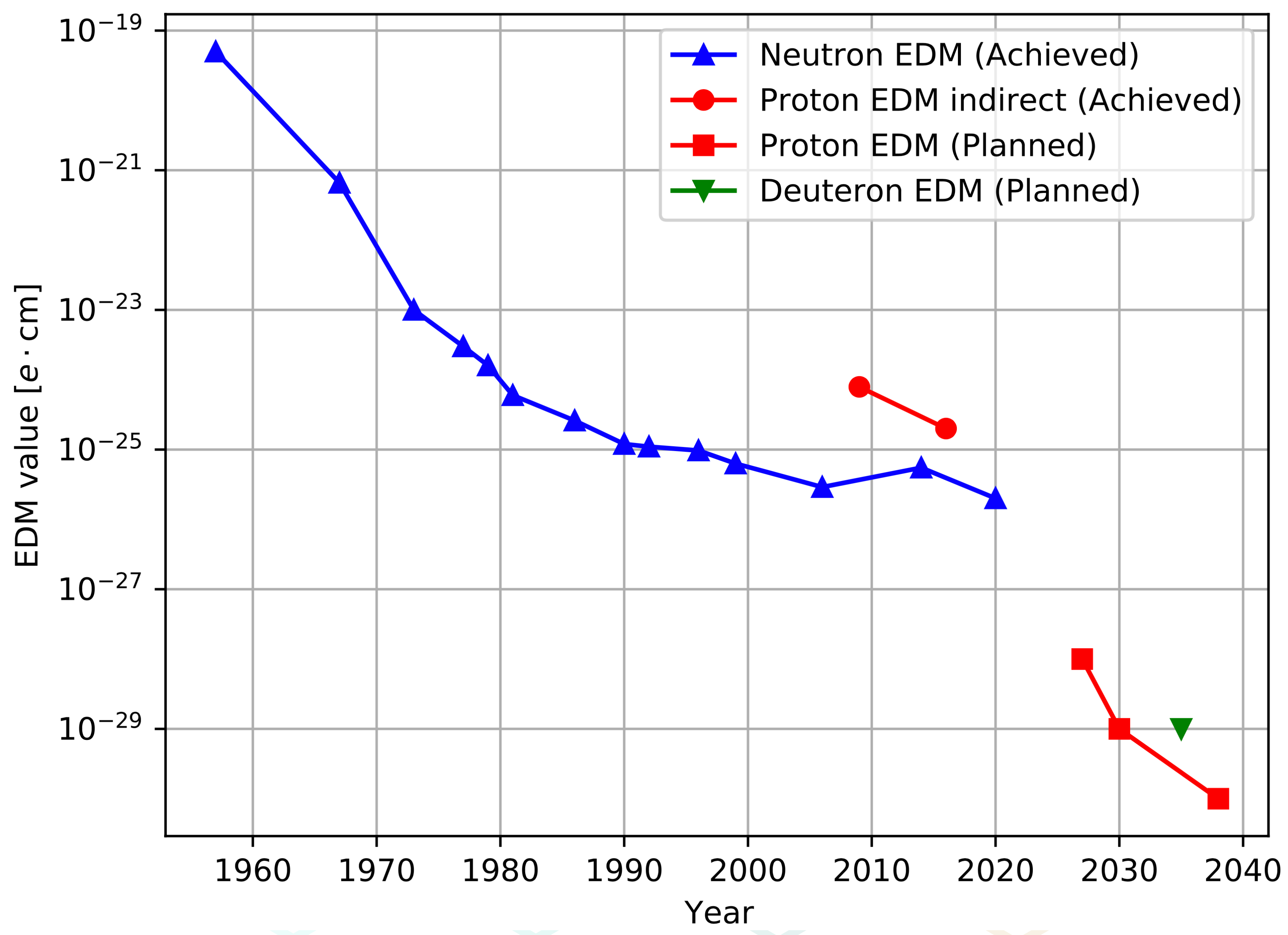
S. Ahmed, *et al.*, *Phys.Rev.C* 99 (2019) 2, 025503

