Generation, Quark/Lepton Mass Hierarchy and Flavor Mixing from Point Interactions in an Extra Dimension

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坂本 眞人（神戸大）
&
soon with 高橋 亮（北海道大）

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  Why so different the structure of flavor mixing is between quark and lepton !?

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in the context of 5d gauge theories on a circle.

Ideas & Features

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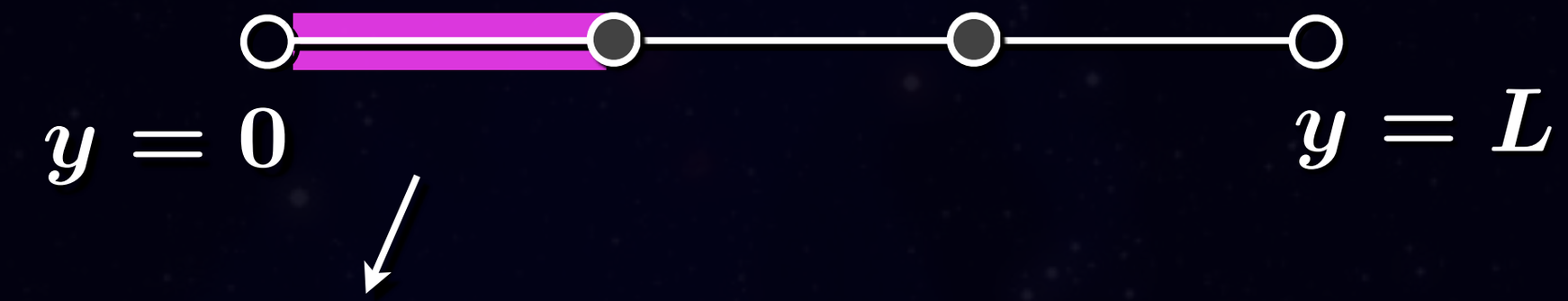

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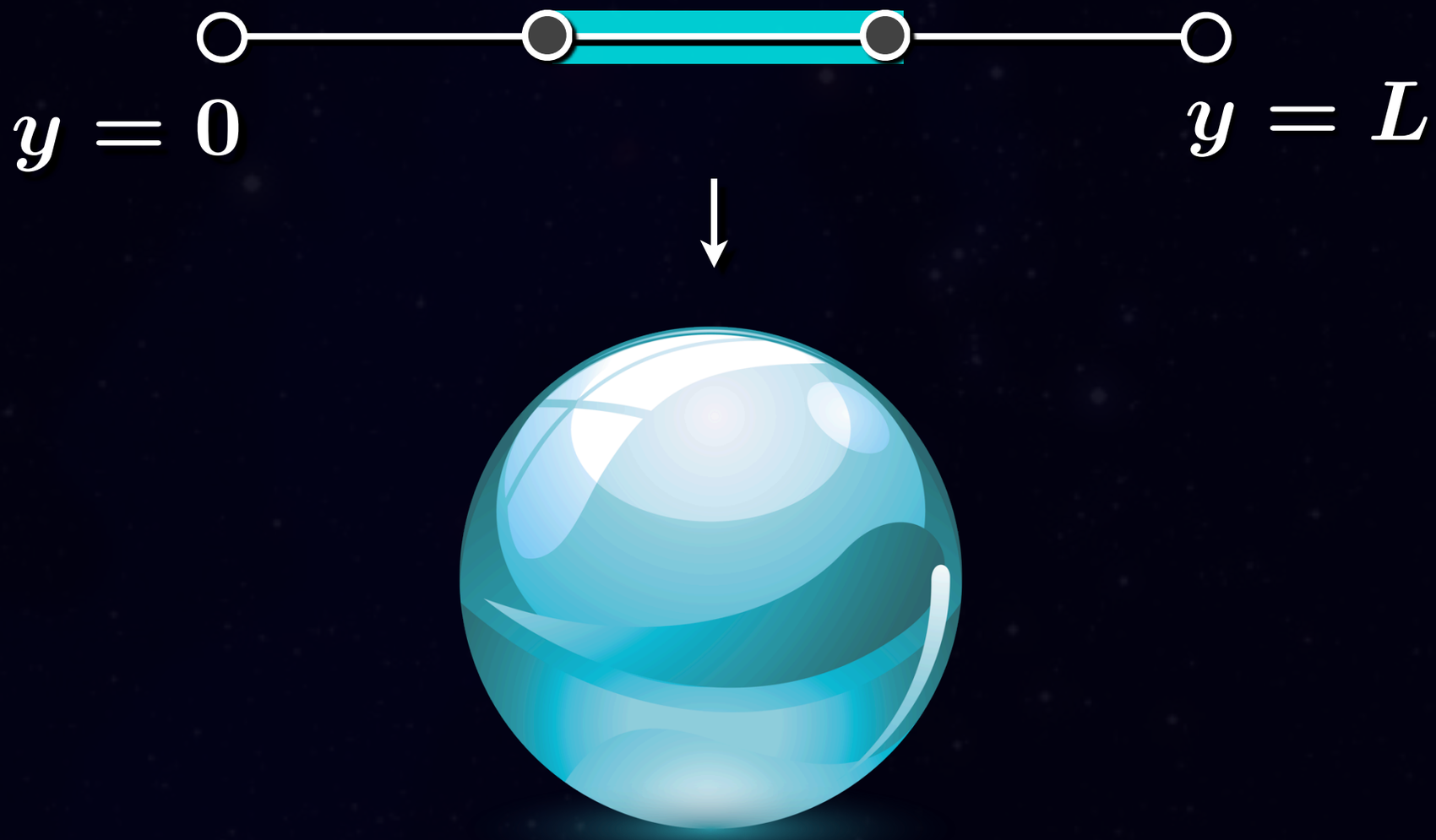

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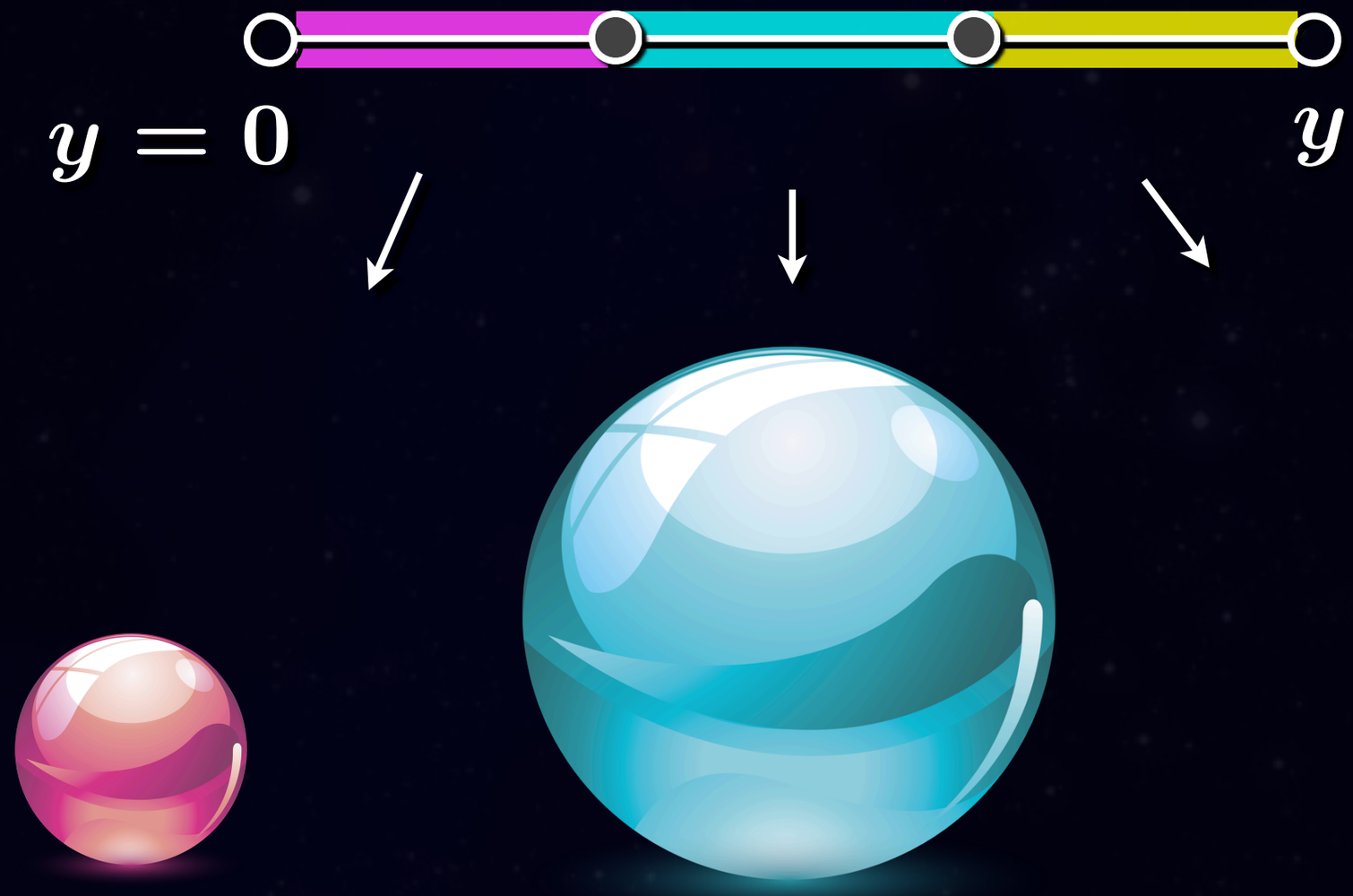

