ウィルソンくりこみ群とホログラフィー

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Phys. Rev.D 106 (2022) 12, 126003
(arXiv:2202.13699)

31.8.2023@PPP2023

Where is Jilin (吉林)?

Challenging problem in QFT

- In strongly correlated systems, particles tend to form bound states.
  - Meson, Baryon in QCD
  - Cooper pair in a superconductor
  - etc..

How to evaluate bound states?

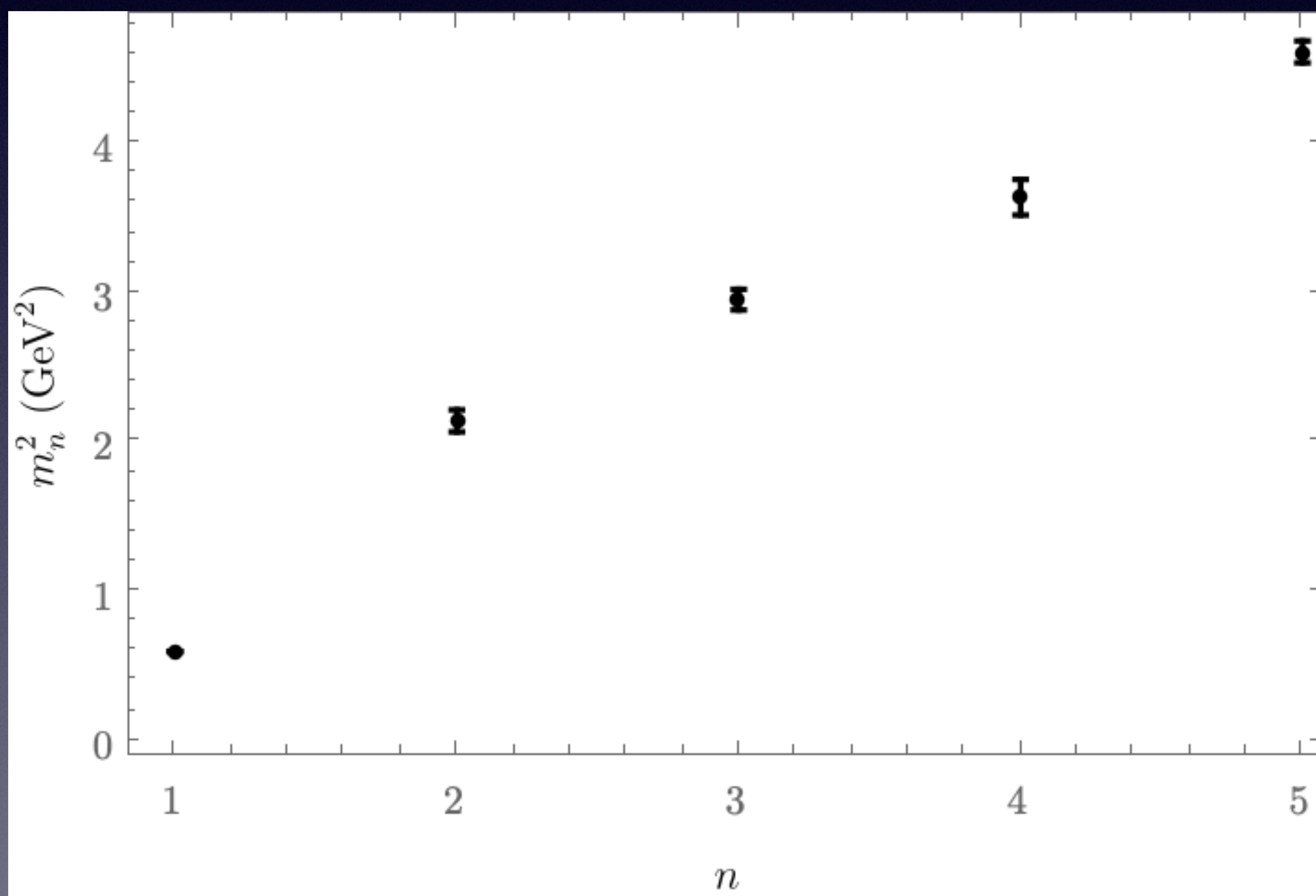
1. Lattice simulation

2. The use of the Bethe-Salpeter equation in QFT.

3. The use of duality

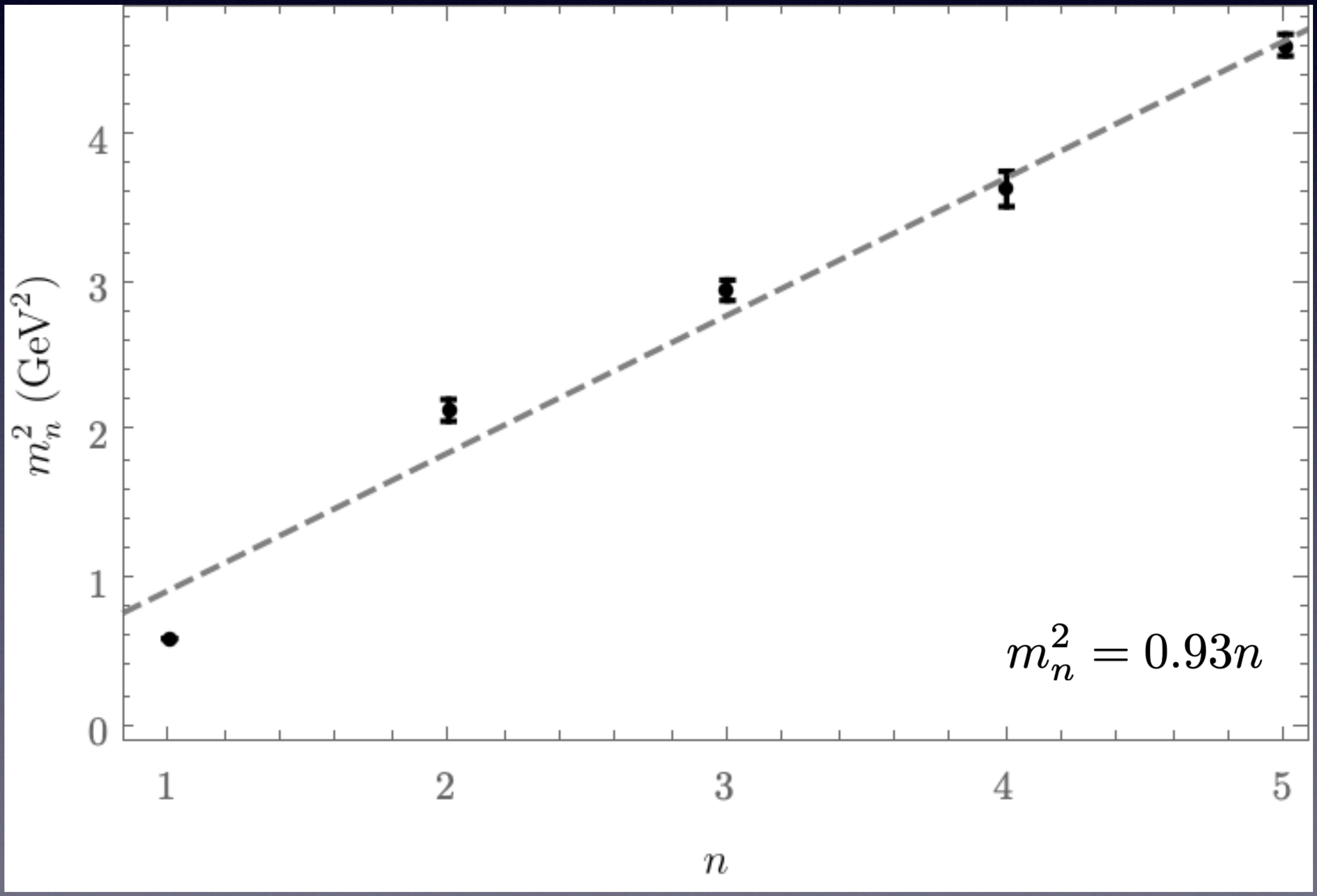
Regge trajectory

- e.g. Excitations of rho meson (vector meson)



Regge trajectory

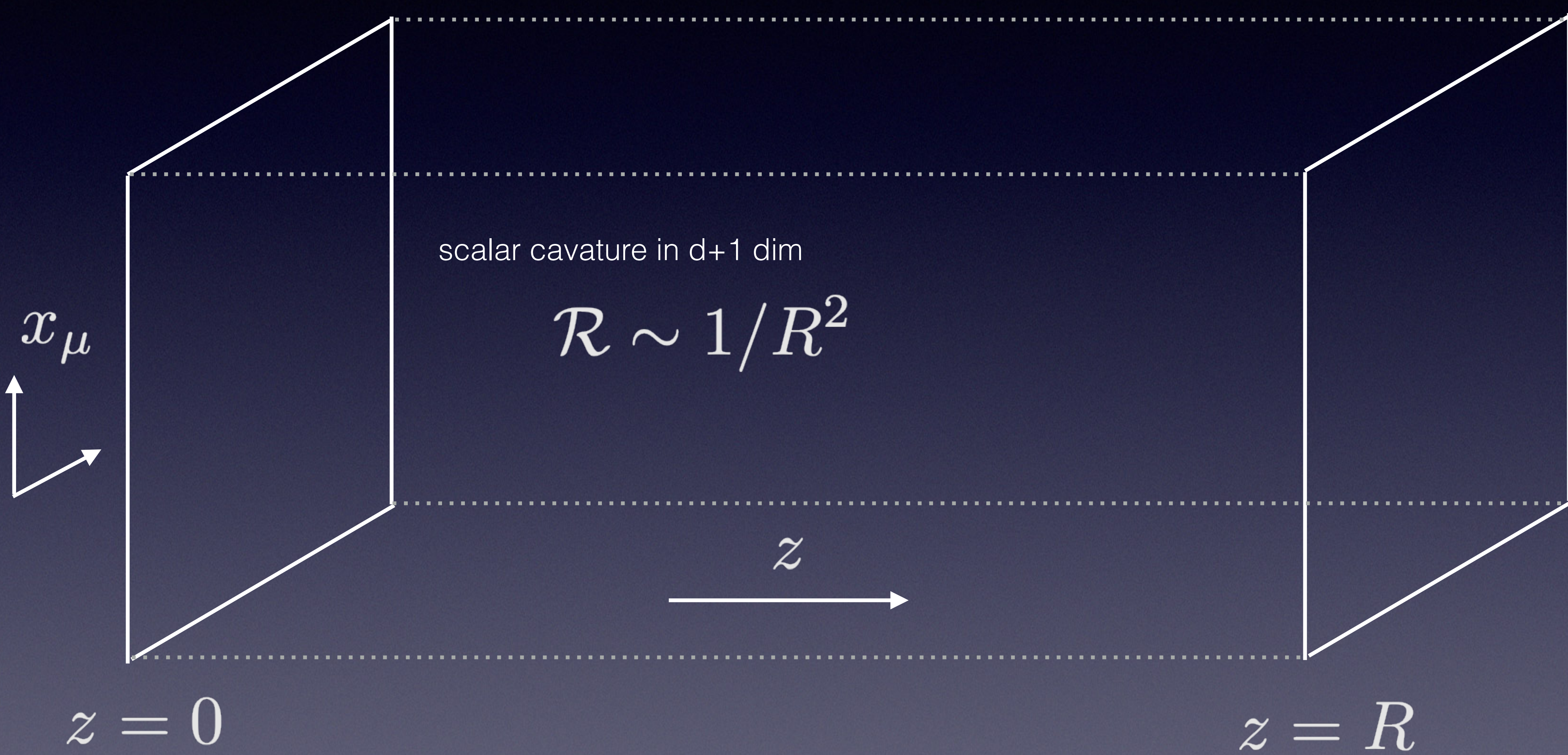
- e.g. Excitations of rho meson (vector meson)