Proton EDM experiment

proton EDM (pEDM) experiment

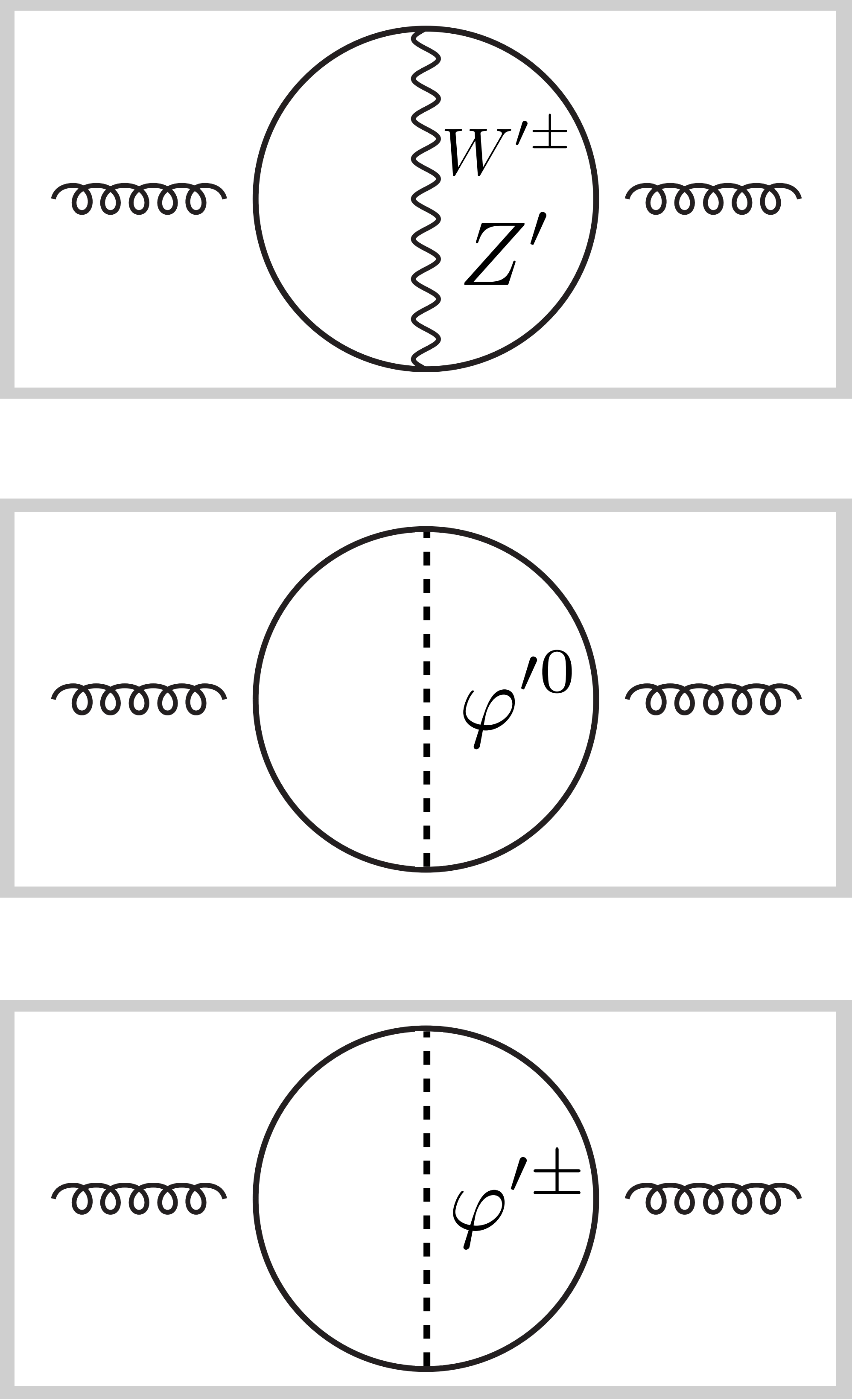
- CERN, CPEDM collaboration
- storage ring $$\frac{d\mathbf{s}}{dt} = \boldsymbol{\mu} \times \mathbf{B} + \mathbf{d} \times \mathbf{E}$$ ($\mathbf{s}$: spin)
- $|d_p| \lesssim 10^{-29} e\,\mathrm{cm}$
- $\bar{\theta}$ parameter
  - improves at three order from nEDM





