量子重イオンビームを利用した、新たなニュートリノ物理

- Neutrino physics using quantum coherence -

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Outline of this talk

Introduction:

remaining important questions in neutrino physics

quantum coherence: an example of adiabatic Raman excitation

De-excitation from quantum ion beam in circular motion

Expected physics outputs in neutrino physics

Present status of neutrino physics

- Oscillation experiments
  - Finite mass
  - Flavor mixing
  - Only mass-squared difference can be measured.

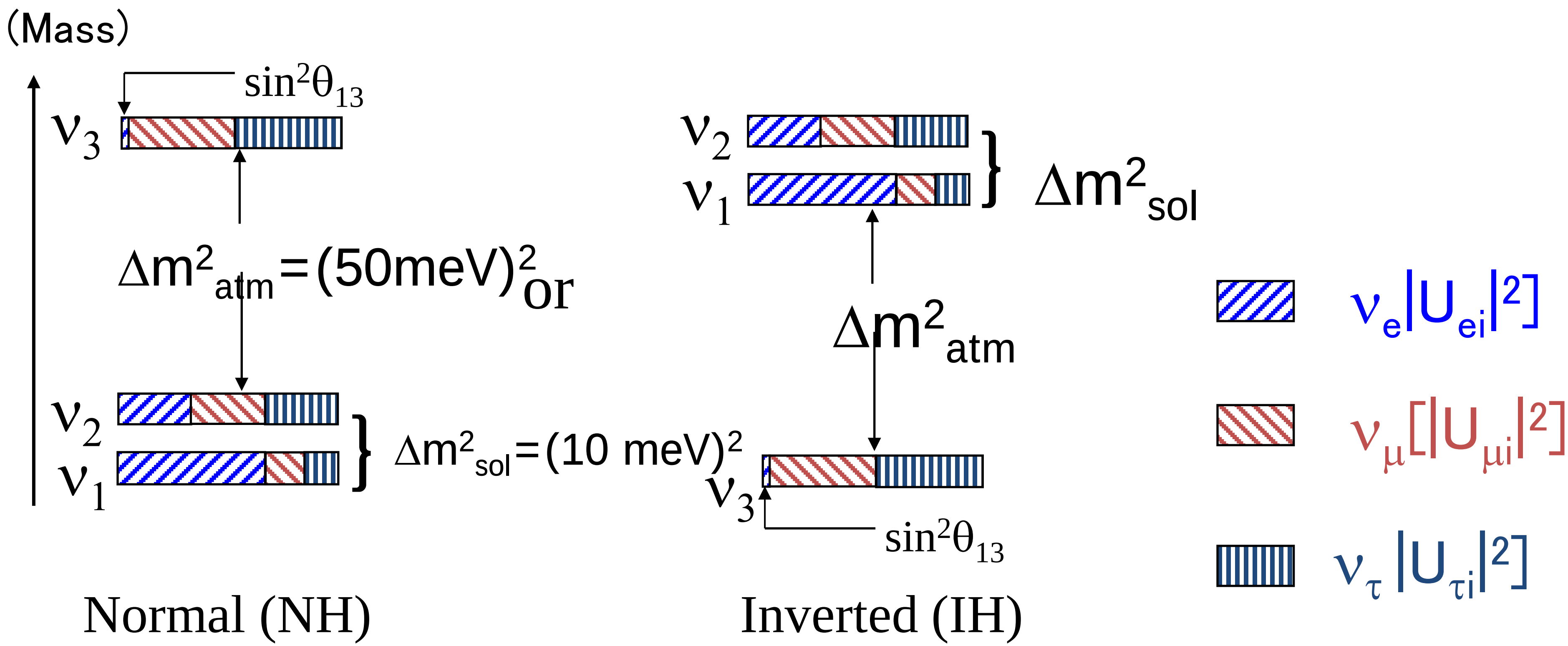
$$U = VP, \tag{A8}$$

where

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{A9}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The diagonal unitary matrix P may be expressed by

$$P = \mathrm{diag.}(1, e^{i\alpha}, e^{i\beta}), \tag{A10}$$



Important questions left in neutrino physics

- Absolute mass scale and the smallest mass (oscillation experiments are sensitive to mass squared differences alone)
- Majorana vs Dirac distinction
- CPV phase （Majorana case has 2 extra phases） $\alpha\,,\ \beta\,,\ \delta\ (\mathrm{KM}-\mathrm{type})$
- Detection of relic 1.9K neutrino

These are relevant to explanation of matter-antimatter imbalance of universe and physics beyond the standard theory.

CP asymmetry in leptogeneis

$$\approx \frac{3y_1^2}{4\pi}\left(-2(\frac{m_3}{m_2})^3 s_{13}^2 \sin 2(\delta+\alpha-\beta) + \frac{m_1}{m_2}\sin(2\alpha)\right)$$

+ (high energy phases inaccessible in low energy experiments)

Significance of Majorana neutrinos

- Theoretical prejudice: Neutral leptons consist of 4 components like all other quarks and leptons, the ordinary massless neutrino and the other 2-component partner having a much larger mass of Majorana-type than the Fermi scale
- -> Seesaw mechanism with a Dirac-type coupling via Higges $\frac{m^2}{M}$
- Plausible scenario of lepto-genesis

Heavy Majorana decay responsible for generation of lepton asymmetry, being converted to baryon asymmetry via strong electroweak B, L violation keeping B-L conserved.

Prerequisite: ordinary neutrinos are massive、but very light Majorana.

New CPV sources related to heavy partners of mass >> Fermi scale

Majorana vs Dirac equations chirally projected solutions

Dirac eq.: degenerate 2 Majorana

$$(i\partial_t - i\vec{\sigma}\cdot\vec{\nabla})\varphi = m\chi\,, \quad (i\partial_t + i\vec{\sigma}\cdot\vec{\nabla})\chi = m\varphi$$

$$\psi_D = (1-\gamma_5)\psi/2$$

$$\psi_D = b(\vec{p},h)e^{-ip\cdot x}u(\vec{p},h) + d^\dagger(\vec{p},h)e^{ip\cdot x}\sqrt{\frac{E_p+hp}{E_p-hp}}(-i\sigma_2)u^*(\vec{p},h)$$

Particle annihilation   Anti-particle creation

2-component in weak process involved

Majorana eq. : particle=antiparticle

$$(i\partial_t - i\vec{\sigma}\cdot\vec{\nabla})\varphi = im\sigma_2\varphi^*$$

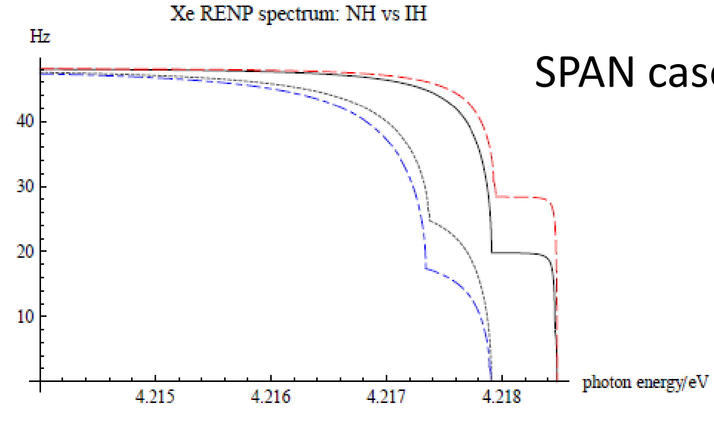
$$\varphi_{\vec{p},h}(x) = c(\vec{p},h)e^{-ip\cdot x}u(\vec{p},h) + c^\dagger(\vec{p},h)e^{ip\cdot x}\sqrt{\frac{E_p+hp}{E_p-hp}}(-i\sigma_2)u^*(\vec{p},h)\,,$$

$$u(\vec{p},h) = \frac{1}{2}\sqrt{\frac{E_p-hp}{pE_p(p+hp_3)}}\begin{pmatrix} p+hp_3 \\ h(p_1+ip_2)\end{pmatrix}.$$

2 neutrino wave functions are anti-symmetrized

Detection principles

１. Majorana/Dirac distinction: identical fermion effects, different effects from SPAN because energy-momentum conservation do not hold and mass threshold regions exist in all photon energy regions



SPAN case

Pair emission probability after helicities summation: MD cases

$$\sum_{h_1 h_2} |j_D \cdot j^e|^2 = \frac{1}{2}\Big(\Big(1 + \frac{\vec{p}_1 \cdot \vec{p}_2}{E_1 E_2}\Big) j_0^e (j_0^e)^\dagger + \Big(1 - \frac{\vec{p}_1 \cdot \vec{p}_2}{E_1 E_2}\Big)$$

$$\times \vec{j}^e (\vec{j}^e)^\dagger + 2\Re \frac{\vec{p}_1 \cdot \vec{j}^e \vec{p}_2 \cdot (\vec{j}^e)^\dagger}{E_1 E_2}$$

