

# 次最小超対称標準模型に対する 宇宙論的制限

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Collaborate with

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based on

- [1] K. Kadota, M. Kawasaki, KS, hep-ph/1503.06998. (accepted in JCAP)
- [2] A. Mazumdar, KS, M. Yamaguchi, J. Yokoyama, work in progress.

# Abstract

- Discuss cosmological aspects of the  $Z_3$ -invariant next-to-minimal supersymmetric standard model (NMSSM)
- Formation of domain walls in the context of primordial inflation Mazumdar, KS, Yamaguchi, Yokoyama, work in progress
  - Can it be avoided ?
  - Under what conditions ?
- Estimate the gravitational wave signatures from domain walls and their parameter dependence

Kadota, Kawasaki, KS, I503.06998

# I. NMSSM as a solution to the $\mu$ -problem

Renormalizable superpotential of the MSSM

$$W_{\text{MSSM}} = \underbrace{\mu H_u H_d}_{\mu\text{-term}} + \lambda_{ij}^e H_d L_i E_j^c + \lambda_{ij}^d H_d Q_i D_j^c - \lambda_{ij}^u H_u Q_i U_j^c$$

$i, j, k = 1, 2, 3$ : family indices

- $\mu$ -problem: Why  $\mu \sim M_{\text{SUSY}}$  rather than  $\mu \sim M_{\text{GUT}}$  or  $M_{\text{Pl}}$ ?
- Introduce a gauge singlet  $S$  and replace the  $\mu$ -term

$$\mu H_u H_d \rightarrow \lambda S H_u H_d$$

- Singlet acquires a VEV to induce an effective  $\mu$ -term

$$\mu_{\text{eff}} = \lambda \langle S \rangle = \frac{\lambda}{\sqrt{2}} v_s \quad \langle S \rangle = \frac{1}{\sqrt{2}} v_s$$

- No dimensionful parameter except for soft SUSY breaking effects  $\sim M_{\text{SUSY}}$

 naively expected that  $\mu_{\text{eff}} \sim \mathcal{O}(M_{\text{SUSY}})$

- Need to forbid any dimensionful parameters like

$$\mu H_u H_d, \quad \mu'^2 S, \quad \text{and} \quad \mu'' S^2$$

- Impose a  $Z_3$  symmetry

$$Z_3 : \Phi \rightarrow e^{2\pi i/3} \Phi$$

$\Phi = (L, E^c, Q, U^c, D^c, H_u, H_d, S)$  : every chiral supermultiplets of the NMSSM

➔  $W_{\text{NMSSM}} = \lambda S H_u H_d + \frac{\kappa}{3} S^3$       Two parameters:  $(\lambda, \kappa)$

$$+ \lambda_{ij}^e H_d L_i E_j^c + \lambda_{ij}^d H_d Q_i D_j^c - \lambda_{ij}^u H_u Q_i U_j^c$$

- $Z_3$  is spontaneously broken when  $S, H_u, H_d$  acquire VEVs

➔ Formation of domain walls


# Decoupling limit

- $v_s \gg v_u, v_d$  is possible if  $\lambda \ll 1$

cf.  $\mu = \frac{1}{\sqrt{2}} \lambda v_s \approx \mathcal{O}(M_{\text{SUSY}})$       $\langle S \rangle = \frac{v_s}{\sqrt{2}}, \langle H_u \rangle = \frac{v_u}{\sqrt{2}}, \langle H_d \rangle = \frac{v_d}{\sqrt{2}}$

- In this limit, the potential can be approximated as

$$V \simeq \kappa^2 |S|^4 + m_S^2 |S|^2 + \left[ \frac{\kappa}{3} A_\kappa S^3 + \text{h.c.} \right]$$

  $v_s \simeq -\frac{\sqrt{2} A_\kappa}{4\kappa} \left( 1 + \sqrt{1 - \frac{8m_S^2}{A_\kappa^2}} \right)$

$A_\kappa, m_S \sim \mathcal{O}(M_{\text{SUSY}})$  : soft SUSY breaking parameters

- $\lambda$  and  $\kappa$  should be of the same order of magnitudes

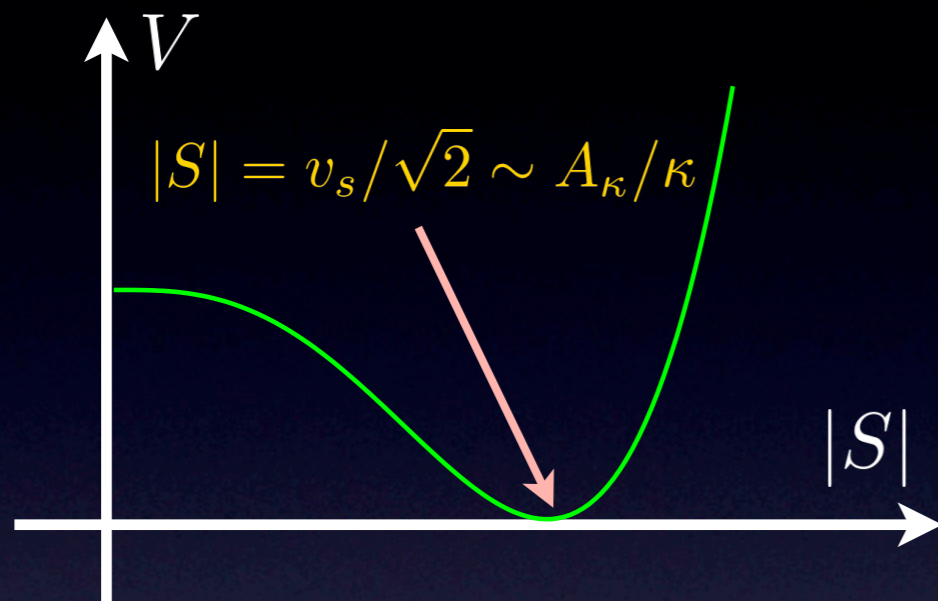
since  $\mu \sim \lambda v_s \sim (\lambda/\kappa) A_\kappa \sim M_{\text{SUSY}}$

- Decoupling limit is given by

$$v_s \sim |A_\kappa/\kappa| \quad \text{for } \lambda \sim \kappa \rightarrow 0$$

