Gravitational Positivity Bounds on Dark Gauge Bosons

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arXiv: 2305.10058
[hep-th]

Introduction

- Unitarity of scattering amplitudes impose strong constraints on gravitational theories (**Gravitational Positivity Bounds**)

- Application of gravitational positivity bounds to **gauge boson models**

- We find that gravitational positivity bounds put constraints on **gauge coupling** and **gauge boson mass**

Outline

- Formulation of positivity bound
- Application to U(1) gauge boson
  - Bounds on Abelian Higgs model
  - Bounds on Stueckelberg model

Positivity Bound w/o gravity

Effective field Theory (EFT)

- Many phenomenological models are **Effective Field Theories(EFTs)**: Effective descriptions @ low energy scale

UV theory: ・contains full spectrum
・valid at all scale

(e.g. massless scalar ϕ
+ massive particles)

Low energy description

EFT: ・described by light particles
・valid only at low energy scale

(e.g. massless scalar EFT
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{a_1}{\Lambda^4}(\partial\phi)^4 + \cdots$)

- EFT parameters are accessible experimentally (bottom-up viewpoint)
- EFT parameters reflect information of UV theory (top-down viewpoint)

Effective field Theory (EFT)

- Many phenomenological models are **Effective Field Theories(EFTs)**: Effective descriptions @ low energy scale

UV theory: ・contains full spectrum
・valid at all scale

(e.g. massless scalar ϕ
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Low energy description

(e.g. massless scalar EFT

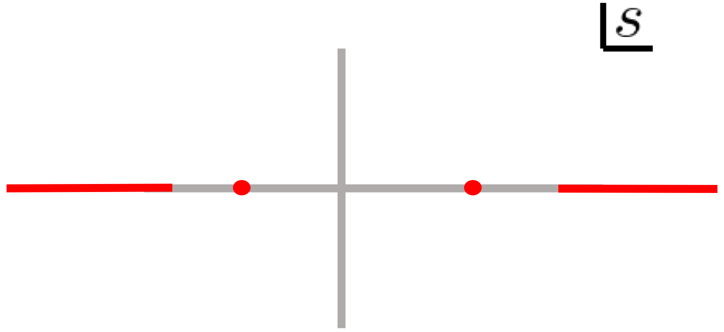
$+ \cdots)$

There are **theoretical bounds on EFT parameters**
which reflect general property (unitarity, causality, …) of UV theories
= **Positivity bounds**!

- **EFT parameters reflect information of UV theory** (top-down viewpoint)

Analytic structure of scattering amplitude

- Structure of 2→2 scattering amplitude $\mathcal{M}(s,t)$ followed from general property of the theory is essential for the derivation of positivity bounds
- Consider analytic structure of $\mathcal{M}(s,t)$ in complex s-plane
  - **Causality** → $\mathcal{M}(s,t)$ is analytic except for real axis (analyticity) Hepp, '63
  - **Unitarity** → In the forward limit $t \to 0$, $\mathrm{Im}\mathcal{M}(s,0) > 0$ (optical theorem)