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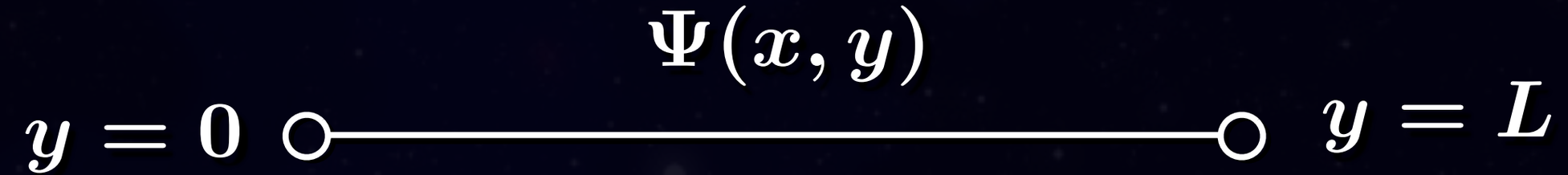

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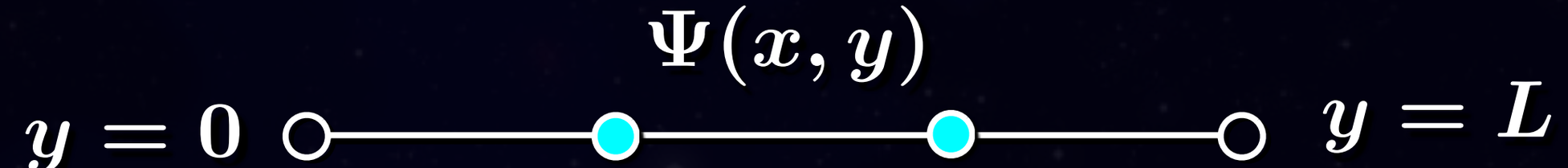

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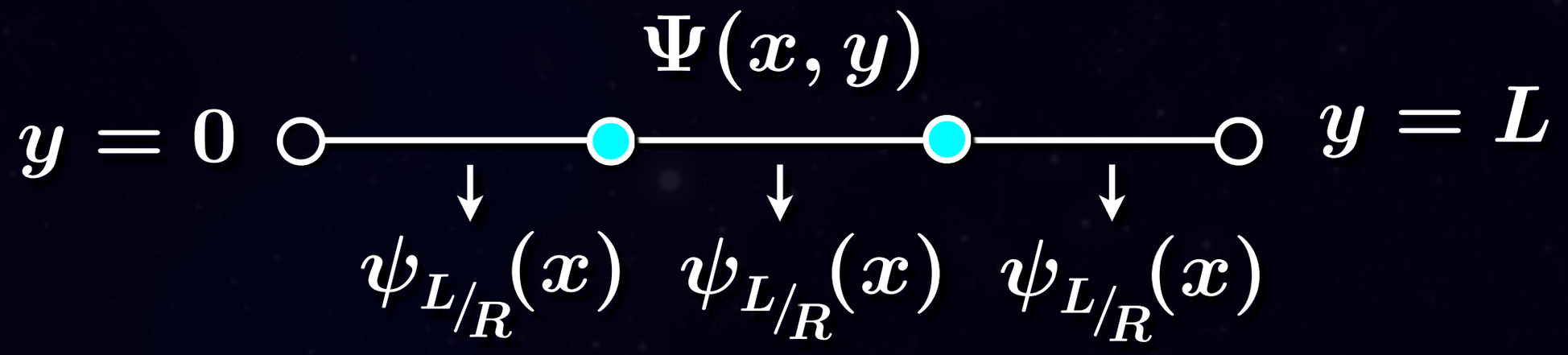

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  - ⋆ **Flavor mixing is determined by a configuration of extra dimension.**

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with { 5d fermions (one generation)
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Gauge fields

$$W_M^a(x,y)$$
$$B_M(x,y)$$

Higgs field Singlet scalar

$$H(x,y)$$
$$\Phi(x,y)$$

Fermions

$$\begin{pmatrix} u(x,y) \\ d(x,y) \end{pmatrix} \quad \begin{pmatrix} e(x,y) \\ \nu(x,y) \end{pmatrix}$$
$$u'(x,y) \qquad e'(x,y)$$
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$H(x,y)$ ← **5D Higgs (break the gauge sym.)**

$\Phi(x,y)$ ← **Gauge singlet scalar field (for fermion mass hierarchy)**

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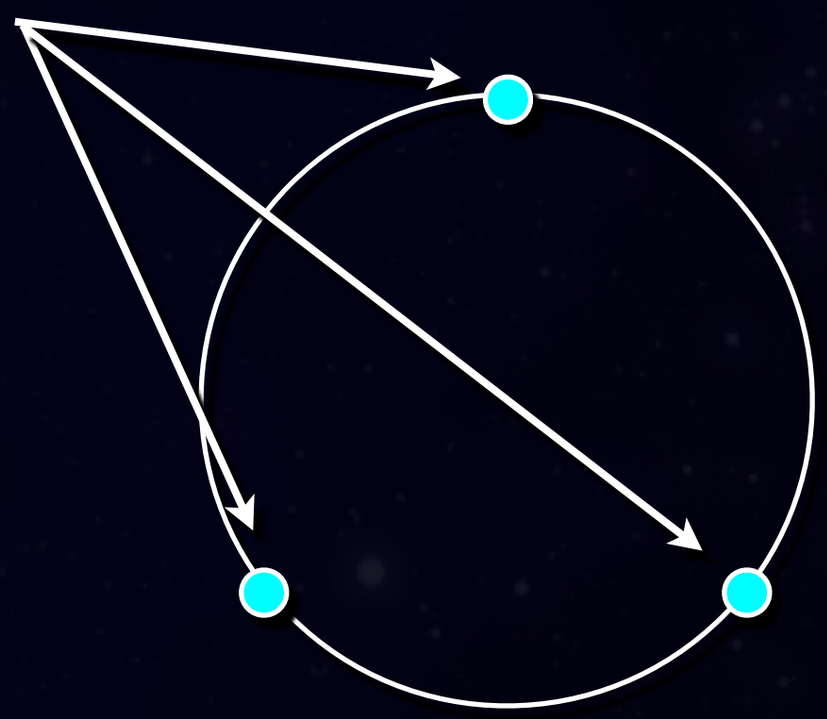

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compatible with { • the action principle
• 5d gauge invariance etc.

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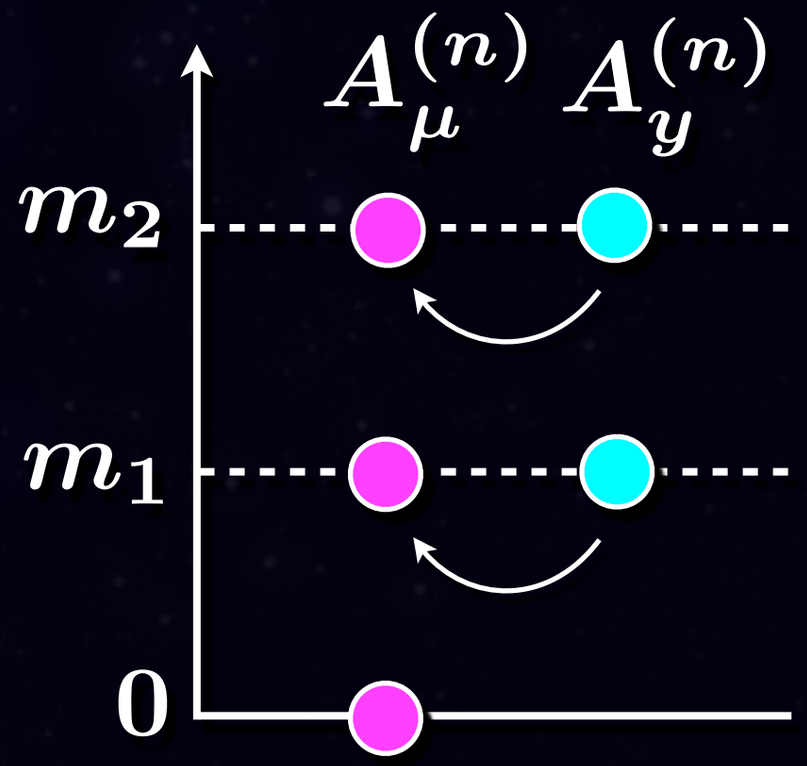
★4d spectrum