Common terms

$$-2\Big(\frac{\vec{p}_1}{E_1} + \frac{\vec{p}_2}{E_2}\Big) \cdot \Re j_0^e (\vec{j}^e)^\dagger + 2\frac{\vec{p}_1 \times \vec{p}_2}{E_1 E_2}$$

$$\cdot \Im j_0^e (\vec{j}^e)^\dagger + 2\Big(\frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2}\Big) \cdot \Re \vec{j}^e \times \Im \vec{j}^e \Big).$$

$$\sum_{h_1 h_2} |j_M \cdot j^e|^2 = \sum_{h_1 h_2} |j_D \cdot j^e|^2 + \frac{m_1 m_2}{2E_1 E_2} (j_0^e (j_0^e)^\dagger - \vec{j}^e \cdot (\vec{j}^e)^\dagger).$$

Majorana term

2. Lepton number violation

can occur either in propagator or as a vertex

$$\langle 0|T\left(\varphi(y)\varphi^\dagger(x)\right)|0\rangle = -\sigma\cdot\partial\int\frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(y-x)}}{p^2-m^2+i\epsilon}\,,$$

$$\langle 0|T\left(\varphi(y)\varphi(x)\right)|0\rangle = im\sigma_2\int\frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(y-x)}}{p^2-m^2+i\epsilon}\,.$$

for $\nu\to\bar\nu$ $\qquad -\frac{m}{2E}\sigma_2 e^{im^2(y_0-x_0)/(2E)}$

Responsible in neutrino-less double beta decay, but see our examples below

References

Neutrino pair and gamma beams from circulating excited ions

arXiv: 1505.07572v2 [hep-ph]

Determination of CP violation parameter using neutrino pair beam

arXiv : 1506.08003v1 [hep-ph]

Majorana/Dirac distinction and neutrino mass determination using circulating heavy ions

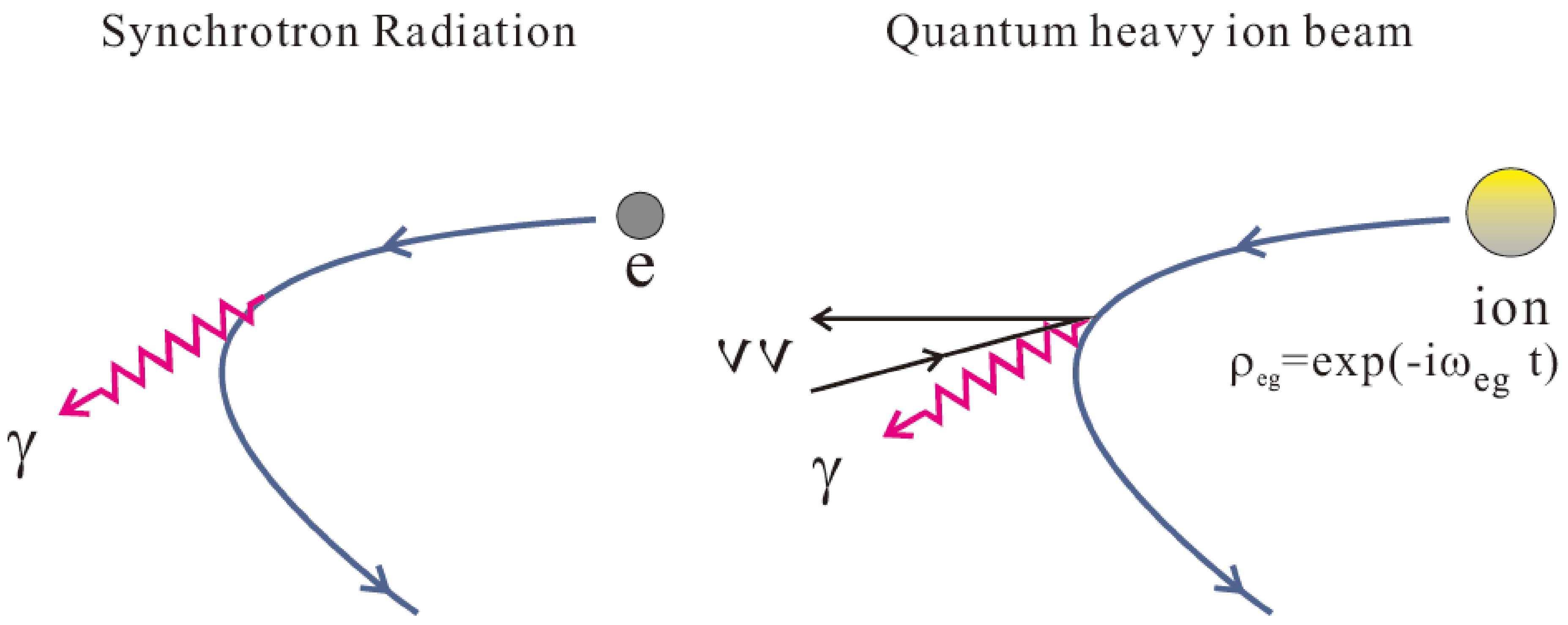
arXiv: 1508.02795v2 [hep-ph] バグあり。以下で修正。

Paper in preparation

Conventional neutrino sources: pi-, mu-, beta-decay

We shall use de-excitation of circulating excited heavy ions, producing pairs of neutrino and anti-neutrino.

Quantum heavy ion beam



直線部分でレーザーを対向照射して励起

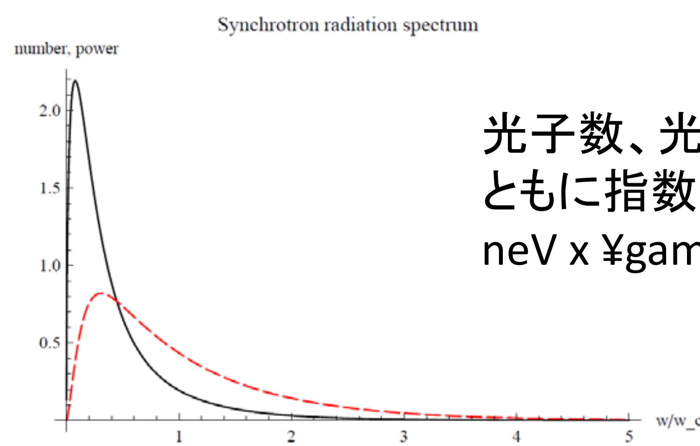
Schwinger's formula for synchrotron radiation

$$\frac{d\mathcal{P}}{d\omega} = \frac{\omega^2}{4\pi^2}\Re\left(\int d\Omega dt'(\vec{j}(\vec{r},t)\cdot(\vec{j}\,'(\vec{r}\,',t') - \rho(\vec{r},t)\rho(\vec{r}\,',t'))e^{-i\omega(t'-t-\vec{k}\cdot(\vec{r}\,'-\vec{r})/\omega)}\right)$$

$$H_{int} = \sqrt{(\vec{p}-e\vec{A}_{ext})^2+m^2} + e\Phi_{ext}$$

- Main results

  Exponential cutoff, both in energy and angular directions, only to keV region available. But flux is much, much larger than decay product.
- Phase integral: same sign phase adds up



