

# Nonlinear-Supersymmetric General Relativity Theory(NLSGR)

- Unification of space, time and matter -

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## OUTLINE

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# 1. Motivation

@ The success of **Two SMs**, i.e. **GR** and **GWS model**,  
**many unsolved fundamental problems are left**, e. g.,

- Unification of two SMs,
- Space-time dimension **four**,
- Three-generations structure of quarks and leptons,
- Tiny Neutrino mass  $M_\nu$ , proton stability in GUT
- Dark Matter, Dark Energy;  $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda\text{CDM}$ , Inflation?

$\Rightarrow$  **SUSY, SUGRA!?**,

- Origin of SUSY breaking ?, ... etc.

@ GR describes geometry of space-time.

However, **unpleasant differences** between **GR** and **SUGRA**:

- **GR**  $\Leftrightarrow$  Geometry of Riemann space-time (**Physical:**  $[x^\mu]$ ,  $\text{GL}(4, \mathbb{R})$ )
- **SUGRA**  $\Leftrightarrow$  Geometry of superspace (**Mathematical:**  $[x^\mu, \theta_\alpha]$ , sPoincaré )

$\Rightarrow$  New **SUSY** paradigm on **physical space-time!**.

@ As for the **three-generations structure** we found:

Among all  $SO(N)$  sP,  $[N = 8$  by M. Gell-Mann]

**only  $SO(10)$  sP** is unique which gives **SM with just 3 generations in one irreducible rep.**

- 10 supercharges  $Q^I, (I = 1, 2, \dots, 10)$  are decomposed as follows:

$$\underline{10}_{SO(10)} = \underline{5}_{SU(5)} + \underline{5}^*_{SU(5)}$$

$$\underline{5}_{SU(5)} = [\{\underline{3}^{*c}, \underline{1}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3}) : Q_a (a = 1, 2, 3)\}, \{\underline{1}^c, \underline{2}^{ew}, (0, -e) : Q_m (m = 4, 5)\}].$$

$\Leftrightarrow$  **Supercharge  $\underline{5}_{SU(5)}$**  has the same quantum numbers as  **$\underline{5}$**  of SU(5) GUT.

- **Massless helicity states of gravity multiplet** of  **$SO(10)$  sP** with **CPT conjugation** are specified by the helicity  $h = (2 - \frac{n}{2})$  and the dimension  $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!}$ :

$$|h \rangle = Q^n Q^{n-1} \dots Q^2 Q^1 |2 \rangle, Q^n (n = a, m, a^*, m^*): \text{supercharge}$$

$ h $	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
			$\underline{1}_{[0]}$	$\underline{10}_{[1]}$	$\underline{45}_{[2]}$	$\underline{120}_{[3]}$	$\underline{210}_{[4]}$
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{45}_{[8]}$	$\underline{120}_{[7]}$	$\underline{210}_{[6]}$	$\underline{252}_{[5]}$	$\underline{210}_{[4]}$

© **Dirac particle survivors** after *tentative* superHiggs-like mechanism

$SU(3)$	$Q_e$	$SU(2) \otimes U(1)$
<u><b>1</b></u>	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad (E)$
<u><b>3</b></u>	5/3 2/3 -1/3 -4/3	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$
<u><b>6</b></u>	4/3 1/3 -2/3	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
<u><b>8</b></u>	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

© **SM Higgs-doublet state survives** in  $h = 0$  state.

- $N > 10$  case is unphysical/ugly and **excluded**.

@ How to construct **N=10 SUSY with gravity**  
beyond well known **No-Go** theorem in **S-matrix** ?

- To circumvent the No-Go theorem degeneracy of space-time is considered.



We show in this talk:

- **N=10 SUSY with gravity** is obtained by the geometric description of General Relativity principle on **specific unstable physical (Riemann) space-time** whose **tangent space possesses global NLSUSY structure**.

## A quick review of NLSUSY:

- Take (flat) space-time specified by  $x^a$  for  $SO(1,3)$  and  $\psi_\alpha$  for  $SL(2,C)$ .
- Consider one form  $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$ ,  
 $\kappa$  is an **arbitrary** constant with the dimension  $l^{+2}$ .

- $\delta\omega^a = 0$  under  $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$  and  $\delta\psi = \zeta$  with a **global** spinor parameter  $\zeta$ .

- An invariant action ( $\sim$  invariant volume) is obtained:

$$S = -\frac{1}{2\kappa^2} \int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA},$$

$L_{VA}$  is **N=1 Volkov-Akulov model of NLSUSY** given by

$$L_{VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2} \left[ 1 - t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \dots \right],$$

$$|w_{VA}| = \det w^a{}_b = \det(\delta_b^a + t^a{}_b), \quad t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi),$$

which is invariant under N=1 NLSUSY transformation:

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_P.$$

- $\psi$  is **Nambu-Goldstone(NG) fermion** for  $\frac{\text{superPoincare}}{\text{Poincare}}$ .
- $\psi$  is quantized **canonically** in compatible with SUSY algebra.

## 2. Nonlinear-Supersymmetric General Relativity (NLSGR)

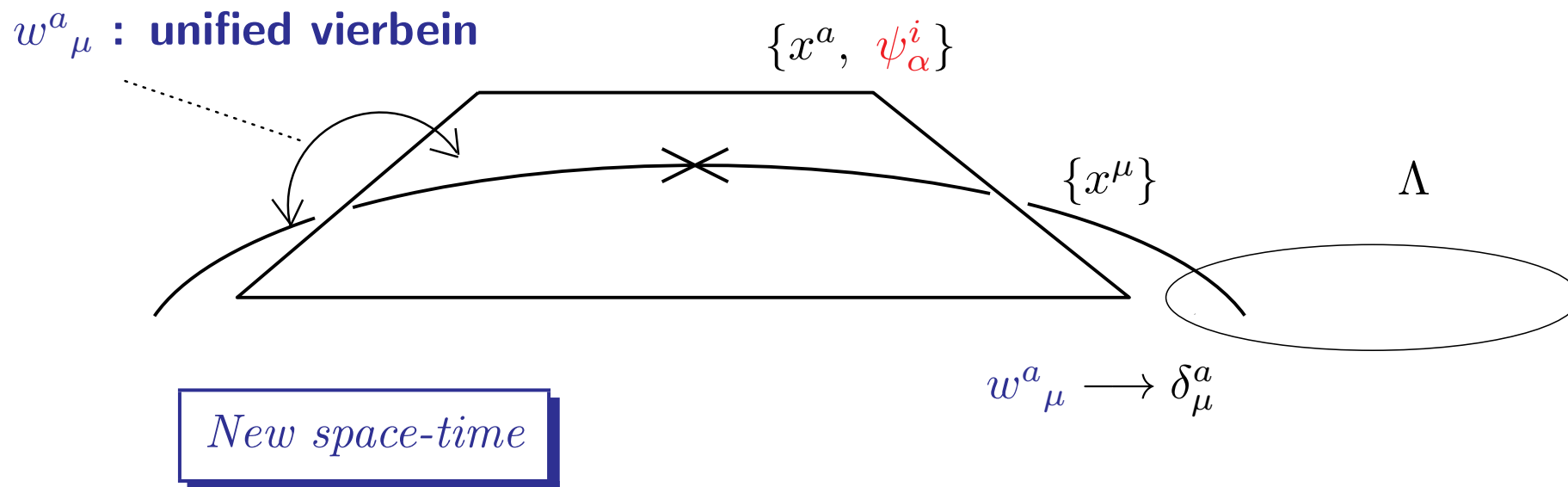
### 2.1. New Space-time as Ultimate Shape of Nature

We consider *new (unstable) physical space-time* inspired by *nonlinear(NL) SUSY*:

The tangent space of new space-time is specified by **Grassmann coordinates**  $\psi_\alpha$  for  $SL(2,C)$  besides the ordinary Minkowski coordinates  $x^a$  for  $SO(1,3)$ ,

i.e., the coordinate  $\psi_\alpha$  of the coset space  $\frac{superGL(4,R)}{GL(4,R)}$  turning to the **NLSUSY NG fermion** (called *superon* hereafter) are attached at every curved space-time point besides  $x^a$ .

- New unstable space(U space):



(Non-compact groups  $SO(1,3)$  and  $SL(2,C)$  for space-time symmetry are analogous to compact groups  $SO(3)$  and  $SU(2)$  for gauge symmetry of 't Hooft-Polyakov monopole.)

- We will see that  $SO(1,3) \cong SL(2,C)$  is crucial for NLSGR scenario.

