Supersymmetric explanation of CP violation in $K \to \pi\pi$ decays

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STRANGE MESONS
$(S=\pm1, C=B=0)$

$K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\overline{K}^0 = \bar{d}s$, $K^- = \bar{u}s$, similarly for K^*'s

K^0

$I(J^P) = \frac{1}{2}(0^-)$

50% K_S, 50% K_L

Mass $m = 497.611 \pm 0.013$ MeV (S = 1.2)

$m_{K^0} - m_{K^\pm} = 3.934 \pm 0.020$ MeV (S = 1.6)

Kaon & CP violation:1

- Kaon and CP transformation

$$CP|K^0\rangle = |\overline{K}^0\rangle, \qquad CP|\overline{K}^0\rangle = |K^0\rangle,$$

$$CP|K^0_\pm\rangle = \pm|K^0_\pm\rangle, \quad \text{where } |K^0_\pm\rangle \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\overline{K}^0\rangle\right)$$

- $|K^0_\pm\rangle$ are CP-eigenstates but not mass-eigenstates, because nature does not respect the CP symmetry

Short lived mass-eigenstate $\quad |K_S\rangle \simeq \frac{1}{\sqrt{1+|\epsilon_K|^2}}\left(|K^0_+\rangle + \epsilon_K|K^0_-\rangle\right)$

Long lived mass-eigenstate $\quad |K_L\rangle \simeq \frac{1}{\sqrt{1+|\epsilon_K|^2}}\left(|K^0_-\rangle + \epsilon_K|K^0_+\rangle\right)$

- The CP violation was measured by

$$\mathcal{A}(\,K_L(\text{almost } CP \text{ odd}) \to \pi\pi(CP \text{ even})\,) \propto \varepsilon_K = \mathcal{O}(10^{-3}) \neq 0$$

[Christenson, Cronin, Fitch, Turlay, 64′ with Nobel prize]

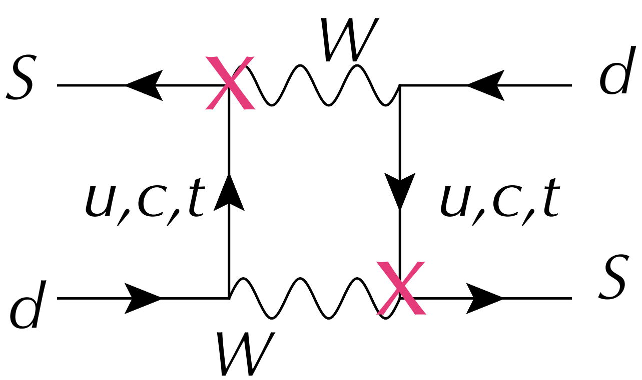
Kaon & CP violation:2

- Precise measurement for Kaon decay discovered the second type of CP violation: Indirect (mixing) (ε_K) & Direct CP violation (ε'_K)

$$\mathcal{A}\left(K_L \to \pi^+\pi^-\right) \propto \varepsilon_K + \varepsilon'_K \qquad \text{with } \varepsilon_K = \mathcal{O}(10^{-3})$$

$$\mathcal{A}\left(K_L \to \pi^0\pi^0\right) \propto \varepsilon_K - 2\varepsilon'_K \qquad \varepsilon'_K = \mathcal{O}(10^{-6})$$

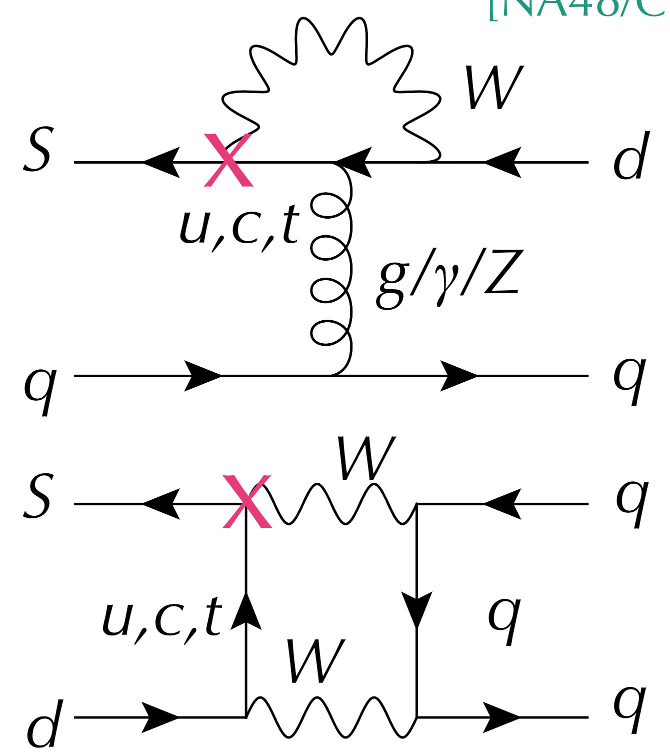
[NA48/CERN and KTeV/FNAL 99']



ΔS=2
Indirect CP violation
Kaon oscillation
W box

$\varepsilon_K \propto \mathrm{Im}[(\mathrm{CKM})^2]$

$$K^0 \longleftrightarrow \overline{K}^0$$



ΔS=1
Direct CP violation
W-box and penguin

$\varepsilon'_K \propto \mathrm{Im}[\mathrm{CKM}]$

- CP violation measures in K→ππ system are only ε_K & ε'_K, which have been measured by experiments very precisely. **Therefore they should be good crosscheck of the CKM phase in the Standard Model**

Kaon & CP violation:3

$$\frac{\epsilon'_K}{\epsilon_K} \simeq \frac{1}{\sqrt{2}|\epsilon_K|}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$
$$= \frac{1}{\sqrt{2}|\epsilon_K|}\frac{\mathrm{Re}A_2}{(\mathrm{Re}A_0)^2}\left(-\mathrm{Im}A_0 + \frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\mathrm{Im}A_2\right)$$

Isospin amplitude

$$\mathcal{A}(K^0 \to (\pi\pi)_I) \equiv \mathcal{A}_I e^{i\delta_I}$$
$$\mathcal{A}(\overline{K}^0 \to (\pi\pi)_I) \equiv \bar{A}_I e^{i\delta_I} = A_I^* e^{i\delta_I}$$

$$\epsilon_K \equiv \frac{2\eta_{+-} + \eta_{00}}{3} \quad \in \mathbb{C}$$

$$\epsilon'_K \equiv \frac{\eta_{+-} - \eta_{00}}{3} \quad \in \mathbb{C}$$

- General remarks
  - This formula is modified by $m_u \neq m_d$ [Cirigliano,Pich,Ecker,Neufeld,PRL 03']
  - Theoretical value of ϵ'_K/ϵ_K is real number
  - $|\epsilon_K|$, $\mathrm{Re}A_0$, and $\mathrm{Re}A_2$ have been measured by experiments very precisely
  - Theorist calculates $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$ for ϵ'_K/ϵ_K
  - Experiments can precisely probe ϵ'_K/ϵ_K by the following combination

$$\mathrm{Re}\left[\frac{\epsilon'_K}{\epsilon_K}\right] \simeq \frac{1}{6}\frac{|\eta_{+-}|^2 - |\eta_{00}|^2}{|\eta_{+-}|^2} = \frac{1}{6}\left(1 - \frac{\frac{\mathrm{Br}(K_L\to\pi^0\pi^0)}{\mathrm{Br}(K_S\to\pi^0\pi^0)}}{\frac{\mathrm{Br}(K_L\to\pi^+\pi^-)}{\mathrm{Br}(K_S\to\pi^+\pi^-)}}\right)$$

$$\eta_{00} \equiv \frac{\mathcal{A}(K_L \to \pi^0\pi^0)}{\mathcal{A}(K_S \to \pi^0\pi^0)}$$
$$\eta_{+-} \equiv \frac{\mathcal{A}(K_L \to \pi^+\pi^-)}{\mathcal{A}(K_S \to \pi^+\pi^-)}$$

Kaon & CP violation:4

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\rm exp}} \frac{\omega_{\rm exp}}{({\rm Re}A_0)_{\rm exp}} \left(-{\rm Im}A_0 + \frac{1}{\omega_{\rm exp}} {\rm Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{{\rm Re}A_0}{{\rm Re}A_2} = 22.46 \ (\text{exp.})$$

- Numerical Remarks
  - *ImA0 (I=0, ΔI=1/2)* term is dominated by gluon-penguin, while *ImA2 (I=2, ΔI=3/2)* term is dominated by EW-penguins ($\propto m_t^2$), and **they have opposite sign contributions**
  - Since *ImA2* is proportional to α but enhanced by *1/ω*, its contribution is comparable to *ImA0*

$$\mathcal{O}(\alpha_s) \overset{!}{\sim} \frac{1}{\omega} \mathcal{O}(\alpha)$$

  - Two terms contribute destructively each other. Actually, ϵ'_K/ϵ_K is canceled out at $m_t \sim 220$ GeV [Paschos,Wu,91′: LO result]
  - The LO QCD contribution does not contribute to *ImA2*. Thus NLO QED corrections are *leading order* to *ImA2* term

