Heterotic Asymmetric Orbifold and E6 GUT Model

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based on [arXiv:1012.1690]

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Introduction

- String → Standard Model
  - String theory
    -- A candidate which describe quantum gravity and unify four forces
    -- Is it possible to realize phenomenological properties of Standard Model ?
  - Minimal Supersymmetric Standard Model (MSSM) -- A candidate BSM
    -- Non-Abelian gauge symmetries ($SU(3) \times SU(2) \times U(1)$)
    -- Three chiral generations (Quarks, Leptons)
    -- Yukawa hierarchy
    -- N=1 SUSY …

- String GUT scenario

  String → GUT → MSSM

  GUT → 4D SUSY-GUT with adjoint representation Higgs

  adjoint representation Higgs → Its VEV breaks GUT symmetry spontaneously

Introduction

- $E_6 \times U(1)_A (\times SU(2)_H)$ GUT models   Maekawa, Yamashita ('01-'04)

  - Unify all SM particles into GUT matter multiplets (E6)
  - Realistic Yukawa hierarchies (E6 , U(1)A)
  - Realize doublet-triplet splitting (U(1)A)
  - Solve SUSY flavor/CP problem (SU(2)H)

Generic interaction : Include all terms allowed by symmetry

→ Matter contents determine the models

Anomalous U(1) symmetry : Often appear in low energy effective theory of string theory.

→ Can we realize these GUT models in string theory ?

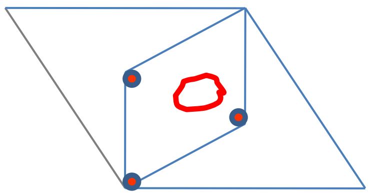
Introduction

- Orbifold compactification of heterotic string theory

- (Symmetric) orbifold compactification   Dixon, Harvey, Vafa, Witten '85,'86

  Advantage
  - E6 GUT gauge symmetry
  - N=1 supersymmetry

  Disadvantage
  - No adjoint representation Higgs



Narain, Sarmadi, Vafa '87

- Asymmetric orbifold compactification, Diagonal embedding method

  Advantage
  - modding out the permutation symmetry of the models

  Ex) $G_1 \times G_1 \times G_1 \to G_3$

  - Realize adjoint representation Higgs(es) !

Goal : Asymmetric orbifold → 4D E6 SUSYGUT with adjoint representation Higgs

E6 Unification

- E6 Unification

Bando, Kugo (1999)

- All quarks and leptons are unified into $\Psi_i(\mathbf{27}) \quad (i = 1, 2, 3)$

$$\Psi_i(\mathbf{27}) = \mathbf{16}_i[\mathbf{10}_i + \bar{\mathbf{5}}_i + \mathbf{1}_i] + \mathbf{10}_i[\mathbf{5}_i + \bar{\mathbf{5}}'_i] + \mathbf{1}_i[\mathbf{1}_i]$$

$$\mathbf{10}(Q, U^c_R, E^c_R) \qquad \bar{\mathbf{5}}(L, D^c_R)$$

$$\begin{pmatrix} \bar{\mathbf{5}}_i : 3 \\ \bar{\mathbf{5}}'_i : 3 \\ \mathbf{5}_i : 3 \end{pmatrix}$$

- Yukawa structure
  - -- Low energy $\bar{\mathbf{5}}$ are from 1, 2 generations $\Psi_1(\mathbf{27}), \Psi_2(\mathbf{27})$

$$\bar{\mathbf{5}}^{\rm low}_1 \sim \bar{\mathbf{5}}_1 \leftarrow \Psi_1(\mathbf{27}) \rightarrow \mathbf{10}_1$$

$$\bar{\mathbf{5}}^{\rm low}_2 \sim \bar{\mathbf{5}}'_1 \qquad \Psi_2(\mathbf{27}) \rightarrow \mathbf{10}_2$$

$$\bar{\mathbf{5}}^{\rm low}_3 \sim \bar{\mathbf{5}}_2 \qquad \Psi_3(\mathbf{27}) \rightarrow \mathbf{10}_3$$

  - -- Mixing of $\bar{\mathbf{5}}$ realize hierarchies

$$\lambda = 0.22$$

Up-type Yukawa

$$(Y_u) \sim \begin{array}{c|ccc} & \mathbf{10}_1 & \mathbf{10}_2 & \mathbf{10}_3 \\ \hline \mathbf{10}_1 & \lambda^6 & \lambda^5 & \lambda^3 \\ \mathbf{10}_2 & \lambda^5 & \lambda^4 & \lambda^2 \\ \mathbf{10}_3 & \lambda^3 & \lambda^2 & 1 \end{array}$$

Down-type, Charged lepton Yukawa (Mild hierarchy)

$$(Y_d) = (Y_e)^{\rm T} \sim \begin{array}{c|ccc} & \bar{\mathbf{5}}^{\rm low}_1 & \bar{\mathbf{5}}^{\rm low}_2 & \bar{\mathbf{5}}^{\rm low}_3 \\ \hline \mathbf{10}_1 & \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \mathbf{10}_2 & \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \mathbf{10}_3 & \lambda^3 & \lambda^{2.5} & \lambda^2 \end{array}$$

