How to go beyond the Higgs: effective operators and dark pions

Hitoshi Murayama (Berkeley, Kavli IPMU)

YITP phenomenology workshop, Sep 6, 2016

A tribute to Yoichiro Nambu

passed away July 5, 2015

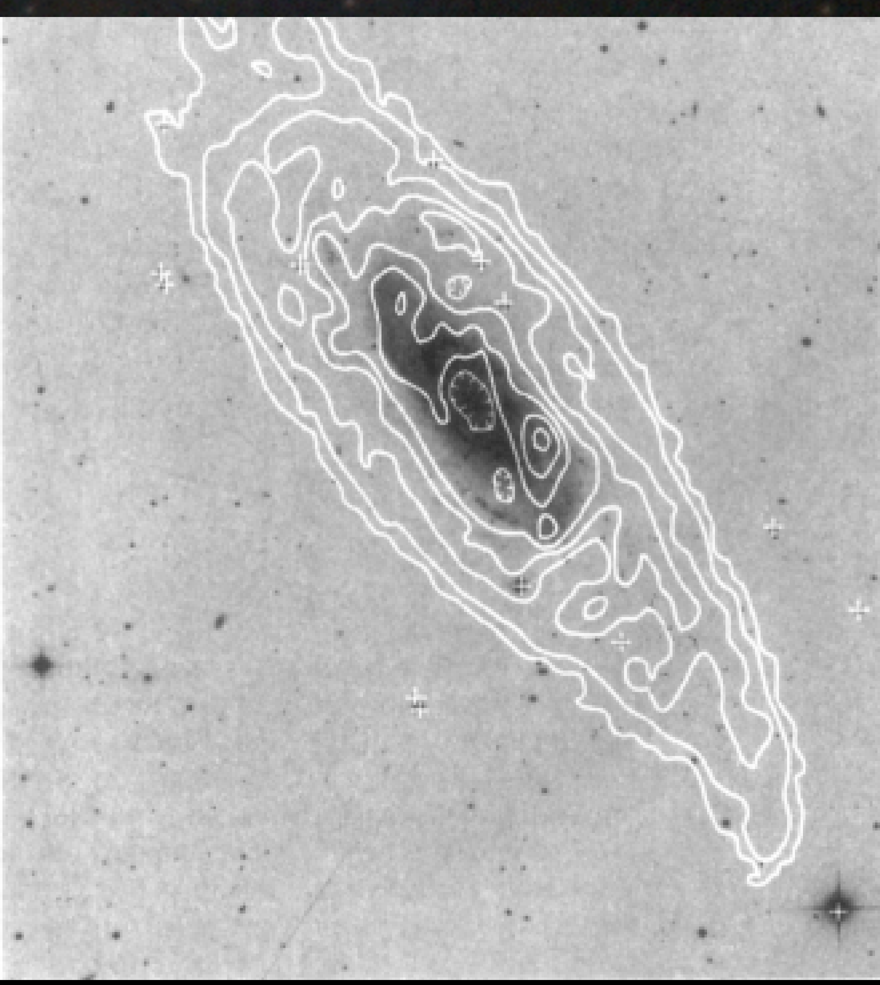
東京大学 THE UNIVERSITY OF TOKYO
UTIAS 東京大学国際高等研究所 THE UNIVERSITY OF TOKYO INSTITUTES FOR ADVANCED STUDY
KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

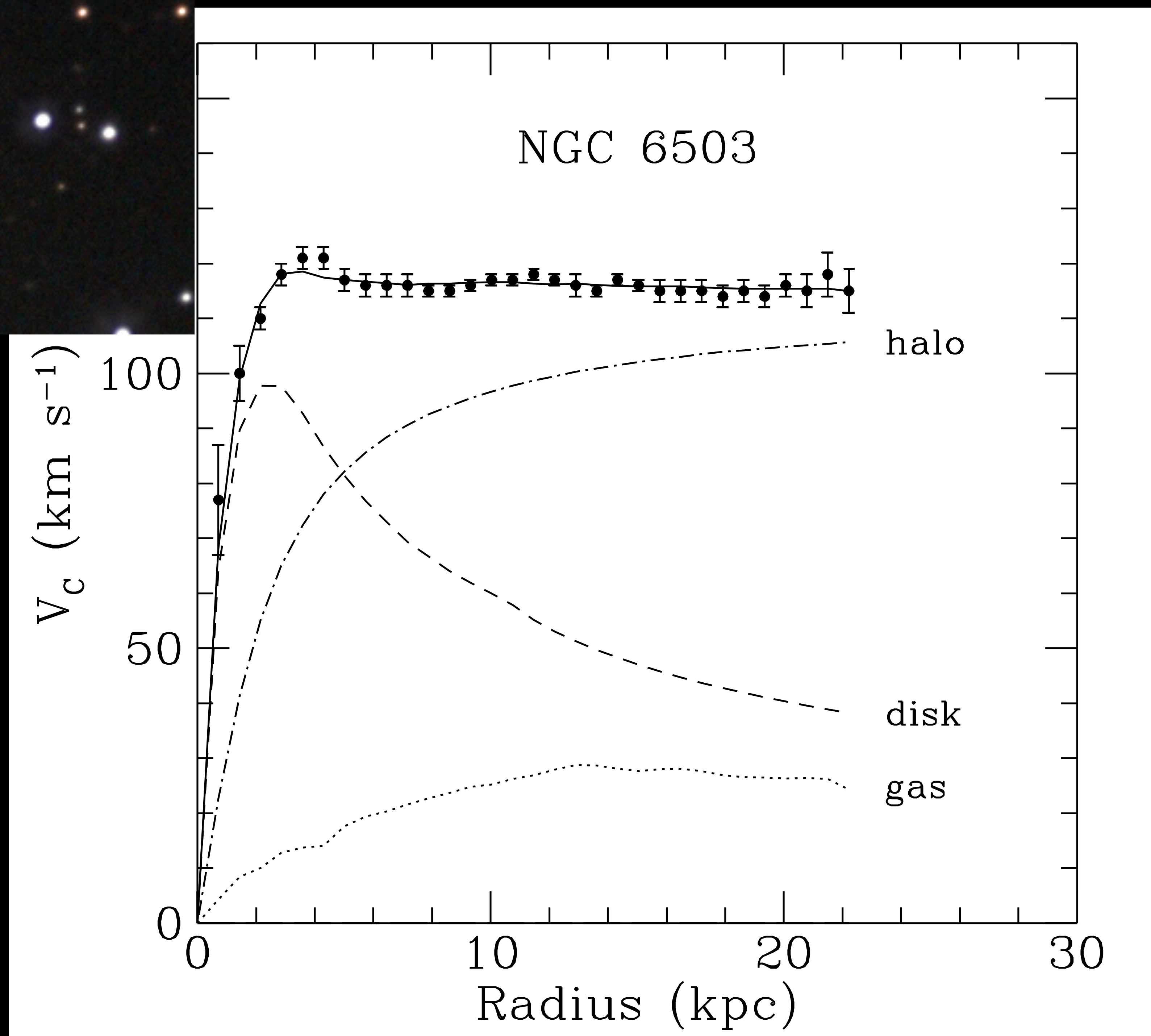
Dark Pions as Dark Matter

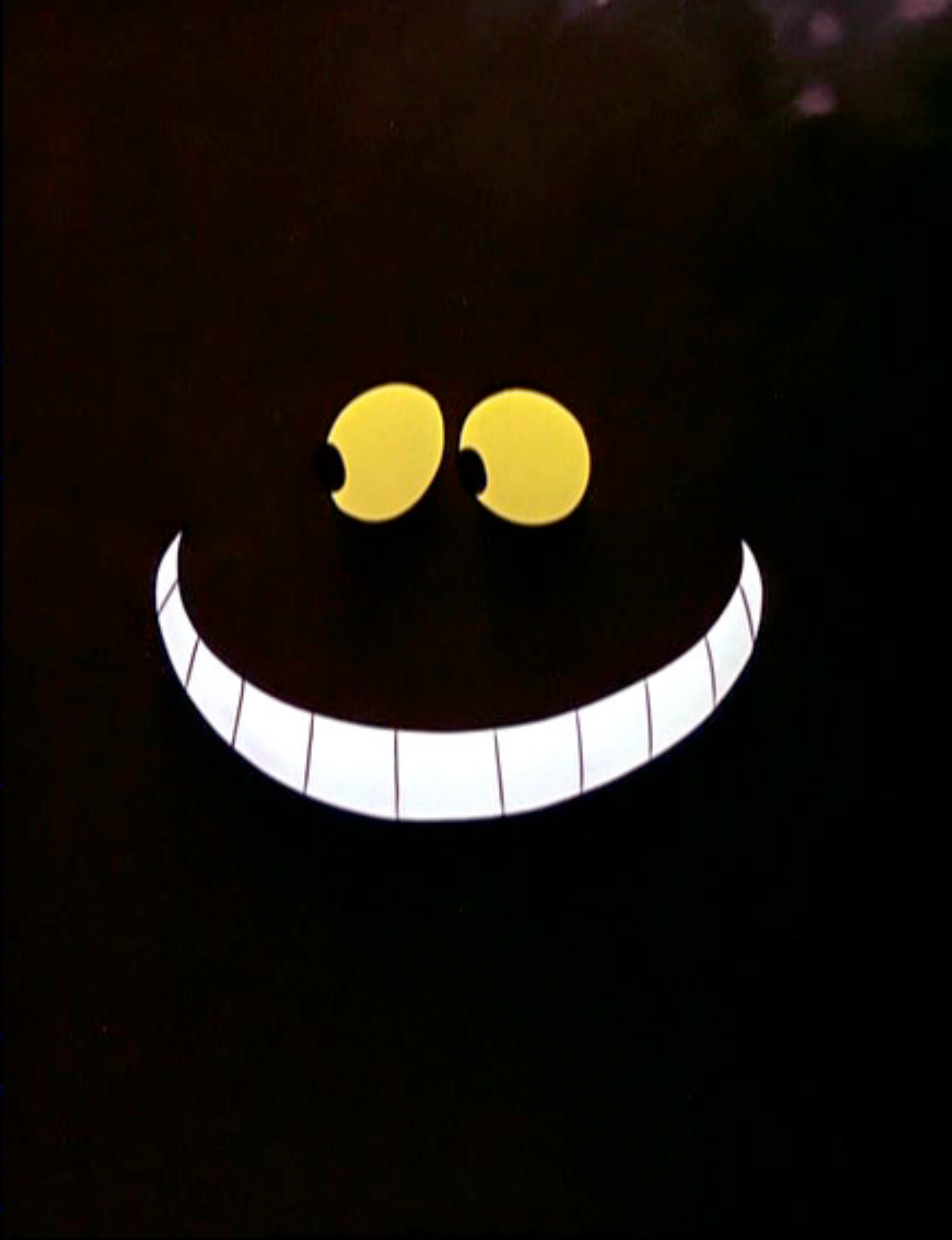
with Yonit Hochberg and Eric Kuflik

arXiv:1411.3727 w/ Tomer Volansky Jay Wacker,
arXiv:1512.07917, 160x.xxxxx

BERKELEY CENTER FOR THEORETICAL PHYSICS
BERKELEY LAB
THE UNIVERSITY OF CALIFORNIA 1868 LET THERE BE LIGHT

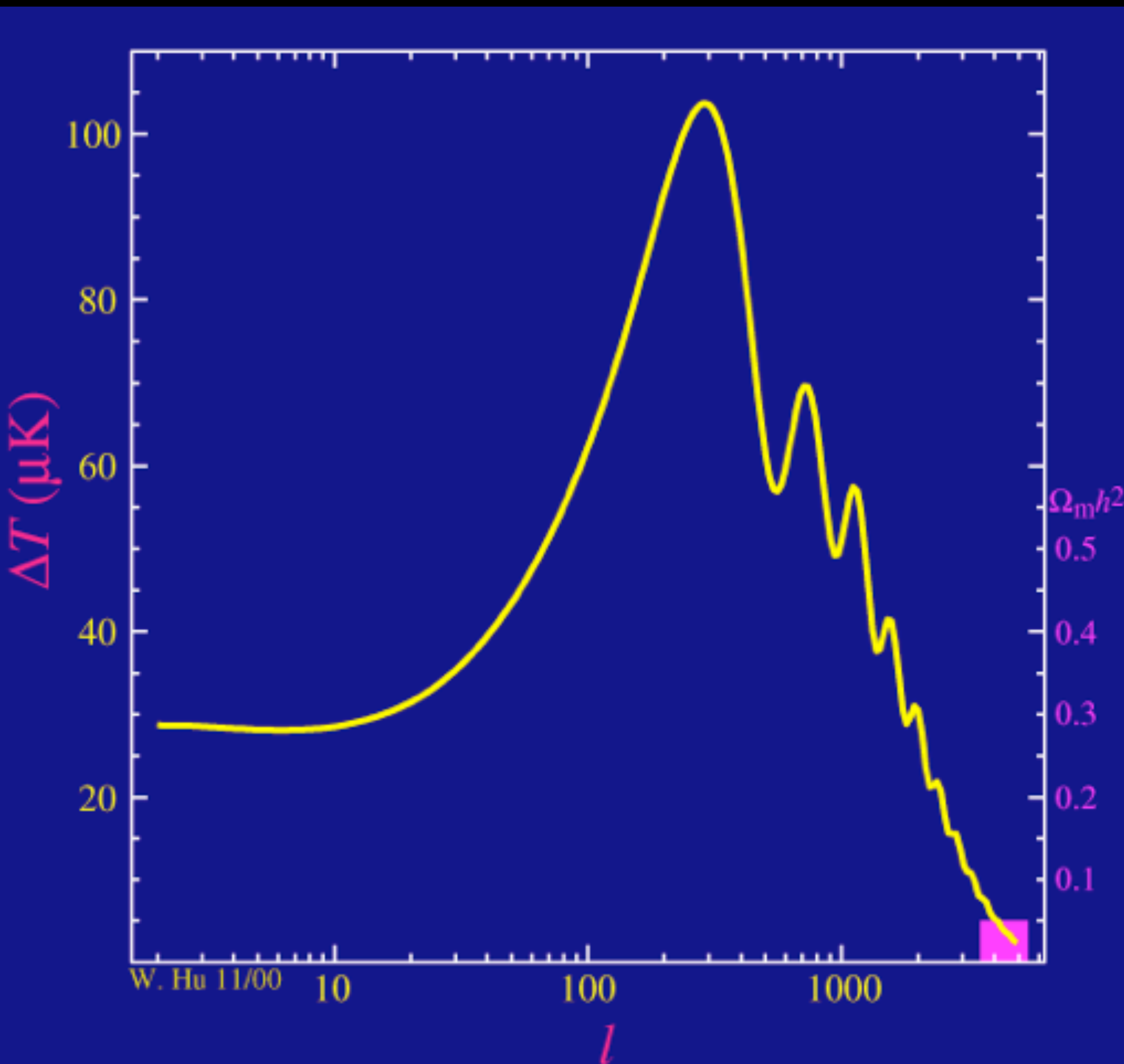


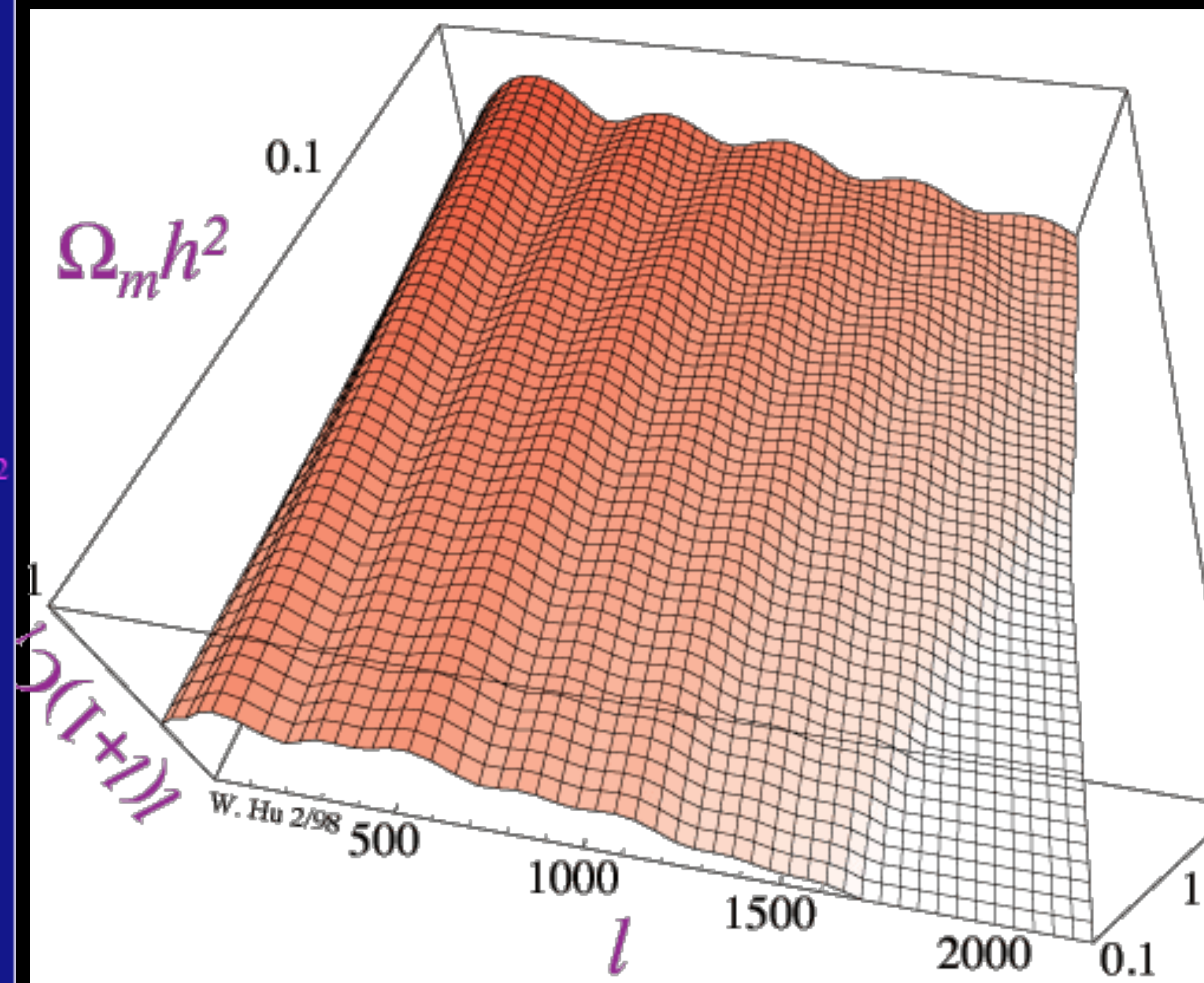

NGC 6503
halo
disk
gas
V_c (km s^{-1})
Radius (kpc)
0
50
100
0
10
20
30

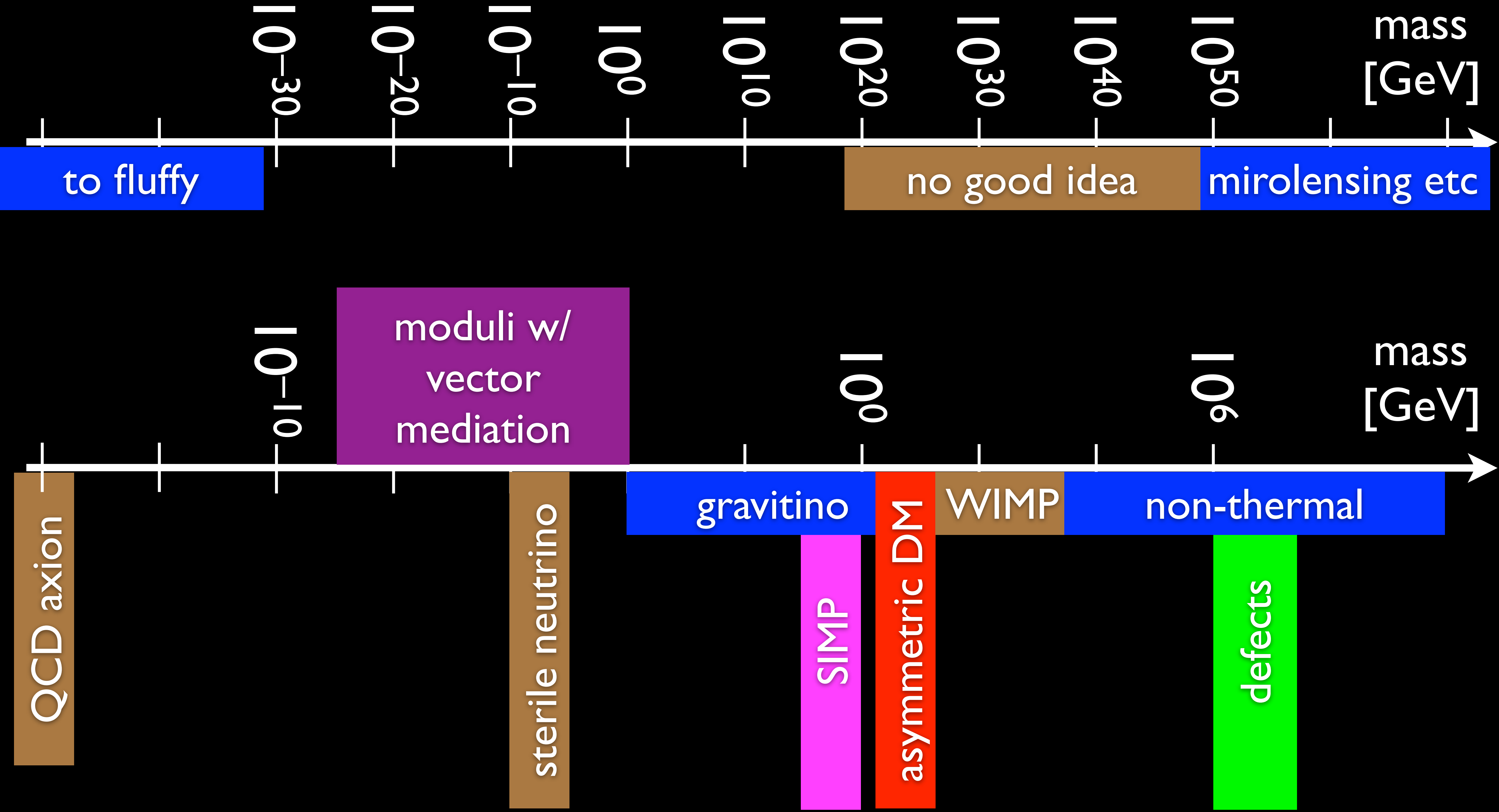
Cheshire cat

matter





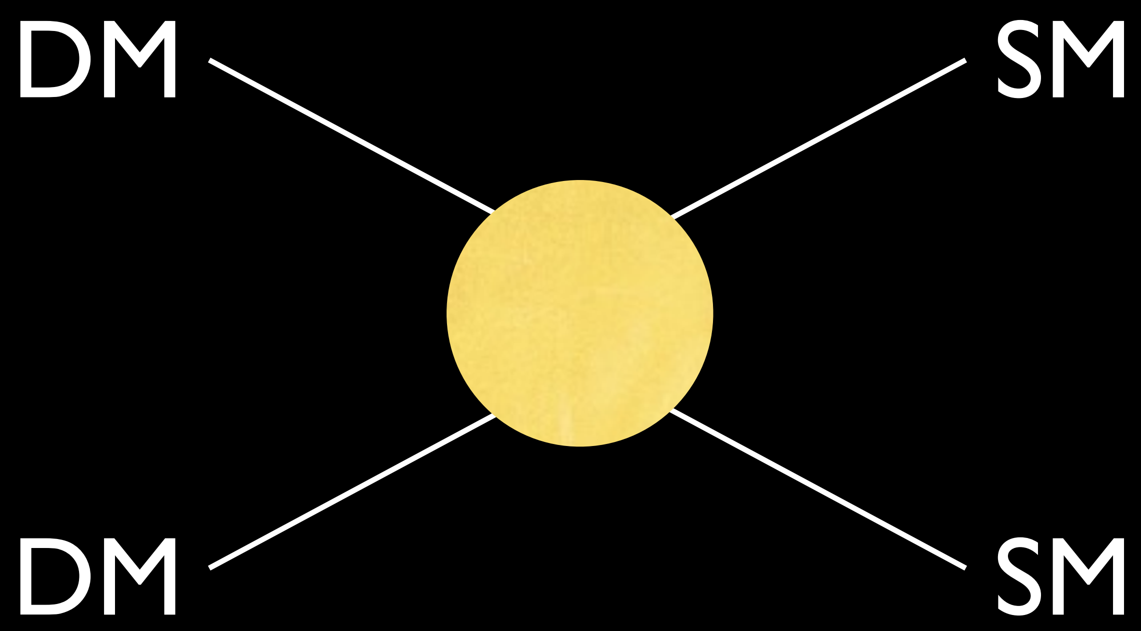
Ω_m changes the overall heights of the peaks



