UVの寄与から現れる Λ_{QCD}の正べき

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Full paper in preparation

Outline

1. Introduction

   摂動QCDに不可避な誤差の問題→UVとIRの分離

2. UVだけから決まる量の計算法

3. Discussion and other quantities

4. まとめと展望

摂動QCD

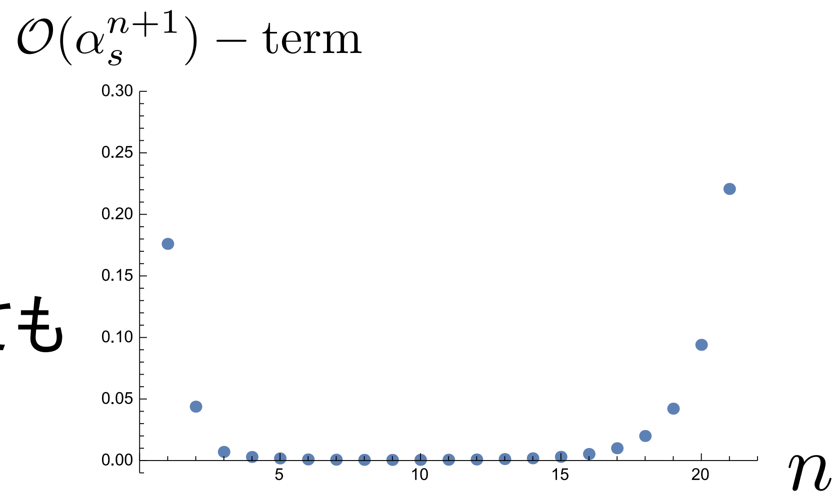
摂動級数は発散する漸近級数

↓

摂動QCDでは高次項をいくら計算しても
予言に不可避な誤差が含まれる

$$\mathcal{O}(\Lambda_{\mathrm{QCD}}/\underline{Q})^n$$

Observableのスケール

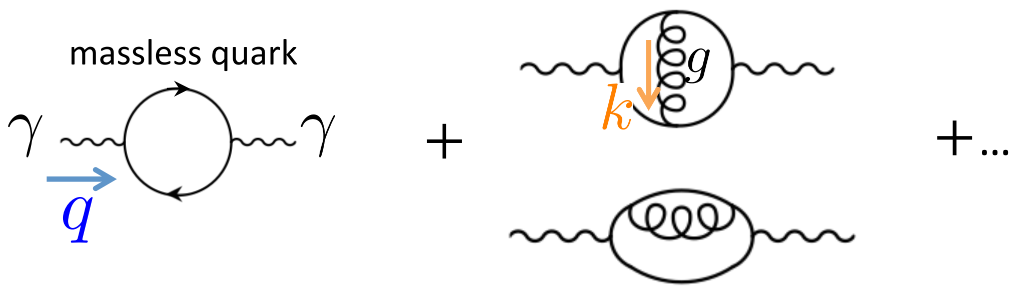


↑

発散的振る舞い

予言向上のためにどうしたら良いか？

PhotonのVacuum polarization (QCD補正)



$\Pi(Q^2)$: muon g-2, R-ratio ($e^+ e^-$->hadrons),... $Q^2 = -q^2 > 0$

$D(Q^2) \sim \dfrac{d\Pi(Q^2)}{d\log Q^2}$ Adler function

摂動論での不可避な誤差 ⟵ 低エネルギーのgluonに起因

$$\Delta_{\rm pert.} D(Q^2) = \mathcal{O}\left(\frac{\Lambda^4_{\rm QCD}}{Q^4}\right)$$

1999 M. Beneke

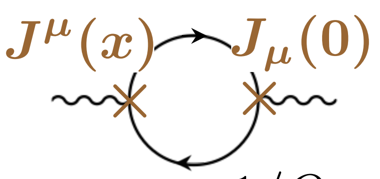
OPE-誤差克服の枠組み-

エネルギーを分割 Factorization scale $\mu_f \gg \Lambda_{\rm QCD}$ （Wilson流）

$k > \mu_f$ UV ⟶ 摂動論で計算 摂動論の誤差は出ない

$k < \mu_f$ IR ⟶ Low energy effective theoryで計算

OPE（演算子積展開）でこの概念が実現可能

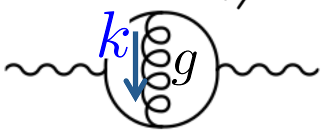


$J^\mu(x) = \bar{q}\gamma^\mu q(x)$

$x \leftrightarrow 1/Q$ で展開

$$D(Q^2) = C_1(Q^2, \mu_f) + C_{GG}(Q^2, \mu_f)\frac{\langle 0|G^{a\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4} + \ldots$$

C_1: UV, $\sim\!\!/$



+ …

C_{GG}: UV

IR
Gluon condensate

$= \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^4}{Q^4}\right)$

これからやること

OPEのLeading order $C_1(Q^2, \mu_f)$　予言がcut-offによって変わってしまう

Cut-offに依らない部分を解析的に構成(一般論)

→ Leading order のreliable part!

