非可換ゲージ理論における
トポロジカル電荷の観測不可能性

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(KMI, Nagoya University / Riken)

Based on
N. Yamanaka, arXiv:2212.10994 [hep-th]
N. Yamanaka, arXiv:2212.11820 [hep-ph]
See also https://www2.yukawa.kyoto-u.ac.jp/~nodoka.yamanaka/topologicalcharge/

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Topological charge

Topological charge

$$\tilde{F}_a^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma}$$

$$K_\mu \equiv \frac{\alpha_s}{8\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu F_a^{\rho\sigma} - \frac{g_s}{3}f_{abc}A_a^\nu A_b^\rho A_c^\sigma\right]$$

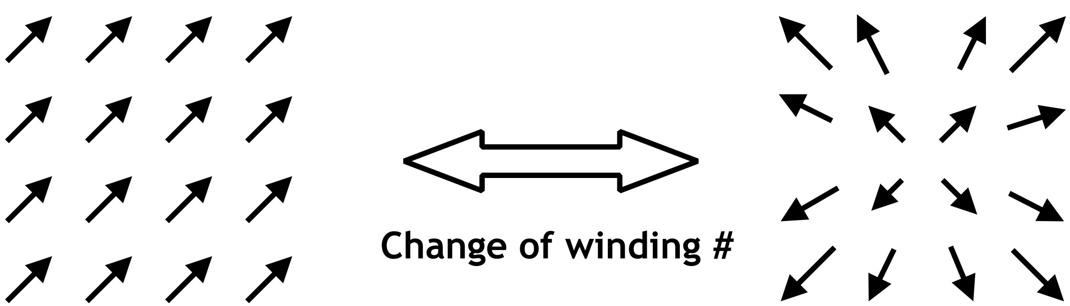
$$\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}_a^{\mu\nu} = \int d^4x\, \partial_\mu K^\mu$$

$$= \frac{ig_s\alpha_s}{24\pi}\int d^3\vec{x}\, f_{abc}\epsilon_{ijk}A_{ia}(\vec{x})A_{jb}(\vec{x})A_{kc}(\vec{x})\Bigg|_{t=-\infty}^{t=+\infty}$$

$$= \Delta n$$ **Integer!**

Integral of total derivative, but nonzero!

⇒ Δ*n* = change of winding number of gauge configurations



n = 0

Change of winding #

n = 1

(3-dimension in reality)

Theta-term and Strong CP problem

QCD vacuum is a coherent superposition of vacua with different winding number (topological charge)

$$|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$$

A correlator in the path integral formulation is written as

$$\langle\theta|O_{\rm phys}|\theta\rangle = \sum_{n,m} e^{-i(n-m)\theta}\langle m|O_{\rm phys}|n\rangle$$
$$= \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\, O_{\rm phys} e^{i\int d^4x\,\mathcal{L}_{\rm QCD} + i\theta\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu a}\tilde{F}^{\mu\nu}_a}$$

(example of one flavor QCD)

If θ is not zero, $\mathcal{L}_\theta = \theta\frac{\alpha_s}{8\pi}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ **appears effectively in Lagrangian**

(so-called θ-term)

There is in principle no symmetry argument to forbid θ-term

Strong CP problem

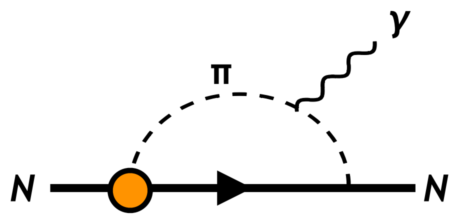
θ-term : $\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$

⇒ A natural CP-odd interaction term of QCD, θ should be O(1)

Neutron EDM (also atoms) is strongly constrained by experiments

$|d_n| < 1.8 \times 10^{-26}$ e cm

C. Abel et al., Phys. Rev. Lett. **124**, 081803 (2020).



Converting to θ via chiral EFT analysis,

$$|\theta| < 1 \times 10^{-10}$$

⇒ Why so small??