**4 dimensional space-time is singled out.**



## 2.2. Nonlinear-Supersymmetric General Relativity (NLSGR)

The geometrical arguments of Einstein general relativity (GR) can be extended to **new (unstable) space-time**.

- Unified vierbein  $w^a{}_\mu(x)$  of new space-time:

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi) \equiv w^a{}_b e^b{}_\mu$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a \equiv e^\mu{}_b w^b{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, \dots, N)$$

(Note that Grassmann d.o.f. induces the imaginary part of  $w^a{}_\mu(x)$  .)

- **$N$ -extended NLSGR action of Einstein-Hilbert (EH)-type for new space-time.  $\implies$**

## $N$ -extended NLSGR action:

$$L_{\text{NLSGR}}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w) + \Lambda\}, \quad (1)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad (2)$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N) \quad (3)$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$  : the unified vierbein of new space-time (*ulvierbein*)
- $e^a{}_\mu(x)$  : the ordinary vierbein for the local  $\text{SO}(1,3)$  d.o.f. of GR,
- $t^a{}_\mu(\psi(x))$  : the mimic vierbein for the local  $\text{SL}(2, \mathbb{C})$  d.o.f. composed of the stress-energy-momentum of NG fermion  $\psi(x)^I$  (called *superons*),
- $\Omega(w)$  : Ricci scalar curvature of new space-time computed in terms of  $w^a{}_\mu$ ,
- $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$ ,  $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) \eta^{ab} w^\nu{}_b(x)$ : metric tensors of new space-time.
- $G$  : the Newton gravitational constant.
- $\Lambda > 0$  : cosmological constant required by the **correct** NLSUSY structure of tangent space.

- **NLSGR scenario fixes the arbitrary constant  $\kappa^2$  to**

$$\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G}\right)^{-1},$$

with the dimension  $(length)^4 \sim (energy)^{-4}$ .

- $\Lambda > 0$  in the action  $L_{\text{NLSGR}}$  allows **negative dark energy density interpretation of  $\frac{\Lambda}{G}$  in the Einstein equation.** → Sec.4.

- **No-go theorem for  $N > 8$  SUSY has been circumvented by the global NLSUSY, i.e. by the vacuum(flat space) degeneracy.**

- **Note that  $SO(1, D - 1) \cong SL(d, C)$ , i.e.  $\frac{D(D-1)}{2} = 2(d^2 - 1)$  holds for **only  $D = 4, d = 2$ .****

**NLSGR scenario predicts 4 dimensional space-time.**

## 2.3. Symmetries of NLSGR(N-extended action)

- **Space-time symmetries** ( $\sim sP$ ):

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}] \quad (4)$$

- **Internal symmetries** for N-extended NLSUSY GR (N-superons  $\psi^I$  ( $I = 1, 2, \dots, N$ )):

$$[\text{global SO}(N)] \otimes [\text{local U}(1)^N] \otimes [\text{chiral}]. \quad (5)$$

For example:

- Invariance under the new NLSUSY transformation;

$$\delta_\zeta \psi^I = \frac{1}{\kappa} \zeta^I - i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_\rho \psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_{[\mu} e^a{}_{\rho]}. \quad (6)$$

induces **GL(4,R) transformations** on  $w^a{}_\mu$  and the unified metric  $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu \partial_\nu w^a{}_\mu + \partial_\mu \xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa}, \quad (7)$$

where  $\zeta$  is a constant spinor parameter,  $\partial_{[\rho} e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$  and  $\xi^\rho = -i\kappa \bar{\zeta}^I \gamma^\rho \psi^I$ .

- Commutators of two new NLSUSY transformations (6) on  $\psi^I$  and  $e^a{}_\mu$  close to **GL(4,R)**,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \quad (8)$$

where  $\Xi^\mu = 2i\bar{\zeta}_1^I \gamma^\mu \zeta_2^I - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$ .  
*q.e.d.*

- New NLSUSY (6) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{\text{GL}(4,\mathbb{R})}, \quad \text{i.e.} \quad \delta \sim \sqrt{\delta_{\text{GL}(4,\mathbb{R})}}.$$

c.f. SUGRA (LSUSY)

$$[\delta_1, \delta_2] = \delta_{\underline{P+L+g}}$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- Invariance under new local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^I \gamma_5 \gamma_d \psi^I (\partial_\mu \epsilon_{bc}) \quad (9)$$

with the local parameter  $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ .

(9) induce the familiar local Lorentz transformation on  $w^a{}_\mu$ :

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \quad (10)$$

with the local parameter  $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,  
e.g., **the new form** on  $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma_d \psi^j (\partial_\mu \beta_{bc}), \quad (11)$$

where  $\beta_{ab} = -\beta_{ba}$  is given by  $\beta_{ab} = \epsilon_{2ac} \epsilon_1^c{}_b - \epsilon_{2bc} \epsilon_1^c{}_a$ .  
*q.e.d.*

### 3. Evolution of NLSGR and linearization of NLSUSY:

#### 3.1. Big Collapse of new space-time toward true vacuum

@  $\Lambda > 0$  allows  $L_{\text{NLSGR}}(w)$  breaks down spontaneously (**Big Collapse**) to ordinary Riemann space-time (graviton) and NG fermion (superon):  $L_{\text{SGM}}(e, \psi)$ .

$$L_{\text{NLSGR}}(w) = L_{\text{SGM}}(e, \psi) = -\frac{c^4}{16\pi G} |e| \{R(e) + |w(\psi^I)|\Lambda + \tilde{T}(e, \psi^I)\}. \quad (12)$$

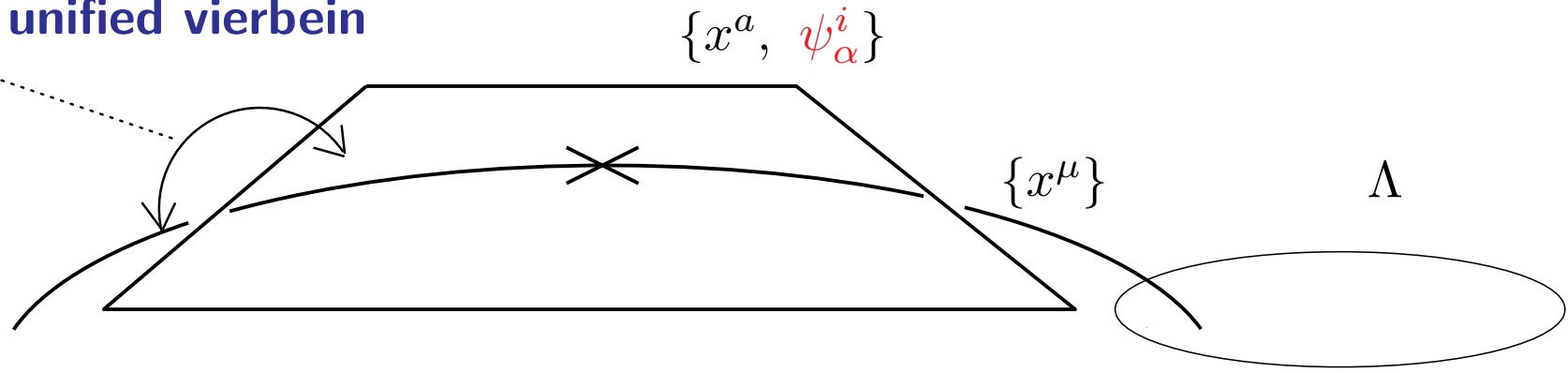
- $R(e)$ : the Ricci scalar curvature of ordinary Riemann space-time
- $\Lambda$  : the cosmological constant
- $|w(\psi^I)| = \det w^a_b = \det \{\delta^a_b + t^a_b(\psi^I)\}$ : NLSUSY action for superon
- $\tilde{T}(e, \psi^I)$  : the gravitational interaction of superon

@ Big Collapse may induce the **rapid spacial expansion** of space-time.