For QCD potential 2004 Y. Sumino
→ Generalization

Impact

UVから $\Lambda_{\rm QCD}^2/Q^2$ が出現!

$$D(Q^2) = \underbrace{C_1(Q^2,\mu_f)}_{\rm UV} + C_{GG}(Q^2,\mu_f)\underbrace{\frac{\langle 0|G^{a\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4}}_{\rm IR} + \cdots$$

OPEに潜む
べき的な構造

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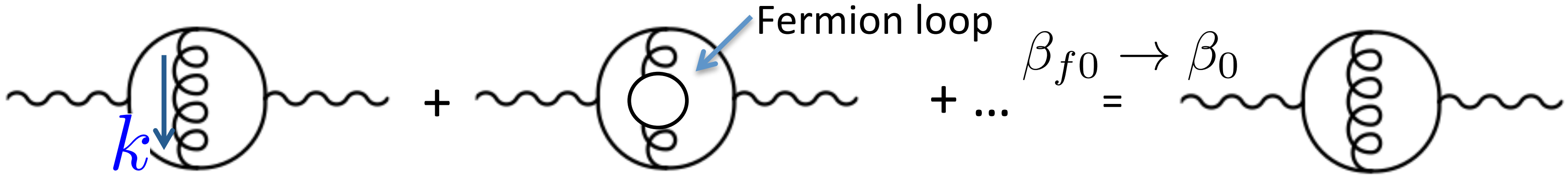
UVの寄与のDiagram計算（Large-β_0近似）

k : UV



$\mathcal{O}(\alpha_s)$ $\mathcal{O}(\alpha_s^2)$

Effective vertex

$\boldsymbol{\alpha_{\beta_0}(\tau)}$

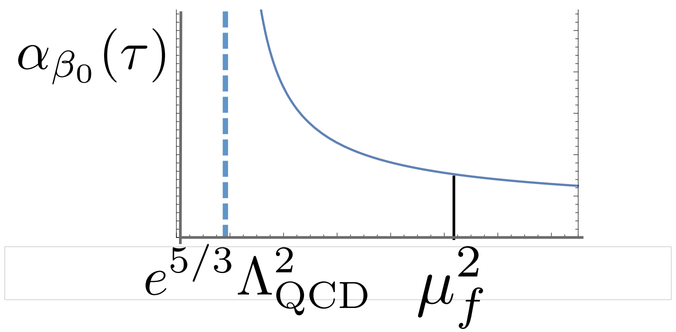
$(\tau = k_E^2)$

$$\boldsymbol{D_{\beta_0}(Q^2;\mu_f^2) = \int_{\mu_f^2}^{\infty} \frac{d\tau}{2\pi\tau} w_D\left(\frac{\tau}{Q^2}\right) \underline{\alpha_{\beta_0}(\tau)}}$$

1995 M. Neubert

Pole around Λ_{QCD}

Cut-offに依らない部分を抜き出す



How to extract cut-off independent part

$$D_{\beta_0}(Q^2;\mu_f^2) = \int_{\mu_f^2}^{\infty} \frac{d\tau}{2\pi\tau} w_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

1. 新しい解析関数Wを考える

$$2\,\mathrm{Im}\,W_D(\tau) = w_D(\tau) \quad (\tau \in \mathbb{R} \text{ and } \tau > 0)$$

1995 M. Neubert

$$W_D(\tau) = \int_0^{\infty} \frac{dx}{2\pi} \frac{w_D(x)}{x - \tau - i0} \qquad \because \ \mathrm{Im}\frac{1}{x-\tau-i0} = \pi\delta(x-\tau)$$

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\int_{\mu_f^2}^{\infty} \frac{d\tau}{\pi\tau} W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

How to extract cut-off independent part

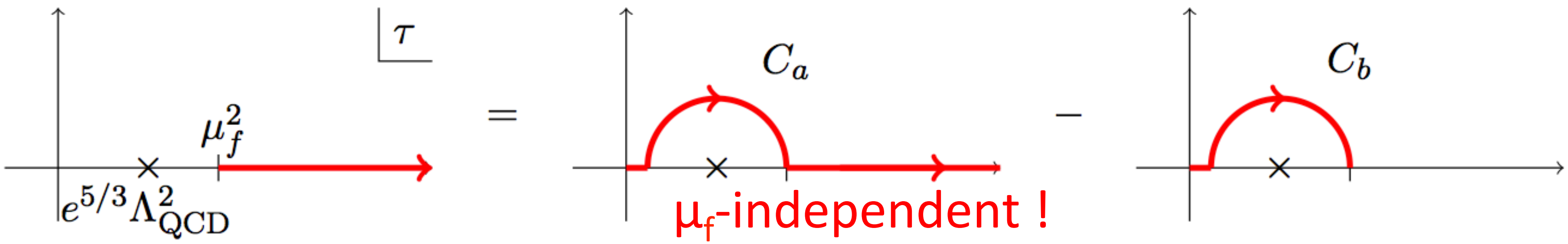
$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\int_{\mu_f^2}^{\infty} \frac{d\tau}{\pi\tau} W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

How to extract cut-off independent part

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\int_{\mu_f^2}^{\infty}\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

2. 複素平面で積分経路を変形

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\left[\int_{C_a} - \int_{C_b}\right]\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