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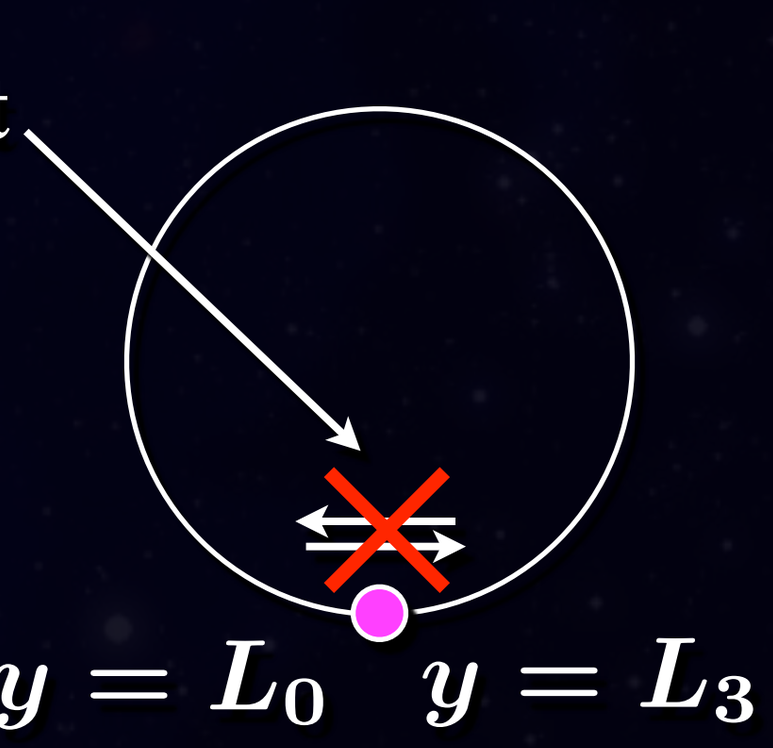

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$$(-\infty \leq L_\pm \leq +\infty)$$

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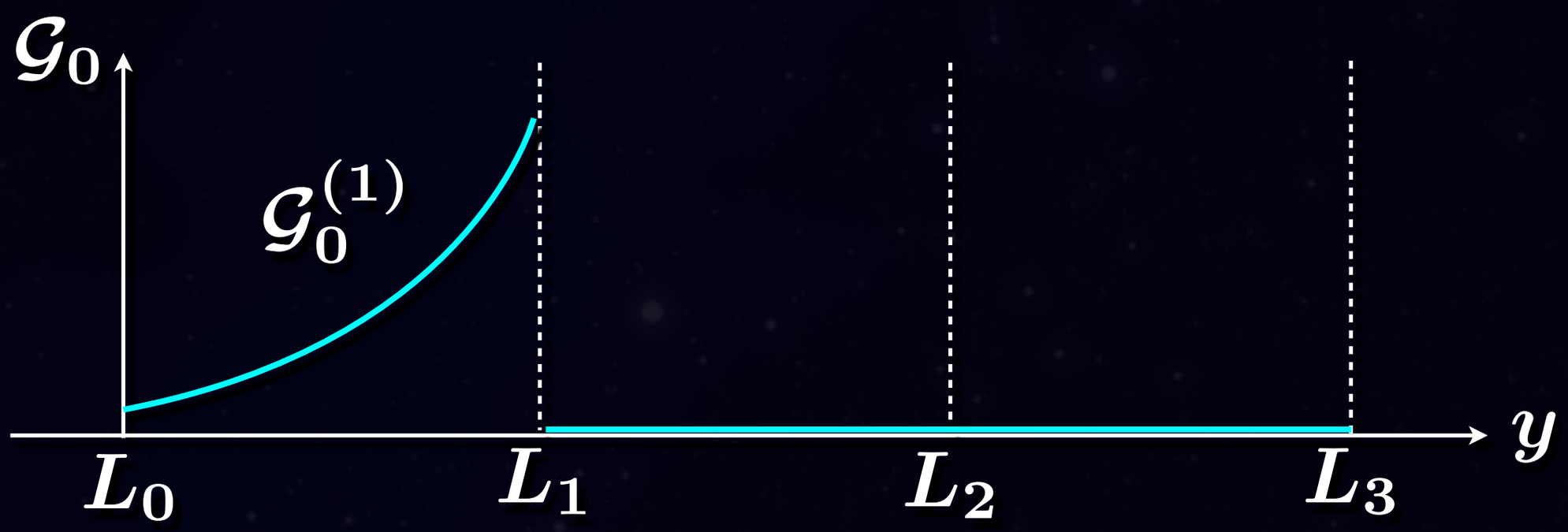

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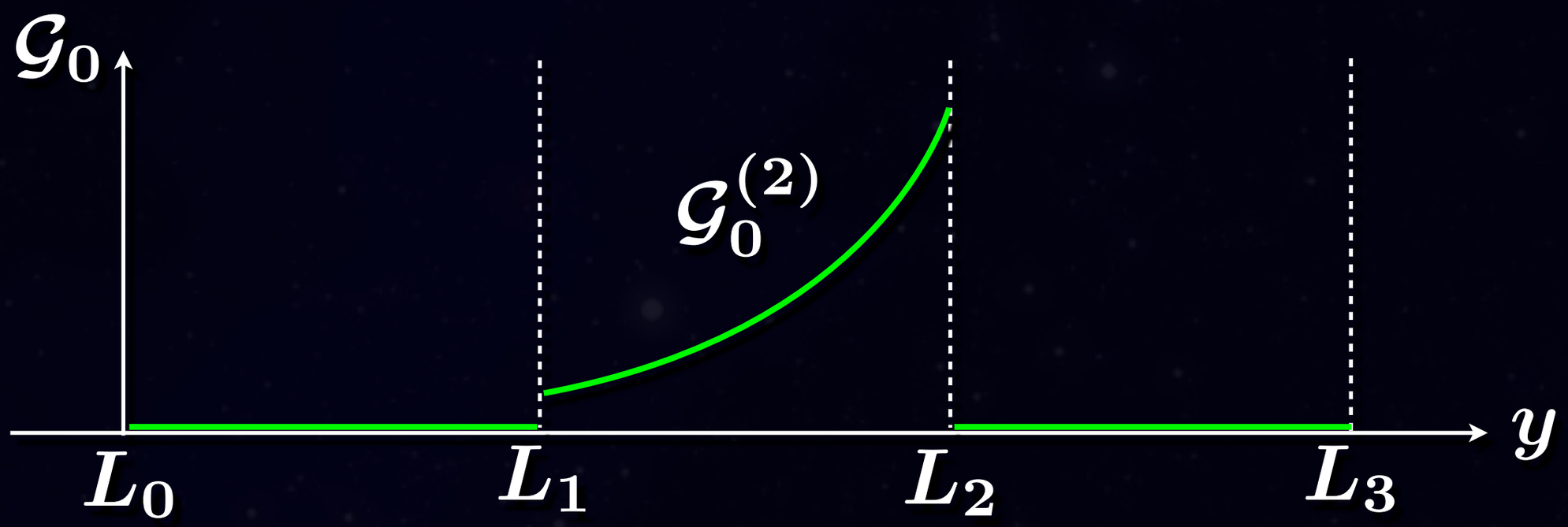

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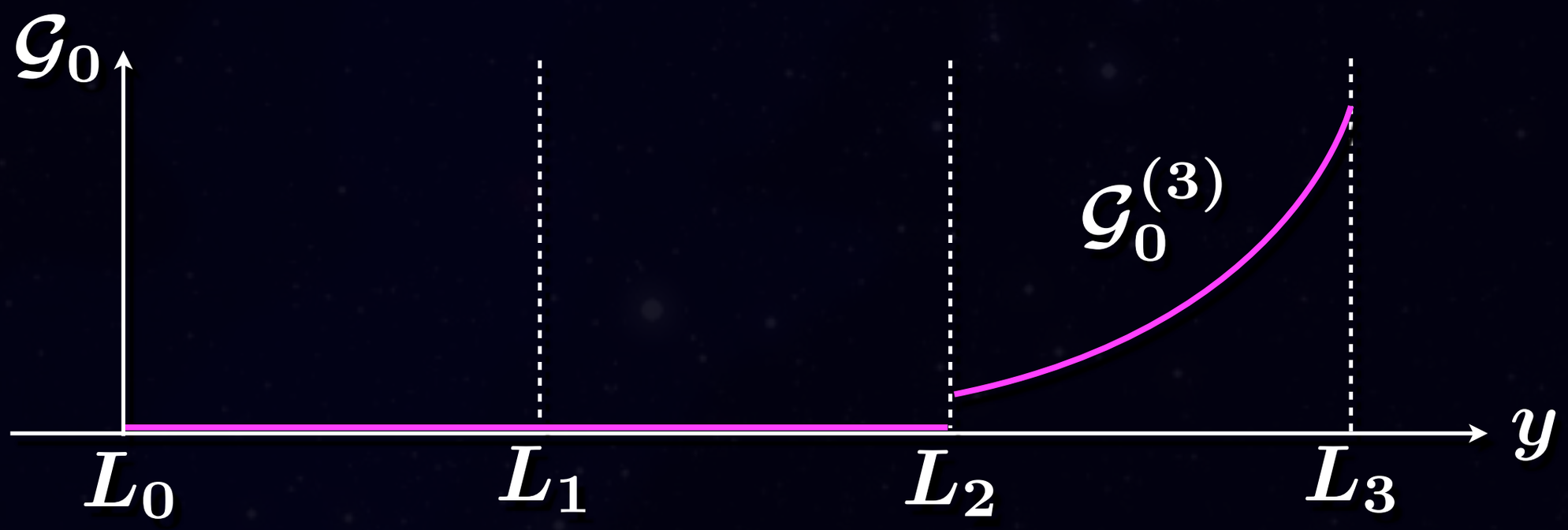