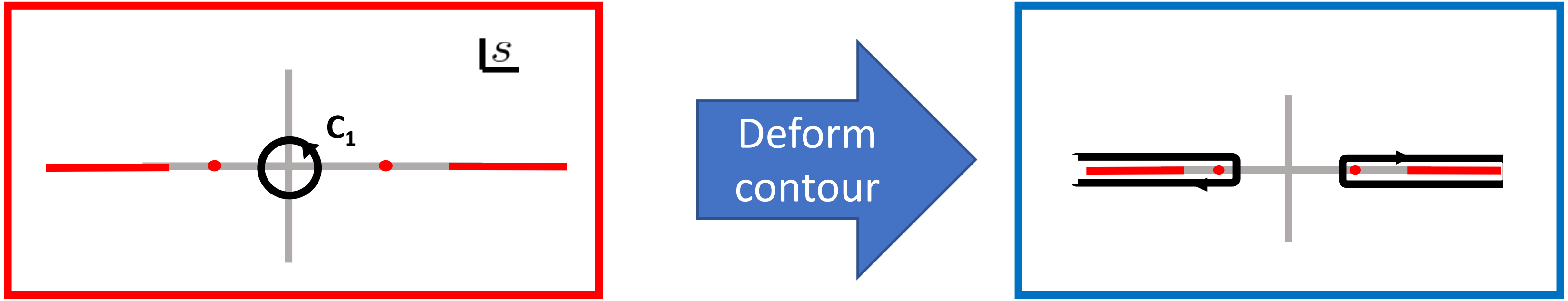


Positivity bound

- Consider low energy expansion of amplitude

$$\mathcal{M}(s,0) = c_0 + c_1 s + c_2 s^2 + \cdots$$

- c_2 is equal to the integral of $\mathrm{Im}\mathcal{M}(s,0)$ → $c_2 > 0$



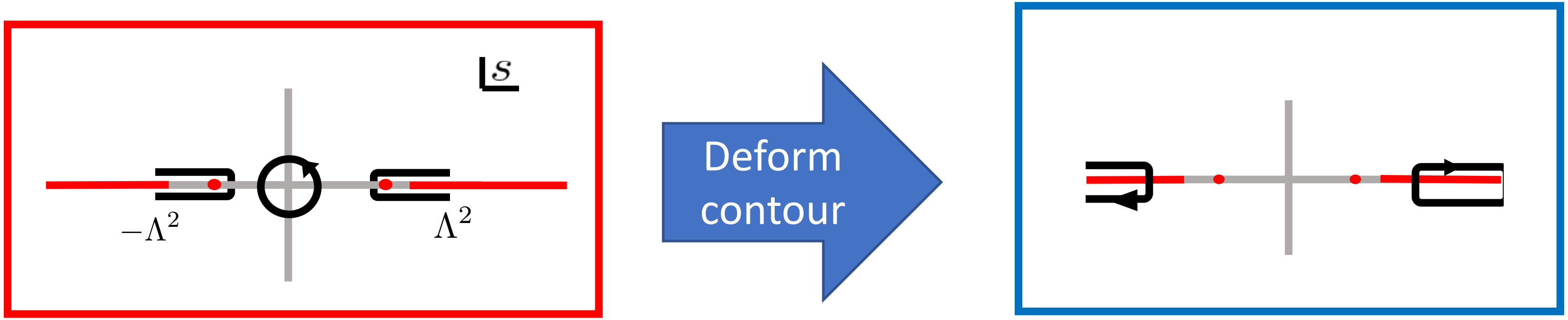
Positivity bound Adams+ '06

$$c_2 = \frac{1}{2\pi i}\oint_{\mathcal{C}_1} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \frac{2}{\pi}\int ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} > 0$$

Improved positivity bound

- If EFT is valid below Λ, integral of Im$\mathcal{M}(s,0)$ is calculable up to Λ^2

$$\underbrace{c_2 = \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3}}_{\text{Calculable}} + \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3}$$



Improved positivity bound Bellazini '16, de Rham+ '17

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3} > 0$$

Improved positivity bound

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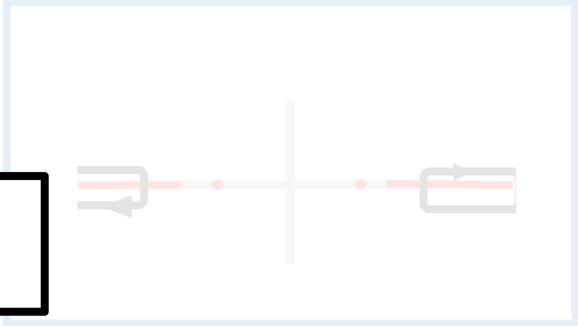
Deform



- Bound is more stringent
- Dependence on Λ ($\sim$ Limits of validity of EFT)