@  $L_{\text{SGM}}(e, \psi^I)$  evolves toward the true vacuum by constituting **gravitational composite (massless) eigenstates of broken LSUSY SO(N) sP**, which is the **ignition of the Big Bang SMs scenario**.  $\implies$



$w^a{}_\mu$  : unified vierbein

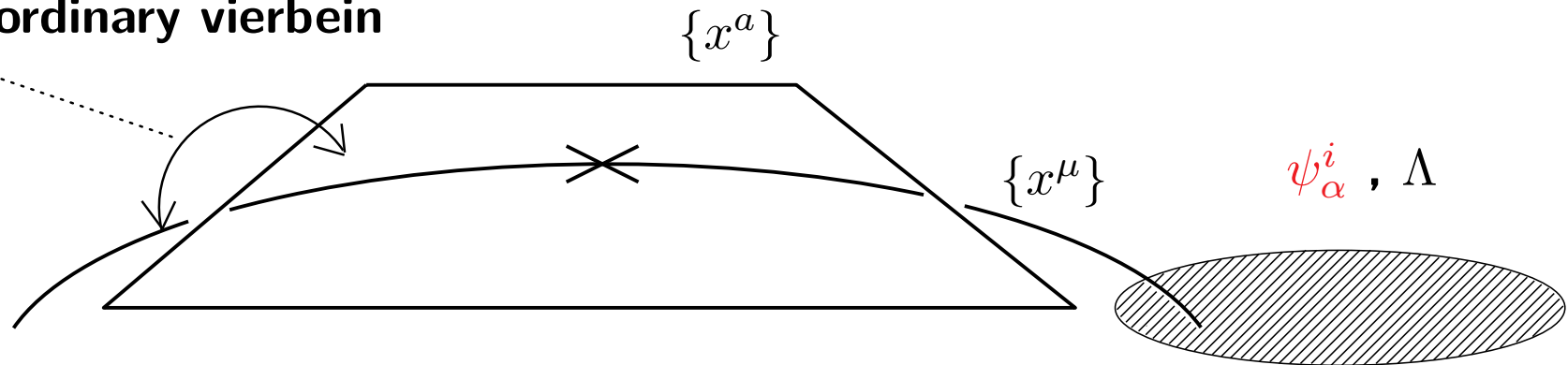


*New space-time*

$$w^a{}_\mu \longrightarrow \delta^a{}_\mu$$

⇓ **Big Collapse**

$e^a{}_\mu$  : ordinary vierbein



*Riemann spacetime*  $\oplus$  **matter**

$$e^a{}_\mu \longrightarrow \delta^a{}_\mu$$

( asymptotic )

**Ignition of the Big Bang**

- The Noether's theorem gives the conserved supercurrent:

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}} e_a{}^\mu \gamma^a \psi^I + \dots \quad (13)$$

- The supercurrent couples the graviton and the superon(NG fermion) to the vacuum

$$\langle e_b{}^\nu \psi_\beta{}^J | S_\alpha{}^{I\mu} | 0 \rangle = i\sqrt{\frac{c^4\Lambda}{8\pi G}} \delta^{\mu\nu} \delta^{IJ} (\gamma_b)_{\alpha\beta} \quad (14)$$

with the strength  $g_{sv} = \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}$ .

## 3.2. Linearization of NLSUSY and vacuum of $L_{\text{SGM}}(e, \psi^I)$

**The graviton is the universal attractive force and creates all possible gravitational composites of superons,**  
which are helicity eigenstates of LSUSY sP algebra of asymptotic space-time symmetry

- By linearizing NLSUSY we anticipate that we will obtain the (SUGRA-analogue) **broken global LSUSY theory expressed in terms of the composite of superon and  $e^a{}_\mu$ :**

$$L_{\text{LSUSY}}(e^a{}_\mu, \psi_\nu(e, \psi), M(e, \psi), N(e, \psi), \dots)$$

which satisfies:

**NL/L SUSY relation (equivalence):**

$$L_{\text{NL SGR}}(w) = L_{\text{SGM}}(e, \psi) = L_{\text{LSUSY}}(e^a{}_\mu, \psi_\nu, M, N, \dots).$$

- LSUSY of higher-spin massless supermultiplet of SGM:

For the massless supermultiplet with the highest-helicity  $(\pm 3, \pm \frac{5}{2})$  of SGM, we take **the symmetric tensor(-spinor) gauge field multiplet**  $(\varphi_{\mu\nu\rho}, \lambda_{\mu\nu})$ .

The equations of motion and the local gauge transformations are given by :

$$\begin{aligned}
\Gamma_{\sigma^{\sigma}, \mu_1 \mu_2 \mu_3} &= \partial^{\rho} \partial_{\rho} \varphi_{\mu_1 \mu_2 \mu_3} - \sum_{\mu(1)} \partial_{\mu_1} \partial_{\rho} \varphi^{\rho}_{\mu_2 \mu_3} + \sum_{\mu(2)} \partial_{\mu_1} \partial_{\mu_2} \varphi^{\rho}_{\rho \mu_3} = 0, \\
\delta_{\xi} \varphi_{\mu_1 \mu_2 \mu_3} &= \sum_{\mu(1)} \partial_{\mu_1} \xi_{\mu_2 \mu_3} \equiv \partial_{\mu_1} \xi_{\mu_2 \mu_3} + \partial_{\mu_2} \xi_{\mu_1 \mu_3} + \partial_{\mu_3} \xi_{\mu_1 \mu_2}, \\
\gamma^{\rho} \Gamma_{\rho, \mu_1 \mu_2}^{(1)} &= \gamma^{\rho} \partial_{\rho} \lambda_{\mu_1 \mu_2} - \gamma^{\rho} \partial_{\mu_1} \lambda_{\rho \mu_2} - \gamma^{\rho} \partial_{\mu_2} \lambda_{\mu_1 \rho} = 0, \\
\delta_{\epsilon} \lambda_{\mu_1 \mu_2} &= \partial_{\mu_1} \epsilon_{\mu_2} + \partial_{\mu_2} \epsilon_{\mu_1}, \\
\xi_{\rho}^{\rho} &= 0, \quad \gamma^{\rho} \epsilon_{\rho} = 0
\end{aligned} \tag{15}$$

(The arguments are systematic and geometric).

The naive extension of the familiar **global** LSUY for the **on-shell** gauge vector supermultiplet:

$$\begin{aligned}\delta_\zeta \varphi_{abc} &= -i\bar{\zeta} \gamma_{\dot{a}} \lambda_{b\dot{c}}, \\ \delta_\zeta \lambda_{ab} &= \sigma^{cd} \partial_c \varphi_{d\dot{a}b} \zeta,\end{aligned}\tag{16}$$

produce **on-shell** for  $\varphi_{abc}$  the familiar **SUSY** algebra:

$$\begin{aligned}[\delta_2, \delta_1] \varphi_{abc} &= \delta_P + \delta_G, \\ \delta_P &= (i\bar{\zeta}_1 \gamma^d \zeta_2) \partial_d \varphi_{abc}, \quad \delta_G = \partial_{\dot{a}} \{ (i\bar{\zeta}_1 \gamma^d \zeta_2) \varphi_{d\dot{b}\dot{c}} \},\end{aligned}\tag{17}$$

while for  $\lambda_{ab}$  similar analogues result with the extra term  $\delta_\Sigma$  containing  $i\bar{\zeta}_1 \sigma^{de} \zeta_2$ :

$$\begin{aligned}[\delta_2, \delta_1] \lambda_{ab} &= \delta_P + \delta_G + \delta_{\tilde{G}} + \delta_\Sigma, \\ \delta_P &= -24(i\bar{\zeta}_1 \gamma^d \zeta_2) \partial_d \lambda_{ab}, \quad \delta_G = \partial_{\dot{a}} \{ (i\bar{\zeta}_1 \gamma^d \zeta_2) (8\lambda_{d\dot{b}} - 4\gamma_{\dot{b}} \gamma^c \lambda_{dc} + \dots) \}, \\ \delta_{\tilde{G}} &= \partial_{\dot{a}} \{ (i\bar{\zeta}_1 \sigma^{cd} \zeta_2) (\gamma_c \lambda_{d\dot{b}} + \dots) \}, \quad \delta_\Sigma = 2(i\bar{\zeta}_1 \sigma^{cd} \zeta_2) (\gamma_{\dot{a}} \partial_c \lambda_{d\dot{b}} + \gamma_c \partial_d \lambda_{\dot{a}\dot{b}}),\end{aligned}\tag{18}$$

The generalization of (16) and the **off shell** extension of the multiplet will remove  $\delta_\Sigma$ .  $\rightarrow$  minimal **off-shell** case:  $[\#40]_F = [\#20]_B + [\#20(20, 10, 6, 4, 1)]_B$

### 3.3. NL/L SUSY relation (equivalence)

We demonstrate **NL/L SUSY relation for  $N=2$  SUSY in flat space:**

$$L_{\text{NLSGR}}(w) = L_{\text{SGM}}(e, \psi) \rightarrow L_{\text{NLSUSY}}(\psi) = L_{\text{LSUSY}}(v^a(\psi), \phi(\psi), \dots)$$

(  $N = 2$  **SGM** reduces to  $N = 2$  **NLSUSY** in  $\Lambda$  term of NLSGR)

