「Theory of Quarks and Leptons」へ向けて

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Here I would like to discuss only

How to understand problems of

- Generations, i.e., chiral generations

  No promising ideas:

  enlarging GUT group – horizontal symmetry,

  Hodge numbers of CY manifold,

  orbifolds, ….

- Dark energy or vacuum condensates

Generations:

- Survival Hypothesis (Georgi)
  Quarks and Leptons are light (compared with Planck scale),
  because they are chiral (complex representation) with respect to the SM group $SU(3) \times SU(2) \times U(1)$

This beautifully explains why they exist, but it also pose a difficulty in naïve trials of

Bigger GUT groups
Higher dimensions
Higher N SUSY

} to include generations

→ vector-like (non-chiral)

Simple Groups do not work (naively)

- SU(n), SO(4k+2), E_6 allow complex rprs.

But SU(n) with n>5, SO(4k+2) with k>2 have only real (vector-like) reprs wrt the subgroup SU(5) or SO(10).

- E_7 and E_8 have only real rprs.

- Needs:

orbifolding, or Kawamura mechanism

complex structure for extra dimensions, CY

Or simply, direct product

horizontal symmetry

other ideas? → Quasi-NG-fermion

Quasi-NG-fermions

- If spontaneous breaking of a global symmetry: $G \to H$, then
  Nambu-Goldstone bosons appear for
  broken generators G/H at around M_{Planck}
- If ∃SUSY, NG bosons are accompanied by massless spin ½ fermions, called Quasi-NG-fermions
  (Buchmüller-Peccei-Yanagida'87, Ong'83)
- Kugo-Yanagida'84
  three generations from $E_7/SU(5)\times SU(3)\times U(1)$

Exceptional Groups: E_n

E_8 248 Maximal: a priori ***Raison d'être*** superstring, …

↓

E_7 133 N=8 SUGRA $E_{7(7)}$

↓

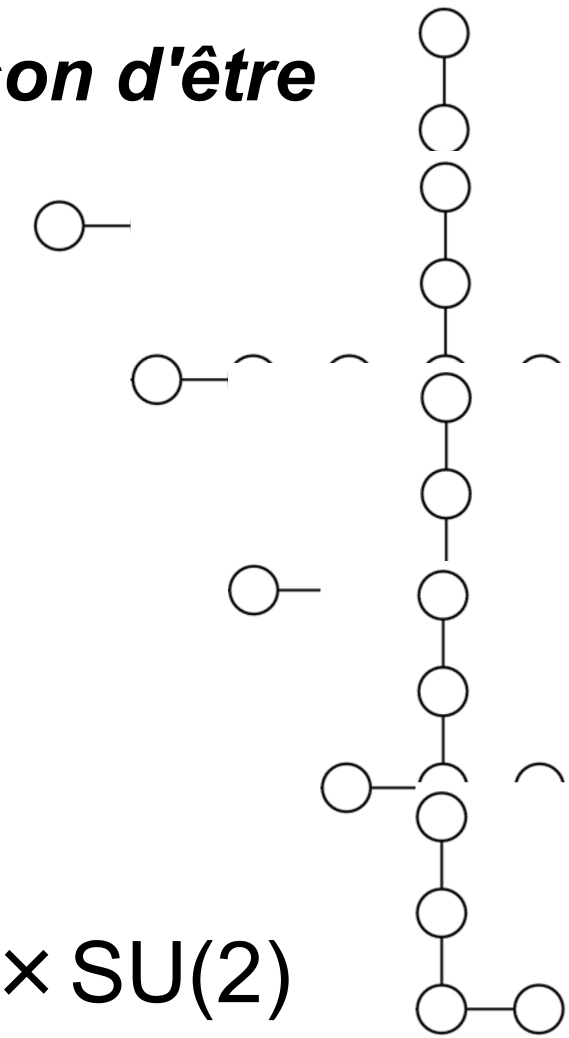
E_6 78

↓

$E_5 = SO(10)$ 45

↓

$E_4 = SU(5)$ 24 → $E_3 = SU(3) \times SU(2)$



Suppose $\exists E_8$ or $E_7 \rightarrow E_4 \times U(1)^n$

$E_8 \rightarrow E_4 \times U(1)^4$

$$\frac{E_8}{E_4 \times U(1)^4} = \frac{E_8}{E_7 \times U(1)} \times \frac{E_7}{E_4 \times U(1)^3}$$

$E_7 \rightarrow E_4 \times U(1)^3$

$$\frac{E_7}{E_4 \times U(1)^3} = \frac{E_7}{E_6 \times U(1)} \times \frac{E_6}{E_5 \times U(1)} \times \frac{E_5}{E_4 \times U(1)}$$

$E_5 = SO(10)$, $E_4 = SU(5)$

Exceptional Groups: E_n

E_8 248 $\supset E_7$

$\downarrow$ 248-(133+1) = 56 +1 + h.c.

