Higgs mechanism without Higgs potential in an extra dimension

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Mysteries of the Standard Model

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> **Symmetry breaking**

Why does nature take M_H^2 negative?

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> Chiral theory

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> Symmetry breaking

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Why chiral theory the Standard Model is?

> Mass hierarchy

Why so different the masses of the fermions are?

Purpose

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In the context of 5d gauge theories on an interval, we want to figure out the mysteries of the standard model

> Symmetry breaking
> Chiral theory
> Mass hierarchy

naturally.

Setting

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> 5d U(1) gauge theory on an interval

with
- 5d fermions
- 5d complex scalar

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> All fields live in the bulk.

> No negative square mass for the scalar.

> General boundary conditions

compatible with { 5d gauge invariance / the action principle

Results

Results

5d gauge theories on an interval

↓ The low energy effective theories

4d gauge theories

- \+ Symmetry breaking
- \+ Chiral theories
- \+ Mass hierarchy

5d U(1) gauge field

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The boundary condition for the U(1) gauge field is restricted uniquely by the 5d gauge invariance.

$$\begin{cases} \partial_y A_\mu(x,0) = 0 \\ A_y(x,0) = 0 \end{cases}$$

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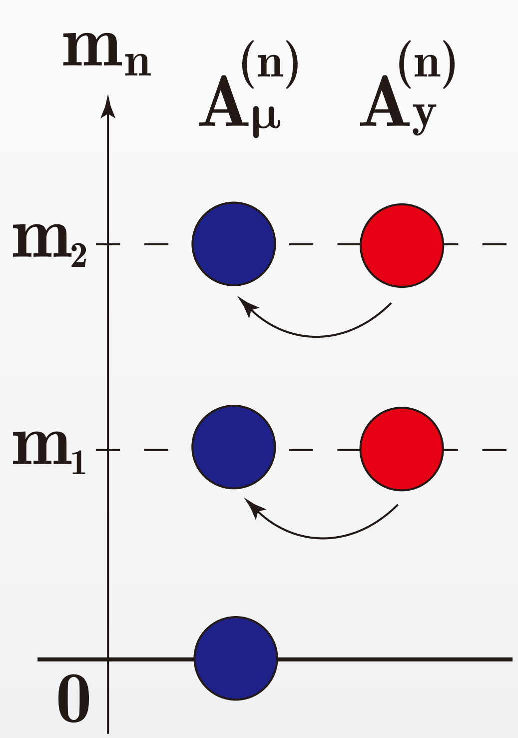
> 4d mass spectrum



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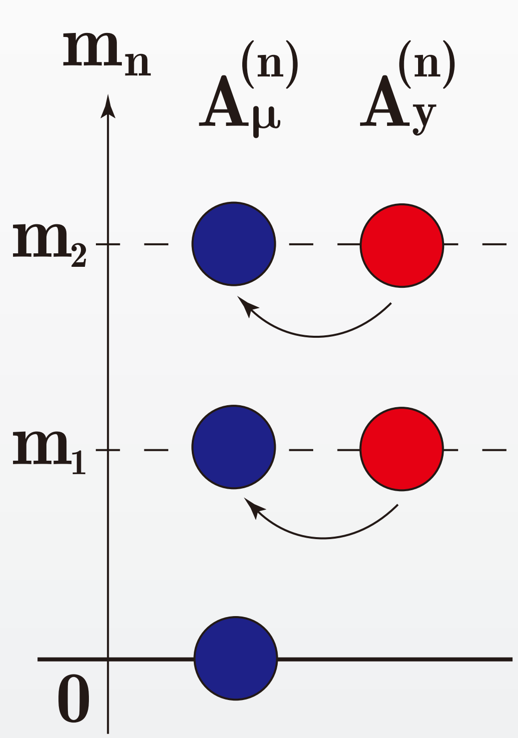

!

A wider class of boundary conditions are allowed to non-abelian gauge fields.

Higgs mechanism with $M^2>0$

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5d complex scalar field

$$S=\int d^4x\int_0^L dy\left[\Phi^*(x,y)(\partial^\mu\partial_\mu+\partial_y^2)\Phi(x,y)-V(\Phi)\right]$$

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>Possitive square mass

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+ **General boundary conditions**

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+ General boundary conditions

(2-parameter family)

$$\begin{cases} \Phi(x,0) + L_+ \partial_y \Phi(x,0) = 0 \\ \Phi(x,L) - L_- \partial_y \Phi(x,L) = 0 \end{cases}$$

No outflow of the probability current.

$$(-\infty \le L_\pm \le +\infty)$$

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> The vacuum of the theory is given by the configuration which minimizes $V_{4d}(\Phi)$.

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> The vacuum of the theory is given by the configuration which minimizes $V_{4d}(\Phi)$.

> What $V_{4d}(\Phi)$ looks like when we see it from the point of view of 4d mass eigenstates?

Higgs mechanism with $M^2>0$

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↓

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4d mass eigenstates (→ $\varphi_n(x)$)

mode functions (→ $f_n(y)$)

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Boundary conditions

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> **Negative square mass** if $E_B + M^2 < 0$ due to the eigenvalue of the bound state !!

Higgs mechanism with $M^2>0$

Non-trivial phase structures

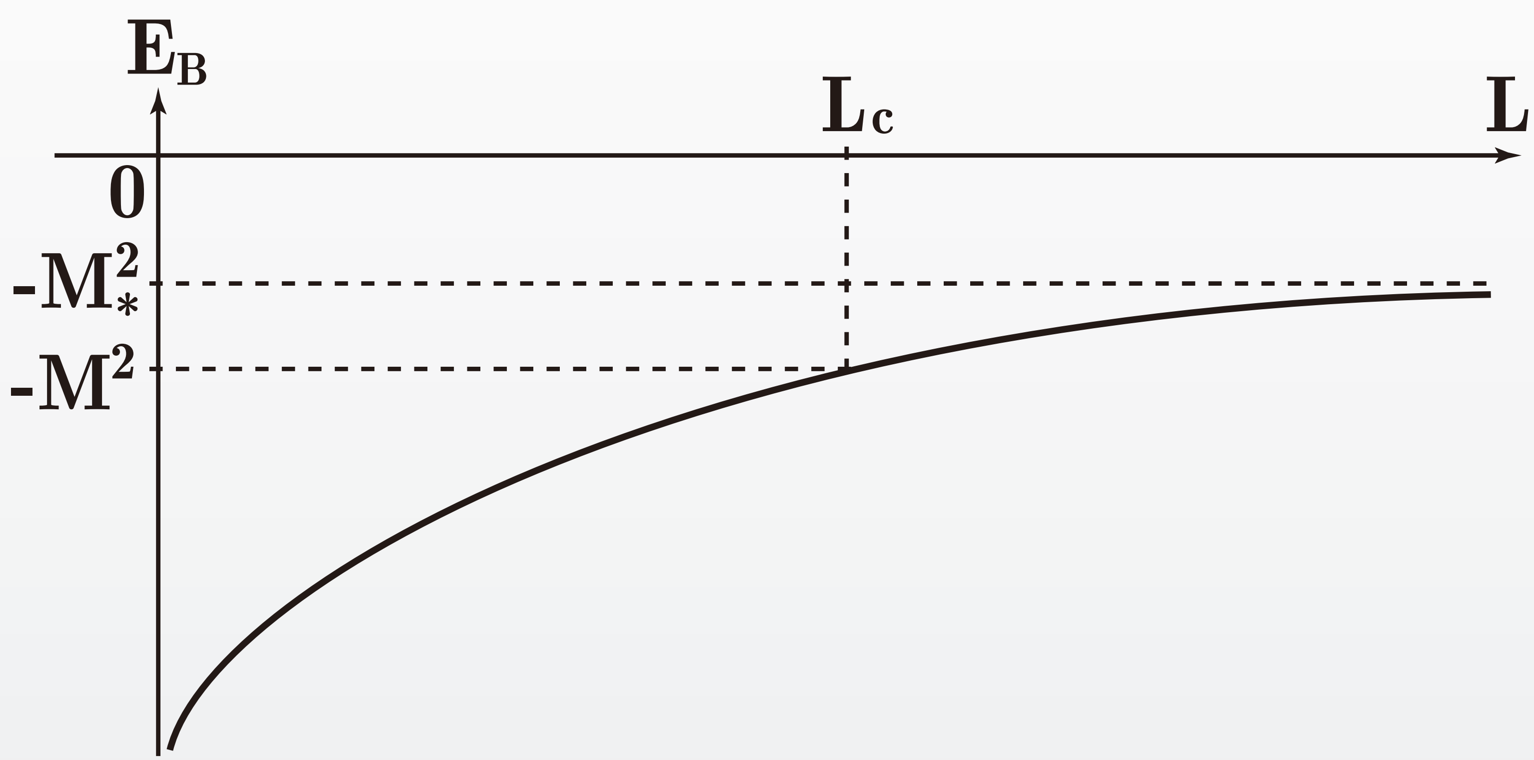
(e.g.) Energy spectrum of the bound state

