Gravitational charge in expanding Universe

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References

S. Aoki and K. Kawana, "Entropy and its conservation in expanding Universe", International Journal of Modern Physics A38 (2023) 2350072 [arXiv:2210.03323 [hep-th]].

S. Aoki, T. Onogi and T. Yamaoka, "Energies and a gravitational charge for massive particles in general relativity", [arXiv:2305.09849 [gr-qc]].

S. Aoki, *et al.,* work in progress.

現在進行中の内容も含む。

I. Introduction

Is energy conserved in general relativity ?

Einstein equation $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 2\kappa T_{\mu\nu}$ $\kappa = 4\pi G_N$

energy momentum tensor (EMT)

covariant conservation $\nabla_\mu(\sqrt{-g} T^\mu{}_\nu) = \partial_\mu(\sqrt{-g} T^\mu{}_\nu) + \Gamma^\mu_{\mu\alpha}(\sqrt{-g} T^\alpha{}_\nu) - \Gamma^\alpha_{\mu\nu}(\sqrt{-g} T^\mu{}_\alpha) = 0$

but what we naively need for a conserved energy in a curved spacetime is $\partial_\mu(\sqrt{-g} T^\mu{}_\nu) = 0$

2nd and 3rd terms are obstructions.

Conclusion from our previous studies

エネルギーは保存しない？

1. There exist no covariant definition of conserved energy in general relativity.

2. The energy covariantly defined in general relativity is not conserved.

S. Aoki, T. Onogi and S. Yokoyama, "Conserved charge in general relativity", Int. J. Mod. Phys. A36 (2021) 2150098.

S. Aoki, T. Onogi and S. Yokoyama, "Charge conservation, Entropy Current, and Gravitation", Int. J. Mod. Phys. A36 (2021)2150201.

S. Aoki and T. Onogi, "Conserved non-Noether charge in general relativity: Physical definition vs. Noether's 2nd theorem", Int. J. Mod. Phys. A36 (2022) 2250129,

This talk

A new conserved charge in general relativity = Gravitational charge

以下の論文の提案にあった問題点を改良し精密化した。

S. Aoki, T. Onogi and S. Yokoyama, "Charge conservation, Entropy Current, and Gravitation", Int. J. Mod. Phys. A36 (2021)2150201.

S. Aoki and T. Onogi, "Conserved non-Noether charge in general relativity: Physical definition vs. Noether's 2nd theorem", Int. J. Mod. Phys. A36 (2022) 2250129,

S. Aoki, "Noether's 1st theorem with local symmetries", PTEP 2023 (2023)1, 013B03.

1. Present a more precise definition of "gravitational charge" . Sec. II
2. Give a physical interpretation: "gravitational charge" = entropy Sec. II
3. Calculate the gravitational charge in a system of massive particles. Sec. III

   S. Aoki, T. Onogi and T. Yamaoka, "Energies and a gravitational charge for massive particles in general relativity", [arXiv:2305.09849 [gr-qc]].

4. Calculate entropy in expanding Universe with constant equation of state (EOS) Sec. IV

S. Aoki and K. Kawana, "Entropy and its conservation in expanding Universe", Int. J. Mod. Phys. A38 (2023) 2350072.

II. Gravitational charge

A new conserved charge in general relativity

重力系の新しい保存量の提案

1. Proposal for a gravitational charge

Let us consider the following decomposition of EMT.

$$T_{\mu\nu} = \rho n_\mu n_\nu + P_{\mu\nu}, \quad P_{\mu\nu} n^\nu = n^\mu P_{\mu\nu} = 0 \qquad \rho\text{: energy density} \qquad P_{\mu\nu}\text{ : pressure tensor}$$

$$n^\mu(x^0, x^i)\text{: a time-like unit vector} \qquad n^\mu n_\mu = -1$$

c.f. This decomposition is not generic but standard for massive classical matters.