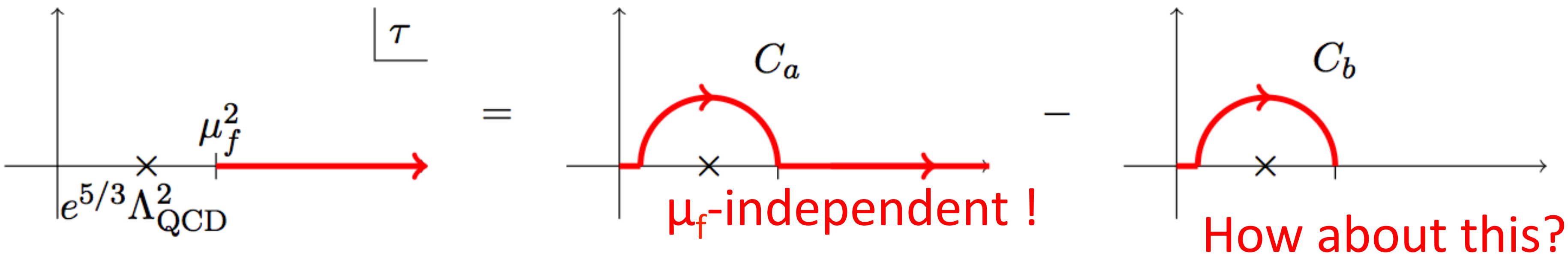


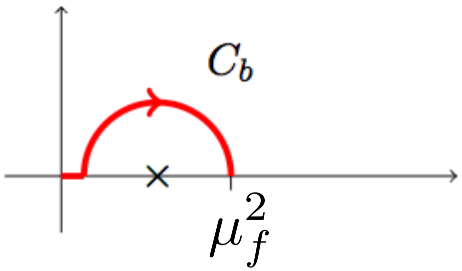
How to extract cut-off independent part

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\int_{\mu_f^2}^{\infty}\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

2. 複素平面で積分経路を変形

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\left[\int_{C_a} - \int_{C_b}\right]\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$





の積分

Expand W(τ) around τ=0: $(\Lambda_{\rm QCD}^2 \ll \mu_f^2 \ll Q^2)$

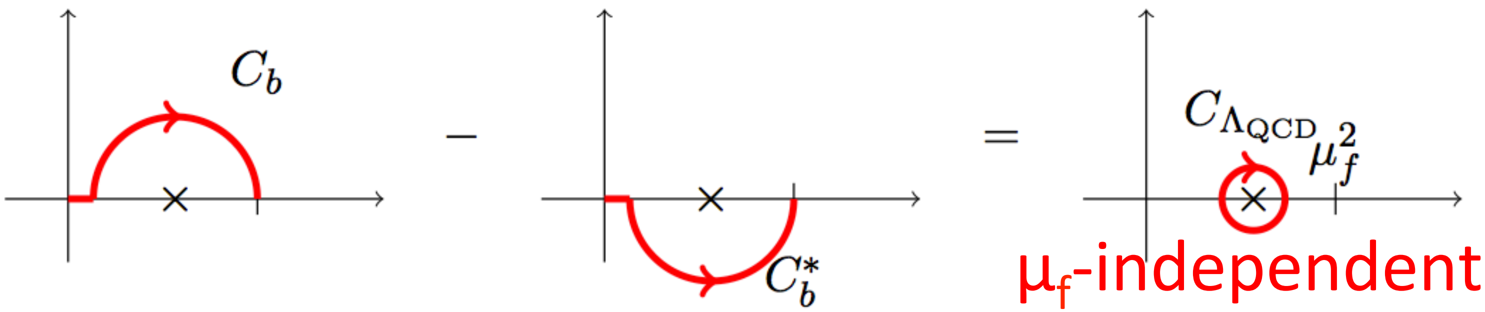
$$\mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)=\sum_n \mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}c_n\left(\frac{\tau}{Q^2}\right)^n\alpha_{\beta_0}(\tau)$$

(i) The case of $c_n \in \mathbb{R}$

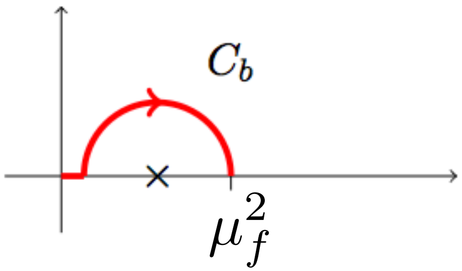
$$\mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}c_n\left(\frac{\tau}{Q^2}\right)^n\alpha_{\beta_0}(\tau)=\frac{1}{2\pi i}\oint_{C_{\Lambda_{\rm QCD}}}=-\frac{4\pi c_n}{\beta_0}\left(\frac{e^{5/3}\Lambda_{\rm QCD}^2}{Q^2}\right)^n$$

{Integrand(τ)}*=Integrand(τ*)

$\Lambda_{\rm QCD}$が出現!



μ_f-independent!



の積分

Expand W(τ) around τ=0: $(\Lambda_{\rm QCD}^2 \ll \mu_f^2 \ll Q^2)$

$$\mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)=\sum_n \mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}c_n\left(\frac{\tau}{Q^2}\right)^n\alpha_{\beta_0}(\tau)$$

(ii) The case of $c_n = a_n + i b_n \in \mathbb{C}$

{Integrand(τ)}*=—Integrand(τ*)

$$\mathrm{Im}\int_{C_b}\frac{d\tau}{\pi\tau}ib_n\left(\frac{\tau}{Q^2}\right)^n\alpha_{\beta_0}(\tau)=\frac{1}{2\pi i}\left[\int_{C_b}+\int_{C_b^*}\right]\frac{d\tau}{\tau}ib_n\left(\frac{\tau}{Q^2}\right)^n\alpha_{\beta_0}(\tau)$$

$$=\mathcal{O}((\mu_f^2/Q^2)^n)$$

μ_f依存性が残る

結果

Wの展開(係数の実・虚)は

Ball, Beneke, Braun
Nucl.Phys. B452 (1995) 563-625

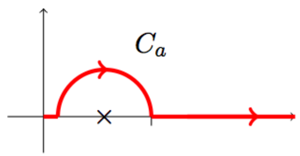
$$W_D\left(\frac{\tau}{Q^2}\right) =$$

$$N_cC_F\left[\frac{1}{4\pi}+\frac{8-6\zeta_3}{3\pi}\frac{\tau}{Q^2}+\frac{10-12\zeta_3-3\log\left(\tau/Q^2\right)+3i\pi}{6\pi}\left(\frac{\tau}{Q^2}\right)^2+\ldots\right]$$

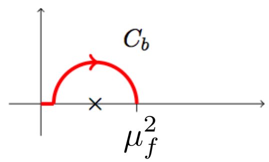
$\mathbb{R}$ Case (i) $\mathbb{C}$ Case (ii)

故に

$$D_{\beta_0}(Q^2;\mu_f^2) = \mathrm{Im}\int_{C_a}\frac{d\tau}{\pi\tau}W_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$



$$+\frac{N_cC_F}{\beta_0}+\frac{8(4-3\zeta_3)e^{5/3}N_cC_F}{3\beta_0}\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}+\mathcal{O}(\mu_f^4/Q^4)$$



μ_f-independent!