10^−30
10^−20
10^−10
10^0
10^10
10^20
10^30
10^40
10^50
mass [GeV]
to fluffy
no good idea
mirolensing etc
moduli w/ vector mediation
10^−10
10^0
10^6
mass [GeV]
QCD axion
sterile neutrino
gravitino
SIMP
asymmetric DM
WIMP
non-thermal
defects

$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\rm GeV}{m_{\rm DM}}$$

WIMP Miracle



$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

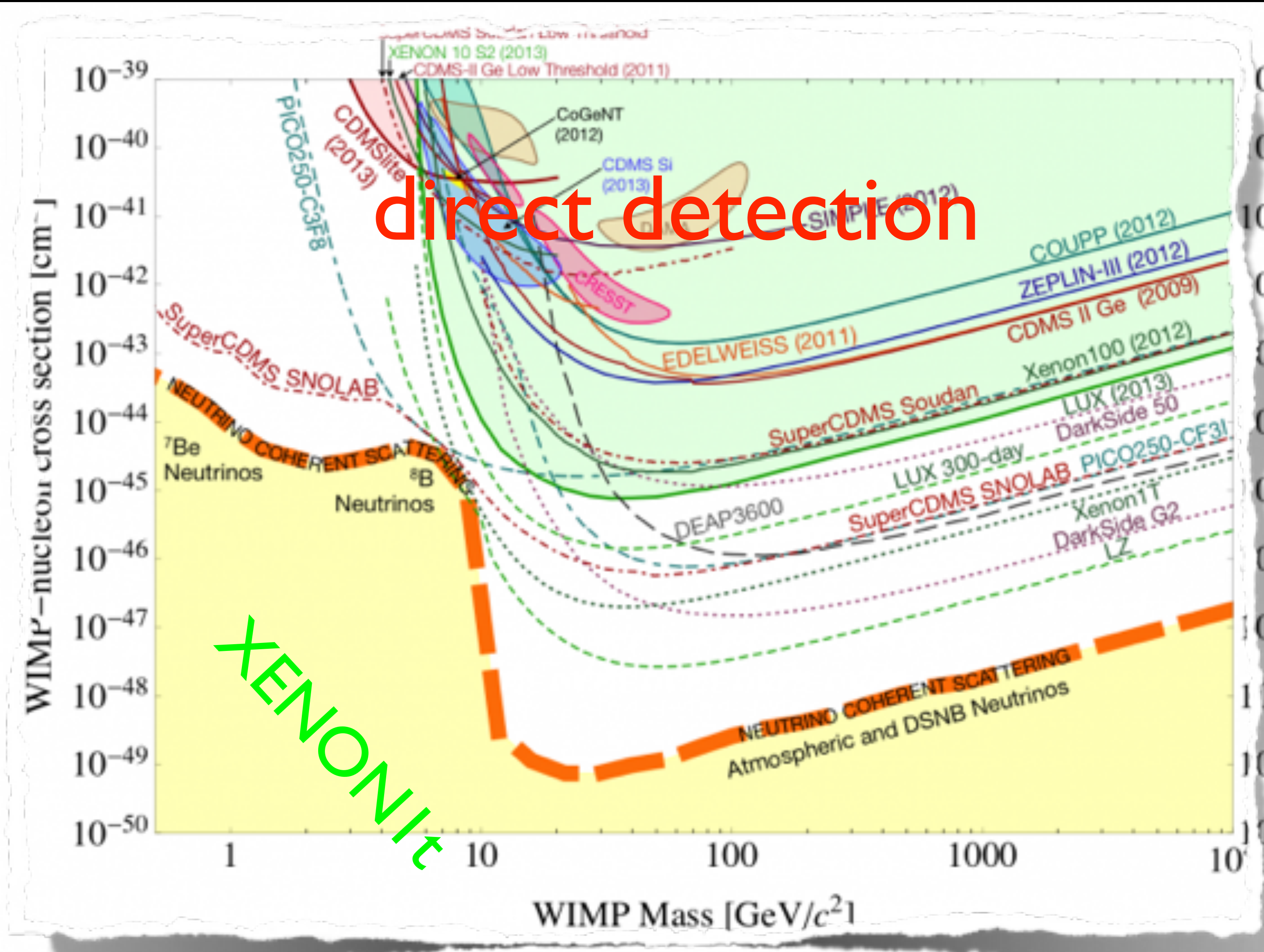
$$m \approx 300 \ {\rm GeV}$$

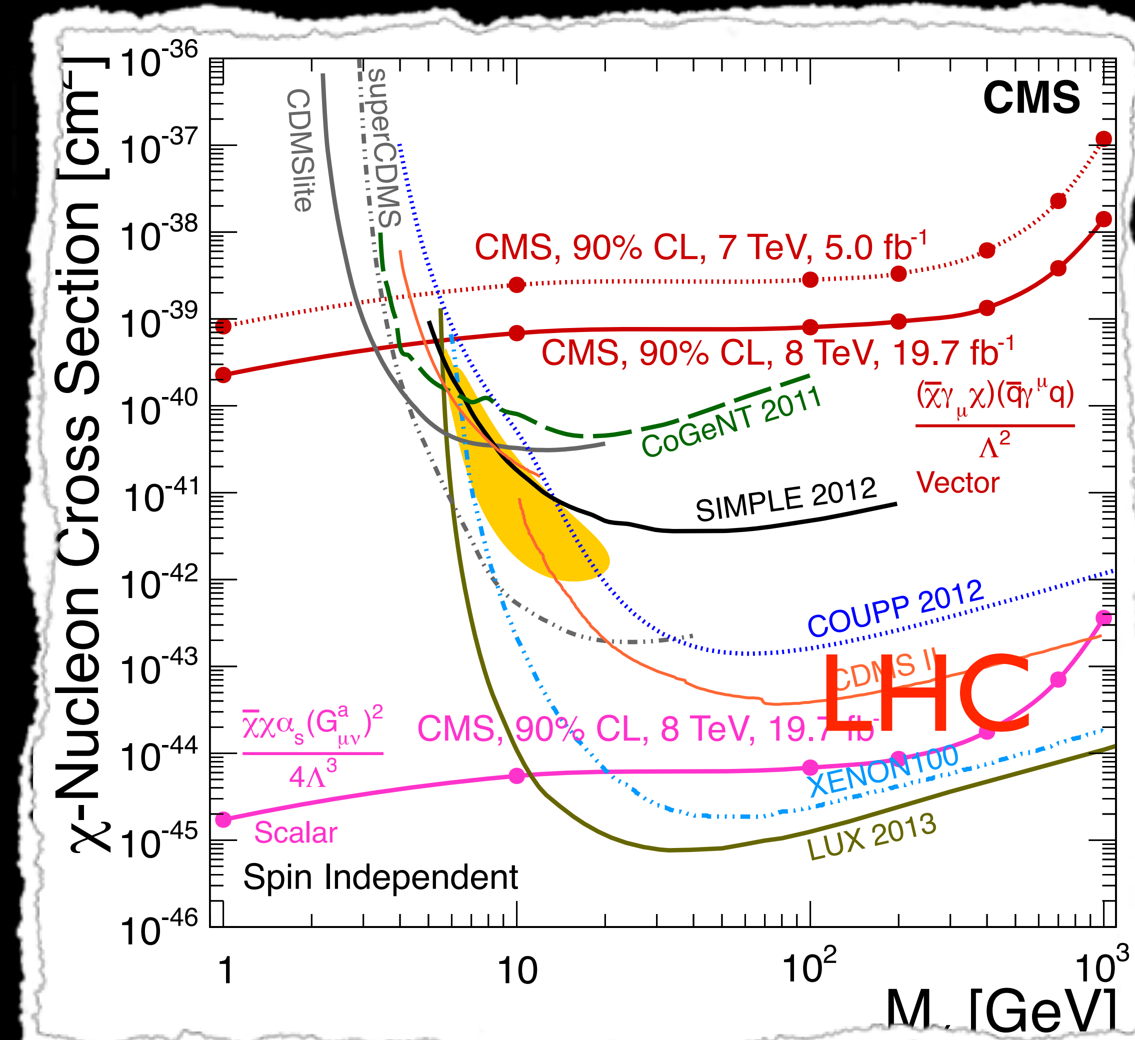
"weak" coupling
"weak" mass scale → correct abundance

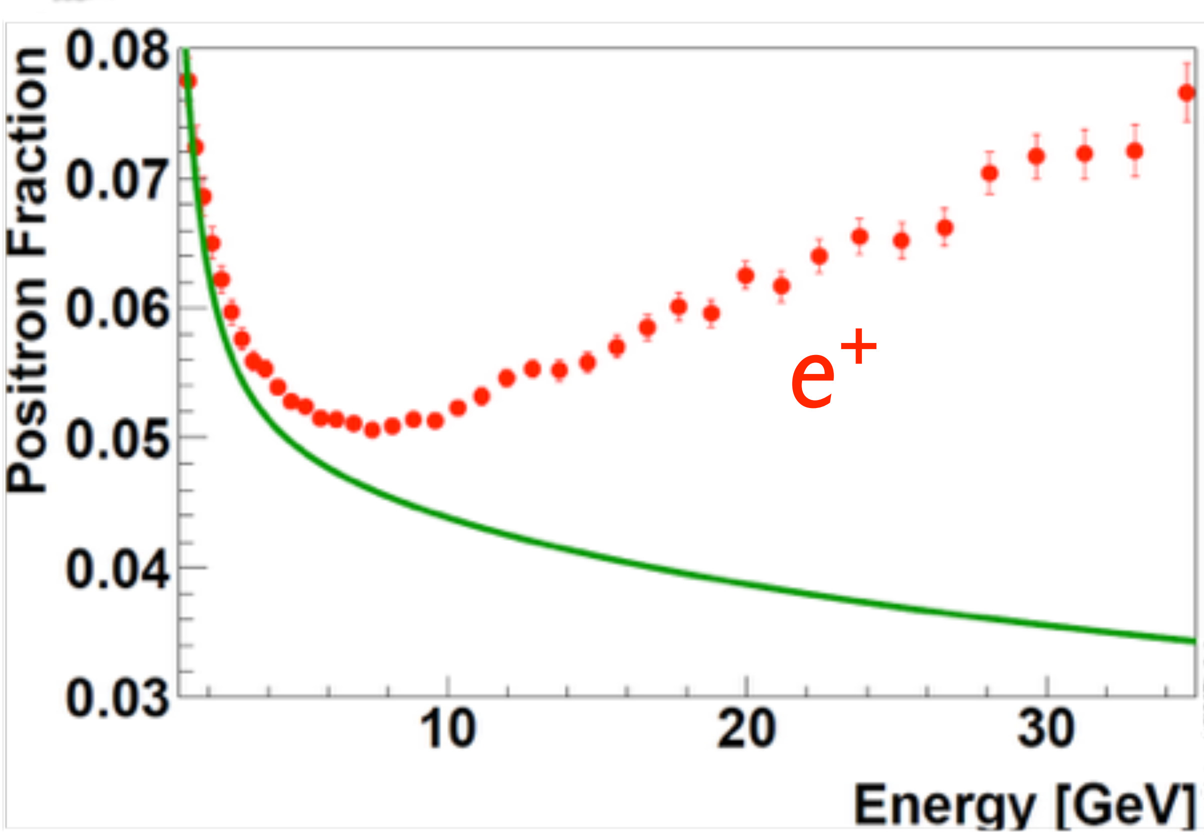
Miracle^2

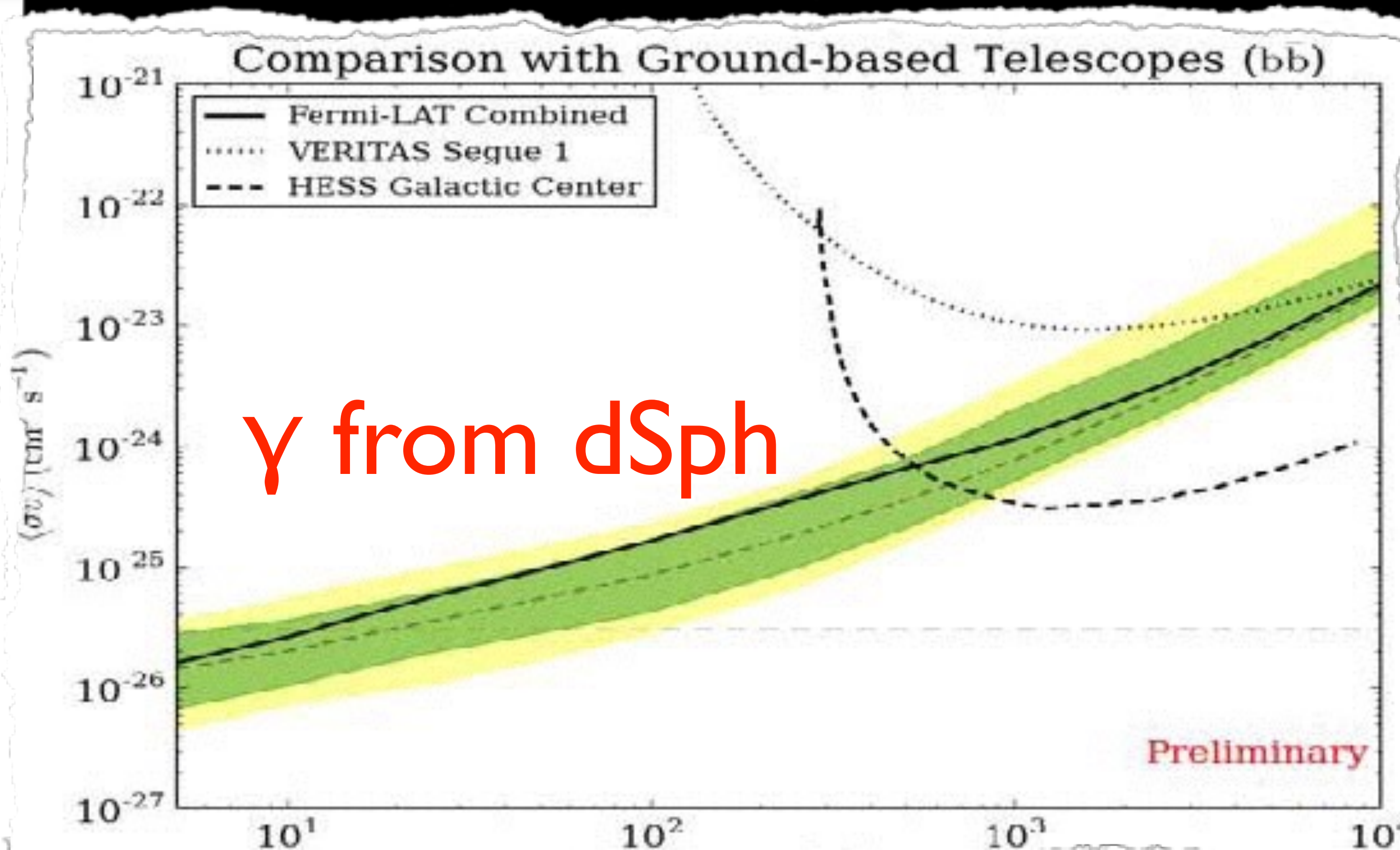
sociology

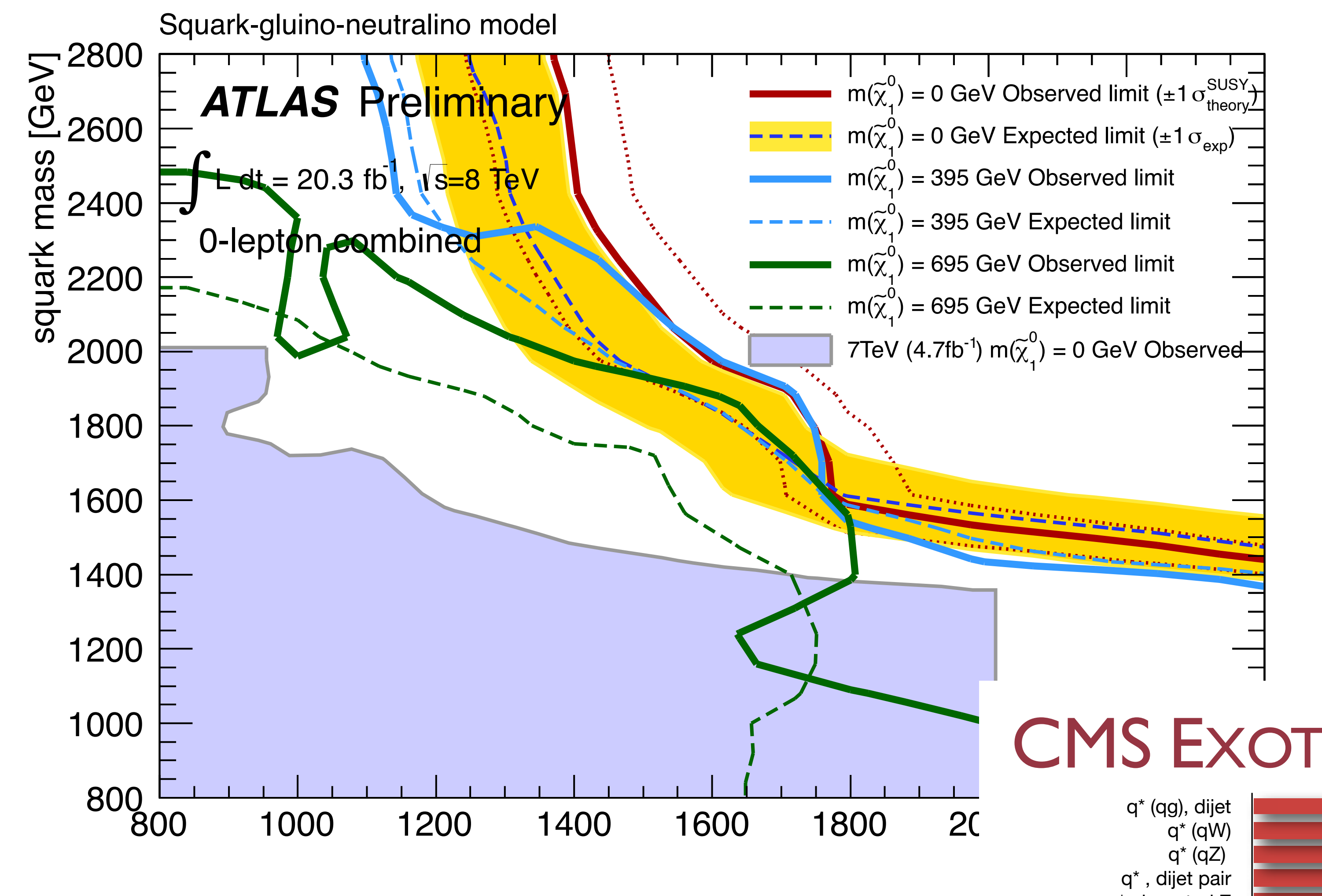
- We used to think
  - need to solve problems with the SM
  - hierarchy problem, strong CP, etc
  - it is great if a solution also gives dark matter candidate as an *option*
  - big ideas: supersymmetry, extra dim
  - probably because dark matter problem was not so established in 80's

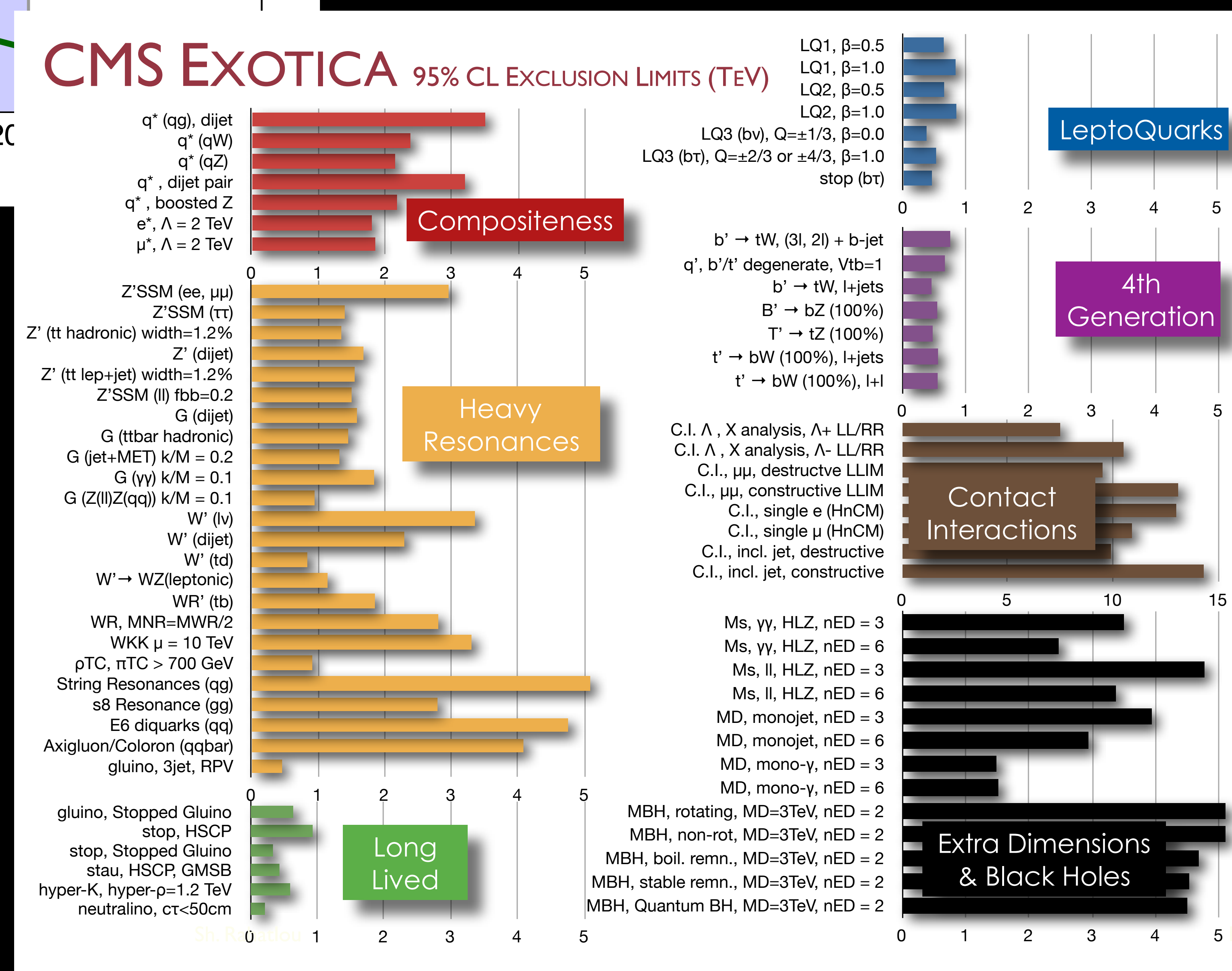

direct detection
XENON1t
WIMP–nucleon cross section [cm²]
WIMP Mass [GeV/c²]
XENON 10 S2 (2013)
CDMS-II Ge Low Threshold (2011)
CoGeNT (2012)
CDMS Si (2013)
PICO250-C3F8
CDMSlite (2013)
SIMPLE (2012)
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
CRESST
EDELWEISS (2011)
Xenon100 (2012)
SuperCDMS SNOLAB
SuperCDMS Soudan
LUX (2013)
DarkSide 50
LUX 300-day
PICO250-CF3I
DEAP3600
SuperCDMS SNOLAB
Xenon1T
DarkSide G2
LZ
NEUTRINO COHERENT SCATTERING
⁷Be Neutrinos
⁸B Neutrinos
NEUTRINO COHERENT SCATTERING
Atmospheric and DSNB Neutrinos