$$L_\pm > 0 > L_\mp$$

with

$$L_\pm + L_\mp < 0$$

$L_\pm\ ,\ M^2$ **: fix**



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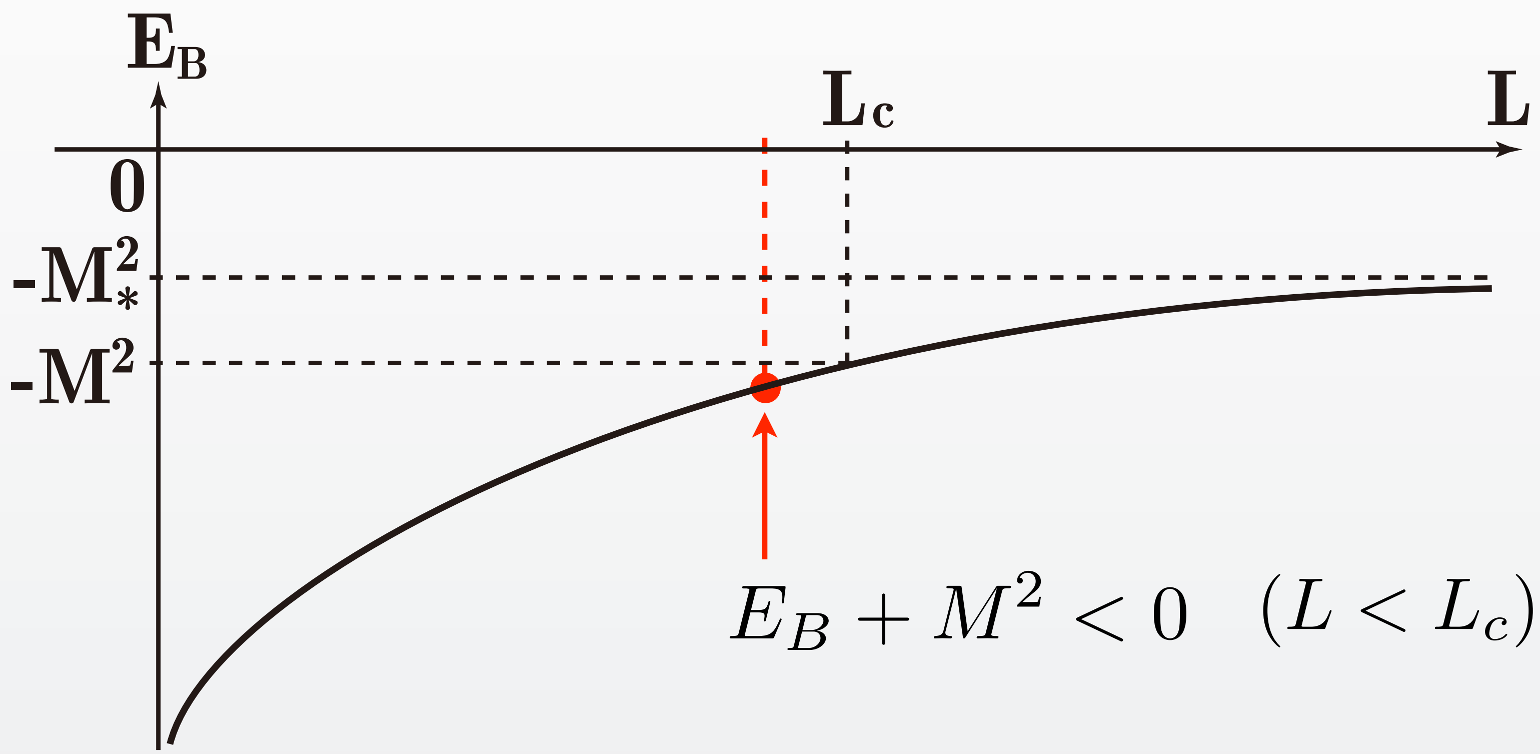

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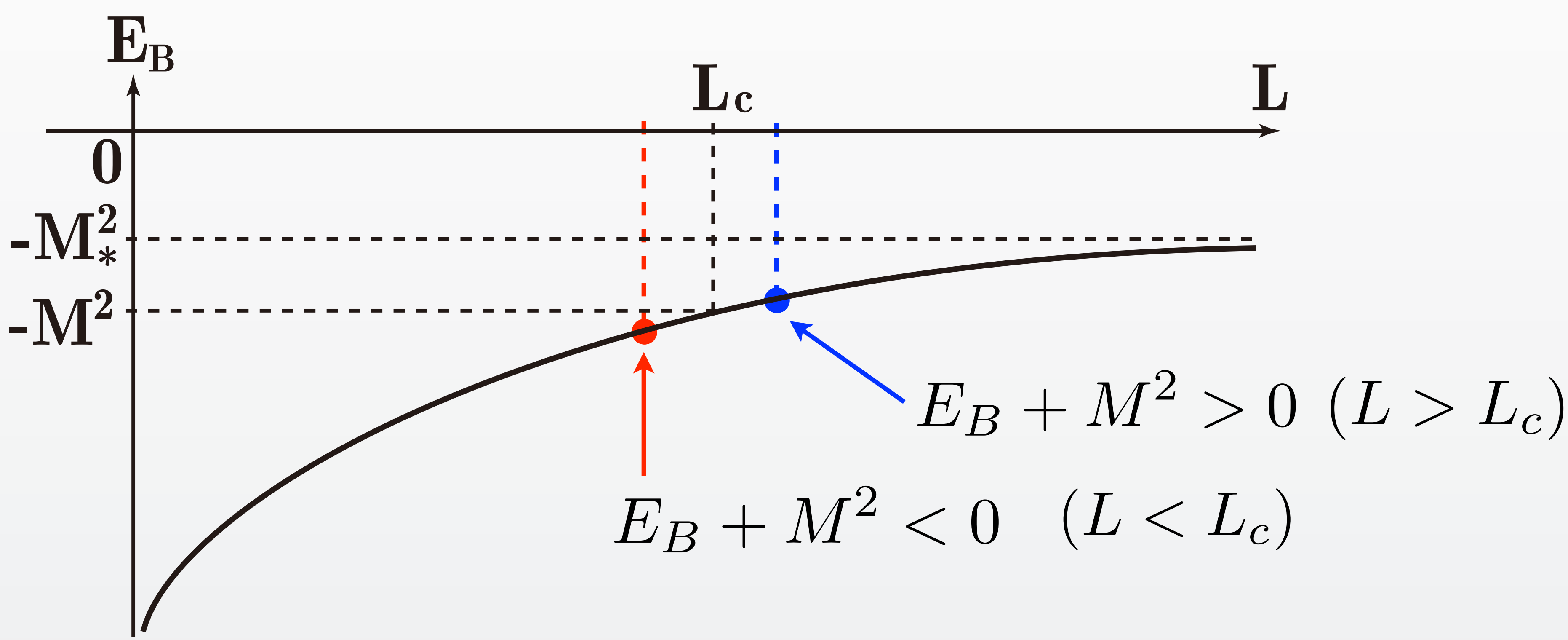