Requirements for String Model Building

Typical E6 X U(1)A (x SU(2)H) GUT models contain :

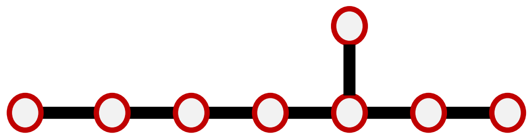
- 4D $\mathcal{N} = 1$ SUSY,
- E6 unification group,
- Net 3 chiral generations ($\mathbf{27}$, $\overline{\mathbf{27}}$),
- Adjoint Higgs fields ($\mathbf{78}$),
- SU(2)H or SU(3)H family symmetry,
- Anomalous U(1)A gauge symmetry,
- Adjoint Higgs fields charged under the anomalous U(1)A gauge symmetry.

(The first four items:) Minimum requirements for 4D-SUSYGUT models ⇒ String

Heterotic String Theory

- Heterotic string theory
  - Heterotic string for our starting point
  - Degrees of freedom
    --- Left mover 26 dim. Bosons X_{L}
    --- Right mover 10 dim. Bosons and fermions X_{R} Ψ_{R}
  - Extra 16 dim. have to be compactified
  - Consistency → If 10D N=1, E8 X E8 or SO(32)

Dynkin Diagram (E8)

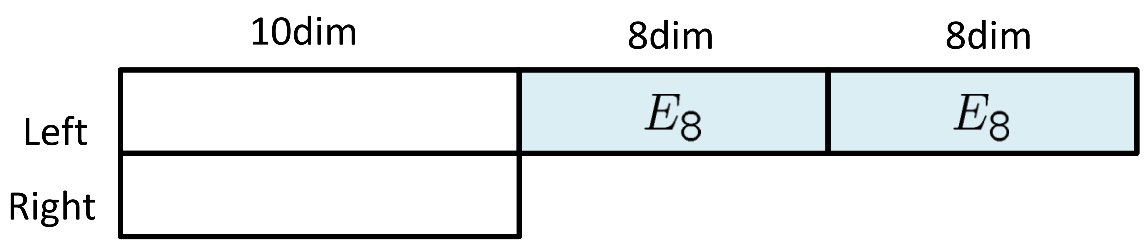


Ex.) E8 Root Lattice **1E8**

$$\mathbf{1_{E8}} \equiv \sum_{i=1}^{8} n_i \alpha_i \quad (n_i \in \mathbf{Z}^8)$$

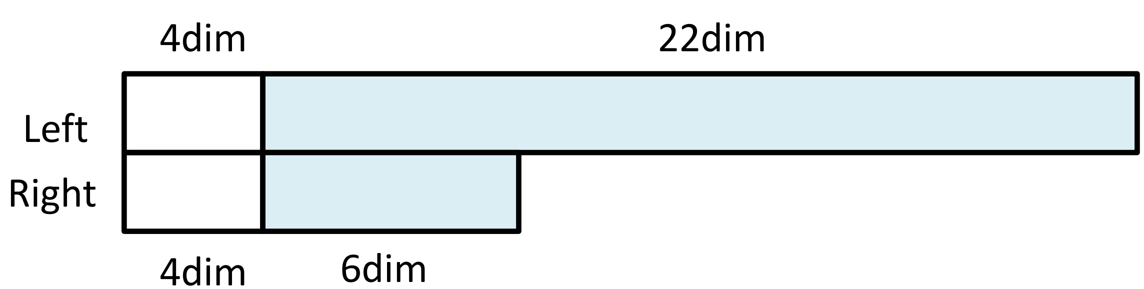
Left-moving momentum $p_{\mathrm{L}} \in$ **1E8**

$\alpha_i \quad (i = 1 \sim 8)$: Simple roots of E8



Even Self-Dual Lattice

- Even self-dual lattice (Narain lattice) $\Gamma_{22,6}$
  - General flat compactification of heterotic string
    --- Left : 22 dim
    --- Right : 6 dim
  - Left-right combined momentum $(p_{\rm L}||p_{\rm R})$ are quantized, and compose a lattice
  - Many possible even self-dual lattices