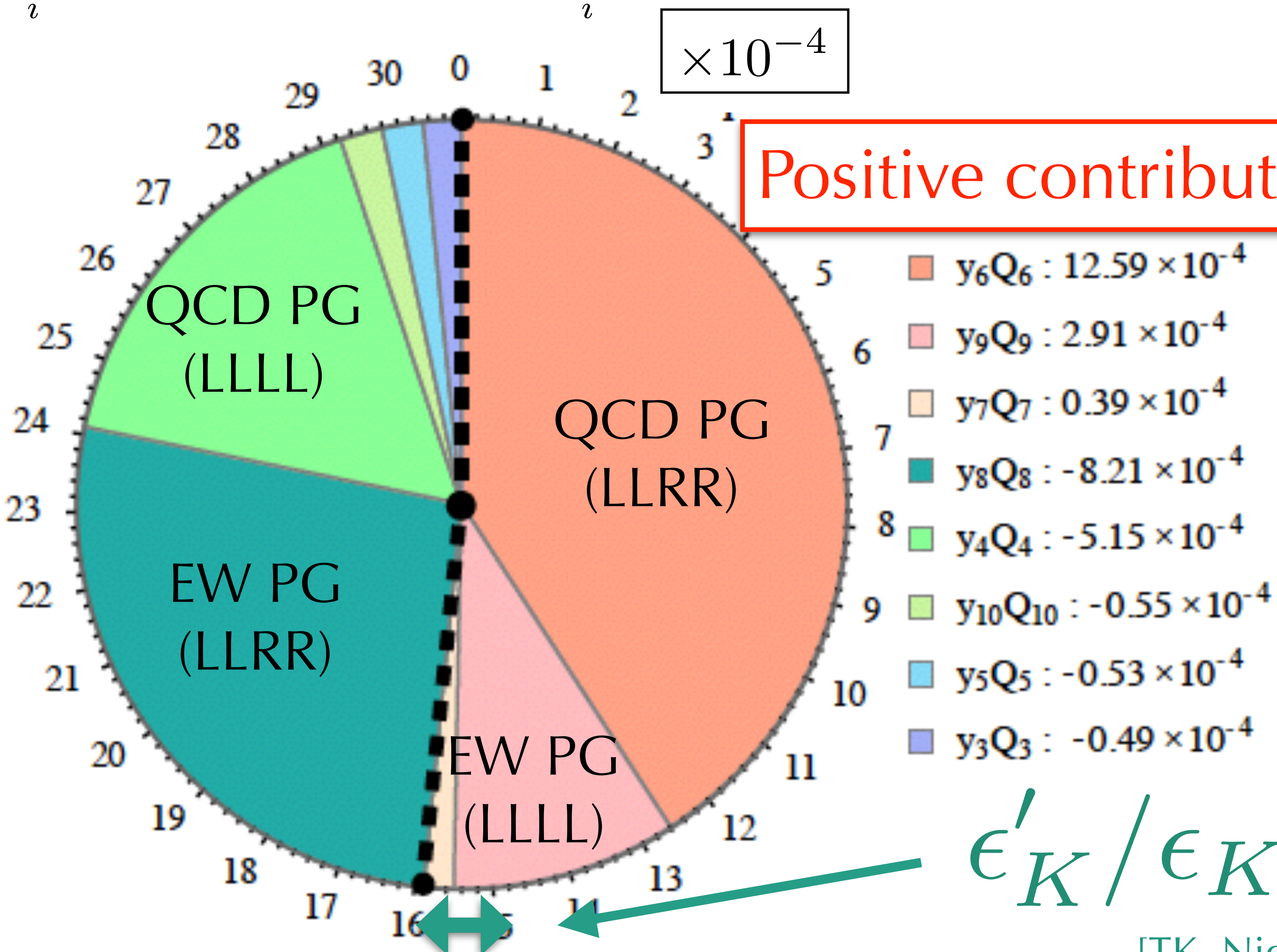
Kaon & CP violation:5

- The Isospin amplitude can be decomposed into Wilson coefficients (C_i) and hadronic matrix elements ($<Q_i>$)

$$A_{I=0,2} = \langle (\pi\pi)_{I=0,2} | \mathcal{H}_{\rm eff}^{|\Delta S|=1} | K^0 \rangle$$
$$= \sum_i C_i \langle (\pi\pi)_{I=0,2} | Q_i | K^0 \rangle \equiv \sum_i C_i \langle Q_i \rangle_{I=0,2}$$

Q_i are four-fermi operators

Composition of ϵ'_K/ϵ_K with respect to the operator basis



[TK, Nierste, Tremper 16']

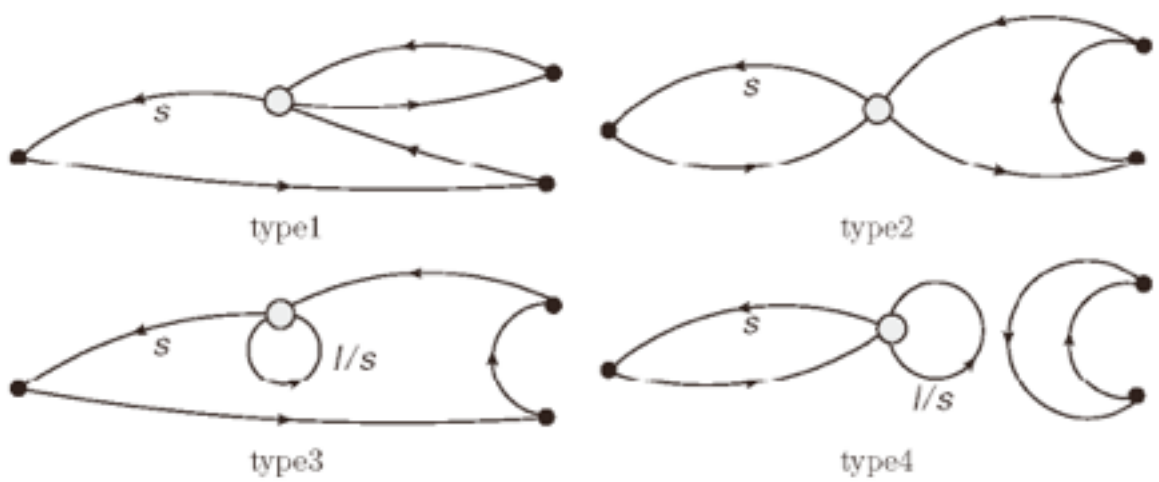
The first lattice result for <Qi>

- The calculation of the hadronic matrix elements (<Q_i>), being non-perturbative quantities, is a major challenge, and have been estimated by the effective theories (e.g. chiPT, dual QCD model, NJL model, …)
- But their results have a tension among each other (next slide)
- Recently, a determination of all hadronic matrix elements by lattice QCD is obtained **with controlled errors** (**first lattice result)**

[RBC-UKQCD, PRL115 (2015)]

initial state

$\mathcal{A}_I = \quad K^0$



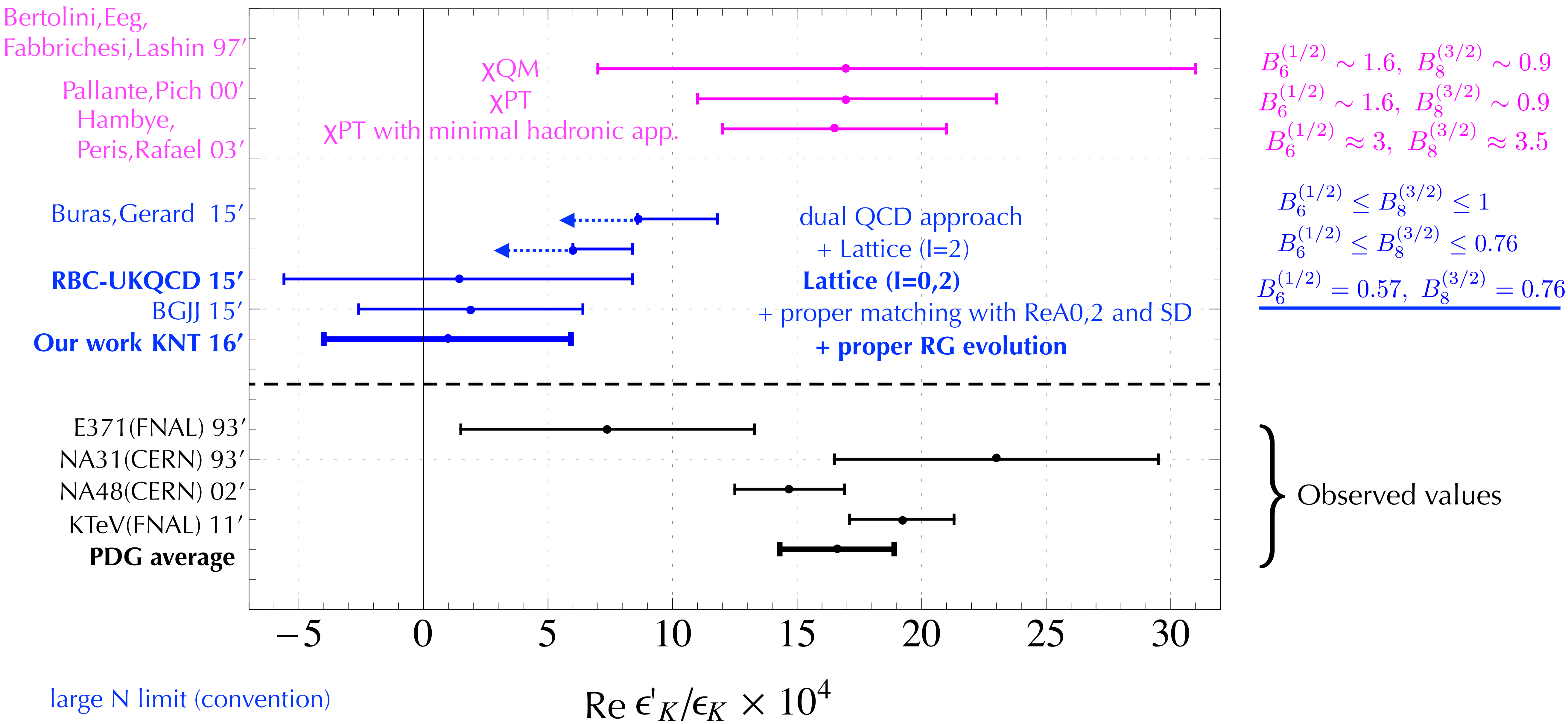
final state

$(\pi\pi)_{I=0,2}$

[Figure in RBC-UKQCD, PRL115 (2015)]

- Now, one can estimate ϵ'_K/ϵ_K without using the effective theories

Current situation of ε'_K $\propto \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} \propto \mathrm{Im}A_0 - \left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)\mathrm{Im}A_2$



large N limit (convention)

$$B_6^{(1/2)} = B_8^{(3/2)} = 1$$

dual QCD prediction

$$B_6^{(1/2)} \leq B_8^{(3/2)} < 1,\ B_8^{(3/2)} = 0.8$$

	Exp.	χPT	dual QCD	**Lattice (I=0,2)**
$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)$	22.45 ± 0.05	~ 14	16.0 ± 1.5	31.0 ± 6.6

Singularity

$$\hat{J}_s - \left[\hat{J}_s, \frac{\hat{\gamma}_s^{(0)T}}{2\beta_0}\right] = \frac{\beta_1}{\beta_0}\frac{\hat{\gamma}_s^{(0)T}}{2\beta_0} - \frac{\hat{\gamma}_s^{(1)T}}{2\beta_0},$$

- Go on the diagonalized basis of $\gamma s^{(0)T}$, the equation becomes

$$\left(\hat{V}^{-1}\hat{J}_{s,e}\hat{V}\right)_{ij} = \frac{\cdots}{2\beta_0 \mp \left((\hat{\gamma}_{s,D}^{(0)T})_{jj} - (\hat{\gamma}_{s,D}^{(0)T})_{ii}\right)}.$$

- Unfortunately, when f=3, $2\beta_0 = 18,\ \hat{\gamma}_{s,D}^{(0)T} \supset +2, -16$, then the denominator vanishes with a generally non-zero numerator -> Singularity
- The other *J* matrices also have similar singularity when f= 3,4,5,6

$$\hat{U}_f(\mu_1,\mu_2) = \hat{K}(\mu_1)\hat{U}_0(\mu_1,\mu_2)\hat{K}'(\mu_2),$$

with

$$\hat{K}(\mu_1) = \left(\hat{1} + \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}\right)\left(\hat{1} + \frac{\alpha_s(\mu_1)}{4\pi}\hat{J}_s\right)\left(\hat{1} + \frac{\alpha_{EM}}{\alpha_s(\mu_1)}\hat{J}_e\right),$$

$$\hat{K}'(\mu_2) = \left(\hat{1} - \frac{\alpha_{EM}}{\alpha_s(\mu_2)}\hat{J}_e\right)\left(\hat{1} - \frac{\alpha_s(\mu_2)}{4\pi}\hat{J}_s\right)\left(\hat{1} - \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}\right),$$