CMS
LHC
χ-Nucleon Cross Section [cm²]
M [GeV]
CDMSlite
superCDMS
CMS, 90% CL, 7 TeV, 5.0 fb⁻¹
CMS, 90% CL, 8 TeV, 19.7 fb⁻¹
CoGeNT 2011
SIMPLE 2012
Vector
COUPP 2012
CDMS II
CMS, 90% CL, 8 TeV, 19.7 fb⁻¹
XENON100
Scalar
LUX 2013
Spin Independent


e+
Positron Fraction
Energy [GeV]


Comparison with Ground-based Telescopes (bb)
Fermi-LAT Combined
VERITAS Segue 1
HESS Galactic Center
γ from dSph
Preliminary



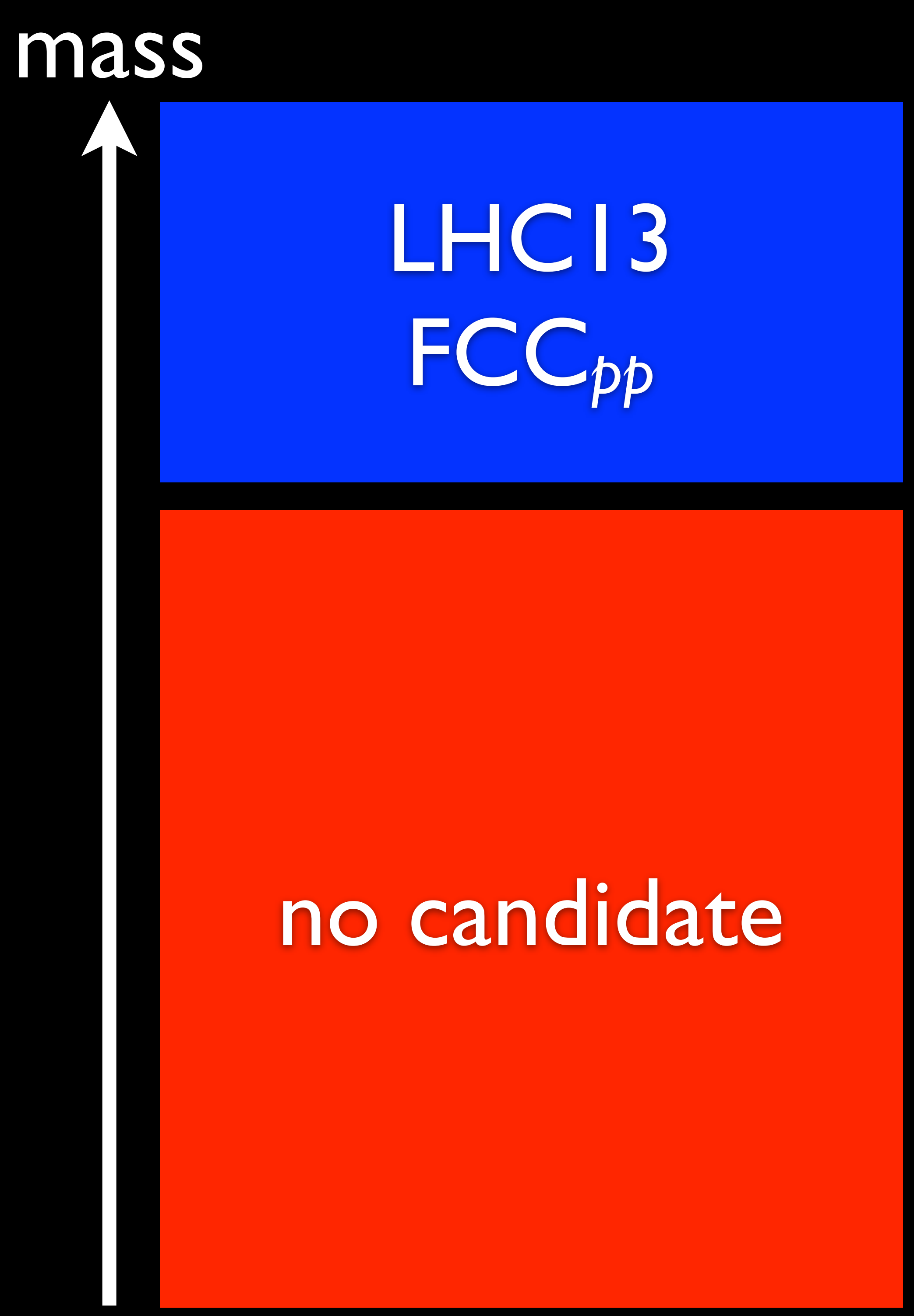


no sign of
new physics
that explains
naturalness!

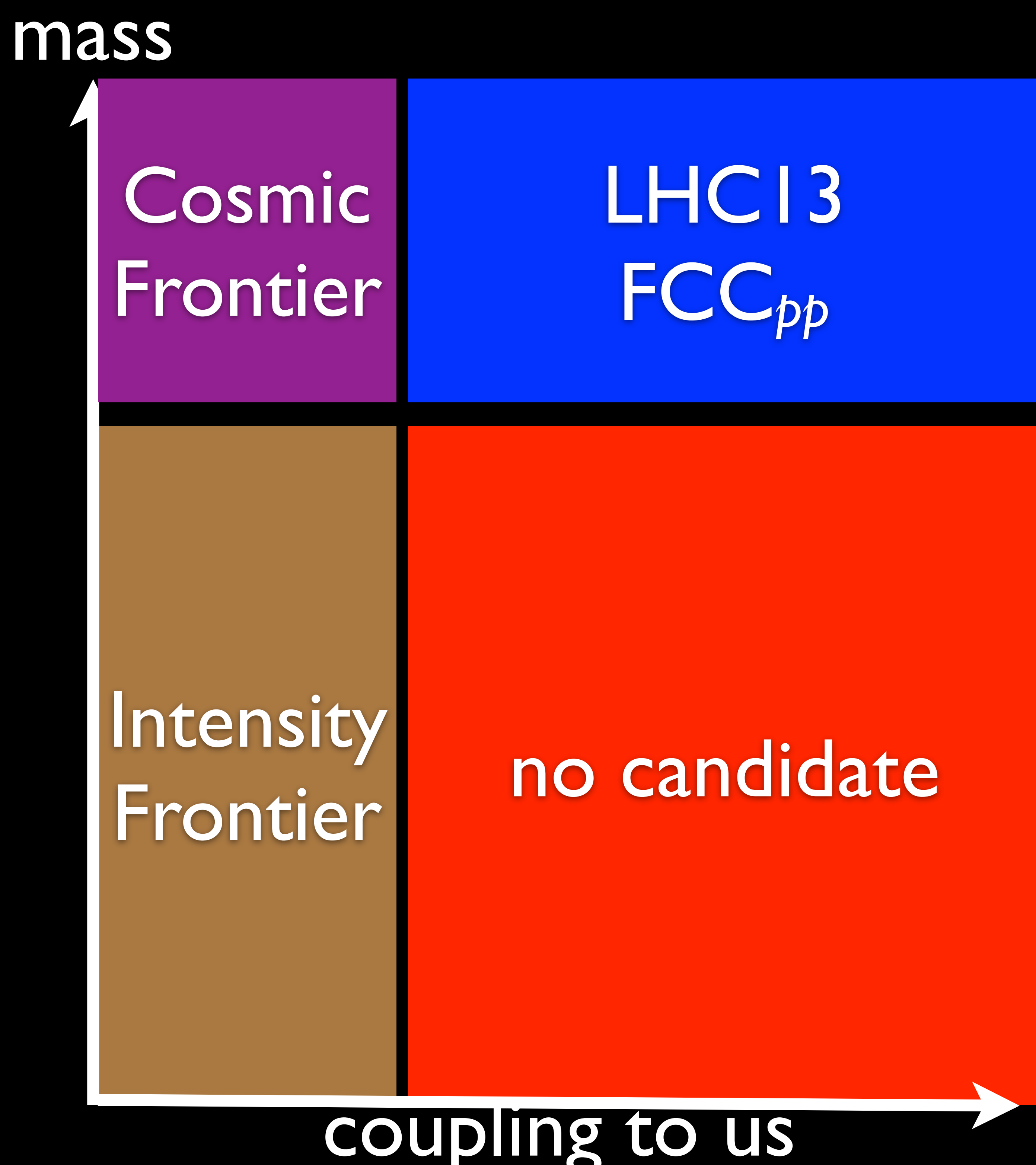
recent thinking

- dark matter definitely exists
  - naturalness problem is optional?
- need to explain dark matter on its own
- perhaps we should decouple these two
- do we really need big ideas like SUSY?
- perhaps we can solve it with ideas more familiar to us?

dark matter



dark matter

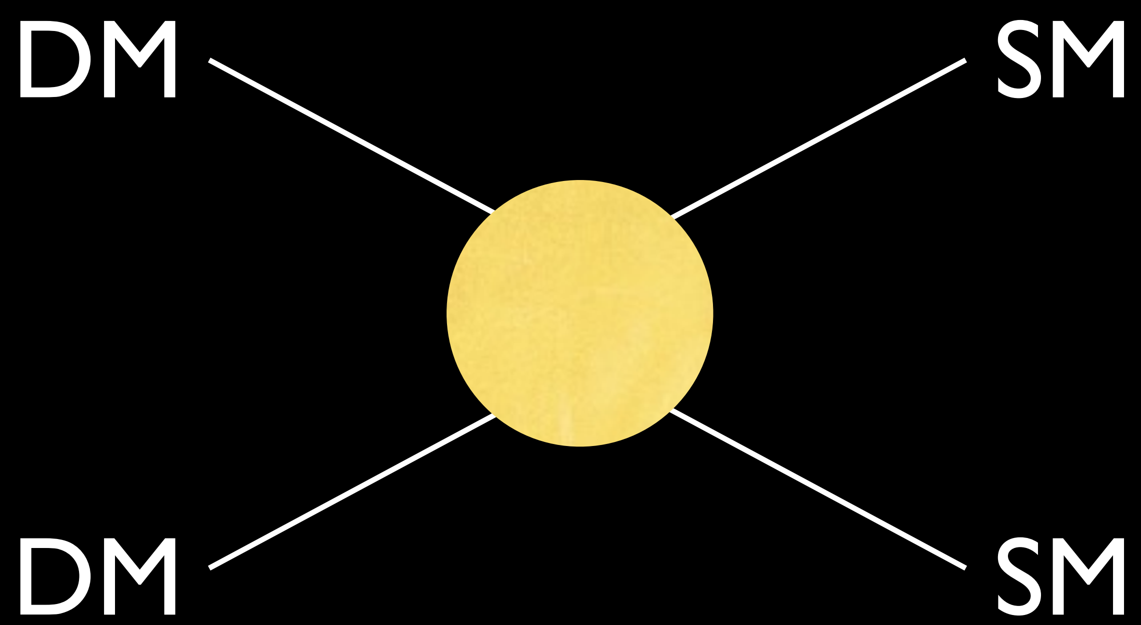

mass
Cosmic Frontier
LHC13
FCC$_{pp}$
Intensity Frontier
no candidate
coupling to us

Seminar in Berkeley
Strongly Interacting Massive Particle (SIMP)

Yonit Hochberg

$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\rm GeV}{m_{\rm DM}}$$

Miracles

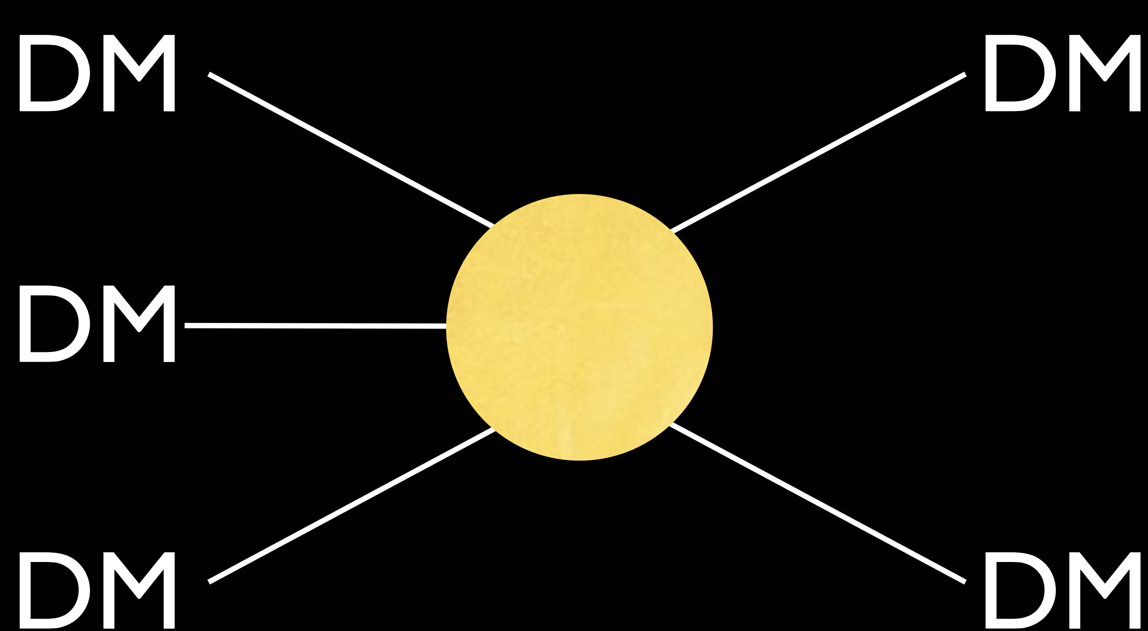


$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

$$m \approx 300 \text{ GeV}$$

WIMP miracle!



$$\langle \sigma_{3\to 2} v^2 \rangle \approx \frac{\alpha^3}{m^5}$$

$$\alpha \approx 4\pi$$

$$m \approx 300 \text{MeV}$$

Hochberg, Kuflik, Volansky, Wacker arXiv:1402.5143

SIMP miracle!

~~LEE WEINBERG~~ 3→2 FREEZE-OUT

Back of the envelope calculation

$$\Gamma_{\rm ann} \simeq H\big|_{\rm freezeout}$$

$$\boldsymbol{\Gamma_{\rm ann} \simeq n_{\rm dm}^2 \langle \sigma v^2 \rangle_{3\to 2}}$$

$$H \simeq \frac{T^2}{M_{\rm pl}}$$

$$m_{\rm dm} n_{\rm dm} \sim m_p n_b$$

$$\langle \sigma v^2 \rangle_{3\to 2} \simeq \frac{\alpha^3}{m_{\rm dm}^5}$$

$$n_b \sim \eta_b s$$

$$\eta_b \simeq T_{\rm eq}/m_p \qquad s \simeq T^3$$

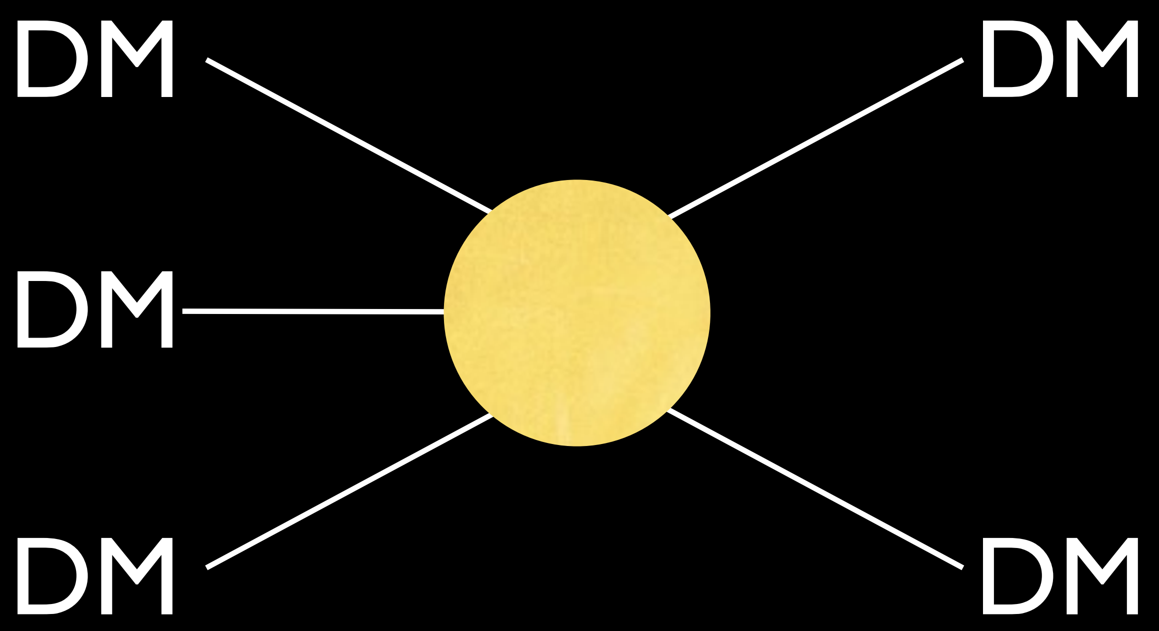
$$\boldsymbol{\Gamma_{\rm ann} \simeq \frac{T_{\rm eq}^2 \alpha^3}{x_F^4 \times m_{\rm dm}}} \qquad = \qquad H \simeq \frac{m_{\rm dm}^2}{M_{\rm pl} x_F^2}$$

Eric Kuflik

$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\rm GeV}{m_{\rm DM}}$$

SIMPlest Miracle

- Not only the mass scale is similar to QCD
- dynamics itself can be QCD! Miracle^3
- DM = pions
- e.g. SU(4)/Sp(4) = S^5



$$\mathcal{L}_{\rm chiral} = \frac{1}{16 f_\pi^2} {\rm Tr} \partial^\mu U^\dagger \partial_\mu U$$

+HM

arXiv:1411.3727

$$\mathcal{L}_{\rm WZW} = \frac{8 N_c}{15 \pi^2 f_\pi^5} \epsilon_{abcde} \epsilon^{\mu\nu\rho\sigma} \pi^a \partial_\mu \pi^b \partial_\nu \pi^c \partial_\rho \pi^d \partial_\sigma \pi^e + O(\pi^7)$$

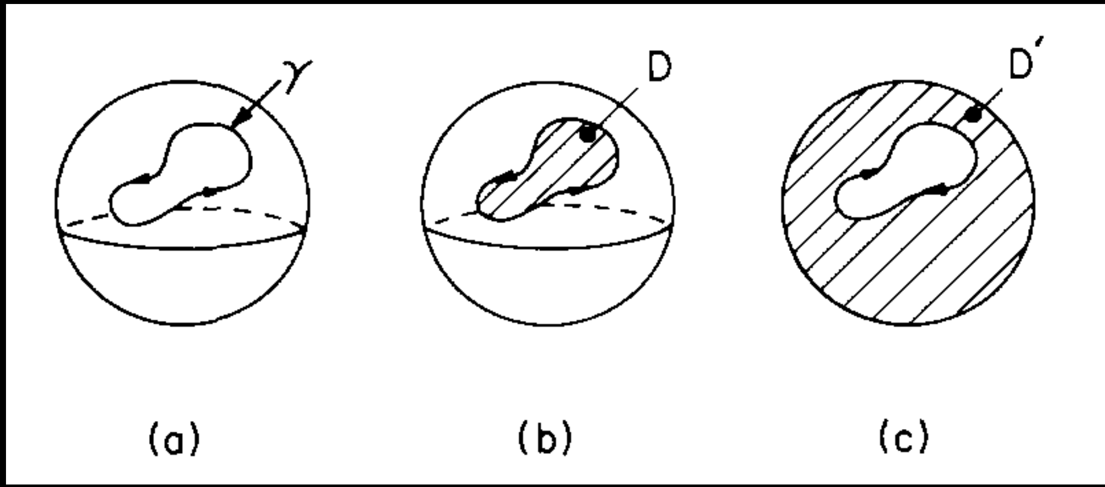
$$\pi_5(G/H) \neq 0$$

SIMPlest Miracle

- SU(2) gauge theory with four doublets
- SU(4)=SO(6) flavor symmetry
- $\langle q^i q^j \rangle \neq 0$ breaks it to Sp(2)=SO(5)
- coset space SO(6)/SO(5)=S^5
- $\pi_5(S^5)=\mathbb{Z} \Rightarrow$ Wess-Zumino term
- $\mathscr{L}_{WZ}=\varepsilon_{abcde}\, \varepsilon^{\mu\nu\rho\sigma}\, \pi^a \partial_\mu \pi^b \partial_\nu \pi^c \partial_\rho \pi^d \partial_\sigma \pi^e$

Wess-Zumino term

- SU(N_c) gauge theory
  - $\pi_5(SU(N_f))=\mathbb{Z}$ ($N_f \geq 3$)
- Sp(N_c) gauge theory
  - $\pi_5(SU(2N_f)/Sp(N_f))=\mathbb{Z}$ ($N_f \geq 2$)
- SO(N_c) gauge theory
  - $\pi_5(SU(N_f)/SO(N_f))=\mathbb{Z}$ ($N_f \geq 3$)