Contents

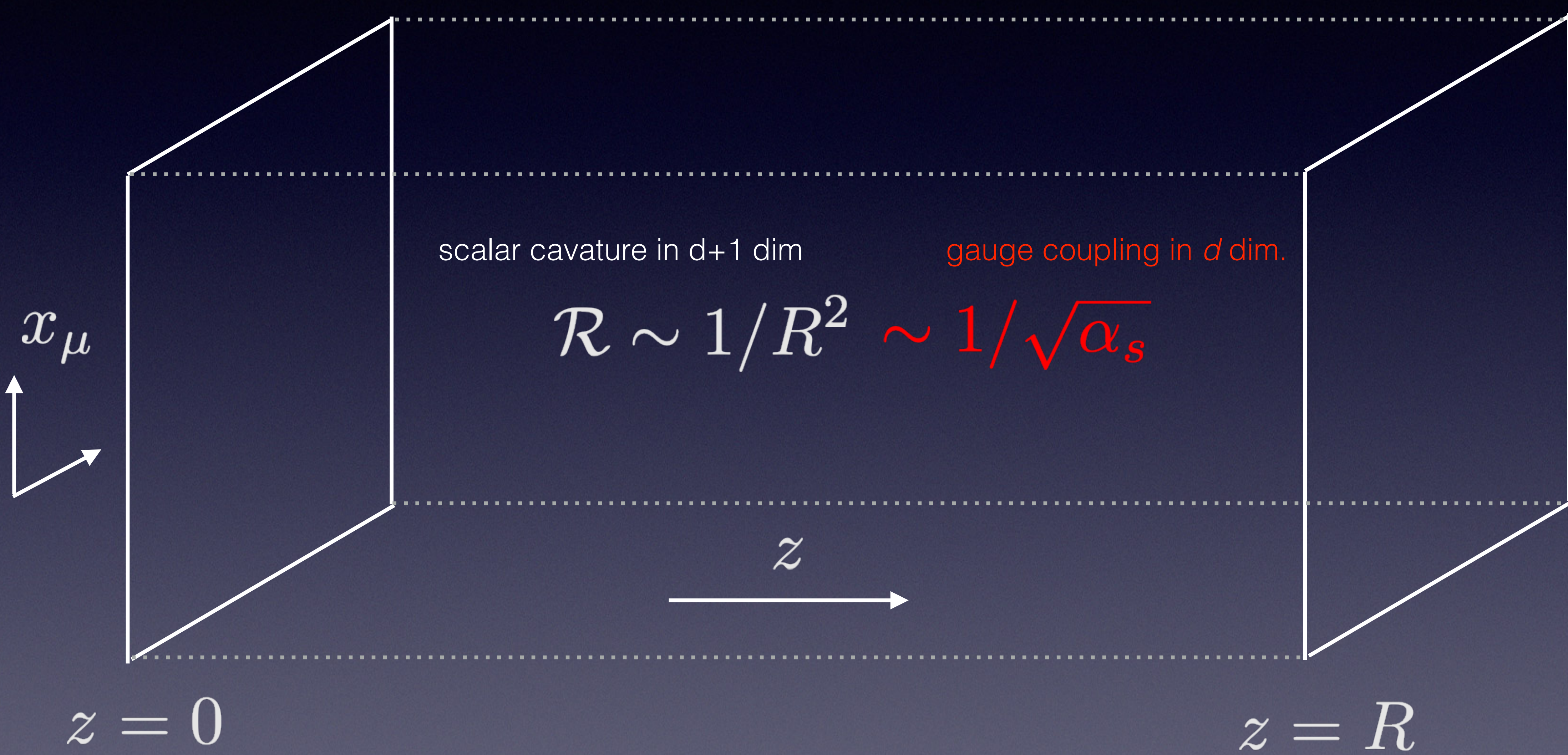
- AdS/CFT correspondence and Regge trajectory
- Wilsonian Renormalization Group
- From Wilsonian RGE to holographic equations

AdS$_{d+1}$ spacetime



$$\mathrm{d}s^2 = g_{MN}\mathrm{d}x^M\mathrm{d}x^N = \frac{R^2}{z^2}\left(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}z^2\right)$$

AdS$_{d+1}$/CFT$_d$ conjecture (gravity/gauge conjecture)



$$ds^2 = g_{MN} dx^M dx^N = \frac{R^2}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

Dynamics on AdS

- Free scalar (spin-0 pion, $\Phi(x,z)$) theory on AdS_{4+1}

$$S = \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}z\sqrt{g}\left(g^{MN}\partial_M\Phi\partial_N\Phi - \mu^2\Phi^2\right)$$

- Equation of motion for $\Phi(x,z) = e^{-iP\cdot x}\phi(z)$

$$[z^2\partial_z^2 - 3z\partial_z + z^2M^2 - (\mu R)^2]\phi(z) = 0$$

$$M^2 = P_\mu P^\mu$$

- This equation of motion is equivalent to

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \frac{1-4L^2}{4z^2}\right)\psi(z) = M^2\psi(z)$$

$$\psi(z) \sim z^{-3/2}\phi(z)$$

$$L^2 = (\mu R)^2 + 4$$

Dynamics on AdS

- Considering dilaton effects, $\Phi(x,z) = e^{-iP\cdot x} e^{-\varphi(z)/2} \phi(z)$

$$[z^2\partial_z^2 - 3z\partial_z + z^2 m_n^2 - (\mu R)^2 - z^2 U(z)]\phi(z) = 0$$

or

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}z^2} - \frac{1-4L^2}{4z^2} + U(z)\right)\psi(z) = m_n^2 \psi(z)$$

- where $U(z)$ is given in terms of $\varphi(z)$.
- Solving the Schrödinger equation with $U(z)$, we can obtain the eigenvalues!