Singularities

Removing the Singularities:1

- In order to eliminate the singularities, we generalize the Roma group's ansatz by adding a logarithmic scale dependence to the *J* matrices

Our singularity-free analytic solution

$$\hat{U}_f(\mu_1,\mu_2) = \hat{K}(\mu_1)\hat{U}_0(\mu_1,\mu_2)\hat{K}'(\mu_2),$$

with

$$\hat{K}(\mu_1) = \left(\hat{1} + \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}(\alpha_s(\mu_1))\right)\left(\hat{1} + \frac{\alpha_s(\mu_1)}{4\pi}\hat{J}_s(\alpha_s(\mu_1))\right) \times \left(\hat{1} + \frac{\alpha_{EM}}{\alpha_s(\mu_1)}\hat{J}_e(\alpha_s(\mu_1)) + \left(\frac{\alpha_{EM}}{\alpha_s(\mu_1)}\right)^2 \hat{J}_{ee}(\alpha_s(\mu_1))\right),$$

$$\hat{K}'(\mu_2) = \left(\hat{1} - \frac{\alpha_{EM}}{\alpha_s(\mu_2)}\hat{J}_e(\alpha_s(\mu_2)) - \left(\frac{\alpha_{EM}}{\alpha_s(\mu_2)}\right)^2 \left(\hat{J}_{ee}(\alpha_s(\mu_2)) - \left(\hat{J}_e(\alpha_s(\mu_2))\right)^2\right)\right) \times \left(\hat{1} - \frac{\alpha_s(\mu_2)}{4\pi}\hat{J}_s(\alpha_s(\mu_2))\right)\left(\hat{1} - \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}(\alpha_s(\mu_2))\right),$$

where

$$\hat{J}_s \to \hat{J}_s(\alpha_s(\mu)) = \hat{J}_{s,0} + \hat{J}_{s,1}\ln\alpha_s(\mu),$$

$$\hat{J}_e \to \hat{J}_e(\alpha_s(\mu)) = \hat{J}_{e,0} + \hat{J}_{e,1}\ln\alpha_s(\mu),$$

$$\hat{J}_{se} \to \hat{J}_{se}(\alpha_s(\mu)) = \hat{J}_{se,0} + \hat{J}_{se,1}\ln\alpha_s(\mu) + \hat{J}_{se,2}\ln^2\alpha_s(\mu).$$

$$\hat{J}_{ee}(\alpha_s(\mu)) = \hat{J}_{ee,0} + \hat{J}_{ee,1}\ln\alpha_s(\mu).$$

[TK, Nierste, Tremper 16′]

Removing the Singularities:2

- Then, J_s matrices are the solution of the following equations

$$\hat{J}_{s,0} - \left[\hat{J}_{s,0}, \frac{\hat{\gamma}_s^{(0)T}}{2\beta_0}\right] = \frac{\beta_1}{\beta_0}\frac{\hat{\gamma}_s^{(0)T}}{2\beta_0} - \frac{\hat{\gamma}_s^{(1)T}}{2\beta_0} - \hat{J}_{s,1}$$

$$\hat{J}_{s,1} - \left[\hat{J}_{s,1}, \frac{\hat{\gamma}_s^{(0)T}}{2\beta_0}\right] = 0,$$

- Overview of our solution
  - All singularity terms are regulated into **logarithmic terms**
  - Some logarithmic terms are consistent with literature
  - Our solution does not rely on a specific basis and permits a much faster, easier and, in particular, more stable computational algorithm
  - Our next-to-leading order RG evolution matrix has an additional **new** correction of $O(\alpha^2/\alpha_s^2)$, which appears only at this order

numerically $\alpha^2/\alpha_s^2 \sim \alpha$, but enhanced by $1/\omega \sim 22$

Summary of Introduction

- In the SM, ϵ'_K/ϵ_K is significantly suppressed by the GIM suppression AND by the accidental cancellation between QCD and EW penguin contributions

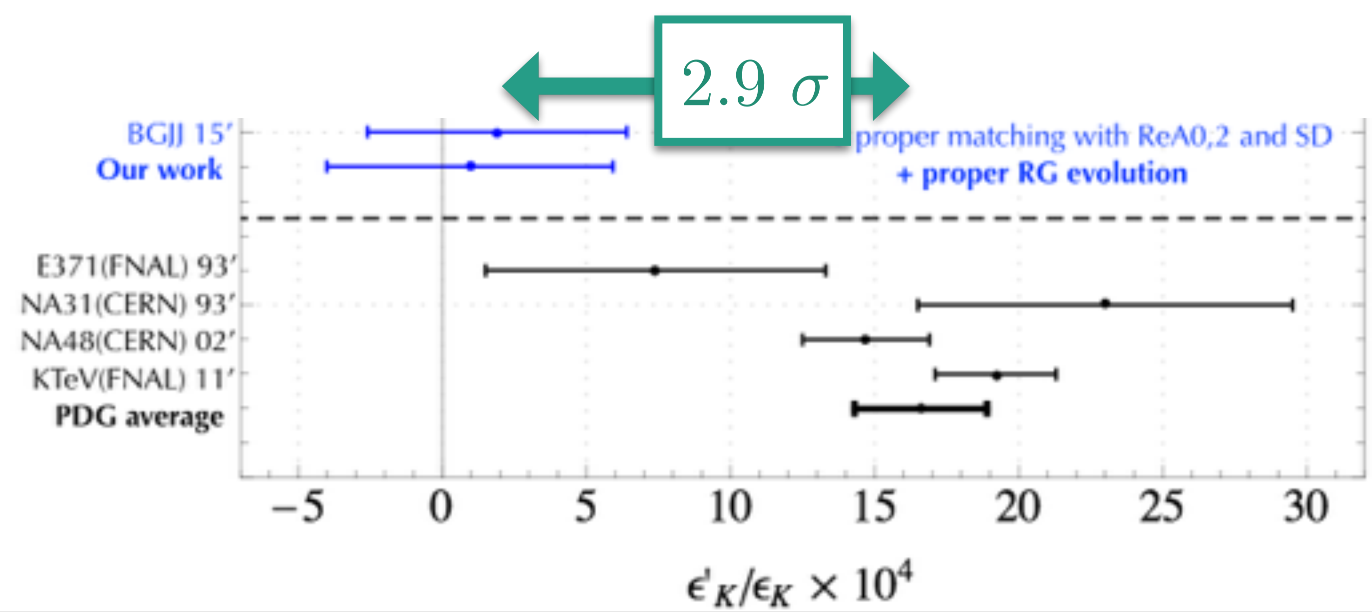
- *SM expectation value at NLO (without effective theory)* [TK, Nierste, Tremper 16']

$$\left(\frac{\epsilon'_K}{\epsilon_K}\right)_{\text{SM-NLO}} = (0.96 \pm 4.68 \pm 1.52 \pm 0.60 \pm 0.24) \times 10^{-4}$$

Lattice NNLO isospin violating mt

- We have calculated ϵ'_K/ϵ_K in the Standard Model at the next-to-leading order. The result is **2.9 sigma** below the experimental measured value. It highlights a tension between the Standard-Model prediction and experiment.

$$\mathrm{Re}\left(\frac{\epsilon'_K}{\epsilon_K}\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}.$$



シン・アノマリー(?)

KAON	GODZILLA
1947:発見	1954:ゴジラ
1964:IDCPV発見	1964:モスラ
1999:DCPV発見	2001:大怪獣総攻撃
2015:アノマリー発見?	2016:シン・ゴジラ

We found a solution in the Minimal Supersymmetric Standard Model

Preliminary for NP part

- The SM prediction of ϵ'_K/ϵ_K is 2.9 sigma below the experimental values, which give strong motivation for searching for NP contributions
- ϵ'_K/ϵ_K is highly sensitive to CP violation of NP

SM	loop suppression *GIM suppression* accidental cancelation
	vs.
NP	**(loop suppression) *(large coupling) * NP scale suppression**

- One should also consider the other flavour constraints
- Actually, some models can explain this discrepancy, e.g. Littlest Higgs model, 331 model, generic Z′ models, 750GeV model (dead?), and SUSY

[Buras,Fazio,Girrbach 14′, Buras,Buttazzo,Knegjens 15, Buras 15′, Buras,Fazio 15′, 16′, Goertz,Kamenik,Katz,Nardecchia 15′, Blanke,Buras,Recksiegel 16′,TK,Nierste,Tremper 16′, Tanimoto, Yamamoto 16′,Endo,Mishima,Ueda,Yamamoto 16′]

Our calculation strategy for MSSM

- Our work
  - CP violating phase in the MSSM
    - CKM matrix
    - squark mass matrix
    - μ (Higgsino mass)
    - gaugino mass
    - A term

→ (CKM matrix, squark mass matrix) **Included**

→ (μ, gaugino mass, A term) **take to be Real**
in light of severe constraint from EDM experiments

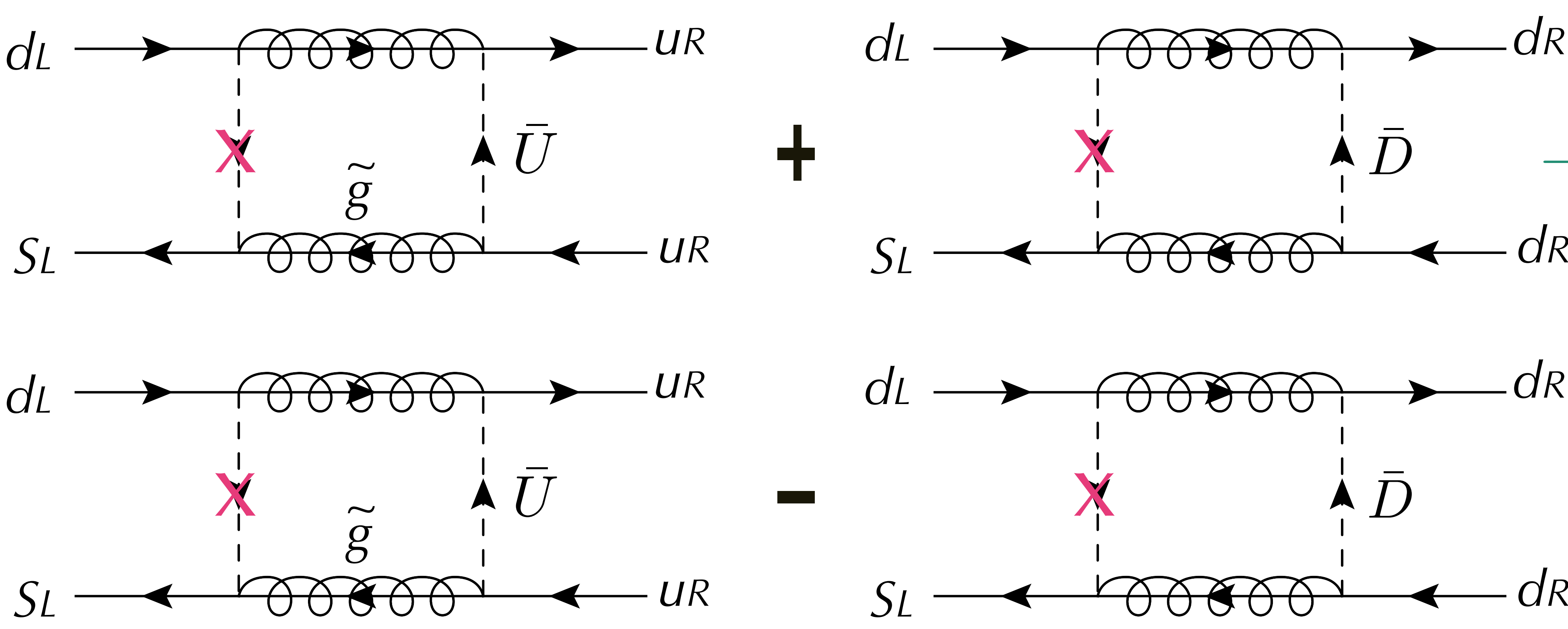
- We calculate SUSY QCD (gluino) corrections and chargino/neutralino-Z penguin contribution in light of strong coupling and Isospin symmetry breaking
- TeV scale SUSY & SUSY scale matching, mass eigenbasis calc., NLO-QCD and QED RGE corrections