Improved positivity bound Bellazini '16, de Rham+ '17

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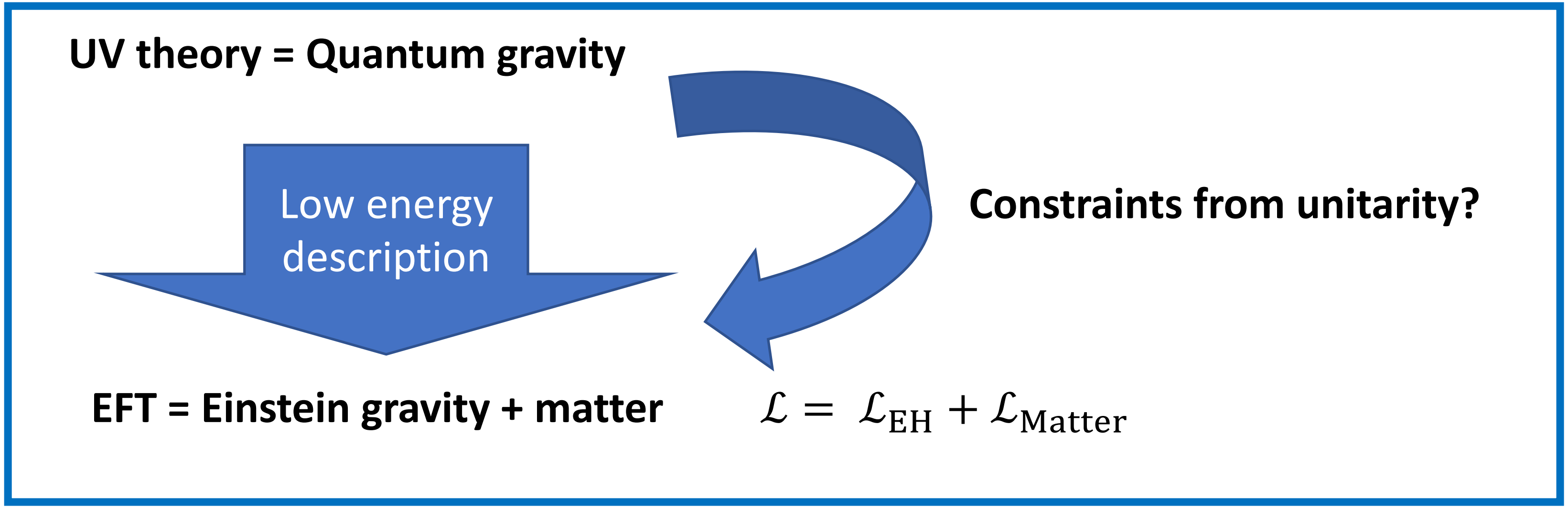
Gravitational positivity bound

Gravitational theory as EFT

- Einstein gravity is not UV complete
  → It is EFT = low energy description of quantum gravity
- What is positivity bound on gravitational theory which reflects unitarity of quantum gravity?



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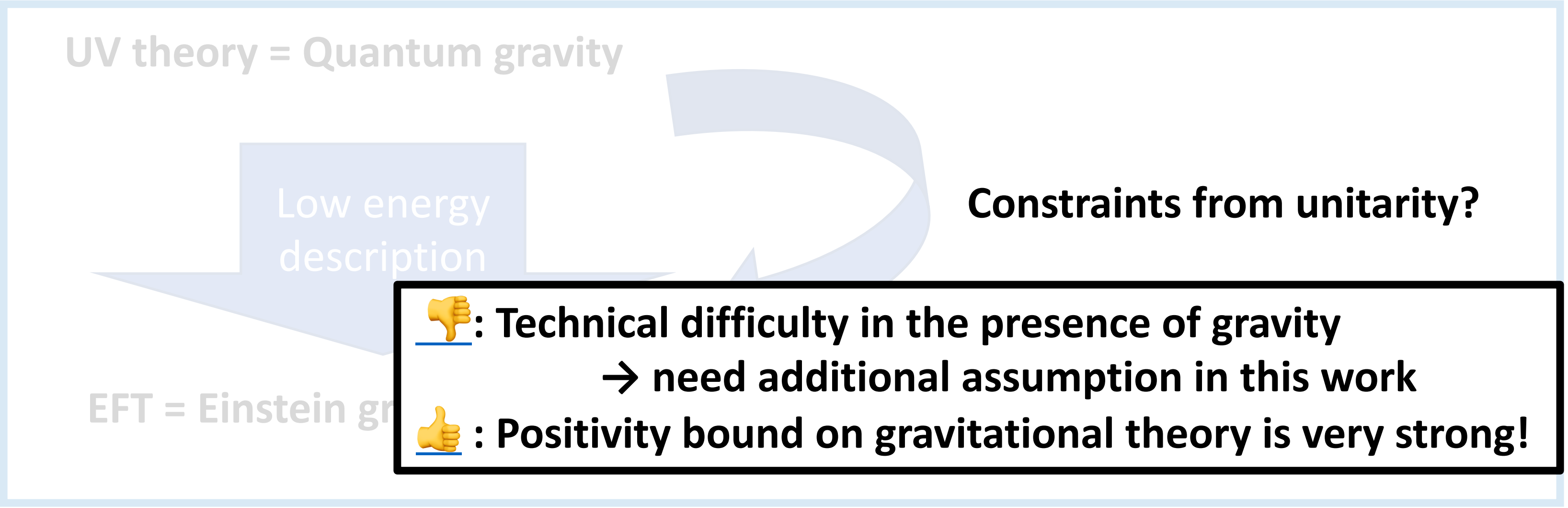