# Potential for $S$

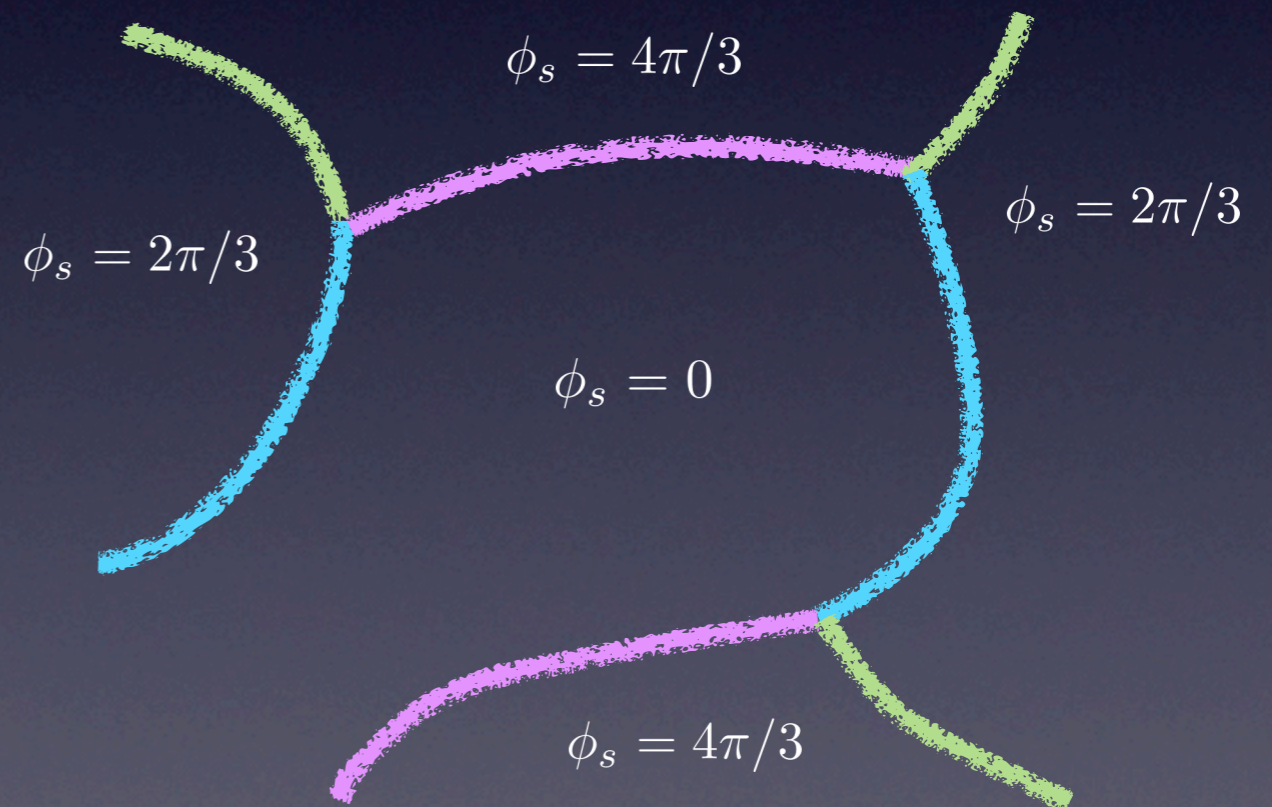
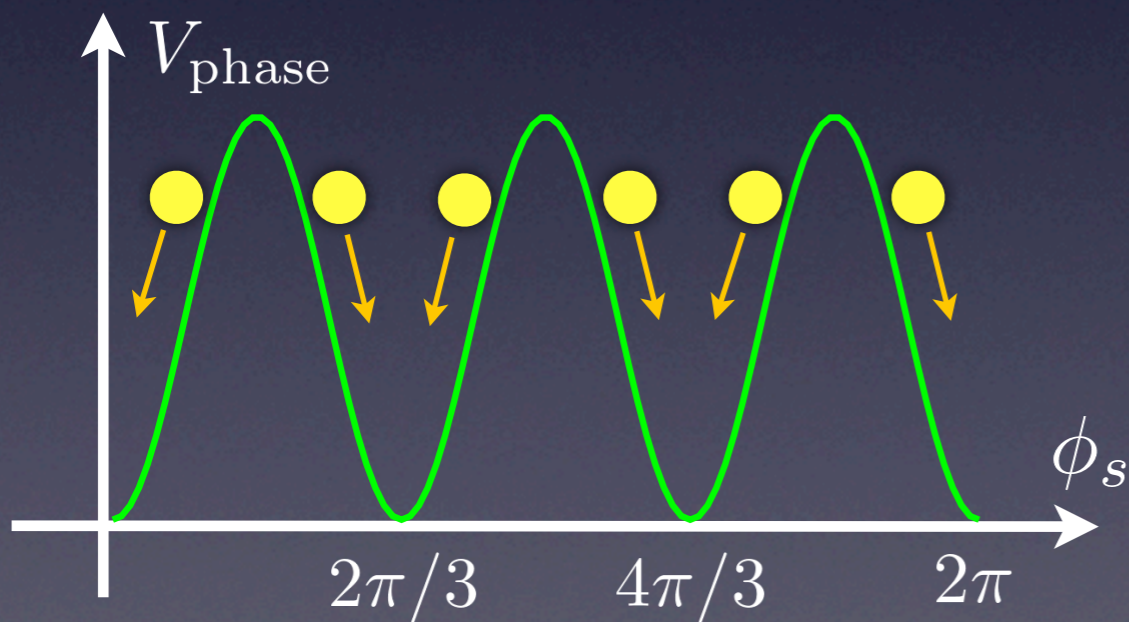
Radial direction  $|S|$



$$\langle S \rangle = v_s e^{i\phi_s} / \sqrt{2}$$

$$V \simeq m_S^2 |S|^2 + \kappa^2 |S|^4 + \underbrace{\left[ \frac{1}{3} \kappa A_\kappa S^3 + \text{c.c.} \right]}_{V_{\text{phase}}}$$

Phase dependent terms



- Three degenerate minima related by  $\phi_s \rightarrow \phi_s + 2\pi k/3$ ,  $k = 0, 1, 2$
- Domain walls are formed at their boundaries

# Properties of domain walls

$$V \simeq \kappa^2 |S|^4 + m_S^2 |S|^2 + \left[ \frac{\kappa}{3} A_\kappa S^3 + \text{h.c.} \right]$$

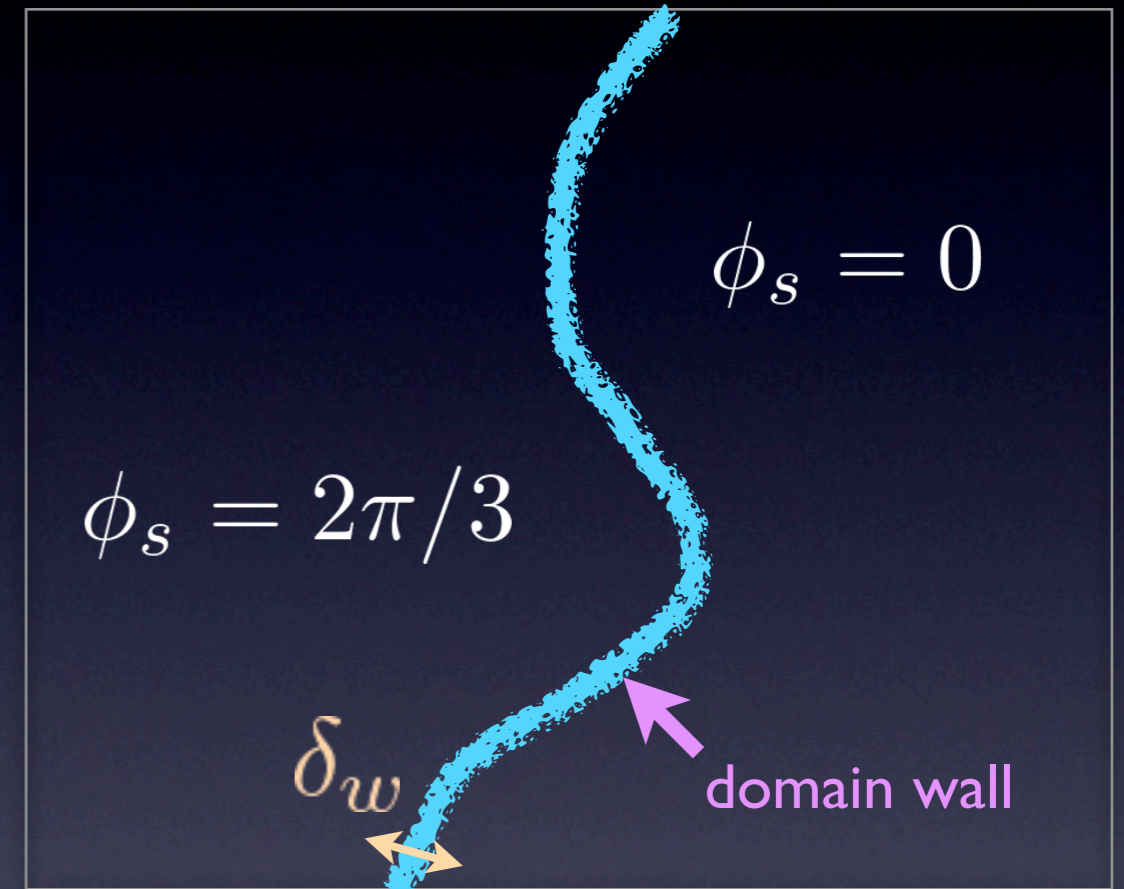
$$S = \frac{1}{\sqrt{2}} v_s e^{i\phi_s} \quad v_s \sim \left| \frac{A_\kappa}{\kappa} \right|$$