Gluino box ("Trojan penguin")

[Kagan, Neubert, PRL83(1999),
Grossman, Kagan, Neubert, JHEP10(1999)]

- In spite of QCD correction, gluino box diagram **can** break isospin symmetry through mass difference between right-handed squark masses

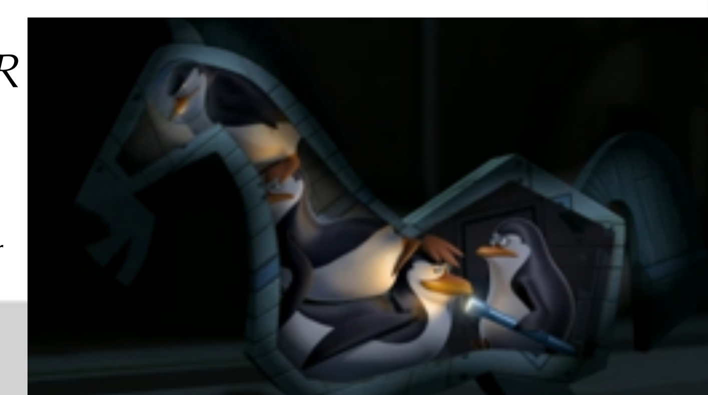
- *"It is neither (pure) penguins nor of electroweak origin. Nevertheless, at low energies their effects are parameterized by an extension of the usual basis of electroweak penguin operators."*



$\rightarrow$ QCD penguin(Q_6)

$\rightarrow$ EW penguin(Q_8)

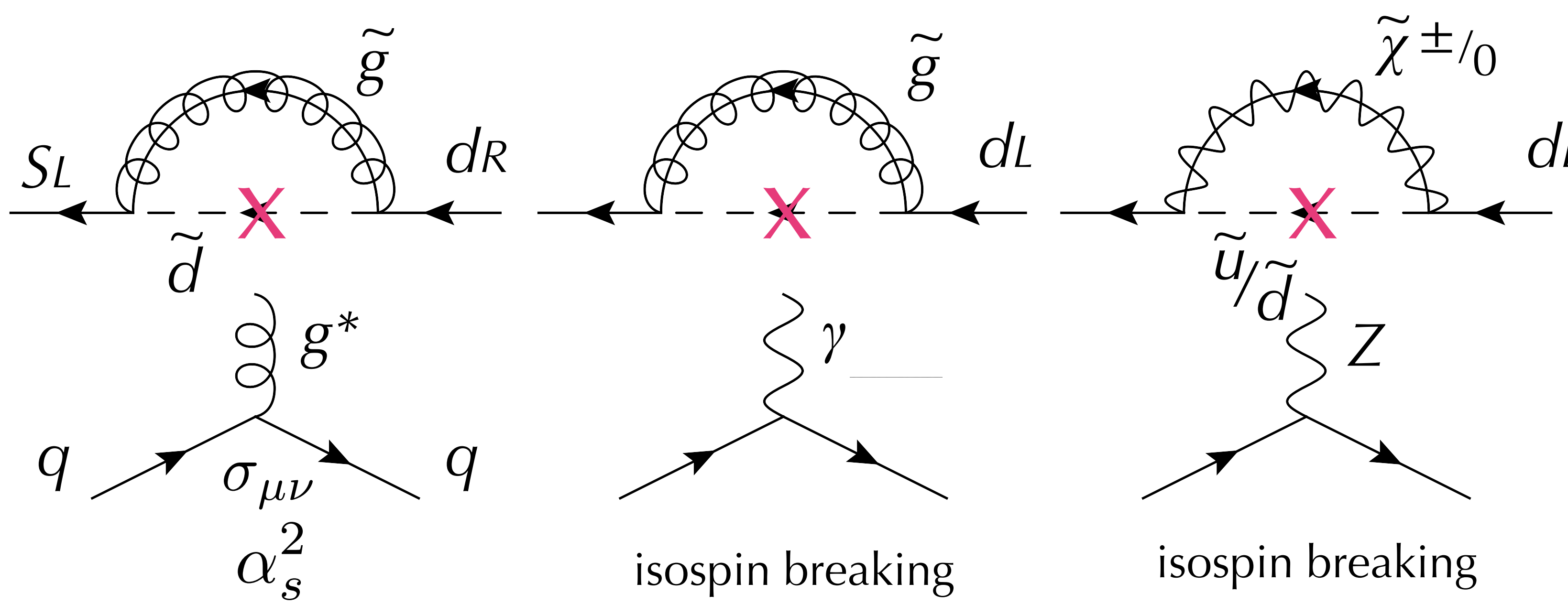
$m_{\bar{U}} \neq m_{\bar{D}} \rightarrow$ contribute to ImA_2



Movie:Penguins of Madagascar

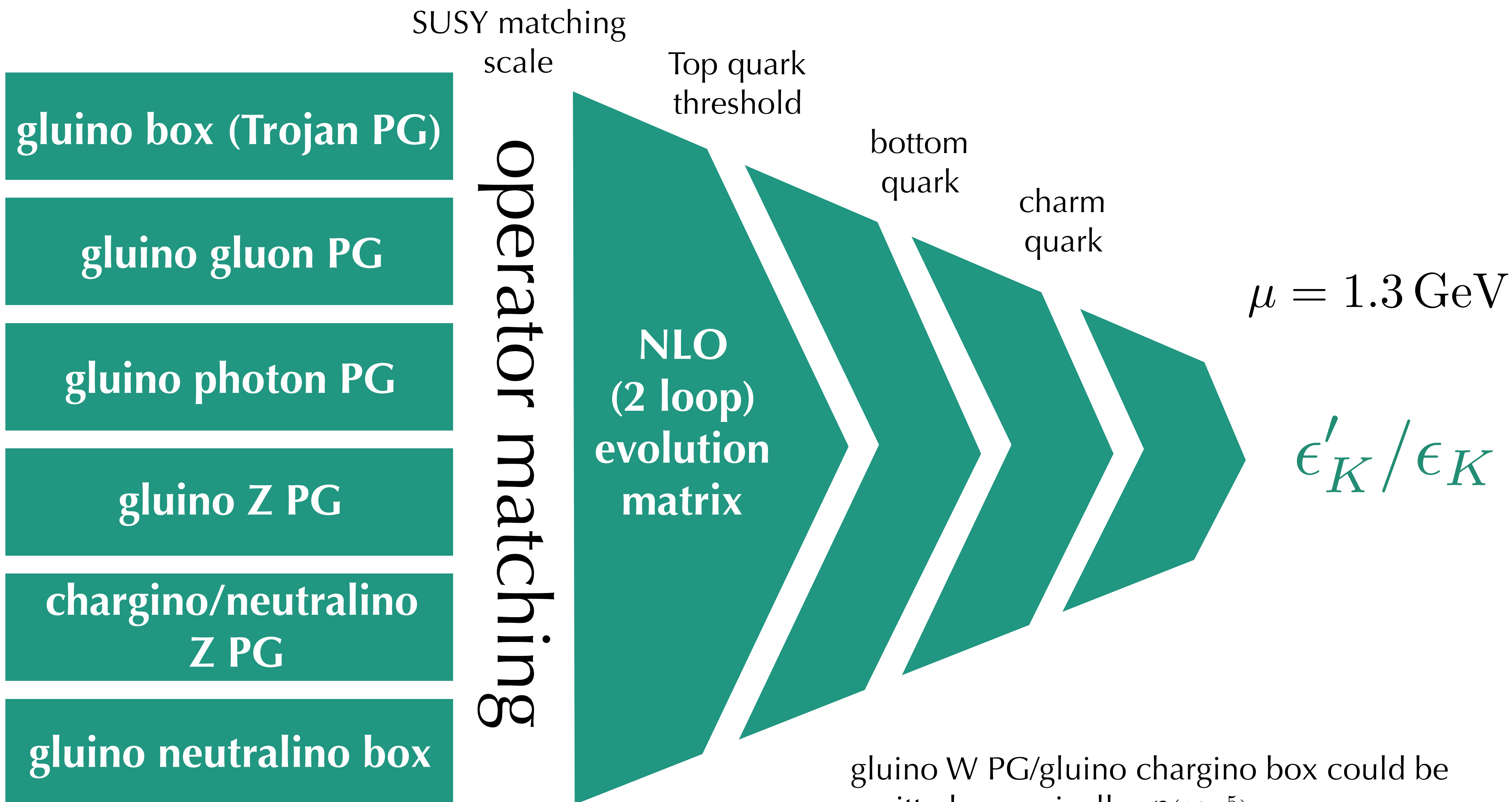
Sub leading contributions

- Gluino chromomagnetic penguin operator can give subleading contribution, but there is no reliable results for hadronic matrix element
  [Buras,Colangero,Ishidori,Romanino,Silvestrini,00′]
- Gluino photon-penguin breaks isospin sym. explicitly, but is suppressed by $\alpha/\alpha s$
  [Langacker,Sathiapalan,84',Grossman,Worah,97',Abel,Cottingham,Whittingham,98']
- Z-penguin contribution needs to break the EW sym. like $\mathcal{L}_{\rm eff} = \frac{\lambda_{ij}}{M^2}|H|^2\bar{d}_i \not{D} d_j$, Hence, chargino Z-penguin contribution is always larger than gluino Z-penguin
  [Colangelo,Isidori,98'@$K\to\pi\nu\nu$]