@ **N=2, d=2 NLSUSY model:**

$$L_{\text{NLSUSY}} = -\frac{1}{2\kappa^2} |w_{\text{NLSUSY}}| = -\frac{1}{2\kappa^2} \left[ 1 + t^a{}_a + \frac{1}{2} (t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \dots \right], \quad (19)$$

$$|w_{\text{NLSUSY}}| = \det w^a{}_b = \det(\delta^a{}_b + t^a{}_b), \quad t^a{}_b = -i\kappa^2 (\bar{\psi}_j \gamma^a \partial_b \psi^j - \bar{\psi}_j \gamma^a \partial_b \psi^j), \quad (j = 1, 2),$$

**which is invariant under N=2 NLSUSY transformation,**

$$\delta_\zeta \psi^j = \frac{1}{\kappa} \zeta^j - i\kappa (\bar{\zeta}_k \gamma^a \psi^k - \bar{\zeta}_k \gamma^a \psi^k) \partial_a \psi^j, \quad (j = 1, 2).$$

## N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\left( \begin{array}{c} +1 \\ +\frac{1}{2}, +\frac{1}{2} \\ 0 \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell **minimal** vector supermultiplet:  $(v^a, \lambda^i, A, \phi, D; i=1,2)$  in *WZ gauge*. ( $A$  and  $\phi$  are two singlets,  $0^+$  and  $0^-$ , scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\left( \begin{array}{c} +\frac{1}{2} \\ 0, 0 \\ -\frac{1}{2} \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets:  $(\chi, B^i, \nu, F^i; i = 1, 2)$ ,  $B^i$  and  $F^i$  are complex.

- The most general  $N = 2, d = 2$  SUSYQED action ( $m = 0$  case) :

$$L_{N=2\text{SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf}, \quad (20)$$

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{\partial} \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi0} = \frac{i}{2}\bar{\chi} \not{\partial} \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu} \not{\partial} \nu + \frac{1}{2}|F^i|^2,$$

$$L_e = e \left\{ i v_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi \bar{\chi} \gamma_5 \nu \right. \\ \left. + B^i (\bar{\lambda}^i \chi - \epsilon^{ij} \bar{\lambda}^j \nu) - \frac{1}{2}|B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2}e^2(v_a^2 - A^2 - \phi^2)|B^i|^2,$$

$$L_{Vf} = f \{ A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j + (A^2 - \phi^2)D - \epsilon^{ab} A \phi F_{ab} \} \quad (21)$$

- Note that

$J = 0$  states in the vector multiplet for  $N \geq 2$  SUSY induce Yukawa coupling.



$L_{N=2\text{SUSYQED}}$  is invariant under  $N = 2$  **LSUSY** transformation:

- For the **minimal vector off-shell supermultiplet**:

$$\begin{aligned}
 \delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
 \delta_\zeta\lambda^i &= (D - i\cancel{\partial}A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\cancel{\partial}\phi\zeta^j, \\
 \delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
 \delta_\zeta\phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
 \delta_\zeta D &= -i\bar{\zeta}^i\cancel{\partial}\lambda^i.
 \end{aligned} \tag{22}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{23}$$

where  $\zeta^i, i = 1, 2$  are constant spinors and  $\delta_g(\theta)$  is the  $U(1)$  gauge transformation for only  $v^a$  with  $\Xi^\mu = 2i\bar{\zeta}_1^I\gamma^\mu\zeta_2^I$ ,  $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$ .

- For the **two scalar off-shell supermultiplets**:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\rlap{/}\partial B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\rlap{/}\partial B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \rlap{/}\partial \chi - i\epsilon^{ij}\bar{\zeta}^j \rlap{/}\partial \nu \\
&\quad - e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]\chi &= \Xi^a \partial_a \chi - e\theta \nu, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]B^i &= \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]\nu &= \Xi^a \partial_a \nu + e\theta \chi, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}]F^i &= \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{24}$$

**with**  $V^i = iv_a \gamma^a \zeta^i - \epsilon^{ij} A \zeta^j - \phi \gamma_5 \zeta^i$  **and the U(1) gauge parameter**  $\theta$ .

$N = 2$  **NL/L SUSY relation**(*equivalence*):

$$L_{\mathbf{N=2SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf} = L_{\mathbf{N=2NLSUSY}} + [\text{surface terms}], \quad (25)$$

is realized by

(I) **commutator-based (heuristic aspect) linearization**

or

(II) **superfield-based (systematic aspect) linearization.**

Z.B.

(I) **Commutator-based linearization:**

@The product of **Lorentz tensors composed of  $\psi^i$**  and  $|w|$  play a basic role.

$$Q^n \sim (\psi + \bar{\psi}\gamma \cdot \partial\psi\psi + \dots)^n \sim (\psi)^n, \quad (\psi)^n \equiv 0, n > 4,$$

• For such **Lorentz tensors (of current) of  $\psi^i$**  multiplied by  $|w|$ ,

$$b^i_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n \left( (\psi^i)^{2(n-1)} |w| \right) = \kappa^{2n-3} \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w|, \quad (26)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\dots n}{}_C{}^p \left( (\psi^i)^{2n-1} |w| \right) = \kappa^{2(n-1)} \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w|, \quad (27)$$

the variations under the NLSUSY transformations become

$$\delta_{\zeta} b^i{}_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n = \kappa^{2(n-1)} \left[ \left\{ (\bar{\zeta}^i \gamma_A \psi^j + \bar{\psi}^i \gamma_A \zeta^j) \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n + \dots \right\} |w| \right. \\ \left. + \kappa \partial_a \left( \xi^a \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w| \right) \right], \quad (28)$$

$$\delta_{\zeta} f^{ij}{}_A{}^{kl}{}_B{}^{ml\dots n}{}_C{}^p = \kappa^{2n-1} \left[ \left\{ \zeta^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p \right. \right. \\ \left. \left. + \psi^i \left( \bar{\zeta}^j \gamma_A \psi^k + \bar{\psi}^j \gamma_A \zeta^k \right) \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p + \dots \right\} |w| \right. \\ \left. + \kappa \partial_a \left( \xi^a \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w| \right) \right], \quad (29)$$

where  $\xi^a = i\kappa \bar{\zeta}^i \gamma^a \psi^i$ .

- These show that the tables of products of **Lorentz tensors (of currents) of  $\psi^i$  multiplied by  $|w|$  give a finite representation of NLSUSY algebra.**
- The **self-contained part of Lorentz-tensor table** corresponds to **smaller LSUSY multiplet**.
- They satisfy the basic commutator under NLSGR tr.

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v),$$

where  $\delta_P(v)$  is a translation with a parameter  $v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i)$

- These results show that the commutator-based linearization closes on the all possible Lorentz tensors composed of  $\psi^i$  and gives a finite dimensional representation of sP algebra.
- Assign each composite Lorentz tensor to the component field of the LSUSY supermultiplet including the auxiliary field (**susy compositeness**), which reproduces the familiar LSUSY transformation among the supermultiplet under the NLSUSY transformations of constituents  $\psi^i$ .
- Substituting **SUSY compositeness** into  $L_{N=2LSUSYQED}$ , we obtain  $L_{N=2NLSUSY}$ , i .e. the **NL/L SUSY relation(equivalence)**.

- **SUSY compositeness** for the vector off-shell **minimal** supermultiplet:

$$v^a = -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|,$$

$$\lambda^i = \xi\psi^i|w|,$$

$$A = \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|,$$

$$\phi = -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|,$$

$$D = \frac{\xi}{\kappa}|w|, \tag{30}$$

where  $\xi$  is a VEV factor of the auxiliary field  $D$ .

- Note that  $\psi^i$  is the low energy leading term of the supercharge  $Q^i$ .

- **SUSY compositeness** for scalar off-shell **minimal** supermultiplets:

$$\begin{aligned}
\chi &= \xi^i \left[ \psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right] \\
B^i &= -\kappa \left( \frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\
\nu &= \xi^i \epsilon^{ij} \left[ \psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right], \\
F^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
&\quad - \frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|. \tag{31}
\end{aligned}$$

- The **quartic fermion self-interaction term** in  $F^i$  realizes the local  $U(1)$  gauge symmetry of LSUSY.
- $\xi^i$  is the VEV factor of the auxiliary field  $F^i$ .

- **SUSY compositeness** produces under **NLSUSY** transformation

a **new off-shell commutator algebra which closes on only a translation:**

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \quad (32)$$

where  $\delta_P(v)$  is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i) \quad (33)$$

- Note that the commutator does not induce the **U(1)** gauge transformation for the off-shell supermultiplet, which is **different from the ordinary LSUSY**.