Application to Regge trajectory

- If $U(z) \sim z^2$, the Schrödinger equation reproduces

$$m_n^2 \sim n$$

- This explains the Regge trajectory!
- But, how to obtain such a potential?
- This is a big open question.
- Our method provides $U(z)$!

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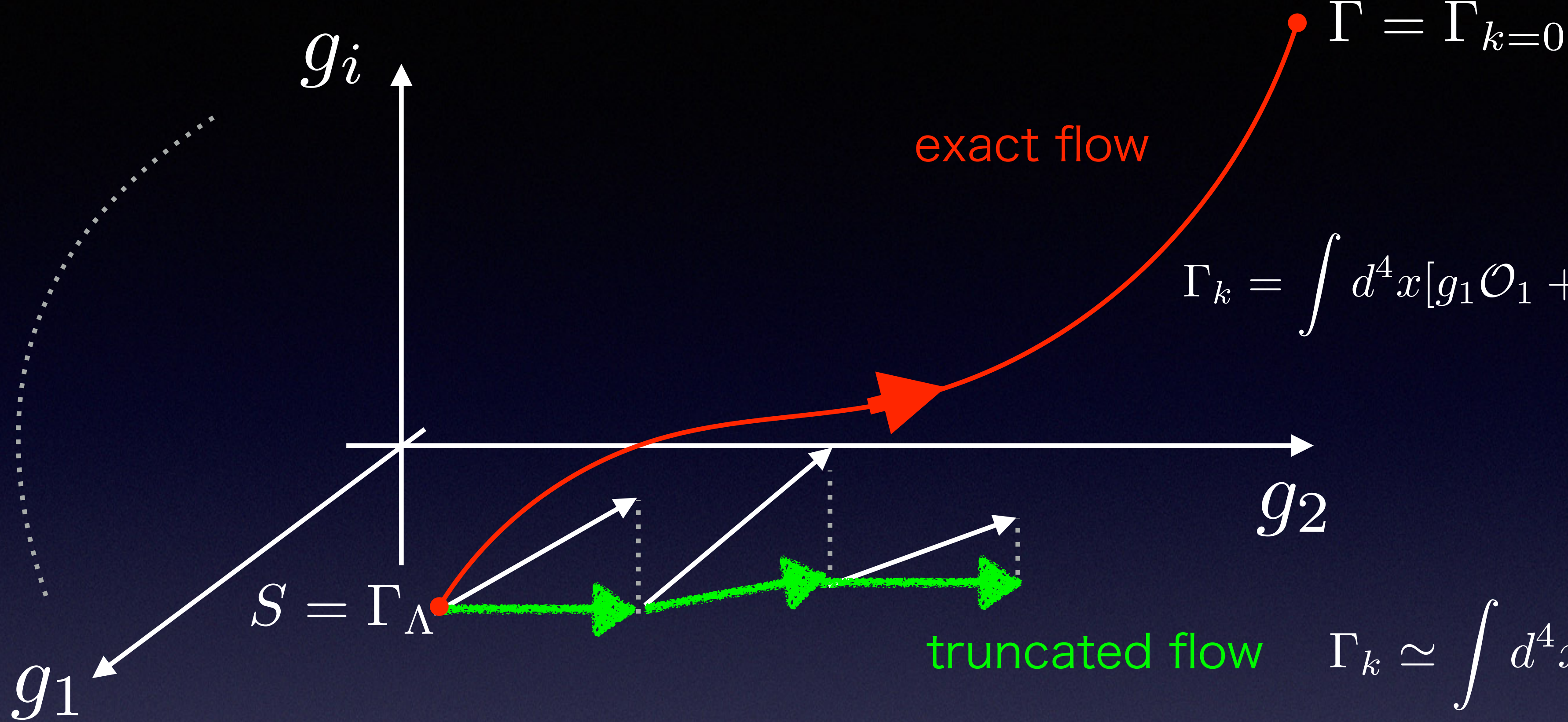
Wilson Renormalization Group



$$\Gamma_k = \int d^4x[g_1\mathcal{O}_1 + g_2\mathcal{O}_2 + \cdots + g_i\mathcal{O}_i + \cdots]$$

projection

$$\Gamma_k \simeq \int d^4x[g_1\mathcal{O}_1 + g_2\mathcal{O}_2]$$

RGE

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Str}[(\Gamma_k^{(2)} + R_k)^{-1}k\partial_k R_k]$$

Fixed point

$$k\partial_k\Gamma_k = \int d^4x[\underbrace{(k\partial_k g_1)}_{\beta_1(g)}\mathcal{O}_1 + \underbrace{(k\partial_k g_2)}_{\beta_2(g)}\mathcal{O}_2 + \cdots + \underbrace{(k\partial_k g_i)}_{\beta_i(g)}\mathcal{O}_i + \cdots]$$