Overview for calculation of SUSY ε'_K

- We calculated the following six-type one-loop SUSY contributions
- SUSY matching scale is given as the input parameter



gluino W PG/gluino chargino box could be omitted numerically: $\mathcal{O}(10^{-5})$

Main Constraint: ε$_K$ (Δ*S*=2, ID-CPV)

- Although ε′$_K$ (Δ*S*=*1*, D-CPV) is sensitive to NP, once ε$_K$ (Δ*S*=*2*, ID-CPV) constraint is taken into account, NP effects in Δ*S*=*1* is highly suppressed
- NP hierarchy in |ΔS| = 1 vs. |ΔS| = 2 transitions;

$$\epsilon_K^{\rm SM} \propto \frac{{\rm Im}(\tau^2)}{M_W^2} \qquad \epsilon_K^{\prime\,\rm SM} \propto \frac{{\rm Im}\tau}{M_W^2} \qquad \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - i0.6)\cdot 10^{-3}$$

If the NP contribution comes with the ΔS = 1 parameter δ and is mediated by heavy particles of mass M, one finds

$$\epsilon_K^{\rm NP} \propto \frac{{\rm Im}(\delta^2)}{M^2} \qquad \epsilon_K^{\prime\,\rm NP} \propto \frac{{\rm Im}\delta}{M^2}$$

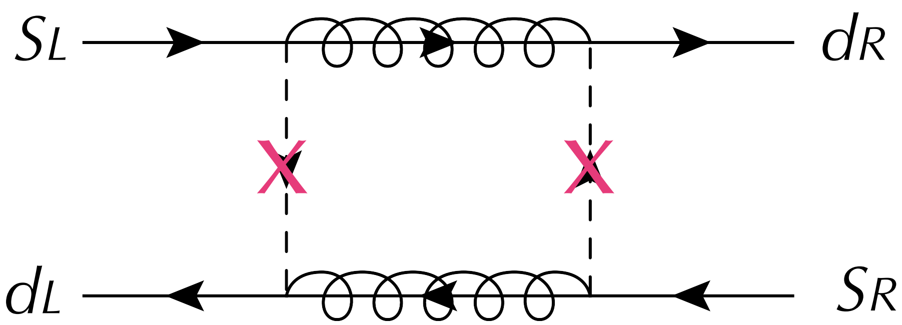
$$\frac{\epsilon_K^{\prime\,\rm NP}}{\epsilon_K^{\prime\,\rm SM}} \leq \frac{\frac{\epsilon_K^{\prime\,\rm NP}}{\epsilon_K^{\rm NP}}}{\frac{\epsilon_K^{\prime\,\rm SM}}{\epsilon_K^{\rm SM}}} = \mathcal{O}\left(\frac{{\rm Re}\tau}{{\rm Re}\delta}\right)$$

↑ $\epsilon_K^{\rm NP} \leq \epsilon_K^{\rm SM}$

With M > 1 TeV, NP effects can only be relevant for |δ| >> |τ| and this equation seemingly forbids detectable NP contributions to ε′K

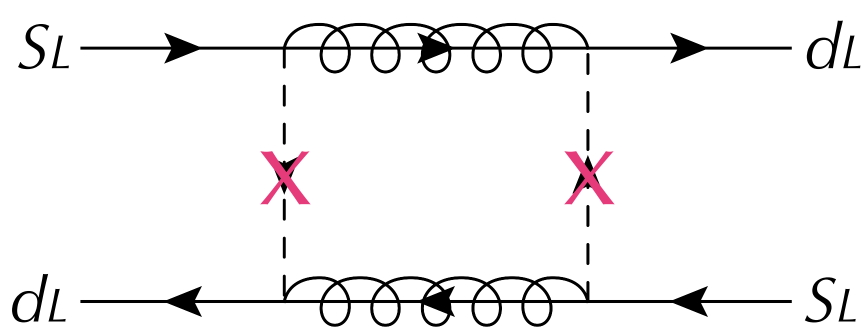
Loophole of constraint from ε_K

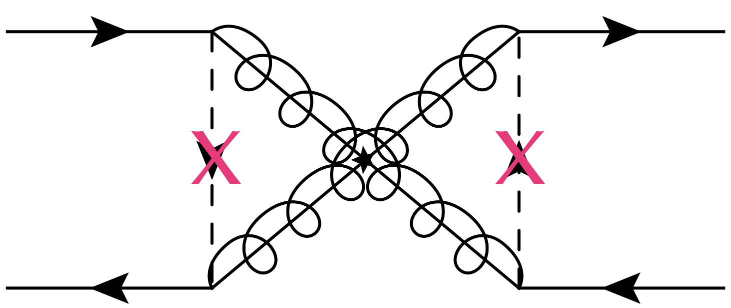
- The leading contribution is given by $\overline{d_L} s_L \overline{d_R} s_R$



$\propto \left(\frac{m_K}{m_s + m_d} \right)^2$ this contribution is suppressed when $\Delta_{\bar{D},12} \simeq 0$

- The next contribution is given by $\overline{d_L} s_L \overline{d_L} s_L$



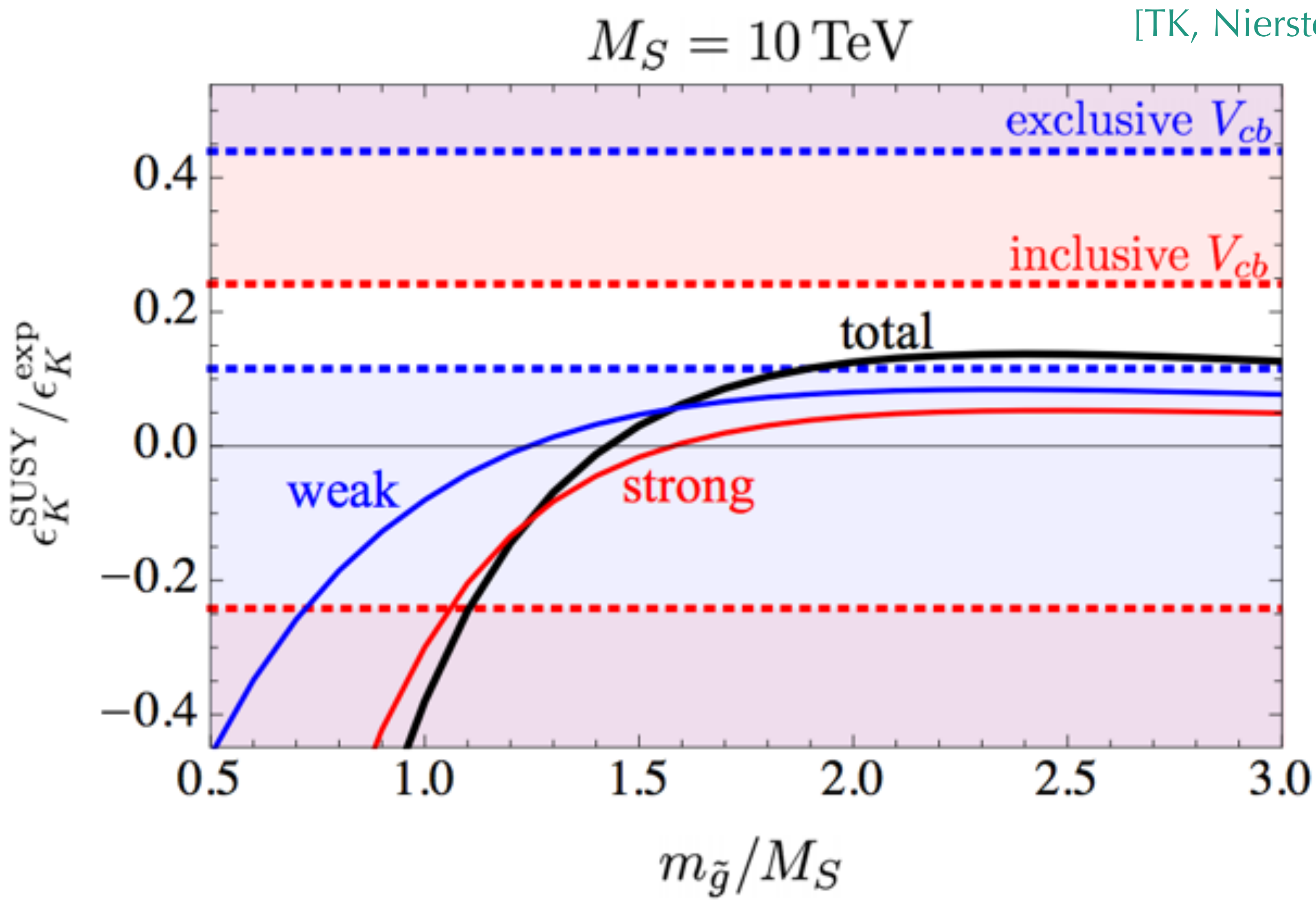


Crossed diagram gives relatively negative contributions

- $m_{\tilde{g}} \gtrsim 1.5\ m_{\tilde{q}}$, these contributions almost cancel out [Crivellin, Davidkov, PRD81(2010)]

Constraint from ε_K

[TK, Nierste, Tremper, PRL(2016)]