Witten

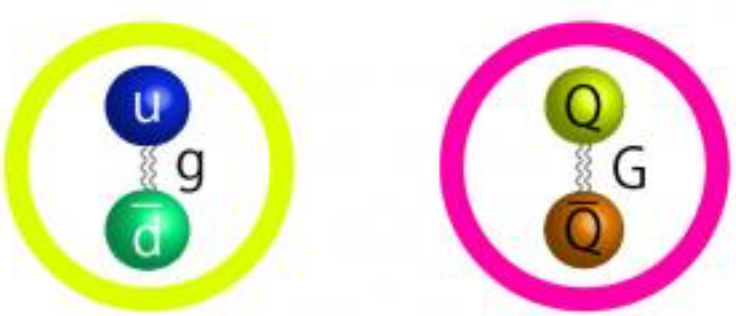
LAGRANGIANS

Quark theory

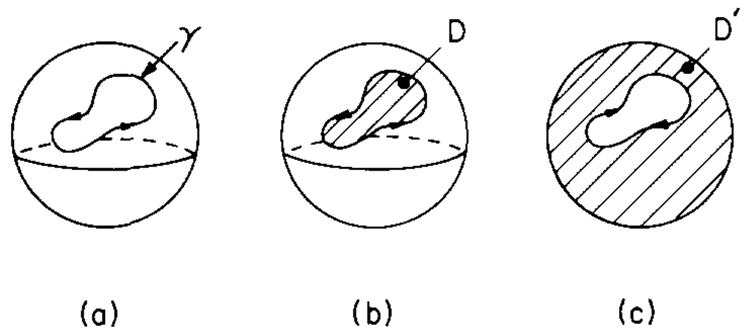
$$\mathcal{L}_{\rm quark} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{q}_i i\not{D} q_i \; -\frac{1}{2}m_Q \; J^{ij}q_i q_j + h.c.$$

Sigma theory

$$\mathcal{L}_{\rm Sigma} = \frac{f_\pi^2}{16}\mathrm{Tr}\partial_\mu\Sigma\;\partial^\mu\Sigma^\dagger - \frac{1}{2}m_Q\mu^3\mathrm{Tr}J\Sigma + h.c. - \frac{iN_c}{240\pi^2}\int \mathrm{Tr}(\Sigma^\dagger d\Sigma)^5$$

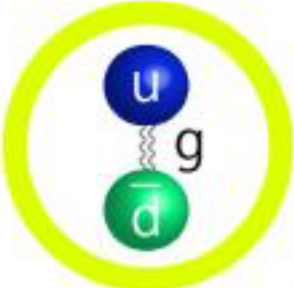


Pion theory

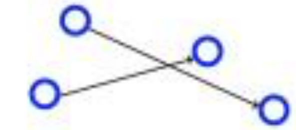


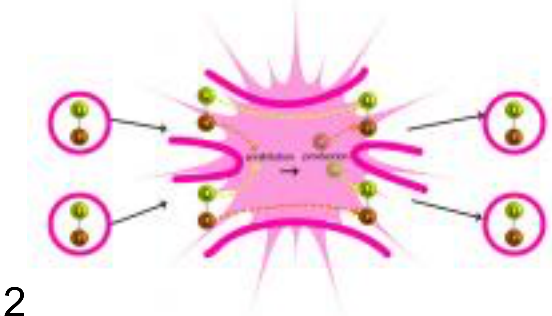
$$\mathcal{L}_{\rm pion} = \frac{1}{4}\mathrm{Tr}\partial_\mu\pi\partial^\mu\pi - \frac{m_\pi^2}{4}\mathrm{Tr}\pi^2 + \frac{m_\pi^2}{12f_\pi^2}\mathrm{Tr}\pi^4 - \frac{1}{6f_\pi^2}\mathrm{Tr}\left(\pi^2\partial^\mu\pi\partial_\mu\pi - \pi\partial^\mu\pi\pi\partial_\mu\pi\right)$$

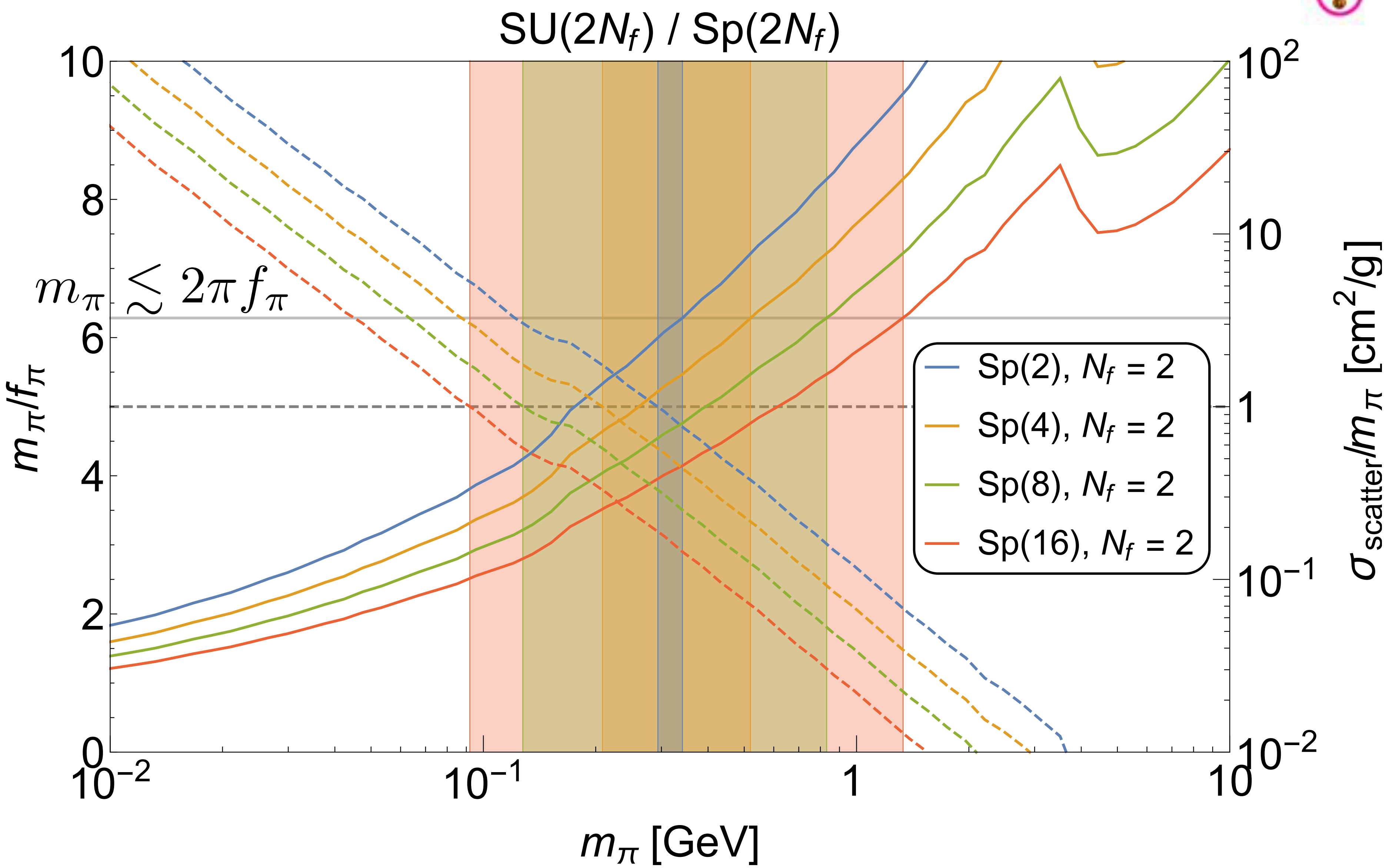
$$+ \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi\right] + \mathcal{O}(\pi^6)$$



The Results





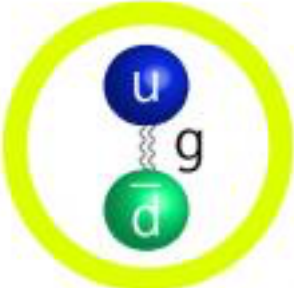


Solid curves: solution to Boltzmann eq.

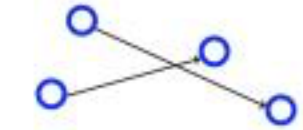
Dashed curves: along that solution

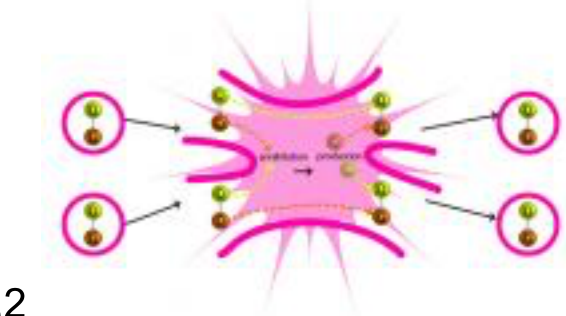
$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

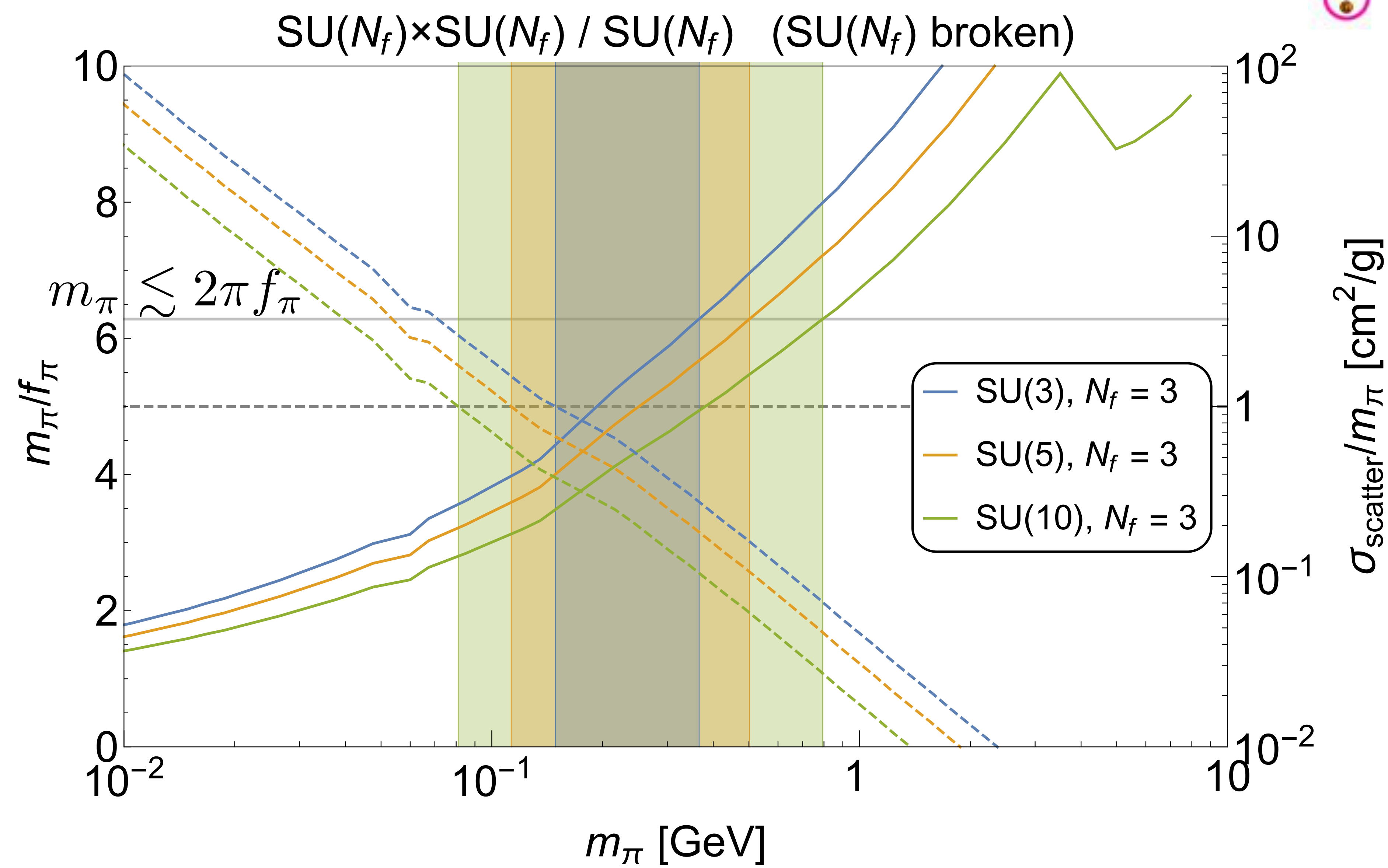
$$\frac{\sigma_{\mathrm{scatter}}}{m_\pi} \propto m_\pi^{-9/5}$$



The Results





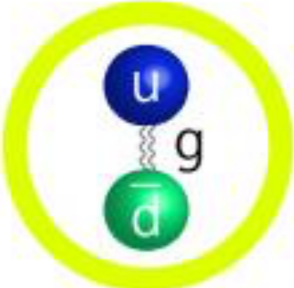


Solid curves: solution to Boltzmann eq.

$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

Dashed curves: along that solution

$$\frac{\sigma_{\rm scatter}}{m_\pi} \propto m_\pi^{-9/5}$$



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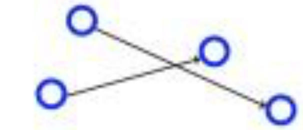
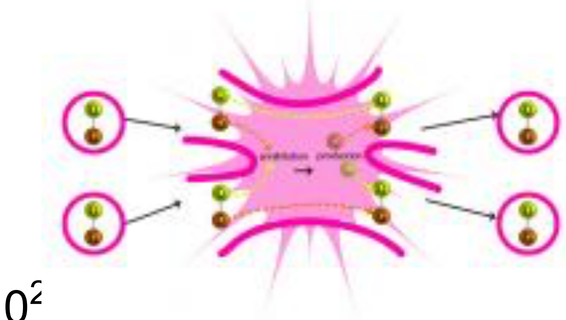

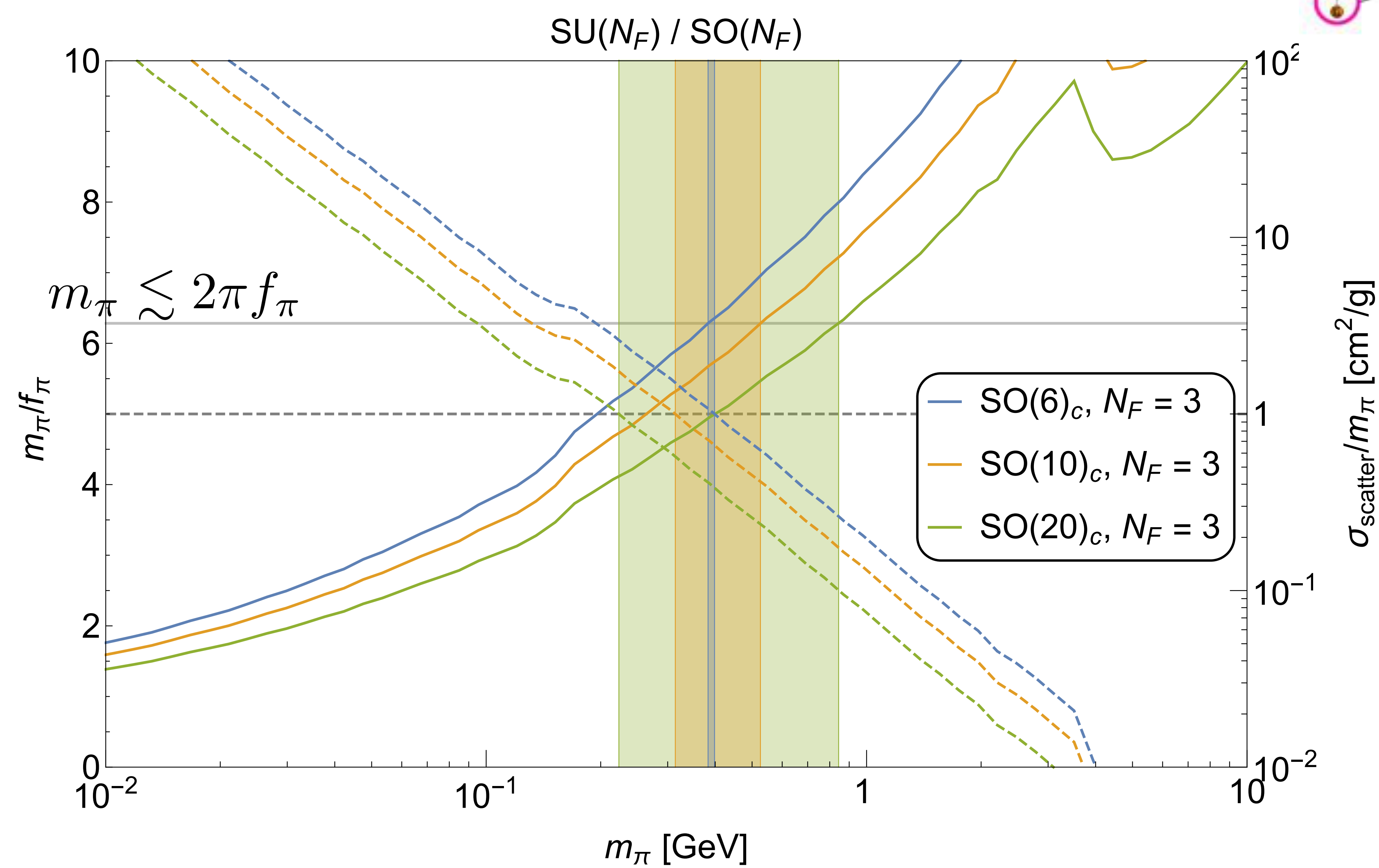

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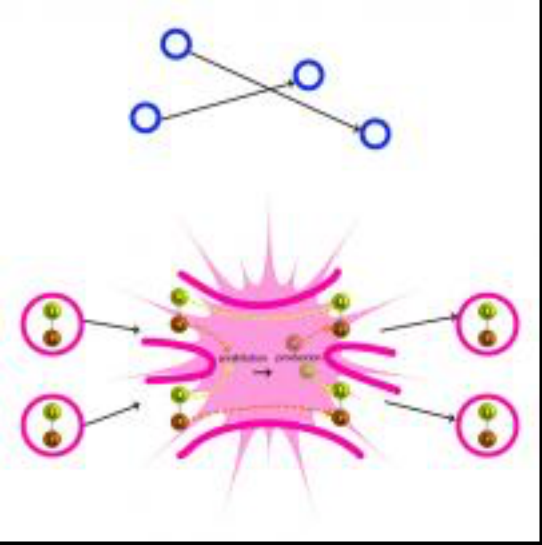
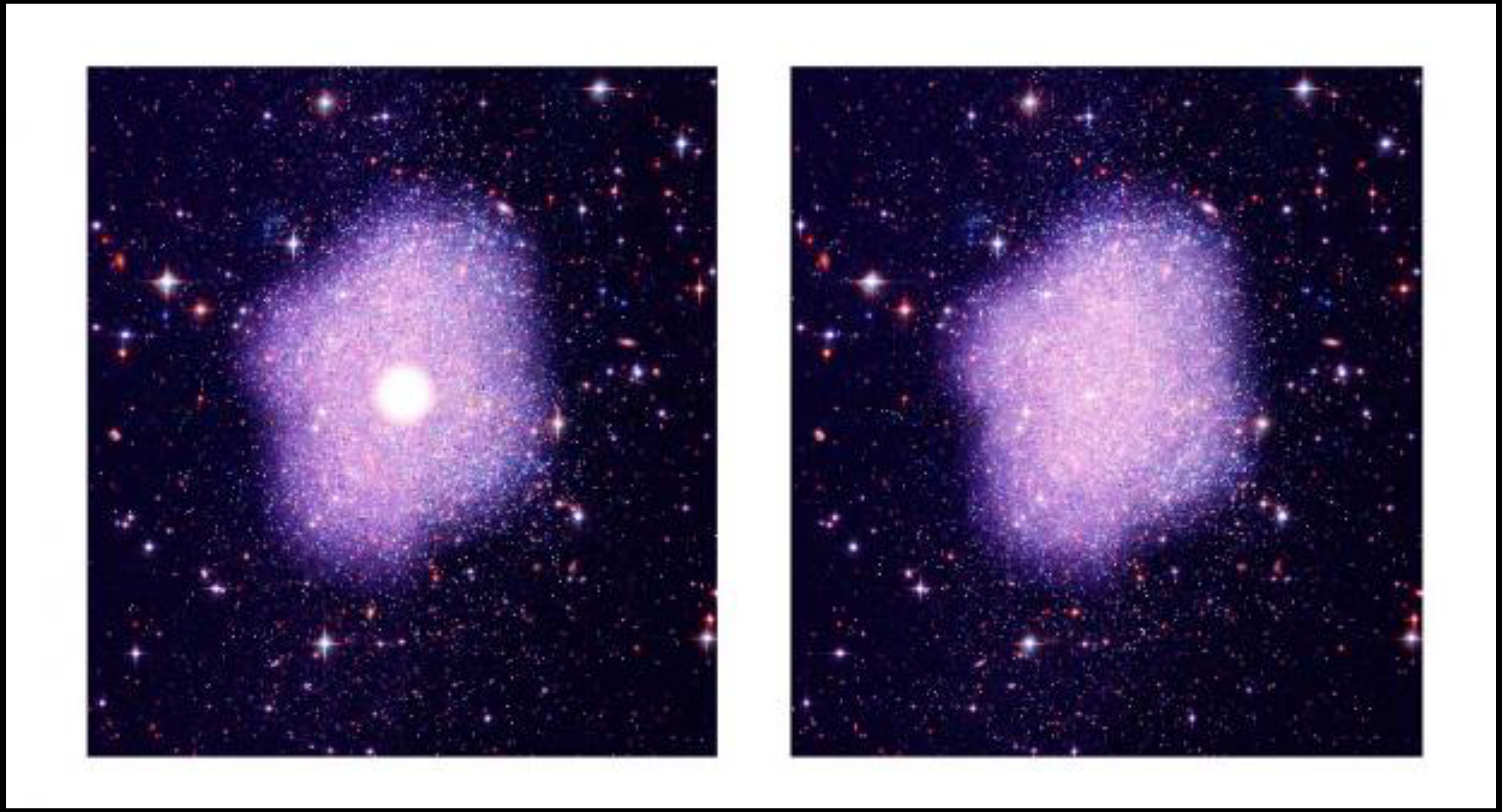
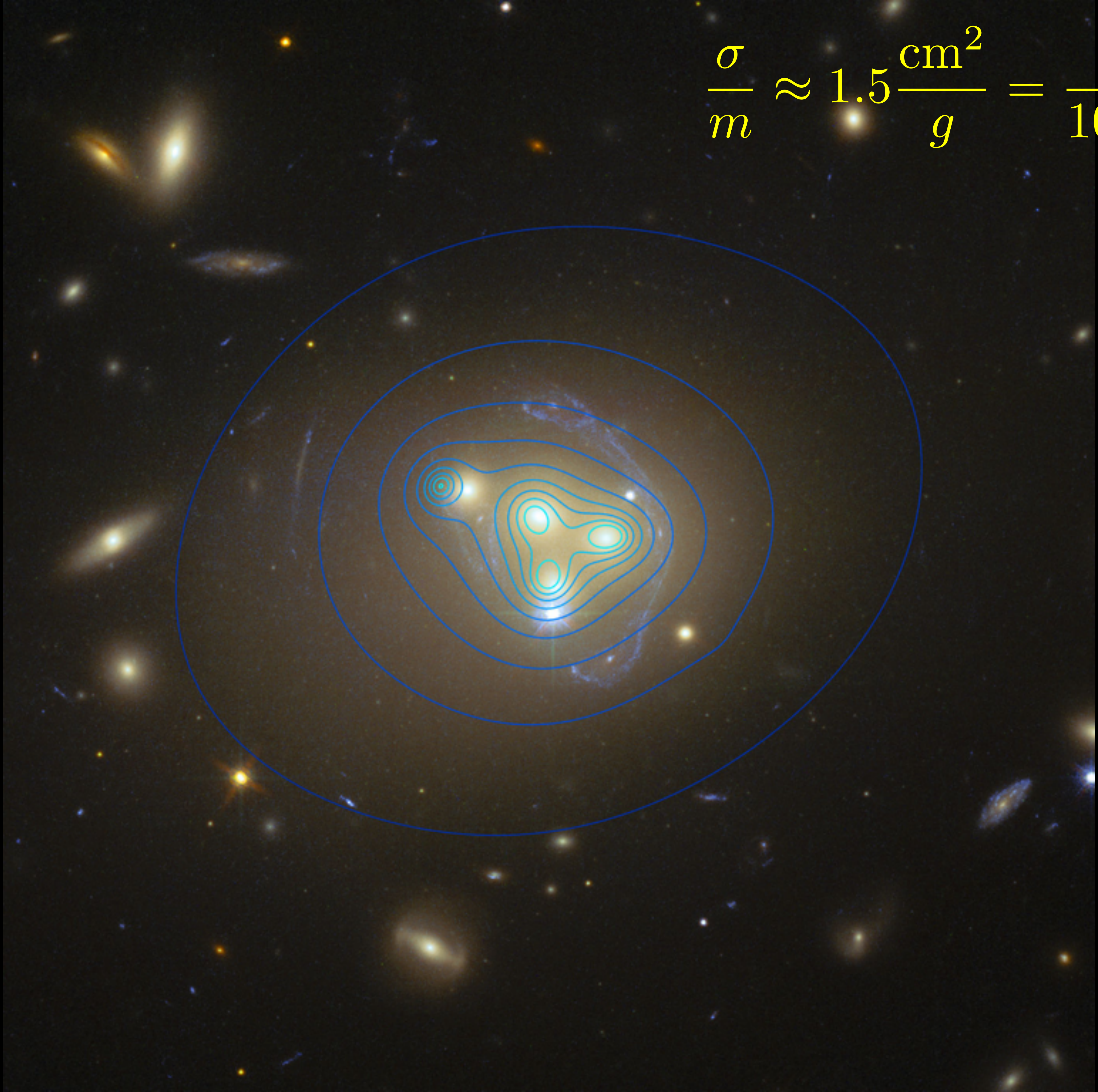
$$\frac{\sigma_{\rm scatter}}{m_\pi} \propto m_\pi^{-9/5}$$

self interaction



- self interaction of $\sigma/m \sim 10^{-24}\mathrm{cm}^2$ / 300MeV
- flattens the cusps in NFW profile
- actually desirable for dwarf galaxies?