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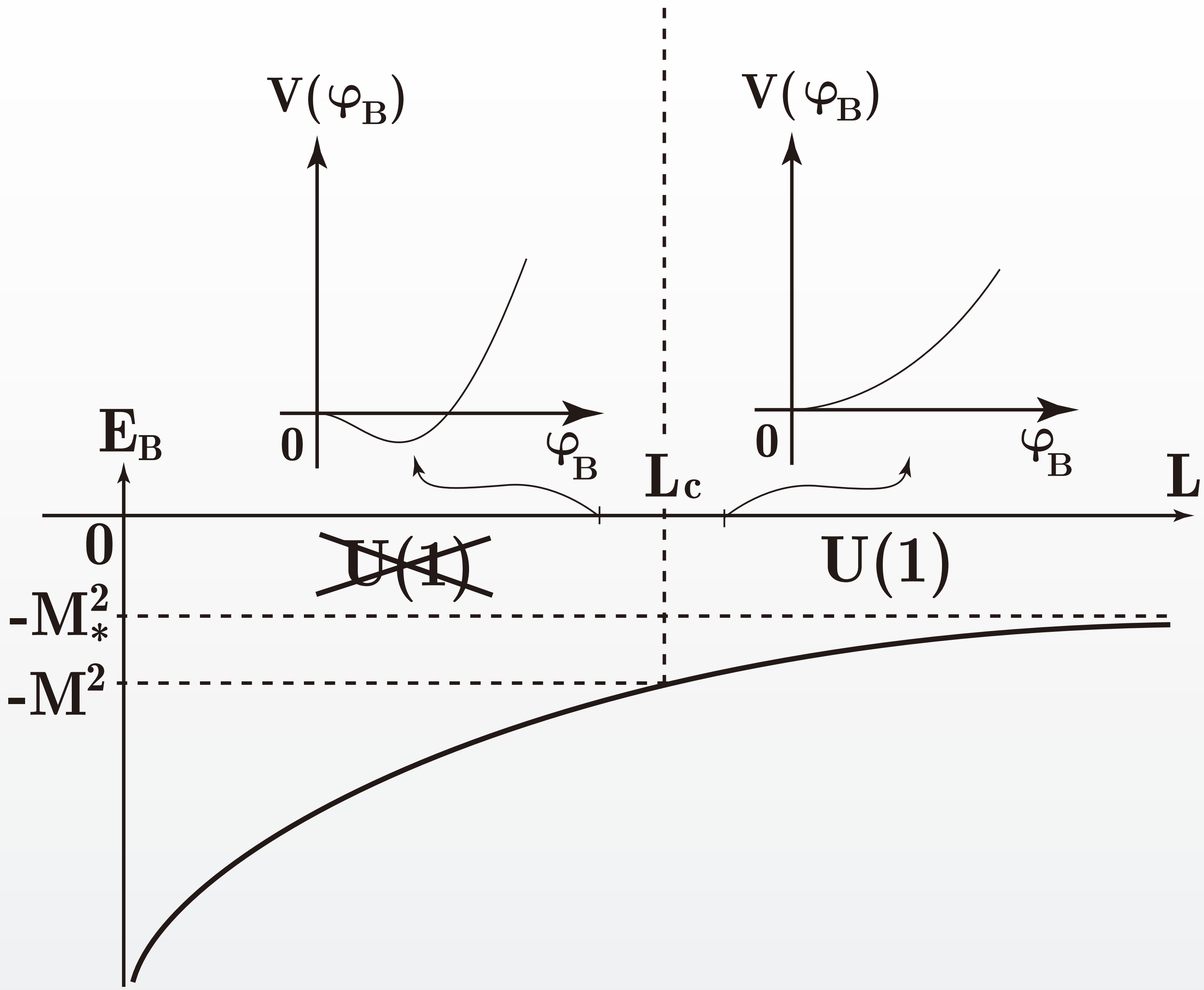

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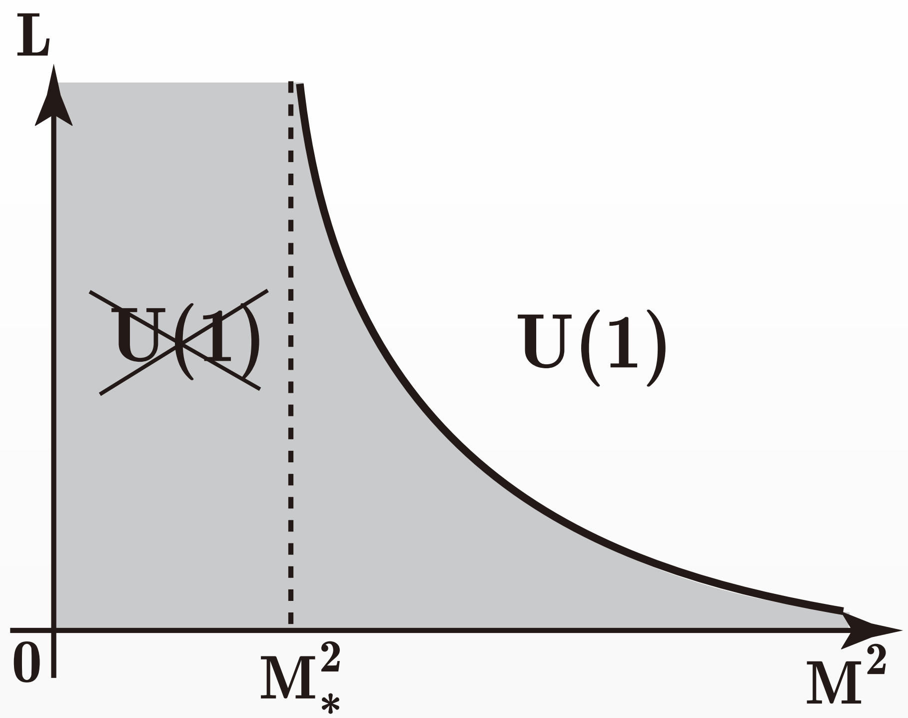