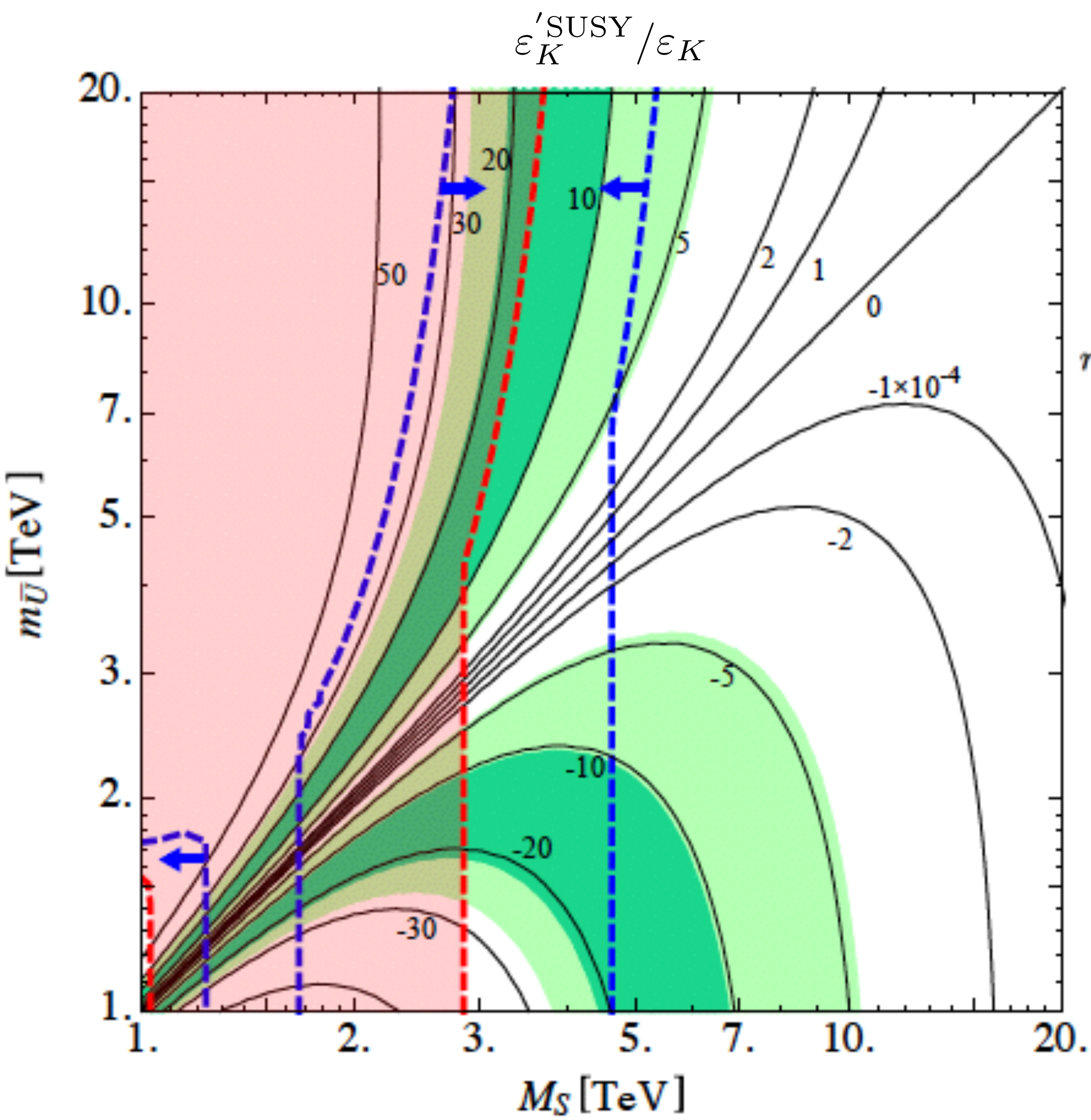
- Actually, there are several expected values of ε_K depending on the input CKM parameters

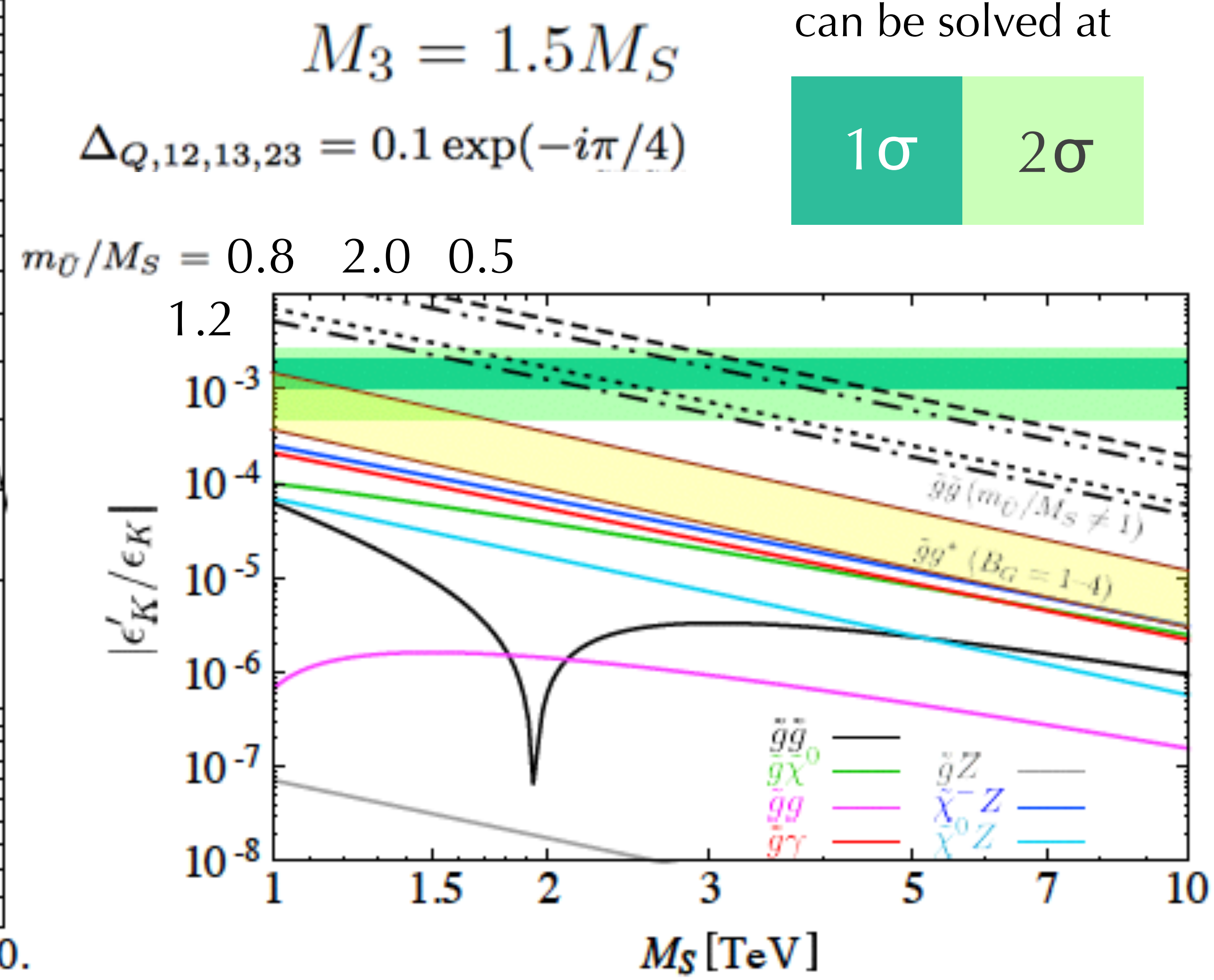
|Vcb|; measured in inclusive b → clν decays….. ε_K is consistent with exp. value

|Vcb|; measured in exclusive B → D(*)lν decays….. ε_K is 3σ below the exp. value

SUSY contributions to ε′K

- We take universal SUSY mass spectrum without gauginos and right-handed up-type squark mass





ε′K discrepancy can be solved at

1σ	2σ

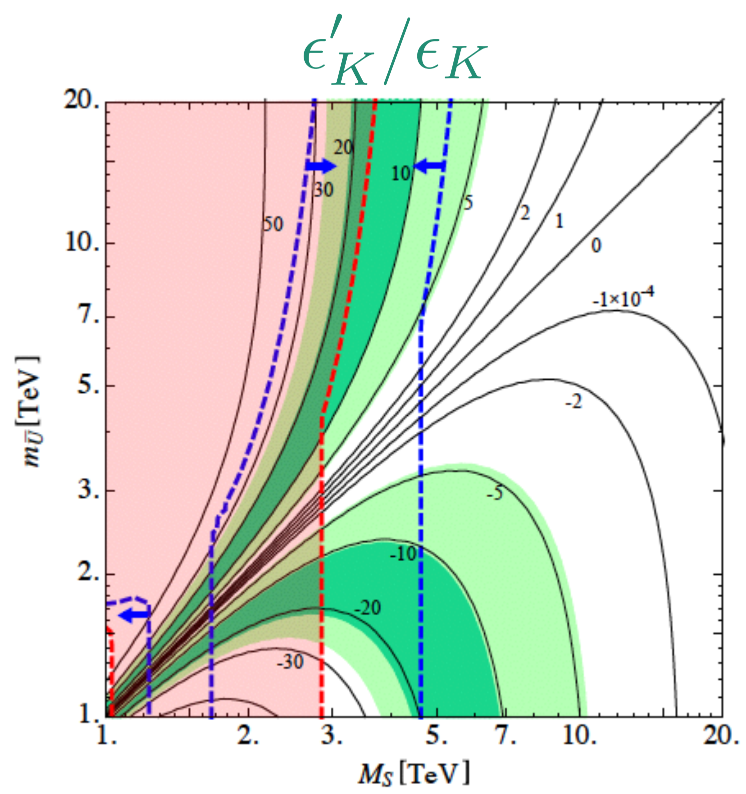
[TK, Nierste, Tremper, PRL(2016)]

- ☑ nEDM, ΔMK, DDbar mixing are weaker constraints than εK

Conclusions

- ϵ'_K/ϵ_K is a good measure of the CP violation from new physics
- The lattice group and our calculation have revealed that the SM expected value deviates significantly from exp. data (~3σ)
- In the MSSM, gluino box diagram with mass different of the right-handed squark contributes ϵ'_K/ϵ_K significantly
- Heavy gluino can relax the constraint from ε_K
- Prospects
  - Correlation with other hadronic channels
  - Higher order corrections:e.g. 2-loop gluino box
  - UV model, GUT?
  - Large A scenario, vacuum stability

TK, Nierste, Tremper, Endo, Mishima, Yamamoto(K) STAY TUNED




Backup
made by
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Kaon & CP violation:3

- Precise definitions of K→ππ system [PDG]

$$\eta_{00} \equiv \frac{\mathcal{A}(K_L \to \pi^0\pi^0)}{\mathcal{A}(K_S \to \pi^0\pi^0)} \overset{exp.}{=} \left(2.220 \cdot 10^{-3}\right) \cdot e^{43.52^\circ i} \qquad \epsilon_K \equiv \frac{2\eta_{+-} + \eta_{00}}{3} \quad \in \mathbb{C}$$

$$\eta_{+-} \equiv \frac{\mathcal{A}(K_L \to \pi^+\pi^-)}{\mathcal{A}(K_S \to \pi^+\pi^-)} \overset{exp.}{=} \left(2.232 \cdot 10^{-3}\right) \cdot e^{43.51^\circ i} \qquad \epsilon'_K \equiv \frac{\eta_{+-} - \eta_{00}}{3} \quad \in \mathbb{C}$$

- Pion isospin decomposition of the physical states

$$|\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}}|\pi\pi\rangle_{I=0} - \sqrt{\frac{2}{3}}|\pi\pi\rangle_{I=2}$$

$$|\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}|\pi\pi\rangle_{I=0} + \sqrt{\frac{1}{3}}|\pi\pi\rangle_{I=2}$$

Two pions (*I*=1) can decompose into *I*=0,2 states with CG coefficients

$$\epsilon_0 \equiv \frac{\mathcal{A}(K_L \to (\pi\pi)_0)}{\mathcal{A}(K_S \to (\pi\pi)_0)} \qquad \epsilon_2 \equiv \frac{1}{\sqrt{2}}\frac{\mathcal{A}(K_L \to (\pi\pi)_2)}{\mathcal{A}(K_S \to (\pi\pi)_0)} \ll \epsilon_0 \qquad \omega \equiv \frac{\mathcal{A}(K_S \to (\pi\pi)_2)}{\mathcal{A}(K_S \to (\pi\pi)_0)} \ll \epsilon_0$$

then $\epsilon_K = \epsilon_0 - \sqrt{2}\epsilon_2\omega + \mathcal{O}(\epsilon_0\omega^2)$ $\qquad \epsilon'_K = \epsilon_2 + \frac{\omega}{\sqrt{2}}(\epsilon_2 - \epsilon_0) + \mathcal{O}(\epsilon_0\omega^2)$