Construct a conserved current from EMT as $S^\mu(x) = T^\mu{}_\nu(x)\beta(x)n^\nu(x) = -\rho(x)\beta(x)n^\mu(x)$

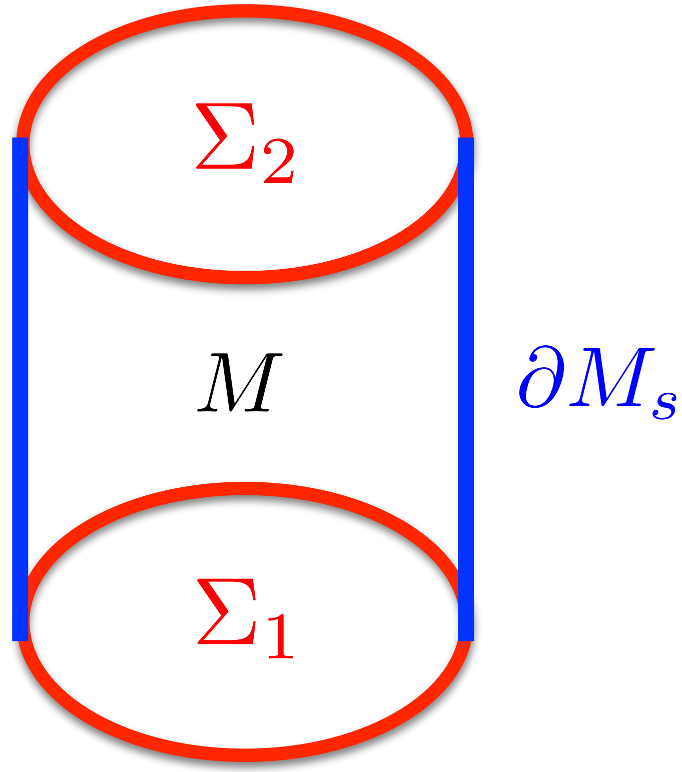
where a scalar function β should satisfy $\nabla_\mu S^\mu = T^\mu{}_\nu \nabla_\mu(\beta n^\nu) = 0$

This definition is coordinate independent.

エネルギー運動量テンソルを用いて方向 n^μ を決めている。

未知関数 β の決定が必要

$$0 = \int_M d^d x \sqrt{-g} \nabla_\mu S^\mu = \int_M d^d x\, \partial_\mu \sqrt{-g} S^\mu = \int_{\partial M} d\Sigma_\mu S^\mu$$



$$\partial M = \partial M_s \oplus \Sigma_2 \oplus \Sigma_1$$

If $d\Sigma_\mu S^\mu = 0$ on ∂M_s ⟶ $S(\Sigma_2) = S(\Sigma_1)$

保存則から保存量を構成

$$S(\Sigma) := \int_\Sigma d\Sigma_\mu S^\mu$$

A conserved gravitational charge

Σ: spacelike-hyper surface

この定義はどのような座標系でも成り立つ。

2. Determination of β

β の決定法

A solution to a linear partial differential equation $\nabla_\mu S^\mu = T^\mu{}_\nu \nabla_\mu (\beta n^\nu) = 0$

generally exists, as shown below.

$$\nabla_\mu S^\mu = -\nabla_\mu(\rho\beta n^\mu) = -n^\mu \partial_\mu(\rho\beta) - \rho\beta K = 0 \qquad K := K^\mu{}_\mu,$$

$$K^\nu{}_\mu := \nabla_\mu n^\nu$$ extrinsic curvature of a hyper-surface normal to n^μ if it exists.