光子数、光子エネルギー
ともに指数関数減衰
neV x ¥gamma^3

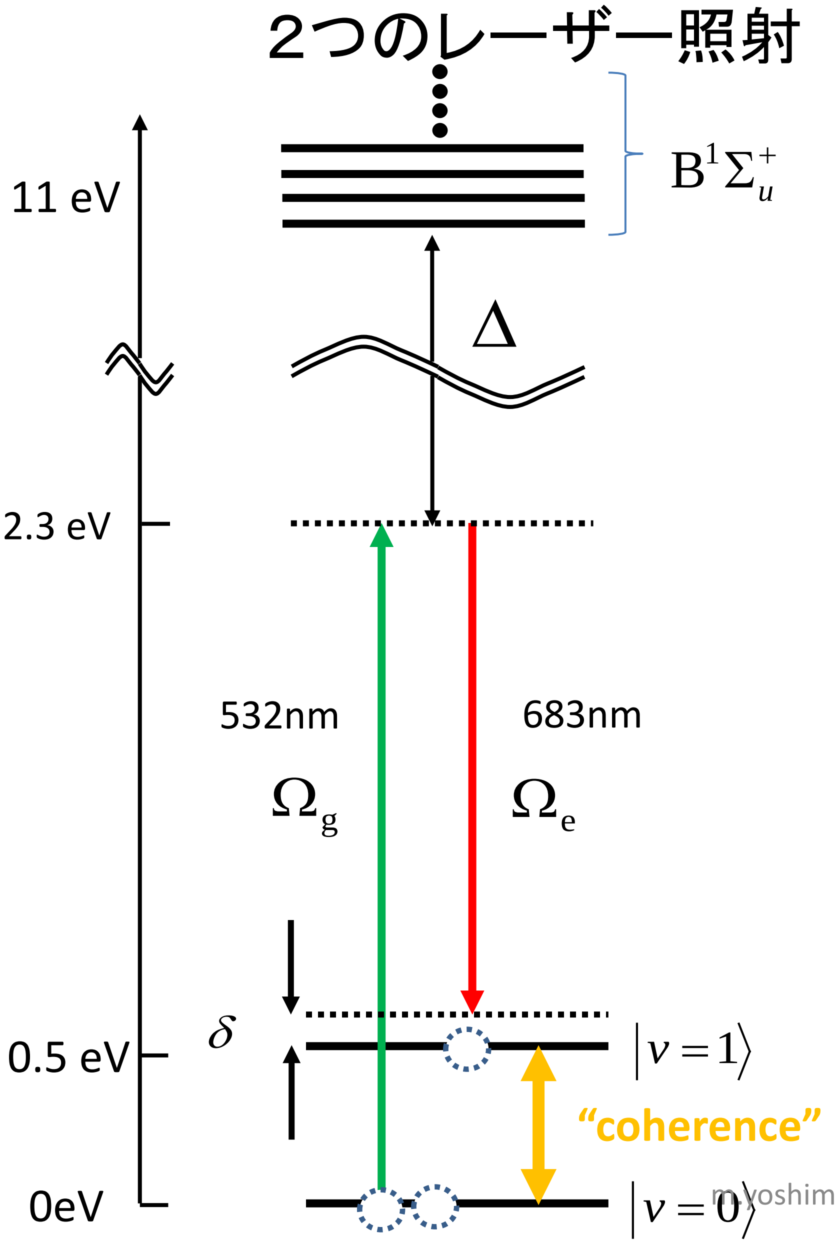
Neutrino pair emission occurs similarly to synchrotron radiation,

But, producing neutrino pairs in the keV energy region with extremely small rates, hence completely negligible for both electron synchrotron and heavy ion in the ground state circulating

New feature for excited ions

Input of excitation energy, leading to a kind of non-linear resonance given by stationary points（positive and negative phases cancellation) in a phase integral over times

Preparation of initial coherence – Adiabatic Raman -



Two laser fields irradiates p-H2

Two photon Rabi frequency $\Omega_{ge} \cong \dfrac{\Omega_g \Omega_e}{\Delta}$

→ |g> and |e> are mixed with an angle

$$\tan\theta \cong \frac{\Omega_{ge}}{\delta}$$

Non-degenerate Superposition States:

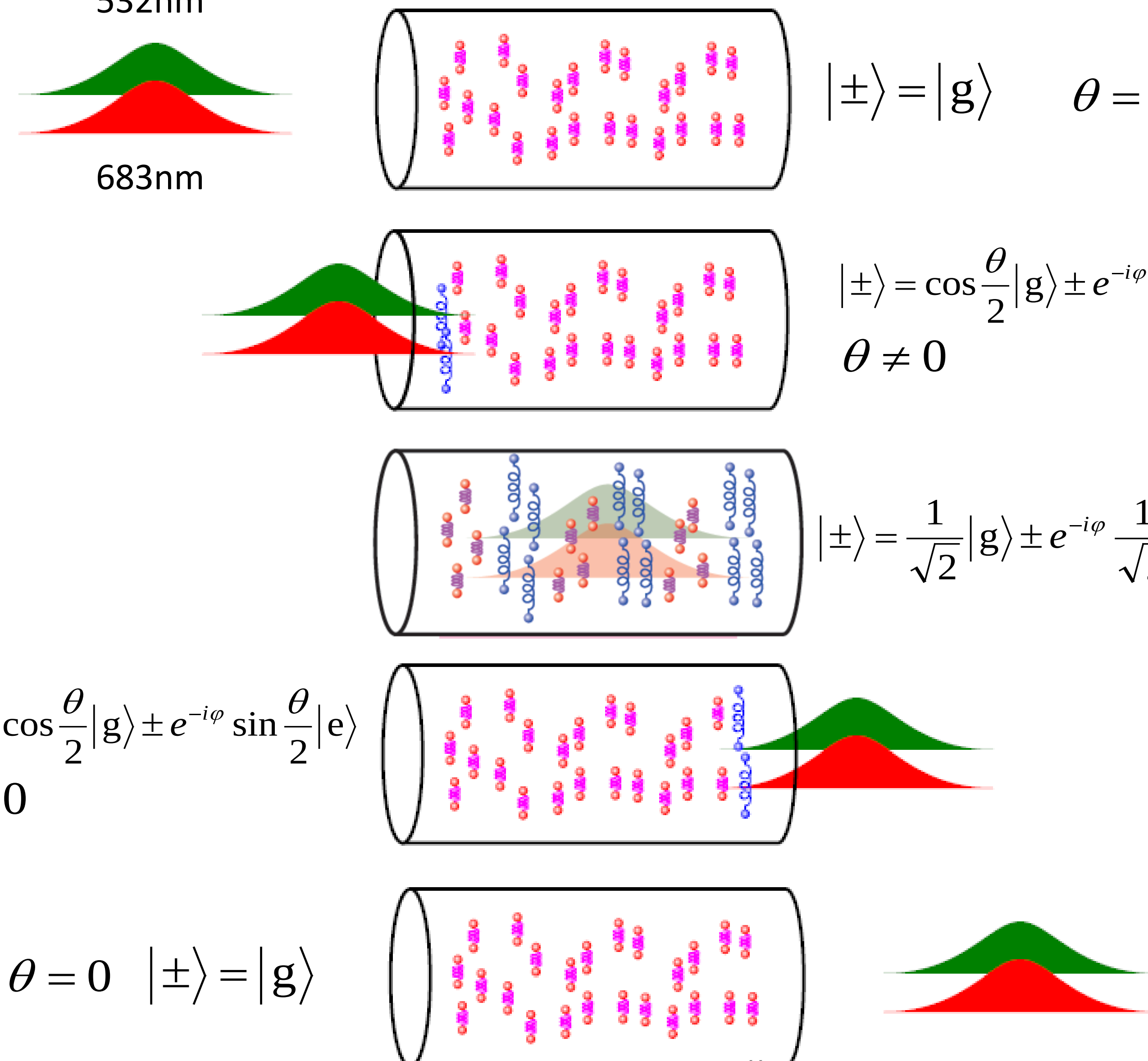
$$|+\rangle = \cos\frac{\theta}{2}|g\rangle + e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

$$|-\rangle = \cos\frac{\theta}{2}|g\rangle - e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

Coherence between |e> and |g>

$$|\rho_{eg}| = \frac{1}{2}\sin\theta$$

p-H2 gas



$$|\pm\rangle = |g\rangle \qquad \theta = 0$$

$$|\pm\rangle = \cos\frac{\theta}{2}|g\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

$$\theta \neq 0$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}|g\rangle \pm e^{-i\varphi}\frac{1}{\sqrt{2}}|e\rangle \qquad \theta = \frac{\pi}{2}$$

$$|\pm\rangle = \cos\frac{\theta}{2}|g\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

$$\theta \neq 0$$

$$\theta = 0 \quad |\pm\rangle = |g\rangle$$

Two useful processes: pair emission and RENP (radiative neutrino pair emission) from circulating excited ions

Without and with a photon emission

$$|e^{\pm}\rangle \to |g^{\pm}\rangle + \nu_i \bar{\nu}_j$$ Neutrino-pair beam

$$|e^{\mp}\rangle \to |g^{\pm}\rangle + \gamma + \nu_i \bar{\nu}_j$$ Beam RENP

1st giving useful and unusual neutrino pair beam for oscillation experiments and 2nd giving opportunities to resolve the Majorana/Dirac distinction and determining the smallest neutrino mass.