J. Alexander, *et al.*, arXiv:2205.00830 [hep-ph]

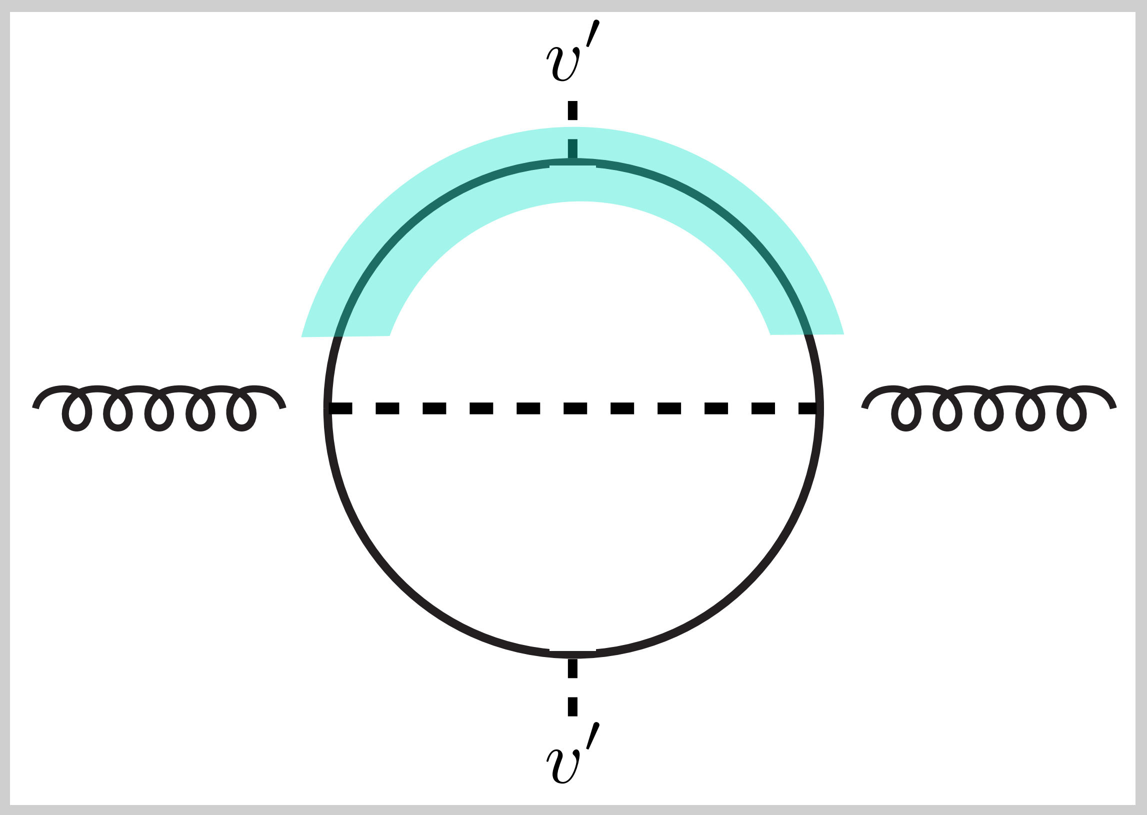
2-loop contributions



$W'^{\pm}$, Z' : gauge bosons from $SU(2)_R$

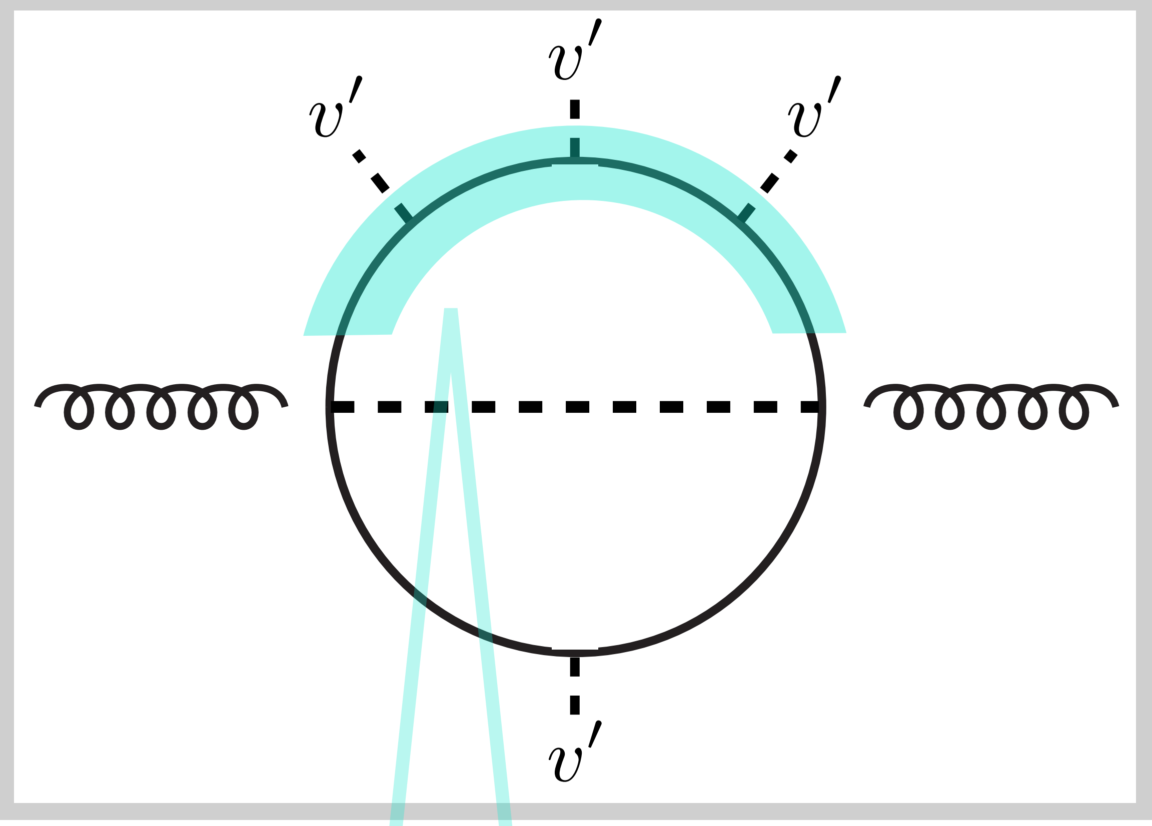
φ'^{0}, $\varphi'^{\pm}$: H''s NG bosons