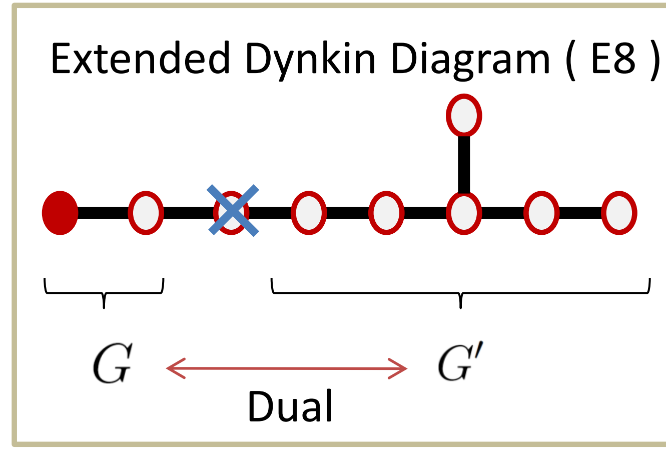


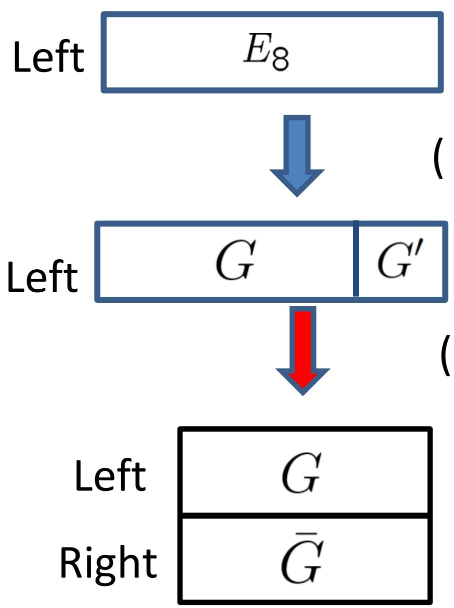
Lattice Engineering Technique

- Lattice engineering technique

  Lerche, Schellekens, Warner '88

  - We can construct new even self-dual lattice from known one.





$p_{\mathrm{L}} \in$ E8 root lat ($\mathbf{1}_{E8}$) (spanned by simple roots of E8)

(Decomposition $E_8 \to G \times G'$)

(Replace left G' → Right $\bar{G}(= G'_{\mathrm{dual}})$)

Lattice Engineering Technique

- Lattice engineering technique

  Lerche, Schellekens, Warner '88

  - We can construct new even self-dual lattice from known one.

(Left / Right) E_6	⟺	(Right / Left) A_2
E6 root lat ($\mathbf{1}_{E6}$)	⟺	A2 root lat ($\mathbf{1}_{A2}$)
E6 fund weight lat ($\mathbf{27}$)	⟺	A2 fund weight lat ($\mathbf{3}$)
E6 anti-fund weight lat ($\overline{\mathbf{27}}$)	⟺	A2 anti-fund weight lat ($\overline{\mathbf{3}}$)

Ex.)

Left: E_8

$p_{\mathrm{L}} \in$ E8 root lat ($\mathbf{1}_{E8}$) (spanned by simple roots of E8)

(Decomposition $E_8 \to E_6 \times A_2$) $A_2 = SU(3)$

Left: E_6 | A_2

$$p_{\mathrm{L}} \in \mathbf{1}_{E8} = (\mathbf{1}_{E6}, \mathbf{1}_{A2}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$$

(Replace left A2 → Right E6)

Left: E_6

Right: $\bar{E}_6$

$$(p_{\mathrm{L}} || p_{\mathrm{R}}) \in (\mathbf{1}_{E6} || \mathbf{1}_{E6}) + (\mathbf{27} || \mathbf{27}) + (\overline{\mathbf{27}} || \overline{\mathbf{27}})$$

$E_6 \times \bar{E}_6$ lattice → E_6 gauge symmetry

Lattice Engineering Technique

- Lattice engineering technique

  Lerche, Schellekens, Warner '88

  - We can construct new even self-dual lattice from known one.

(Left / Right) E_6	⟷	(Right / Left) A_2
E6 root lat ($\mathbf{1}_{E6}$)	⟷	A2 root lat ($\mathbf{1}_{A2}$)
E6 fund weight lat ($\mathbf{27}$)	⟷	A2 fund weight lat ($\mathbf{3}$)
E6 anti-fund weight lat ($\overline{\mathbf{27}}$)	⟷	A2 anti-fund weight lat ($\overline{\mathbf{3}}$)

Ex.)

Left [E_8]

$p_{\mathrm{L}} \in$ E8 root lat ($\mathbf{1}_{E8}$) (spanned by simple roots of E8)

(Decomposition $E_8 \to E_6 \times A_2$) $A_2 = SU(3)$

Left [E_6 | A_2]

$$p_{\mathrm{L}} \in \mathbf{1}_{E8} = (\mathbf{1}_{E6}, \mathbf{1}_{A2}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}})$$

(Replace left E6 → Right A2)

[A_2 / $\bar{A}_2$]

$$(p_{\mathrm{L}} || p_{\mathrm{R}}) \in (\mathbf{1}_{A2} || \mathbf{1}_{A2}) + (\mathbf{3} || \mathbf{3}) + (\overline{\mathbf{3}} || \overline{\mathbf{3}})$$