Adler fn.の構造

$$D_{\beta_0}(Q^2;\mu_f^2) = \int_{\mu_f^2}^{\infty} \frac{d\tau}{2\pi\tau} w_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

$$= D_0(Q^2) + \frac{8(4-3\zeta_3)e^{5/3}N_c C_F}{3\beta_0}\frac{\Lambda_{\rm QCD}^2}{Q^2} + \mathcal{O}(\mu_f^4/Q^4)$$

μfに依らない

Adler fn.の構造

$$D_{\beta_0}(Q^2;\mu_f^2) = \int_{\mu_f^2}^{\infty} \frac{d\tau}{2\pi\tau} w_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

$$= D_0(Q^2) + \frac{8(4-3\zeta_3)e^{5/3}N_c C_F}{3\beta_0}\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2} + \underline{\mathcal{O}(\mu_f^4/Q^4)}$$

Gluon condensateと関わる項
IRの情報と関係

$$D(Q^2) = \underbrace{C_1(Q^2,\mu_f)}_{\mathrm{UV}} + \underbrace{C_{GG}(Q^2,\mu_f)}_{\mathrm{UV}}\underbrace{\frac{\langle 0|G^{a\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4}}_{\mathrm{IR}} + \dots$$

μ_f依存性がキャンセル

Adler fn.の構造

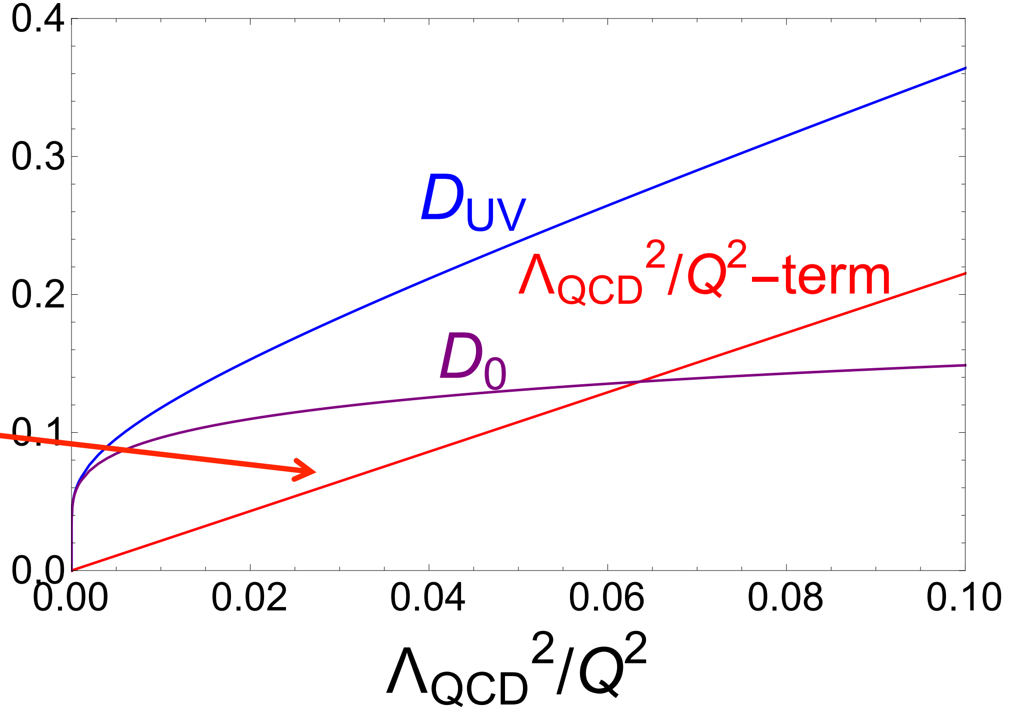
$$D_{\beta_0}(Q^2;\mu_f^2) = \int_{\mu_f^2}^{\infty} \frac{d\tau}{2\pi\tau} w_D\left(\frac{\tau}{Q^2}\right)\alpha_{\beta_0}(\tau)$$

$$= D_0(Q^2) + \frac{8(4-3\zeta_3)e^{5/3}N_cC_F}{3\beta_0}\frac{\Lambda_{\rm QCD}^2}{Q^2} + \mathcal{O}(\mu_f^4/Q^4)$$

$D_{\rm UV}(Q^2)$

UVのみで決まる

Gluon condensateより
支配的なべき



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Wの一意性

$$2\,\mathrm{Im}\,W_D(\tau) = w_D(\tau) \quad (\tau \in \mathbb{R} \text{ and } \tau > 0)$$

このようなWは一意ではない　(Real partは自由)