R. G. Crewther et al., Phys. Lett. B **88** (1979) 123.

➡ Strong CP problem

Resolutions proposed for the Strong CP problem

Conventional resolutions:

- Massless up quark
- Spontaneous CP breaking
- Axion mechanism

} Need new physics beyond standard model

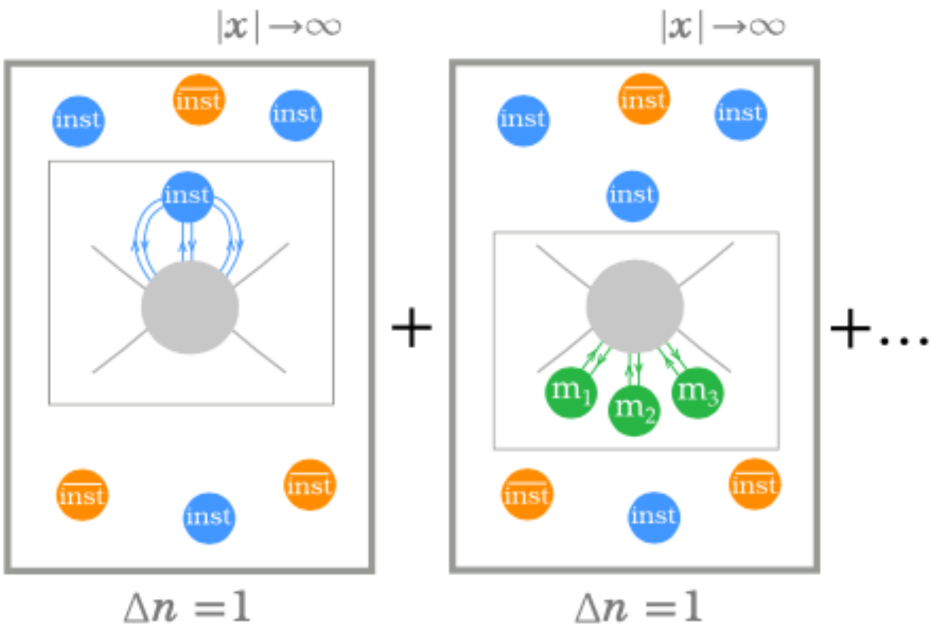
Recently, several intrinsic QCD resolutions were also proposed:

- CP conserving correlations under instanton background
  W.-Y. Ai, J. S. Cruz, B. Garbrecht, and C. Tamarit, Phys. Lett. B **822**, 136616 (2021).
- Color field screening by finite θ
  Y. Nakamura and G. Schierholz, Nucl. Phys. B **986**, 116063 (2023).
- Unphysical topological charge
  NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

CP conserving correlations under instanton background

Basic idea :

Calculate quark correlators with CP-odd mass within dilute instanton gas



Take first infinite volume (take all instantons)

Then, sum over topology

(order of limits is important, relying on semi-classical expansion?)

⇒ Effect of θ cancels with fermion mass complex phase!

⇒ Effective θ is zero!

W.-Y. Ai, J. S. Cruz, B. Garbrecht, C. Tamarit, Phys. Lett. B **822**, 136616 (2021).

Color field screening by finite θ

Basic idea :

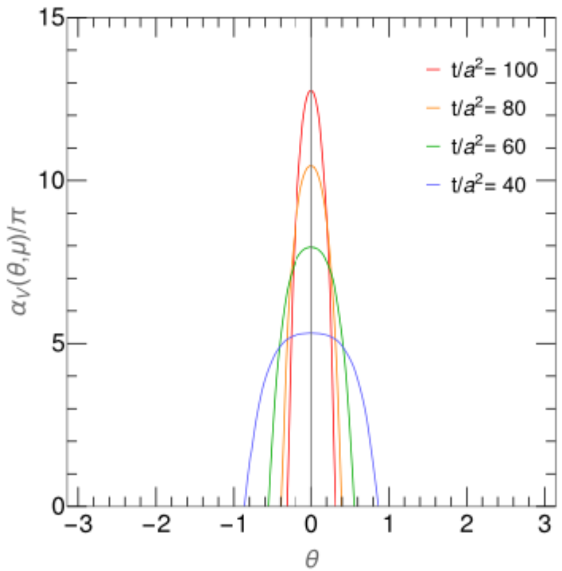
Magnetic monopole condenses in color confining vacuum (dual superconductor)

Magnetic monopoles get electric charge under finite θ

E. Witten, Phys. Lett. B **86** (1979) 283.

Colored particles form bound-state with magnetic monopoles?

If yes, deconfined colored particles with |θ| > 0 !