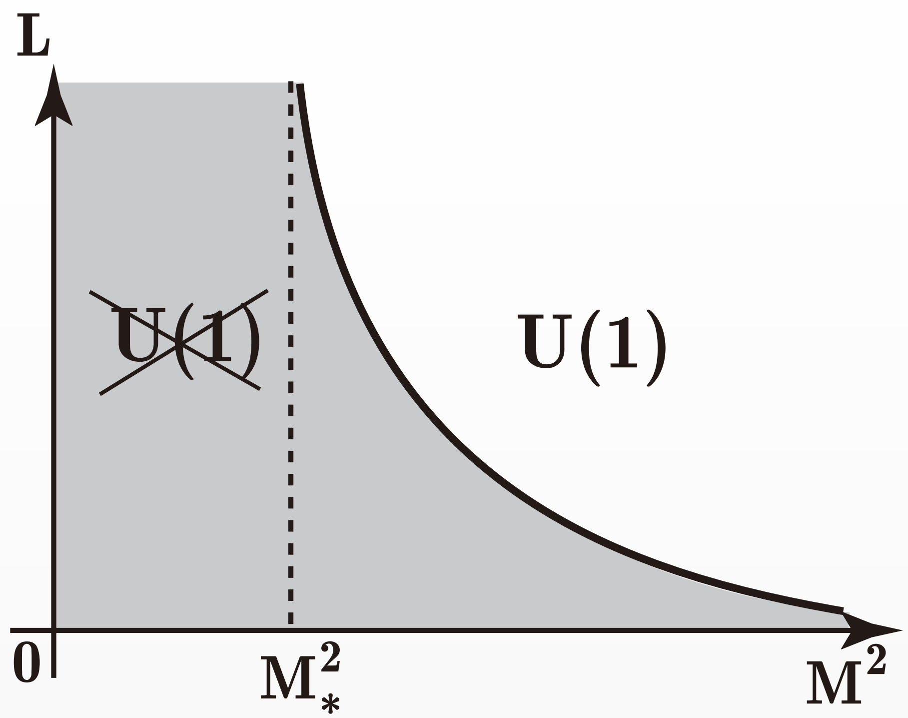
(i) $L_\pm > L_\mp > 0$

(ii) $L_\pm > 0 > L_\mp$

with $L_\pm + L_\mp < 0$

Higgs mechanism with M^2>0

Non-trivial phase structures



(i) $L_\pm > L_\mp > 0$

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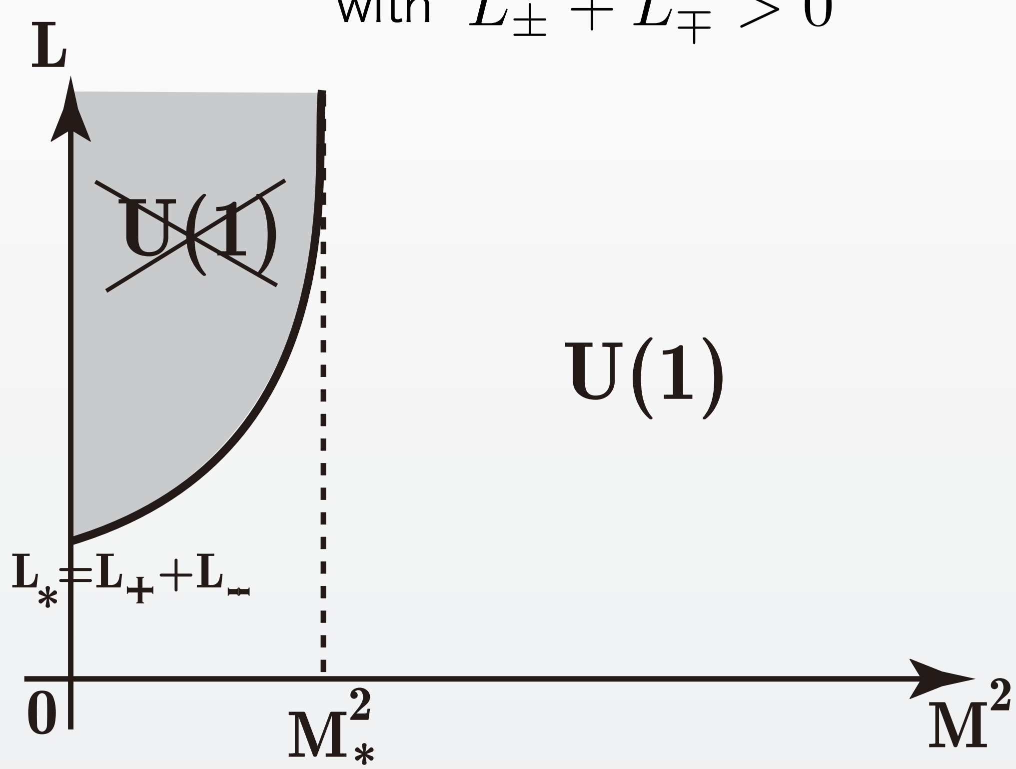

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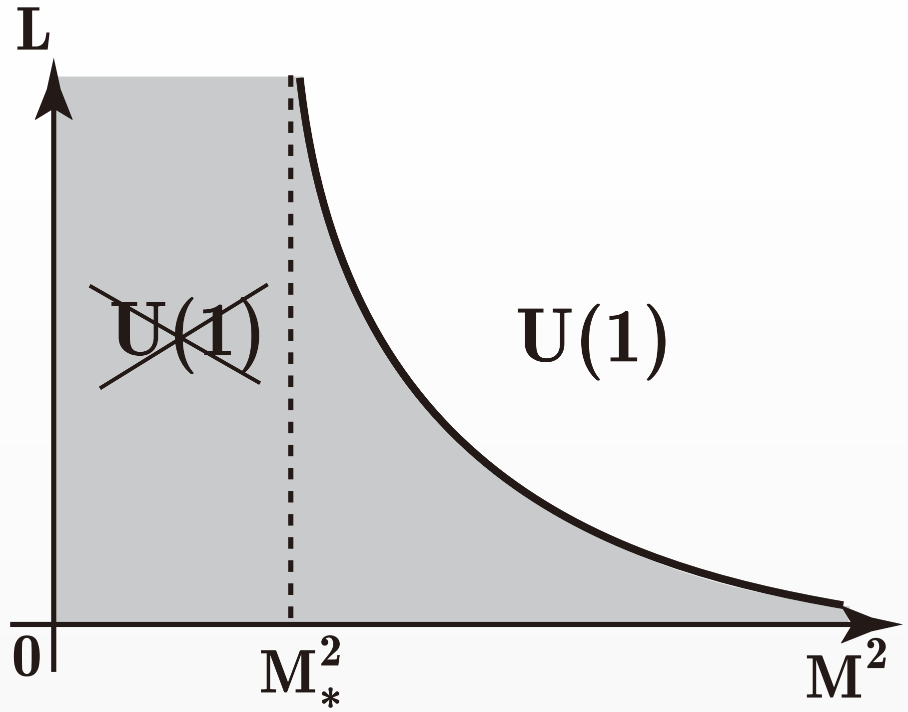

(i) $L_{\pm} > L_{\mp} > 0$
(ii) $L_{\pm} > 0 > L_{\mp}$
with $L_{\pm} + L_{\mp} < 0$

(iii) $L_{\pm} > 0 > L_{\mp}$
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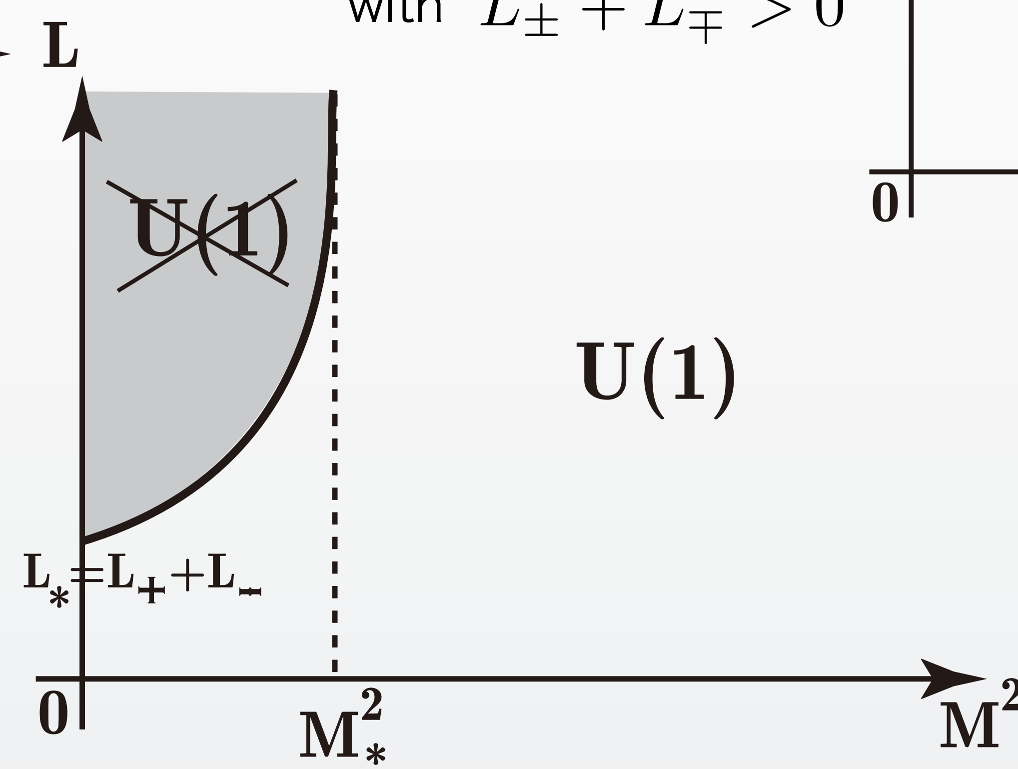