the lowest order: $O(x^4)$



mass insertion

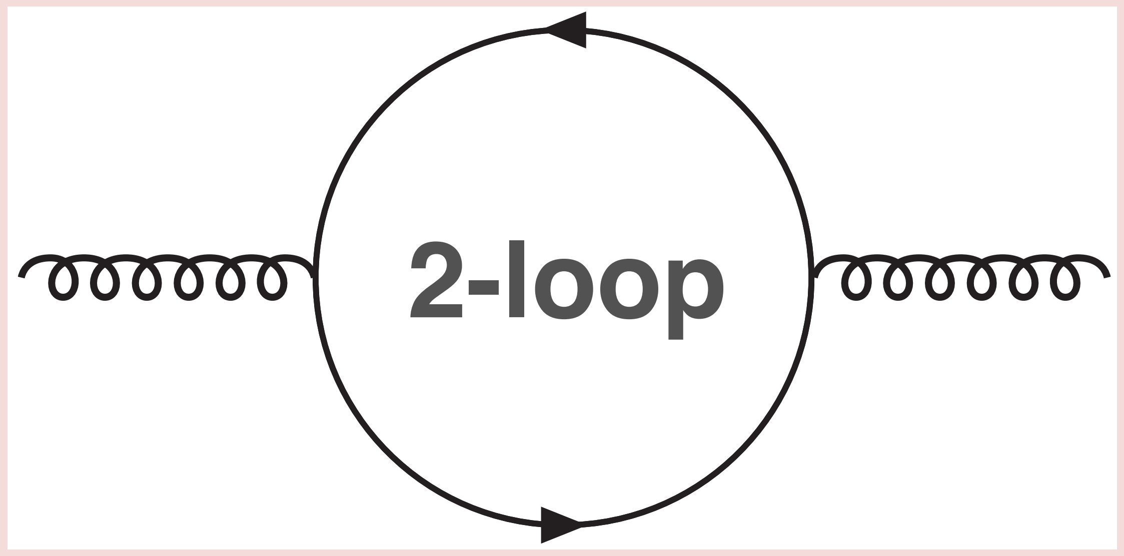
the next order: $O(x^6)$



$$P_L \frac{i(\not{p} + M_u^a)}{p^2 - (M_u^a)^2} (-i x_u^{\dagger aj} v' P_R) \frac{i\not{p}}{p^2} (-i x_u^{jb} v' P_L) \frac{i(\not{p} + M_u^b)}{p^2 - (M_u^b)^2} (-i x_u^{\dagger bi} v' P_R) \frac{i\not{p}}{p^2} P_L$$

$$= i v'^3 P_L \frac{1}{p^2 - (M_u^a)^2} x_u^{\dagger aj} x_u^{jb} \frac{1}{p^2 - (M_u^b)^2} x_u^{\dagger bi}$$