Adler fn.の$\Lambda_{QCD}{}^2/Q^2$ はWのreal partから出てきた

一意的に決まるものではない？

条件

Adler fn. (Correlation fn.)をQ^2-複素平面で解析関数として表示できるようなW に限る → Wは一意的！

$\Lambda_{QCD}{}^2/Q^2$の自然な係数は一意に決まる

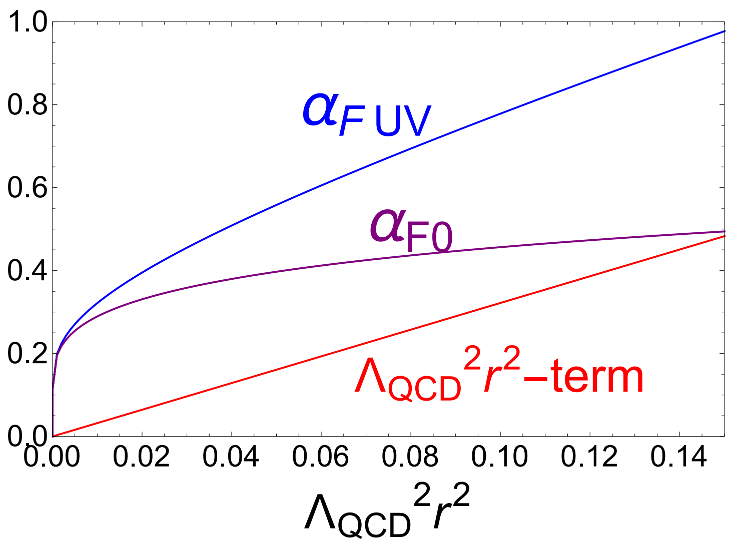
Other quantities

Force between $Q\bar{Q}$

$$F(r^2) = -\frac{d}{dr}V_{\beta_0}(r) = -C_F\frac{\alpha_F(1/r^2)}{r^2}$$

$$\alpha_{F\,\beta_0}(1/r^2;\mu_f) = \alpha_{F\,\mathrm{UV}}(1/r^2) + \mathcal{O}(\mu_f^3 r^3)$$

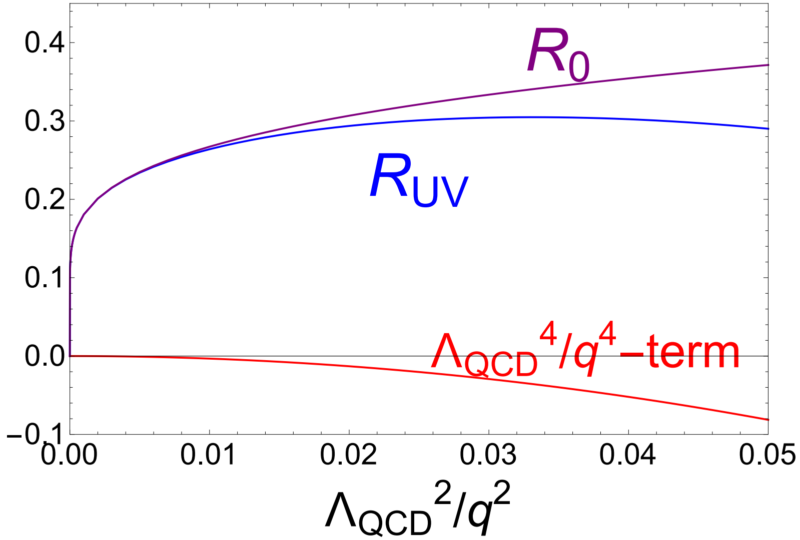
$$\alpha_{F\,\mathrm{UV}}(1/r^2) = \alpha_{F0}(1/r^2) + \boxed{\frac{2\pi e^{5/3}}{\beta_0}\Lambda_{\mathrm{QCD}}^2 r^2}$$



R-ratio $R_{\beta 0}(q^2) \sim \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

$$R_{\beta 0}(q^2,\mu_f) = R_{\mathrm{UV}}(q^2) + \mathcal{O}(\mu_f^6/q^6)$$

$$R_{\mathrm{UV}}(q^2) = R_0(q^2) - \boxed{\frac{3N_c C_F e^{10/3}}{\beta_0}\frac{\Lambda_{\mathrm{QCD}}^4}{q^4}}$$



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まとめ

- 摂動QCDでは低エネルギーのグルーオンに起因する誤差がつきまとう
- UVの寄与だけを考え、cut-offに依らない部分を抜き出す一般的方法

  ⟶ Leading orderのreliable part

- UV由来のΛ_{QCD}^2/Q^2がC_1に存在

$$D(Q^2) = \boxed{C_1(Q^2,\mu_f)} + C_{GG}(Q^2,\mu_f)\frac{\langle 0|G^{a\mu\nu}G^a_{\mu\nu}|0\rangle}{Q^4} + \ldots$$

高エネルギーでの振る舞いはOPEから
素朴に予想されるべき的振る舞いと異なる

展望

課題

- Systematic improvement of calculation of UV contributions → Beyond large-β_0 approx.

Ongoing and future works

- α_s の高精度決定

  QCD potential　Lattice v.s. UV contribution

- Gluon condensate の高精度決定

$$D(Q^2) = \underline{C_1(Q^2, \mu_f)} + C_{GG}(Q^2, \mu_f) \underline{\frac{\langle 0 | G^{a\mu\nu} G^a_{\mu\nu} | 0 \rangle}{Q^4}} + \ldots$$