$$k\partial_k\Gamma_k^* = 0$$

$$\beta_i(g^*) = 0$$

Critical exponent

$$k\frac{dg_i}{dk} = \beta_i(g)$$

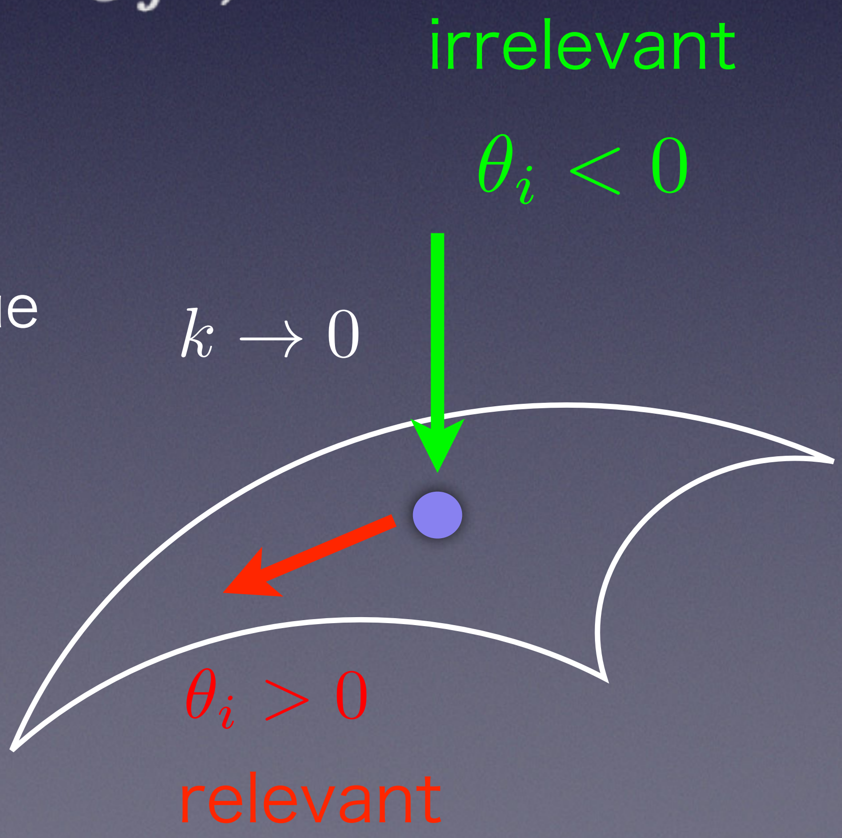
- RG eq. around FP g*

$$k\frac{dg_i}{dk} \simeq \cancel{\beta_i(g_*)} + \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g_*}(g_j - g_{j*})$$

negative eigenvalue

- Solution of RG eq.

$$g_i(k) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{k}{\Lambda}\right)^{-\theta_j}$$



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Equivalence

- Identification I: 5 dim coordinate = cutoff scale

$$k = 1/z$$

- Identification II: n-point functions = wave functions

$$\Gamma_k^{(n)} \sim [z^{d-\eta}\phi(z)]^{-1}$$

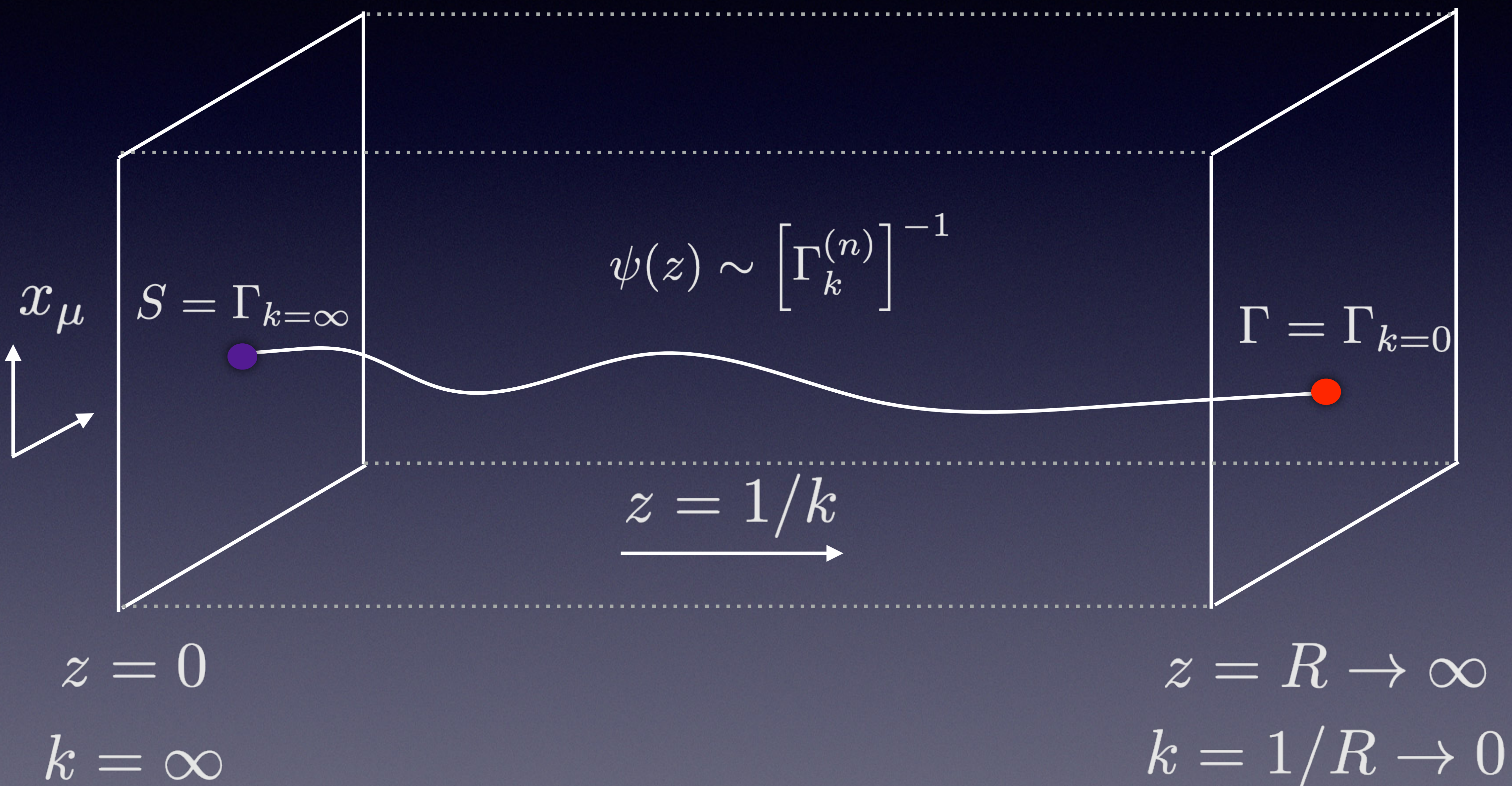
$$[z^2\partial_z^2 - 3z\partial_z + z^2 m_n^2 - (\mu R)^2 - z^2 U(z)]\phi(z) = 0$$