How to calculate RENP emission rate

- Semi-classical approximation: classical ion CM motion and quantum internal state
- Spin current dominance from valence electron transition

hamiltonian

$$H_{eff} = \frac{1}{\sqrt{\gamma}} \frac{1}{\Delta\epsilon} \int d^3x \int dt \rho_{eg}(t) \frac{G_F}{\sqrt{2}} S_e^{\alpha} \sum_{ij} a_{ij} \nu_i^{\dagger}(x) \sigma_{\alpha} \nu_j(x) \vec{d}_{eg} \cdot \vec{E}(x)\, \delta^{(4)}(x - x_A(t)\,)$$

CP-even

Mixture of well-defined phase

$$a_{ij} = U_{ei}^* U_{ej} - \frac{1}{2}\delta_{ij}$$

coherence

$$\rho_{eg}(t) = \rho_{eg}(0) \exp[-(i\epsilon_{eg} + \frac{1}{T_2})\frac{t}{\gamma}]\,,$$

Spin factor

$$(S_{\alpha}) = (\gamma \vec{\beta} \cdot \vec{S}_e\,, \vec{S}_e + \frac{\gamma^2}{\gamma + 1}(\vec{\beta} \cdot \vec{S}_e)\vec{\beta}\,) \sim \gamma(\vec{\beta} \cdot \vec{S}_e\,, (\vec{\beta} \cdot \vec{S}_e)\vec{\beta}\,)\,,$$

$$-> \qquad \gamma^2 \frac{S_e^2}{3} \left(1 + \frac{1}{3} \frac{\vec{p}_1 \cdot \vec{p}_2}{E_1 E_2} - \frac{m_1 m_2}{2E_1 E_2} \delta_M \right)$$

Ion trajectory

$$\vec{r}_A(t) = \rho \left(\sin\frac{vt}{\rho}\,, 1 - \cos\frac{vt}{\rho}\,, 0 \right)$$

Some details of calculation

$$i\partial_t \mathcal{A}_{ij}(\vec{k}\epsilon_k, p_1h_1, p_2h_2; t) = \langle 0|[a_k d_i(p_2h_2;t) b_j(p_1h_1;t)\,,\, H_{eff}]|0\rangle\,,$$

$$\mathcal{A}_{ij}(\vec{k}\epsilon_k, p_1h_1, p_2h_2; t) = -i\sqrt{2}G_F\frac{1}{\sqrt{\gamma}}C_{ij}\int_{-\infty}^{t} dt' e^{i(\omega+E+E')t'}\,\tilde{J}_A^{\dagger}(\vec{k}, \vec{p}_1+\vec{p}_2; t')\cdot j_\nu\,,$$

$$\tilde{J}_A^{\alpha}(\vec{k}, (\vec{p}_1+\vec{p}_2); t) = \rho_{eg}(t) S^\alpha e^{-i(\vec{k}+\vec{p}_1+\vec{p}_2)\cdot\vec{r}_A(t)}\,, \qquad j_\nu = u^\dagger(p_1h_1)\sigma v(p_2h_2)\,,$$

$$P_{ij}(t; p_1h_1, p_2h_2) = \partial_t|\mathcal{A}_{ij}(p_1h_1, p_2h_2; t)|^2 =$$

$$4G_F^2|\rho_{eg}(0)|^2\frac{1}{\gamma}|C_{ij}|^2\int_{-\infty}^{0} dt S^\alpha \Re\left(\mathcal{N}_{\alpha\beta}(p_1h_1, p_2h_2)e^{i(\Delta(0)-\Delta(t))}\right)S^\beta$$

$$= 4G_F^2|\rho_{eg}(0)|^2\frac{1}{\gamma}|C_{ij}|^2\frac{d^2}{3}\int_{0}^{\infty} dt S^\alpha \Re\left(\mathcal{N}_{\alpha\beta}(p_1h_1, p_2h_2)e^{i(\Delta(0)-\Delta(-t))}\right)S^\beta\,,$$

$$\Delta(t) = (\omega + E_1 + E_2 - \frac{\epsilon_{eg}}{\gamma})t - (\vec{k}+\vec{p}_1+\vec{p}_2)\cdot\vec{r}_A(t)\,,$$

$$\mathcal{N}^{\alpha\beta}(p_1h_1, p_2h_2) = j_\nu^\alpha(p_1h_1, p_2h_2)(j_\nu^\dagger)^\beta(p_1h_1, p_2h_2)\,,$$

In the large radius (¥rho) limit

位相因子の積分で停留点近似を行う

$$\int_0^\infty dt \cos\Phi(t)\,, \qquad \Phi(t) = \frac{\omega + E_1 + E_2}{2\rho\gamma}\sqrt{D}(t - \frac{\rho}{\gamma}D)^2$$

$$D = 1 - \frac{2\epsilon_{eg}\gamma + \gamma^2(m_i^2/E_1 + m_j^2/E_2)}{\omega + E_1 + E_2} - (\text{quadratic function of angles})\,,$$

$$\text{resonance in time domain :} \quad t_r = \frac{\rho}{\gamma}D \approx 10\text{ps}\,,$$

$$\text{width;} \quad \Delta t_r = \rho\sqrt{\frac{2}{(E_1 + E_2)t_r}} \gg t_r$$

$$\int_0^\infty dt \cos\Phi(t) \sim \int_0^\infty dt \cos\frac{(t - t_r)^2}{(\Delta t_r)^2} \sim \sqrt{\frac{2\pi}{3}}\Delta t_r = \sqrt{\frac{\pi}{3}}(\frac{\rho\gamma}{E_1 + E_2})^{1/2}D^{-1/4}$$

for GeV neutrinos

Difference from usual synchrotron radiation

Ground state ion

X = rescaled time

$$\int_0^\infty dx h(x) \cos\xi(\frac{1}{2}x^3 + \frac{3}{2}x) \rightarrow \sqrt{\frac{\pi}{6}} e^{-\xi} \frac{h(0)}{\sqrt{\xi}} .$$

$$\xi = \rho(E_1 + E_2) \times \text{ a function of } \left(\frac{E_1}{E_2}, \frac{\epsilon_{eg}}{E_1 + E_2}, \gamma, \text{angles} \right)$$

- Always the same sign phase added, leading to exponential dampling

Excited ion with coherence

$$\int_0^\infty dx h(x) \cos\xi(\frac{1}{2}x^3 - \frac{3}{2}x) \rightarrow \sqrt{\frac{2\pi}{3}} \cos(\xi - \frac{\pi}{4}) \frac{h(1)}{\sqrt{\xi}}$$

Cancellation of positive and negative phases
Energy input leads to resonance-like behavior

Intuitive understanding

- A kind of non-linear resonance: orbital energy balanced against internal ion energy, giving non-linear resonance oscillation. Its width around the stationary point gives a sharp resonance-like behavior in time domain.

- Key concept for its success: quantum coherence typically realized by ionic system under laser irradiation, but may persist without phase relaxation.

Simple example of quantum coherence: adiabatic Raman process

$$\omega_m \frac{d\Gamma_{ij}}{d\omega} = RF_{ij}(\frac{\omega}{\omega_m}) \times \left(|a_{ij}^2|^2 - \delta_M \frac{3}{8} \frac{m_i m_j}{\omega_m^2 x_1 x_2} \Re(a_{ij}^2) \right) , \quad a_{ij} = U_{ei}^* U_{ej} - \frac{1}{2}\delta_{ij} ,$$

$$R = \frac{\sqrt{\pi}}{2\sqrt{3}(2\pi)^8} v_5 G_F^2 d_{pe}^2 \gamma^6 N |\rho_{eg}(0)|^2 \sqrt{\rho} \epsilon_{eg}^{19/2} \frac{1}{\epsilon_{pe}^2}$$

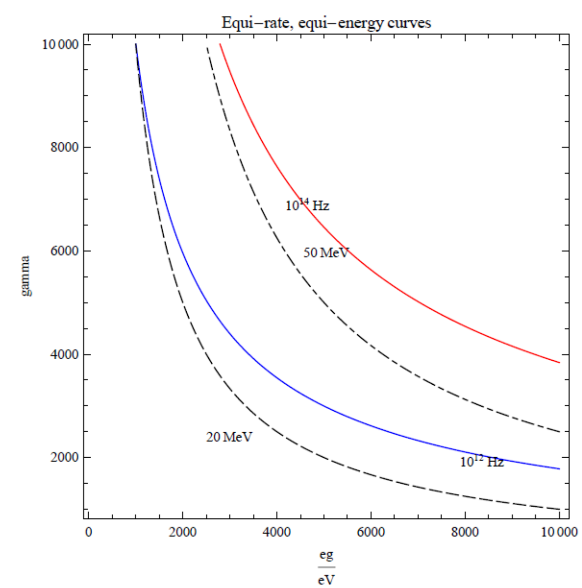
$$\sim 4.6 \times 10^{19}\text{Hz} \frac{N|\rho_{eg}(0)|^2}{10^8} \frac{\gamma_{pe}}{100\text{MHz}} \sqrt{\frac{\rho}{4\text{km}}} (\frac{\gamma}{10^4})^6 (\frac{\epsilon_{eg}}{10\text{keV}})^{15/2} ,$$