$A_2 \times \bar{A}_2$ lattice → A_2 gauge symmetry

Lattice Engineering Technique

- Lattice engineering technique

  Lerche, Schellekens, Warner '88

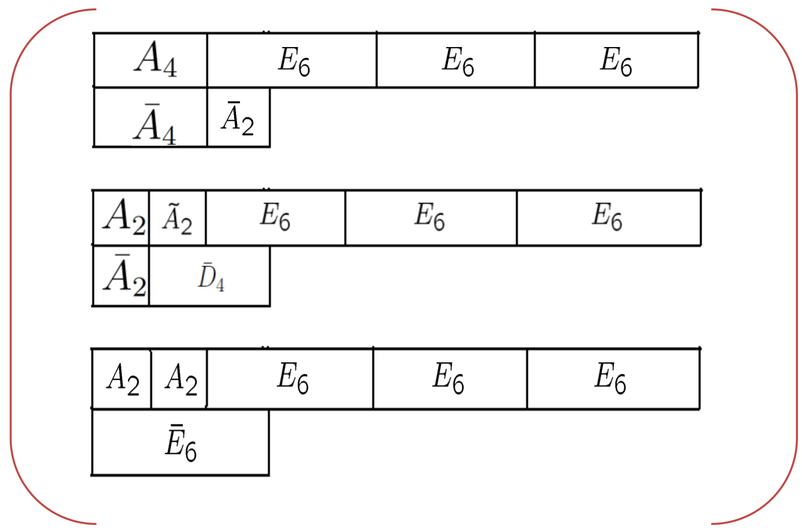
  - We can construct new even self-dual lattice from known one.
  - In repeating fashion, we can construct various even self-dual lattices $\Gamma_{22,6}$

In general

$G_{\rm L} \times \bar{G}_{\rm R}$ lattice

↓

$G_{\rm L}$ gauge symmetry



- Advantage : Various gauge symmetries.

  Easy to find out discrete symmetries of the lattices. → Orbifold

Asymmetric Orbifold Compactification

- Asymmetric orbifold compactification
  - Orbifold action $\theta = (\theta_{\mathsf{L}}, \theta_{\mathsf{R}})$ (Twist, Shift, Permutation)

    Left mover : $X_{\mathrm{L}} \to \theta_{\mathrm{L}} X_{\mathrm{L}}$

    Right mover : $X_{\mathrm{R}} \to \theta_{\mathrm{R}} X_{\mathrm{R}}$
    $\Psi_{\mathrm{R}} \to \theta_{\mathrm{R}} \Psi_{\mathrm{R}}$

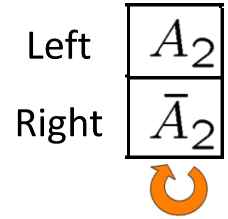
Modding out the lattice in left-right independent way

$$\theta = (\theta_{\mathsf{L}}, \theta_{\mathsf{R}}) \quad \theta_{\mathsf{L}} \neq \theta_{\mathsf{R}}$$

- No geometric picture

Ex.) $\mathbf{Z}_3$ action

θ_{L} None



θ_{R} 1/3 rotation

Diagonal Embedding Method

● Diagonal embedding method

Modding out the permutation symmetry of the lattice

Ex.) $(G)_1 \times (G)_1 \times (G)_1 \rightarrow (G)_3$

Combining with phaseless right-moving states

$|\mathbf{Adj.}\rangle_1 \otimes |\mathrm{Right}\rangle_1$ } G gauge fields

Combining with right-moving states with opposite phases

$|\mathbf{Adj.}\rangle_{\omega^*} \otimes |\mathrm{Right}\rangle_\omega$

$|\mathbf{Adj.}\rangle_\omega \otimes |\mathrm{Right}\rangle_{\omega^*}$ } An adjoint representation Higgs

θ : permutation

$(\omega = e^{2\pi i/3})$

Eigen states of θ :

$$|\mathbf{Adj.}\rangle_1 \equiv |\mathbf{Adj.}, \mathbf{1}, \mathbf{1}\rangle + |\mathbf{1}, \mathbf{Adj.}, \mathbf{1}\rangle + |\mathbf{1}, \mathbf{1}, \mathbf{Adj.}\rangle$$
$$|\mathbf{Adj.}\rangle_\omega \equiv |\mathbf{Adj.}, \mathbf{1}, \mathbf{1}\rangle + \omega^*|\mathbf{1}, \mathbf{Adj.}, \mathbf{1}\rangle + \omega|\mathbf{1}, \mathbf{1}, \mathbf{Adj.}\rangle$$
$$|\mathbf{Adj.}\rangle_{\omega^*} \equiv |\mathbf{Adj.}, \mathbf{1}, \mathbf{1}\rangle + \omega|\mathbf{1}, \mathbf{Adj.}, \mathbf{1}\rangle + \omega^*|\mathbf{1}, \mathbf{1}, \mathbf{Adj.}\rangle$$

→

$$\theta|\mathbf{Adj.}\rangle_1 = |\mathbf{Adj.}\rangle_1$$
$$\theta|\mathbf{Adj.}\rangle_\omega = \omega|\mathbf{Adj.}\rangle_\omega$$
$$\theta|\mathbf{Adj.}\rangle_{\omega^*} = \omega^*|\mathbf{Adj.}\rangle_{\omega^*}$$