$$U_J(z) = -\frac{z^{-1-\delta}}{\Gamma_z^{(n)}}\partial_z\left(z^\delta \beta_{\Gamma_k}^{(n)}\right)$$

$$\delta = 2J + d - 2\eta$$

$$(\mu R)^2 = (d-\eta-\delta)(d-\eta) = (J-d/2)^2 - \delta^2/4$$

AdS spacetime



Application to 3d scalar theory

- We demonstrate bound states of scalar fields.

- Two-point function

$$\Gamma_k^{(2)}(p,-p) = \langle \phi(p)\phi(-p) \rangle_{1\mathrm{PI}}$$

- Zero momentum limit

$$\Gamma_k^{(2)}(0,0) \sim m_k^2$$

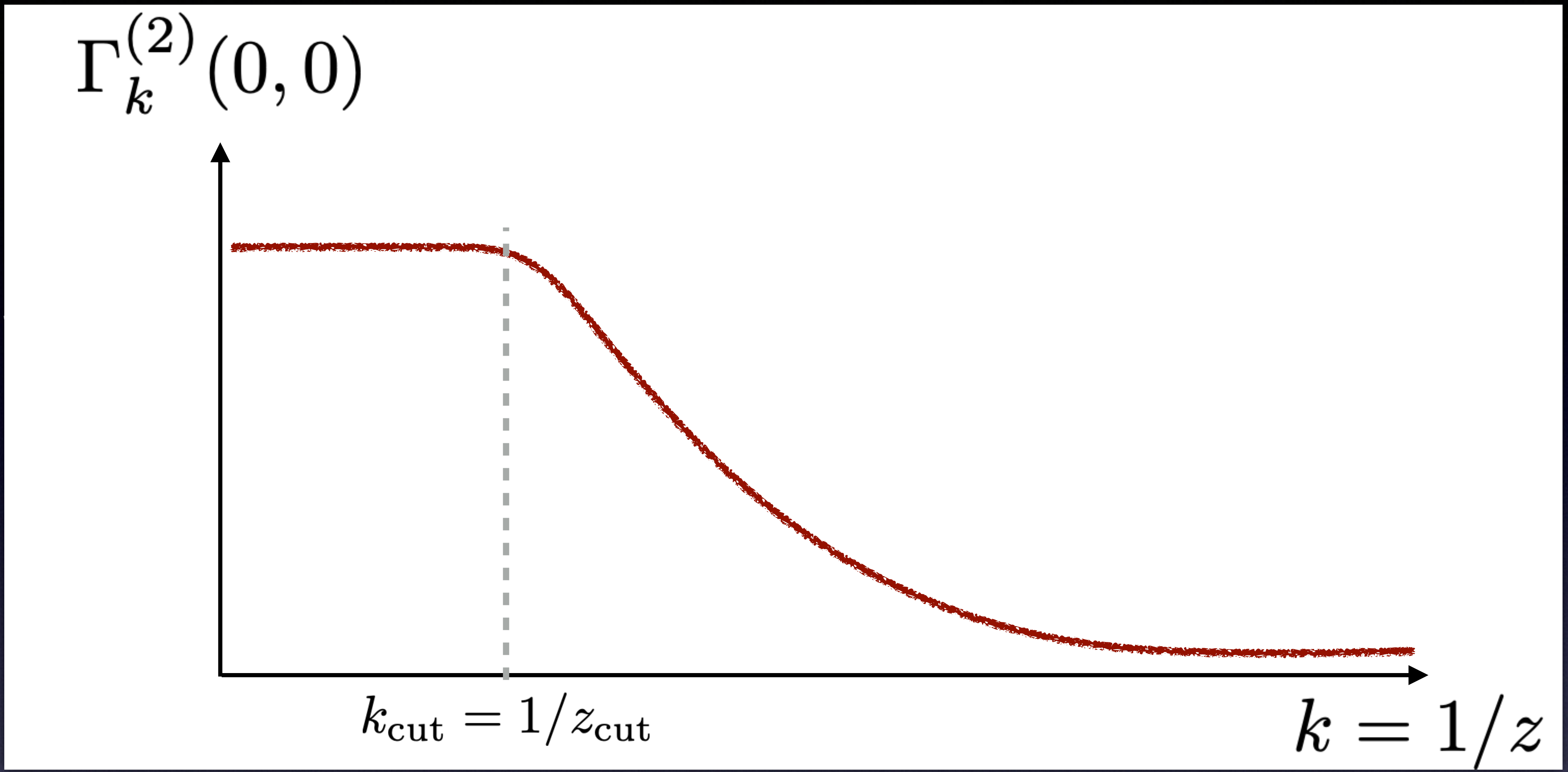
Demonstration

- 3D scalar theory
  - Gaussian fixed point (UV FP)
    - $\Gamma_k^{(2)}(0,0) \sim k^{-2}$ $\Rightarrow$ $U(z) = 0$
  - Wilson-Fisher fixed point (IR FP)
    - $\Gamma_k^{(2)}(0,0) \sim k^{-\theta_m}$ $\Rightarrow$ $U(z) = \gamma z^{-2+\theta_m}$