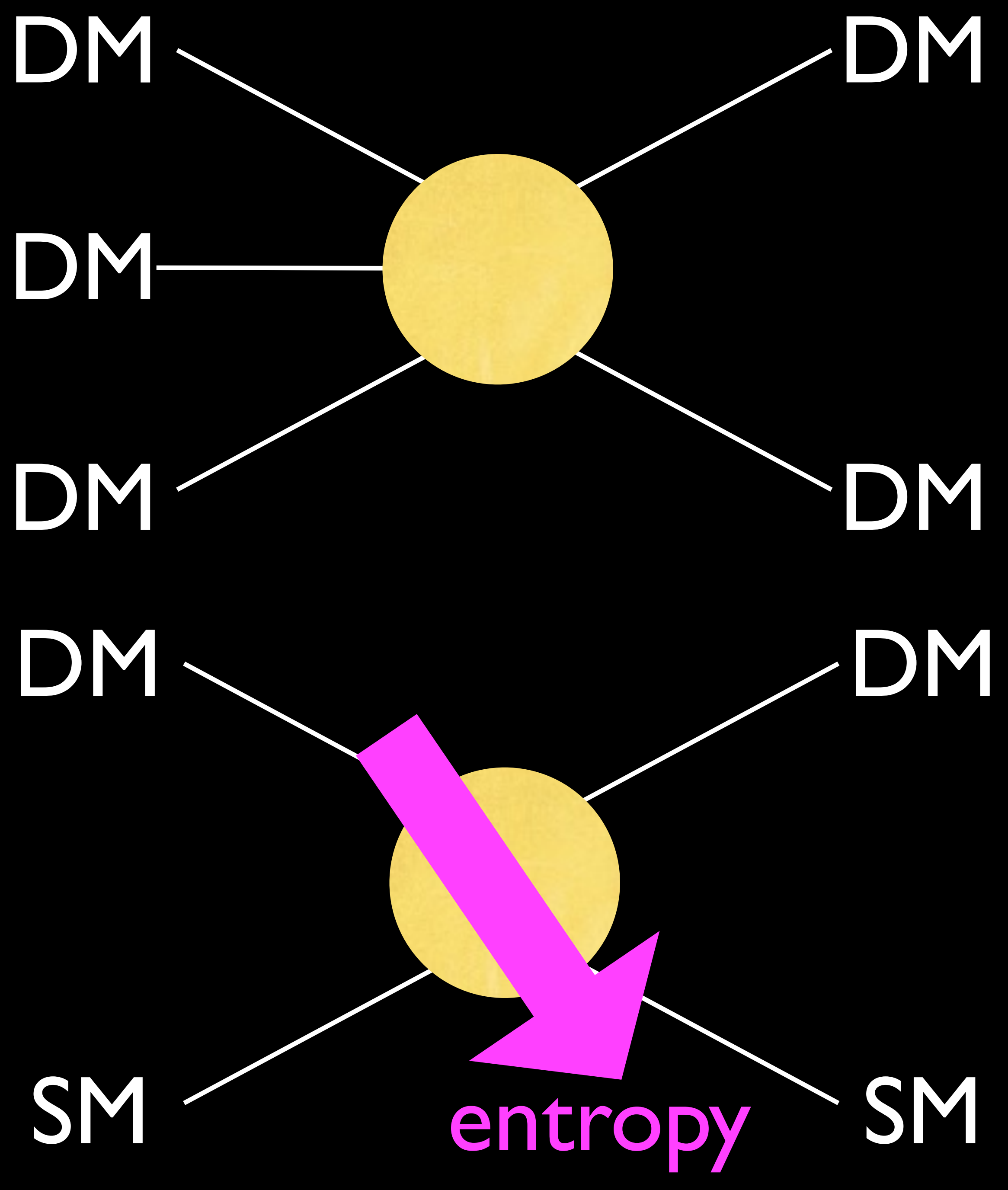




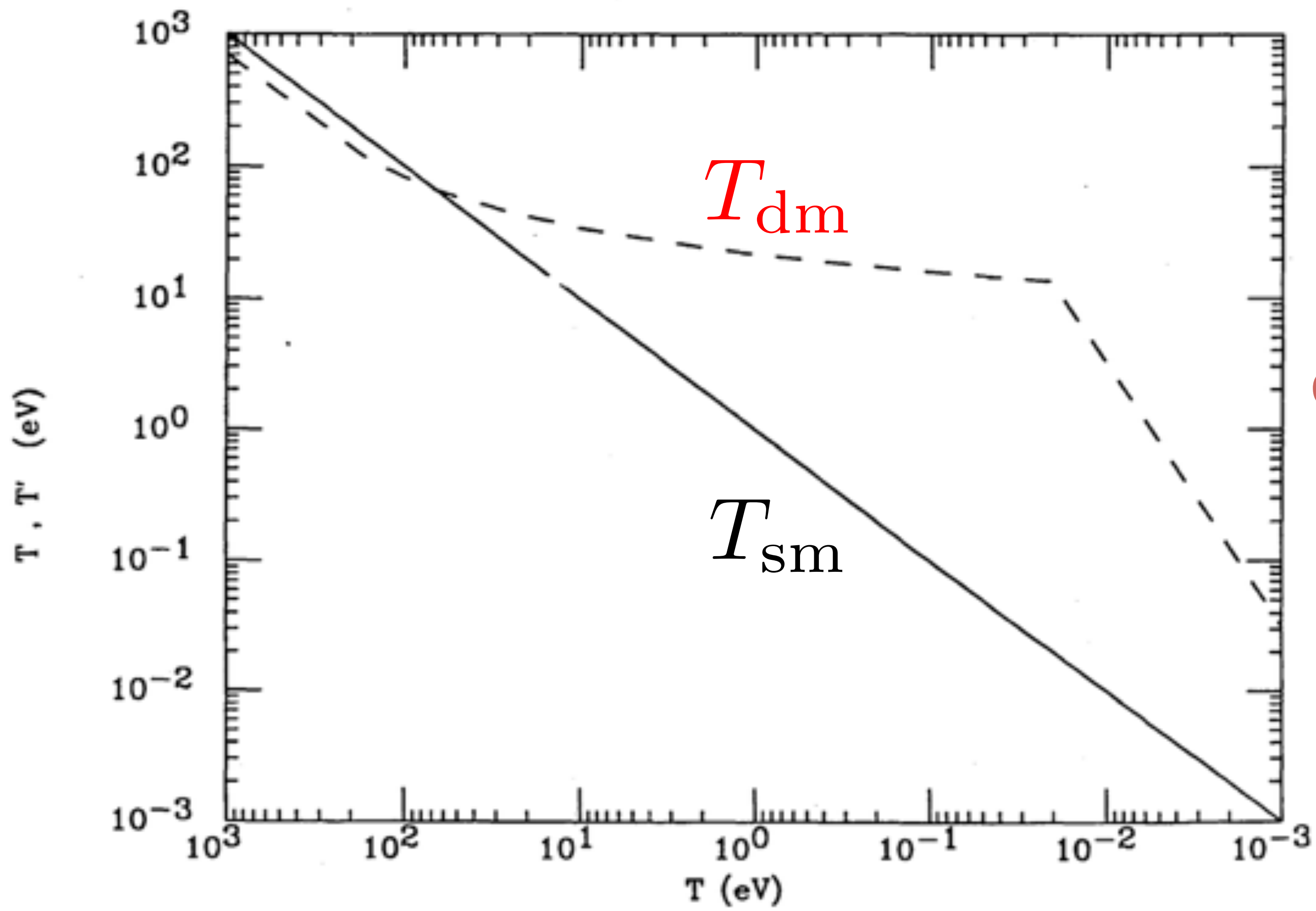
$$\frac{\sigma}{m} \approx 1.5\frac{\mathrm{cm}^2}{g} = \frac{0.27\mathrm{b}}{100\mathrm{MeV}}$$

communication

- 3 to 2 annihilation
- excess entropy *must* be transferred to $e^{\pm}$, γ
- need communication at some level
- leads to experimental signal



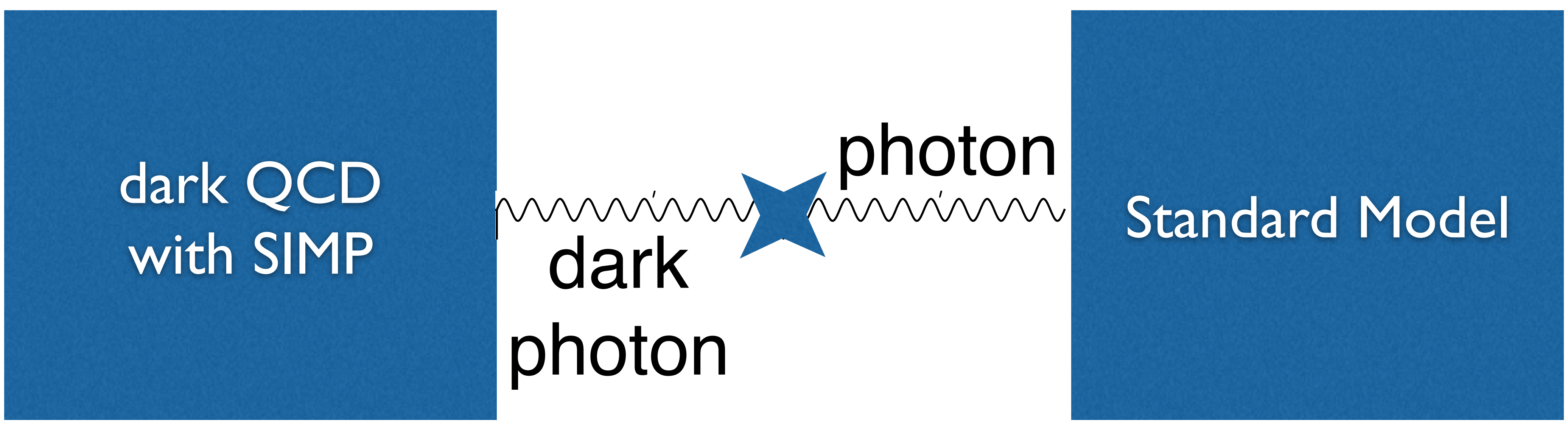
if totally decoupled



Carlson, Hall and Machacek, Astrophys. J. 398, 43 (1992)

- 3→2 annihilations without heat exchange is excluded by structure formation, [de Laix, Scherrer and Schaefer, Astrophys. J. 452, 495 (1995)]

vector portal



$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

Kinetically mixed U(1)

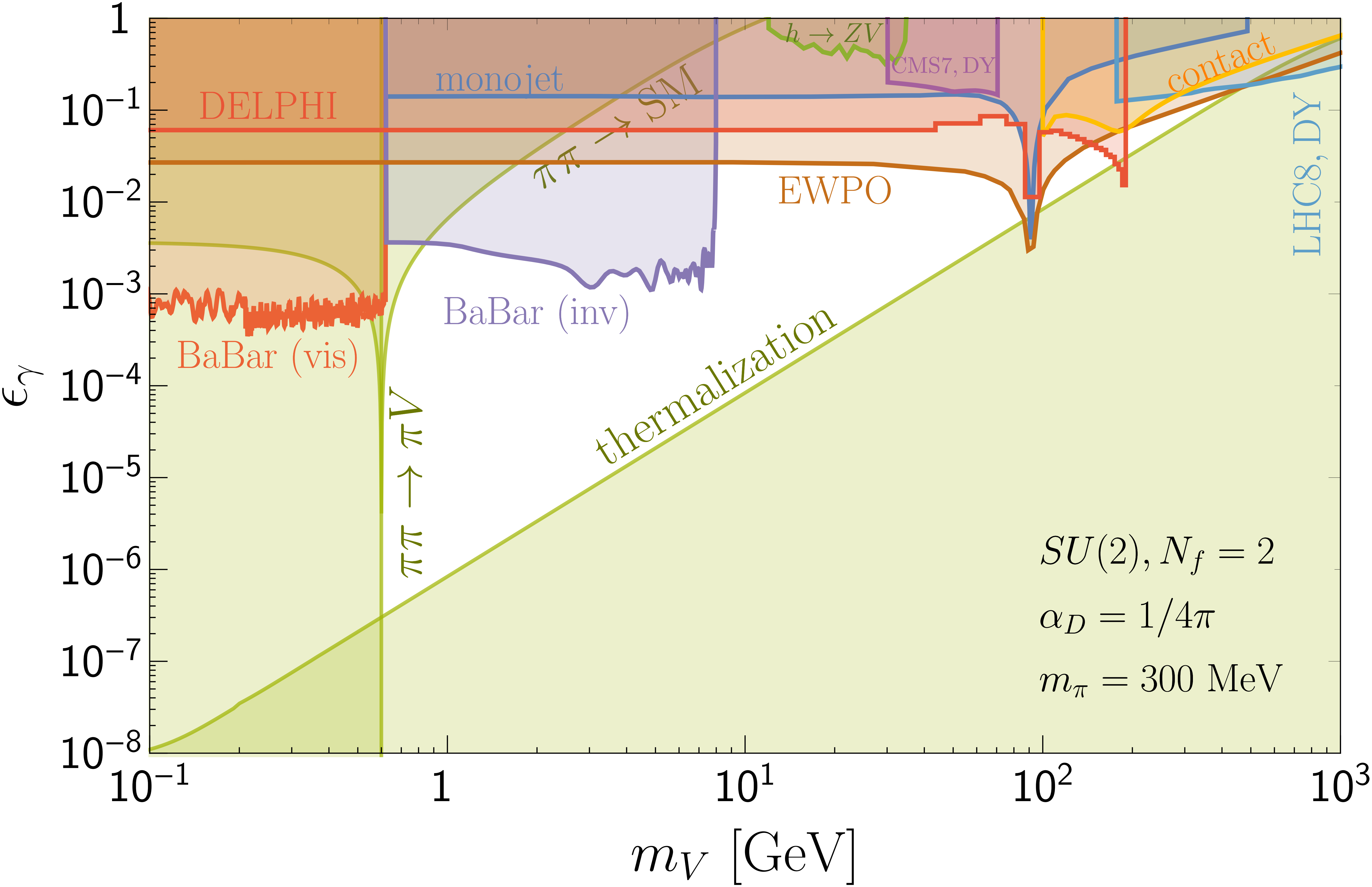
- e.g., the SIMPlest model SU(2) gauge group with N_f=2 (4 doublets)
- gauge U(1)=SO(2) ⊂ SO(2) ×SO(3) ⊂ SO(5)=Sp(4)
- maintains degeneracy of quarks
- near degeneracy of pions for co-annihilation

$$SU(4)/Sp(4) = S^5$$

$$(q^+, q^+, q^-, q^-)$$

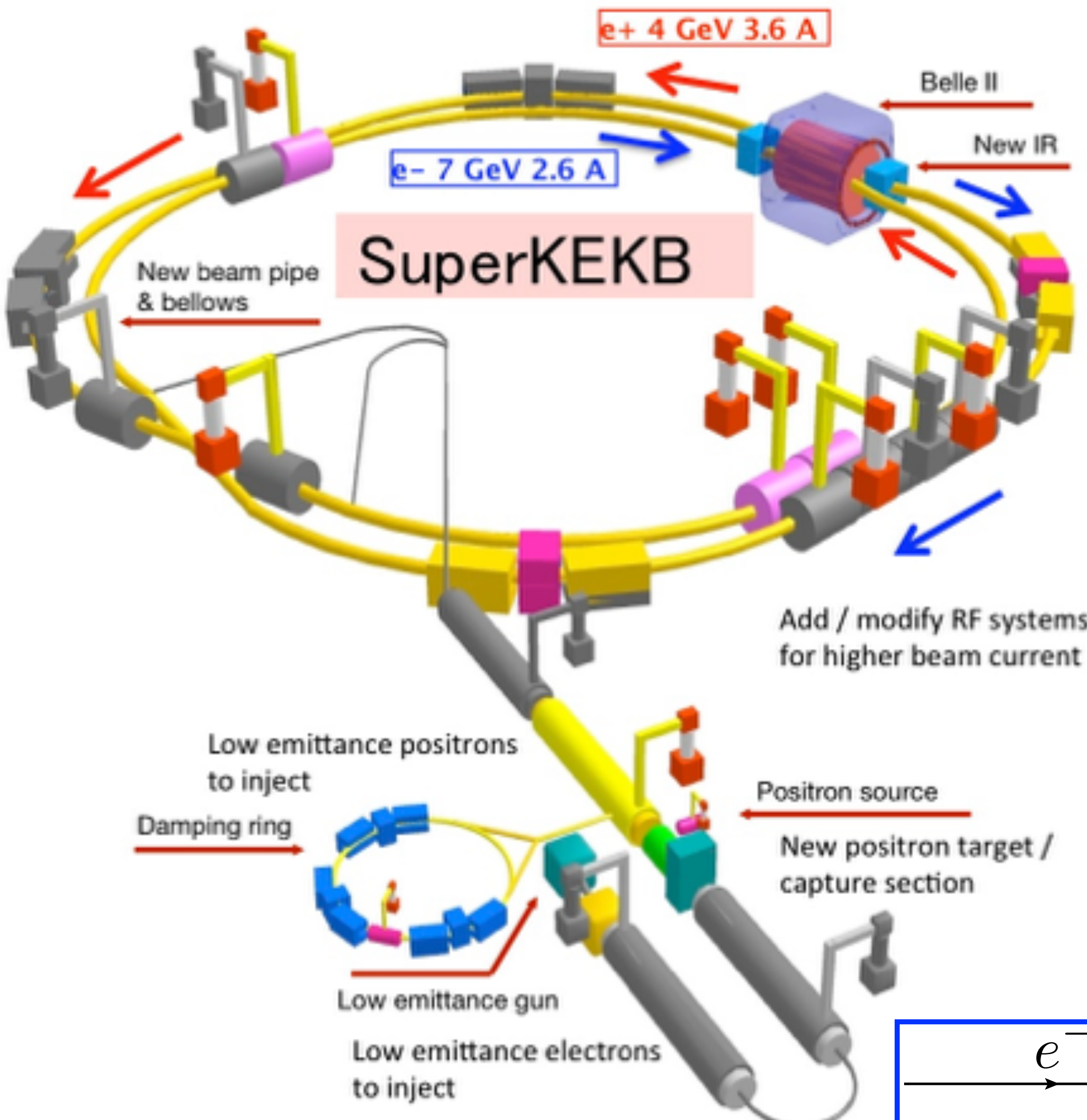
$$(\pi^{++}, \pi^{--}, \pi_x^0, \pi_y^0, \pi_z^0)$$