Kaon & CP violation:4

- Then, $$\frac{\epsilon'_K}{\epsilon_K} = \left(\epsilon_2 + \frac{\omega}{\sqrt{2}}(\epsilon_2 - \epsilon_0)\right)\left(\epsilon_0 - \sqrt{2}\epsilon_2\omega\right)^{-1} + \mathcal{O}(\omega^2)$$ $$= \frac{1}{\sqrt{2}}\left[\frac{\mathcal{A}(K_L \to (\pi\pi)_2)}{\mathcal{A}(K_L \to (\pi\pi)_0)} - \frac{\mathcal{A}(K_S \to (\pi\pi)_2)}{\mathcal{A}(K_S \to (\pi\pi)_0)}\right] + \mathcal{O}(\omega^2)$$

- K_L and K_S also can be decomposed into isospin eigenstates ($K^0,\ \overline{K}^0$)

$$|K_S\rangle \equiv \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\delta_\epsilon|^2}}\left((1+\delta_\epsilon)|K^0\rangle + (1-\delta_\epsilon)|\overline{K}^0\rangle\right)$$
$$|K_L\rangle \equiv \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\delta_\epsilon|^2}}\left((1+\delta_\epsilon)|K^0\rangle - (1-\delta_\epsilon)|\overline{K}^0\rangle\right)$$

- Let us define *isospin amplitudes*

$$\mathcal{A}(K^0 \to (\pi\pi)_I) \equiv \mathcal{A}_I e^{i\delta_I}$$
$$\mathcal{A}(\overline{K}^0 \to (\pi\pi)_I) \equiv \bar{A}_I e^{i\delta_I} = A_I^* e^{i\delta_I}$$

δ_I is a strong phase, which comes from the final pion state re-scattering

then $$\frac{\mathcal{A}(K_L \to (\pi\pi)_2)}{\mathcal{A}(K_L \to (\pi\pi)_0)} = e^{i(\delta_2-\delta_0)}\frac{i\mathrm{Im}(A_2) + \delta_\epsilon \mathrm{Re}(A_2)}{i\mathrm{Im}(A_0) + \delta_\epsilon \mathrm{Re}(A_0)}$$

$$\frac{\mathcal{A}(K_S \to (\pi\pi)_2)}{\mathcal{A}(K_S \to (\pi\pi)_0)} = e^{i(\delta_2-\delta_0)}\frac{\mathrm{Re}(A_2) + i\delta_\epsilon \mathrm{Im}(A_2)}{\mathrm{Re}(A_0) + i\delta_\epsilon \mathrm{Im}(A_0)}$$

Kaon & CP violation:5

- Using $\epsilon_K = |\epsilon_K| e^{i\phi_\epsilon} = \epsilon_0 - \sqrt{2}\epsilon_2 \omega + \mathcal{O}(\epsilon_0 \omega^2) \simeq \frac{i\mathrm{Im}(A_0) + \delta_\epsilon \mathrm{Re}(A_0)}{\mathrm{Re}(A_0) + i\delta_\epsilon \mathrm{Im}(A_0)}$

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{i}{\sqrt{2}|\epsilon_K|} e^{i(\delta_2 - \delta_0 - \phi_\epsilon)} \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \left(\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right) + \mathcal{O}((\delta_\epsilon, \omega) \cdot \text{1st term})$$

- Using the fact that the total phase is excellently real

$$\begin{aligned} ie^{i(\delta_2 - \delta_0 - \phi_\epsilon)} &= 0.9990 + 0.04i \ (\delta_0 = 37^\circ,\ \delta_2 = -7^\circ,\ \phi_\epsilon = (43.52 \pm 0.05)^\circ \ (exp.)) \\ &= 0.98 + 0.19i \ (\delta_0 = (23.8 \pm 5.0)^\circ,\ \delta_2 = (-11.6 \pm 2.8)^\circ \ (Lattice)) \end{aligned}$$

$$\begin{aligned} \frac{\epsilon'_K}{\epsilon_K} &\simeq \frac{1}{\sqrt{2}|\epsilon_K|} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right) \\ &= \frac{1}{\sqrt{2}|\epsilon_K|} \frac{\mathrm{Re}A_2}{(\mathrm{Re}A_0)^2} \left(-\mathrm{Im}A_0 + \frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \mathrm{Im}A_2 \right) \end{aligned}$$

Singularity:1

- The renormalization group (RG) evolution matrix *Uf* plays a central role

$$A_I = \sum_i C_i(\mu)\langle Q_i(\mu)\rangle_I$$ renormalization scale μ: $\Lambda_{QCD} < \mu < m_c$

WC: $\vec{C}(\mu) = \hat{U}_3(\mu,\mu_c)\hat{M}_c(\mu_c)\hat{U}_4(\mu,m_b)\hat{M}_b(m_b)\hat{U}_5(m_b,M_W)\vec{C}(M_W)$

HME: $\langle\vec{Q}(\mu)^T\rangle_I = \langle\vec{Q}(\mu_{lat})^T\rangle_I\left(\hat{U}_3(\mu,\mu_{lat})\right)^{-1}$

$$\hat{U}_f(\mu_1,\mu_2) = T_{g_s}\exp\int_{g_s(\mu_2)}^{g_s(\mu_1)} dg_s' \frac{\hat{\gamma}^T(g_s')}{\beta(g_s')},$$

Anomalous dimension matrix of 4-fermi operators: $\hat{\gamma}(g_s(\mu)) = \frac{\alpha_s(\mu)}{4\pi}\hat{\gamma}_s^{(0)} + \frac{\alpha_{EM}}{4\pi}\hat{\gamma}_e^{(0)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\hat{\gamma}_s^{(1)} + \frac{\alpha_{EM}\alpha_s(\mu)}{(4\pi)^2}\hat{\gamma}_{se}^{(1)},$

QCD β-function: $\beta(g_s(\mu)) = -g_s(\mu)\left(\frac{\alpha_s(\mu)}{4\pi}\beta_0 + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\beta_1\right),$

Singularity:2

- LO RG evolution is known its analytic formula, thereby it is no-problem

$$\hat{U}_0(\alpha_1,\alpha_2) = \hat{V}\,\mathrm{diag}\left(\left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\left(\hat{\gamma}^{(0)T}_{s,D}\right)_{1,1}}{2\beta_0}}, \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\left(\hat{\gamma}^{(0)T}_{s,D}\right)_{2,2}}{2\beta_0}}, \ldots, \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\left(\hat{\gamma}^{(0)T}_{s,D}\right)_{10,10}}{2\beta_0}}\right)\hat{V}^{-1}$$

- When one calculates **NLO** RG evolution with **f=3 analytically, singularities** appear! [Ciuchini,Franco,Martinelli,Reina, 93′, 94′, Buras,Jamin,Lautenbacher 93′]

usual analytic form $\hat{U}_f(\mu_1,\mu_2) = \hat{K}(\mu_1)\hat{U}_0(\mu_1,\mu_2)\hat{K}'(\mu_2),$

with

$$\hat{K}(\mu_1) = \left(\hat{1} + \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}\right)\left(\hat{1} + \frac{\alpha_s(\mu_1)}{4\pi}\hat{J}_s\right)\left(\hat{1} + \frac{\alpha_{EM}}{\alpha_s(\mu_1)}\hat{J}_e\right),$$

$$\hat{K}'(\mu_2) = \left(\hat{1} - \frac{\alpha_{EM}}{\alpha_s(\mu_2)}\hat{J}_e\right)\left(\hat{1} - \frac{\alpha_s(\mu_2)}{4\pi}\hat{J}_s\right)\left(\hat{1} - \frac{\alpha_{EM}}{4\pi}\hat{J}_{se}\right),$$

- Here, *Js* is the solution of the following equation

$$\hat{J}_s - \left[\hat{J}_s, \frac{\hat{\gamma}_s^{(0)T}}{2\beta_0}\right] = \frac{\beta_1}{\beta_0}\frac{\hat{\gamma}_s^{(0)T}}{2\beta_0} - \frac{\hat{\gamma}_s^{(1)T}}{2\beta_0},$$

Singularity:4

- Points of view of literatures
  - One can avoid some of singularities by using different NLO analytic formula, but singularities still remain [Buras,Jamin,Lautenbacher 93']
  - Using small shift of eigenvalue *by hand*, all these singularities cancel and the evolution matrix becomes finite
- Our opinion
  - Statement of literature is right. But singularities make a computational evaluation highly laborious and complicated
  - We want to use this RG evolution for NP calc. therefore the singularities should be dropped not by hand but *automatically*
  - Singularity-free analytical solution would be exist…

Numerical results:1

- Wilson coefficients $@\mu = 1.3$ GeV $\quad C_i(\mu) \equiv z_i(\mu) - \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} y_i(\mu)$ new results

i	$z_i(\mu)$	$y_i(\mu)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_{EM}/\alpha_s)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_{EM})$	$\mathcal{O}(\alpha_{EM}^2/\alpha_s^2)$
1	−0.3903	0	0	0	0	0	0
2	1.200	0	0	0	0	0	0
3	0.0044	0.0274	0.0254	0.0001	0.0007	0.0012	0
4	−0.0131	−0.0566	−0.0485	−0.0003	−0.0069	−0.0009	0
5	0.0039	0.0068	0.0124	0.0001	−0.0059	0.0001	0
6	−0.0128	−0.0847	−0.0736	−0.0003	−0.0099	−0.0008	0
$7/\alpha_{EM}$	0.0042	−0.0344	0	−0.1120	0	0.0757	0.0019
$8/\alpha_{EM}$	0.0020	0.1158	0	−0.0222	0	0.1373	0.0007
$9/\alpha_{EM}$	0.0053	−1.3834	0	−0.1269	0	−1.2582	0.0017
$10/\alpha_{EM}$	−0.0013	0.4877	0	0.0214	0	0.4668	−0.0004

- Hadronic matrix elements $@\mu = 1.3$ GeV

i	$\langle Q_i(\mu)\rangle_0^{\overline{\text{MS}}\text{–NDR}}(\text{GeV})^3$
1	-0.145 ± 0.046
2	0.105 ± 0.015
3	-0.041 ± 0.066
4	0.209 ± 0.066
5	-0.180 ± 0.068
6	-0.342 ± 0.122
7	0.160 ± 0.065
8	1.556 ± 0.376
9	-0.197 ± 0.069
10	0.053 ± 0.037

i	$\langle Q_i(\mu)\rangle_2^{\overline{\text{MS}}\text{–NDR}}(\text{GeV})^3$
1	0.01006 ± 0.00002
2	0.01006 ± 0.00002
3	—
4	—
5	—
6	—
7	0.135 ± 0.012
8	0.874 ± 0.054
9	0.01509 ± 0.00003
10	0.01509 ± 0.00003