Calculate running of α_s with gradient flow

α_s damps for |θ| > 0 with decreasing scale!
(α_s only grows for θ = 0)

⇒ Confinement only at θ = 0 !

From RG flow of θ-term (fitted from fig.), θ → 0 in IR limit

Y. Nakamura and G. Schierholz, Nucl. Phys. B **986**, 116063 (2023).

Object of study

We propose a new approach:
the strong CP problem is resolved if the topological charge is unphysical.

Objective:

We show that the topological charge of nonabelian gauge theory is unphysical, and inspect the phenomenological consequences.

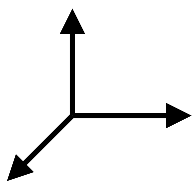
N. Yamanaka, arXiv:2212.10994 [hep-th]
N. Yamanaka, arXiv:2212.11820 [hep-ph]

Unphysical topological charge

Topological charge is a triple product of gauge fields

$$\frac{ig_s\alpha_s}{24\pi}\int d^3\vec{x}\, f_{abc}\epsilon_{ijk}A_{ia}(\vec{x})A_{jb}(\vec{x})A_{kc}(\vec{x})\Big|_{t=-\infty}^{t=+\infty}$$

Time is frozen, restricted to 3-dimensional space

ε_{ijk} means  **⇒ Covers all 3-dimensional directions!**

⇒ Also covers the unphysical gauge freedom direction!

$$(\vec{A}_a \to \vec{A}_a + \vec{\nabla}\chi_a)$$

Topological charge is unphysical !!

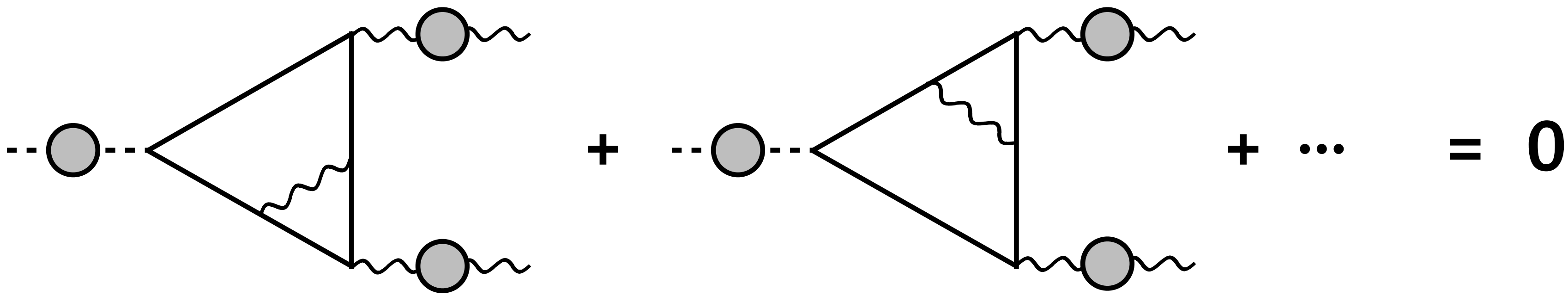
This only holds in perturbation theory … Is it really OK ??

NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

Adler-Bardeen theorem

Radiative corrections do not contribute to the chiral anomaly up to renormalization of external fields

⇒ Works for nonabelian gauge theories, even with nonrenormalizable interaction



S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969);
D. Anselmi, Phys. Rev. D **91**, 105016 (2015).

Chiral anomaly = topological charge does not change at any scale, even if external fields are "dressed" by nonperturbative effects ('t Hooft anomaly matching)

⇒ Topological charge is strictly perturbation finite!

⇒ We can FULLY use the argument of unphysical gauge component!

(To be precise, it is possible to show the irrelevance of Gribov copies)

For detail, see NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph]

Ward-Takahashi identity (WTI) of BRST symmetry

Let us also show the unphysicalness of topological charge using WTI

Assume the following BRST algebra

$$\begin{cases} [Q_B, A(x)] = iC(x) \\ \{Q_B, \bar{C}(y)\} = B(y) \\ \{Q_B, C(x)\} = 0 \\ [Q_B, B(x)] = 0 \end{cases}$$

The following WTI then holds:

$$\langle 0|\{Q_B, T[A(x), \bar{C}(y)]\}|0\rangle$$

$$= \boxed{\langle 0|T[A(x)B(y) - iC(x)\bar{C}(y)]|0\rangle = 0}$$

Here operators A and C̄ may be chosen arbitrarily.