symmetric about a and b



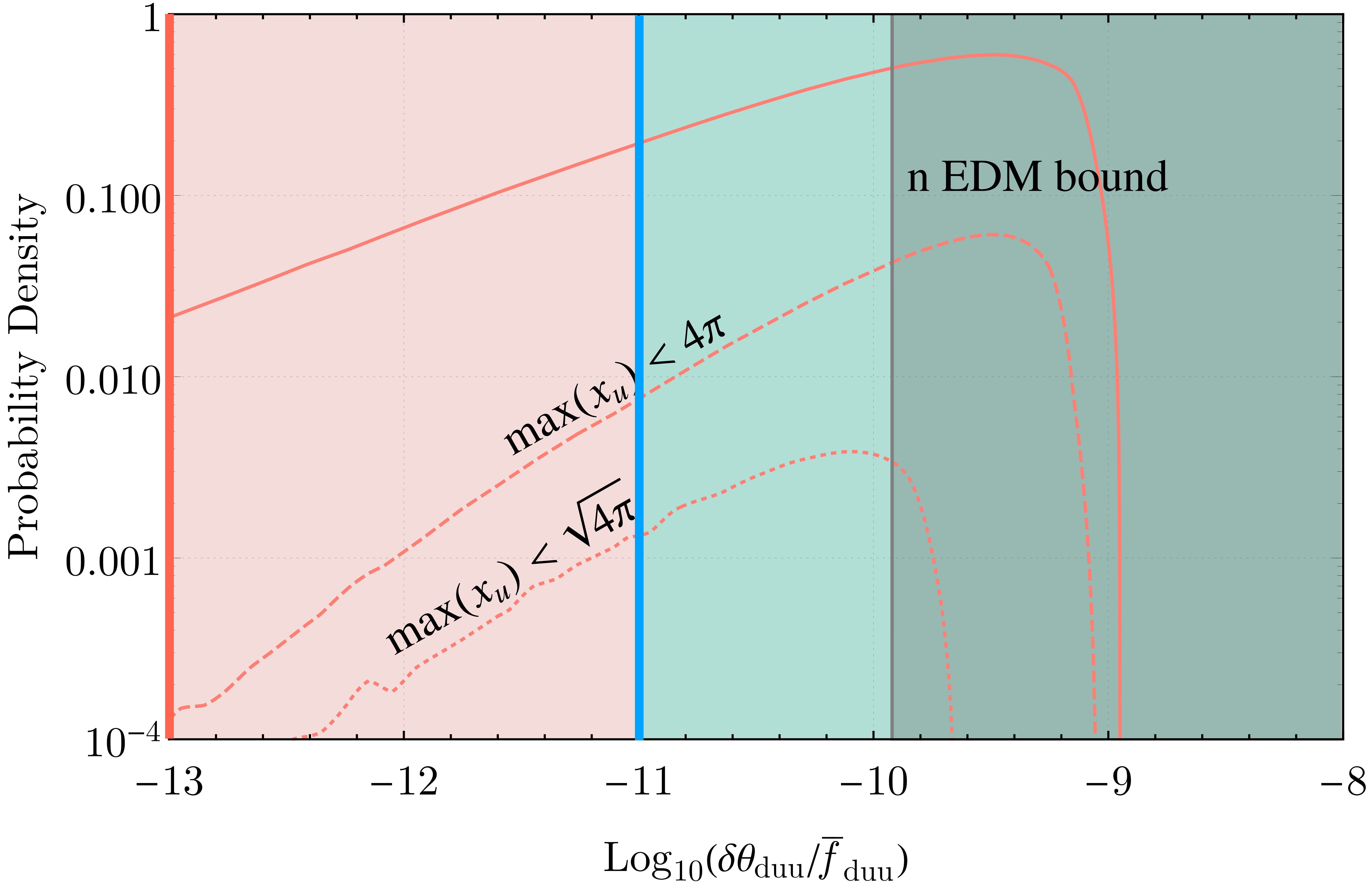
symmetric

$$\mathrm{Im}\,\mathrm{Tr}\left(A_q^a \left[A_{q'}^b, A_{q''}^c\right]\right) f(M_q^a, M_{q'}^b, M_{q''}^c)$$

$$= 0$$

(analytically & numerically checked!)

PDF and future experiments



$$M_u^3/M_u^1 = 10^{-3}\,,\ M_u^1 = M_u^2$$

excluded region

nEDM ($|d_n| \lesssim 1 \times 10^{-27} e\,\mathrm{cm}$)

$\max(x_u) < 4\pi$	94.0%
$\max(x_u) < \sqrt{4\pi}$	82.5%

pEDM ($|d_p| \lesssim 1 \times 10^{-29} e\,\mathrm{cm}$)

$\max(x_u) < 4\pi$	99.9%
$\max(x_u) < \sqrt{4\pi}$	99.8%

θ parameter respect to $SU(2)_L, SU(2)_R$

$$\mathcal{L} \ni \theta_2 \frac{\alpha_2}{8\pi} W^{\hat{a}}_{\mu\nu} \tilde{W}^{\hat{a}\mu\nu} + \theta_2 \frac{\alpha_2}{8\pi} W'^{\hat{a}}_{\mu\nu} \tilde{W}'^{\hat{a}\mu\nu}$$