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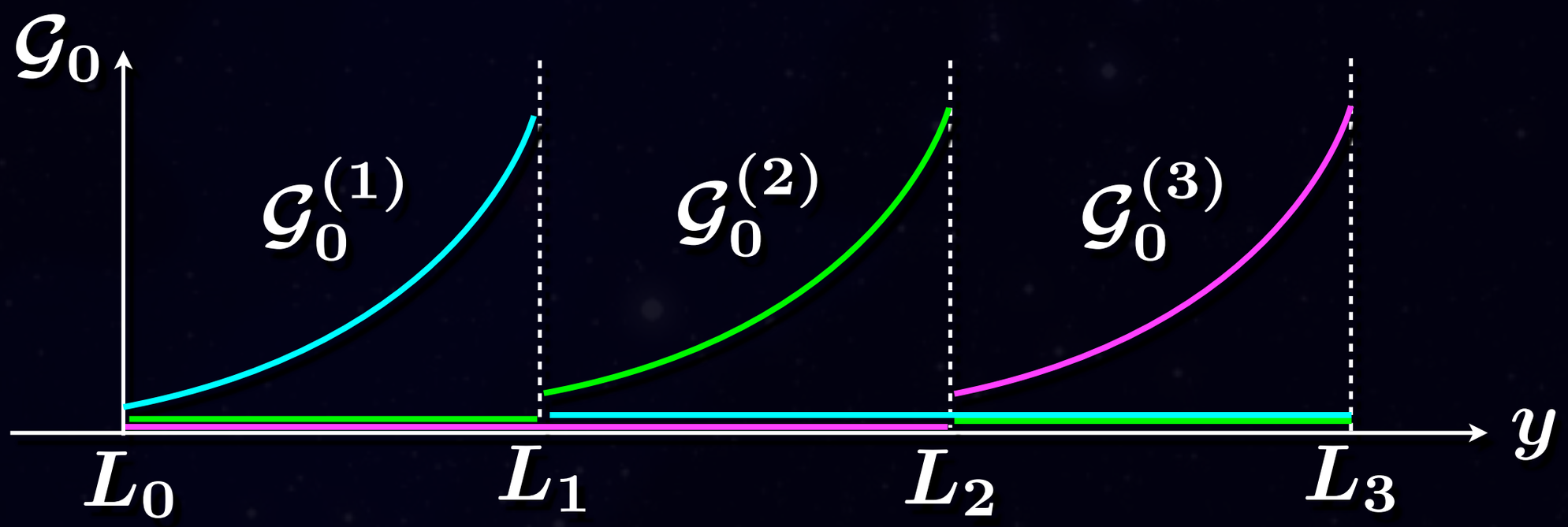

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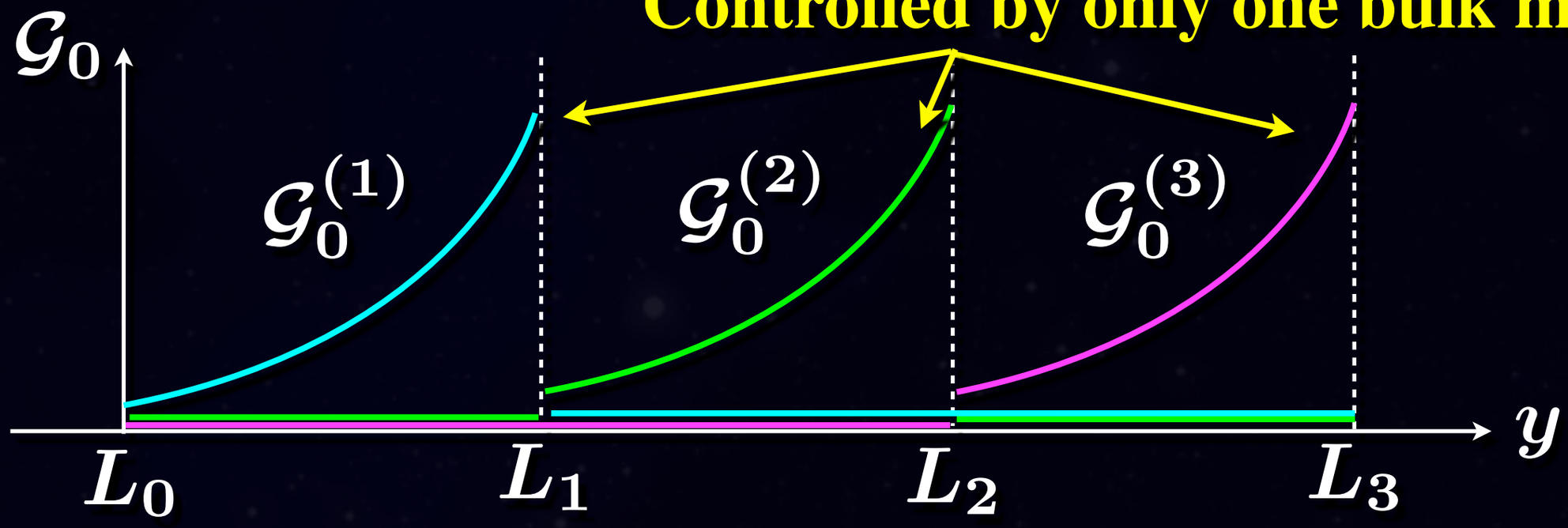
Controlled by only one bulk mass !!



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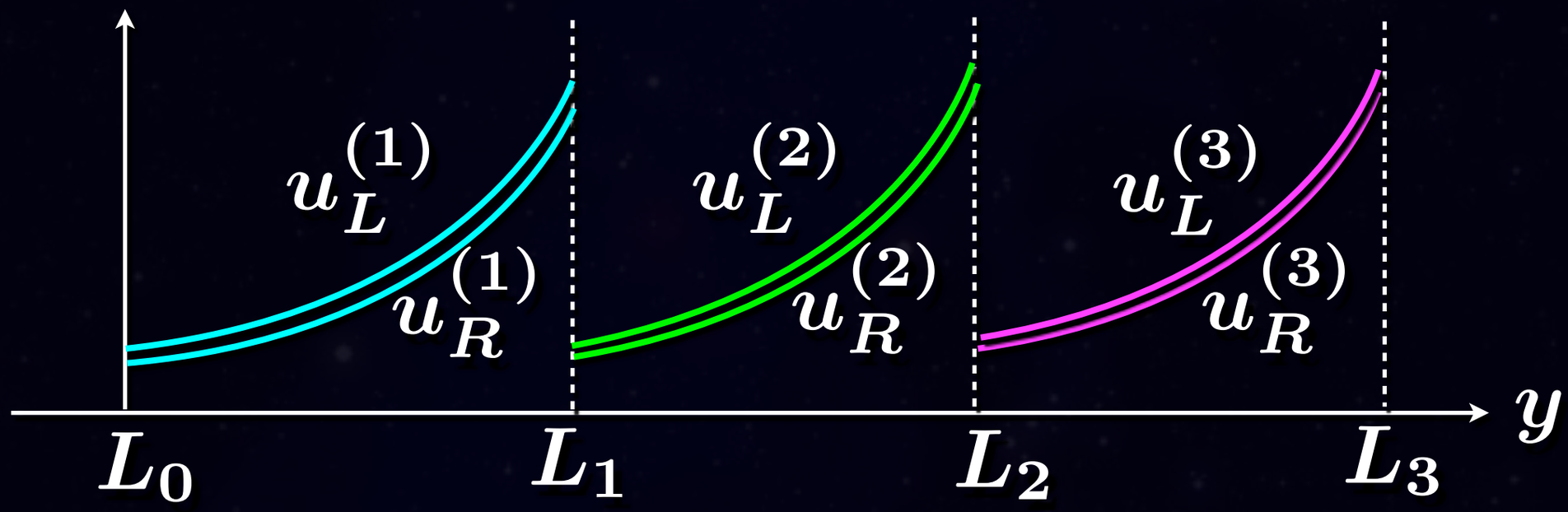