$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

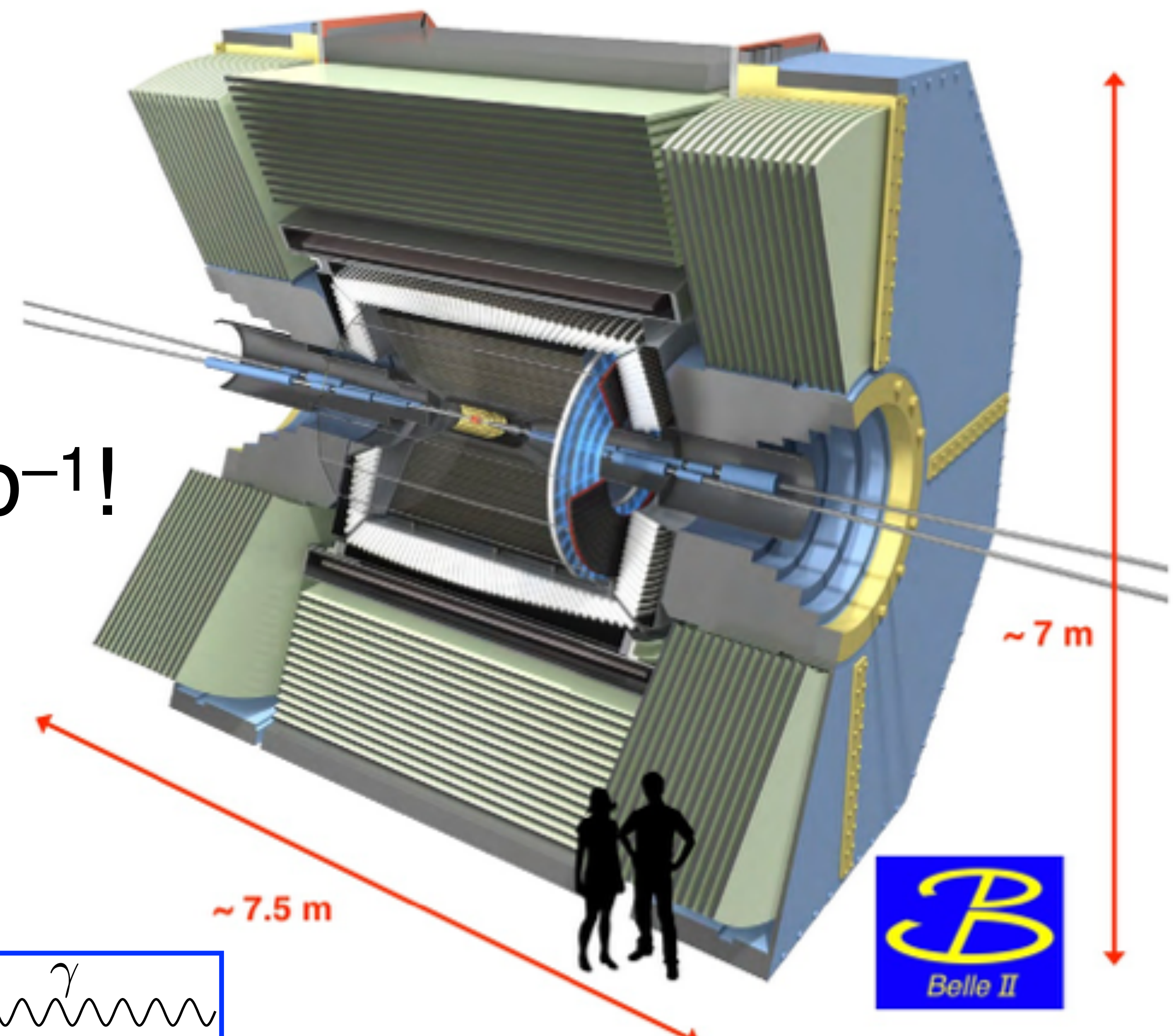


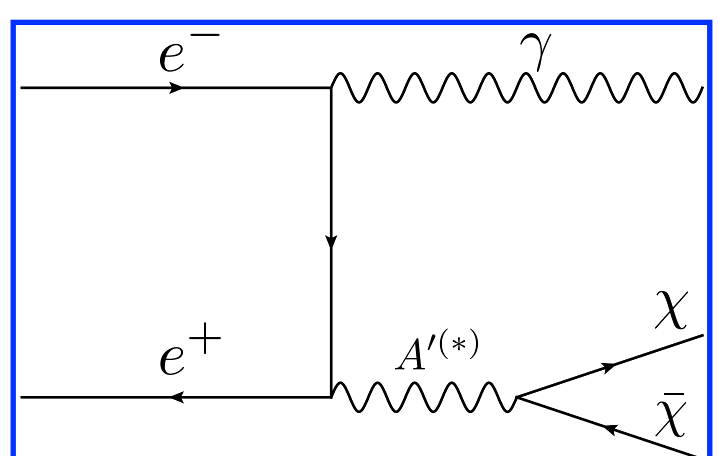
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^{1}
10^{2}
10^{3}
m_V [GeV]
DELPHI
monojet
$\pi\pi \to SM$
$h \to ZV$
CMS7, DY
contact
LHC8, DY
EWPO
BaBar (inv)
BaBar (vis)
$\pi\pi \to \pi V$
thermalization
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

Super KEK B & Belle II

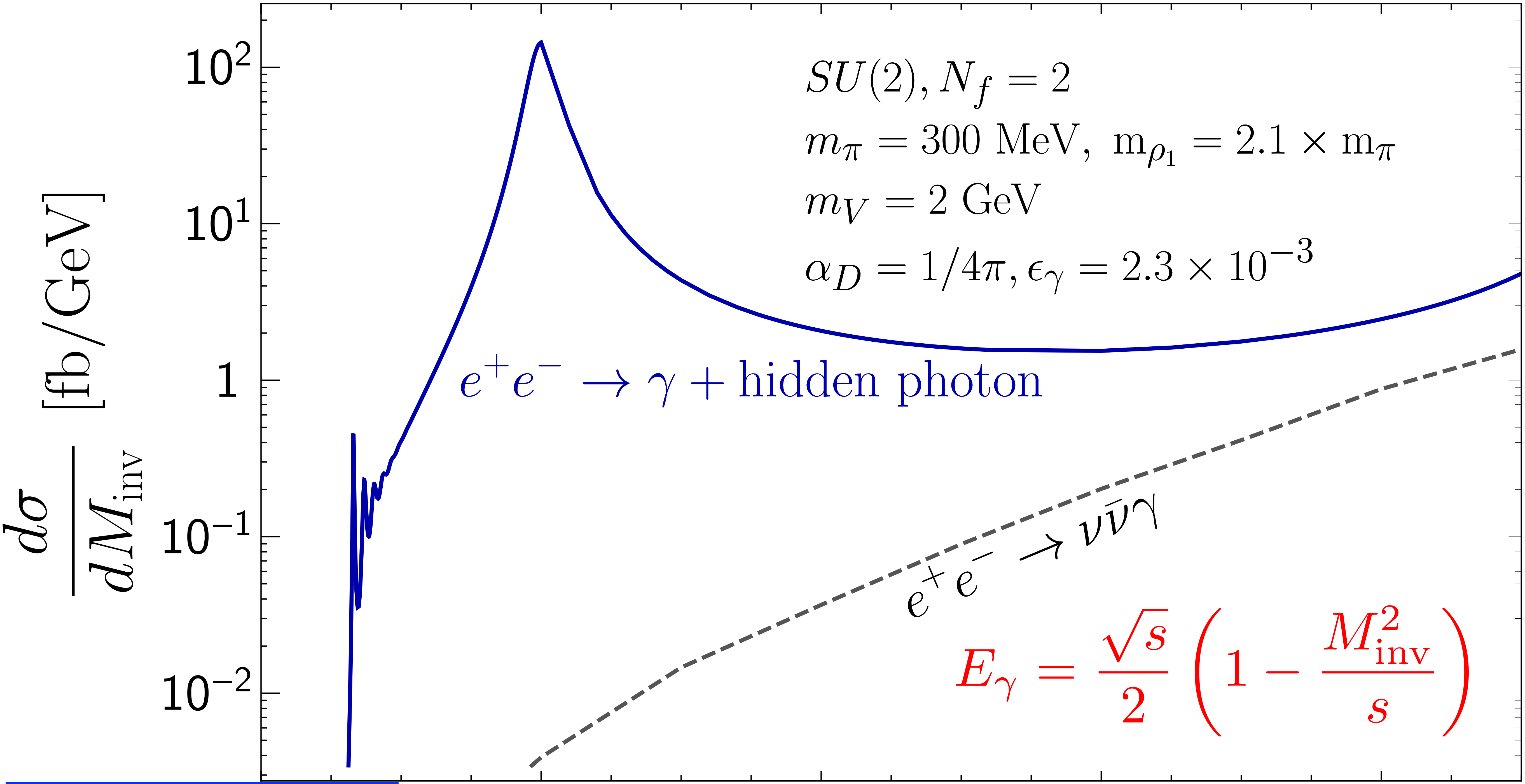


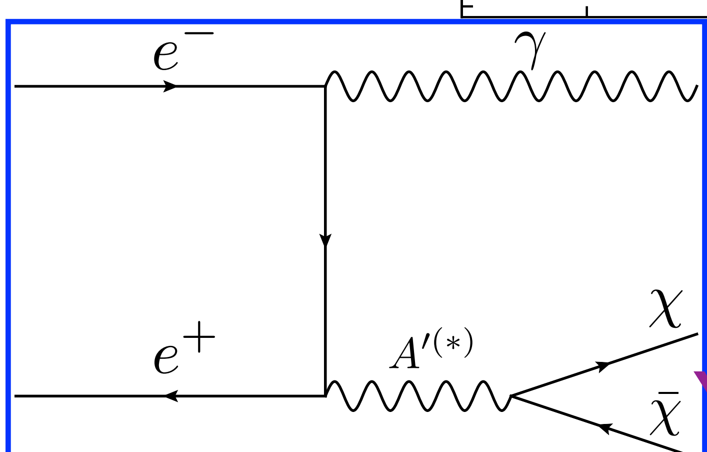
50 ab^{-1}!



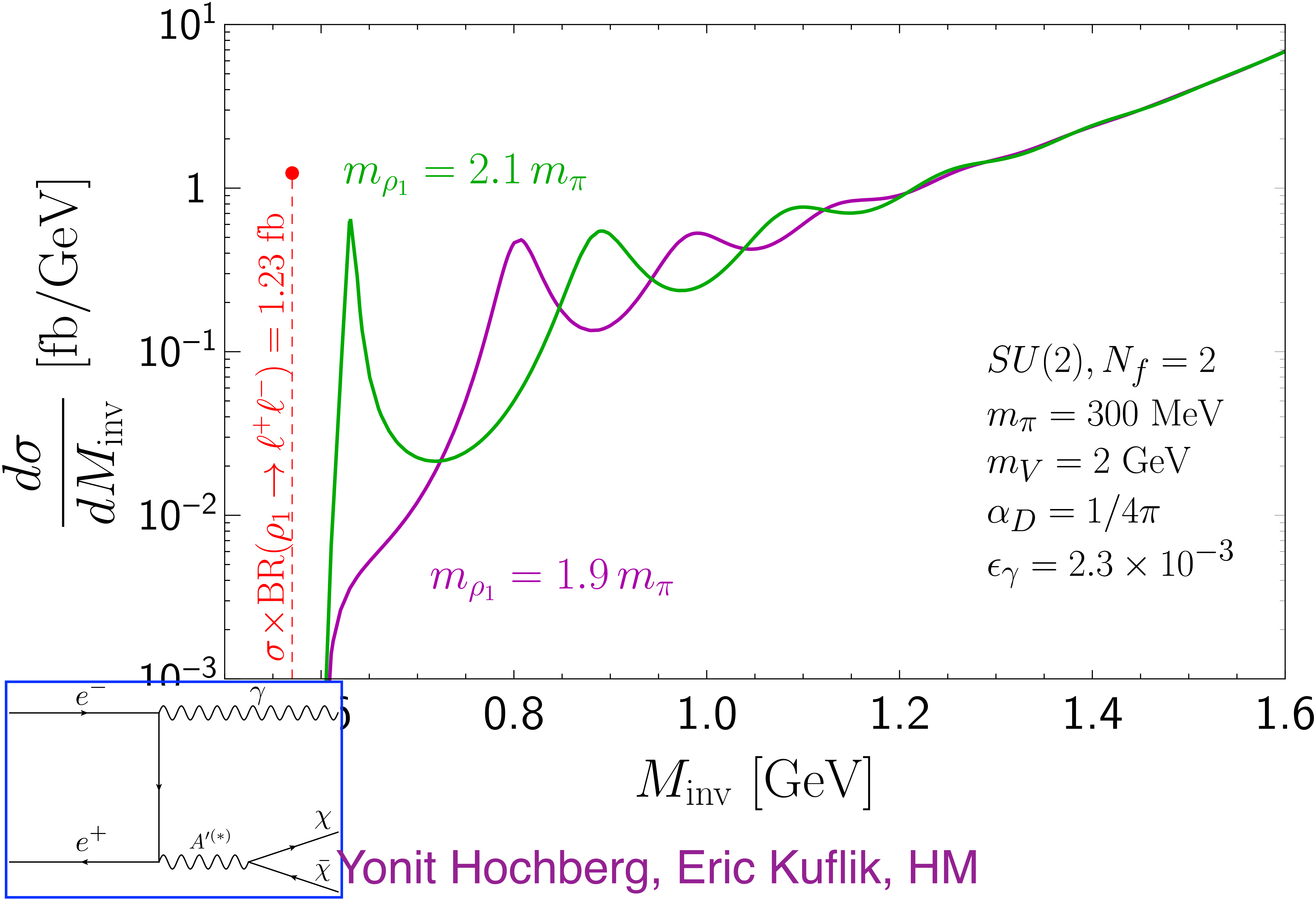


$$E_\gamma = \frac{\sqrt{s}}{2}\left(1 - \frac{M_{\mathrm{inv}}^2}{s}\right)$$

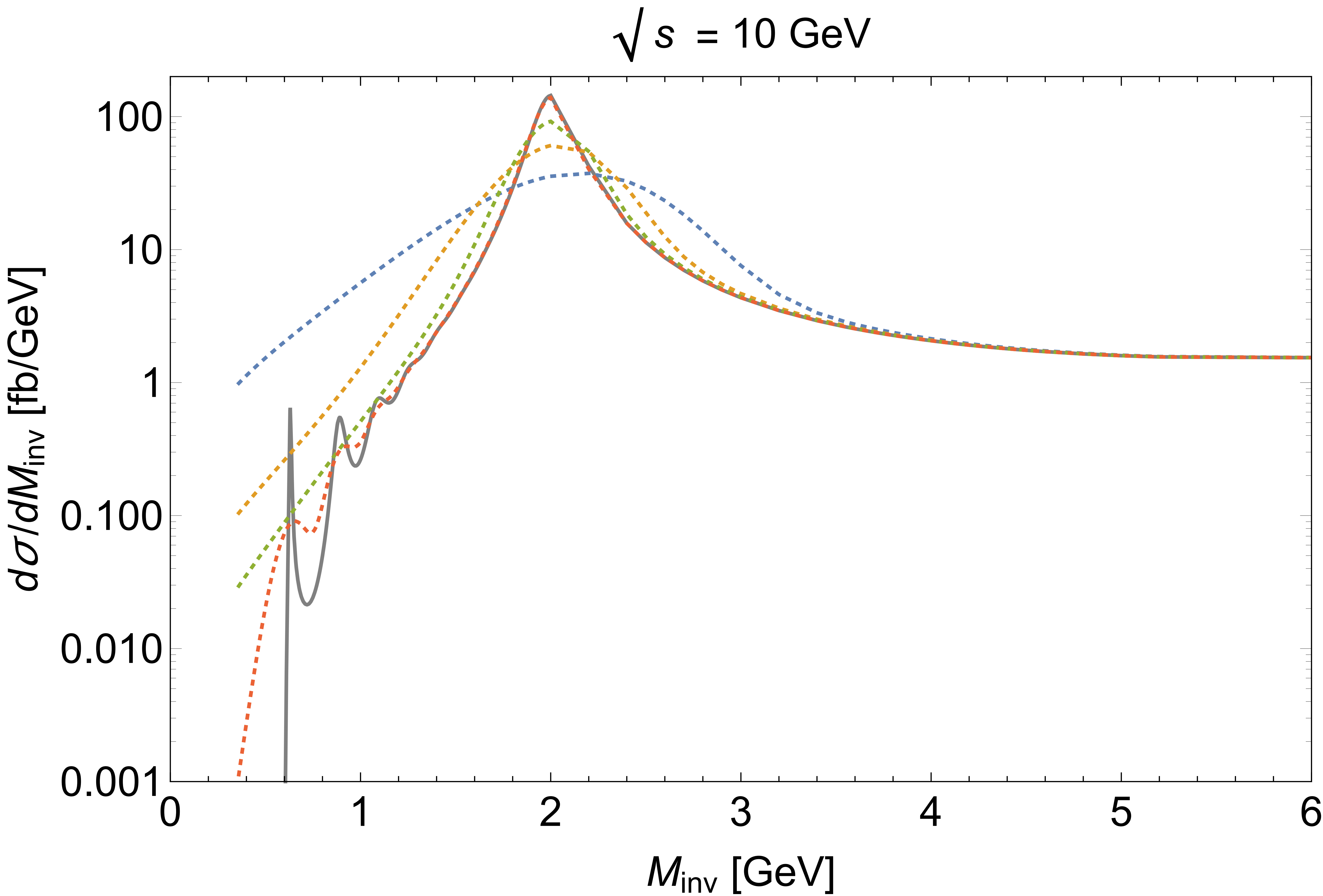




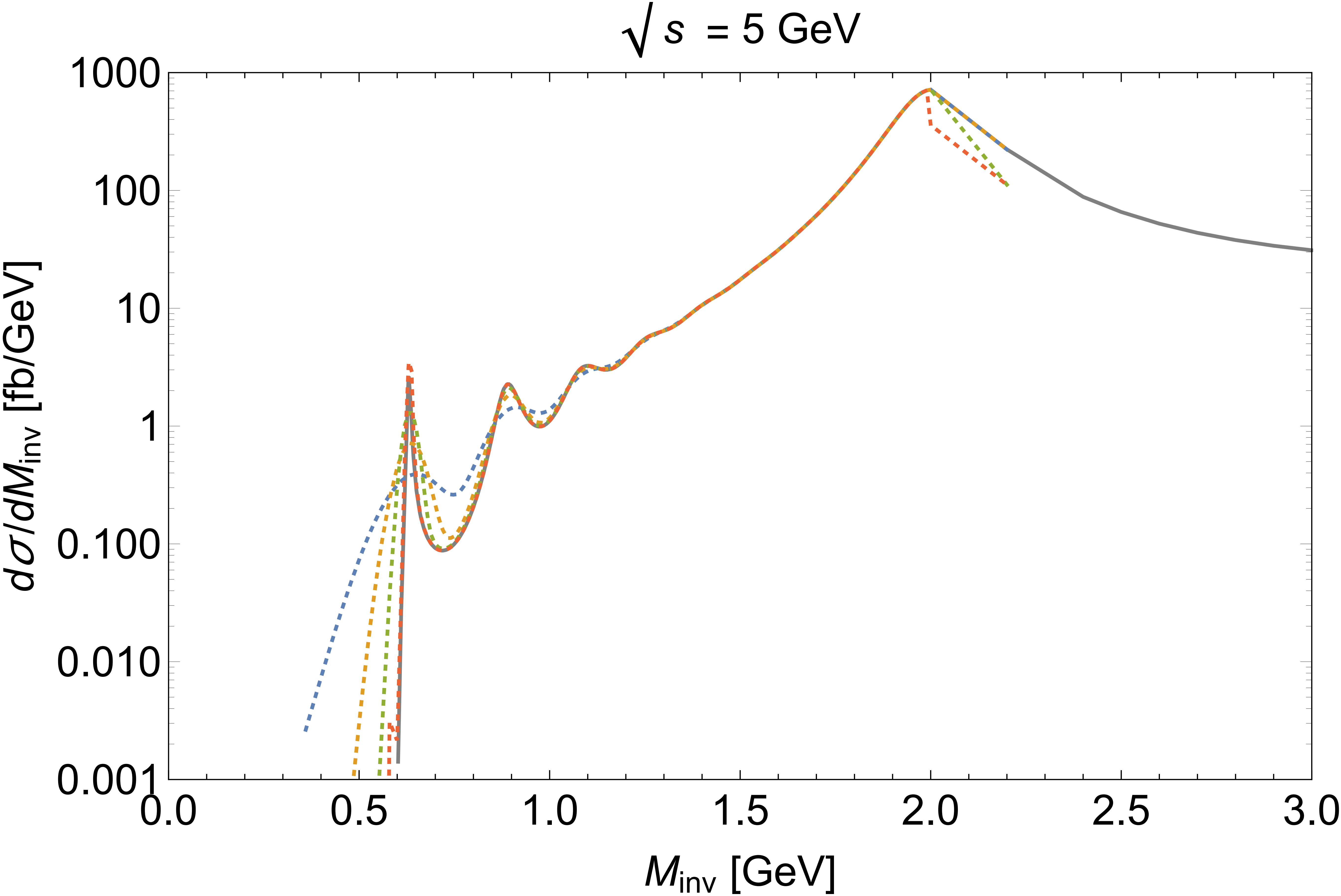
Yonit Hochberg, Eric Kuflik, HM



Yonit Hochberg, Eric Kuflik, HM



Yonit Hochberg, Eric Kuflik, HM



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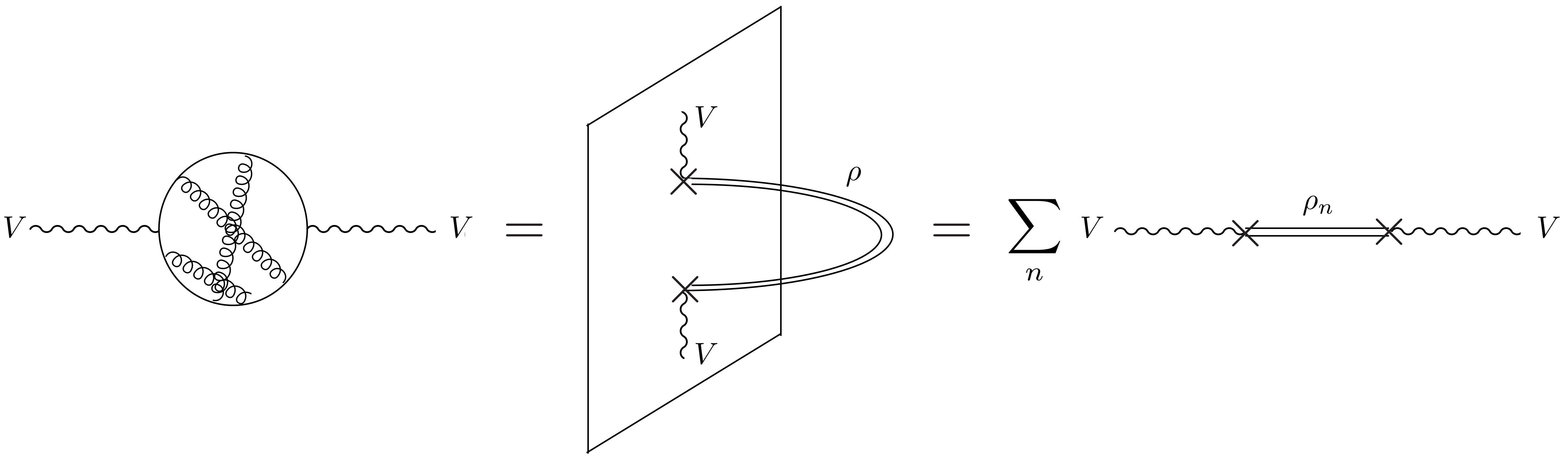
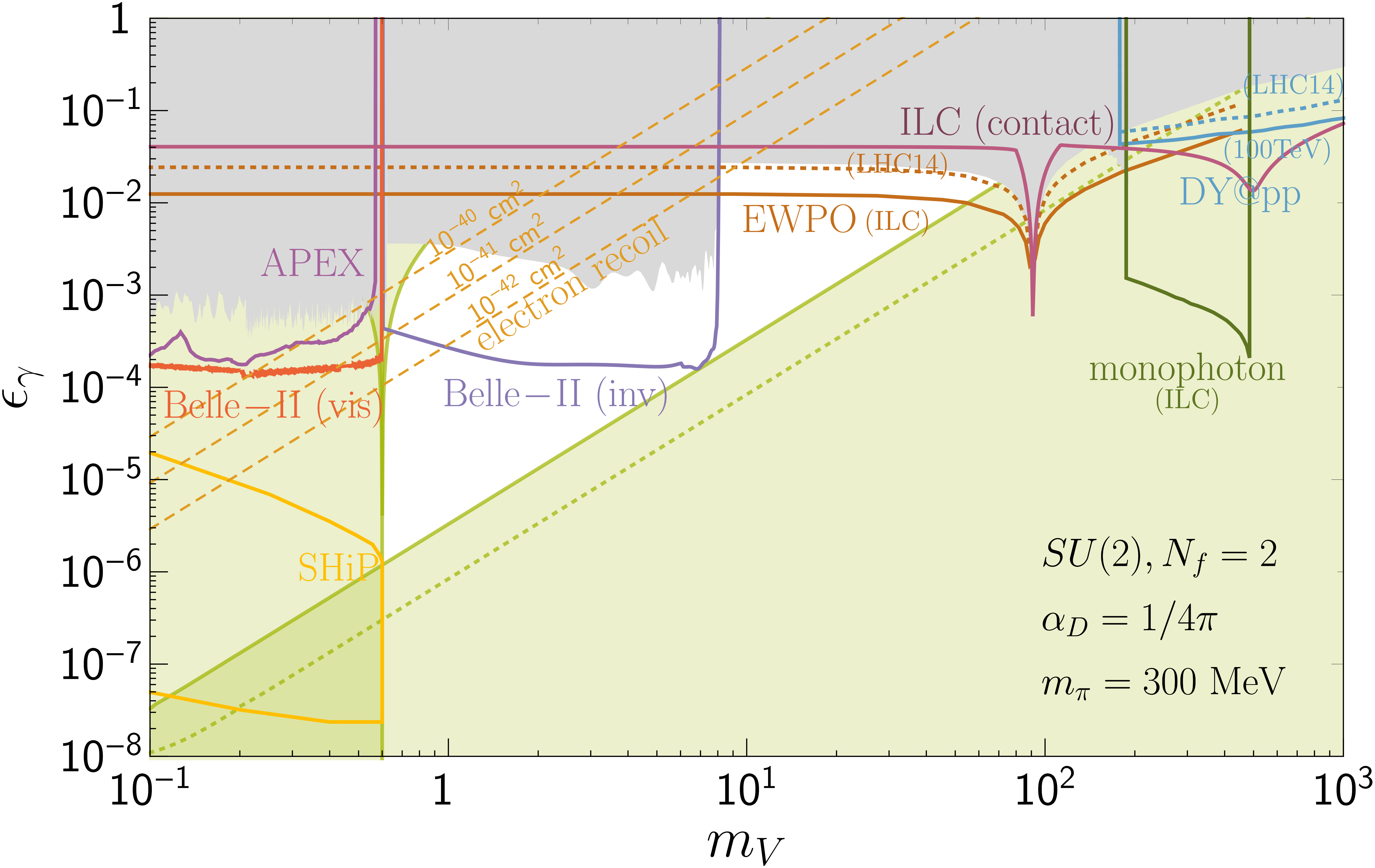
Holographic QCD



inspired by AdS/CFT from string theory



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^1
10^2
10^3
m_V
APEX
Belle−II (vis)
Belle−II (inv)
SHiP
10^{-40} cm^2
10^{-41} cm^2
10^{-42} cm^2
electron recoil
ILC (contact)
(LHC14)
EWPO (ILC)
(LHC14)
(100TeV)
DY@pp
monophoton
(ILC)
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

Conclusion

- surprising an *old* theory for dark matter
- SIMP Miracle^3
    - mass ~ QCD
    - coupling ~ QCD
    - theory ~ QCD
- can solve problem with DM profile
- very rich phenomenology
- Exciting *dark spectroscopy*!