Summary of Our Method

- Summary of our method

1. (22, 6) – dimensional even self-dual lattice with $(E_6)^K$ ⇒ 4 D

2. Orbifold identification by

   (a). Permutation among the E6 factors ⇒ Adjoint Higgs

   (b). Rotations of right-moving six dimensions ⇒ N = 1 SUSY

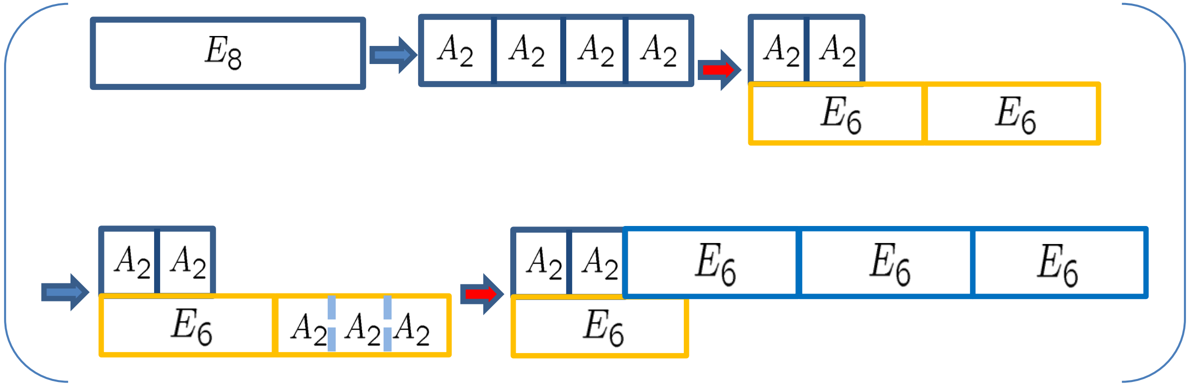
3. Number of generations ⇒ 3 generations ?

Empirically, $\sharp$ of generations $\propto K$

⇒ So we consider the case of $K = 3$

E_6^3 Lattice

- E_6^3 models (starting point)
  - Our starting point: $E_6^3 \times A_2^2 \times \bar{E}_6$ even self-dual lattice



$A_2 = SU(3)$

--- 4D N=4 $E_6^3 \times SU(3)^2$ model

$\mathbf{Z}_{12}$ Asymmetric Orbifold Model

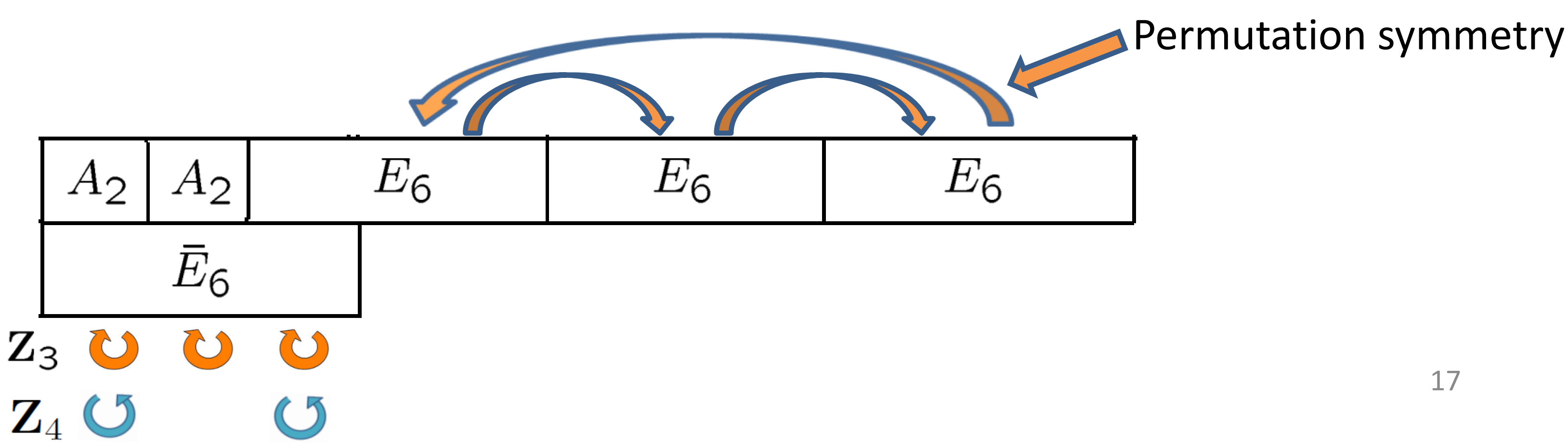
- $\mathbf{Z}_{12} = \mathbf{Z}_3 \times \mathbf{Z}_4$ asymmetric orbifold construction

For E_6^3 part and right mover:

$\mathbf{Z}_3$ action : Permutation for the three E_6 factors.
Rotation for right mover.

$\mathbf{Z}_4$ action : Rotation for right mover. (Twist vector $t_{\mathrm{R}} = (1, 4, -5)/12$)

Previous study claimed : Only Z3 x Z2 is possible Kakushadze, Tye '97
(Simple compactification + A fields + B fields)
However, we find no reasons to exclude Z12 orbifold.