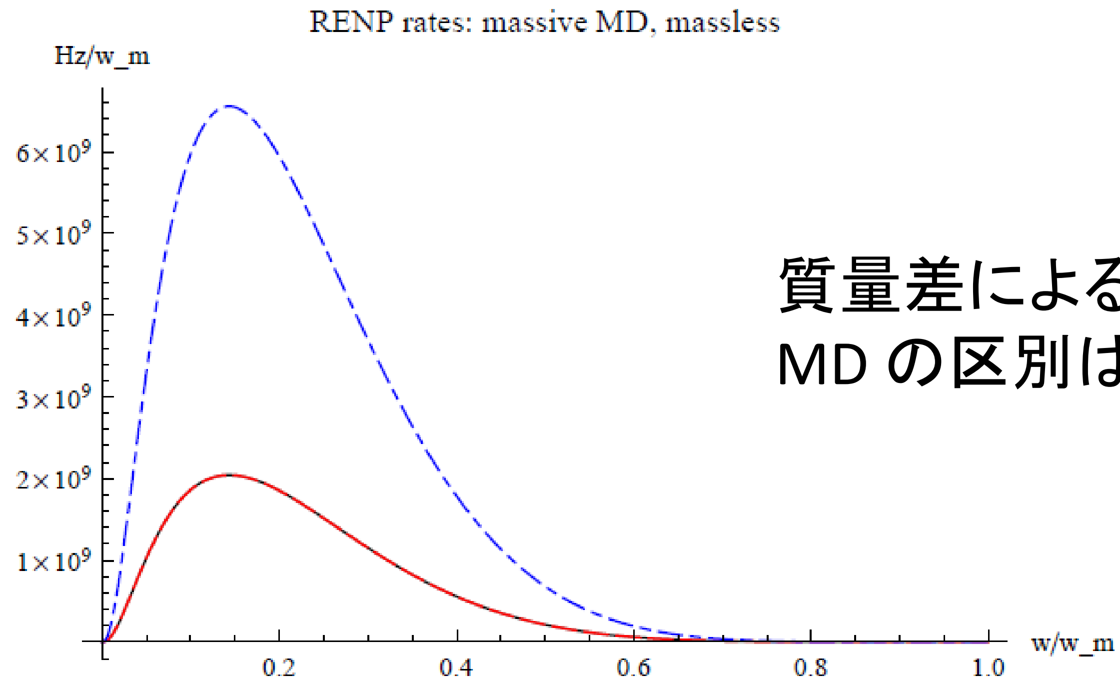
$$v_5 = \int dV_5 (1 - r^2)^{-1/4} \sim 9.1 \times 10^{-6} , \quad F_{ij}(y) = \int_0^1 dx_1 \int_0^1 dx_2 H_{ij}(y, x_1, x_2) ,$$

$$H_{ij}(y, x_1, x_2) = y^{5/2} (1 + \frac{2\epsilon_{eg}}{\epsilon_{pe}} y)^{-2} x_1 x_2 (x_1 + x_2 + y)^{1/4} G_{ij}(x_1, x_2, y)^{9/4} \Theta(G_{ij}(x_1, x_2, y)) ,$$

$$G_{ij}(x_1, x_2, y) = 1 - x_1 - x_2 - y - \frac{1}{4\epsilon_{eg}^2} (\frac{m_i^2}{x_1} + \frac{m_j^2}{x_2}) ,$$

Candidate ion: Pb^72+ (Ne-like)
LHCで既にPb^82+を7TeV に加速済み

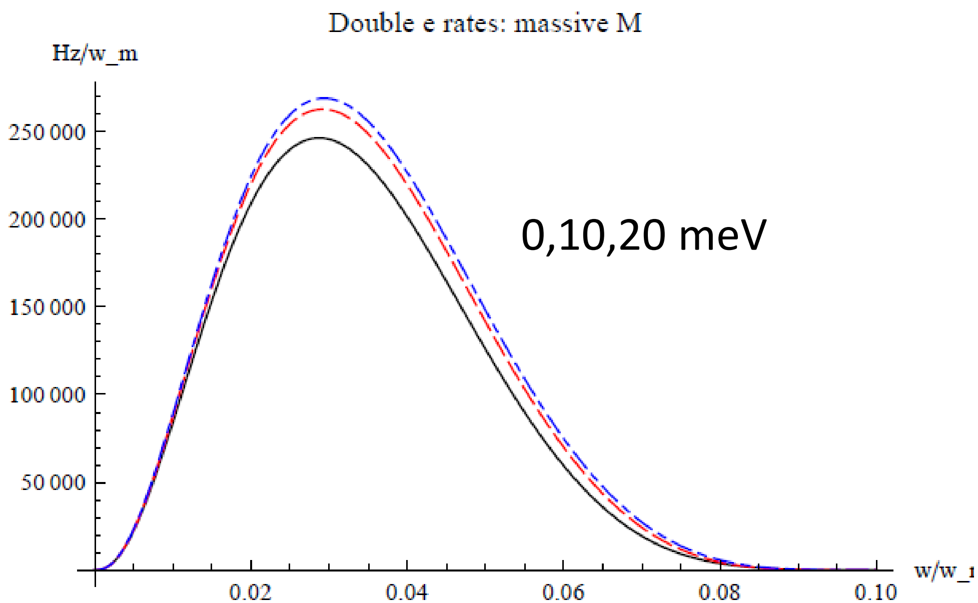




質量差による効果は大きい
MD の区別は難しそう

Figure 3: Single photon spectral rates: the massive (NH of zero smallest mass) Majorana case in solid black, the massive Dirac (NH of zero smallest mass) case in dashed-red, and the massless case in dash-dotted blue. $N|\rho_{eg}(0)|^2 = 10^8, \gamma = 5 \times 10^4, \gamma_{pe} = 100\text{MHz}, \rho = 4\text{km}$, vanishing CPV parameters and $\epsilon_{eg} = 1, \epsilon_{pe} = 0.1\text{keV}$ are assumed. The maximum photon energy $\omega_m = 100\text{MeV}$.

Neutrino mass determination



Sensitivity to smallest 1 meV

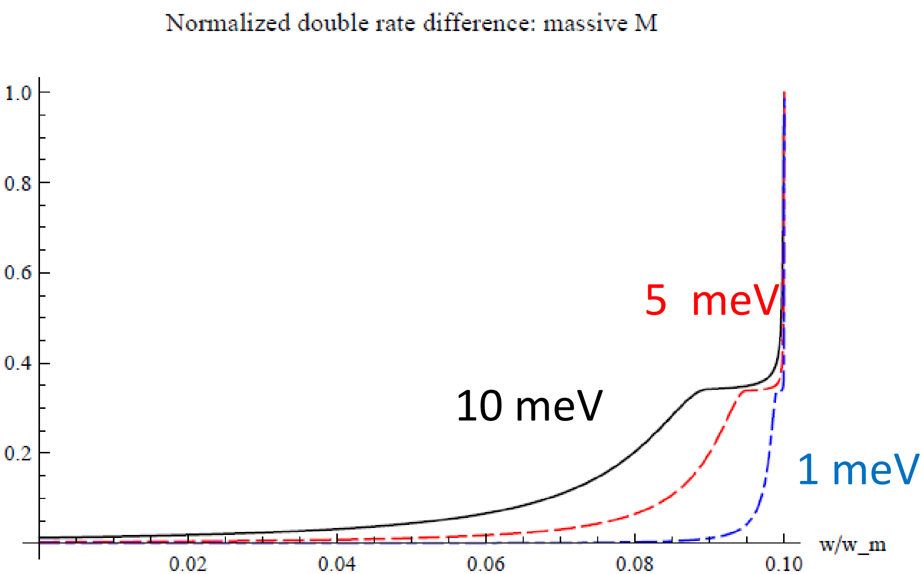


Figure 5: Double spectral rates of photon with a detected ν_e of fixed energy $0.9\omega_m = 90$MeV: the smallest neutrino mass 20 meV in solid black, 10 meV in dashed red, and 0 meV in dashed-dotted blue, all for the Majorana NH cases. $N|\rho_{eg}(0)|^2 = 10^8, \gamma_{pe} = 100\text{MHz}, \rho = 4\text{km}$ and $\gamma = 5\times 10^4, \epsilon_{eg} = 1\text{keV}, \epsilon_{pe} = 0.1\text{keV}$ are assumed.