Classification of Effective Operators

with Brian Henning, Xiaochuan Lu, Thomas Melia

arXiv:1507.07240, 1512.03433, 160x.xxxxx

why effective operators

- In the absence of any concrete signal of new particles, we need to discuss effective operators to *go beyond Higgs*, *i.e.*probe physics at higher energies or weaker couplings
  - precision Higgs
  - precision flavor
  - B, L violation
- similar to four-fermion operators in weak interactions

Nambu Jona-Lasinio

- D=6 four-fermion operator
  - can cause fermion bi-linear condensate
  - inspired by the BCS theory
  - gap equation to solve for the condensate
- Simple but important question:
  - In a given field theory, what is the complete set of higher dimension operators?

Effective Operators

- Surprisingly difficult question
- In the case of the Standard Model
  - Weinberg (1980) on D=6 $\not{B}$, D=5 $\not{L}$
  - Buchmüller-Wyler (1986) on D=6 ops
    - 80 operators for N_f=1, B, L conserving
  - Grzadkowski et al (2010) removed redundancies and discovered one missed
    - 59 operators for N_f=1, B, L conserving
  - Mahonar et al (2013) general N_f
  - Lehman-Martin (2014,15) D=7 for general N_f, D=8 for N_f=1 (incomplete)

Repeating this at order ϵ^6 we obtain the Hilbert series for dimension-six operators of the SM EFT:

$\not{B}, \not{L}$

$$
\begin{aligned}
\widehat{H}_6 \; = \; & H^3 H^{\dagger 3} + u^\dagger Q^\dagger H H^{\dagger 2} + 2Q^2 Q^{\dagger 2} + \boxed{Q^{\dagger 3} L^\dagger + Q^3 L} + 2QQ^\dagger L L^\dagger + L^2 L^{\dagger 2} + uQH^2 H^\dagger \\
& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger 2} + \boxed{e^\dagger u^\dagger Q^2} + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + eLHH^{\dagger 2} + \boxed{euQ^{\dagger 2}} \\
& + 2euQL + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger 2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger 2} + \boxed{d^\dagger u^\dagger QL} \\
& + \boxed{d^\dagger e^\dagger u^{\dagger 2}} + d^\dagger e Q^\dagger L + dQHH^{\dagger 2} + 2duQ^2 + \boxed{duQ^\dagger L^\dagger} + de^\dagger QL^\dagger + \boxed{deu^2} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger 2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + uQHG_L \\
& + dQH^\dagger G_L + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
& + uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
& + d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L + HH^\dagger B_L W_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + du^\dagger H^{\dagger 2} \mathcal{D} \\
& + dd^\dagger HH^\dagger \mathcal{D} + 2H^2 H^{\dagger 2} \mathcal{D}^2 \, .
\end{aligned} \tag{3.16}
$$

Setting all of the spurions equal to unity gives $\widehat{H}_6 = 84$, the total number of independent local operators at dimension 6, but more information is contained in eq.(3.16). For instance, the counting can easily be further decomposed by baryon number violation, $76 + 8$. The perhaps more familiar '$59 + 4$' counting is one in which hermitian conjugates of fermionic operators are not counted separately (such counting can of course also be obtained from eq. (3.16)).

Repeating this at order ϵ^6 we obtain the Hilbert series for dimension-six operators of the SM EFT:

$$
\begin{aligned}
\widehat{H}_6 \; = \; & H^3H^{\dagger\,3} + u^\dagger Q^\dagger H H^{\dagger\,2} + 2Q^2Q^{\dagger\,2} + \text{[redacted]} + 2QQ^\dagger LL^\dagger + L^2L^{\dagger\,2} + \boxed{uQH^2H^\dagger} \\
& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2u^{\dagger\,2} + \text{[redacted]} + e^\dagger L^\dagger H^2H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + \boxed{eLHH^{\dagger\,2}} + \text{[redacted]} \\
& + \boxed{2euQL} + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2e^{\dagger\,2} + d^\dagger Q^\dagger H^2H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + \text{[redacted]} \\
& + \text{[redacted]} + d^\dagger eQ^\dagger L + \boxed{dQHH^{\dagger\,2} + 2duQ^2} + \text{[redacted]} + \boxed{de^\dagger QL^\dagger} + \text{[redacted]} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger HG_R + HH^\dagger G_R^2 + G_R^3 + \boxed{uQHG_L} \\
& + \boxed{dQH^\dagger G_L} + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger HW_R + d^\dagger Q^\dagger HW_R + HH^\dagger W_R^2 + W_R^3 \\
& + \boxed{uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L} + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger HB_R \\
& + d^\dagger Q^\dagger HB_R + HH^\dagger B_RW_R + HH^\dagger B_R^2 + \boxed{uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L} + HH^\dagger B_LW_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger uH^2\mathcal{D} + \boxed{du^\dagger H^{\dagger\,2}\mathcal{D}} \\
& + dd^\dagger HH^\dagger \mathcal{D} + 2H^2H^{\dagger\,2}\mathcal{D}^2 \,. \qquad (3.16)
\end{aligned}
$$

Hermitian conjugates

Setting all of the spurions equal to unity gives $\widehat{H}_6 = 84$, the total number of independent local operators at dimension 6, but more information is contained in eq.(3.16). For instance, the counting can easily be further decomposed by baryon number violation, $76 + 8$. The perhaps more familiar '$59 + 4$' counting is one in which hermitian conjugates of fermionic operators are not counted separately (such counting can of course also be obtained from eq. (3.16)).

Repeating this at order ϵ^6 we obtain the Hilbert series for dimension-six operators of the SM EFT:

59 operators

$$
\begin{aligned}
\widehat{H}_6 \;=\;& H^3 H^{\dagger\,3} + u^\dagger Q^\dagger H H^{\dagger\,2} + 2Q^2 Q^{\dagger\,2} + \text{[illegible]} + 2QQ^\dagger L L^\dagger + L^2 L^{\dagger\,2} + \text{[illegible]} \\
&+ 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger\,2} + \text{[illegible]} + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + \text{[illegible]} + \text{[illegible]} \\
&+ \text{[illegible]} + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger\,2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + \text{[illegible]} \\
&+ \text{[illegible]} + d^\dagger e Q^\dagger L + \text{[illegible]} + \text{[illegible]} + \text{[illegible]} + \text{[illegible]} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
&+ 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + \text{[illegible]} \\
&+ \text{[illegible]} + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
&+ \text{[illegible]} + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
&+ d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + \text{[illegible]} + HH^\dagger B_L W_L \\
&+ HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + \text{[illegible]} \\
&+ dd^\dagger HH^\dagger \mathcal{D} + 2H^2 H^{\dagger\,2} \mathcal{D}^2 \,.
\end{aligned}
\tag{3.16}
$$

Setting all of the spurions equal to unity gives $\widehat{H}_6 = 84$, the total number of independent local operators at dimension 6, but more information is contained in eq.(3.16). For instance, the counting can easily be further decomposed by baryon number violation, $76 + 8$. The perhaps more familiar '$59 + 4$' counting is one in which hermitian conjugates of fermionic operators are not counted separately (such counting can of course also be obtained from eq. (3.16)).

redundancies

- effective operators are invariants under the gauge group, Lorentz group, etc
- their classifications go back to Hilbert, Weyl
- applied to superpotentials, Standard Model
- but so far no general discussions on operators with derivatives
- two sources of redundancies
  - equation of motion (EOM)
  - integration by parts (IBP)

Simple Example

- scalars four-point at $O(\partial^2)$: 4(4+1)/2=10

$$(\partial_\mu \partial_\mu \varphi_i)\varphi_j \varphi_k \varphi_l \qquad (\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l$$

- $\partial^2 \varphi_i = m_i^2$ removes the first class: 4
- We know only 2 out of 6 are independent
  - $s, t, u, s+t+u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

$$(\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l - \varphi_i \varphi_j (\partial_\mu \varphi_k)(\partial_\mu \varphi_l) = \frac{1}{2}\partial^2(\varphi_i \varphi_j)(\varphi_k \varphi_l) - \frac{1}{2}(\varphi_i \varphi_j)\partial^2(\varphi_k \varphi_l) \approx 0$$

$$\partial_\mu \varphi_i \partial_\mu \varphi_j \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \partial_\mu \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \varphi_k \partial_\mu \varphi_l = \partial_\mu \varphi_i \partial_\mu (\varphi_j \varphi_k \varphi_l) \approx 0$$

- In addition, there are only *d* linearly independent momenta in *d*-dimensions

Main idea

- Take kinetic terms as the zeroth order Lagrangian $(\partial\phi)^2,\ \bar{\psi}i\partial\!\!\!/\psi,\ (F_{\mu\nu})^2$
- Classically, it is conformally invariant under SO(4,2)≃SO(6,C)
- Operator-State correspondence tells us that operators fall into representations of the conformal group
  - equation of motion: short multiplets
  - remove total derivatives: primary states

Master formula

- Define a multi-variate Hilbert series

$$H(p,\phi_1,\cdots,\phi_n) = \int d\mu_{\rm conformal} d\mu_{\rm gauge} \sum_{n=1}^{\infty} p^n \chi^*_{[n;0]} \prod_i PE[\phi_i \chi_i(q,\alpha,\beta)]$$

- PE are (anti-)symmetric products of characters for each field ϕ_i of dimension d_i
- integration over the gauge groups pick up gauge invariants
- integration over the conformal group picks only the primary states and Lorentz scalars
- expand it in power series in ϕ_i and p to find

*There are corrections for operators d≤4 due to lack of orthonormality among characters for short multiplets

Hilbert series

- ring freely generated by φ:
  - 1, φ, φ^2, φ^3, φ^4, …

$$H(\varphi) = \frac{1}{1-\varphi}$$

- mod out by ideal, e.g. $\varphi^2=0$

$$H(\varphi) = \frac{1-\varphi^2}{1-\varphi} = 1+\varphi$$

- convenient way to encode all possible operators in a given theory
- basically a "generating function"

characters

- character $\chi(x_1, x_2, \ldots, x_r)=\mathrm{Tr}_R\, g$
- e.g., SU(2)

$$e^{i\theta T_3} = \mathrm{diag}(e^{ij\theta}, e^{i(j-1)\theta}, \cdots, e^{i(-j)\theta}) = (y^{2j}, y^{2j-2}, \cdots y^{-2j})$$

$$y = e^{i\theta/2}$$

$$\chi = y^{2j} + y^{2j-2} + \cdots + y^{-2j} = y^{2j}\frac{1-y^{-4j-2}}{1-y^{-2}} = \frac{y^{2j+1}-y^{-2j-1}}{y-y^{-1}}$$

- orthonormality on Haar measure

$$\delta_{R_i,R_j} = \int d\mu_{SU(2)}\chi^*_{R_i}\chi_{R_j} = \oint_{|y|=1}\frac{dy}{2\pi i}\frac{(1-y^2)(1-y^{-2})}{y}\chi^*_{R_i}\chi_{R_j}$$

conformal characters

- Primary field characterized by its spin $s=(j_1,j_2)$ and conformal weight Δ

$$\chi_{[\Delta,s]}(q,\alpha,\beta) = q^{\Delta} P(q;\alpha,\beta)\chi_s(\alpha,\beta)$$

$$P(q;\alpha,\beta) = \frac{1}{(1-q\alpha\beta)(1-q\alpha\beta^{-1})(1-q\alpha^{-1}\beta)(1-q\alpha^{-1}\beta^{-1})}$$

- For $\Delta=1+j_1+j_2$ which saturates the unitarity bound, there are "short multiplets" for EoM

$$\chi_0(\alpha,\beta) = 1-t^2 \qquad \phi$$

$$\chi_{(\frac{1}{2},0)}(\alpha,\beta) = \alpha+\alpha^{-1} - t(\beta+\beta^{-1}) = \chi_{(0,\frac{1}{2})}(\beta,\alpha) \qquad \psi_\alpha$$

$$F_{\mu\nu}$$

$$\chi_{(1,0)}(\alpha,\beta) = \alpha^2+1+\alpha^{-2} - t(\alpha+\alpha^{-1})(\beta+\beta^{-1}) + t^2 = \chi_{(0,1)}(\beta,\alpha)$$

Plethystic Exponential

- symmetric tensor product R^n of R

$$PE[u\chi_R](x_1, x_2, \cdots, x_r) \equiv \frac{1}{\det_R(1-ug)}$$

$$= \sum_n u^n \chi_{R^n} = \exp\left[-\mathrm{Tr}_R \log(1-ug)\right]$$

$$= \exp\left[\sum_{n=1}^{\infty} \frac{u^n}{n} \chi_R(x_1^n, \cdots, x_r^n)\right]$$

$$PE[u\chi_{1/2}] = \frac{1}{\det\begin{pmatrix} 1-uy & 0 \\ 0 & 1-uy^{-1} \end{pmatrix}}$$

$$= \frac{1}{(1-uy)(1-uy^{-1})} = 1 + u(y+y^{-1}) + u^2(y^2+1+y^{-2}) + u^3(y^3+y+y^{-1}+y^{-3}) + \cdots$$

Plethystic Exponential

- anti-symmetric tensor product R^n of R

$$PE[u\chi_R](x_1, x_2, \cdots, x_r) \equiv \det{}_R(1+ug)$$

$$= \sum_n u^n \chi_{R^n} = \exp\left[\mathrm{Tr}_R \log(1+ug)\right]$$

$$= \exp\left[-\sum_{n=1}^{\infty} \frac{(-u)^n}{n} \chi_R(x_1^n, \cdots, x_r^n)\right]$$

$$PE[u\chi_{1/2}] = \det\begin{pmatrix} 1+uy & 0 \\ 0 & 1+uy^{-1} \end{pmatrix}$$

$$= (1+uy)(1+uy^{-1}) = 1 + u(y+y^{-1}) + u^2$$

```
χH[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[3, x] * su2f[y];
χHd[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[-3, x] * su2fb[y];
χQ[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[1, x] * su2f[y] * su3f[z1, z2];
χQd[t_, α_, β_, x_, y_, z1_, z2_] :=
  χfermR[t, α, β] * u1[-1, x] * su2fb[y] * su3fb[z1, z2];
χu[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-4, x] * su3fb[z1, z2];
χud[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[4, x] * su3f[z1, z2];
χd[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[2, x] * su3fb[z1, z2];
χdd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-2, x] * su3f[z1, z2];
χL[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-3, x] * su2f[y];
χLd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[3, x] * su2fb[y];
χe[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[6, x];
χed[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-6, x];
χBl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β];
χBr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β];
χWl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su2ad[y];
χWr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su2ad[y];
χGl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su3ad[z1, z2];
χGr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su3ad[z1, z2];
```

Master formula

- Define a multi-variate Hilbert series

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- PE are (anti-)symmetric products of characters for each field ϕ_i of dimension d_i
- integration over the gauge groups pick up gauge invariants
- integration over the conformal group picks only the primary states and Lorentz scalars
- expand it in power series in ϕ_i and p to find

*There are corrections for operators d≤4 due to lack of orthonormality among characters for short multiplets