- Substituting **SUSY copositeness** into  $L_{N=2LSUSYQED}$ , we find **NL/L SUSY relation for the minimal supermultiplet**:

$$L_{N=2LSUSYQED} = f(\xi, \xi^i) L_{N=2NLSUSY} + [\text{suface terms}], \quad (34)$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \quad (35)$$

⇒ **LSUSY** may be regarded as composite eigenstates of **space-time** symmetries.

- **NL/L SUSY relation** bridges naturally **the cosmology and the low energy particle physics in NLSGR**. (⇒ Sec. 4).

- **The direct linearization** of highly nonlinear SGM action (12) in curved space **remains to be carried out**.

→

**In Riemann flat space-time of SGM,  
ordinary LSUSY gauge theory with the spontaneous SUSY breaking  
emerges  
from  
the cosmological term  $\Lambda$  of SGM and materializes the true vacuum of SGM  
( as gravitational composites of NG fermion )**

*SM can be a low energy effective theory of SGM/NLSGR.*

## (II) Linearization of NLSUSY by the superfield formulation( $d = 2$ )

- **General superfields** are given for the  $N = 2$  vector supermultiplet by

$$\begin{aligned} \mathcal{V}(x, \theta^i) = & C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x) \\ & - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j v^a(x) - \frac{1}{2} \bar{\theta}^i \theta^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8} \bar{\theta}^i \theta^i \bar{\theta}^j \theta^j D(x), \end{aligned} \quad (36)$$

and for the  $N = 2$  scalar supermultiplet by

$$\begin{aligned} \Phi^i(x, \theta^i) = & B^i(x) + \bar{\theta}^i \chi(x) - \epsilon^{ij} \bar{\theta}^j \nu(x) - \frac{1}{2} \bar{\theta}^j \theta^j F^i(x) + \bar{\theta}^i \theta^j F^j(x) - i \bar{\theta}^i \not{\partial} B^j(x) \theta^j \\ & + \frac{i}{2} \bar{\theta}^j \theta^j (\bar{\theta}^i \not{\partial} \chi(x) - \epsilon^{ik} \bar{\theta}^k \not{\partial} \nu(x)) + \frac{1}{8} \bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \end{aligned} \quad (37)$$

- Extend the superspace to the following superspace  $(x'^a, \theta')$  with  $-\kappa\psi(x)$ ,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \quad (38)$$

and denotes **the extended superfields** on  $(x'^a, \theta'^i)$  and their  $\theta$ -expansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \quad (39)$$

- **Extended** global SUSY transformations  $\tilde{\delta} = \delta^L(x.\theta) + \delta^{NL}(\psi)$  on  $(x'^a, \theta'^i)$  give:

$$\tilde{\delta}\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \tilde{\delta}\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \quad (40)$$

- Therefore, the conditions (**SUSY invariant constraints**):

$$\tilde{\varphi}_{\mathcal{V}}^I(x) = \xi_{\mathcal{V}}^I(\text{constant}) \quad \tilde{\varphi}_{\Phi}^I(x) = \xi_{\Phi}^I(\text{constant}), \quad (41)$$

make extended superfields invariant under the extended SUSY transformations.  
which provide **SUSY compositeness**.

- Putting constants in the most general case as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad (42)$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (43)$$

- We obtain straightforwardly **SUSY compositeness**  $\varphi_V^I = \varphi_V^I(\psi)$  for the vector supermultiplet

$$C = \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2} \kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4} \xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4} \xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\ - \frac{1}{2} \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8} \xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j,$$

$$\Lambda^i = \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2} \xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2} \xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \\ - \frac{1}{2} \xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2} \kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi_\lambda^j) \\ - \frac{1}{2} \xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i \kappa \not{\partial} C(\psi) \psi^i,$$

$$\begin{aligned}
M^{ij} &= \xi_M^{ij} + \kappa \bar{\psi}^i \xi_\lambda^j + \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^j + i \kappa \epsilon^{(i|k| \epsilon^{j)l} \bar{\psi}^k \not{\partial} \Lambda^l(\psi) - \frac{1}{2} \kappa^2 \epsilon^{ik} \epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi_\lambda^j - \frac{1}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \not{\partial} \Lambda^j(\psi) + \frac{1}{2} \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a &= \xi_v^a - i \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi_\lambda^j - \frac{i}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \not{\partial} \gamma^a \Lambda^j(\psi) + \frac{i}{2} \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
&\quad - i \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi \psi^i - i \kappa \not{\partial} M^{ij}(\psi) \psi^j + \frac{i}{2} \kappa \epsilon^{ab} \epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
&\quad - \frac{1}{2} \kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2} \epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
&\quad - \frac{1}{2} \kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j \\
&\quad - \gamma_a \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j + 2 \not{\partial} \partial_a \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j \} - \frac{i}{2} \kappa^3 \not{\partial} \partial^2 C(\psi) \psi^i \bar{\psi}^j \psi^j, \\
D &= \frac{\xi}{\kappa} - i \kappa \bar{\psi}^i \not{\partial} \lambda^i(\psi)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \kappa^2 \left\{ \bar{\psi}^i \psi^j \partial^2 M^{ij}(\psi) - \frac{1}{2} \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 \phi(\psi) \right. \\
& \left. + \frac{i}{2} \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial^2 v^a(\psi) - i \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b(\psi) \right\} \\
& - \frac{i}{2} \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \not{\partial} \partial^2 \Lambda^j(\psi) + \frac{1}{8} \kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \partial^4 C(\psi), \tag{44}
\end{aligned}$$

and **SUSY compositeness for the scalar multiplet**  $\varphi_{\Phi}^I = \varphi_{\Phi}^I(\psi)$ :

$$\begin{aligned}
B^i &= \xi_B^i + \kappa(\bar{\psi}^i \xi_{\chi} - \epsilon^{ij} \bar{\psi}^j \xi_{\nu}) - \frac{1}{2} \kappa^2 \{ \bar{\psi}^j \psi^j F^i(\psi) - 2 \bar{\psi}^i \psi^j F^j(\psi) + 2i \bar{\psi}^i \not{\partial} B^j(\psi) \psi^j \} \\
&\quad - i \kappa^3 \bar{\psi}^j \psi^j \{ \bar{\psi}^i \not{\partial} \chi(\psi) - \epsilon^{ik} \bar{\psi}^k \not{\partial} \nu(\psi) \} + \frac{3}{8} \kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 B^i(\psi), \\
\chi &= \xi_{\chi} + \kappa \{ \psi^i F^i(\psi) - i \not{\partial} B^i(\psi) \psi^i \} \\
&\quad - \frac{i}{2} \kappa^2 [ \not{\partial} \chi(\psi) \bar{\psi}^i \psi^i - \epsilon^{ij} \{ \psi^i \bar{\psi}^j \not{\partial} \nu(\psi) - \gamma^a \psi^i \bar{\psi}^j \partial_a \nu(\psi) \} ] \\
&\quad + \frac{1}{2} \kappa^3 \psi^i \bar{\psi}^j \psi^j \partial^2 B^i(\psi) + \frac{i}{2} \kappa^3 \not{\partial} F^i(\psi) \psi^i \bar{\psi}^j \psi^j + \frac{1}{8} \kappa^4 \partial^2 \chi(\psi) \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j,
\end{aligned}$$

$$\begin{aligned}
\nu &= \xi_\nu - \kappa \epsilon^{ij} \{ \psi^i F^j(\psi) - i \not{\partial} B^i(\psi) \psi^j \} \\
&\quad - \frac{i}{2} \kappa^2 [ \not{\partial} \nu(\psi) \bar{\psi}^i \psi^i + \epsilon^{ij} \{ \psi^i \bar{\psi}^j \not{\partial} \chi(\psi) - \gamma^a \psi^i \bar{\psi}^j \partial_a \chi(\psi) \} ] \\
&\quad + \frac{1}{2} \kappa^3 \epsilon^{ij} \psi^i \bar{\psi}^k \psi^k \partial^2 B^j(\psi) + \frac{i}{2} \kappa^3 \epsilon^{ij} \not{\partial} F^i(\psi) \psi^j \bar{\psi}^k \psi^k + \frac{1}{8} \kappa^4 \partial^2 \nu(\psi) \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\
F^i &= \frac{\xi^i}{\kappa} - i \kappa \{ \bar{\psi}^i \not{\partial} \chi(\psi) + \epsilon^{ij} \bar{\psi}^j \not{\partial} \nu(\psi) \} \\
&\quad - \frac{1}{2} \kappa^2 \bar{\psi}^j \psi^j \partial^2 B^i(\psi) + \kappa^2 \bar{\psi}^i \psi^j \partial^2 B^j(\psi) + i \kappa^2 \bar{\psi}^i \not{\partial} F^j(\psi) \psi^j \\
&\quad + \frac{1}{2} \kappa^3 \bar{\psi}^j \psi^j \{ \bar{\psi}^i \partial^2 \chi(\psi) + \epsilon^{ik} \bar{\psi}^k \partial^2 \nu(\psi) \} - \frac{1}{8} \kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 F^i(\psi). \tag{45}
\end{aligned}$$



- Choosing the following **Lorentz invariant** and **SUSY invariant** constraints of the component fields in  $\tilde{\mathcal{V}}$  and  $\tilde{\Phi}$ ,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (46)$$

give **previous SUSY compositeness** for the minimal supermultiplet.