It is possible to derive the unphysicalness of topological charge by choosing suitable operators (next page).

T. Kugo and I. Ojima, Phys. Lett. B **73**, 459 (1978);
T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**,1 (1979).

WTI for topological charge

Consider the following (topological) BRST quartet

$$\begin{cases} K_\mu = \frac{\alpha_s}{4\pi}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\partial^\rho A_a^\sigma + \frac{1}{3}g_s f_{abc}A_a^\nu A_b^\rho A_c^\sigma\right] \\ \mathcal{C}_\mu = \frac{\alpha_s}{4\pi}\epsilon_{\mu\nu\rho\sigma}(\partial^\nu c_a)(\partial^\rho A_a^\sigma) \\ \bar{\mathcal{C}}_\mu = \frac{\alpha_s}{8\pi}g_s f_{abc}\epsilon_{\mu\nu\rho\sigma}(\partial^{-2}\partial^\nu\bar{c}_a)A_b^\rho A_c^\sigma \\ \mathcal{B}_\mu = \frac{\alpha_s}{8\pi}g_s f_{abc}\epsilon_{\mu\nu\rho\sigma}\left[(\partial^{-2}\partial^\nu B_a)A_a^\rho A_b^\sigma + (\partial^{-2}\partial^\nu\bar{c}_a)F_b^{\rho\sigma}c_c\right] \end{cases}$$

← **Topological current** (K_μ)

T. Kugo, Nucl. Phys. B **155**, 368 (1979).

← **We have freedom to choose it** ($\bar{\mathcal{C}}_\mu$)

$F_a^{\mu\nu} \equiv \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_s f_{abc}A_b^\mu A_c^\nu$

After taking total derivatives and taking $|x-y|\to\infty$, we obtain the following "topological" WTI

$$\sum_{|\Omega\rangle\neq|0\rangle}\langle 0|\partial_\mu K^\mu(x)|\Omega\rangle\langle\Omega|\partial_\nu\mathcal{B}^\nu(y)|0\rangle = 0$$

(ghost term cancels)

The only surviving operator is $F\tilde{F}$ thanks to Adler-Bardeen theorem!

$$\propto \sum_{|\Omega\rangle\neq|0\rangle}\langle 0|F\tilde{F}(x)|\Omega\rangle\langle\Omega|F\tilde{F}(y)|0\rangle = 0$$

$|\Omega\rangle$ **:vacua belonging to different topological sectors**

⇒ Effect of $F\tilde{F}$ vanishes at the level of observables even if amplitude is finite!

For detail, see NY, arXiv:2212.11820 [hep-ph]

Operator product expansion (OPE)

Is it possible to probe the topological sector of physical states?

⇒ What is the value of $\langle \mathrm{phys}' | F\tilde{F} | \mathrm{phys} \rangle \equiv \langle 0 | F\tilde{F}(x) \phi(x') | 0 \rangle$?

Operator product expansion:

Operators separated by finite distances may be rewritten as a sum of local operators O_i

$$F\tilde{F}(x)\phi(y) = \sum_{O_i \neq F\tilde{F}} C_i O_i \left(\frac{x+y}{2} \right)$$

ϕ : arbitrary operator

Important point:
Adler-Bardeen theorem forbids generation of single topological charge density operator

⇒ We lose the information of topological sector for finitely distanced and correlated operators

We must therefore separate with infinite distance (or isolate them to not make them interact)

➡ Finitely separated operators do not change topology (correlated)

Generalized topological WTI

To avoid OPE, we must isolate operators by infinite distances (or uncorrelate)

The topological WTI for arbitrary Green's function:

$$\sum_{|\Omega\rangle\neq|0\rangle} \langle 0|\phi(x')|0\rangle \underline{\langle 0|F\tilde{F}(x)|\Omega\rangle\langle\Omega|F\tilde{F}(y)|0\rangle} \langle 0|\phi(y')|0\rangle = 0$$

ϕ : arbitrary scalar BRST singlet function

⇒ Factorized into topological WTI thanks to operator product expansion

The topological WTI with higher power of $F\tilde{F}$ for arbitrary Green's function:

$$\sum_{\Omega_1,\cdots,\Omega_{2n-1}} \langle 0|\phi(x')|0\rangle\langle 0|F\tilde{F}(x_1)|\Omega_1\rangle\langle\Omega_1|F\tilde{F}(x_2)|\Omega_2\rangle \times \cdots$$
$$\times\langle\Omega_{2n-2}|F\tilde{F}(y_2)|\Omega_{2n-1}\rangle\langle\Omega_{2n-1}|F\tilde{F}(y_1)|0\rangle\langle 0|\phi(y')|0\rangle = 0$$

➡ Topological sectors and θ are unphysical !