$$+ \bar{Q}^i_L x^{ia}_u U^a_R \tilde{H} + \bar{Q}^i_R x^{ia}_u U^a_L \tilde{H}' + M^a_u \bar{U}^a_L U^a_R$$
$$+ \bar{Q}^i_L x^{ia}_d D^a_R H + \bar{Q}^i_R x^{ia}_d D^a_L H' + M^a_d \bar{D}^a_L D^a_R + \text{h.c.}$$

1: Two θ terms has a same coupling θ_2 due to P_{gen}

2: axial $U(1)$ angle is $\beta = \frac{1}{2}(\theta_R - \theta_L)$

chiral rotation: $Q_L \to e^{i\theta_L} Q_L, \quad Q_R \to e^{i\theta_R} Q_R$

$SU(2)_L \qquad SU(2)_R$

3: U, D have vector-like symmetry

proceeding study

sphaleron proces

$$\exp\left(-\frac{8\pi^2}{g_L^2} - \frac{8\pi^2}{g_R^2}\right) = 10^{-169}$$

$$g_L = g_R = 0.637$$

A. A. Anselm, A. A. Johansen, Nucl. Phys. *B* 412 (1994) 553-573

The θ parameter of $SU(2)_L, SU(2)_R$ can be an observables, but it is too small to verify.