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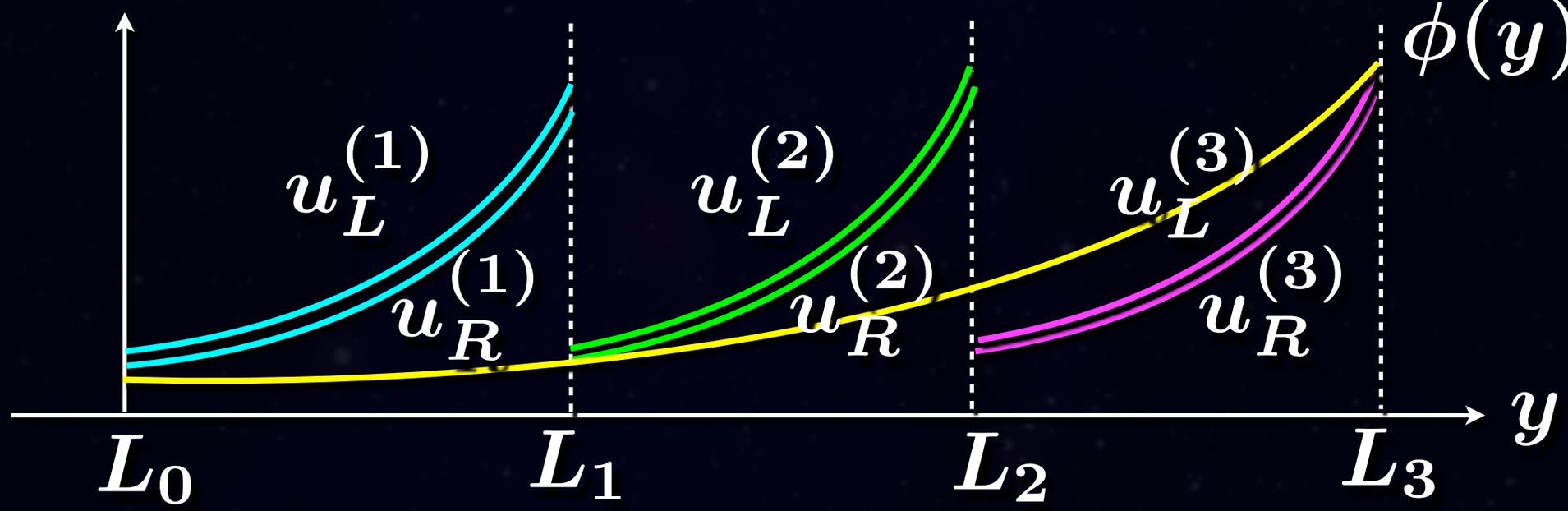

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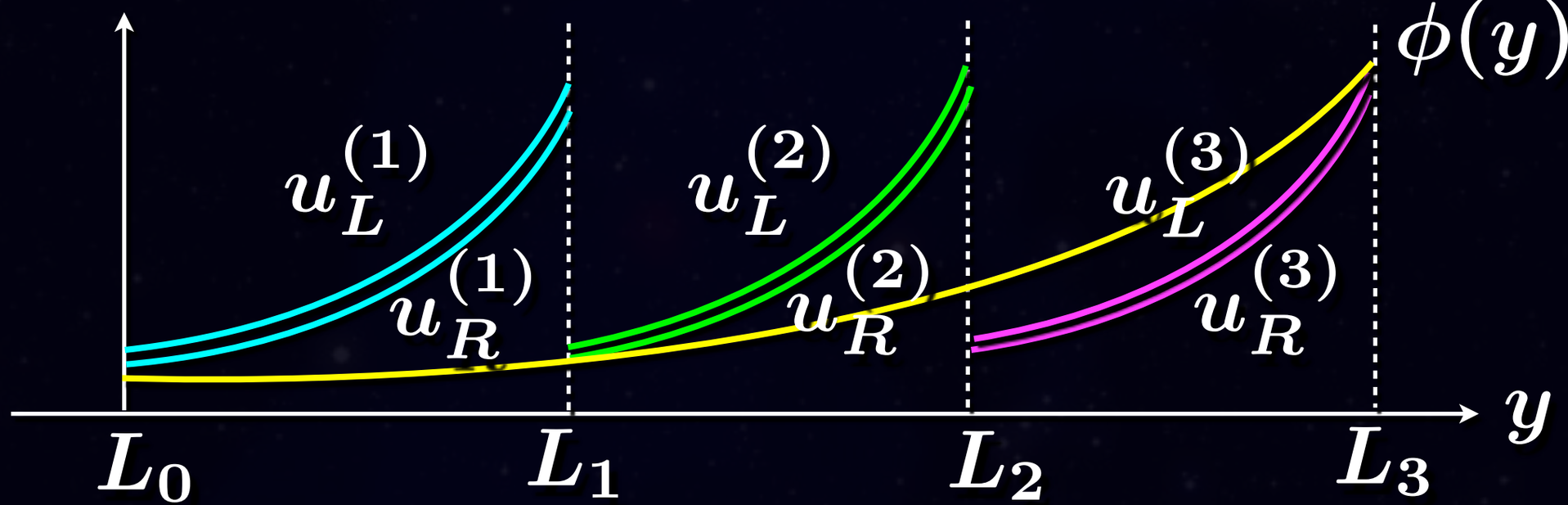
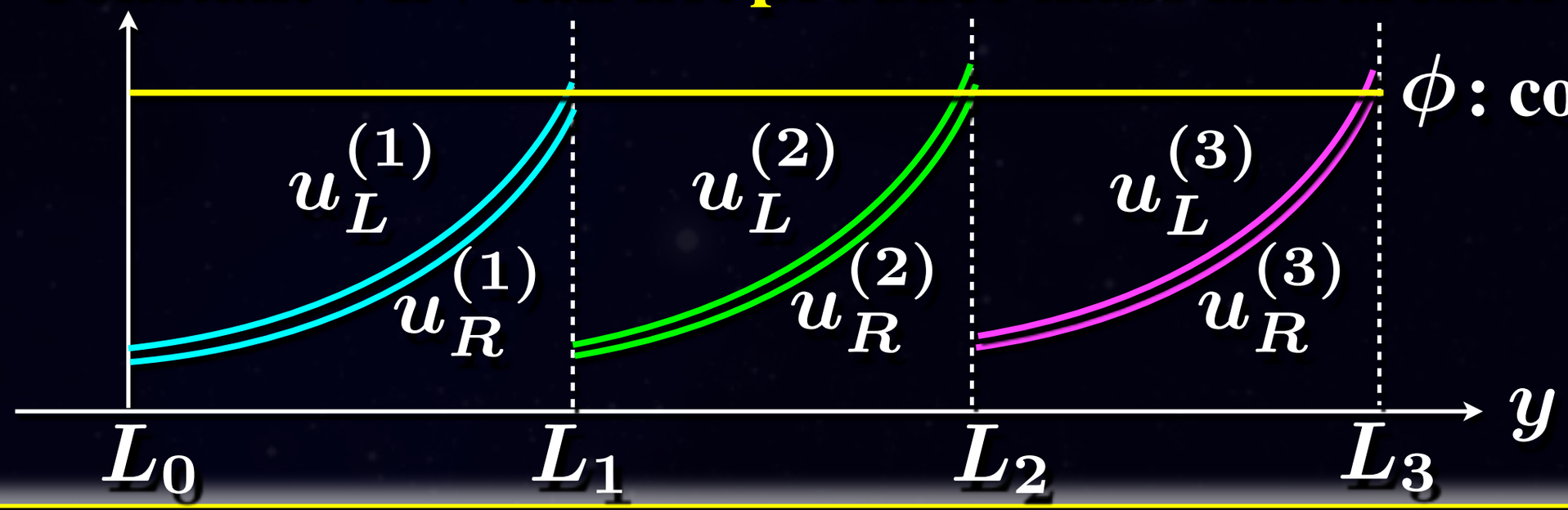


★ Constant VEV **can not produce** mass hierarchies.



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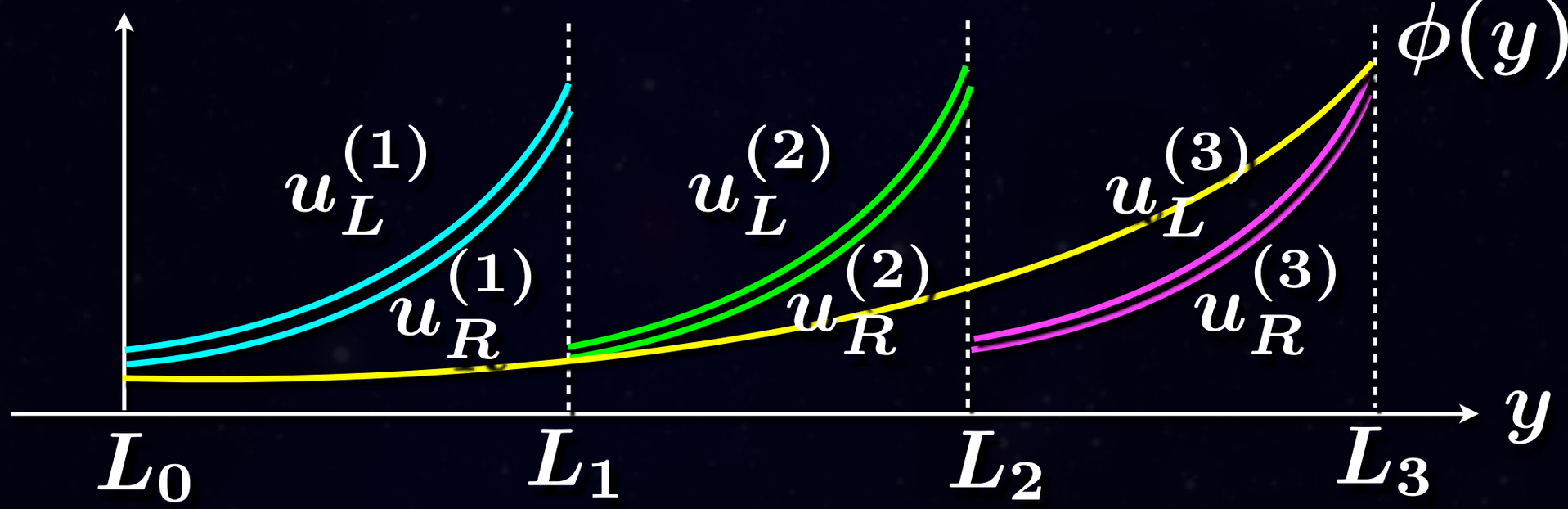