- **Width of the wall**

$$\begin{aligned} \delta_w &\sim \left| \frac{\partial^2 V}{\partial (v_s \phi_s)^2} \right|^{-1/2} \\ &\sim |\kappa A_\kappa v_s|^{-1/2} \sim \mathcal{O}(M_{\text{SUSY}}^{-1}) \end{aligned}$$

- **Surface mass density**

$$\begin{aligned} \sigma_{\text{wall}} &= \int dz \rho_{\text{wall}}(z) \\ &\sim \delta_w \times V \sim \mathcal{O}(\kappa v_s^3) \end{aligned}$$



$\rho_{\text{wall}}(z)$  : energy density of the wall  
 $z$  : coordinate perpendicular to the surface of the wall

Note that  $\sigma_{\text{wall}} \sim \mathcal{O}(\kappa v_s^3) \gg \mathcal{O}(M_{\text{SUSY}}^3)$  for  $\kappa \ll 1$

We expect that domain walls are formed if the singlet scalar  $S$  takes different phases from place to place.

How ? and under what conditions ?

Let us carefully see the evolution of  $S$  after inflation.



# 2. Conditions for (non-)formation of domain walls

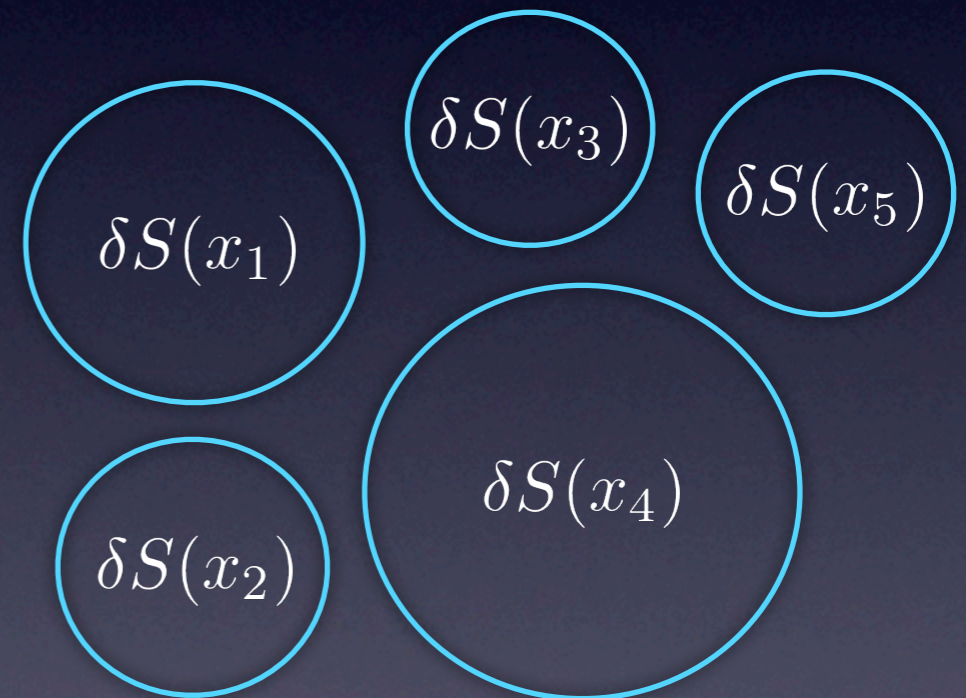
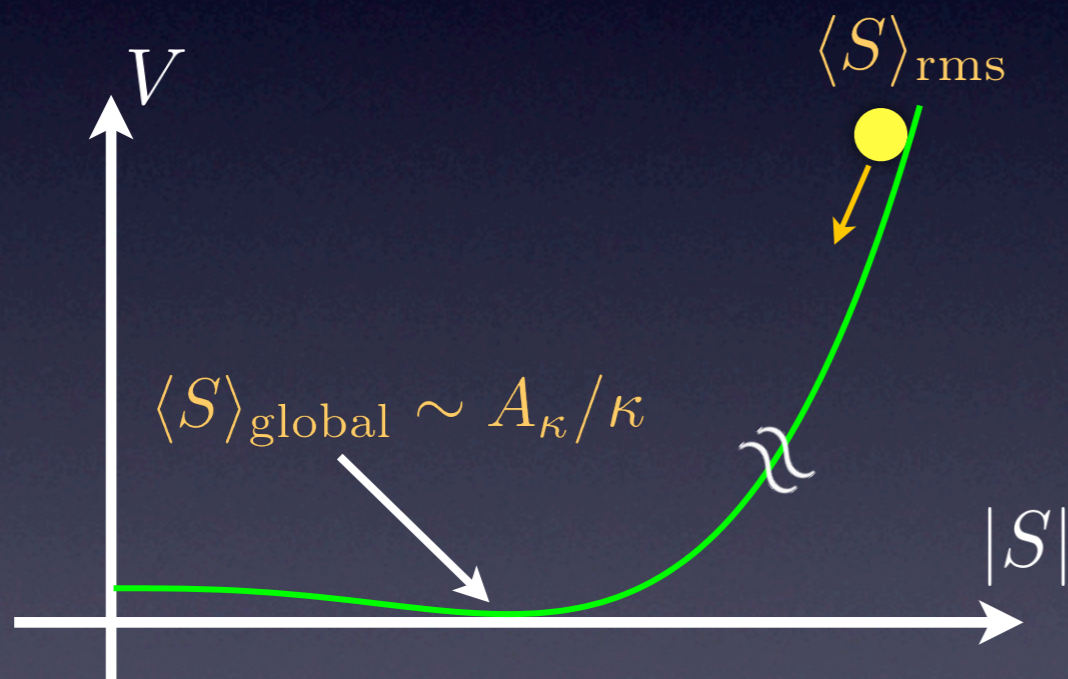
Mazumdar, KS, Yamaguchi, Yokoyama, work in progress

During inflation,  $S$  is effectively massless  $m_S \ll H_{\text{inf}}$

$H_{\text{inf}}$  : Hubble parameter during inflation

➔  $S$  is easily displaced from the global minimum due to the quantum fluctuations  $\delta S \sim \mathcal{O}(H_{\text{inf}})$

$$\kappa^2 |S|^4 \simeq H_{\text{inf}}^4 \quad \Rightarrow \quad \langle S \rangle_{\text{rms}} \sim H_{\text{inf}} / \sqrt{\kappa} \gg \langle S \rangle_{\text{global}}$$



After inflation,  $S$  oscillates around  $S = 0$ , reducing its amplitude

➔ Fluctuations  $\delta S$  are enhanced due to the parametric resonance, which results in the formation of defects