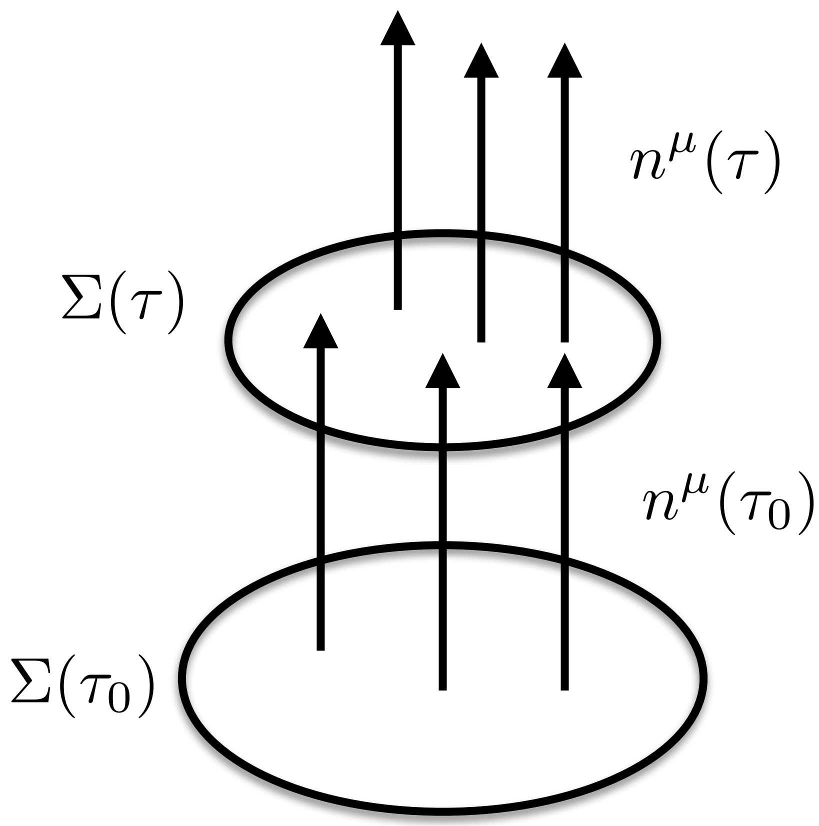
Introduce a parameter τ such that $n^\mu(x(\tau)) := \dfrac{dx^\mu(\tau)}{d\tau}$ ⟶ $n^\mu \partial_\mu = \dfrac{dx^\mu}{d\tau}\dfrac{\partial}{\partial x^\mu} = \dfrac{d}{d\tau}$

⟶ $$\frac{d}{d\tau}\rho[x(\tau)]\beta[x(\tau)] = -\rho[x(\tau)]\beta[x(\tau)]K[x(\tau)]$$ 偏微分方程式を常微分方程式に

⟶ explicit solution on each $x^\mu(\tau)$

$$\rho[x(\tau)]\beta[x(\tau)] = \rho[x(\tau_0)]\beta[x(\tau_0)] \exp\left[-\int_{\tau_0}^{\tau} d\eta\, K[x(\eta)]\right]$$

initial condition of β : $\rho[x(\tau_0)]\beta[x(\tau_0)] = \text{constant}$ この取り方で良いか？



$$\beta(\tau) = \beta(\tau_0)\frac{\rho(\tau_0)}{\rho(\tau)} \exp\left[-\int_{\tau_0}^{\tau} d\eta\, K(\eta)\right]$$

$\beta(\tau)$ is given along a each $n^\mu(x(\tau))$ as a function of τ.

β は軌道毎に決まる

A solution has been known as a Kodama vector for a spherically symmetric system.

Kodama'80

A conserved gravitational charge $$S(\Sigma) = -\int_\Sigma d\Sigma_\mu n^\mu(x)\rho(x)\beta(x)$$

where $$\rho[x(\tau)]\beta[x(\tau)] = C \exp\left[-\int_{\tau_0}^{\tau} d\eta\, K[x(\eta)]\right]$$

What is a physical meaning of the gravitational charge ?

この保存量の物理的意味は何か？

3. Gravitational charge = entropy

Our interpretation: gravitational charge = entropy 保存量はエントロピー？

一般相対論の時間発展は断熱的かつ準静的

fundamental relation of thermodynamics

$$S(U, V, \{N_i\}) = UG\left(\frac{U}{V}, \left\{\frac{N_i}{V}\right\}\right)$$

清水明「熱力学の基礎」

internal energy U volume V other extensive parameters N_i $(i = 1, 2, \cdots)$

baryon number, charge, etc.

S is concave in each argument 上に凸

We will check whether this interpretation is reasonable.

いくつかの例にこの解釈を適用してみる。

III. Free massive particles with gravitational interaction

質量を持った自由粒子、重力相互作用のみ

S. Aoki, T. Onogi and T. Yamaoka, “Energies and a gravitational charge for massive particles in general relativity”, [arXiv:2305.09849 [gr-qc]].