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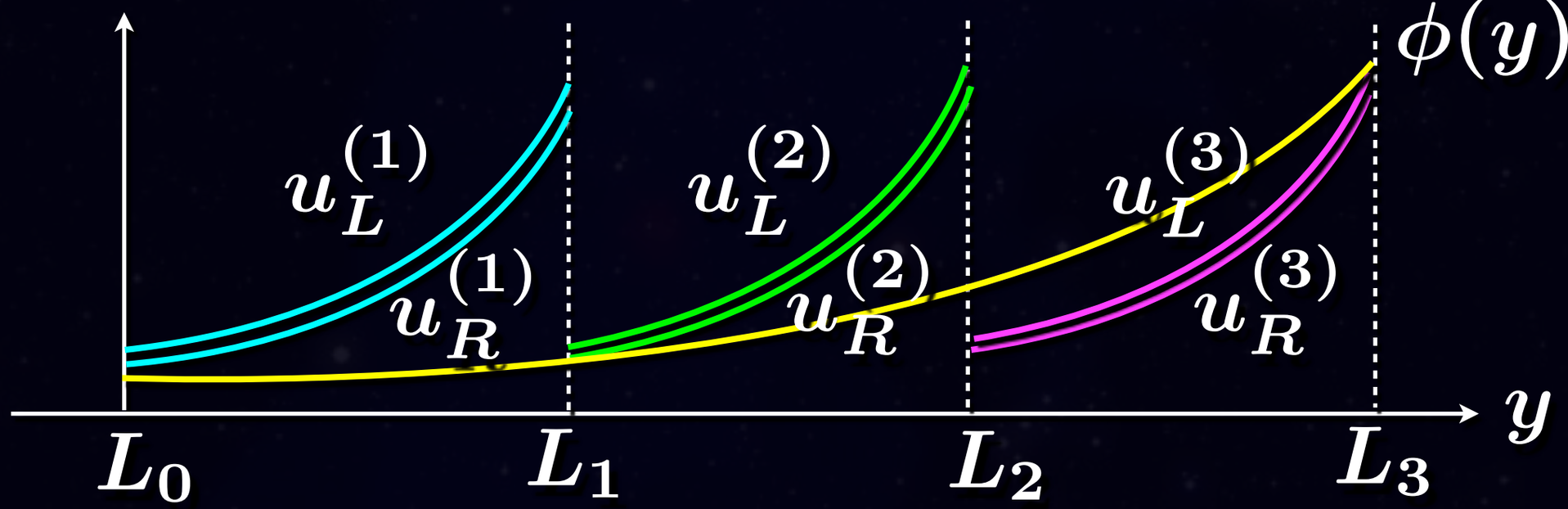

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$$\langle \Phi(x, y) \rangle = \phi(y)$$

Y.F., T.Nagasawa, S.Ohya and M.Sakamoto, PTP 126 (2011) 841

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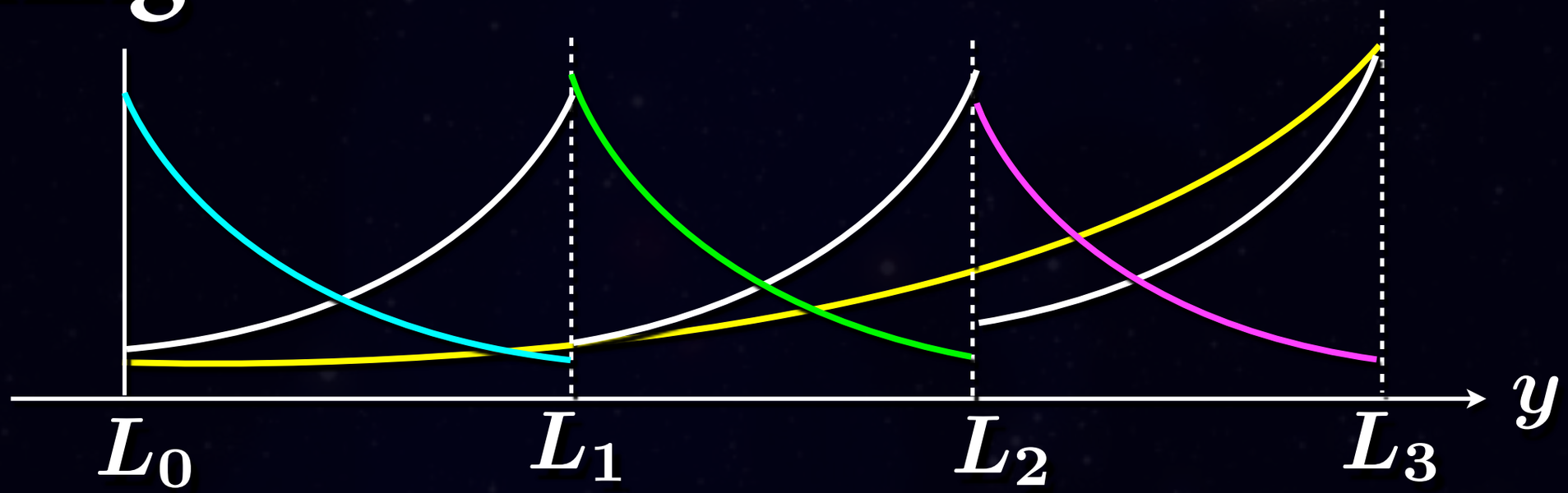

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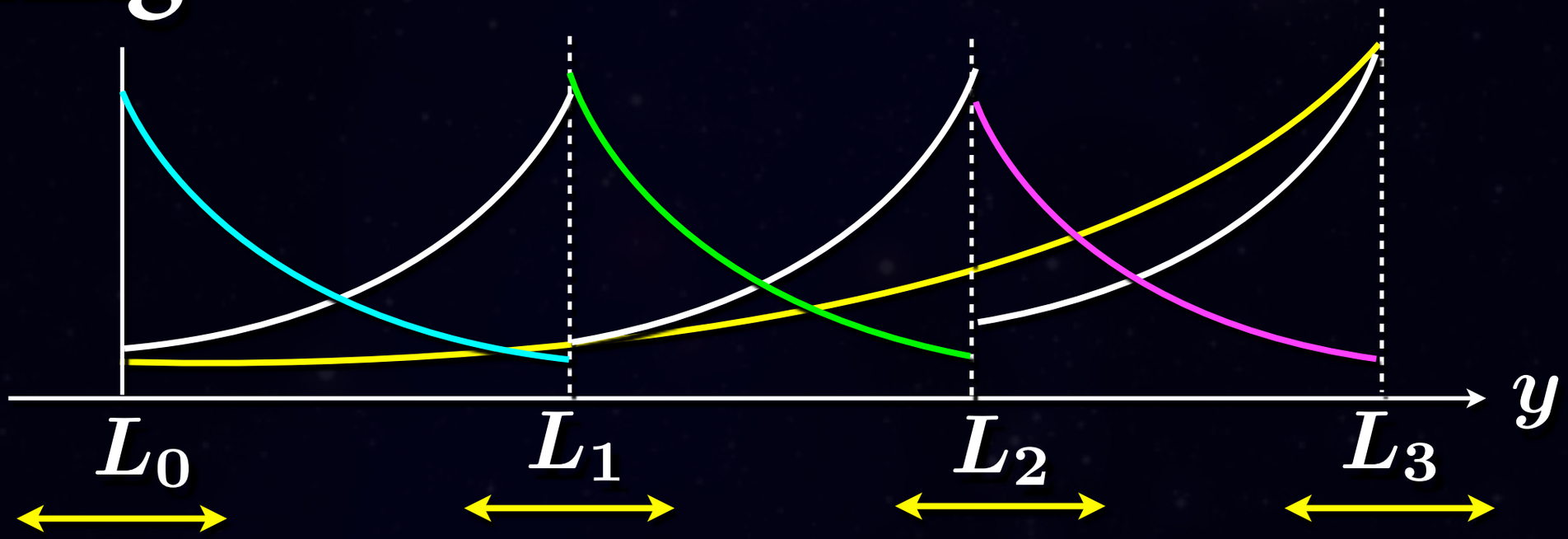