$\rightarrow$ NG chiral multiplet 56 of E_7

but any reprs of E_7 are real

56 = 27+1+27*+1* $\rightarrow$ 0 generations

E_7 133

$\downarrow$ 133-(78+1) = 27 + h.c. $\rightarrow$

Branching rules for E groups

E_8

$$\begin{aligned} E_8 &\supset E_7 \times SU(2) \\ \mathbf{248} &= (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{133}) + (\mathbf{2},\mathbf{56}) \\ 3875 &= (1,1) + (2,56) + (3,133) + (1,1539) + (2,912) \end{aligned}$$

E_7

$$\begin{aligned} E_7 &\supset E_6 \times U(1) \\ 56 &= 1_3 + 1_{-3} + 27_1 + \overline{27}_{-1} \\ \mathbf{133} &= \mathbf{1}_0 + \mathbf{78}_0 + \mathbf{27}_{-2} + \overline{\mathbf{27}}_2 \\ 912 &= 27_1 + \overline{27}_{-1} + 78_3 + 78_{-3} + 351_1 + \overline{351}_{-1} \end{aligned}$$

E_6

$$
\begin{aligned}
E_6 &\supset E_5[=SO(10)] \times U(1) \\
27 &= 1_4 + 10_{-2} + 16_1 \\
\mathbf{78} &= \mathbf{1_0} + \mathbf{45_0} + \mathbf{16_{-3}} + \overline{\mathbf{16}}_{\mathbf{3}} \\
351 &= 10_{-2} + \overline{16}_{-5} + 16_1 + 45_4 + 120_{-2} + 144_1 \\
351' &= 1_{-8} + 10_{-2} + 16_{-5} + 54_4 + \overline{126}_{-2} + 144_1
\end{aligned}
$$

$E_5 = SO(10)$

$$
\begin{aligned}
E_5 &\supset E_4[=SU(5)] \times U(1) \\
10 &= 5_{-2} + \bar{5}_2 \\
16 &= 1_5 + \bar{5}_{-3} + 10_1 \\
\mathbf{45} &= \mathbf{1_0} + \mathbf{24_0} + \mathbf{10_{-4}} + \overline{\mathbf{10}}_{\mathbf{4}} \\
54 &= 24_0 + 15_{-4} + \overline{15}_4 \\
120 &= 5_{-2} + \bar{5}_2 + 10_6 + \overline{10}_{-6} + 45_{-2} + \overline{45}_2 \\
126 &= 1_{10} + \bar{5}_2 + 10_6 + \overline{15}_{-6} + 45_{-2} + \overline{50}_2
\end{aligned}
$$

Sequential Symmetry Breaking: E_n

E_7 133 $\supset E_6$

$\downarrow$ 133-(78+1) = 27 + h.c. $\rightarrow$ E_6 27

E_6 78 $\supset E_5$

$\downarrow$ 78-(45+1) = 16 +h.c. $\rightarrow$ E_5 16

E_5 = SO(10) 45 $\supset E_4$

$\downarrow$ 45-(24+1) = 10 +h.c. $\rightarrow$ E_4 10

E_4 = SU(5) 24

Appearing NG chiral multiplets

$E_7 \supset E_5$

$\downarrow \rightarrow E_6\ \mathbf{27} = 16 + 10 + 1$

$= (10+5^*+1)+(5+5^*)+1$

$E_6 \supset E_4$

$\downarrow \rightarrow E_5\ \mathbf{16} = 10+5^*+1$

$E_5 = SO(10) \supset E_4$

$\downarrow \rightarrow E_4\ \mathbf{10} = 10$

$E_4 = SU(5)$

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$E_5 = SO(10) \supset E_4$

$\downarrow \;\rightarrow\; E_4 \; 10 = 10$

$E_4 = SU(5)$

Three generations $3 \times (10+5^*) + 5$ + singlets !

But this has an SU(5) anomaly

The original theory is anomaly free (E_7, E_8 are safe)

’t Hooft’s anomaly matching condition

→ There should appear a massless 5*
as a non-NG chiral matter field

Many problems to be considered

- Low energy effective Lagrangian
- Coupling to SUGRA
- Explicit G-symmetry breaking

  to make NG bosons massive

  in particular, e.g., breaking by gauging H=SU(5)

  → quasi-NG fermions remain massless SM symmetry

  → if SUSY remains, NG bosons are also massless

  not get $m^2 \sim g^2 v^2$ so $m^2 \sim g^2 \not{v}^2_{\text{SUSY}}$?

  also to have superpotential terms
- Spontaneous GUT symmetry breaking
- Spontaneous or explicit soft SUSY breaking
- Yukawa coupling

Effective Theory for the NG superfields: BKMU non-linear Lagrangian

$$\mathcal{G} = \{ \overbrace{H, \quad X,}^{\hat{H}} \quad \overbrace{\bar{X}}^{\mathcal{G}^{\mathcal{C}}/\hat{H}} \}$$
$$= 0, \quad > 0, \quad < 0$$

$$\begin{aligned} E_7 &\supset E_6 \times U(1) \\ 56 &= 1_3 + 1_{-3} + 27_1 + \overline{27}_{-1} \\ 133 &= \underbrace{1_0 + 78_0}_{H} + \underbrace{27_{-2}}_{\bar{X}} + \underbrace{\overline{27}_2}_{X} \end{aligned}$$