EMT for free massive particles with gravitational interaction

$$T^{\mu\nu}(x) = \frac{1}{\sqrt{-g(x)}} \sum_{n=1}^{N} m_n \int ds_n \, v_n^\mu(s_n) v_n^\nu(s_n) \delta^{(4)}(x - x_n(s_n)) \qquad v_n^\mu(s_n) := \frac{dx_n^\mu(s_n)}{ds_n},$$

$$g_{\mu\nu} v_n^\mu v_n^\nu = -1$$

velocity

at $x \simeq x_n(s_n)$

$$\beta(x) n^\mu(x) \simeq \left[\beta_n(s_n) + \gamma_{n,\nu}(s_n)(x^\nu - x_n^\nu(s_n))\right] v_n^\mu(s_n) + O\left((x - x_n)^2\right)$$

v^μ が n^μ に対応

conserved current density

$$\sqrt{-g} S^\mu = - \sum_{n=1}^{N} m_n \int ds_n \, \beta(s_n) v_n^\mu(s_n) \delta^{(4)}(x - x_n(s_m))$$

$$0 = \partial_\mu(\sqrt{-g} S^\mu) = \sum_{n=1}^{N} m_n \int ds_n \, \beta(s_n) \frac{d}{ds_n} \delta^{(4)}(x - x_n(s_n))$$

$$\longrightarrow \quad \frac{d\beta_n(s_n)}{ds_n} = 0 \quad \longrightarrow \quad \beta_n(x^0) = -\beta_n^0$$

β が各粒子ごとに決まる

conserved charge

$$S := \int d^{d-1}x \sqrt{-g} S^0 = \sum_n m_n \beta_n^0 \int ds_n v_n^0(s_n) \delta(x^0 - x_n^0(s_n)) = \sum_n m_n \beta_n^0$$

initial condition $\beta_n^0 m_n = \text{constant} = 1$ $\rho[x(\tau_0)]\beta[x(\tau_0)] = \text{constant}$

⟶ $S = N$ a number of particles

S counts a total "gravitational charge"

c.f. total electric charge

この結果は自明に見えるが、ニュートン定数のall orderで成り立つ。

電荷と同様に「単位重力荷」g があれば、全質量 $M = gN$

IV. Entropy in expanding Universe

S. Aoki and K. Kawana, "Entropy and its conservation in expanding Universe",
International Journal of Modern Physics A38 (2023) 2350072 [arXiv:2210.03323 [hep-th]].

1. Homogeneous and isotropic expansing Universe

$$ds^2 = -(dx^0)^2 + a^2(x^0)\tilde{g}_{ij}dx^i dx^j$$ Freedman-Lemaitre-Robertson-Walker metric

EMT (perfect fluid) $T^0{}_0 = -\rho(x^0),\ T^i{}_j = P(x^0)\delta^i_j,\ T^0{}_j = T^i{}_0 = 0$

covariant conservation $\nabla_\mu T^\mu{}_\nu = 0$ $\longrightarrow$ $\dot{\rho} + (d-1)(\rho + P)\dfrac{\dot{a}}{a} = 0$

energy $E(x^0) := -\displaystyle\int d^{d-1}x\,\sqrt{-g}\,T^0{}_0 = V_{d-1}a^{d-1}\rho, \quad V_{d-1} := \int d^{d-1}x\,\sqrt{\tilde{g}}.$

$\longrightarrow$ $\dfrac{\dot{E}}{E} = -(d-1)\dfrac{\dot{a}}{a}\dfrac{P}{\rho} \neq 0$ not conserved 確かにエネルギーは保存しない

condition $T^\mu{}_\nu \nabla_\mu(-\beta(x^0)\delta^\mu_0) = 0$ $\longrightarrow$ $\rho\dot{\beta} - (d-1)\beta\dfrac{\dot{a}}{a}P = 0$

charge $S(x^0) := -\displaystyle\int d^{d-1}x\,\sqrt{-g}\,T^0{}_0\beta = V_{d-1}a^{d-1}\rho\beta = E(x^0)\beta(x^0)$

$\longrightarrow$ $\dfrac{\dot{S}}{S} = \dfrac{\dot{E}}{E} + \dfrac{\dot{\beta}}{\beta} = -(d-1)\dfrac{\dot{a}}{a}\dfrac{P}{\rho} + (d-1)\dfrac{\dot{a}}{a}\dfrac{P}{\rho} = 0$ conserved !