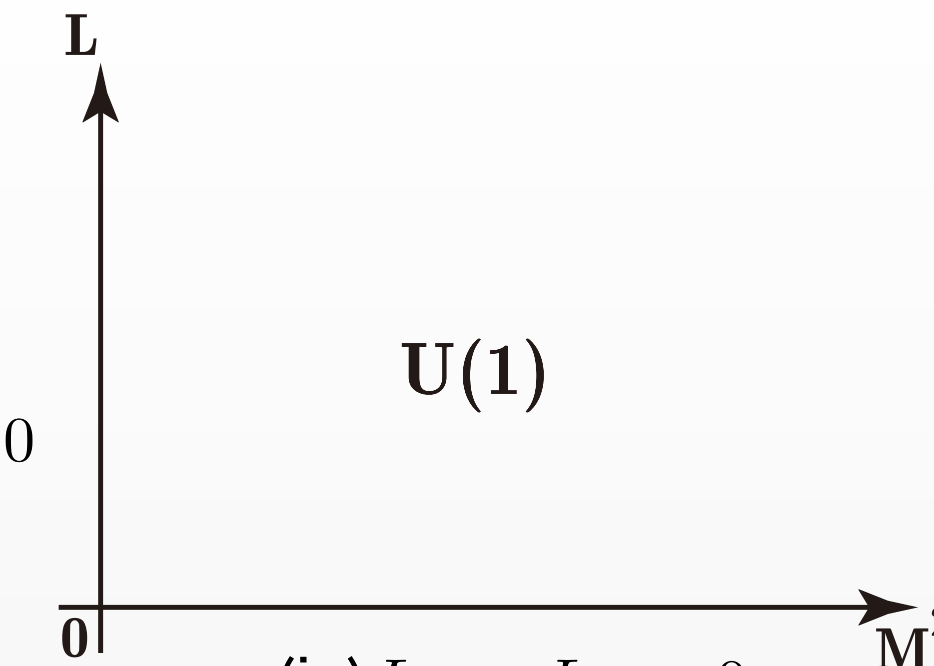

(iv) $L_{\pm} < L_{\mp} < 0$

Chiral theory

Chiral theory

Boundary conditions for the fermion are restricted to 3-cases by the action principle.

$$\begin{cases} (\partial_y + M_F)\Psi_R(x,0) = 0 \\ \Psi_L(x,0) = 0 \end{cases}$$

or

$$\begin{cases} \Psi_R(x,0) = 0 \\ (-\partial_y + M_F)\Psi_L(x,0) = 0 \end{cases}$$

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y=0 y=L

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Bulk mass (→ M_F)

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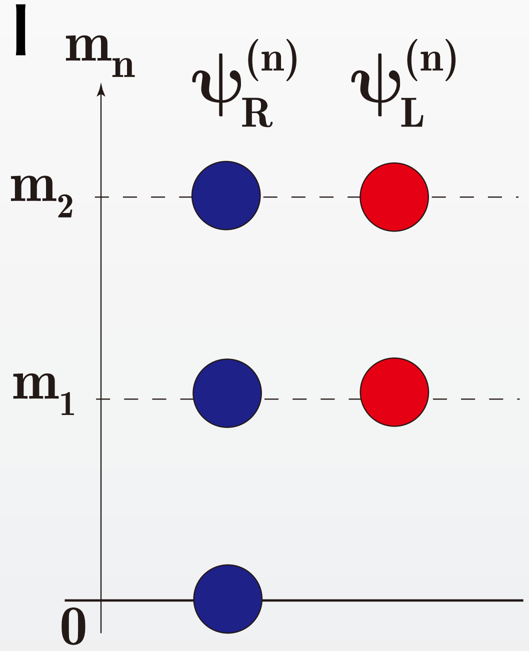
type I



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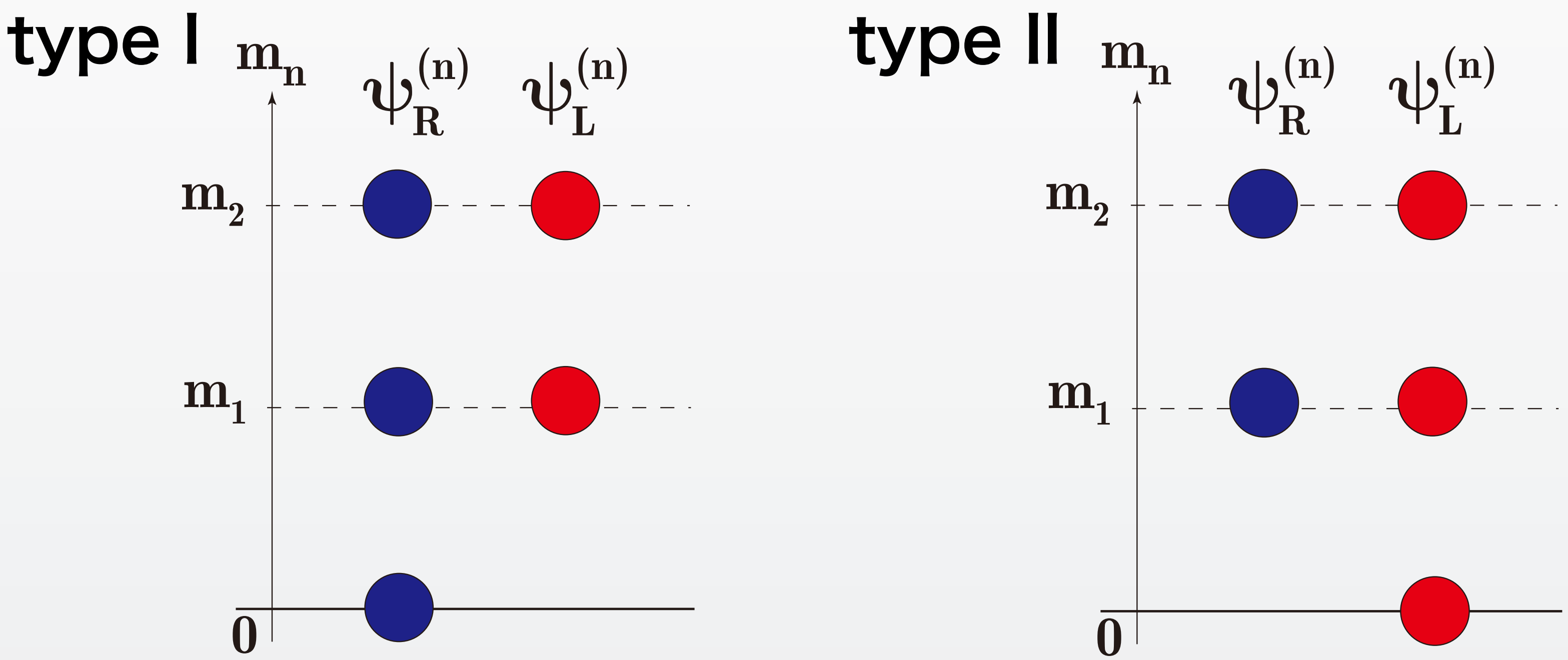
type II



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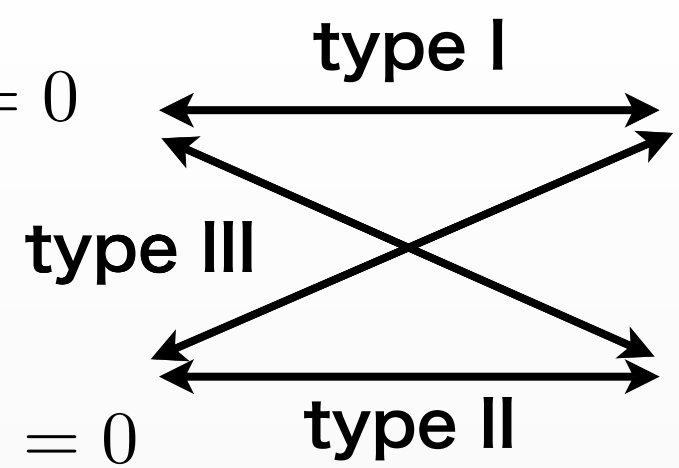

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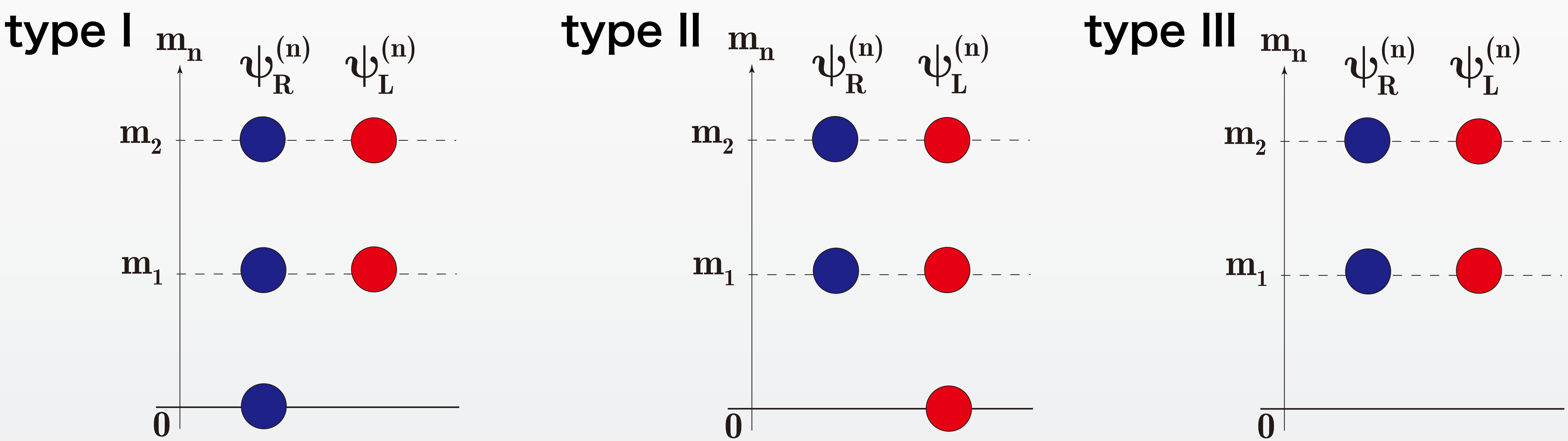