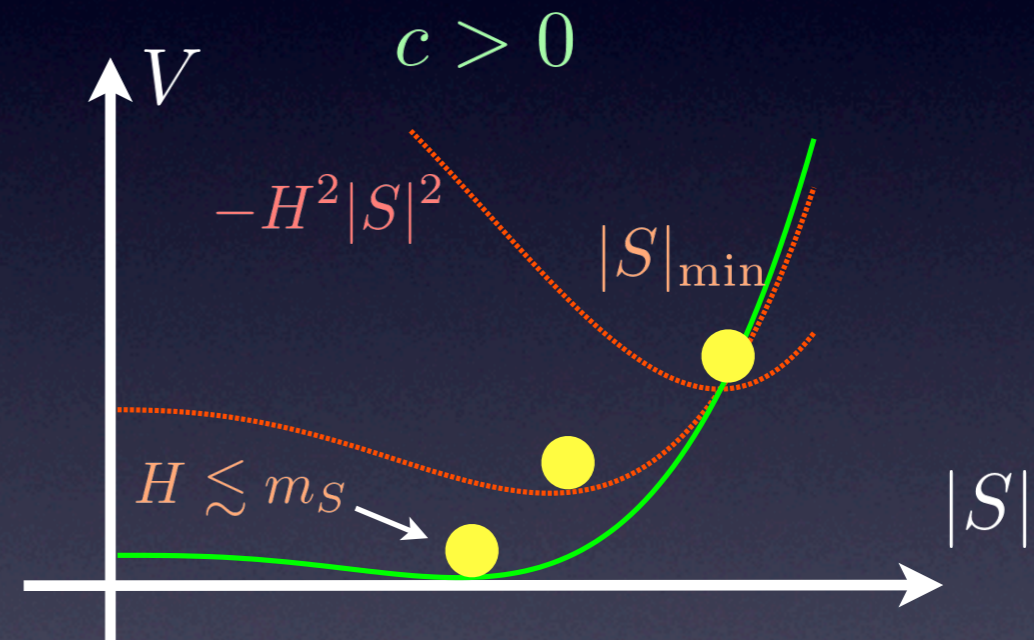
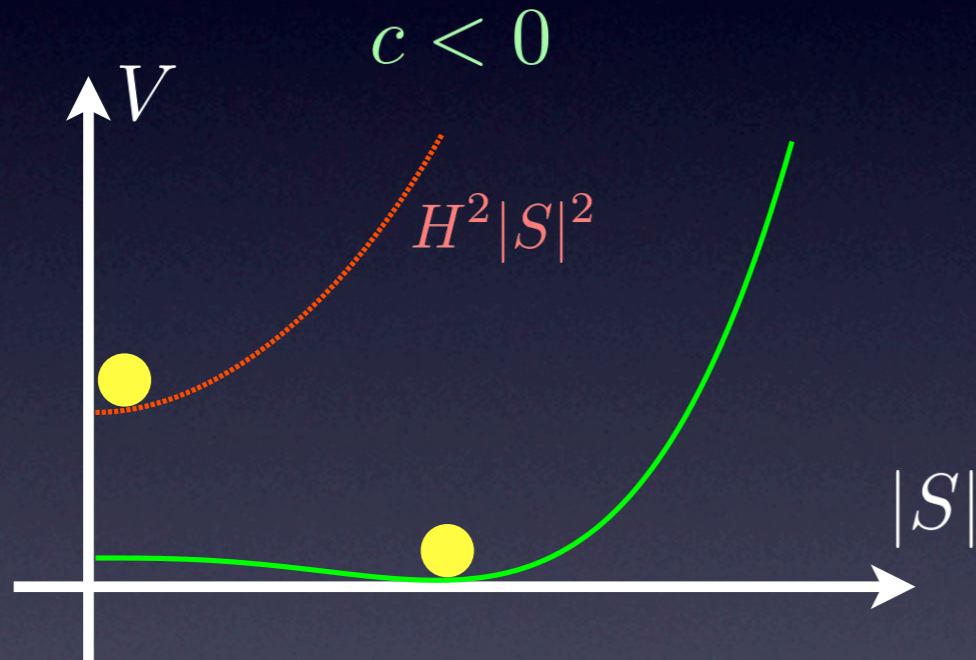
# Supergravity effects

$S$  acquires the effective mass of  $\mathcal{O}(H)$

$$\mathcal{L} \supset \frac{|S|^2}{M_{\text{Pl}}^2} \partial_\mu I^* \partial^\mu I, \quad \frac{|S|^2}{M_{\text{Pl}}^2} V(I) \quad \longrightarrow \quad m_{S,\text{eff}}^2 = -cH^2$$

$I$  : inflaton field

$c \sim \mathcal{O}(1)$  : model-dependent parameter



If  $c > 0$ ,  $S$  tracks minimum of the effective potential after inflation

$$V(S) \simeq -cH^2|S|^2 + \kappa^2|S|^4 \quad \longrightarrow \quad |S|_{\text{min}} \simeq \frac{\sqrt{c}H(t)}{\sqrt{2\kappa}}$$

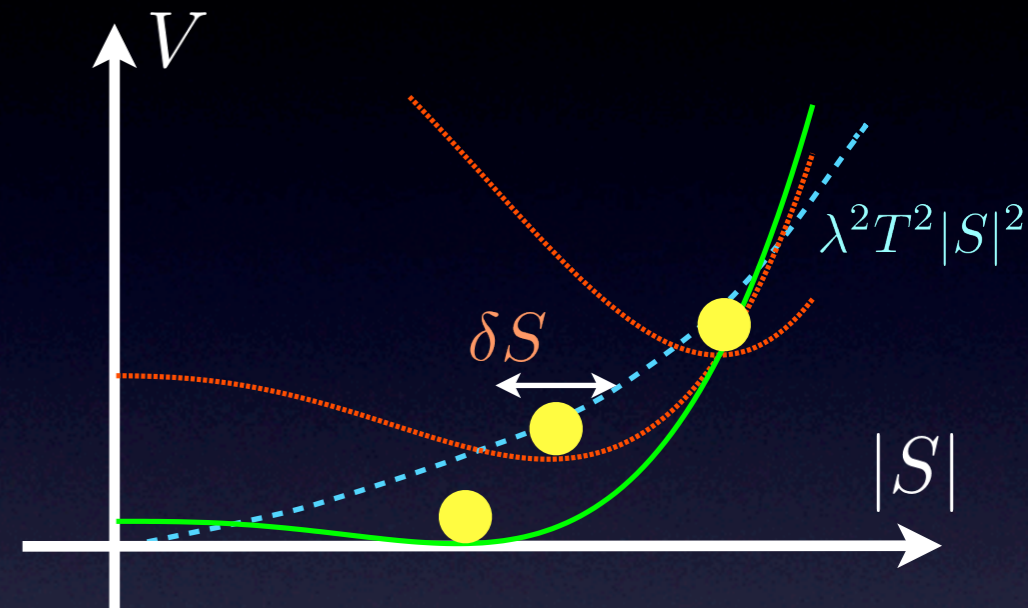
No oscillation  $\rightarrow$  the defect formation can be avoided

# Finite temperature effects

- Following conditions must be satisfied until  $S$  reaches the global minimum ( $cH^2 \gtrsim m_S^2$ )

- Correction term  $\Delta V \propto \lambda^2 T^2 |S|^2$  in the effective potential should not alter the tracking behavior:

$$\lambda^2 T^2 \ll cH^2$$



- Thermal fluctuations  $\delta S(x) \sim T$  should remain small:

$$|S|_{\min} \gg \delta S(x) \sim T$$

- Above two requirements lead to the same condition (recall that  $|S|_{\min} \propto \sqrt{cH}/\kappa$  and  $\kappa \simeq \lambda$ )