BKMU basic variable $\quad \xi(\phi) = e^{i\phi\cdot\bar{X}}$

変換則 $\quad g\xi(\phi) = \xi(\phi')\hat{h}(g,\phi)$

$$\xi(\phi) \to \xi(\phi') = g\xi(\phi)\hat{h}^{-1}(g,\phi)$$

どの表現で計算しても良いが56で計算すると

$$56 = 1_3 + 1_{-3} + 27_1 + \overline{27}_{-1}$$

$$133 = \underbrace{1_0 + 78_0}_{H} + \underbrace{27_{-2}}_{\overline{X}} + \underbrace{\overline{27}_2}_{X}$$

$$\begin{pmatrix} 1_0 & \bar{27}_2 & & \\ 27_{-2} & 1+78 & \bar{27}_2 & \\ & 27_{-2} & 1+78 & \bar{27}_2 \\ & & 27_{-2} & 1_0 \end{pmatrix} \begin{pmatrix} 1_3 \\ 27_1 \\ \bar{27}_{-1} \\ 1_{-3} \end{pmatrix}$$

$$\xi(\phi) = \exp \begin{pmatrix} 0 & & & \\ \phi_{27} & 0 & & \\ & \phi_{27} & 0 & \\ & & \phi_{27} & 0 \end{pmatrix} = \begin{pmatrix} 1 & & & \\ \phi_{27} & 1 & & \\ \phi_{27}^2 & \phi_{27} & 1 & \\ \phi_{27}^3 & \phi_{27}^2 & \phi_{27} & 1 \end{pmatrix}$$

$$\hat{h} = \begin{pmatrix} 1 & \bar{27}_2 & (\bar{27})^2 & (\bar{27})^3 \\ & 1+78 & \bar{27} & (\bar{27})^2 \\ & & 1+78 & \bar{27} \\ & & & 1 \end{pmatrix}$$

BKMU non-linear Lagrangian

$$K(\phi, \bar{\phi}) = \sum_i c_i \ln \det_{\eta_i} \left(\xi^\dagger(\phi)\xi(\phi) \right)$$

η は $\hat{H}$-closed projection s.t. $\hat{H}\eta = \eta\hat{H}\eta$

$$\eta = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$$ の形

Sequential breaking

$$\begin{array}{ccccccc} E_7 & \underset{\substack{\phi_{27} \\ \xi_{56}(\phi_{27})}}{\to} & E_6 & \underset{\substack{\phi_{16} \\ \xi_{27}(\phi_{16})}}{\to} & E_5 & \underset{\substack{\phi_{10} \\ \xi_{16}(\phi_{10})}}{\to} & E_4 \end{array}$$

Sequential breaking

$$
\begin{array}{ccccccc}
E_7 & \rightarrow & E_6 & \rightarrow & E_5 & \rightarrow & E_4 \\
 & \phi_{27} & & \phi_{16} & & \phi_{10} & \\
 & \xi_{56}(\phi_{27}) & & \xi_{27}(\phi_{16}) & & \xi_{16}(\phi_{10}) &
\end{array}
$$

$$
\begin{aligned}
&\xi(\phi_{27}, \phi_{16}, \phi_{10}) \\
&= \xi_{56}(\phi_{27}) \times \begin{pmatrix} 1 & & & \\ & \xi_{27}(\phi_{16}) & & \\ & & \xi_{\overline{27}}(\phi_{16}) & \\ & & & 1 \end{pmatrix} \\
&\times \begin{pmatrix} 1 & & & \\ & \begin{pmatrix} 1 & & \\ & \xi_{16}(\phi_{10}) & \\ & & \xi_{10}(\phi_{10}) \end{pmatrix} & & \\ & & \begin{pmatrix} \xi_{10}(\phi_{10}) & & \\ & \xi_{\overline{16}}(\phi_{10}) & \\ & & 1 \end{pmatrix} & \\ & & & 1 \end{pmatrix}
\end{aligned}
$$

Matter fields

$$\overbrace{\Psi}^{\text{linear base}} = \xi(\phi) \overbrace{\psi}^{\text{non-linear}}$$

$$\Psi' = g\Psi \qquad \xi(\phi') = g\xi(\phi)\hat{h}^{-1}(g,\phi)$$

$$\psi' = \hat{h}(g,\phi)\psi$$

$$\begin{pmatrix} H & * & * \\ 0 & H & * \\ 0 & 0 & H \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad \begin{pmatrix} \psi_1 \\ 0 \\ 0 \end{pmatrix} \text{は} \quad H\text{-変換のみ}$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ 0 \end{pmatrix} \text{は} \quad \begin{cases} \psi_1' = H\psi_1 + *\psi_2 \\ \psi_2' = H\psi_2 \end{cases}$$

でnonlinear baseで直接Invariant を作るのは難しい