Neutrino

If the LR symmetry is generated to the lepton sector,

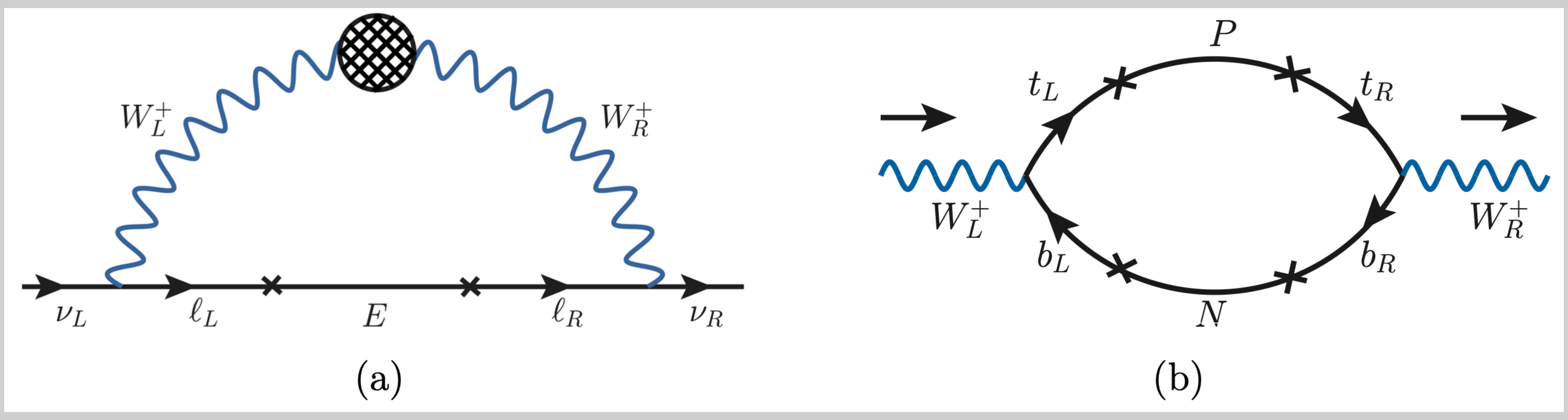
- no neutral VL lepton

$$\Psi_L(1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \Psi_R(1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \quad E_{L/R}, \quad N_{L/R}$$

- charged lepton mass matrix

$$\mathcal{M}_e = \begin{pmatrix} 0 & x_e v \\ x_e^\dagger v' & M_E \end{pmatrix}$$

Dirac neutrino mass



Certain benchmark points can explain the neutrino oscillation.

Oscillation parameters	3σ range NuFit5.1 [48]	Model prediction			
		BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	6.82 - 8.04	7.42	7.38	7.35	7.35
$\Delta m^2_{23}(10^{-3}\ \mathrm{eV}^2)$(IH)	2.410 - 2.574	-	-	2.48	2.52
$\Delta m^2_{31}(10^{-3}\ \mathrm{eV}^2)$(NH)	2.43 - 2.593	2.49	2.51	-	-
$\sin^2\theta_{12}$	0.269 - 0.343	0.324	0.301	0.306	0.310
$\sin^2\theta_{23}$ (IH)	0.410 - 0.613	-	-	0.510	0.550
$\sin^2\theta_{23}$ (NH)	0.408 - 0.603	0.491	0.533	-	-
$\sin^2\theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0219	0.0213
$\sin^2\theta_{13}$(NH)	0.02060 - 0.02435	0.0234	0.0213	-	-
δ_{CP} (IH)	192 - 361	-	-	236°	279°
δ_{CP} (NH)	105 - 405	199°	280°	-	-
m_{light} (10^{-3}) eV		0.66	2.04	14.1	8.50
M_{E_1}/M_{W_R}		917	45.5	1936	1990
M_{E_2}/M_{W_R}		0.650	0.43	0.12	0.11
M_{E_3}/M_{W_R}		0.019	0.029	0.015	0.012

K. S. Babu, *et al.*, JHEP 08 (2022) 140