$$\lambda T \ll \sqrt{cH} \quad \text{for} \quad cH^2 \gtrsim m_S^2$$

# Conditions to avoid the domain wall formation

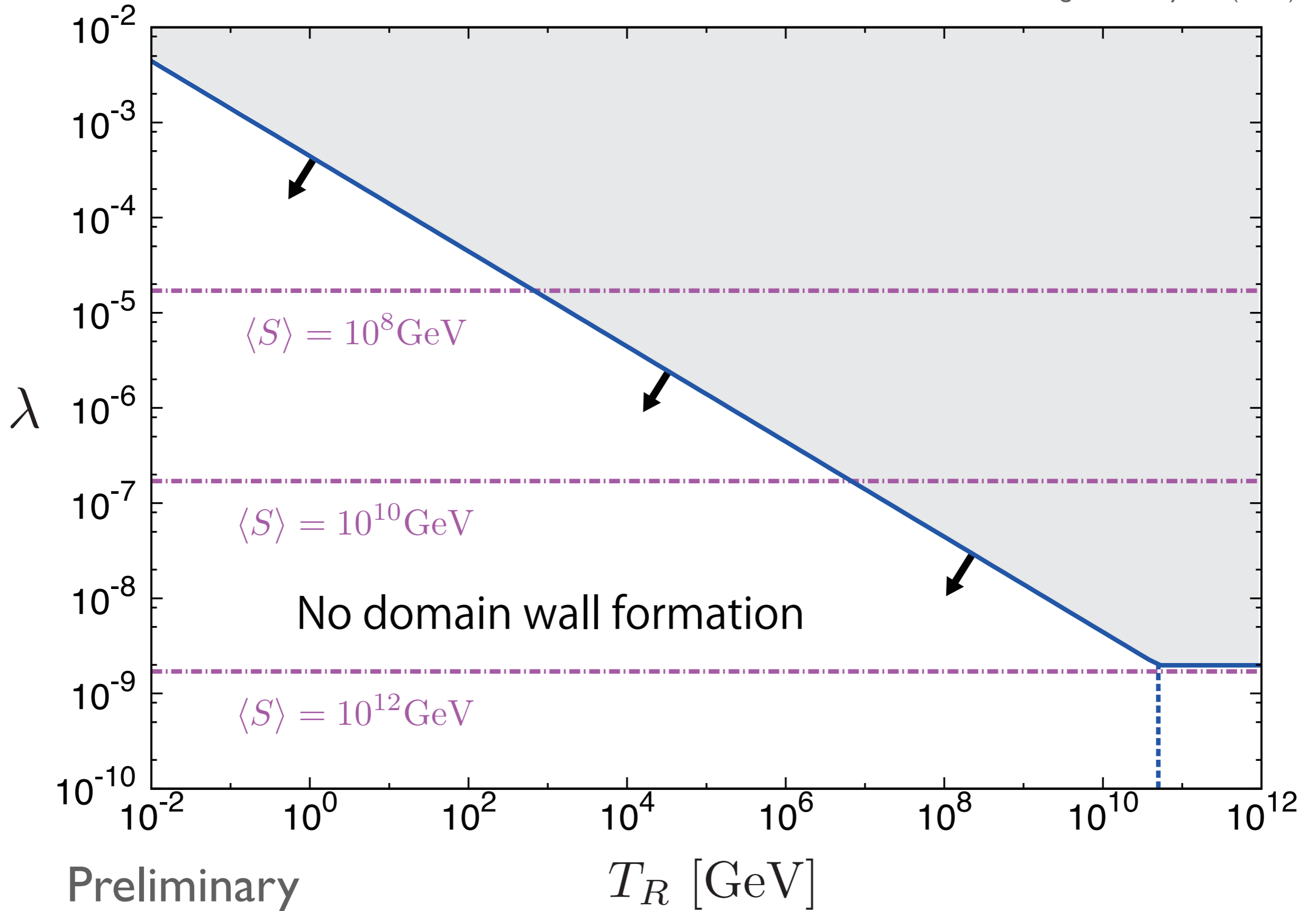
1. Existence of the **negative effective mass**  $-cH^2|S|^2$  during and after inflation
2.  $A_\kappa^2/m_S^2 \gtrsim \mathcal{O}(10)$  must be satisfied to prevent the  $S$  field from rotating in the phase direction at late times (This is automatically satisfied if  $v_S \neq 0$ )
3. Thermal effects must remain irrelevant

$$\lambda T \ll \sqrt{cH} \quad \text{for} \quad cH^2 \gtrsim m_S^2$$

 puts a **constraint on the couplings** ( $\lambda, \kappa$ ) and **reheating temperature**  $T_R$

4. Initial field value should be uniquely determined during inflation

$$\kappa^2|S|^4 \sim c^2 H_{\text{inf}}^4 / \kappa^2 > \mathcal{O}(H_{\text{inf}}^4) \quad \img alt="blue arrow" data-bbox="615 910 685 955" \quad c > \kappa$$



Preliminary

$T_R$  [GeV]

- Formation of domain walls is likely to occur if  $T_R$  and/or couplings  $(\lambda, \kappa)$  are sufficiently large.
- What occurs if they are formed ?
  - If they are absolutely stable, they come to overclose the universe.  
(conflict with standard cosmology)  
Zel'dovich, Kobzarev, Okun, JETP 40, 1 (1975)
  - They must collapse at some early time.
  - If they lived for sufficiently long time, they can be a source of the gravitational wave background.