Therefore,

- under **SUSY invariant constraints**,

the  $N = 2$  **NLSUSY** action  $S_{N=2\text{NLSUSY}}$  is related to  $N = 2$  **SUSY QED** action:

$$L_{N=2\text{SUSYQED}} \equiv L_{\mathcal{V}\text{free}} + L_{\mathcal{V}f} + L_{\text{gauge}} = f(\xi, \xi^i) S_{N=2\text{NLSUSY}} \quad (47)$$

when  $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$ .

- **NL/L SUSY relation** bridges **the cosmology** and **the low energy particle physics** in **NLSGR** scenario  $\implies$  **Sec. 4.**

@ SGM scenario predicts the magnitude of the bare gauge coupling constant.

For more general SUSY invariant constraints (vev of  $0^+$  auxiliary field):

$$\underline{\tilde{C}} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (48)$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \quad e = \frac{\ln\left(\frac{\xi^{i2}}{\xi^2 - 1}\right)}{4\xi_c}, \quad (49)$$

where  $e$  is the bare gauge coupling constant.

- This mechanism is natural and favorable for SGM scenario as a theory of everything.

Broken LSUSY(QED) gauge theory is encoded  
in the vacuum of NLSUSY theory  
as composites of NG fermion.

### 3.3. $N = 3$ NL/L SUSY relation and SUSY Yang-Mills theory

- Physical helicity states of  $N = 3$  LSUSY vector supermultiplet:

$$\left[ \underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right) \right] + [\text{CPT conjugate}], \quad (50)$$

where  $\underline{n}(\lambda)$  means the dimension  $\underline{n}$  and the helicity  $\lambda$ , are accommodated in  $N = 3$  off-shell vector supermultiplet ( $d = 2$ ):

- $N = 3$  superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), \chi_{\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \dots, \dim.G) \quad (51)$$

Each component field belongs to the adjoint representation of the YM gauge group  $G$ :

$$[T^I, T^J] = if^{IJK}T^K \text{ and denoted as } \varphi^i = \varphi^{iI}T^I, \text{ etc..}$$

- $N = 3$  (pure) SUSYM action:

$$\begin{aligned}
S_{\text{SYM}} = \int d^2x \operatorname{tr} \left\{ & -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{D} \lambda^i + \frac{1}{2}(D_a A^i)^2 + \frac{i}{2}\bar{\chi} \not{D} \chi + \frac{1}{2}(D_a \phi)^2 + \frac{1}{2}(D^i)^2 \right. \\
& -ig\{\epsilon^{ijk} A^i \bar{\lambda}^j \lambda^k - [A^i, \bar{\lambda}^i] \chi + \phi(\bar{\lambda}^i \gamma_5 \lambda^i + \bar{\chi} \gamma_5 \chi)\} \\
& \left. + \frac{1}{4}g^2([A^i, A^j]^2 + 2[A^i, \phi]^2) \right\}, \tag{52}
\end{aligned}$$

where  $g$  is the gauge coupling constant,  $D_a$  and  $F_{ab}$  are the covariant derivative and the YM gauge field strength defined as

$$\begin{aligned}
D_a \varphi &= \partial_a \varphi - ig[v_a, \varphi], \\
F_{ab} &= \partial_a v_b - \partial_b v_a - ig[v_a, v_b]. \tag{53}
\end{aligned}$$

• **SUSY YM action is invariant under  $N = 3$  LSUSY transformations:**

$$\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i \gamma^a \lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk} (D^j - i\mathcal{D}A^j) \zeta^k + \frac{1}{2} \epsilon^{ab} F_{ab} \gamma_5 \zeta^i - i\gamma_5 \mathcal{D}\phi \zeta^i \\
&\quad + ig([A^i, A^j] \zeta^j + \epsilon^{ijk} [A^j, \phi] \gamma_5 \zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk} \bar{\zeta}^j \lambda^k - \bar{\zeta}^i \chi, \\
\delta_\zeta \chi &= (D^i + i\mathcal{D}A^i) \zeta^i + ig(\epsilon^{ijk} A^i A^j \zeta^k - [A^i, \phi] \gamma_5 \zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i \gamma_5 \lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk} \bar{\zeta}^j \mathcal{D}\lambda^k - i\bar{\zeta}^i \mathcal{D}\chi + ig(\bar{\zeta}^i [\lambda^j, A^j] + \bar{\zeta}^j [\lambda^i, A^j] - \bar{\zeta}^j [\lambda^j, A^i] \\
&\quad - \epsilon^{ijk} \bar{\zeta}^j [\chi, A^k] + \epsilon^{ijk} \bar{\zeta}^j \gamma_5 [\lambda^k, \phi] + \bar{\zeta}^i \gamma_5 [\chi, \phi]), \tag{54}
\end{aligned}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{55}$$

**where  $\delta_G(\theta)$  means  $\delta_G(\theta)\varphi = ig[\theta, \varphi]$  and  $\delta_g(\theta)$  is the  $U(1)$  gauge transformation only for  $v^a$  with  $\theta = -2(i\bar{\zeta}_1^i \gamma^a \zeta_2^i v_a - \epsilon^{ijk} \bar{\zeta}_1^i \zeta_2^j A^k - \bar{\zeta}_1^i \gamma_5 \zeta_2^i \phi)$ .**

• **SUSY invariant(composite) relations for  $N = 3$  YM off-shell gauge supermultiplet**

$$\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) \\
&\quad + \frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\psi^k)\} \\
&\quad - \frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \tag{56}
\end{aligned}$$

- **Arbitrary real constants  $\xi^{iI}$  of auxiliary fields  $D^{iI}$  bridge  $N = 3$  SUSY and the YM gauge group  $G$ .**

- **Substituting (56) into the SYM action (52), we can show the NL/L SUSY relation for  $N = 3$  SUSY:**

$$L_{\text{SUSYYM}}(\psi) = -(\xi^{iI})^2 L_{\text{NLSUSY}} + [\text{surface terms}]. \quad (57)$$

### 3.4. Linearization of $L_{SGM}(e, \psi)$ in curved space-time

@ Commutator-based linearization:

• Show

**NL/L SUSY relation** (*equivalence*):

$$L_{NLSGR}(w) = L_{SGM}(e, \psi) = L_{LSUSY}(e^a_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \dots).$$

• NLSUSY transformations

$$\delta_\zeta \psi^i = \frac{1}{\kappa} \zeta^i + \xi^\mu \partial_\mu \psi^i, \quad \delta_\zeta e^a_\mu = 2i\kappa \bar{\zeta}^i \gamma^\nu \psi^i \partial_{[\mu} e^a_{\nu]}, \quad \xi^\mu = i\kappa \bar{\psi}^j \gamma^\mu \zeta^j.$$

• Based commutator algebra, **LSUSY** as well,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{GL(4,R)}(\Xi^\mu), \quad \Xi^\mu = 2(i\bar{\zeta}_1^i \gamma^\mu \zeta_2^i - \xi^\nu_1 \xi^\kappa_2 e^\mu_a \partial_{[\mu} e^a_{\nu]})$$

• In curved space-time, Lorentz- and Riemann-tensor functionals with  $\gamma$ -matrices

$$F_M^{IA} = F_M^{IA}(\psi^i, e^a_\mu, \partial\psi^i, \partial_{[\nu} e^a_{\kappa]})$$

satisfies the commutator algebra, favourable to **the SUGRA-like LSUSY** structure.



## 4. Low energy particle physics and Cosmology of NLSGR

### 4.1. Low Energy Particle Physics of NLSGR :

@ As we have seen that

$N = 2$  SGM is essentially  $N=2$  NLSUSY action in tangent(flat)) space-time, we focus on  $N=2$  NLSUSY action for extracting physical implications of SGM.