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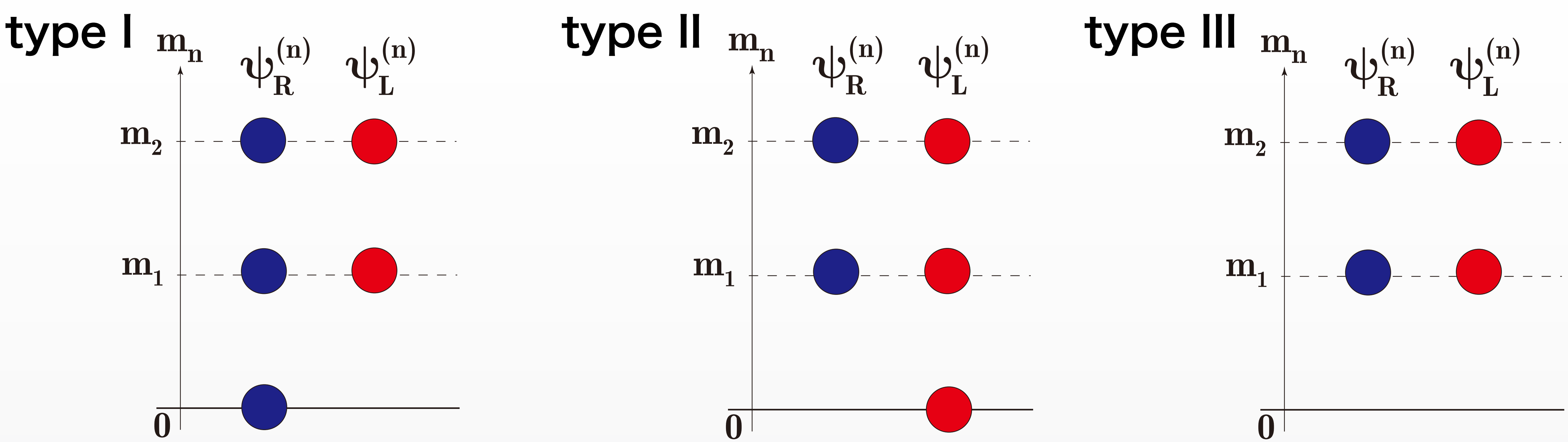

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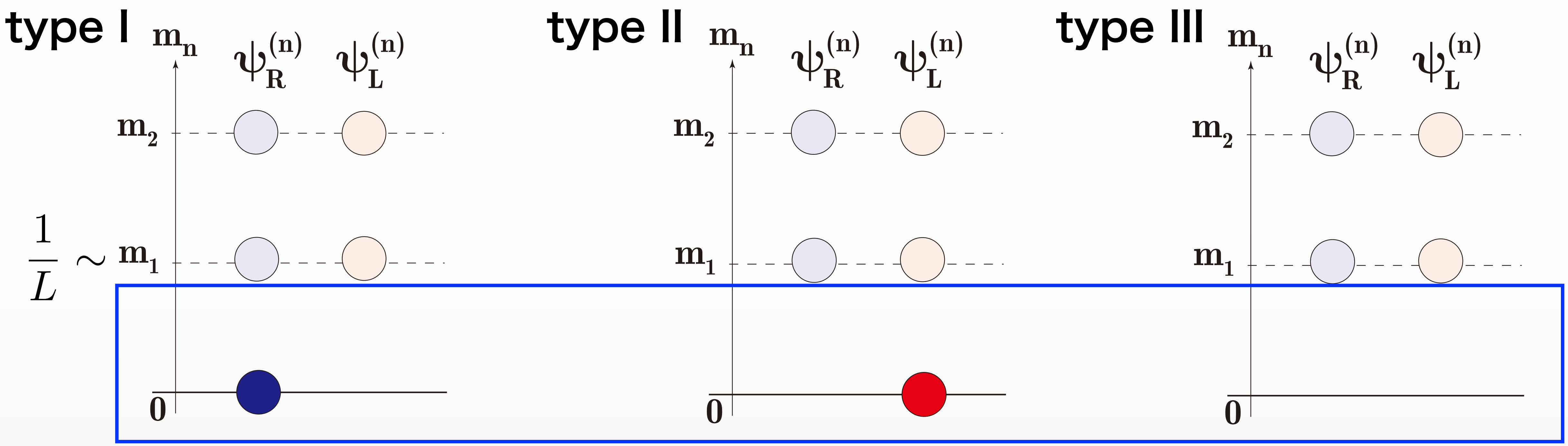

> 4d low energy efferctive theory at E<1/L is chiral theory, irrespective of the bulk mass.

Mass hierarchy

Mass hierarchy

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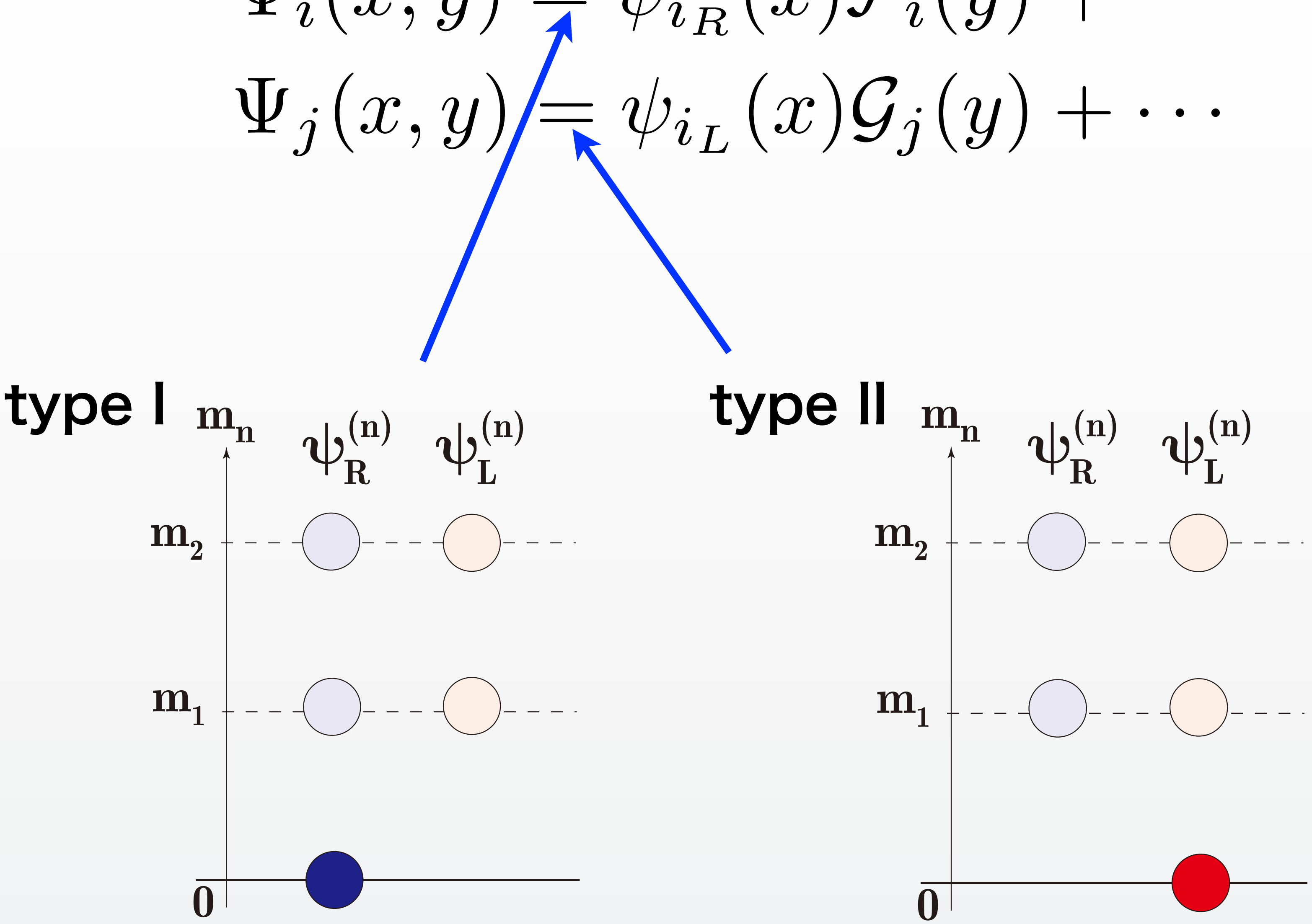
Mass hierarchy