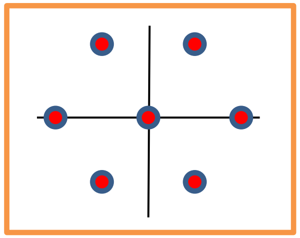
$\mathbf{Z}_{12}$ Asymmetric Orbifold Model

- $\mathbf{Z}_{12} = \mathbf{Z}_3 \times \mathbf{Z}_4$ asymmetric orbifold construction

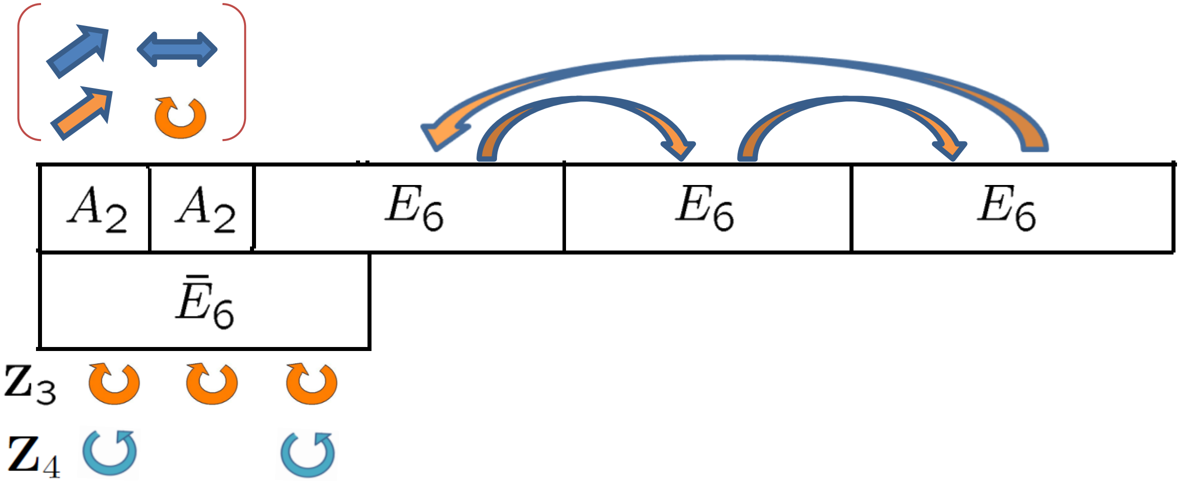
For two A2 parts $(i = 1, 2)$:

Possible orbifold actions are

1. Shift
2. 1/3 (or 2/3) rotation
3. Weyl reflection



We consider all possible $\mathbf{Z}_{12}$ orbifold actions for the two A2 parts.



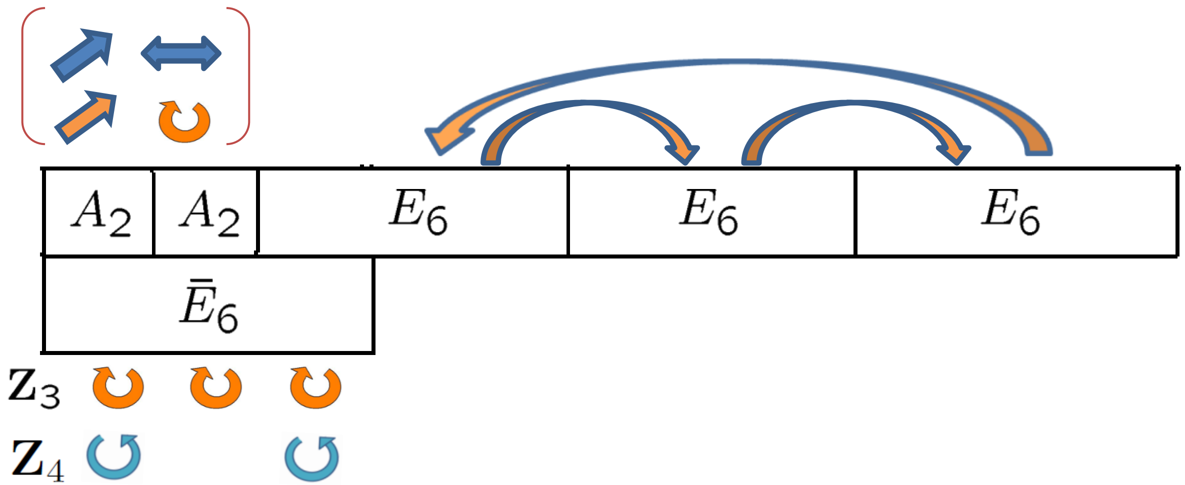
Possible Models

We find 8 possible choices which satisfy the consistency.