For detail, see NY, arXiv:2212.11820 [hep-ph]

Now let us include fermions

Chiral Ward-Takahashi identity

The well-known chiral (or anomalous) WTI:

$$\partial_\mu J_5^\mu \equiv \partial_\mu \sum_\psi^{N_f} \bar{\psi}\gamma^\mu\gamma_5\psi = -2\sum_\psi^{N_f}\left[m_\psi \bar{\psi} i\gamma_5\psi\right] - \frac{N_f\alpha_s}{8\pi}F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}$$

Right-hand side has $F\tilde{F}$, we saw that its integral is unphysical

What is the unphysical part of the quark?

<u>Atiyah-Singer's theorem:</u>

$$\mathrm{ind}(\not{D}) = -\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}$$

(number of chiral Dirac zero-modes = topological charge)

Since the topological charge is unphysical,

chiral Dirac zero-modes of quarks are also unphysical !

$U(1)_A$ symmetry is physical

We remove the unphysical topological charge and chiral Dirac zero-modes from the known chiral WTI to obtain the "physical chiral WTI":

$$\sum_{\psi}^{N_f}\left[\partial^\mu \bar{\psi}\gamma_\mu\gamma_5\psi + 2m_\psi\bar{\psi}i\gamma_5\psi\right]_{\lambda\neq 0} = -\frac{N_f\alpha_s}{8\pi}F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}\Big|_{\Delta n=0}$$

Remove only chiral zero-modes (λ=0)

Right-hand side vanishes at zero momentum inflow since no topological charge (just a total divergence, without nontrivial effect at long distance)

⇒ Global conservation of "physical" $U(1)_A$ current!

(up to current quark mass m_ψ)

⇒ "Physical" chiral (=$U(1)_A$) rotation does not change topological charge !

⇒ $U(1)_A$ symmetry is physical at the "physical" Lagrangian level !

$U(1)_A$ of QCD is only broken spontaneously, restores at $T > T_c$

(Consistent with lattice calculations)

JLQCD Collaboration, PRD **87**, 114514 (2013), etc.

Chiral anomaly is physical

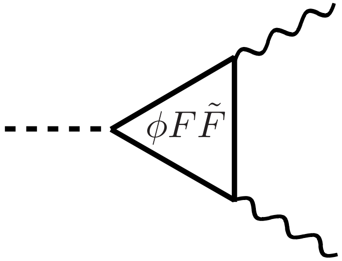
We saw that $U(1)_A$ is not explicitly broken

Is then the chiral anomaly unphysical?

⇒ No, the chiral anomaly can be physical,

only its 4-dimensional integral (=topological charge) is unphysical

Local anomaly interaction



⇒ Physical!

Anomaly only violates $U(1)_A$ locally!

(If we assign a charge to (pseudo)scalar ϕ, conservation law is OK)

⇒ η' decay, axion, CP-odd Higgs ... are physical!

Topological charge

$$\int d^4x\, F\tilde{F}$$

⇒ Unphysical!

This looks like a global violation of $U(1)_A$, but does not contribute to observables!

Chiral anomaly is physical

We saw that $U(1)_A$ is not explicitly broken

Is then the chiral anomaly unphysical?

⇒ No, the chiral anomaly can be physical,

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Local anomaly interaction

Important phenomenological consequence:

Axion may exist!

You can use axions in BSM models!

(axions are just pseudoscalar bosons, like CP-odd Higgs)

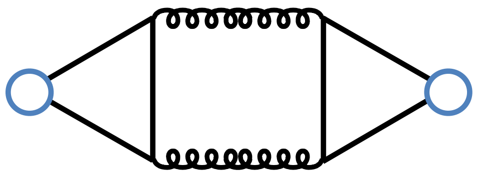
It is not anymore motivated by the Strong CP problem, but still attractive as candidate of dark matter

but does not contribute to observables!