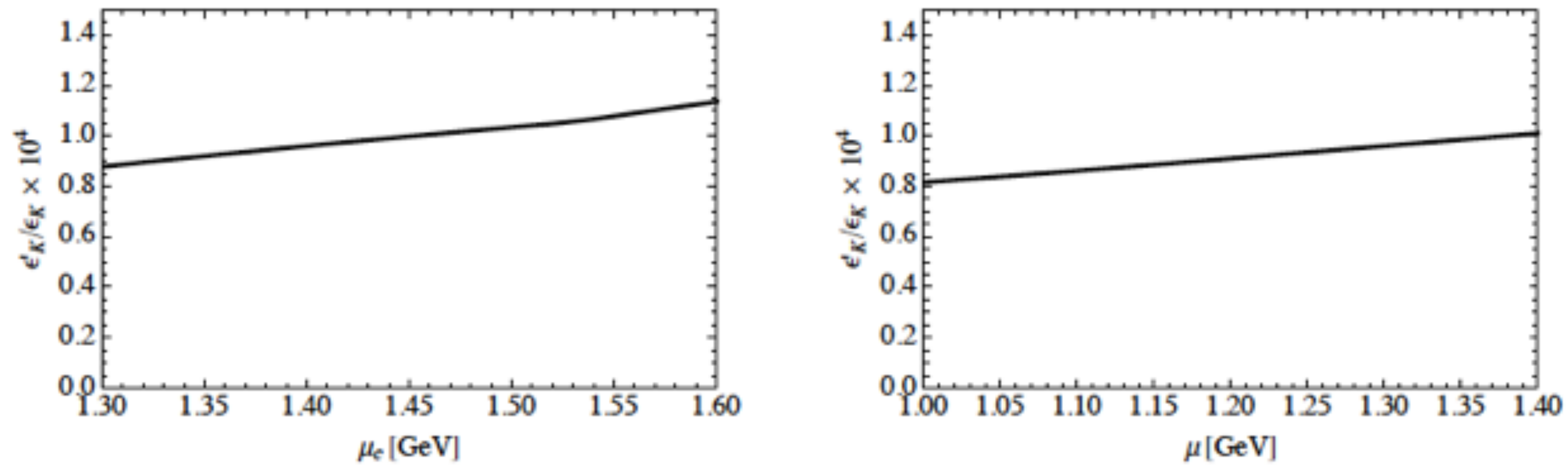
Lattice simulation is calculated at μ=1.5 GeV (I=0) and μ=3.0 GeV (I=2) with 2+1 flavour

We exploit CP-conserving data (with z_i) to reduce hadronic uncertainties

[TK, Nierste, Tremper 16′]

Numerical results:2

- μc and μ dependence [TK, Nierste, Tremper 16′]



(a) μ_c dependence of ϵ'_K/ϵ_K (b) μ dependence of ϵ'_K/ϵ_K

- *Final result*

$$\left(\frac{\epsilon'_K}{\epsilon_K}\right)_{\text{SM–NLO}} = (0.96 \pm 4.68 \pm 1.52 \pm 0.60 \pm 0.24) \times 10^{-4}$$

Lattice NNLO isospin violating mt

… **2.9 sigma** below from exp.

cf. $= -0.52 \times 10^{-4}$ using numerical RG evolution

$$\frac{d\vec{v}(\mu)}{d\ln\mu} = \hat{\gamma}^T(g_s(\mu))\vec{v}(\mu), \qquad \frac{d\vec{z}(\mu)}{d\ln\mu} = \hat{\gamma}^T(g_s(\mu))\vec{z}(\mu).$$

Overview of effective models

- Chiral perturbation theory
  - Effective theory of the QCD Goldstone bosons: $\Phi = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(g_8 f^4 \mathrm{tr}\left(\lambda L_\mu L^\mu\right) + g_{27} f^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + \mathcal{O}(g_E W) \right)$$

with $\quad L_\mu = -iU^\dagger D_\mu U \qquad U = \exp\left(i \frac{\sqrt{2}\Phi}{f} \right)$

- dual QCD method [Bardeen, Buras, Gerard 87′, 14′]
  - Effective theory of the truncated pseudo-scalar and vector mesons:

$$\mathcal{L} = \frac{f^2}{4} \mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{4} \mathrm{tr}\left(V_{\mu\nu} V^{\mu\nu}\right) - \frac{f^2}{2} \mathrm{tr}\left(\partial_\mu \xi^\dagger \xi + \partial_\mu \xi \xi^\dagger - 2igV_\mu\right)^2 \quad \text{with} \quad U = \xi\xi$$

- Chiral quark model
  - Mean-field approximation of the full extended NJL model

$$\mathcal{L} = \mathcal{L}_{QCD} - M\left(\bar{q}_R U q_L + \bar{q}_L U^\dagger q_R\right)$$

Operator basis

- In general BSM, there are 24 four-fermi and 4 dipole operators for $\Delta S=1$
- At not large LR mixing region, many operators are suppressed, in

$$
\begin{aligned}
Q_1^{q=u,c,t} &= (\bar{s}_\alpha q_\beta)_{V-A}\,(\bar{q}_\beta d_\alpha)_{V-A}\,, & Q_2^{q=u,c,t} &= (\bar{s}q)_{V-A}\,(\bar{q}d)_{V-A}\,,\\
Q_1^{\prime q} &= (\bar{s}d)_{V-A}\,(\bar{q}q)_{V+A}\,, & Q_2^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V-A}\,(\bar{q}_\beta q_\alpha)_{V+A}\,,\\
Q_3^{\prime q} &= (\bar{s}d)_{V-A}\,(\bar{q}q)_{V-A}\,, & Q_4^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V-A}\,(\bar{q}_\beta q_\alpha)_{V-A}\,,\\
\tilde{Q}_1^{q=u,c,t} &= (\bar{s}_\alpha q_\beta)_{V+A}\,(\bar{q}_\beta d_\alpha)_{V+A}\,, & \tilde{Q}_2^{q=u,c,t} &= (\bar{s}q)_{V+A}\,(\bar{q}d)_{V+A}\,,\\
\tilde{Q}_1^{\prime q} &= (\bar{s}d)_{V+A}\,(\bar{q}q)_{V-A}\,, & \tilde{Q}_2^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V+A}\,(\bar{q}_\beta q_\alpha)_{V-A}\,,\\
\tilde{Q}_3^{\prime q} &= (\bar{s}d)_{V+A}\,(\bar{q}q)_{V+A}\,, & \tilde{Q}_4^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V+A}\,(\bar{q}_\beta q_\alpha)_{V+A}\,,\\
Q_5^{\prime q=u,c,t} &= (\bar{s}_\alpha q_\beta)_{V-A}\,(\bar{q}_\beta d_\alpha)_{V+A}\,, & Q_6^{\prime q=u,c,t} &= (\bar{s}q)_{V-A}\,(\bar{q}d)_{V+A}\,,\\
\tilde{Q}_5^{\prime q=u,c,t} &= (\bar{s}_\alpha q_\beta)_{V+A}\,(\bar{q}_\beta d_\alpha)_{V-A}\,, & \tilde{Q}_6^{\prime q=u,c,t} &= (\bar{s}q)_{V+A}\,(\bar{q}d)_{V-A}\,,\\
Q_5^{\prime b} &= (\bar{s}_\alpha b_\beta)_{V-A}\,(\bar{b}_\beta d_\alpha)_{V+A}\,, & Q_6^{\prime b} &= (\bar{s}b)_{V-A}\,(\bar{b}d)_{V+A}\,,\\
\tilde{Q}_5^{\prime b} &= (\bar{s}_\alpha b_\beta)_{V+A}\,(\bar{b}_\beta d_\alpha)_{V-A}\,, & \tilde{Q}_6^{\prime b} &= (\bar{s}b)_{V+A}\,(\bar{b}d)_{V-A}\,,\\
Q_7^{\prime q} &= (\overline{s_L} d_R)\,(\overline{q_L} q_R)\,, & Q_8^{\prime q} &= (\overline{s_L}_\alpha d_{R\beta})\,(\overline{q_L}_\beta q_{R\alpha})\,,\\
Q_9^{\prime q=u,c,t} &= (\overline{s_L} q_R)\,(\overline{q_L} d_R)\,, & Q_{10}^{\prime q=u,c,t} &= (\overline{s_L}_\alpha q_{R\beta})\,(\overline{q_L}_\beta d_{R\alpha})\,,\\
\tilde{Q}_7^{\prime q} &= (\overline{s_R} d_L)\,(\overline{q_R} q_L)\,, & \tilde{Q}_8^{\prime q} &= (\overline{s_R}_\alpha d_{L\beta})\,(\overline{q_R}_\beta q_{L\alpha})\,,\\
\tilde{Q}_9^{\prime q=u,c,t} &= (\overline{s_R} q_L)\,(\overline{q_R} d_L)\,, & \tilde{Q}_{10}^{\prime q=u,c,t} &= (\overline{s_R}_\alpha q_{L\beta})\,(\overline{q_R}_\beta d_{L\alpha})\,,\\
Q_9^{\prime b} &= (\overline{s_L} b_R)\,(\overline{b_L} d_R)\,, & Q_{10}^{\prime b} &= (\overline{s_L}_\alpha b_{R\beta})\,(\overline{b_L}_\beta d_{R\alpha})\,,\\
\tilde{Q}_9^{\prime b} &= (\overline{s_R} b_L)\,(\overline{b_R} d_L)\,, & \tilde{Q}_{10}^{\prime b} &= (\overline{s_R}_\alpha b_{L\beta})\,(\overline{b_R}_\beta d_{L\alpha})\,,\\
Q'_{7\gamma} &= \frac{e m_b}{4\pi^2}\overline{s_L}\sigma_{\mu\nu}F^{\mu\nu}d_R, & Q'_{8g} &= \frac{g_s m_b}{4\pi^2}\overline{s_L}\sigma_{\mu\nu}G^{\mu\nu}d_R,\\
\tilde{Q}'_{7\gamma} &= \frac{e m_b}{4\pi^2}\overline{s_R}\sigma_{\mu\nu}F^{\mu\nu}d_L, & \tilde{Q}'_{8g} &= \frac{g_s m_b}{4\pi^2}\overline{s_R}\sigma_{\mu\nu}G^{\mu\nu}d_L,
\end{aligned}
$$

Not large LR mixing regime

$$Q_{1,2}^{u},\ Q_{1,2,3,4}^{\prime u,d},\ \tilde{Q}_{1,2,3,4}^{\prime u,d},\ \underline{Q_{5,6}^{\prime b},\ \tilde{Q}_{5,6}^{\prime b}}$$

They do not contribute to ε′K

↓ Linear combination

SM 4-fermi operator basis $Q_{1,2,\ldots,10}$

with $\langle(\pi\pi)_I|\tilde{Q}|K^0\rangle = -\langle(\pi\pi)_I|Q|K^0\rangle$

[Gabbiani,Gabrielli,Masiero,Silvestrini,96']