3 - generation models X 3

9 - generation models X 2

0 - generation models X 3



Three Generation Models

- Result : 3-generation E_6 models

Ito et.al. (2010)

	Model 1	Model 2	Model 3
	$E_6 \times SU(2) \times U(1)^3$	$E_6 \times SU(2) \times U(1)^3$	$E_6 \times U(1)^4$
U	$(\mathbf{1},\mathbf{1},+6,0,0)_{\rm L}$ $(\mathbf{78},\mathbf{1},0,0,0)_{\rm L}$	$(\mathbf{1},\mathbf{1},+6,\pm3,0)_{\rm L}$ $(\mathbf{78},\mathbf{1},0,0,0)_{\rm L}$	$(\mathbf{1},-6,0,0,0)_{\rm L}$ $(\mathbf{1},+3,\pm6,0,0)_{\rm L}$ $(\mathbf{78},0,0,0,0)_{\rm L}$
T_1	$(\mathbf{27},\mathbf{1},+1,0,\pm1)_{\rm L}$	—	$(\mathbf{27},-1,-1,+1,0)_{\rm L}$
T_2	$(\overline{\mathbf{27}},\mathbf{1},-1,\pm1,0)_{\rm L}$	$(\overline{\mathbf{27}},\mathbf{1},+2,0,-2)_{\rm L}$	$(\overline{\mathbf{27}},+1,0,0,\pm1)_{\rm L}$
T_3	$2(\mathbf{1},\mathbf{1},-3,0,\pm3)_{\rm L}$	$(\mathbf{1},\mathbf{1},-3,\pm3,-3)_{\rm L}$	$(\mathbf{1},+3,-3,+3,0)_{\rm L}$ $(\mathbf{1},+3,+3,-3,0)_{\rm L}$
T_4	$(\mathbf{27},\mathbf{1},-2,0,0)_{\rm L}$	$(\mathbf{27},\mathbf{1},-2,\pm1,0)_{\rm L}$	$(\mathbf{27},+2,0,0,0)_{\rm L}$ $(\mathbf{27},-1,\pm2,0,0)_{\rm L}$
T_5	$(\mathbf{27},\mathbf{1},+1,0,\pm1)_{\rm L}$	$(\mathbf{27},\mathbf{1},+1,\pm1,+1)_{\rm L}$	$(\mathbf{27},-1,+1,-1,0)_{\rm L}$
T_6	$(\mathbf{1},\mathbf{2},0,0,\pm3)_{\rm L}$ $(\mathbf{1},\mathbf{1},+3,\pm3,0)_{\rm L}$	$(\mathbf{1},\mathbf{2},0,\pm3,0)_{\rm L}$ $(\mathbf{1},\mathbf{1},-6,0,+6)_{\rm L}$	$(\mathbf{1},-3,0,0,\pm3)_{\rm L}$ $(\mathbf{1},0,+6,-2,0)_{\rm L}$ $(\mathbf{1},0,-6,+2,0)_{\rm L}$

- Gauge symmetry : $E_6 \times SU(2) \times U(1)^3$ or $E_6 \times U(1)^4$
- Net 3 chiral generations (5 – 2 = 3 or 4 – 1 = 3)
- 1 adjoint representation Higgs

Three Generation Models

- Result : 3-generation E_6 models

Ito et.al. (2010)

	Model 1
	$E_6 \times SU(2) \times U(1)^3$
U	$(\mathbf{1}, \mathbf{1}, +6, 0, 0)_{\mathrm{L}}$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_{\mathrm{L}}$
T_1	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_{\mathrm{L}}$
T_2	$(\overline{\mathbf{27}}, \mathbf{1}, -1, \pm 1, 0)_{\mathrm{L}}$
T_3	$2(\mathbf{1}, \mathbf{1}, -3, 0, \pm 3)_{\mathrm{L}}$
T_4	$(\mathbf{27}, \mathbf{1}, -2, 0, 0)_{\mathrm{L}}$
T_5	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_{\mathrm{L}}$
T_6	$(\mathbf{1}, \mathbf{2}, 0, 0, \pm 3)_{\mathrm{L}}$ $(\mathbf{1}, \mathbf{1}, +3, \pm 3, 0)_{\mathrm{L}}$

- Model 1 : Same mass spectrum with $\mathbf{Z}_6$ asymmetric orbifold model although orbifold action is different.

Kakushadze and Tye,1997

Three Generation Models

- Result : 3-generation E_6 models

Ito et.al. (2010)

- Model 2, 3 : New models !

Model 2	Model 3
$E_6 \times SU(2) \times U(1)^3$	$E_6 \times U(1)^4$
$(\mathbf{1}, \mathbf{1}, +6, \pm 3, 0)_L$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_L$	$(\mathbf{1}, -6, 0, 0, 0)_L$ $(\mathbf{1}, +3, \pm 6, 0, 0)_L$ $(\mathbf{78}, 0, 0, 0, 0)_L$
—	$(\mathbf{27}, -1, -1, +1, 0)_L$
$(\overline{\mathbf{27}}, \mathbf{1}, +2, 0, -2)_L$	$(\overline{\mathbf{27}}, +1, 0, 0, \pm 1)_L$
$(\mathbf{1}, \mathbf{1}, -3, \pm 3, -3)_L$	$(\mathbf{1}, +3, -3, +3, 0)_L$ $(\mathbf{1}, +3, +3, -3, 0)_L$
$(\mathbf{27}, \mathbf{1}, -2, \pm 1, 0)_L$	$(\mathbf{27}, +2, 0, 0, 0)_L$ $(\mathbf{27}, -1, \pm 2, 0, 0)_L$
$(\mathbf{27}, \mathbf{1}, +1, \pm 1, +1)_L$	$(\mathbf{27}, -1, +1, -1, 0)_L$
$(\mathbf{1}, \mathbf{2}, 0, \pm 3, 0)_L$ $(\mathbf{1}, \mathbf{1}, -6, 0, +6)_L$	$(\mathbf{1}, -3, 0, 0, \pm 3)_L$ $(\mathbf{1}, 0, +6, -2, 0)_L$ $(\mathbf{1}, 0, -6, +2, 0)_L$