重力荷は保存

2. Constant equation of motion (EOS)

空間は平坦とする

Einstein equation

$$\frac{(d-2)(d-1)}{2}H^2 = 2\kappa\rho \qquad (d-2)\left[\dot{H} + \frac{d-1}{2}H^2\right] = -2\kappa P \qquad H := \frac{\dot{a}}{a}$$

constant EOS $P(x^0) = w\rho(x^0)$

$$a(x^0) = (1 + C_0 H_0 x^0)^{1/C_0} \qquad a(x^0 = 0) = 1 \qquad C_0 := \frac{(d-1)(1+w)}{2}$$

$$\rho(x^0) = \rho_0 \left(\frac{1}{a(x^0)}\right)^{(d-1)(1+w)} \qquad \beta(x^0) = \beta_0 a^{(d-1)w}(x^0)$$

β が時間の関数として決まる

internal energy $U(x^0) = V_{d-1}\dfrac{\rho_0}{a^{(d-1)w}(x^0)}$ volume $V(x^0) = V_{d-1}a^{(d-1)}(x^0)$

entropy $S(x^0) = V_{d-1}\rho_0\beta_0$ エントロピーが保存

エントロピーの形　$S(U,V,N)$

1次同次式　$S(\lambda U, \lambda V, \lambda N) = \lambda S(U,V,N)$

各変数に関して上に凸

2階微分がnegative semi-definite

以下の項に正の係数かけたものの線形結合

$$U^{a_1} V^{a_2} N^{a_3}, \quad \sum_i a_i = 1,\ 0 \le a_i \le 1$$

$$-U \log \frac{U}{V},\ -U \log \frac{U}{N}$$

$$-V \log \frac{V}{U},\ -V \log \frac{V}{N}$$

$$-N \log \frac{N}{U},\ -N \log \frac{N}{V}$$

2-1. Radiation

輻射優勢期

$$P(x^0) = \frac{1}{d-1}\rho(x^0) \longrightarrow a(x^0) = \left(1 + \frac{d}{2}H_0 x^0\right)^{1/d}$$

$$\longrightarrow \rho(x^0) = \frac{\rho_0}{a^d(x^0)}, \quad \beta(x^0) = \beta_0 a(x^0), \quad \sqrt{-g} = a^{d-1}(x^0)\sqrt{\tilde{g}}$$

$$\longrightarrow U = V_{d-1}\frac{\rho_0}{a(x^0)}, \quad V = V_{d-1}a^{d-1}(x^0), \quad S = V_{d-1}\rho_0\beta_0$$

fundamental relation for radiation $S = UG(U/V) = cU\left(\frac{U}{V}\right)^{\alpha}$ $\because$ 輻射なので。 no N_i

これが重要

$S \sim 1$ $U \sim a^{-1}(x^0), \quad U/V \sim a^{-d}(x^0) \longrightarrow \alpha = -\frac{1}{d}$ 冪が決まる。

$$\longrightarrow S = cV_{d-1}\rho_0^{\frac{d-1}{d}} \longrightarrow c = \rho_0^{\frac{1}{d}}\beta_0$$

$$\longrightarrow S(U,V) = \beta_0\rho_0^{\frac{1}{d}}U^{1-\frac{1}{d}}V^{\frac{1}{d}}$$

fundamental relation is determined.

concave conditions are satisfied.

various thermodynamic quantities

(Inverse) temperature $$\frac{1}{T(x^0)} := \frac{\partial S}{\partial U} = \rho_0^{\frac{1}{d}} \beta_0 \frac{d-1}{d} \left(\frac{V}{U}\right)^{\frac{1}{d}} = \frac{d-1}{d}\beta_0 a(x^0) = \frac{d-1}{d}\beta(x^0)$$

Pressure $$\frac{P}{T} := \frac{\partial S}{\partial V} = \rho_0^{\frac{1}{d}} \beta_0 \frac{1}{d} \left(\frac{U}{V}\right)^{1-\frac{1}{d}} = \frac{1}{d-1}\frac{\rho}{T} \longrightarrow \boxed{P(x^0) = \frac{1}{d-1}\rho(x^0)}$$

consistency

Entropy density $$s := \frac{S}{V} = \rho_0^{\frac{1}{d}} \beta_0 \left(\frac{U}{V}\right)^{1-\frac{1}{d}} = \frac{d}{d-1}\frac{\rho}{T} = \frac{\rho + P}{T}$$ thermodynamic relation