👎: **Technical difficulty in the presence of gravity**
→ need additional assumption in this work

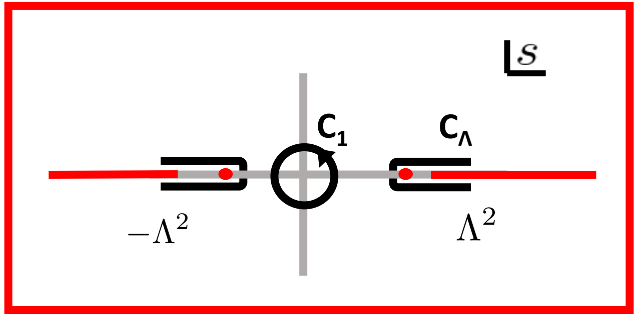
👍 : **Positivity bound on gravitational theory is very strong!**

Technical problem with gravity

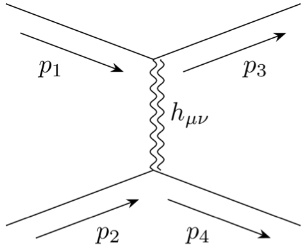
- Positivity bound w/ gravity: non-trivial consistency condition with quantum gravity (relation with swampland program)
- Technical problem due to massless spin-2 particle i.e. graviton:

  <u>Divergence in the forward limit</u>

$$\frac{1}{2\pi i}\oint_{\mathcal{C}_1+\mathcal{C}_\Lambda} ds' \frac{\mathcal{M}(s',0)}{s'^3} = \lim_{t\to 0}\left(-\frac{1}{M_{\rm Pl}^2 t} + B^{(2)}(\Lambda)\right) = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\,\mathcal{M}(s',0)}{s'^3}$$



t-channel graviton exchange → diverge in forward limit



$$\sim -\frac{s^2}{M_{\rm Pl}^2 t} \xrightarrow[t\to 0]{} \infty$$

Gravitational positivity bound

- **Additional assumptions on high energy behavior** of scattering amplitude to remove the divergence in the forward limit

Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)} \quad \text{for } s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

- Positivity bound holds approximately:

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\left(s',0\right)}{s^3} \gtrsim 0$$

Gravitational positivity bound

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Regge behavior at the high energy
Cancel out the divergent term

$$B^{(2)}(\Lambda) > \underline{\frac{1}{M_{\rm Pl}^2}\left[\frac{f'}{f} + j' \ln\left(\frac{\alpha' M_*^2}{4}\right) - \frac{j''}{j'}\right]}$$

The remaining term

Gravitational positivity bound

Tokuda, Aoki, Hirano '20

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Gravitational positivity bound

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Assumption(1) $\mathrm{Im}\,\mathcal{M}(s,t) \sim f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)}$ for $s > M_*^2$

Regge behavior at the high energy
Cancel out the divergent term

Assumption(2) $\left|\frac{f'}{f}\right|, \left|\frac{j''}{j'}\right|, |j'| \ll \frac{1}{\Lambda^2}$

The remaining term is small

- Positivity bound holds approximately:

Gravitational positivity bound

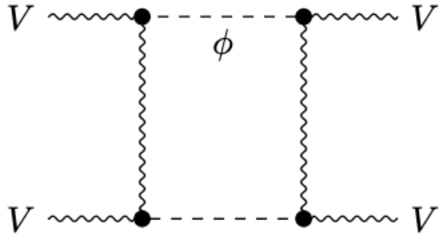
Tokuda, Aoki, Hirano '20

$$B^{(2)}(\Lambda) := c_2 - \frac{2}{\pi}\int^{\Lambda^2} ds' \frac{\mathrm{Im}\,\mathcal{M}\,(s', 0)}{s^3} \gtrsim 0$$

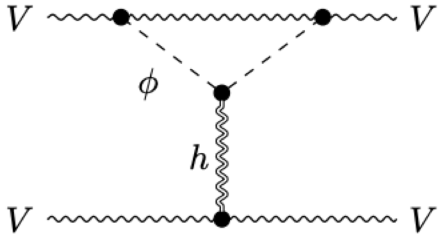
Structure of $B^{(2)}(\Lambda)$

$$B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$$

"non-gravitational part"
= no graviton exchange, positive



"gravitational part"
= graviton exchange diagram, negative



Implication of gravitational positivity bound

- Non-gravitational interaction is bounded below by gravitational interaction

→ gravity should be weak!

$$\boxed{B^{(2)}_{\text{non-grav}}(\Lambda) > \left| B^{(2)}_{\text{grav}}(\Lambda) \right|}$$

- Positivity bounds gives weak gravity conjecture-like constraints on gravitational EFT

Cheung+ '14, Hamada+ '18, Tolley+ '20

- What are implications for various models? useful for phenomenology?

→ **This work: Positivity bounds on U(1) gauge boson models**

Application to U(1) Gauge Boson

- Application to Higgs gauge theory
- Application to Stueckelberg gauge theory

Abelian Higgs model

- Lagrangian: $\mathcal{L}_{AH} = -\frac{1}{4}F^2 + |D_\mu\Phi|^2 - \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$

$$F = \partial_\mu V_\nu - \partial_\nu V_\mu \qquad D_\mu = \partial_\mu - i g_\Phi V_\mu$$

- Three independent parameters: $m_V\ , m_\Phi\ , g_\Phi$

$$\rightarrow \quad v = \frac{m_V}{\sqrt{2} g_\Phi} \qquad \lambda = \frac{2 g_\Phi^2 m_\Phi^2}{m_V^2}$$

- We consider the simple model: Abelian Higgs + gravity

$$S = S_{\text{AH}} + S_{\text{Einstein–Hilbert}}$$

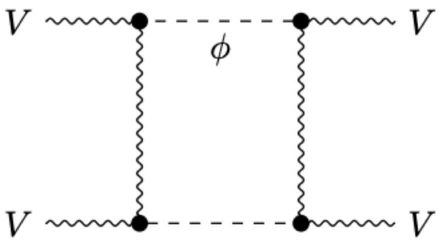
Application of positivity

- Calculate scattering of gauge bosons @ 1-loop
- Gauge bosons have Transverse mode & Longitudinal mode

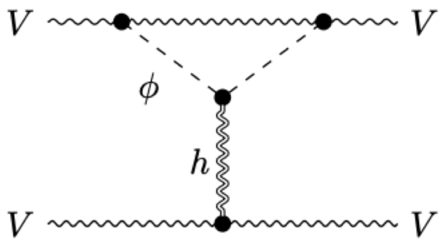
  → Three helicity amplitudes: TT , TL , LL

- Calculate $B^{(2)}(\Lambda) = B^{(2)}_{\text{non-grav}}(\Lambda) - \left| B^{(2)}_{\text{grav}}(\Lambda) \right|$

non-gravitational part



gravitational part



Bounds on Abelian Higgs model

TT $$\frac{g_\Phi^4}{4\pi^2\Lambda^4} - \frac{g_\Phi^2}{72\pi^2 M_{Pl}^2 m_\Phi^2} > 0 \quad \Rightarrow \quad m_\Phi > \frac{\Lambda^2}{3\sqrt{2} g_\Phi M_{\mathrm{Pl}}}$$

Lower bound on Higgs mass

TL $$\frac{g_\Phi^4}{2\pi^2\Lambda^4} - \frac{g_\Phi^2}{144\pi^2 M_{Pl}^2 m_V^2} > 0 \quad \Rightarrow \quad m_V > \frac{\Lambda^2}{6\sqrt{2} g_\Phi M_{\mathrm{Pl}}}$$