• The low energy theorem for NLSUSY gives the following **superon(massless NG fermion)-vacuum coupling**

$$\langle \psi^j_\alpha(x) | J^{k\mu}_\beta | 0 \rangle = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} (\gamma^\mu)_{\alpha\beta} \delta^{jk} + \dots, \quad (58)$$

where  $J^{k\mu} = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} \gamma^\mu \psi^k + \dots$  is the conserved supercurrent.

$\sqrt{\frac{c^4 \Lambda}{8\pi G}} = \frac{1}{\sqrt{2\kappa}}$  is the coupling constant ( $\equiv g_{sv}$ ) of superon with the vacuum.

$$\implies m_\nu^4 \sim \rho_{DE},$$

- How to extract the vacuum configuration of SGM.

In Riemann-flat space-time, **NL/L SUSY relation(equivalence)** gives:

$$L_{N=2\text{SGM}} \longrightarrow L_{N=2\text{NLSUSY}} + [\text{suface terms}] = L_{N=2\text{SUSYQED}}. \quad (59)$$

- We study vacuum structures of  $N = 2$  **LSUSY QED** action in stead of  $N = 2$  SGM.

The vacuum is given by the minimum of the potential  $V(A, \phi, B^i, D)$  of  $L_{N=2\text{LSUSYQED}}$ ,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \quad (60)$$

- **Substituting the solution of the equation of motion for the auxiliary field  $D$  we obtain**

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)|B^i|^2 \geq 0. \quad (61)$$

- **Two different types of vacua  $V = 0$  exist in  $(A, \phi, B^i)$ -space:**

$$(I) \quad A = \phi = 0, \quad |\tilde{B}^i|^2 = -k^2 \quad \left( \tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (62)$$

and

$$(II) \quad |\tilde{B}^i|^2 = 0, \quad A^2 - \phi^2 = k^2. \quad (63)$$

- **Expansions of  $A, \phi, \tilde{B}^i$  around vacuum values give low energy particles  $\hat{A}, \hat{\phi}, \hat{B}^i$  in the true vacuum.**

- For the type (I) vacuum with  $SO(2)$  symmetry for  $(\tilde{B}^1, \tilde{B}^2)$ ,  $e\xi < 0$ ,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a\rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a\hat{A})^2 + (\partial_a\hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i\partial\lambda^i + \frac{i}{2}\bar{\chi}\partial\chi + \frac{i}{2}\bar{\nu}\partial\nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \dots,
\end{aligned}
\tag{64}$$

and following mass spectra

$$\begin{aligned} m_\rho^2 &= m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\ m_{\lambda^i} &= m_\chi = m_\nu = 0. \end{aligned} \tag{65}$$

- The **vacuum** breaks **both SUSY and the local  $U(1)(O(2))$  spontaneously** and Higgs-Kibble mechanism works.
- All bosons have the same mass and **all fermions remain massless**.
- $\lambda^i$  are **NG fermions**.

- For the type (II) vacuum with  $SO(1, 1)$  symmetry for  $(A, \phi)$ , e.g.  $f\xi > 0$ ,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2 k^2 \hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2 k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \not{\partial} \lambda^i - 2fk\bar{\lambda}^i \lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi} \not{\partial} \chi + \bar{\nu} \not{\partial} \nu) - ek(\bar{\chi} \chi + \bar{\nu} \nu)\} + \dots.
\end{aligned} \tag{66}$$

and following mass spectra:

$$\begin{aligned}
 m_{\hat{A}}^2 &= m_{\lambda^i}^2 = 4f^2 k^2 = \frac{4\xi f}{\kappa}, \\
 m_{\hat{B}^1}^2 &= m_{\hat{B}^2}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2 k^2 = \frac{\xi e^2}{\kappa f}, \\
 m_{\nu_a} &= m_{\hat{\phi}} = 0,
 \end{aligned}
 \tag{67}$$

which produces mass hierarchy by the factor  $\frac{e}{f}$  independent of  $\kappa$ . ( $\kappa^{-2} = \frac{c^4 \Lambda}{16\pi G}$ )

• The vacuum breaks both SUSY and  $SO(1,1)$  for  $(A, \phi)$   
 and restores(maintains)  $SO(2)(U(1))$  for  $(\tilde{B}^1, \tilde{B}^2)$ , spontaneously,

which produces NG-Boson  $\hat{\phi}$  and massless photon  $\nu_a$   
 and gives soft masses  $\langle A \rangle$  to  $\lambda^i$ .

- We have shown explicitly that

**N=2 LSUSY QED, i.e. the matter sector(  $\Lambda$  term) of  $N = 2$  SGM (in flat-space), possesses a true vacuum type (II).**

- The resulting model describes qualitatively

**lepton-Higgs-U(1) sector analogue of SM:**

**one massive charged Dirac fermion ( $\psi_D^c \sim \chi + i\nu$ ),**

**one massive neutral Dirac fermion ( $\lambda_D^0 \sim \lambda^1 - i\lambda^2$ ),**

**one massless vector (a photon) ( $v_a$ ),**

**one charged scalar ( $\hat{B}^1 + i\hat{B}^2$ ),**

**one neutral complex scalar ( $\hat{A} + i\hat{\phi}$ ),**

**which are composites of superons.**



**In Riemann flat space-time of SGM,  
ordinary LSUSY gauge theory with the spontaneous SUSY breaking  
emerges  
from  
the NLSUSY cosmological term of SGM  
as composites of NG fermion in the true vacuum.**

## 4. Cosmological implications of SGM scenario

The variation of SGM action  $L_{\text{SGM}}(e, \psi)$  with respect to  $e^a{}_\mu$  yields **Einstein equation equipping with matter and cosmological term:**

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4} \left\{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\}. \quad (68)$$

where  $\tilde{T}_{\mu\nu}(e, \psi)$  abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

- Note that the cosmological term  $-\frac{c^4 \Lambda}{8\pi G}$  can be interpreted as **the negative energy density of space-time**, i.e. **the dark energy density  $\rho_D$** .

- **Big collapse** may induce **3 dimensional expansion** of space-time by **Pauli principle**:

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \Phi_{\mu\nu}(e, \psi)\}dx^\mu dx^\nu.$$

$$\{\psi(x), \bar{\psi}(y)\} = 0 \Rightarrow \{\psi(x), \bar{\psi}(y)\} = \delta^{(3)}(\mathbf{x} - \mathbf{y})$$

- **Big Collapse** produces **composite (massless) eigenstates of SO(N) sP algebra** due to the **universal gravitational force**,

which is **the ignition of the Big Bang(BB) SM scenario**.

- As shown in the toy model, the vacuum of **the composite SGM scenario** may explain naturally **observed mysterious (numerical) relations**:

$$\text{dark energy density } \rho_D \sim O(\kappa^{-2}) \sim m_\nu^4 \sim (10^{-12} \text{GeV})^4 \sim g_{sv}^2,$$

provided  $\lambda_D^0$  is identified with neutrino and  $f\xi \sim O(1)$ .

## 5. Rarita-Schwinger NLSGR

- New SUSY algebra containing **spinor-vector generators**  $Q_\alpha^\mu$ :

$$\{Q_\alpha^\mu, Q_\beta^\nu\} = \varepsilon^{\mu\nu\lambda\rho} P_\lambda (\gamma_\rho \gamma_5 C)_{\alpha\beta}, \quad (69)$$

$$[Q_\alpha^\mu, P^\nu] = 0, \quad (70)$$

$$[Q_\alpha^\mu, J^{\lambda\rho}] = \frac{1}{2}(\sigma^{\lambda\rho} Q^\mu)_\alpha + i\eta^{\lambda\mu} Q_\alpha^\rho - i\eta^{\rho\mu} Q_\alpha^\lambda, \quad (71)$$

where  $Q_\alpha^\mu$  are vector-spinor generators satisfying Majorana condition  $Q_\alpha^\mu = C_{\alpha\beta} \bar{Q}_\alpha^\mu$ .

- Consider the following global (3/2 super)translations:

$$\psi_\alpha^a \longrightarrow \psi_\alpha^a + \zeta_\alpha^a. \quad (72)$$

$$x_a \longrightarrow x_a + i\kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 \zeta^d, \quad (73)$$

where  $\zeta_\alpha^a$  is a constant Majorana tensor-spinor parameter.