Stefan-Boltzmann $$\rho(x^0) = \rho_0 \left(\frac{d-1}{d}\beta_0 T(x^0)\right)^d = \sigma_d T^d(x^0) \qquad \sigma_d := \rho_0 \left(\frac{d-1}{d}\beta_0\right)^d$$

various thermodynamic quantities are correctly reproduced from the fundamental relation

$$S(U,V) = \beta_0 \rho_0^{\frac{1}{d}} U^{1-\frac{1}{d}} V^{\frac{1}{d}}$$

輻射の熱力学的性質が再現された

2-2. Dark energy (Inflation)

$$P(x^0) = -\rho(x^0) \longrightarrow \underline{a(x^0) = e^{H_0 x^0}}, \quad \rho(x^0) = \rho_0, \quad \beta(x^0) = \frac{\beta_0}{a^{d-1}(x^0)}$$

exponential expansion = inflation

The metric is equivalent to (static) de Sitter spacetime

$$ds^2 = -(dx^0)^2 + e^{2H_I t}(dR^2 + R^2 d\Omega_{d-2}^2)$$

Hubble constant $H_I := H_0$

$$x^0 = t + \frac{1}{2H_I}\log(1 - H_I^2 r^2), \quad R = \frac{r e^{-H_I t}}{\sqrt{1 - H_I^2 r^2}}.$$

$$ds^2 = -(1 - H_I^2 r^2)dt^2 + \frac{dr^2}{1 - H_I^2 r^2} + r^2 d\Omega_{d-2}^2$$

cosmological constant $\Lambda = \frac{(d-1)(d-2)}{2} H_I^2 := \frac{(d-1)(d-2)}{2R_H^2}$

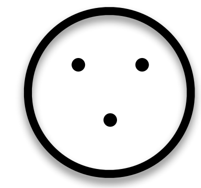
$R_H = \frac{1}{H_I}$ radius of de Sitter horizon =radius of Hubble horizon

unform matter with $w = -1$ (dark energy) $\longleftrightarrow$ de Sitter spacetime

$$P(x^0) = -\rho(x^0) \longrightarrow U = V_{d-1}\rho_0 a^{d-1}(x^0), \quad V = V_{d-1}a^{d-1}(x^0), \quad S = V_{d-1}\rho_0\beta_0$$

fundamental relation

$$S(U,V,N) = UG(U/V, N/V) = c\,U\left(\frac{N}{V}\right)^{\beta}\left(\frac{U}{V}\right)^{\alpha}$$



$$\frac{U}{V} = \rho_0$$

$$\longrightarrow \frac{1}{T} = c\left(\frac{N}{V}\right)^{\beta}(\alpha+1)\left(\frac{U}{V}\right)^{\alpha}, \qquad \frac{P}{T} = -c\,\rho\left(\frac{N}{V}\right)^{\beta}(\alpha+\beta)\left(\frac{U}{V}\right)^{\alpha},$$

consistency $P = -\rho \longrightarrow \beta = 1$

$$N := V_{d-1}n_0 a^{\gamma}(x^0) \longrightarrow S \sim a^{d-1}(x^0)\frac{a^{\gamma}(x^0)}{d^{d-1}(x^0)} \sim 1 \longrightarrow \gamma = 0$$

concave condition $\longrightarrow \alpha = -1$

$$S = c\,N = c\,V_{d-1}n_0$$

fundamental relation

$$\longrightarrow \rho_0\beta_0 = c\,n_0$$

various thermodynamic quantities

(inverse) temperature $$\frac{1}{T(x^0)} = 0$$

entropy density $$s = \rho_0 \beta(x^0)$$

ダークエネルギーは熱力学的には奇妙

chemical potential $$\frac{\mu}{T} := -\frac{\partial S}{\partial N} = -c \quad \longrightarrow \quad \mu = -cT \to -\infty$$

dark energy $$P(x^0) = -\rho(x^0)$$

V. Conclusion

The conserved charge in general relativity (gravitational charge) = entropy

if $T^{\mu}{}_{\nu}$ is given in general relativity

$\longrightarrow$ the fundamental relation of thermodynamics (S) is obtained.

Gravity (general relativity) is a machinery to provide a fundamental relation of thermodynamics for matters if the system is in equilibrium.

エントロピーの基本関係式は物質の固有な性質である。重力と結合させるとその時間は点が一定であるということから基本関係式を決めることができる。関係式は重力がない状況にも適用できる。

If the system is out of equilibrium, this generalizes a definition of entropy to non-equilibrium cases.

A conservation of entropy indicates that the dynamics of GR is adiabatic and quasi-static.

Inversely, the entropy is the source of the gravitational interaction in general relativity.

We will collect more examples in future studies.

Thank you !