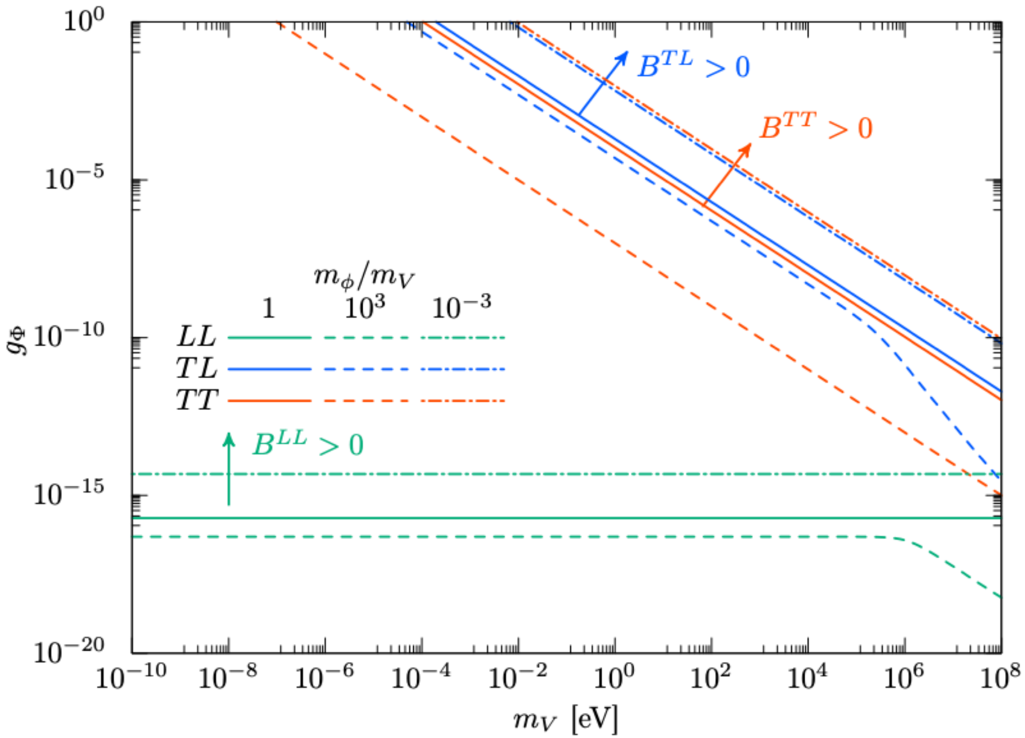
Lower bound on gauge boson mass

LL $$\underbrace{\frac{g_\Phi^4}{\pi^2\Lambda^2 m_V^2}}_{\text{non-gravitational}} - \underbrace{\frac{g_\Phi^2}{72\pi^2 M_{Pl}^2 m_V^2}}_{\text{gravitational}} > 0 \quad \Rightarrow \quad g_\Phi > \frac{\Lambda}{6\sqrt{2} M_{\mathrm{Pl}}}$$

Lower bound on gauge coupling

Bounds on Abelian Higgs model

- Bounds from TL scattering are the most stringent



$\Lambda = 1\text{TeV}$

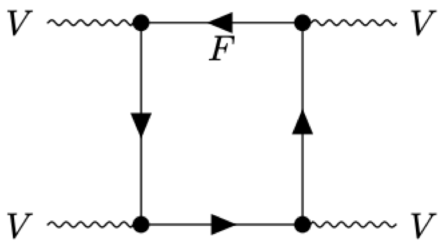
Stueckelberg + Fermion

- Lagrangian: $\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{2}m^2V^2 + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi \; - m_F\bar{\psi}\psi$

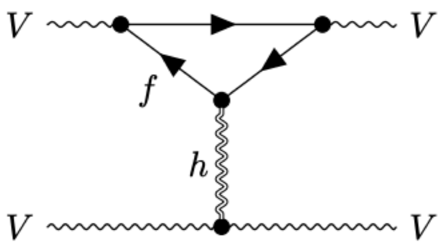
$$F = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu} \quad D_{\mu} = \partial_{\mu} - ig_F V_{\mu}$$