光子とニュートリノ同時測定もできる

Majorana/Dirac distinction in RENP

- Difficult in the usual ways
- Best is to discover doubly charged nu-nu events
  : Lepton Number Violating (LNV) process

$$\langle 0|T\left(\varphi(y)\varphi^\dagger(x)\right)|0\rangle = -\sigma\cdot\partial\int\frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(y-x)}}{p^2-m^2+i\epsilon}\,,$$

$$\langle 0|T\left(\varphi(y)\varphi(x)\right)|0\rangle = im\sigma_2\int\frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot(y-x)}}{p^2-m^2+i\epsilon}\,.$$

for $\nu\to\bar\nu$ $\qquad -\frac{m}{2E}\sigma_2 e^{im^2(y_0-x_0)/(2E)}$

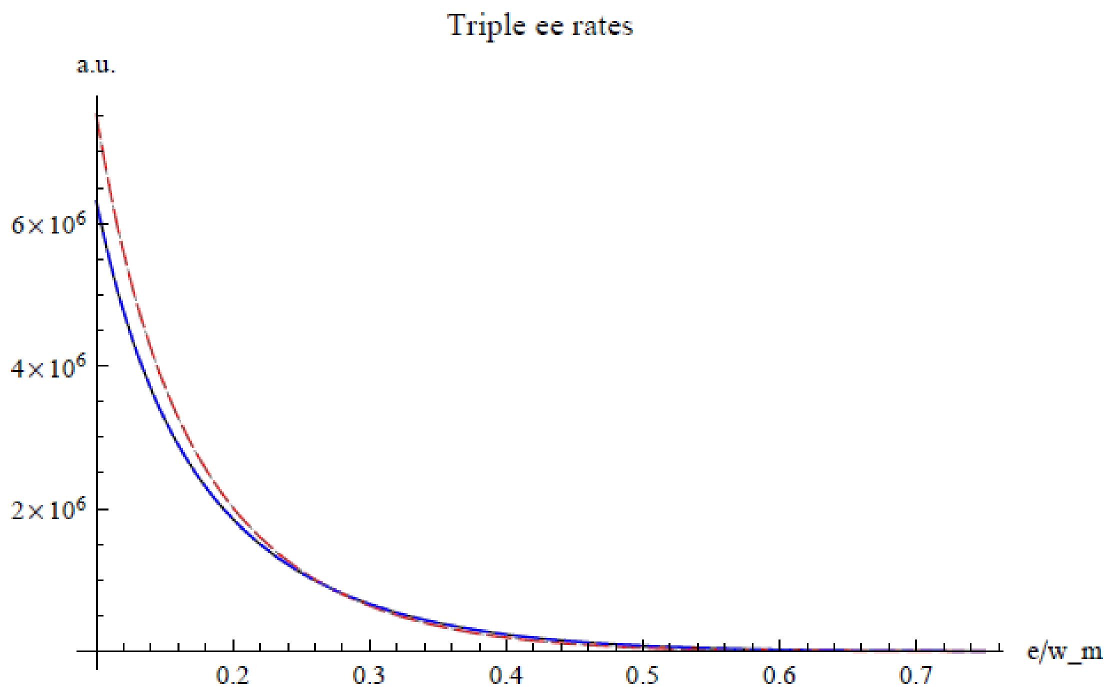
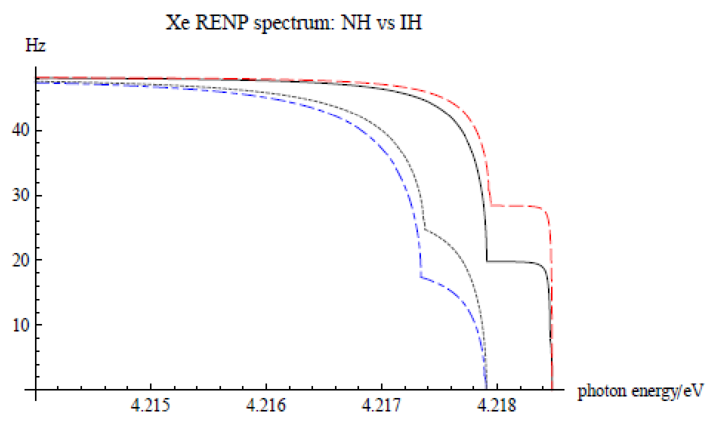


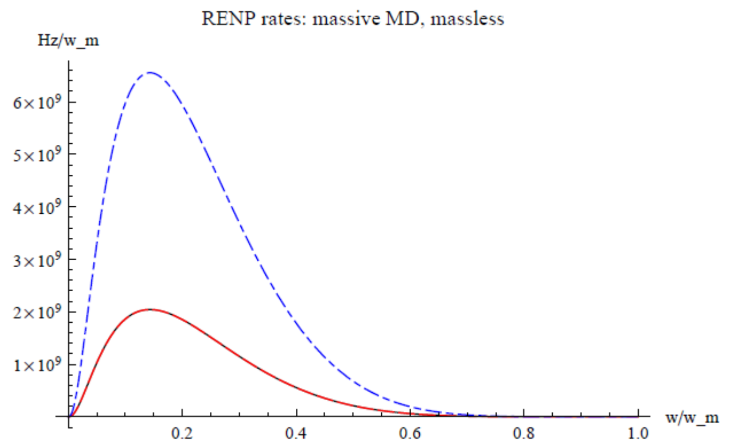
Figure 6: Neutrino energy spectrum in triple detection of doubly charged LNV process γ, ν_e, ν_e. The photon energy is fixed at $\omega/\omega_m = 0.1$ in solid black, 0.2 dashed red, with $(\alpha, \beta, \delta) = (0, 0, 0)$, and $\omega/\omega_m = 0.1$ in dash-dotted blue, 0.2 dotted black, with $(\alpha, \beta, \delta) = (0, \pi/2, 0)$. The smallest neutrino mass is taken vanishing in NH. $\gamma = 5 \times 10^4, \epsilon_{eg} = 1\text{keV}, \epsilon_{pe} = 0.1\text{keV}$ are assumed.

rate computations at LHC to be done

RENP using pair beam：　まとめ

- Absolute mass determination and MD distinction expected
- Kinematics different from SPAN: energy and momentum conservation not obeyed, and only the energy sum of photon and two neutrinos limited
- Rate scales with gamma^6





Comparisons

Comparison of radiative and non-radiative neutrino pair emission

	oscillation	absolute ν mass	CPV	parameter dependence
beam RENP	n	y	?	$\gamma^6 \epsilon_{eg}^{15/2} N$
non-radiative NP	y	y	y	$\gamma^4 \epsilon_{eg}^{11/2} N$
SPAN	n	y	y	N^2

Choice of ion and atom targets not to be forgotten.

Comparison of neutrino-pair beams

	process	rough flux/Hz m^{-2}	reference
neutrino factory	$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$	$\sim 10^4$	S.Geer, PRD
quantum ion beam	$\|e\rangle \to \|g\rangle \nu_a \bar{\nu}_a$	$\sim 10^{15}(N/10^8)^*$	YS

* Effect of collimated beam size considered by $1/m^2$.

開発実験研究

- 量子重イオンビームの実現:
  標的イオンの選定。対向照射レーザーの作成。
  理研で低エネルギーイオンビームによるガンマ線またはX線放出を測定するのがよい。(目標はコヒーラントガンマ線ビーム)

- 現存LHCおよびそのアップグレードで何ができるか:
  Pb原子核衝突を既に 7TeV で実現

- 新たなFCC加速器の最適パラメータは？

- 最適化した検出器の設計:　e^+- の区別必要

- 理論の協力が必要。

Another application of coherent quantum beam

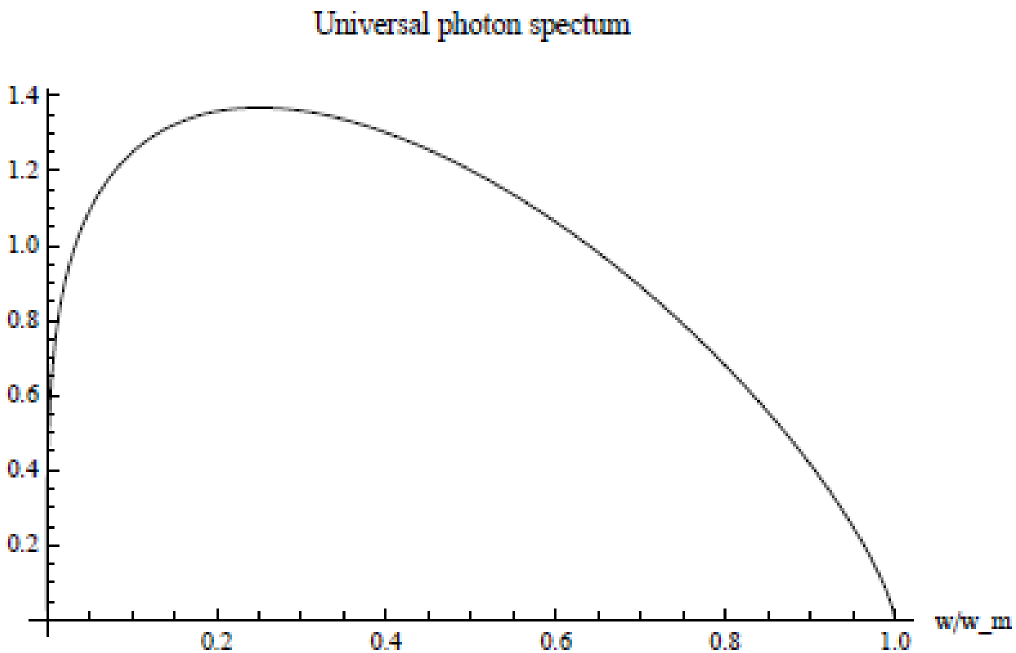
- When coherence exists between two levels related by E1 transition, exponential cutoff of synchrotron radiation