摂動論の誤差を持たない　　数値的に高精度で決定できるか

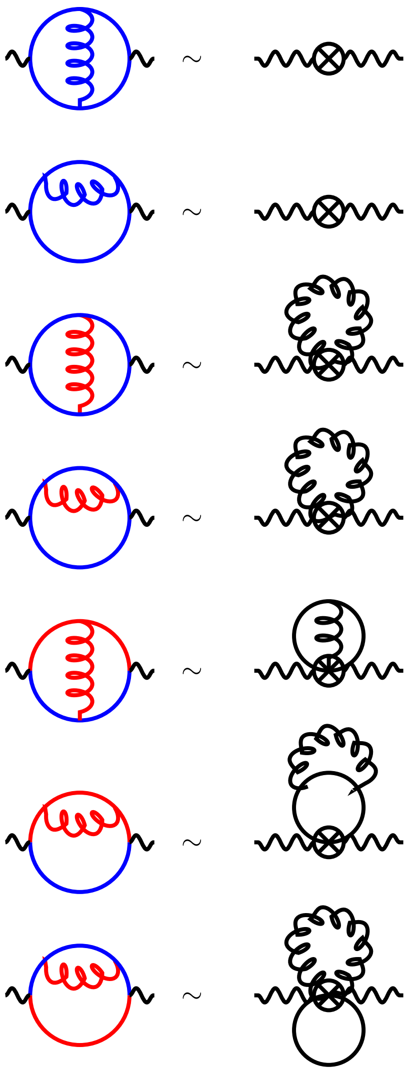
Back up

W_Dの構成法

$$W_D(\tau) = \frac{N_c C_F}{12\pi}\Big[3 + 16\tau(\tau+1)H(\tau) - 14\tau^2 \log(-\tau)$$
$$+ 8\tau(\tau+1)\{-\log(-\tau)\mathrm{Li}_2(-\tau) + \mathrm{Li}_3(\tau) + \mathrm{Li}_3(-\tau)\}$$
$$+ 4\{2\tau^2 + 2\tau + 1 - 4\tau(\tau+1)\log(1+\tau)\}\mathrm{Li}_2(\tau)$$
$$+ 2(7\tau^2 - 4\tau - 3)\log(1-\tau) - 8\zeta_2\tau(\tau+1)\log(1+\tau)$$
$$+ 4\{\tau^2 - \tau(\tau+1)\log(1+\tau)\}\log^2(-\tau)$$
$$+ 2(4\zeta_2 - 7\zeta_3)\tau^2 + 2(11 - 7\zeta_3)\tau\Big], \qquad (13)$$

$$H(\tau) = \int_\tau^1 dx \frac{\log(1+x)\log(1-x)}{x}$$

Relation between diagramatic calculation and OPE



$$W_D^{(H,H)A} \sim \sim \langle \mathbf{1} \rangle$$

$$W_D^{(H,H)B} \sim \sim \langle \mathbf{1} \rangle$$

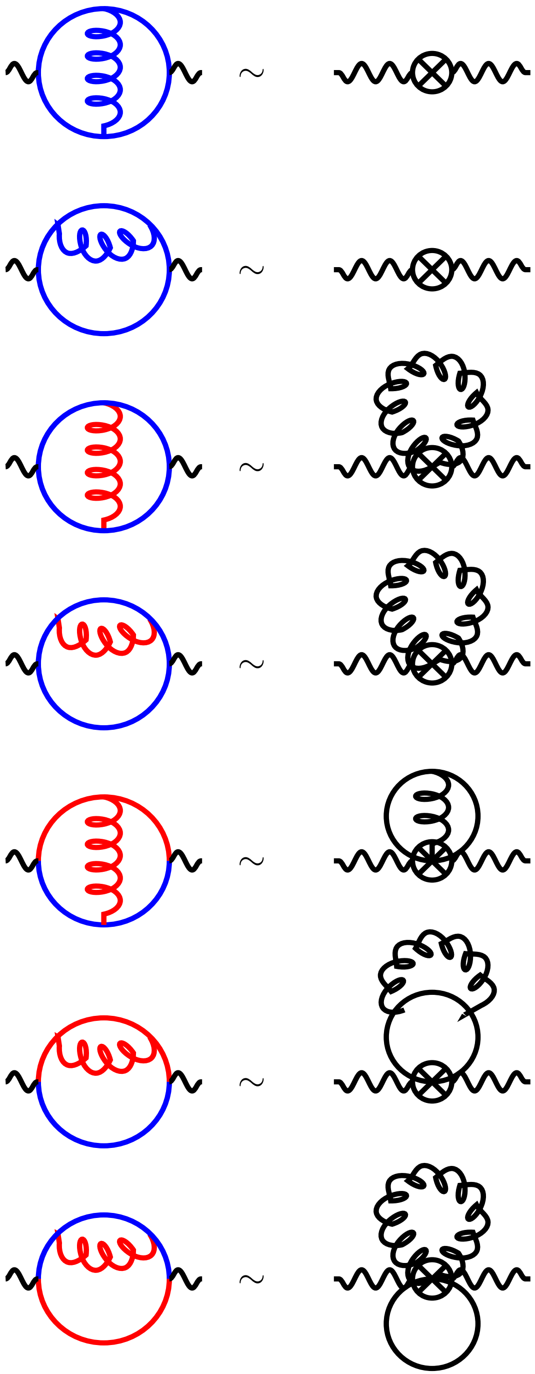
$$W_D^{(S,H)A} \sim \sim \frac{\langle G^2 \rangle}{Q^4}$$

$$W_D^{(S,H)B} \sim \sim \frac{\langle G^2 \rangle}{Q^4} \qquad (1)$$

$$W_D^{(S,S)A} \sim \sim \frac{\langle \bar{\psi} \not{D} \psi \rangle}{Q^4}$$

$$W_D^{(S,S)B} \sim \sim \frac{\langle \bar{\psi} \not{D} \psi \rangle}{Q^4}$$

$$W_D^{(S,S)C} \sim \sim \frac{\langle \bar{\psi} \partial^\mu \partial^\nu \psi G^{\mu\nu} \rangle}{Q^6}$$