- Gauge boson acquires mass by Stueckelberg mechanism

non-gravitational part



gravitational part



Bounds on Stuckelberg + Fermion

TT $$\frac{g_F^4\left(2\log\frac{\Lambda^2}{m_F^2}+1\right)}{4\pi^2\Lambda^4}-\frac{11g_F^2}{360\pi^2M_{Pl}^2m_F^2}>0 \quad \Rightarrow \quad g_F > 0.2\frac{\Lambda^2}{m_F M_{\mathrm{Pl}}\sqrt{\log(\Lambda m_F^{-1})}}$$

Lower bound on gauge coupling

TL $$\frac{4g_F^4m_V^2}{3\pi^2\Lambda^6}-\frac{11g_F^2}{720\pi^2M_{Pl}^2m_F^2}>0 \quad \Rightarrow \quad m_V > 0.1\frac{\Lambda^3}{g_F m_F M_{\mathrm{Pl}}}$$

Lower bound on gauge boson mass

LL $$\frac{g_F^4m_V^4\left(4\log\frac{\Lambda^2}{m_F^2}+7\right)}{\pi^2\Lambda^8}-\frac{g_\Phi^2m_V^2}{420\pi^2M_{Pl}^2m_F^4}>0 \quad \Rightarrow \quad m_V > 0.02\frac{\Lambda^4}{g_F m_F^2 M_{\mathrm{Pl}}\sqrt{\log(\Lambda m_F^{-1})}}$$

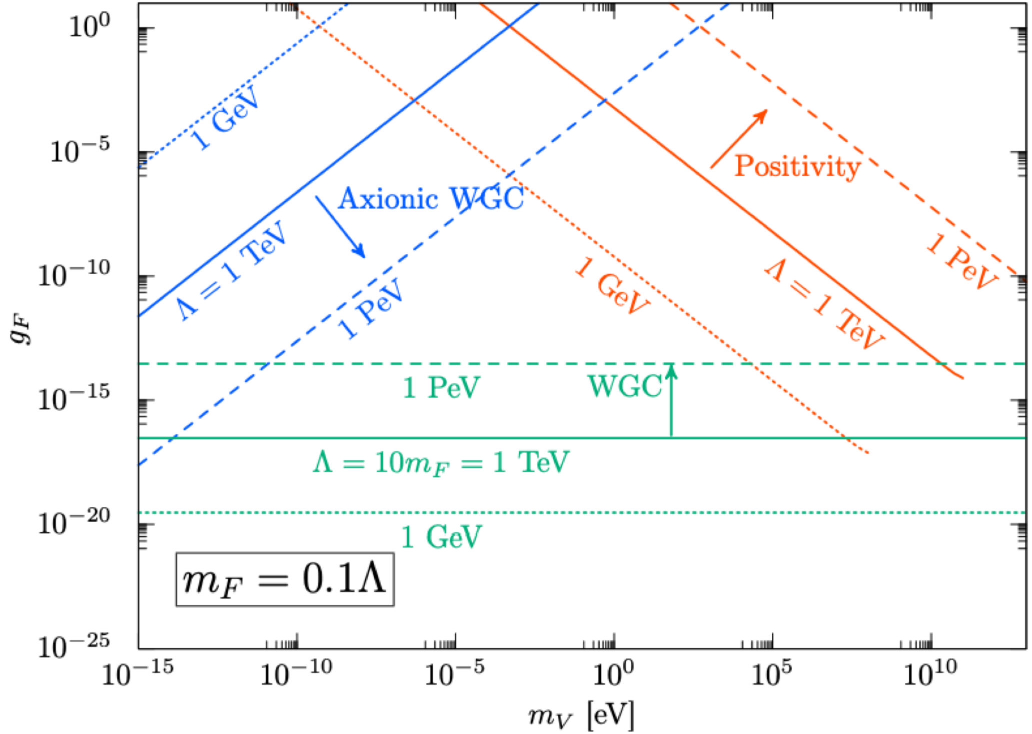
Lower bound on gauge boson mass

non-gravitational

gravitational

Comparison with other QG constraints

- Positivity bounds are stronger than other quantum gravity constraints for light vector boson