$$(e^{-\omega/\omega_c} , \omega_c = \frac{3}{2}\frac{1}{\rho}\gamma^3)$$

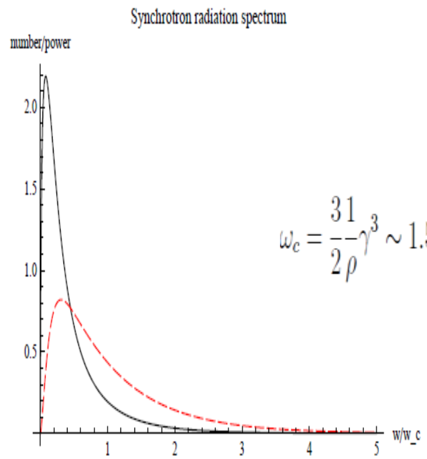
  does not exist, and gamma ray energy is only limited by the same boosted level spacing

- Coherence among many ions (macro-coherence) may lead to coherent gamma ray beam (gamma ray “laser”)

Energy spectrum: comparison



Synchrotron radiation



$$\omega_c = \frac{3}{2}\frac{1}{\rho}\gamma^3 \sim 1.5\mathrm{keV}\frac{200\mathrm{m}}{\rho}(\frac{\gamma}{10^4})^3$$

$$\Gamma \sim 1.6\times 10^{29}\mathrm{Hz}\,\frac{\gamma_{eg}}{100\mathrm{MHz}}\sqrt{\frac{\rho}{200\mathrm{m}}}(\frac{\epsilon_{eg}}{50\mathrm{keV}})^{1/2}(\frac{\gamma}{10^4})^2\frac{N|\rho_{eg}(0)|^2}{10^8}$$

$$\propto\ \gamma^2 d_{eg}^2\epsilon_{eg}^3\sqrt{\rho\epsilon_{eg}}$$

Summary of this talk

- We should maximally exploit quantum coherence towards the ultimate clarification of mysteries of neutrino.
- Coherent quantum heavy ion synchrotron is excellent for the smallest mass measurement, NH/IH hierarchy distinction, MD distinction, and CPV parameter determination .
- Accelerator R & D works crucial to obtain a high coherence beam.

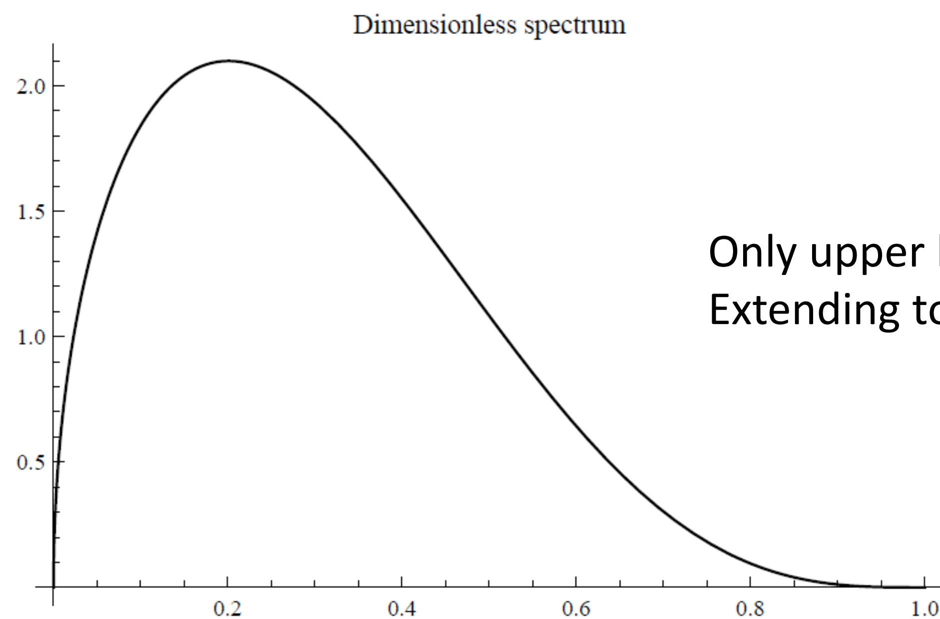
Backup

Neutrino pair beam and Neutrino oscillation experiments

Differential and total production rates

$$\Gamma = \sum_i \Gamma_i \sim 3.1 \times 10^{21}\text{Hz}\left(\frac{\rho}{4\text{km}}\right)^{1/2} \frac{S_e^2 N |\rho_{eg}(0)|^2}{10^8} \left(\frac{\gamma}{10^4}\right)^4 \left(\frac{\epsilon_{eg}}{50\text{keV}}\right)^{11/2},$$

$$\text{with } E_m = 2\epsilon_{eg}\gamma = 1\text{GeV}\frac{\epsilon_{eg}}{50\text{keV}}\frac{\gamma}{10^4}.$$



Only upper bound of total energy
Extending to GeV region

Detection of neutrino pair away from synchrotron

$$\sum_b (\frac{G_F}{\sqrt{2}})^2 \bar{\nu}_a \gamma_\alpha (1-\gamma_5) l_a J^\alpha \bar{l}_c \gamma_\beta (1-\gamma_5)\nu_c (J^\beta)^\dagger \langle \bar{a}|e^{-iHL}|\bar{b}\rangle \langle c|e^{-iHL}|b\rangle \mathcal{P}_{\bar{b}b}(1,2)$$

$$H = U \begin{pmatrix} \frac{m_1^2}{2E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} \end{pmatrix} U^\dagger \mp \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Single neutrino detection eliminates oscillation pattern

$$\langle c|e^{-iHL}|b\rangle = \sum_i V_{ci}^* V_{bi} e^{-i\lambda_i L}\,, \qquad \langle \bar{a}|e^{-iHL}|\bar{b}\rangle = \sum_i \bar{V}_{ai}^* \bar{V}_{bi} e^{-i\bar{\lambda}_i L}\,,$$

$$\sum_b \langle \bar{a}|e^{-iHL}|\bar{b}\rangle \langle c|e^{-iHL}|b\rangle c_b = \sum_{ij} V_{ci}^* \bar{V}_{aj}^* \xi_{ij} e(\lambda_j, \lambda_i)\,, \qquad (c_b) = \frac{1}{2}(1,-1,-1)\,,$$

$$\xi_{ij} = \bar{V}_{ej} V_{ei} - \bar{V}_{\mu j} V_{\mu i} - \bar{V}_{\tau j} V_{\tau i}\,, \qquad e(\bar{\lambda}_j, \lambda_i) = \exp[-iL(\lambda_i + \bar{\lambda}_j)]\,.$$

$$\sum_c |\sum_{ij} V_{ci}^* \bar{V}_{\mu j}^* \xi_{ij} e(\lambda_j, \lambda_i)|^2 = \sum_{ijkl} \sum_c V_{ci}^* V_{ck} \bar{V}_{\mu j} \bar{V}_{\mu l}^* \xi_{ij} \xi_{kl}^* e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_k)$$

$$= \sum_{jl} \bar{V}_{\mu j} \bar{V}_{\mu l}^* e(\lambda_j, \lambda_i) e^*(\lambda_l, \lambda_i) \sum_i \xi_{ij} \xi_{il}^* = \frac{1}{4} \sum_{jl} \bar{V}_{\mu j} \bar{V}_{\mu l}^* e(m_j, m_i) e^*(m_l, m_i) \delta_{jl} = \frac{1}{4}\,,$$

Short baseline experiments

$$\sqrt{2}G_F n_e L \sim 1 \times \frac{n_e}{1.4 \times 6 \times 10^{23}\mathrm{cm}^{-3}} \frac{L}{1860\mathrm{km}}$$

$$|\sum_{ij} V_{ci}^* \bar{V}_{\mu j}^* \xi_{ij} e(\bar{\lambda}_j, \lambda_i)|^2 \frac{d^4\Gamma}{dE_1 dE_2 d\Omega_1 d\Omega_2} \frac{d^2\sigma}{dE_+ d\sin\psi_+} \frac{d^2\sigma}{dE_- d\sin\psi_-}$$