$$W_D^{(H,H)A} \sim \langle \mathbf{1} \rangle$$

$$W_D^{(H,H)B} \sim \langle \mathbf{1} \rangle$$

$$W_D^{(S,H)A} \sim \frac{\langle G^2 \rangle}{Q^4}$$

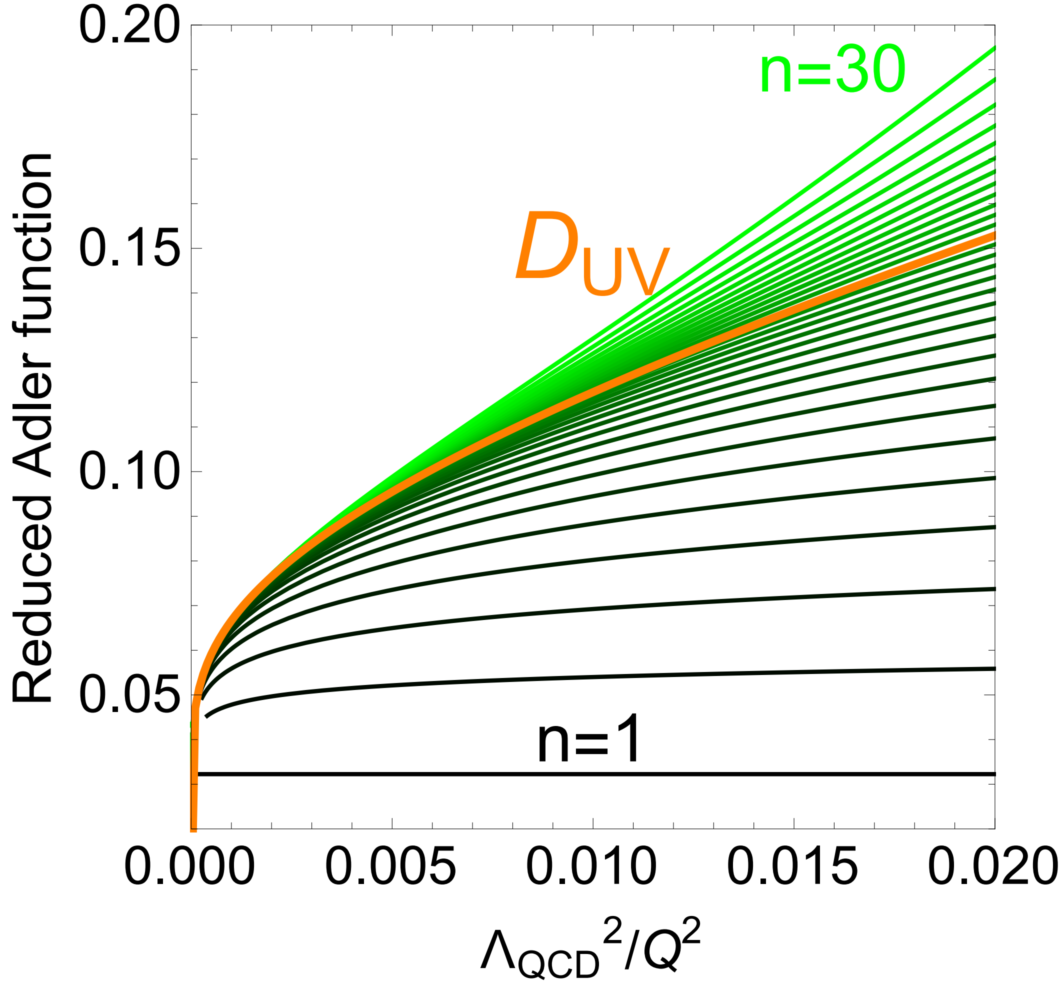
$$W_D^{(S,H)B} \sim \frac{\langle G^2 \rangle}{Q^4} \qquad (1)$$

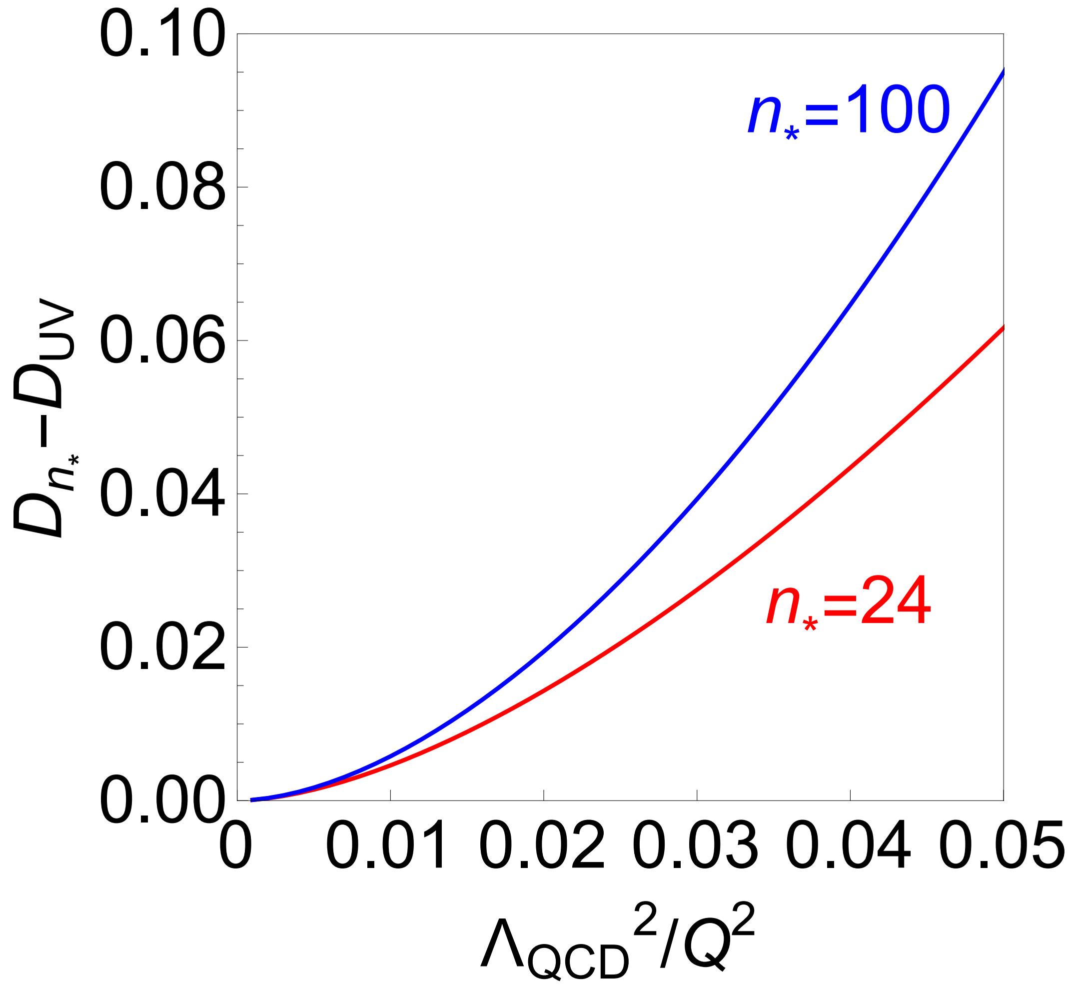
$$W_D^{(S,S)A} \sim \frac{\langle \bar{\psi} \not{D} \psi \rangle}{Q^4}$$