Positivity (This work, LL scattering)

$$g_F m_V > 0.02 \frac{\Lambda^4}{m_F^2 M_{\rm Pl}\sqrt{\log(\Lambda m_F^{-1})}}$$

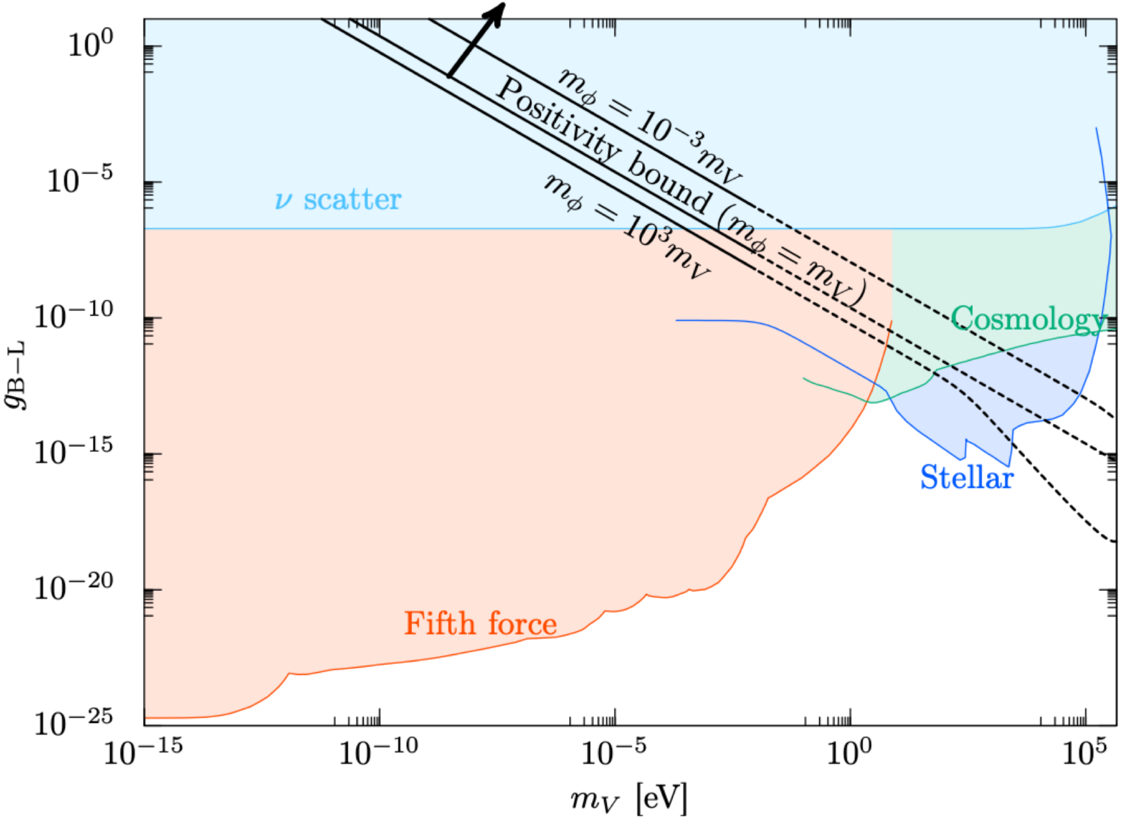
WGC

$$g_F > \frac{m_F}{M_{\rm Pl}}$$

Axionic WGC [Reece '18]

$$\frac{m_V}{g_F} > \frac{\Lambda^2}{M_{\rm Pl}}$$

Implication to $U(1)_{B-L}$ gauge boson



Positivity bound on Abelian Higgs Model (from TL scattering) gives strong constraint

Assumptions

- SM particles are neglected
- $g_{B-L} = g_\Phi$

$\Lambda = 1\mathrm{GeV}$

Summary

- Gravitational positivity bound: Unitarity of scattering amplitudes impose swampland-like constraints on gravitational theories

- Application to U(1) gauge boson

  → Lower bound on gauge coupling and gauge boson mass

- Gravitational positivity bounds Potentially put stringent constraints on phenomenological models