# 3. Gravitational waves from domain walls

Kadota, Kawasaki, KS, 1503.06998



# Collapse of domain walls

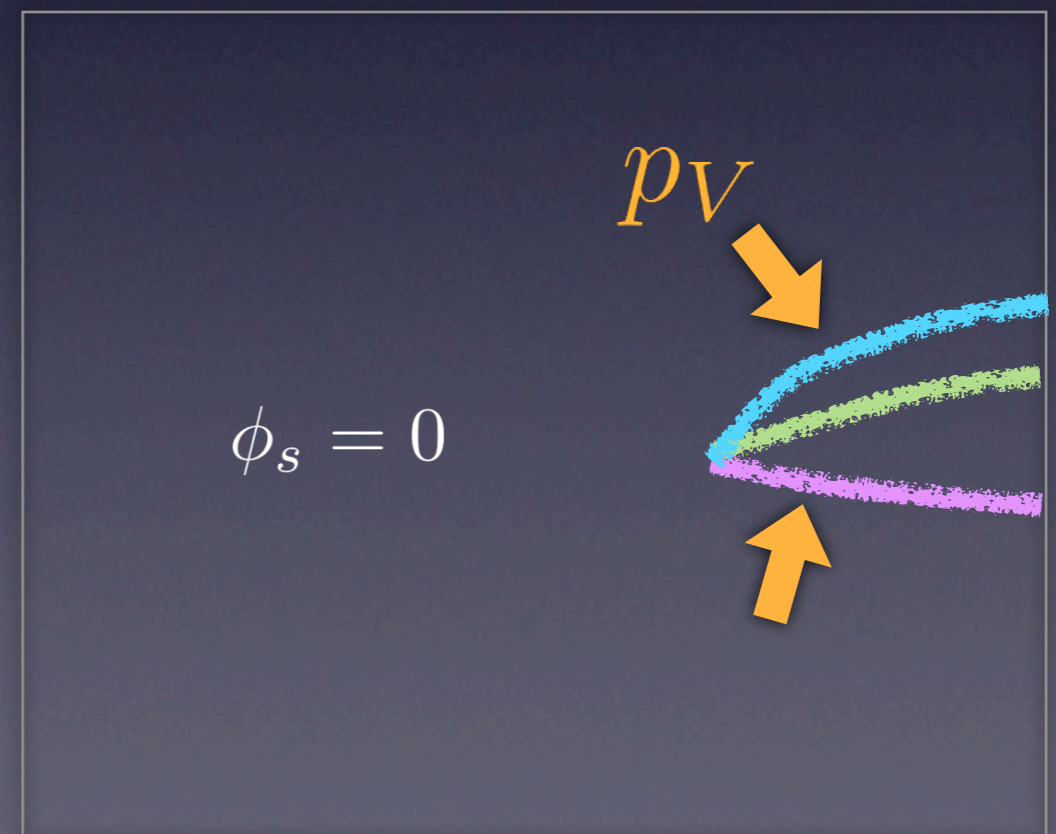
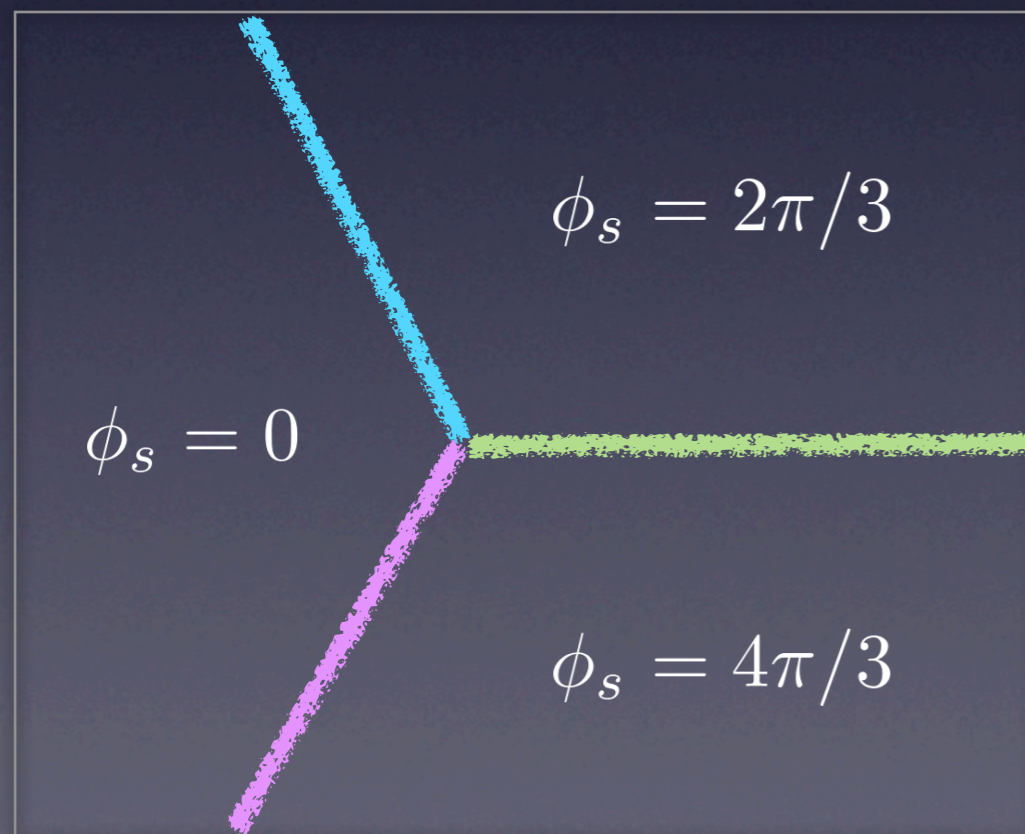
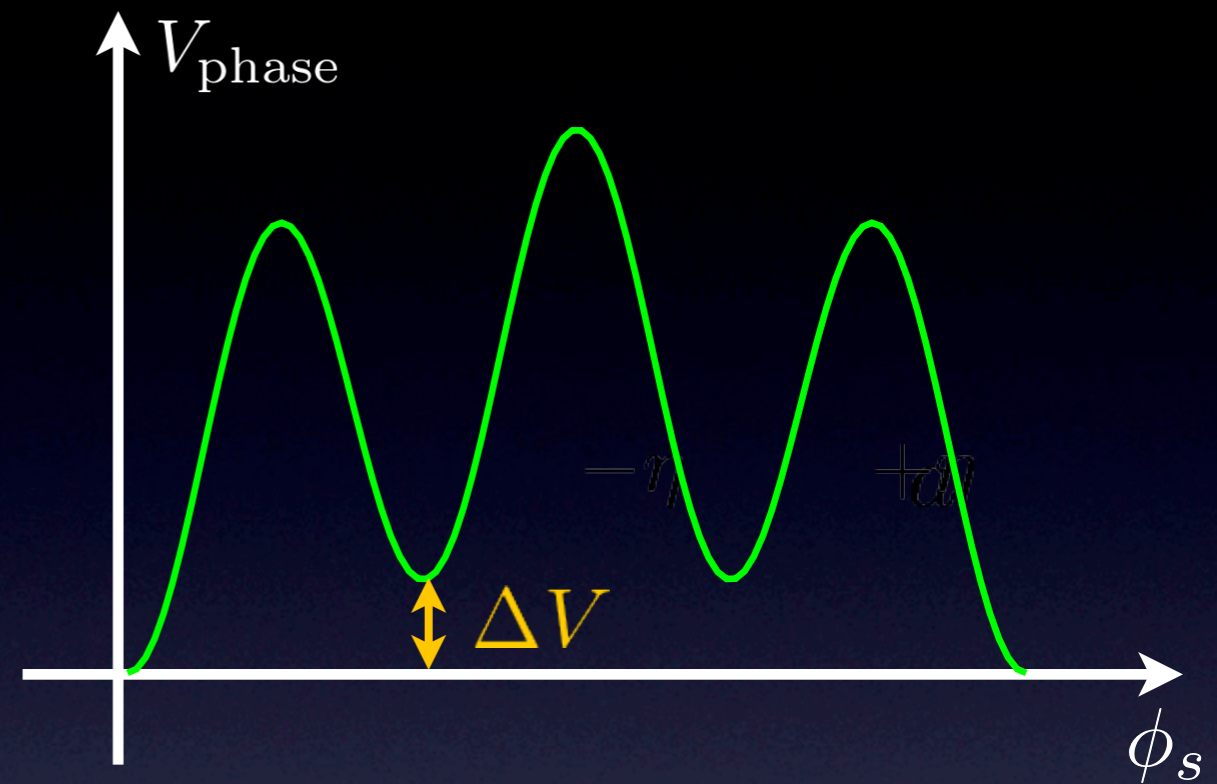
Approximate discrete symmetry (bias)

Vilenkin, PRD23, 852 (1981)

$$\Delta V \sim \Lambda^4 \quad \Lambda \ll v_s$$

➔ Degenerate vacua are lifted

➔ Domain walls are annihilated due to the pressure  $p_V \sim \Delta V$



Annihilation occurs when

$$p_V \sim p_T$$



Decay time

$$t_{\text{dec}} \sim R|_{p_V=p_T} \sim \frac{\sigma_{\text{wall}}}{\Lambda^4}$$
$$\sim 7\text{sec} \left( \frac{\sigma_{\text{wall}}}{1\text{TeV}^3} \right) \left( \frac{0.1\text{MeV}}{\Lambda} \right)^4$$

$$p_T \sim \sigma_{\text{wall}}/R : \text{tension}$$

$R$  : curvature radius of walls

$\sigma_{\text{wall}}$  : surface mass density of walls

- Decay products of domain walls may dissociate light element created during Big Bang Nucleosynthesis (BBN)