Consistency with U(1) problem

U(1) problem: η' is heavy (960MeV), no $U(1)_A$ Nambu-Goldstone mode?

Conventionally, resolved by increasing η' mass by the topological susceptibility



$$\left(\frac{\alpha_s}{8\pi}\right)^2 \int d^4x\, e^{ik\cdot x} \langle 0|F_{\mu\nu,a}\tilde{F}_a^{\mu\nu}(x) F_{\rho\sigma,b}\tilde{F}_b^{\rho\sigma}(0)|0\rangle$$

$$= \sum_h \langle 0|F\tilde{F}|h(k)\rangle \frac{i}{k^2 - m_h^2} \langle h(k)|F\tilde{F}|0\rangle$$

$|h(k)\rangle$ **: hadronic state generated by FF̃**

However,

in zero momentum limit k=0, $\langle 0|F\tilde{F}|h(k=0)\rangle \propto \langle 0|F\tilde{F}|\Omega\rangle$

⇒ $\langle 0|F\tilde{F}|h(k=0)\rangle$ **and topological susceptibility at k=0 are unphysical!**

In the chiral limit, NG mode with $k^2=0$ has no mass shift ⇒ Massless!

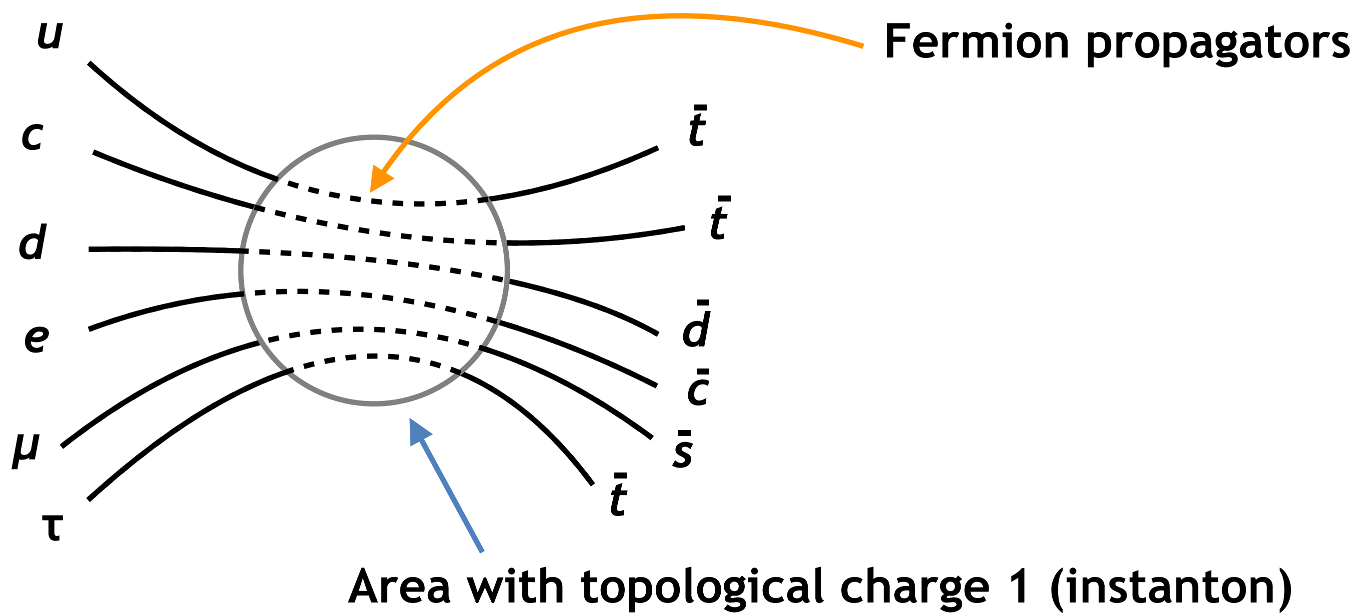
Why does η' have a large mass in real QCD?

The mass shift is probably due to nonzero quark mass which brings finite momentum inflow to topological susceptibility. This probably has a momentum dependence (to be checked with lattice).