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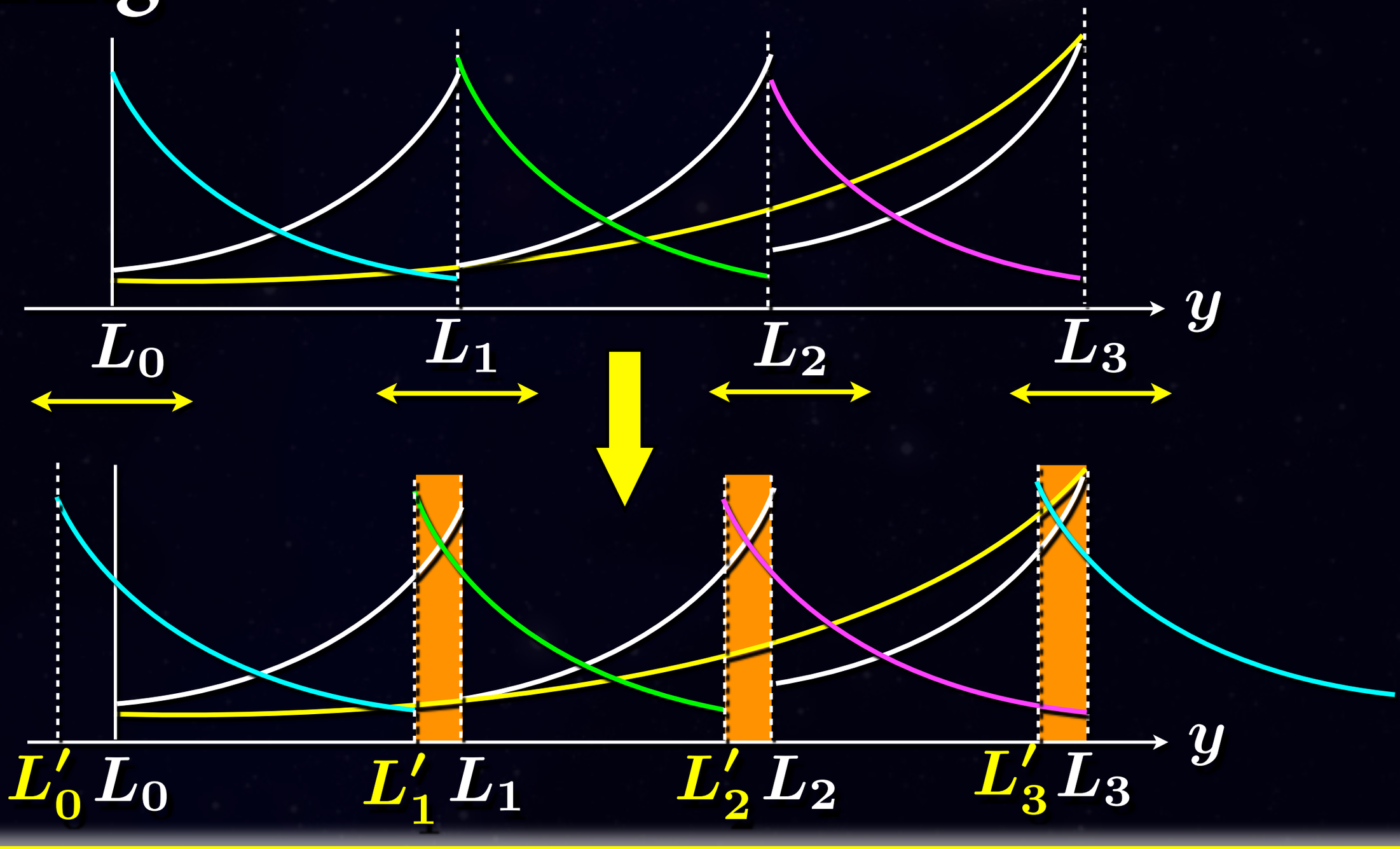

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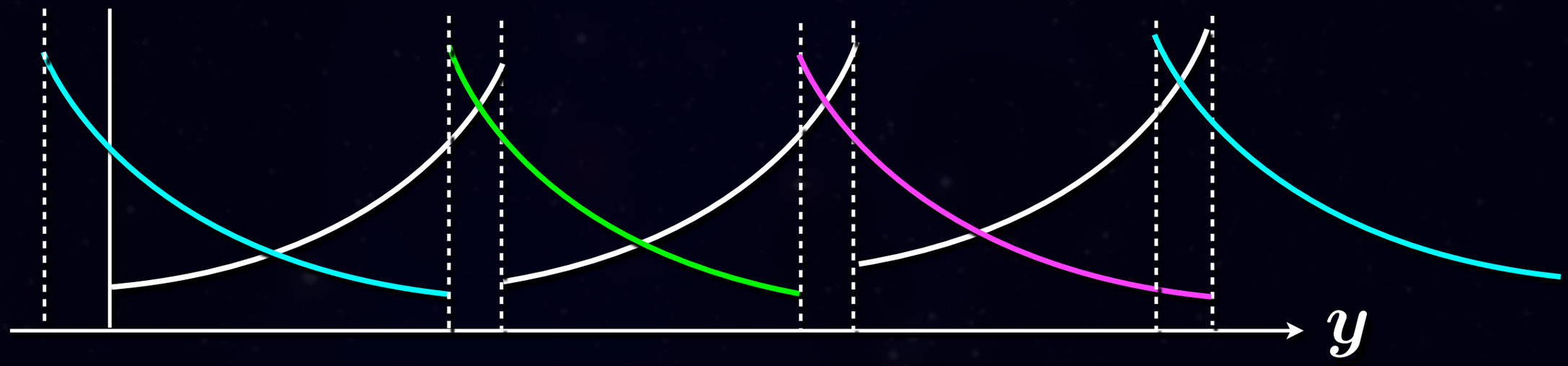

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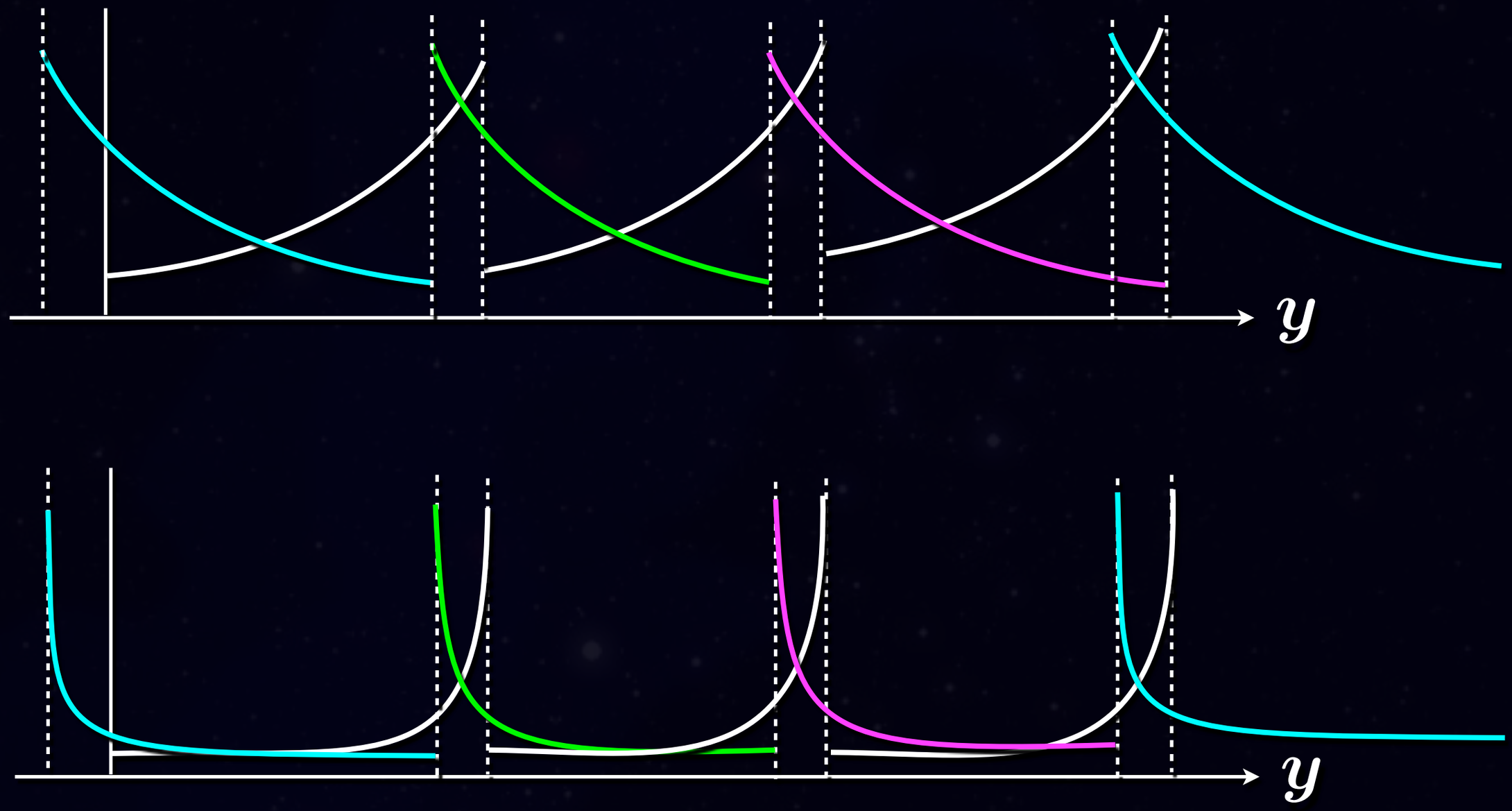