> **Chiral zero mode of the fermion is localized at one of the boundaries.**

$$\Psi_i(x,y) = \psi_{i_R}(x)\mathcal{F}_i(y) + \cdots$$
$$\Psi_j(x,y) = \psi_{i_L}(x)\mathcal{G}_j(y) + \cdots$$



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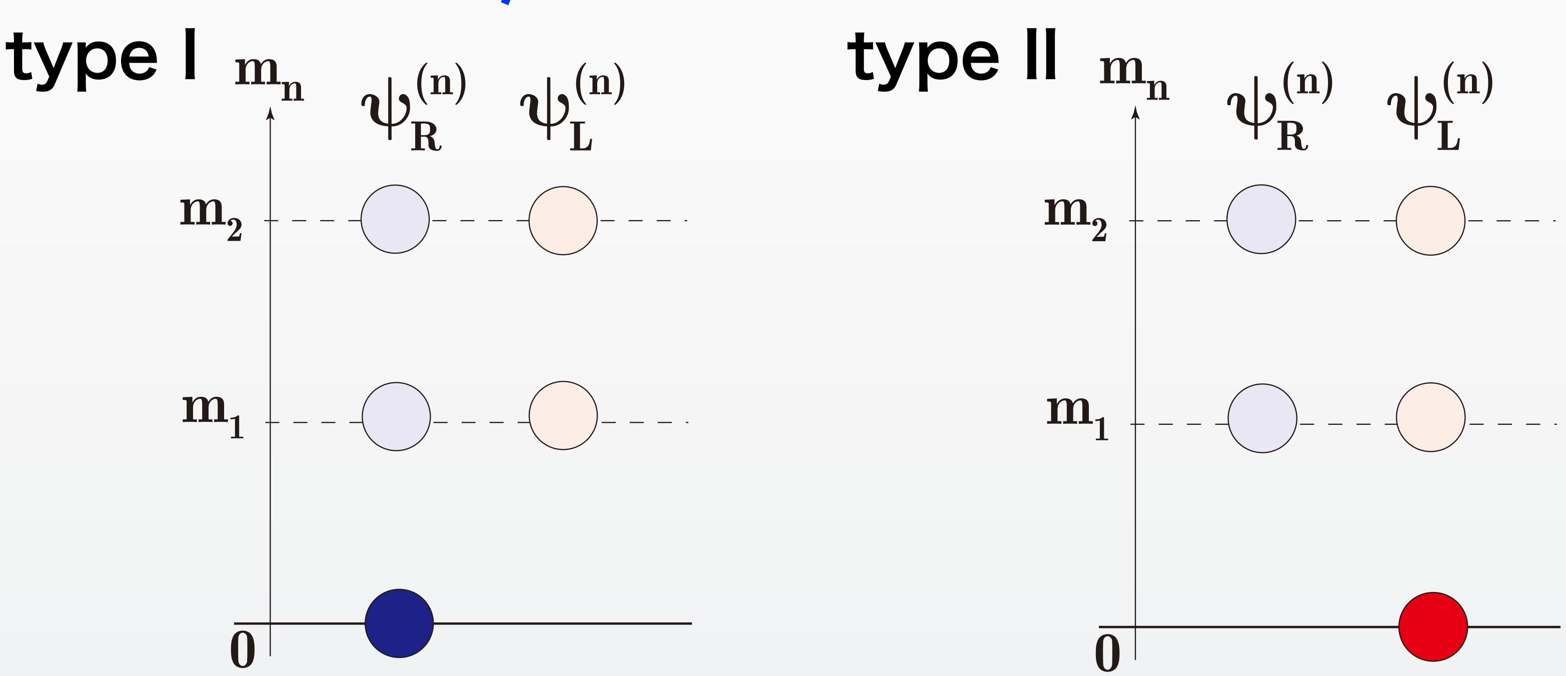
mode functions of the zero modes.



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Bulk mass

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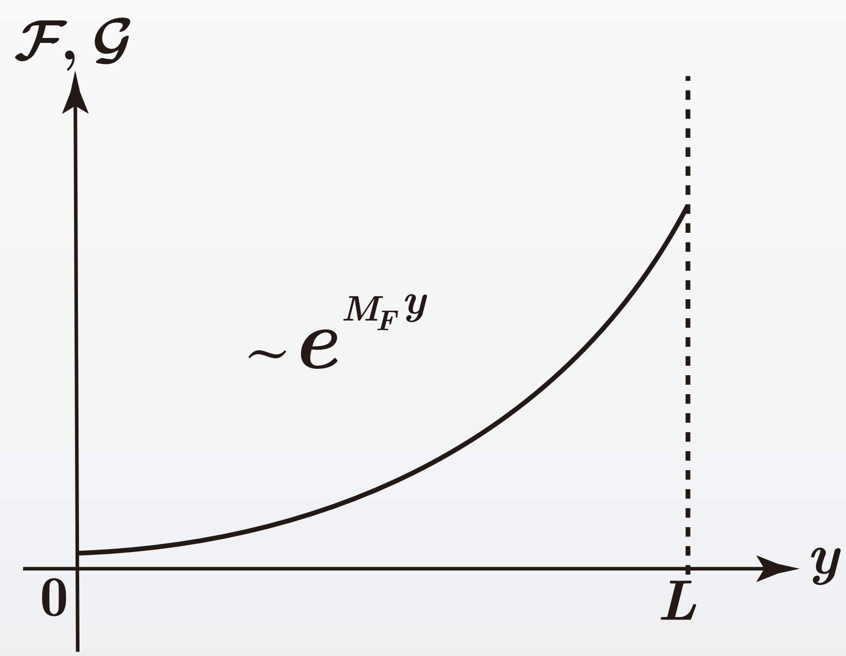

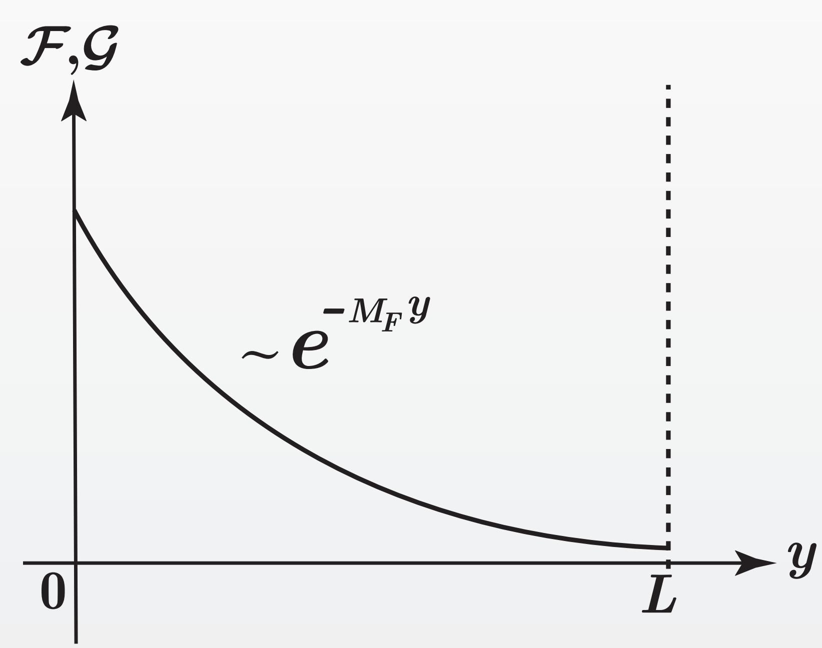

Mass hierarchy

> Chiral zero mode of the fermion is localized at one of the boundaries.