$\theta_m = 1.59$

$\gamma = -0.15$

$\psi(z_{\text{cut}} = 10) = 0$



- Wilson-Fisher fixed point (IR FP)
  - $\Gamma_k^{(2)}(0,0) \sim k^{-\theta_m}$ → $U(z) = \gamma z^{-2+\theta_m}$

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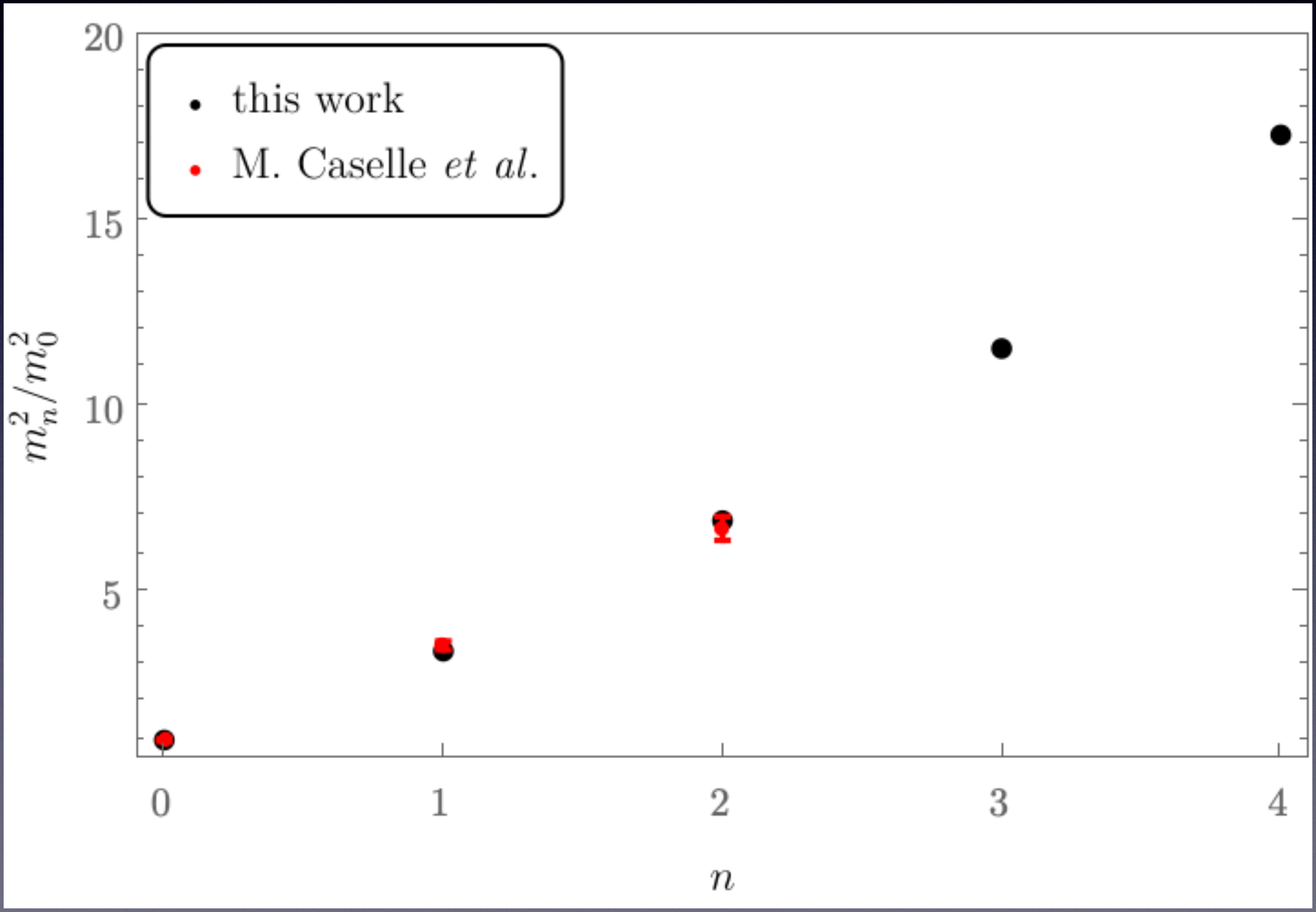
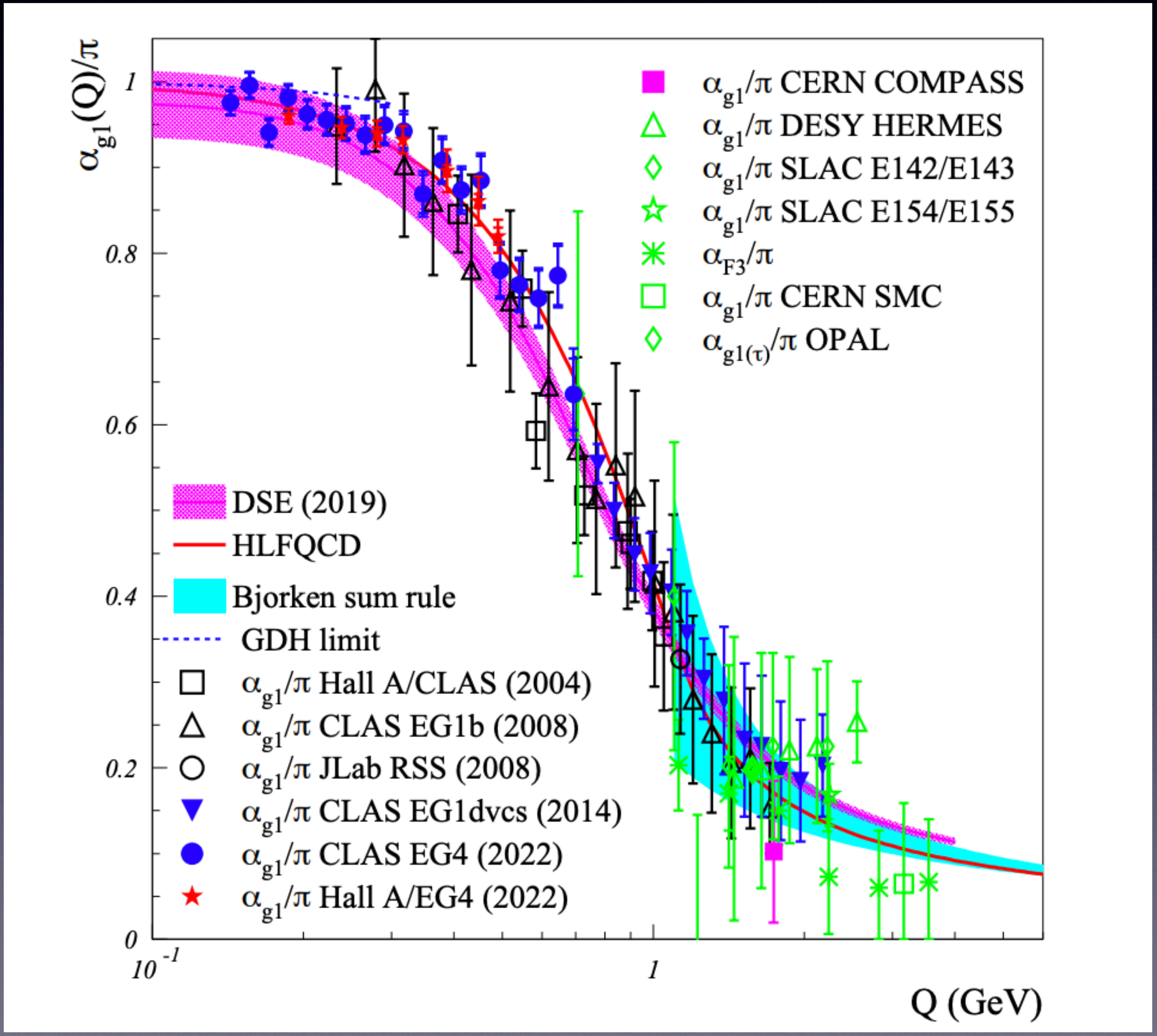
Excited mass spectra of bound state