Three Generation Models

- Result : 3-generation E_6 models

Ito et.al. (2010)

	Model 1	Model 2	Model 3
	$E_6 \times SU(2) \times U(1)^3$	$E_6 \times SU(2) \times U(1)^3$	$E_6 \times U(1)^4$
U	$(\mathbf{1}, \mathbf{1}, +6, 0, 0)_{\rm L}$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_{\rm L}$	$(\mathbf{1}, \mathbf{1}, +6, \pm 3, 0)_{\rm L}$ $(\mathbf{78}, \mathbf{1}, 0, 0, 0)_{\rm L}$	$(\mathbf{1}, -6, 0, 0, 0)_{\rm L}$ $(\mathbf{1}, +3, \pm 6, 0, 0)_{\rm L}$ $(\mathbf{78}, 0, 0, 0, 0)_{\rm L}$
T_1	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_{\rm L}$	—	$(\mathbf{27}, -1, -1, +1, 0)_{\rm L}$
T_2	$(\overline{\mathbf{27}}, \mathbf{1}, -1, \pm 1, 0)_{\rm L}$	$(\overline{\mathbf{27}}, \mathbf{1}, +2, 0, -2)_{\rm L}$	$(\overline{\mathbf{27}}, +1, 0, 0, \pm 1)_{\rm L}$
T_3	$2(\mathbf{1}, \mathbf{1}, -3, 0, \pm 3)_{\rm L}$	$(\mathbf{1}, \mathbf{1}, -3, \pm 3, -3)_{\rm L}$	$(\mathbf{1}, +3, -3, +3, 0)_{\rm L}$ $(\mathbf{1}, +3, +3, -3, 0)_{\rm L}$
T_4	$(\mathbf{27}, \mathbf{1}, -2, 0, 0)_{\rm L}$	$(\mathbf{27}, \mathbf{1}, -2, \pm 1, 0)_{\rm L}$	$(\mathbf{27}, +2, 0, 0, 0)_{\rm L}$ $(\overline{\mathbf{27}}, -1, \pm 2, 0, 0)_{\rm L}$
T_5	$(\mathbf{27}, \mathbf{1}, +1, 0, \pm 1)_{\rm L}$	$(\mathbf{27}, \mathbf{1}, +1, \pm 1, +1)_{\rm L}$	$(\mathbf{27}, -1, +1, -1, 0)_{\rm L}$
T_6	$(\mathbf{1}, \mathbf{2}, 0, 0, \pm 3)_{\rm L}$ $(\mathbf{1}, \mathbf{1}, +3, \pm 3, 0)_{\rm L}$	$(\mathbf{1}, \mathbf{2}, 0, \pm 3, 0)_{\rm L}$ $(\mathbf{1}, \mathbf{1}, -6, 0, +6)_{\rm L}$	$(\mathbf{1}, -3, 0, 0, \pm 3)_{\rm L}$ $(\mathbf{1}, 0, +6, -2, 0)_{\rm L}$ $(\mathbf{1}, 0, -6, +2, 0)_{\rm L}$

- These models satisfy the minimum requirements for 4D E6 GUT models.
- No non-Abelian family symmetry
- No anomalous U(1) symmetry

Summary

- Summary

  - We construct 3-generation E6 models with an adjoint representation Higgs in the framework of $\mathbf{Z}_{12}$ asymmetric orbifold construction.
  - Systematical search by utilizing lattice engineering technique for asymmetric orbifold.
  - We obtain 2 more three-generation E6 models which satisfy the minimum requirements, although we do not obtain non-Abelian family symmetry nor anomalous U(1) symmetry.

- Outlook

  - We have to search another possible even self-dual lattice and orbifolds.
  - Possibility of the charged adjoint Higgs ? $(E_6)_1 \times (E_6)_1 \to (E_6)_2$
  - Phenomenology.
  - Another gauge symmetries (SM, MSSM, SU(5), SO(10), …).

Kac–Moody Algebra and Diagonal Embedding

- Kac-Moody algebra

$$[J^a_m, J^b_n] = \underline{k}\,\delta^{ab} m \delta_{m+n,0} + i f^{abc} J^c_{m+n} \qquad J^a = \sum_{n\in\mathbf{Z}} J^a_n z^{-n-1}$$

k : Kac–Moody level
(Ordinary string : Level k=1)

Diagonal
Embedding

$$[J^a_{\mathrm{diagm}}, J^b_{\mathrm{diagn}}] = \underline{N}\,\delta^{ab} m \delta_{m+n,0} + i f^{abc} J^c_{\mathrm{diagm+n}}$$

Diagonal combination

$$J_{\mathrm{diag}}(z) = J_G(z) + J_G(z) + \cdots + J_G(z)$$