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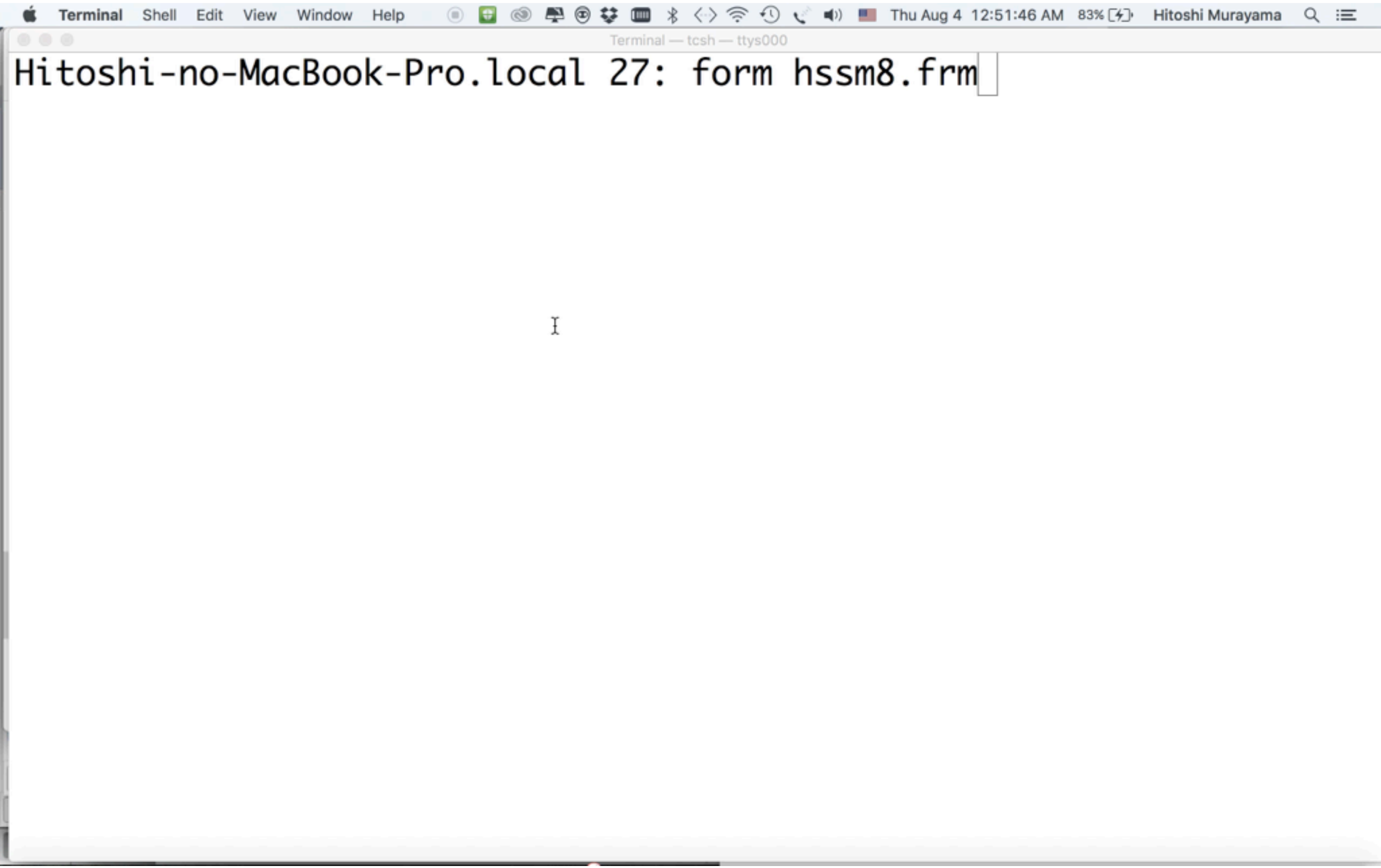
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Thu Aug 4 12:51:46 AM 83% Hitoshi Murayama
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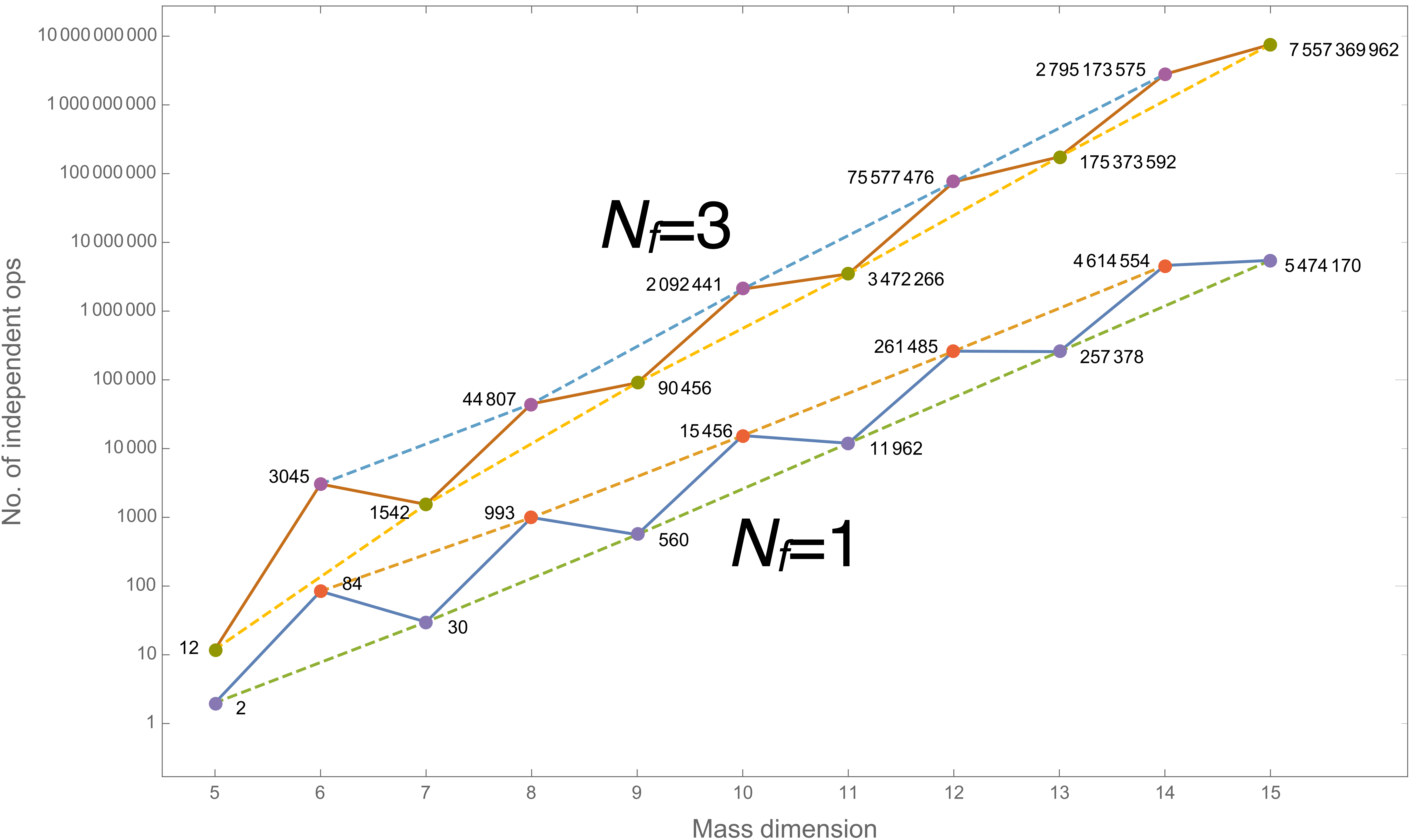
D=8 operators

```
f =
   2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2 + 2*d*dd*L*Ld*t^2 + 2*
   d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 + 2*u*ud*L*Ld*t^2 + 2*u
   *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2*u^2*ud^2*t^2 + 2*Qd*
   dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2 + 3*Qd^2*ud*dd*t^2 +
   Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*Q*ud*dd*L*t^2 + 3*Q*u*
   ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*Q*Qd*d*dd*t^2 + 4*Q*Qd
   *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*Qd^2*t^2 + Q^3*L*t^2
    + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr*u*ud*L*Ld + Wr*Qd*dd*
   ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2*ud*dd + Wr*Qd^2*u*ee
    + 2*Wr*Qd^3*Ld + Wr*Q*d*ed*Ld + Wr*Q*ud*dd*L + 3*Wr*Q*Qd*L*Ld + Wr*Q*Qd
   *ee*ed + 2*Wr*Q*Qd*d*dd + 2*Wr*Q*Qd*u*ud + 2*Wr*Q^2*Qd^2 + Wr^2*L*Ld*t
    + Wr^2*Q*Qd*t + 2*Wr^4 + Wl*L^2*Ld^2 + Wl*ee*ed*L*Ld + Wl*d*dd*L*Ld +
   Wl*u*ud*L*Ld + Wl*Qd*dd*ee*L + Wl*Qd*u*d*Ld + Wl*Q*d*ed*Ld + Wl*Q*ud*dd*
   L + 3*Wl*Q*u*ee*L + 3*Wl*Q*Qd*L*Ld + Wl*Q*Qd*ee*ed + 2*Wl*Q*Qd*d*dd + 2*
   Wl*Q*Qd*u*ud + Wl*Q^2*ud*ed + 3*Wl*Q^2*u*d + 2*Wl*Q^2*Qd^2 + 2*Wl*Q^3*L
    + 2*Wl*Wr*L*Ld*t + Wl*Wr*ee*ed*t + Wl*Wr*d*dd*t + Wl*Wr*u*ud*t + 2*Wl*
   Wr*Q*Qd*t + Wl^2*L*Ld*t + Wl^2*Q*Qd*t + 2*Wl^2*Wr^2 + 2*Wl^4 + Gr*d*dd*L
   *Ld + Gr*d*dd*ee*ed + Gr*d^2*dd^2 + 3*Gr*ud^2*dd*ed + Gr*u*ud*L*Ld + Gr*
   u*ud*ee*ed + 4*Gr*u*ud*d*dd + Gr*u^2*ud^2 + Gr*Qd*dd*ee*L + 3*Gr*Qd*ud*
   ed*Ld + 2*Gr*Qd*u*d*Ld + 6*Gr*Qd^2*ud*dd + Gr*Qd^2*u*ee + 2*Gr*Qd^3*Ld
    + Gr*Q*d*ed*Ld + 2*Gr*Q*ud*dd*L + 2*Gr*Q*Qd*L*Ld + Gr*Q*Qd*ee*ed + 4*Gr
   *Q*Qd*d*dd + 4*Gr*Q*Qd*u*ud + Gr*Q^2*ud*ed + 2*Gr*Q^2*Qd^2 + Gr*Wr*Q*Qd*
   t + Gr*Wl*Q*Qd*t + Gr^2*d*dd*t + Gr^2*u*ud*t + Gr^2*Q*Qd*t + 2*Gr^2*Wr^2
    + Gr^2*Wl^2 + 3*Gr^4 + Gl*d*dd*L*Ld + Gl*d*dd*ee*ed + Gl*d^2*dd^2 + Gl*
   u*ud*L*Ld + Gl*u*ud*ee*ed + 4*Gl*u*ud*d*dd + 3*Gl*u^2*d*ee + Gl*u^2*ud^2
    + Gl*Qd*dd*ee*L + 2*Gl*Qd*u*d*Ld + Gl*Qd^2*u*ee + Gl*Q*d*ed*Ld + 2*Gl*Q
   *ud*dd*L + 3*Gl*Q*u*ee*L + 2*Gl*Q*Qd*L*Ld + Gl*Q*Qd*ee*ed + 4*Gl*Q*Qd*d*
   dd + 4*Gl*Q*Qd*u*ud + Gl*Q^2*ud*ed + 6*Gl*Q^2*u*d + 2*Gl*Q^2*Qd^2 + 2*Gl
   *Q^3*L + Gl*Wr*Q*Qd*t + Gl*Wl*Q*Qd*t + Gl*Gr*L*Ld*t + Gl*Gr*ee*ed*t + 3*
   Gl*Gr*d*dd*t + 3*Gl*Gr*u*ud*t + 3*Gl*Gr*Q*Qd*t + Gl*Gr*Wl*Wr + Gl^2*d*dd
   *t + Gl^2*u*ud*t + Gl^2*Q*Qd*t + Gl^2*Wr^2 + 2*Gl^2*Wl^2 + 3*Gl^2*Gr^2
    + 3*Gl^4 + Br*ee*ed*L*Ld + Br*d*dd*L*Ld + Br*d*dd*ee*ed + 2*Br*ud^2*dd*
   ed + Br*u*ud*L*Ld + Br*u*ud*ee*ed + 2*Br*u*ud*d*dd + Br*Qd*dd*ee*L + 3*
   Br*Qd*ud*ed*Ld + Br*Qd*u*d*Ld + 3*Br*Qd^2*ud*dd + Br*Qd^3*Ld + Br*Q*d*ed
   *Ld + Br*Q*ud*dd*L + 2*Br*Q*Qd*L*Ld + Br*Q*Qd*ee*ed + 2*Br*Q*Qd*d*dd + 2
   *Br*Q*Qd*u*ud + Br*Q^2*ud*ed + Br*Wr*L*Ld*t + Br*Wr*Q*Qd*t + Br*Wl*L*Ld*
   t + Br*Wl*Q*Qd*t + Br*Gr*d*dd*t + Br*Gr*u*ud*t + Br*Gr*Q*Qd*t + Br*Gr^3
    + Br*Gl*d*dd*t + Br*Gl*u*ud*t + Br*Gl*Q*Qd*t + Br*Gl^2*Gr + 2*Br^2*Wr^2
    + Br^2*Wl^2 + 2*Br^2*Gr^2 + Br^2*Gl^2 + Br^4 + Bl*ee*ed*L*Ld + Bl*d*dd*
   L*Ld + Bl*d*dd*ee*ed + Bl*u*ud*L*Ld + Bl*u*ud*ee*ed + 2*Bl*u*ud*d*dd + 2
   *Bl*u^2*d*ee + Bl*Qd*dd*ee*L + Bl*Qd*u*d*Ld + Bl*Qd^2*u*ee + Bl*Q*d*ed*
   Ld + Bl*Q*ud*dd*L + 3*Bl*Q*u*ee*L + 2*Bl*Q*Qd*L*Ld + Bl*Q*Qd*ee*ed + 2*
   Bl*Q*Qd*d*dd + 2*Bl*Q*Qd*u*ud + 3*Bl*Q^2*u*d + Bl*Q^3*L + Bl*Wr*L*Ld*t
    + Bl*Wr*Q*Qd*t + Bl*Wl*L*Ld*t + Bl*Wl*Q*Qd*t + Bl*Gr*d*dd*t + Bl*Gr*u*
   ud*t + Bl*Gr*Q*Qd*t + Bl*Gl*d*dd*t + Bl*Gl*u*ud*t + Bl*Gl*Q*Qd*t + Bl*Gl
   *Gr^2 + Bl*Gl^3 + Bl*Br*L*Ld*t + Bl*Br*ee*ed*t + Bl*Br*d*dd*t + Bl*Br*u*
   ud*t + Bl*Br*Q*Qd*t + Bl*Br*Wl*Wr + Bl*Br*Gl*Gr + Bl^2*Wr^2 + 2*Bl^2*
   Wl^2 + Bl^2*Gr^2 + 2*Bl^2*Gl^2 + Bl^2*Br^2 + Bl^4 + 3*Hd*ee*L^2*Ld*t +
   Hd*ee^2*ed*L*t + 3*Hd*d*dd*ee*L*t + 3*Hd*ud*d*ed*Ld*t + 2*Hd*ud^2*dd*L*t
    + 2*Hd*u*d^2*Ld*t + 3*Hd*u*ud*ee*L*t + 6*Hd*Qd*ud*L*Ld*t + 3*Hd*Qd*ud*
   ee*ed*t + 6*Hd*Qd*ud*d*dd*t + 3*Hd*Qd*u*d*ee*t + 3*Hd*Qd*u*ud^2*t + 3*Hd
   *Qd^2*d*Ld*t + Hd*Qd^3*ee*t + 6*Hd*Q*d*L*Ld*t + 3*Hd*Q*d*ee*ed*t + 3*Hd*
   Q*d^2*dd*t + 2*Hd*Q*ud^2*ed*t + 6*Hd*Q*u*ud*d*t + 6*Hd*Q*Qd*ee*L*t + 6*
   Hd*Q*Qd^2*ud*t + 3*Hd*Q^2*ud*L*t + 6*Hd*Q^2*Qd*d*t + Hd*Wr*ee*L*t^2 + 2*
   Hd*Wr*Qd*ud*t^2 + Hd*Wr*Q*d*t^2 + Hd*Wr^2*ee*L + 2*Hd*Wr^2*Qd*ud + Hd*
   Wr^2*Q*d + 2*Hd*Wl*ee*L*t^2 + Hd*Wl*Qd*ud*t^2 + 2*Hd*Wl*Q*d*t^2 + 2*Hd*
   Wl^2*ee*L + Hd*Wl^2*Qd*ud + 2*Hd*Wl^2*Q*d + 2*Hd*Gr*Qd*ud*t^2 + Hd*Gr*Q*
   d*t^2 + 2*Hd*Gr*Wr*Qd*ud + Hd*Gr*Wr*Q*d + Hd*Gr^2*ee*L + 3*Hd*Gr^2*Qd*ud
    + 2*Hd*Gr^2*Q*d + Hd*Gl*Qd*ud*t^2 + 2*Hd*Gl*Q*d*t^2 + Hd*Gl*Wl*Qd*ud +
   2*Hd*Gl*Wl*Q*d + Hd*Gl^2*ee*L + 2*Hd*Gl^2*Qd*ud + 3*Hd*Gl^2*Q*d + Hd*Br*
   ee*L*t^2 + 2*Hd*Br*Qd*ud*t^2 + Hd*Br*Q*d*t^2 + Hd*Br*Wr*ee*L + 2*Hd*Br*
   Wr*Qd*ud + Hd*Br*Wr*Q*d + 2*Hd*Br*Gr*Qd*ud + Hd*Br*Gr*Q*d + Hd*Br^2*ee*L
    + Hd*Br^2*Qd*ud + Hd*Br^2*Q*d + 2*Hd*Bl*ee*L*t^2 + Hd*Bl*Qd*ud*t^2 + 2*
   Hd*Bl*Q*d*t^2 + 2*Hd*Bl*Wl*ee*L + Hd*Bl*Wl*Qd*ud + 2*Hd*Bl*Wl*Q*d + Hd*
   Bl*Gl*Qd*ud + 2*Hd*Bl*Gl*Q*d + Hd*Bl^2*ee*L + Hd*Bl^2*Qd*ud + Hd*Bl^2*Q*
   d + Hd^2*ee^2*L^2 + Hd^2*ud*d*t^3 + Hd^2*ud*d*L*Ld + Hd^2*Qd*ud*ee*L + 2
   *Hd^2*Qd^2*ud^2 + 2*Hd^2*Q*d*ee*L + 2*Hd^2*Q*Qd*ud*d + 2*Hd^2*Q^2*d^2 +
   Hd^2*Wr*ud*d*t + Hd^2*Wl*ud*d*t + Hd^2*Gr*ud*d*t + Hd^2*Gl*ud*d*t + Hd^2
   *Br*ud*d*t + Hd^2*Bl*ud*d*t + 3*H*ed*L*Ld^2*t + H*ee*ed^2*Ld*t + 3*H*d*
   dd*ed*Ld*t + 2*H*ud*dd^2*L*t + 3*H*u*dd*ee*L*t + 3*H*u*ud*ed*Ld*t + 2*H*
   u^2*d*Ld*t + 6*H*Qd*dd*L*Ld*t + 3*H*Qd*dd*ee*ed*t + 3*H*Qd*d*dd^2*t + 6*
   H*Qd*u*ud*dd*t + 2*H*Qd*u^2*ee*t + 3*H*Qd^2*u*Ld*t + 3*H*Q*ud*dd*ed*t +
   6*H*Q*u*L*Ld*t + 3*H*Q*u*ee*ed*t + 6*H*Q*u*d*dd*t + 3*H*Q*u^2*ud*t + 6*H
   *Q*Qd*ed*Ld*t + 6*H*Q*Qd^2*dd*t + 3*H*Q^2*dd*L*t + 6*H*Q^2*Qd*u*t + H*
   Q^3*ed*t + 2*H*Wr*ed*Ld*t^2 + 2*H*Wr*Qd*dd*t^2 + H*Wr*Q*u*t^2 + 2*H*Wr^2
   *ed*Ld + 2*H*Wr^2*Qd*dd + H*Wr^2*Q*u + H*Wl*ed*Ld*t^2 + H*Wl*Qd*dd*t^2
    + 2*H*Wl*Q*u*t^2 + H*Wl^2*ed*Ld + H*Wl^2*Qd*dd + 2*H*Wl^2*Q*u + 2*H*Gr*
   Qd*dd*t^2 + H*Gr*Q*u*t^2 + 2*H*Gr*Wr*Qd*dd + H*Gr*Wr*Q*u + H*Gr^2*ed*Ld
    + 3*H*Gr^2*Qd*dd + 2*H*Gr^2*Q*u + H*Gl*Qd*dd*t^2 + 2*H*Gl*Q*u*t^2 + H*
   Gl*Wl*Qd*dd + 2*H*Gl*Wl*Q*u + H*Gl^2*ed*Ld + 2*H*Gl^2*Qd*dd + 3*H*Gl^2*Q
   *u + 2*H*Br*ed*Ld*t^2 + 2*H*Br*Qd*dd*t^2 + H*Br*Q*u*t^2 + 2*H*Br*Wr*ed*
   Ld + 2*H*Br*Wr*Qd*dd + H*Br*Wr*Q*u + 2*H*Br*Gr*Qd*dd + H*Br*Gr*Q*u + H*
   Br^2*ed*Ld + H*Br^2*Qd*dd + H*Br^2*Q*u + H*Bl*ed*Ld*t^2 + H*Bl*Qd*dd*t^2
    + 2*H*Bl*Q*u*t^2 + H*Bl*Wl*ed*Ld + H*Bl*Wl*Qd*dd + 2*H*Bl*Wl*Q*u + H*Bl
   *Gl*Qd*dd + 2*H*Bl*Gl*Q*u + H*Bl^2*ed*Ld + H*Bl^2*Qd*dd + H*Bl^2*Q*u + 4
   *H*Hd*L*Ld*t^3 + 2*H*Hd*L^2*Ld^2 + 2*H*Hd*ee*ed*t^3 + 2*H*Hd*ee*ed*L*Ld
    + H*Hd*ee^2*ed^2 + 2*H*Hd*d*dd*t^3 + 2*H*Hd*d*dd*L*Ld + H*Hd*d*dd*ee*ed
    + H*Hd*d^2*dd^2 + H*Hd*ud^2*dd*ed + 2*H*Hd*u*ud*t^3 + 2*H*Hd*u*ud*L*Ld
    + H*Hd*u*ud*ee*ed + 2*H*Hd*u*ud*d*dd + H*Hd*u^2*d*ee + H*Hd*u^2*ud^2 +
   2*H*Hd*Qd*dd*ee*L + 4*H*Hd*Qd*ud*ed*Ld + 2*H*Hd*Qd*u*d*Ld + 4*H*Hd*Qd^2*
   ud*dd + H*Hd*Qd^2*u*ee + 2*H*Hd*Qd^3*Ld + 2*H*Hd*Q*d*ed*Ld + 2*H*Hd*Q*ud
   *dd*L + 4*H*Hd*Q*u*ee*L + 4*H*Hd*Q*Qd*t^3 + 5*H*Hd*Q*Qd*L*Ld + 2*H*Hd*Q*
   Qd*ee*ed + 4*H*Hd*Q*Qd*d*dd + 4*H*Hd*Q*Qd*u*ud + H*Hd*Q^2*ud*ed + 4*H*Hd
   *Q^2*u*d + 3*H*Hd*Q^2*Qd^2 + 2*H*Hd*Q^3*L + 6*H*Hd*Wr*L*Ld*t + 2*H*Hd*Wr
   *ee*ed*t + 2*H*Hd*Wr*d*dd*t + 2*H*Hd*Wr*u*ud*t + 6*H*Hd*Wr*Q*Qd*t + 2*H*
   Hd*Wr^2*t^2 + H*Hd*Wr^3 + 6*H*Hd*Wl*L*Ld*t + 2*H*Hd*Wl*ee*ed*t + 2*H*Hd*
   Wl*d*dd*t + 2*H*Hd*Wl*u*ud*t + 6*H*Hd*Wl*Q*Qd*t + 2*H*Hd*Wl*Wr*t^2 + 2*H
   *Hd*Wl^2*t^2 + H*Hd*Wl^3 + 2*H*Hd*Gr*d*dd*t + 2*H*Hd*Gr*u*ud*t + 4*H*Hd*
   Gr*Q*Qd*t + H*Hd*Gr^2*t^2 + H*Hd*Gr^3 + 2*H*Hd*Gl*d*dd*t + 2*H*Hd*Gl*u*
   ud*t + 4*H*Hd*Gl*Q*Qd*t + H*Hd*Gl*Gr*t^2 + H*Hd*Gl^2*t^2 + H*Hd*Gl^3 + 4
   *H*Hd*Br*L*Ld*t + 2*H*Hd*Br*ee*ed*t + 2*H*Hd*Br*d*dd*t + 2*H*Hd*Br*u*ud*
   t + 4*H*Hd*Br*Q*Qd*t + 2*H*Hd*Br*Wr*t^2 + H*Hd*Br*Wr^2 + H*Hd*Br*Wl*t^2
    + H*Hd*Br^2*t^2 + 4*H*Hd*Bl*L*Ld*t + 2*H*Hd*Bl*ee*ed*t + 2*H*Hd*Bl*d*dd
   *t + 2*H*Hd*Bl*u*ud*t + 4*H*Hd*Bl*Q*Qd*t + H*Hd*Bl*Wr*t^2 + 2*H*Hd*Bl*Wl
   *t^2 + H*Hd*Bl*Wl^2 + H*Hd*Bl*Br*t^2 + H*Hd*Bl^2*t^2 + 6*H*Hd^2*ee*L*t^2
    + 6*H*Hd^2*Qd*ud*t^2 + 6*H*Hd^2*Q*d*t^2 + 2*H*Hd^2*Wr*Qd*ud + 2*H*Hd^2*
   Wl*ee*L + 2*H*Hd^2*Wl*Q*d + H*Hd^2*Gr*Qd*ud + H*Hd^2*Gl*Q*d + H*Hd^2*Br*
   Qd*ud + H*Hd^2*Bl*ee*L + H*Hd^2*Bl*Q*d + H*Hd^3*ud*d*t + H^2*ed^2*Ld^2
    + H^2*u*dd*t^3 + H^2*u*dd*L*Ld + 2*H^2*Qd*dd*ed*Ld + 2*H^2*Qd^2*dd^2 +
   H^2*Q*u*ed*Ld + 2*H^2*Q*Qd*u*dd + 2*H^2*Q^2*u^2 + H^2*Wr*u*dd*t + H^2*Wl
   *u*dd*t + H^2*Gr*u*dd*t + H^2*Gl*u*dd*t + H^2*Br*u*dd*t + H^2*Bl*u*dd*t
    + 6*H^2*Hd*ed*Ld*t^2 + 6*H^2*Hd*Qd*dd*t^2 + 6*H^2*Hd*Q*u*t^2 + 2*H^2*Hd
   *Wr*ed*Ld + 2*H^2*Hd*Wr*Qd*dd + 2*H^2*Hd*Wl*Q*u + H^2*Hd*Gr*Qd*dd + H^2*
   Hd*Gl*Q*u + H^2*Hd*Br*ed*Ld + H^2*Hd*Br*Qd*dd + H^2*Hd*Bl*Q*u + 3*H^2*
   Hd^2*t^4 + 4*H^2*Hd^2*L*Ld*t + H^2*Hd^2*ee*ed*t + H^2*Hd^2*d*dd*t + H^2*
   Hd^2*u*ud*t + 4*H^2*Hd^2*Q*Qd*t + 2*H^2*Hd^2*Wr*t^2 + 2*H^2*Hd^2*Wr^2 +
   2*H^2*Hd^2*Wl*t^2 + 2*H^2*Hd^2*Wl^2 + H^2*Hd^2*Gr^2 + H^2*Hd^2*Gl^2 +
   H^2*Hd^2*Br*t^2 + H^2*Hd^2*Br*Wr + H^2*Hd^2*Br^2 + H^2*Hd^2*Bl*t^2 + H^2
   *Hd^2*Bl*Wl + H^2*Hd^2*Bl^2 + H^2*Hd^3*ee*L + H^2*Hd^3*Qd*ud + H^2*Hd^3*
   Q*d + H^3*Hd*u*dd*t + H^3*Hd^2*ed*Ld + H^3*Hd^2*Qd*dd + H^3*Hd^2*Q*u + 2
   *H^3*Hd^3*t^2 + H^4*Hd^4;
```

993 of them



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2 795 173 575
4 614 554
7 557 369 962
5 474 170

Conclusions

- Nailed the question of classifying effective operators in a given Lorentz-inv theory
- Connections to amplitudes?
- perturbation around non-free theories?
- EFT important in many other contexts
  - condensed matter physics
  - nuclear physics
  - cosmological density fluctuations