QCD?



arXiv: 2303.00723

arXiv:2303.00723v1 [hep-ph] 1 Mar 2023

QCD Running Couplings and Effective Charges

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2023 January 20

arXiv:2303.00723v1 [hep-ph] 1 Mar 2023

QCD Running Couplings and Effective Charges

All methods indicated above lead to the result that $U(\zeta^2)$ has the form of a harmonic oscillator [476]:

$$U(\zeta^2) = \kappa^4\zeta^2 + b, \tag{4.15}$$

where κ depends only on AdS space dimension and b is determined by the spin J representations in AdS space. For a 2-body system in AdS_5, $b = 2\kappa(J-1)$. With $m_q = 0$ and Eq. (4.15), a wide range of phenomena are reproduced and predicted, including the following.

(i) Nonperturbative running of $\alpha_s(Q^2)$ with the prediction of freezing in the IR.

(ii) Regge trajectory descriptions of hadron spectra [338, 477]. (Within HLFQCD, this property might be linked with the IR-freezing of $\alpha_s(Q^2)$ [475].)

(iii) A connection between the soft and hard pomerons [382].

(iv) Prediction of a symmetry between the masses of baryons, mesons, and tetraquarks with universal Regge trajectories [338, 477].

(v) Predictions for hadron form factors [118, 380], PDFs [339, 340, 342], and GPDs [341].

[475] F. Gao and M. Yamada, Phys. Rev. D **106**, 126003 (2022).

SUSY??

H. G. Dosch et. al.
arXiv: 1801.00607
Phys.Rev.D 91 (2015) 8, 085016

For the baryons we had obtained, see (4.37):

$$\left(-\partial_z^2 + \frac{4L^2-1}{4z^2} + \lambda_F^2 z^2 + 2(L+1)\lambda_F\right)\Psi^+(q,z) \;=\; M^2\Psi^+(q,z) \qquad (5.39)$$

$$\left(-\partial_z^2 + \frac{4(L+1)^2-1}{4z^2} + \lambda_F^2 z^2 + 2L\,\lambda_F\right)\Psi^-(q,z) \;=\; M^2\Psi^-(q,z) \qquad (5.40)$$

and for mesons with $J = L + S$, see (4.28)

$$\Big(-\partial_z^2 + \frac{4L^2-1}{4\zeta^2} + \lambda^2\zeta^2 + 2(J-1)\lambda\Big)\tilde{\Phi}_{L,J}(q,z) = q^2\tilde{\Phi}_{L,J}(q,z) \qquad (5.41)$$

- Meson/Baryon mass spectra are well-reproduced from superconformal QM.
- Emergence of SUSY from QCD dynamics?

Summary and discussions

- The flow equation can be regarded as the Schrödinger equation on AdS spacetimes,
  - where $k = 1/z$
- A duality is encoded in the fRG!
- Fixed point = scale (conformal) invariant point
- Scaling law (Critical exponent) determines the potential.
- The Regge trajectory implies the existence of an IR fixed point in QCD, at which the scaling law gives $U(z) \sim z^2$.
- Light-front quantized QCD: $\phi_J(z)$ is associated to spin-J mesons.
- Emergence of SUSY from QCD dynamics?