Plotted quantities

$$P_{\bar{a}c} = |\sum_{ij} U_{ci}^* U_{aj} \xi_{ij} e^{-iL(m_j^2/E_1 + m_i^2/E_2)}|^2$$

$$A(\delta) = \frac{d\Gamma(\delta : G_F) - d\Gamma(-\delta : -G_F)}{d\Gamma(\delta : G_F) + d\Gamma(-\delta : -G_F)} .$$

Double rates $10 \sim 100$ mHz for a 100 kt class

$\sigma n_N l \sim 10^{-11} \sim 10^{-10}$ for a single detection

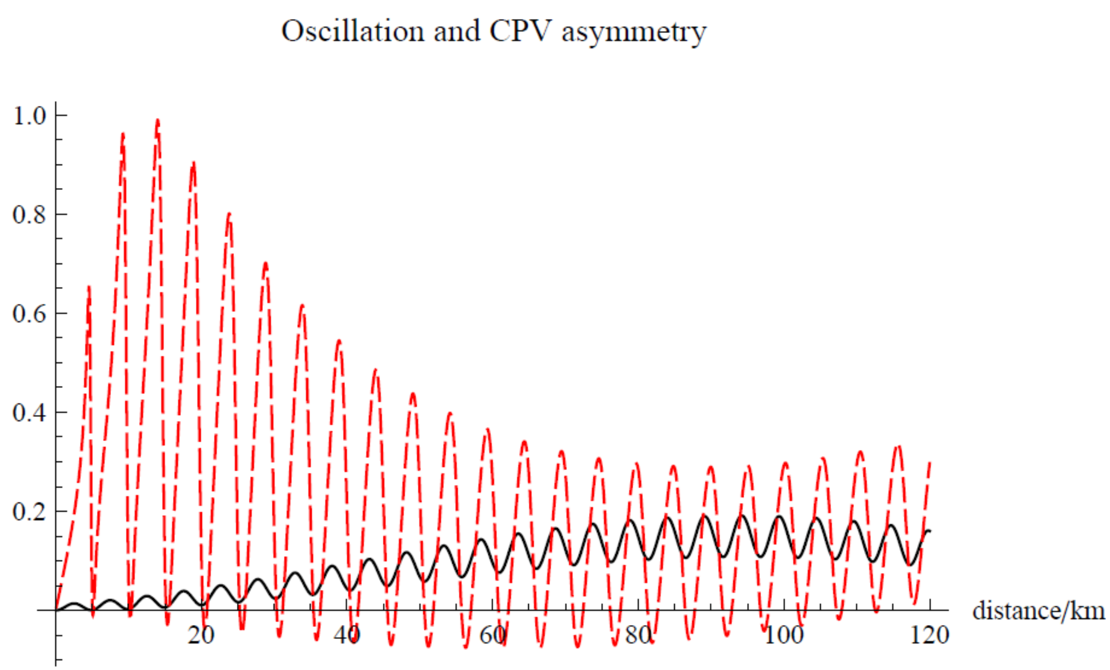


Figure 1: Oscillation pattern given by $P_{\bar{\mu}e}$ of eq.(13) (in solid black) and asymmetry (in dashed red) at various distances for $\bar{\nu}_\mu \nu_e$ CC double events. $\delta = \pi/4$, $E_{\bar{\nu}_\mu} = 500\text{MeV}$, $E_{\nu_e} = 5\text{MeV}$.

How short baseline exp. became effective

- Two factors of L/E, one 10km/10MeV instead of 500km/500 MeV giving the same oscillation pattern

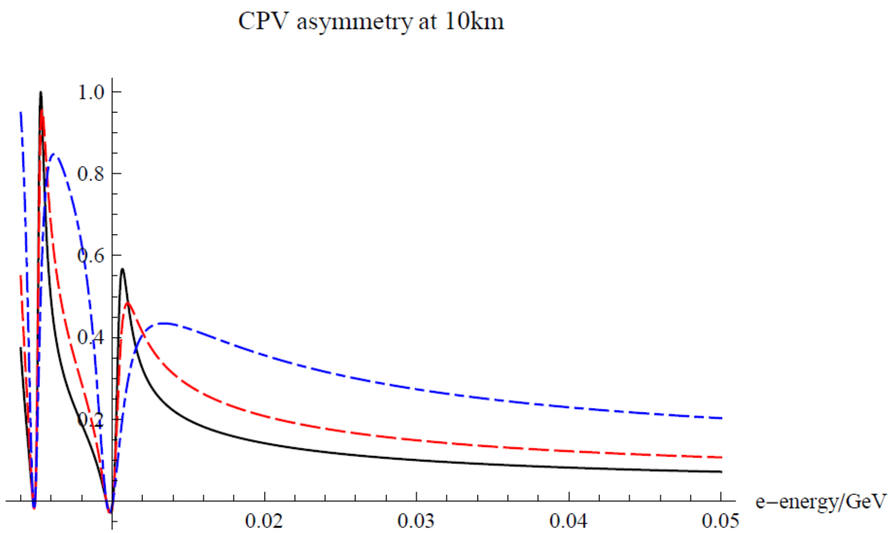


Figure 3: Asymmetry vs electron neutrino energy for $\bar{\nu}_\mu \nu_e$ CC double events. $E_{\bar{\nu}_\mu} = 500\text{MeV}$ and $\delta = \pi/6$ in solid black, $\pi/4$ in dashed red, and $\pi/2$ in dash-dotted blue. NH of smallest mass zero is assumed.

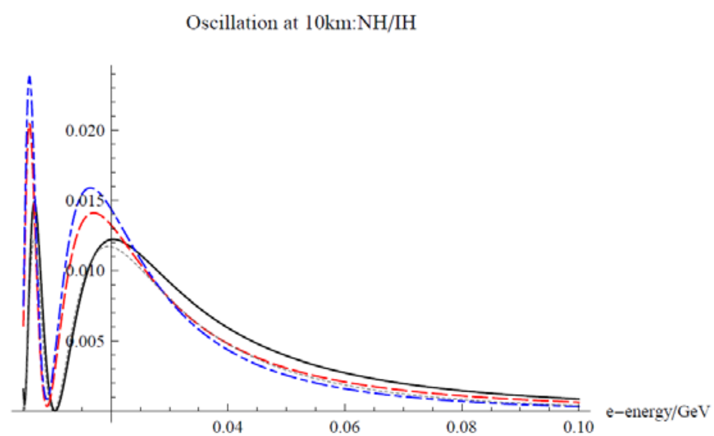


Figure 5: NH vs IH distinction at 10 km away from the synchrotron, given by asymmetric energy combinations: $P_{\bar{\mu}e}$ is plotted for $E_{\bar{\nu}_\mu} = 500, 200\text{MeV}$, fixed and variable E_{ν_e}. NH in blacks, 500 MeV in solid and 200 MeV in dotted lines, and IH in colored, 500MeV, in dashed red and 200 MeV in dash-dotted blue. $\delta = 0$.

Features of pair beam

- Double detection required for oscillation experiments
- Short baseline experiments recommended to avoid the earth matter effect
- Excellent opportunity for ¥delta and NH/IH

Competition with QED photon emission

an example of He-like ion , Pb^{80+}

$|e\rangle = ((2s)(1s))^3_{J=1}$ (a spin triplet state described in jj coupling scheme)

level spacing $\epsilon_{eg} \sim 70\mathrm{keV}$

M1 photon emission $\quad \Gamma_\gamma = \gamma_{M1} N \rho_{ee}(0)$

$$\gamma_{M1} \sim 3.4 \times 10^{13}\mathrm{Hz}$$

-> $$|\rho_{eg}(0)|^2 > O(0.1)\rho_{ee}(0)\left(\frac{\gamma}{10^4}\right)^{-4}$$

Pair emission build-up time should be shorter than 2nu production time.

$$\Delta t < 1/\Gamma_{2\nu}$$

$$\Delta t = \sqrt{\frac{3\xi}{2}\frac{E_1+E_2}{F}}\,, \quad \sqrt{\xi} = \sqrt{\frac{2\sqrt{2}}{3}}\sqrt{\rho(E_1+E_2)}\left(\frac{F}{(E_1+E_2)^2}\right)^{3/4}$$

$$\Gamma_{2\nu} \sim 3.1\times 10^{13}\text{Hz}\left(\frac{\rho}{4\text{km}}\right)^{1/2}\left(\frac{\gamma}{10^4}\right)^4\left(\frac{\epsilon_{eg}}{50\text{keV}}\right)^{11/2}$$