> Vacuum Expectation Value of the scalar field necessarily possesses y-dependence.

$$\langle \Phi(x,y) \rangle = \phi(y)$$

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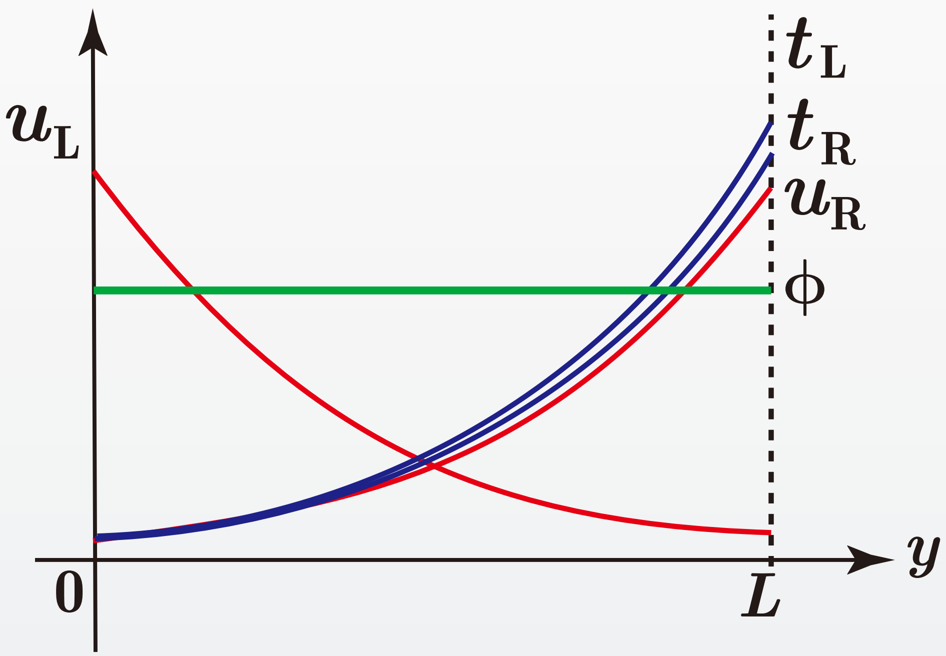

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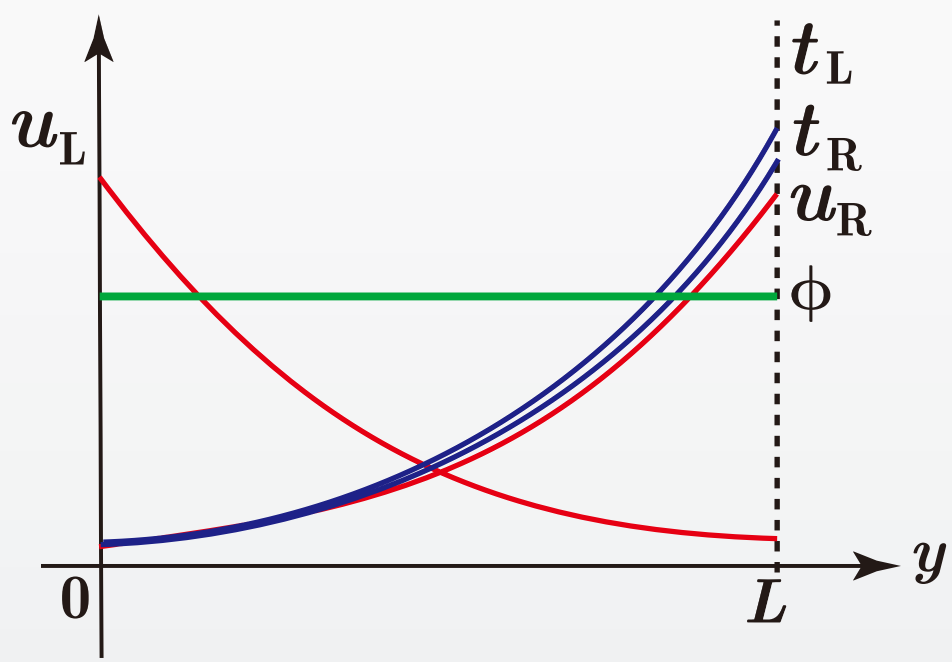
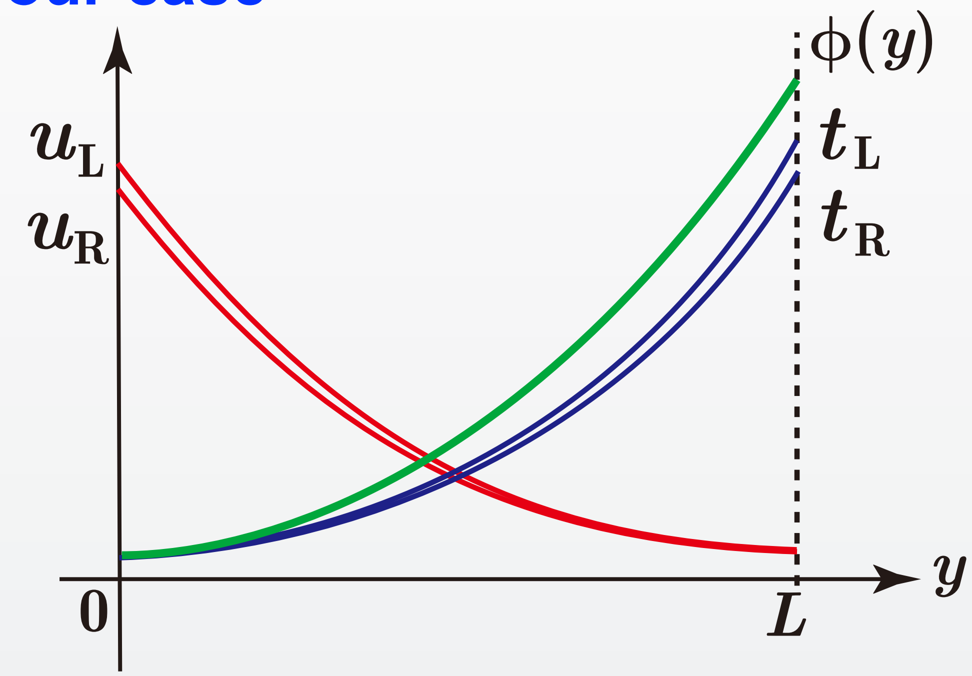


(ii) In our case



This can lead the hierarchical masses.

Conslusions and Discussions

Conslusions and Discussions

5d gauge theories on an interval

↓ The low energy efferctive theories

4d gauge theories

+ Symmetry breaking
+ Chiral theories
+ Mass hierarchy

Conslusions and Discussions

> Challenges for the future

5d gauge theories on an interval

$$\begin{pmatrix} E \\ \nu_E \end{pmatrix}, \quad E', \quad B_M, \quad \boldsymbol{W}_M, \quad \Phi, \quad \cdots \quad \cdots$$

$$\downarrow$$

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \quad e_R \quad B_\mu \quad \boldsymbol{W}_\mu \quad h$$

↓ The low energy efferctive theories

the Standard Model

>Mass of the Higgs is different from the SM.
>Mass hierarchy appears naturally.

Appendix

Mass of the Higgs

> **Mass of the Higgs is given by the eigenvalue of the following eigenvalue equation.**

$$\left(-\frac{d^2}{dy^2} + M^2 + 3\lambda\phi^2(y)\right) J(y) = m^2 J(y)$$

$$+\begin{cases} J(0) + L_+ \partial_y J(0) = 0 \\ J(L) - L_- \partial_y J(L) = 0 \end{cases}$$