- The invariant differential forms become:

$$\omega_a = dx_a + i\kappa \varepsilon_{abcd} \bar{\psi}^b \gamma^c \gamma_5 d\psi^d. \quad (74)$$

- **Invariant action of nonlinear representation of vector-spinor SUSY:**

$$S = \frac{1}{\kappa} \int \omega_0 \wedge \omega_1 \wedge \omega_2 \wedge \omega_3 = \frac{1}{\kappa} \int \det w_{ab} d^4x, \quad (75)$$

$$w_{ab} = \delta_{ab} + t_{ab}, \quad t_{ab} = i\kappa \varepsilon_{acde} \bar{\psi}^c \gamma^d \gamma_5 \partial_b \psi^e, \quad (76)$$

- **By similar geometrical arguments to SGM**  
we obtain **vector-spinor NLSUSY GR:**

$$L_{vsNLSUSYGR} = -\frac{c^3}{16\pi G} |w| \{ \Omega(w^a{}_\mu) + \Lambda \}, \quad (77)$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu), \quad (78)$$

- **Unified vierbeins are:**

$$w^a{}_\mu(x) = e^a{}_\mu(x) + t^a{}_\mu(x), \quad t^a{}_\mu(x) = i\kappa \varepsilon^{abcd} \bar{\psi}_b \gamma_c \gamma_5 \partial_\mu \psi_d, \quad (79)$$

- $L_{vsNLSUSYGR}$  **possesses similar symmetry properties as SGM.**

## 6. Summary

**NLSGR(SGM)** for unity of nature:

- **Ultimate entity; Unstable  $d = 4$  space-time:**  $[x^a, \psi_\alpha^N; x^\mu]$  described by  $[L_{\text{NLSGR}}(w^a{}_\mu)]$  : **NLSGR on New space-time with  $\Lambda > 0$**

⇒ **Big Collapse (BC)**

- **The creation of Riemann space-time [graviton  $e^a{}_\mu$ ] and massless fermionic matter [superon  $\psi_\alpha^N$ ]**  
 $[L_{\text{SGM}} = L_{\text{EH}}(e) - \Lambda + T(\psi.e)]$  : **Einstein GR with  $\Lambda > 0$  and  $N$  superon**

- **The universal attractive force graviton dictates the phase transition by forming gravitational composite LSUSY supermultiplet corresponding to (massless) eigenstates of space-time symmetry  $\text{SO}(10)$  sP.**

⇒ **Ignition of Big Bang Scenario toward  $\Lambda$ CDMSM scenario**

- **In flat space-time, broken  $N$ -LSUSY theory emerges from the  $N$ -NLSUSY cosmological term of  $L_{\text{SGM}}(e, \psi)$  [NL/L SUSY relation].  $\longleftrightarrow$  BCS vs GL**

**The cosmological term is the origin of everything!**

## Predictions and Speculations

@SO(10) sP algebra with  $\underline{10} = \underline{5}_{\text{SU}(5)} + \underline{5}^*_{\text{SU}(5)}$ :  
**superon-quintet model(SQM) of matter**

- **Two new  $1^{\text{C}}$  particles besides SM particles:**

One double-charge spin 1/2 fermion  $E^{2\pm}$ :  $\epsilon^{abc} Q_a Q_b Q_c \epsilon^{mn} Q^*_m Q^*_n$

One neutral vector boson  $S$ :  $\delta^{ab} Q_a Q^*_b$

- Proton decay diagrams of SU(5) GUT **in composite SQM view** are forbidden by **superon slection rule**.  $\Rightarrow$  **stable proton, FCC suppression,**

@Field theory via Linearization:

- NLSGR(SGM) scenario **predicts** 4 dimensional space-time.
- The bare gauge coupling constant is determined (unified).
- **$N$ -L/NL SUSY relation  $\iff$  superon-quintet hypothesis of matter**

cosmological term  $\iff$  dark energy density  $\iff$  SUSY Br.  $\rightarrow m_\nu$

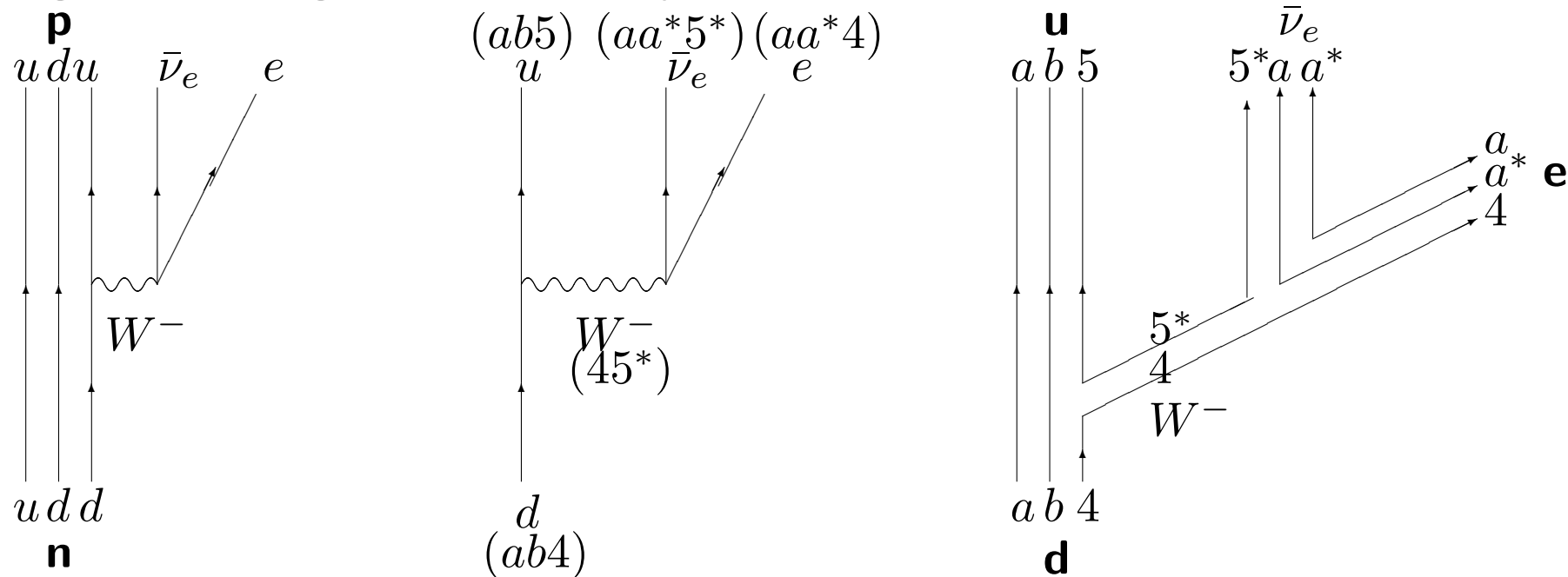
# Many Open Problems! e.g.,

- Direct linearization of **SGM action**,  
i.e. **NL/L SUSY relation in curved space-time, LSUSY of high spin states** .

- Revisit SM and GUT from  $N = 10$  **SQM composite diagram viewpoints**:

$(e, \nu_e): \delta^{ab} Q_a Q_b^* Q_m, (\mu, \nu_\mu): \delta^{ab} Q_a Q_b^* \epsilon^{lm} Q_l Q_m Q_n^*, (\tau, \nu_\tau): \epsilon^{abc} Q_b Q_c \epsilon^{ade} Q_d^* Q_e^* Q_m$   
 $(u, d): \epsilon^{abc} Q_b Q_c Q_m, (c, s): \epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c Q_n^*, (t, b): \epsilon^{abc} Q_a Q_b Q_c Q_d^* Q_m,$   
*HiggsBoson*:  $\delta^{ab} Q_a Q_b^* Q_l Q_m^*,$  *GaugeBoson*:  $Q_a Q_b^*, \dots$

e.g. SQM diagram for  $\beta$ -decay.





- SQM diagram interpretation of dangerous Feynman diagrams of SM and GUT (e.g., proton decay, FCNC, ...).
- The **thermodynamics of the universe** in NLSGR/SGM.
- Physical consequences of spin  $\frac{3}{2}$  NLSGR.

**[Ref.] K. Shima, Invited talk at *Conference on Cosmology, Gravitational Waves and Particles*, 6-10, January, 2017, NTU, Singapore(Uploaded at YouTube by IAS). Proceeding of *CCGWP*, ed. Harald Fritzsch, (World Scientific, Singapore, 2017), 301.**

**K. Shima Invited talk at *14th Rencontres du Vietnam on Windows on the Universe, 25th Anniversary*, 8/5-11, 2018, Quy Nhon, Vietnam. Proceedings of *Windows on the Universe*, ed. Jacques Dumarchez. at press.**