Require that

$$t_{\text{dec}} \lesssim 0.01\text{sec}$$

- Domain walls exist at early epoch and decay before the epoch of BBN

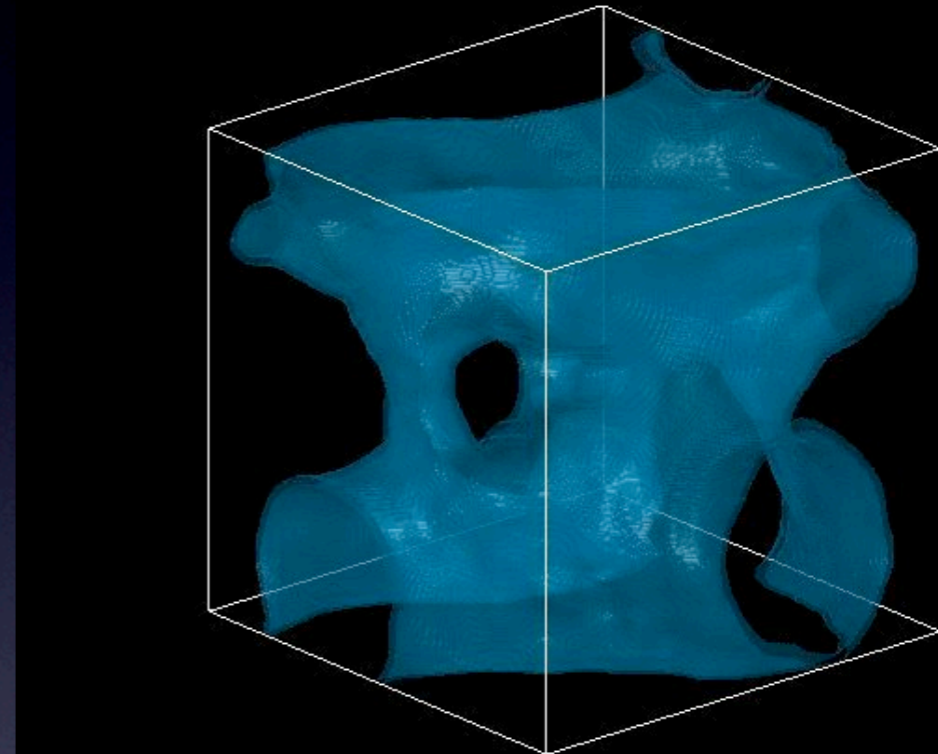
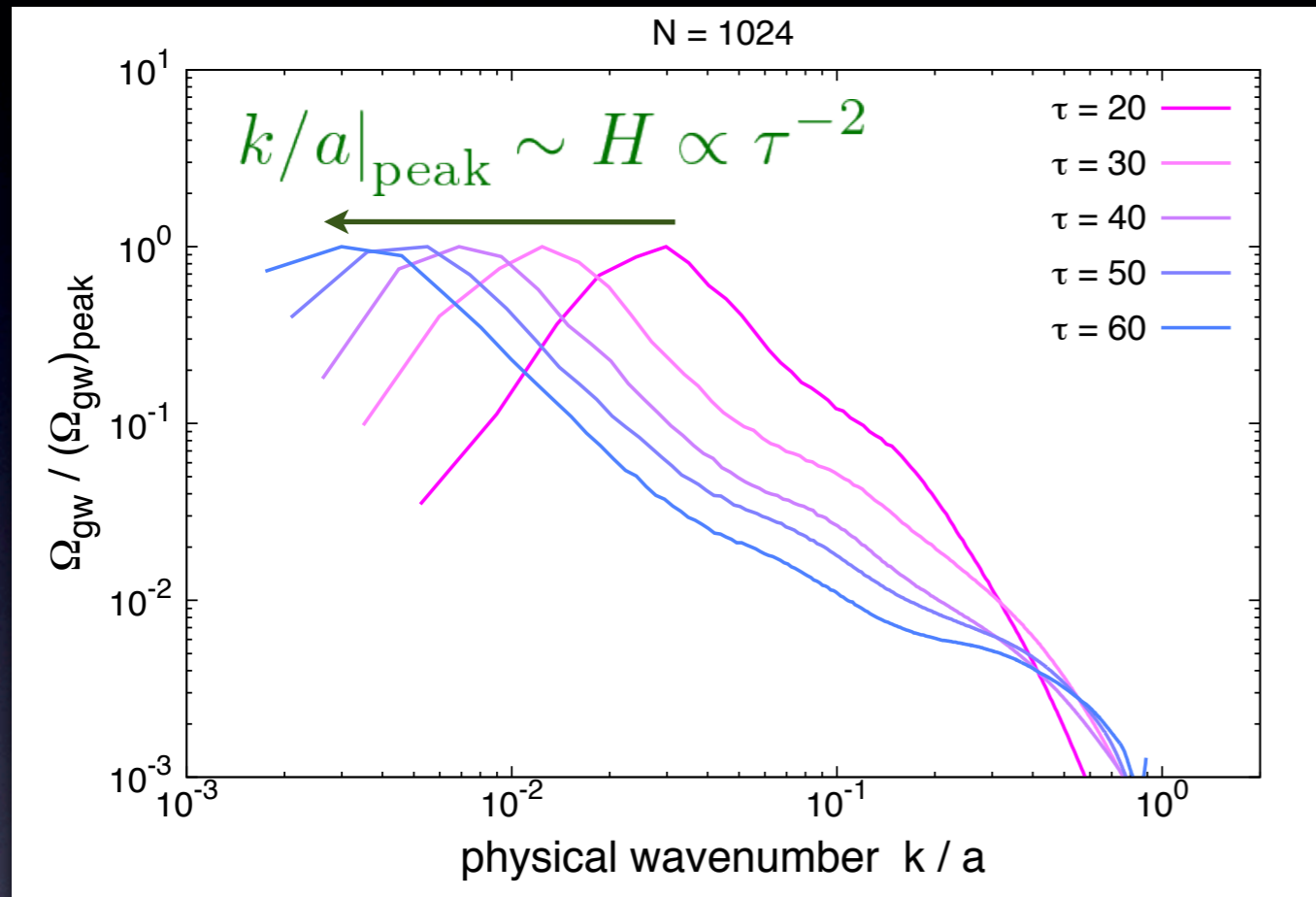


Gravitational waves are expected to be produced

# Gravitational waves from domain walls

Hiramatsu, Kawasaki, KS, JCAP02(2014)031

- Simulation of scalar fields in 3D lattice with  $512^3$  and  $1024^3$



- Amplitude

$$\Omega_{\text{gw}} h^2(t_0) \simeq 1.7 \times 10^{-17} \left( \frac{\sigma_{\text{wall}}}{1\text{TeV}^3} \right)^2 \left( \frac{t_{\text{dec}}}{0.01\text{sec}} \right)^2$$

$$\Omega_{\text{gw}} = \frac{1}{\rho_c(t)} \frac{d\rho_{\text{gw}}}{d \ln k}$$

- Peak frequency

$$f_{\text{peak}}(t_0) = \frac{a(t_{\text{dec}})}{a(t_0)} H(t_{\text{dec}}) \simeq 10^{-9} \text{Hz} \left( \frac{0.01\text{sec}}{t_{\text{dec}}} \right)^{1/2}$$

$$\rho_c(t) = \frac{3H^2}{8\pi G}$$

- Decay before BBN:  $t_{\text{dec}} \lesssim 0.01\text{sec} \rightarrow f \gtrsim 10^{-9}\text{Hz}$

cf. pulsar timing  $\Omega_{\text{gw}} h^2 \sim 10^{-8}$  at  $f \sim 10^{-9} - 10^{-8}\text{Hz}$

# Cosmological constraints

- Gravitational waves

$$\Omega_{\text{gw}} h^2 < \mathcal{O}(10^{-8})$$

from pulsar timing observations

$$\sigma_{\text{wall}} \sim \kappa v_s^3 \quad \mu = \lambda v_s / \sqrt{2} \approx \mathcal{O}(100 \text{ GeV})$$

➔  $\Omega_{\text{gw}} h^2 \propto \sigma_{\text{wall}}^2 t_{\text{dec}}^2 \propto \kappa^2 v_s^6 t_{\text{dec}}^2 \propto \kappa^2 \lambda^{-6} \mu^6 t_{\text{dec}}^2$