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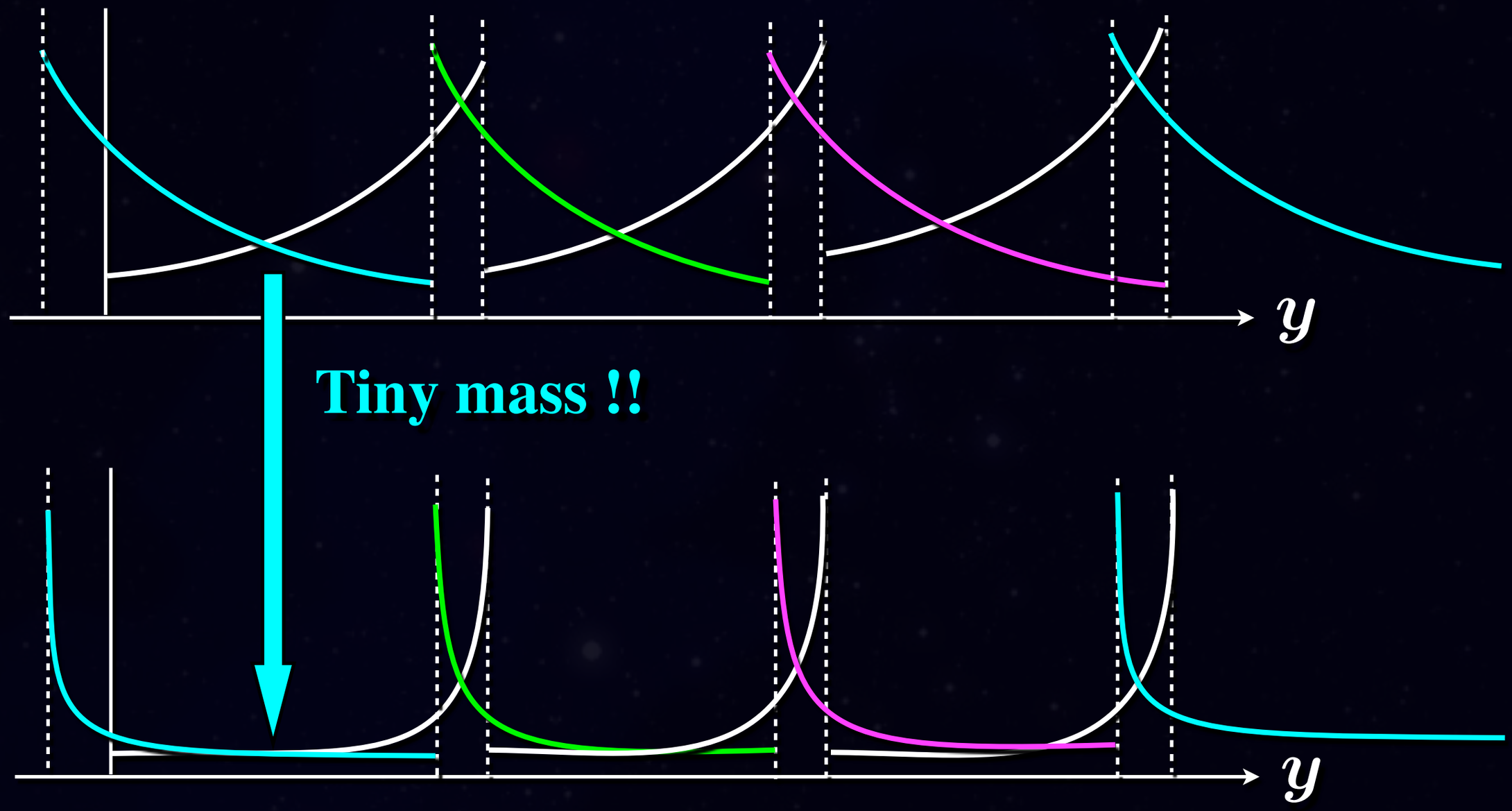

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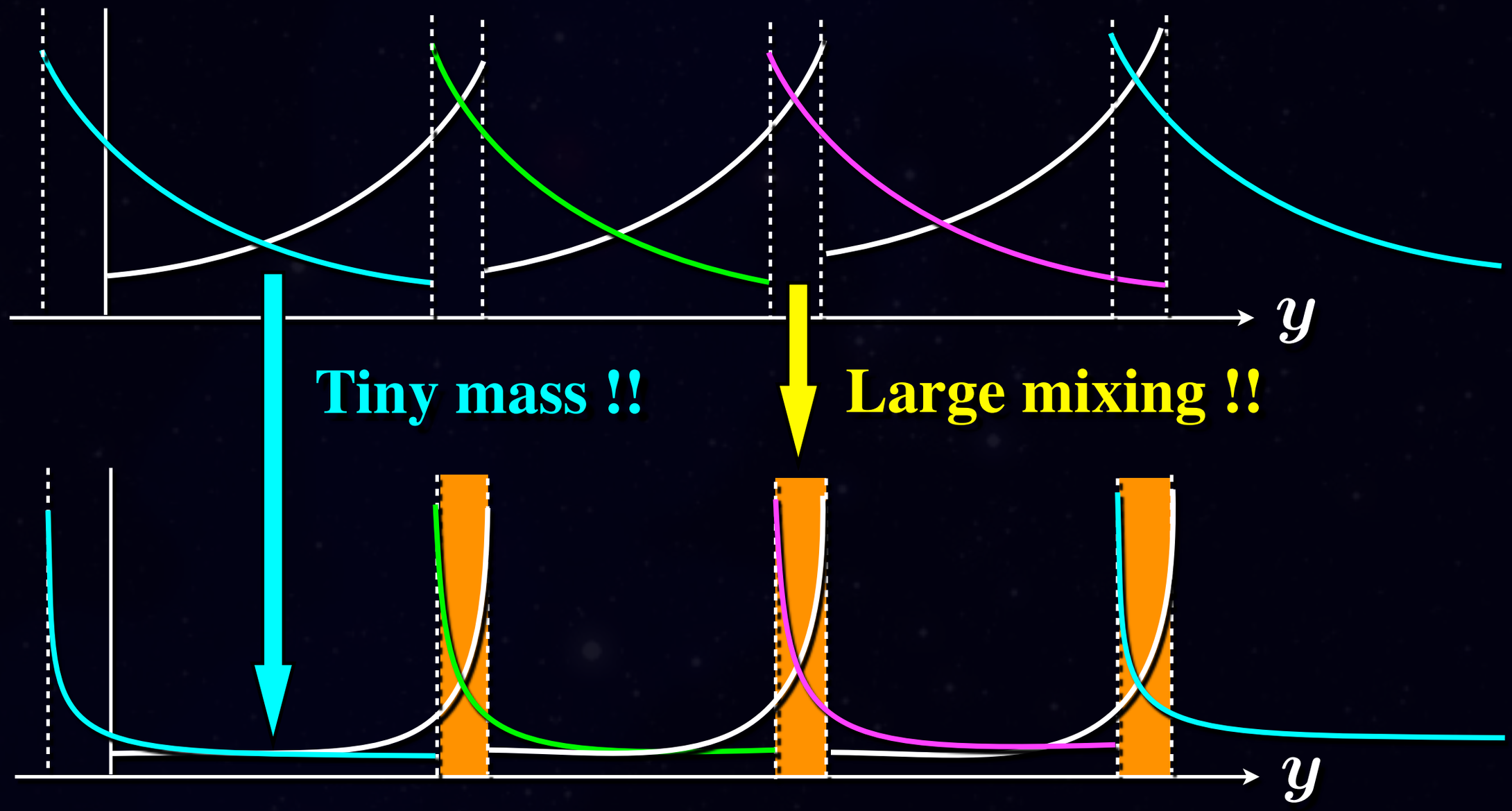

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Good agreement !! But....

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The opposite sign appears...

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60% larger than real CKM.....

Conclusion and Discussion

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5d gauge theories on a circle
with specified boundary conditions

↓ **The low energy effective theory**

4d gauge theories

+ Generation
+ Large mass hierarchy
+ Large/Small mixing

Conclusion and Discussion

- **Challenges for the future**

Conclusion and Discussion

◆ Challenges for the future

- ⋆ **Reproduce PMNS matrix**
- ⋆ **Warped metric**
- ⋆ **CP phase from BCs**
- ⋆ **Constraint for extra dim. from FCNC**

⋮