$$W_D^{(S,S)B} \sim \frac{\langle \bar{\psi} \not{D} \psi \rangle}{Q^4}$$

$$W_D^{(S,S)C} \sim \frac{\langle \bar{\psi} \partial^\mu \partial^\nu \psi G^{\mu\nu} \rangle}{Q^6}$$

D_{UV} v.s. Higher order pert. series




0.10
0.08
0.06
0.04
0.02
0.00
0
0.01
0.02
0.03
0.04
0.05
$D_{n_*} - D_{UV}$
$\Lambda_{QCD}{}^2/Q^2$
n_*=100
n_*=24

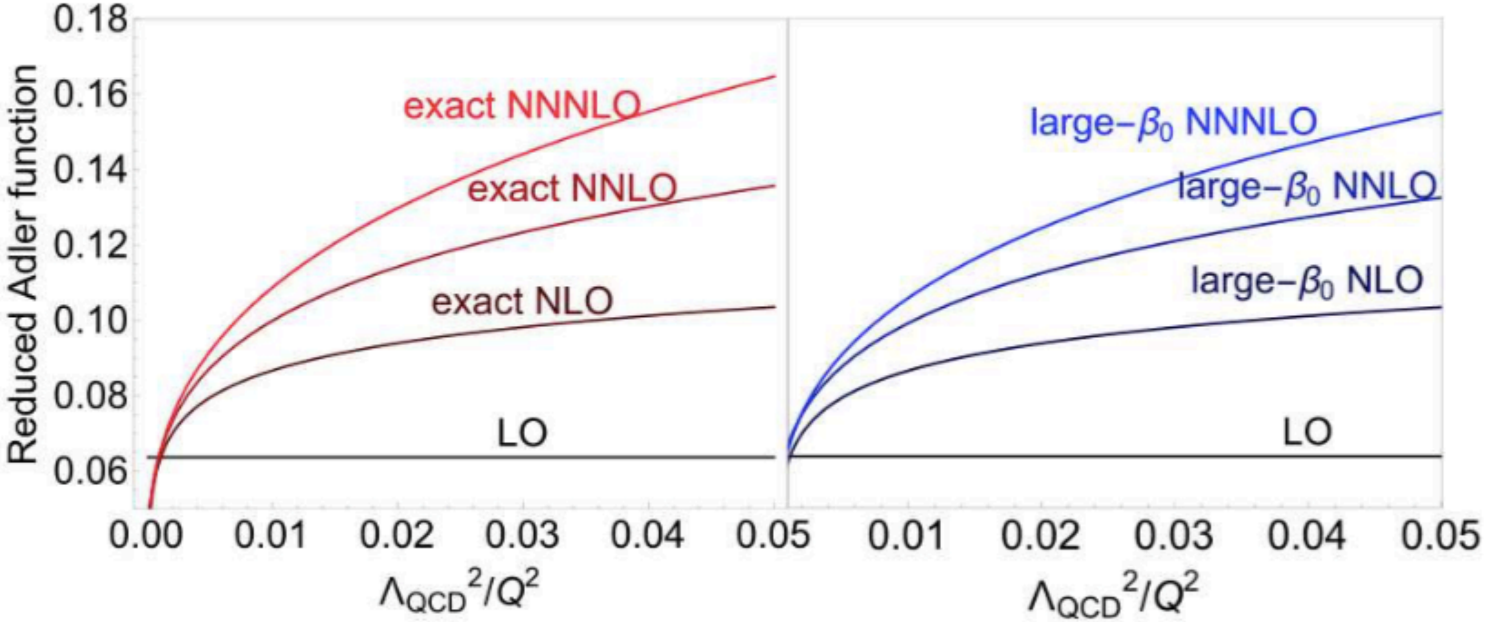


Figure 4: Perturbative series of $D(Q^2)$: exact result for the non-singlet part (left) and large-β_0 approximation (right). N^{k}LO line represents the sum of the series up to $\mathcal{O}(\alpha_s^{k+1})$. The input is taken as $\alpha_s(\mu) = 0.2$.