Implications to particle physics phenomenology

’t Hooft vertex (conventional)

Consider the following multi-fermion amplitude in the standard model



Conventionally, amplitude of this process is finite in the chiral limit if
* all fermions have zero-modes
* all fermions are combined so as to form $U(1)_{B+L}$ anomaly

G. ’t Hooft, PRL **37**, 8 (1976), PRD **14**, 3432 (1976).

This gives rise to a (nonlocal) contact 12-fermion interaction (’t Hooft vertex)

⇒ Baryon and lepton numbers may be generated! (Previous understanding)

V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985), 36;
M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

Implication to baryogenesis

’t Hooft vertex is generated by the propagation of zero-modes

⇒ ’t Hooft vertex is unphysical !

(it may exist, but does not contribute to observables)

Important phenomenological consequences:

⇒ No instanton/sphaleron induced baryogenesis

⇒ Standard leptogenesis does not work (lepton # does not lead to baryon #)

⇒ We need explicit local baryon # violating interactions

such as Grand unified theories, leptoquarks, R-parity violation, …

A comment on lepton number violation:

Sterile neutrino, $0\nu\beta\beta$ decay are not mandatory for baryogenesis

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A comment on lepton number violation:

Sterile neutrino, $0\nu\beta\beta$ decay are not mandatory for baryogenesis

(However, you will certainly obtain the Nobel prize if you discover them because BSM!)

Lepton number asymmetry as a probe of our statement??

We saw that sphaleron induced B+L violation is unphysical

⇒ No wash-out

Baryon number of Universe is known : $B / s = 10^{-10}$

What about lepton number ?

➡ $L / s \sim 10^{-3}$ (from recent analysis of metal poor galaxies)

A. Matsumoto et al., Astrophys. J. **941** (2022) 167

L >> B ??

No wash-out, very unlikely to have had sphaleron processes

⇒ Support our statement!

There are other scenarios using “L-balls” to avoid wash-out, admitting sphaleron process

Kawasaki and Murai, JCAP 08 (2022) 041

What do CP phases of quark mass become?

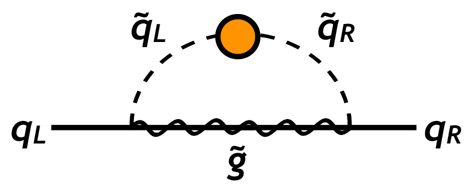
$$\mathcal{L}_{\rm odd} = -m_{\rm odd}\bar{\psi}i\gamma_5\psi$$

⇒ Introduce complex phase to quark mass

Generated in new physics beyond standard model (and also in standard model)

Example of supersymmetry:

Squarks have CP-odd transition



Physical particles are in mass eigenstates : real mass

CP-odd mass may be "rotated away" by $U(1)_A$ transformation:

$$m_{\rm even}\bar{\psi}\psi + m_{\rm odd}\bar{\psi}i\gamma_5\psi \to m\bar{\psi}'\psi'$$

If θ-term is physical, chiral rotation cannot erase both θ and m_{odd}

From our discussion, θ-term is unphysical ⇒ Chiral rotation removes m_{odd} !

m_{odd} does not decouple with increasing BSM scale, but it is unphysical:
Our result (unphysical m_{odd}) leads to correct decoupling of BSM scale

Modification of the BSM phenomenology after our work

- Modification procedure:
  - Neglect θ-term
  - Neglect CP-odd mass of quarks
- Comparison with axion mechanism:
  - No axions
  - Induced θ-term is unphysical
- Impact on particle physics phenomenology:
  - The only source of CP violation of SM is the CP phase of CKM matrix
  - BSM contribution all decouples with increasing BSM energy scale (scales as power of $1/\Lambda_{BSM}$)
  - Baryogenesis is impossible in SM

Summary

- Configurations of nonabelian gauge theory have nontrivial topology, θ-vacua are their superpositions.
- θ is tightly constrained by EDM experiments: Strong CP problem.
- Recently, several resolutions of the Strong CP problem have been proposed within QCD.
- My claim : the topological charge is unphysical. Demonstrated in 2 ways using Adler-Bardeen theorem.
- From Atiyah-Singer theorem, chiral Dirac zero-modes are also unphysical ⇒ 't Hooft vertex is unphysical.
- Theoretical consequence: chiral anomaly does not break global symmetry, but only locally → No U(1) problem.
- Phenomenological consequences: no need for axions, sphaleron induced baryogengesis is forbidden.

Backup slides