- Avoiding unrealistic minima of the potential

- There might be some unrealistic minima on which

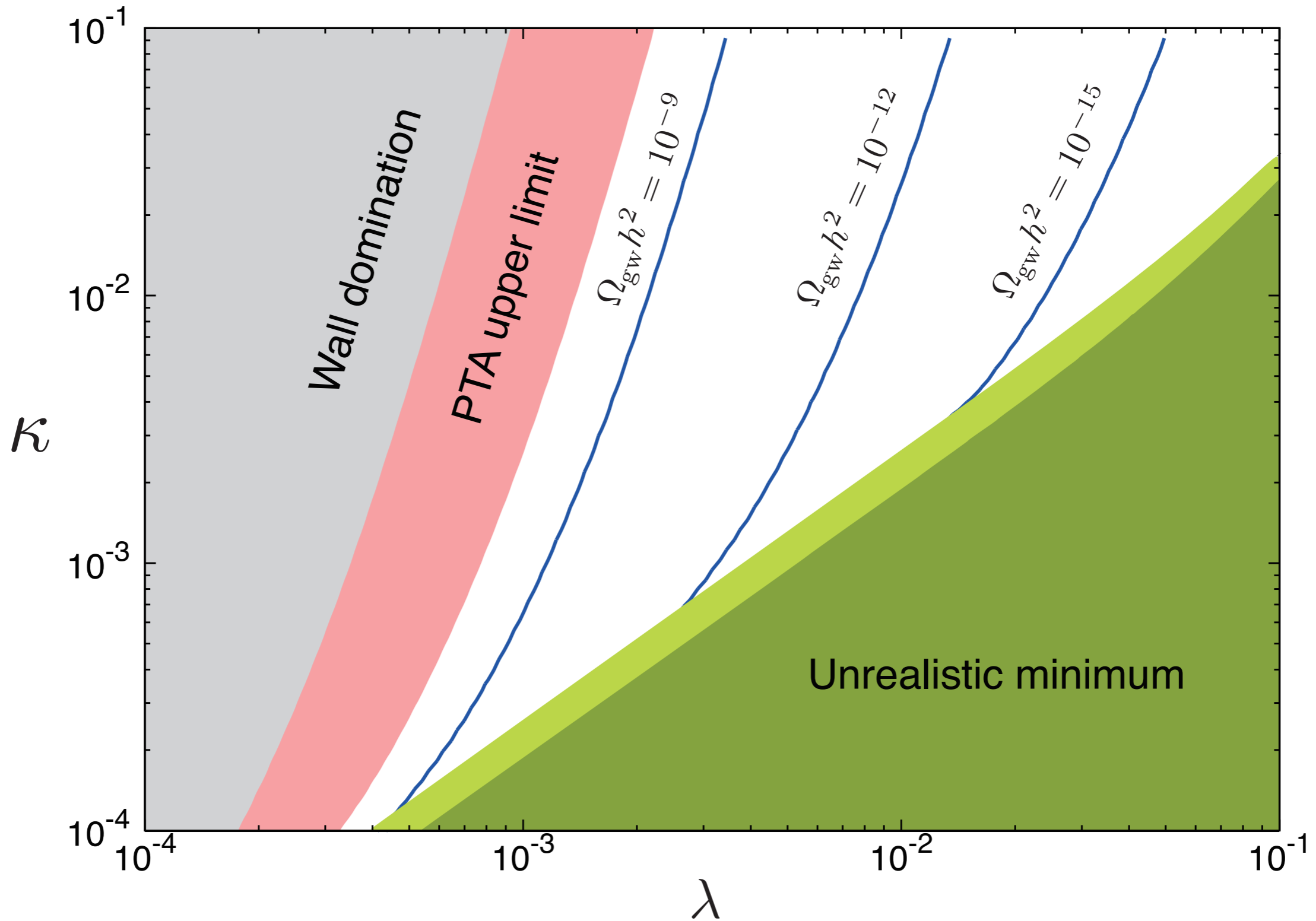
$$\langle H_u \rangle, \langle H_d \rangle \neq \begin{pmatrix} \text{correct} \\ \text{electroweak} \\ \text{value} \end{pmatrix} \quad \text{and} \quad \langle S \rangle \neq \begin{pmatrix} \text{correct value} \\ \text{for } \mu\text{-term} \end{pmatrix}$$

- For the height of the potential  $V_{\text{min}}$ , we should confirm that

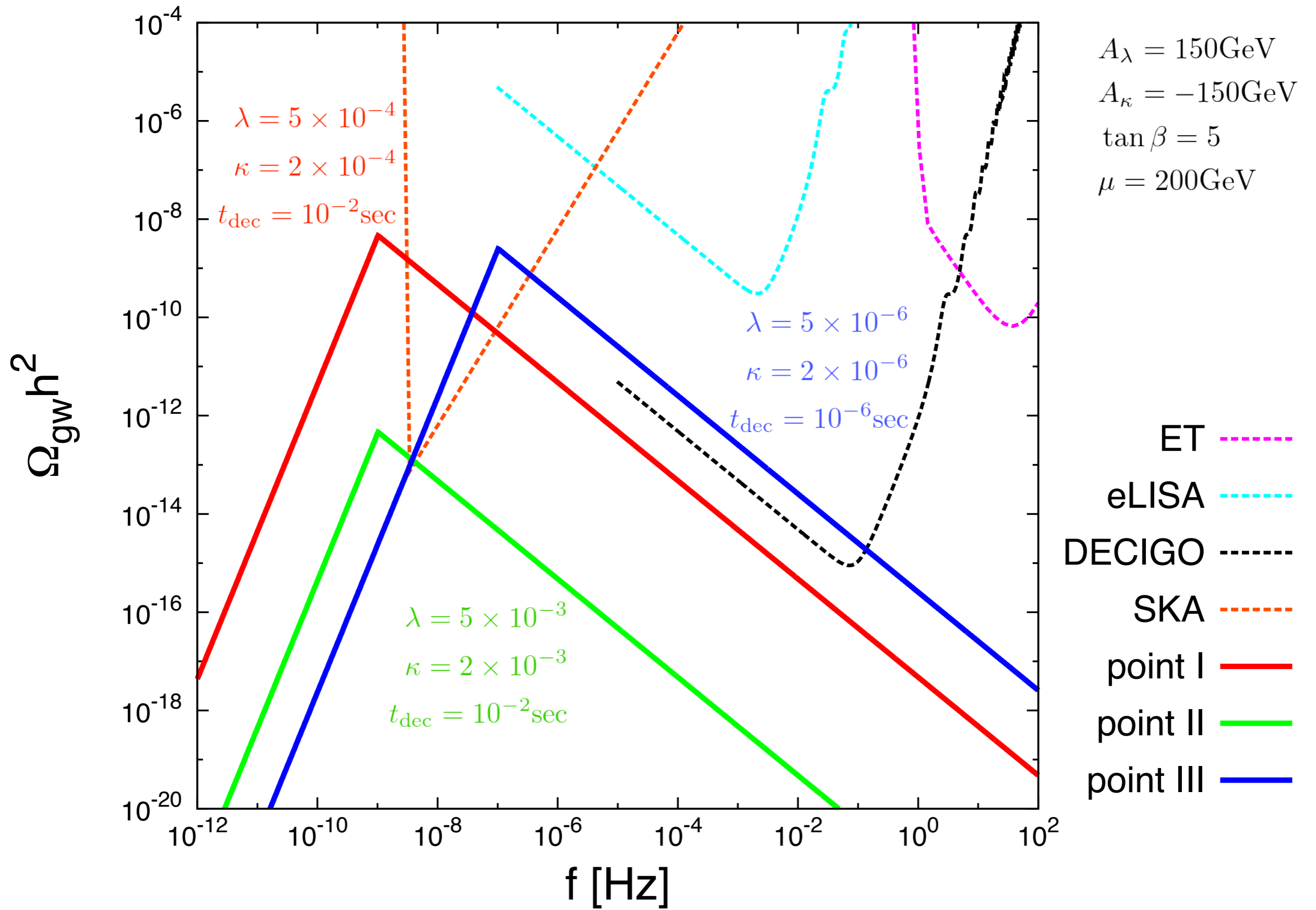
$$V_{\text{min,true}} < V_{\text{min,unrealistic}}$$

$$t_{\text{dec}} = 0.01\text{sec}$$

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$$\Omega_{\text{gw}} h^2 \propto \kappa^2 v_s^6 t_{\text{dec}}^2 \propto \kappa^2 \lambda^{-6} \mu^6 t_{\text{dec}}^2 \quad \text{for } \lambda, \kappa \ll 1$$



# 4. Conclusions

- Domain wall formation in the NMSSM can be avoided if
  - There exists a negative Hubble mass for the singlet scalar
  - Reheating temperature  $T_R$  is sufficiently low and/or Higgs-singlet couplings  $(\lambda, \kappa)$  are sufficiently small
- If domain walls are formed, they can produce gravitational waves
  - typically probed by pulsar timing observations
- Collider experiments will probe large  $(\lambda, \kappa)$  region:  
Cosmology has a complementary role to probe the model