現実的なインフレーション模型は何か

Thoughts on realistic inflation models 2016

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DISCLAIMER

・ 主にインフレーションのレビューです。
・ 限られた経験/知識に基づき、偏見に満ちています。
・ 個々の模型では、議論に色々な抜け道があります。
・ 皆様の模型が出なくても怒らないでください。
（コメントは歓迎。）
・ 寄り道して関連した自分の仕事を紹介します。

Outline

1. **Introduction**: inflation in a nutshell

2. **Universality classes of inflation**
   Realization by “pole inflation”

3. **Initial conditions**: Small-field or Large-field?

4. **Shift symmetry** and **its origin**
   U(1): pNGB or Wilson line
   Weak Gravity Conjecture
   R: scale invariant models

5. **Summary** & **Conclusion**

Introduction:

inflation in a nutshell

動機・利点

- 指数関数的膨張によって、一様性問題、平坦性問題、モノポール問題を解決する。
- インフラトンの量子揺らぎにより、宇宙の大規模構造の「種」をつくる。

宇宙の加速膨張

Einstein eq.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$T^{\mu}_{\nu} = \mathrm{diag}(-\rho, P, P, P)$$

一様等方宇宙

FLRW universe

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

Friedmann eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

Slow-roll

$$\rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$P = \frac{1}{2}\dot{\phi}^2 - V$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

slow-roll 近似

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \left|\frac{V''}{V}\right| \ll 1$$

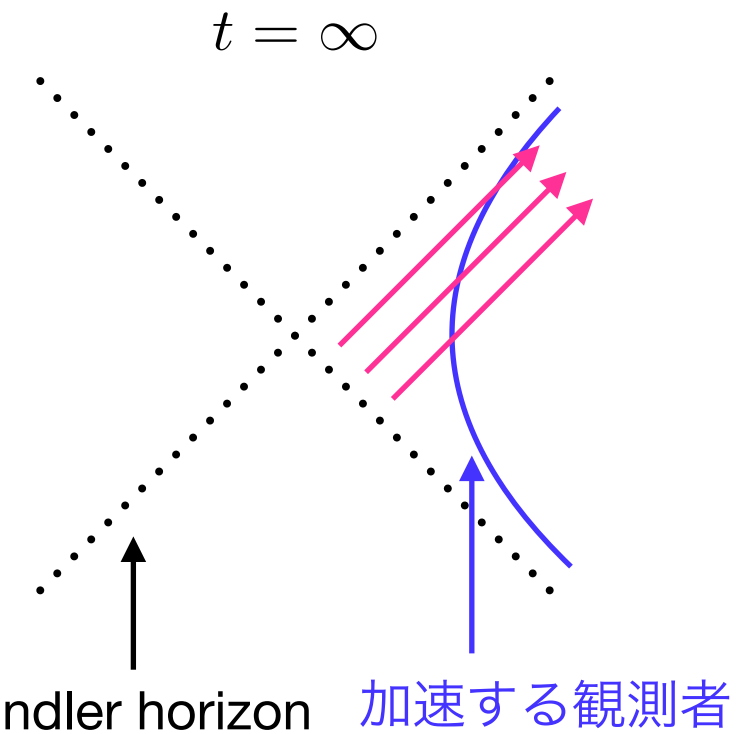
$$P \simeq -\rho \simeq \mathrm{const.}$$

$$a(t) \simeq e^{Ht}$$

$$N = \int_t^{t_{\mathrm{end}}} H \mathrm{d}t = \int_{\phi_{\mathrm{end}}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon}}$$

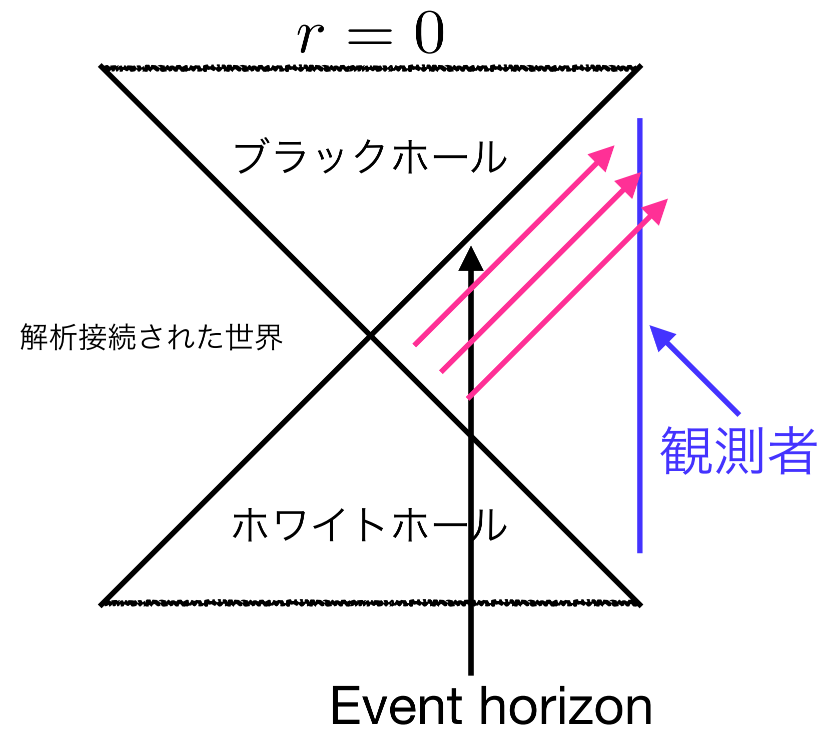
[Unruh, PRD14 (1976) 870]
[Hawking, Commun.Math.Phys. 43 (1975) 199, Erratum: ibid. 46 (1976) 206]
[Gibbons, Hawking, PRD15 (1977) 2738]

Minkowski 時空



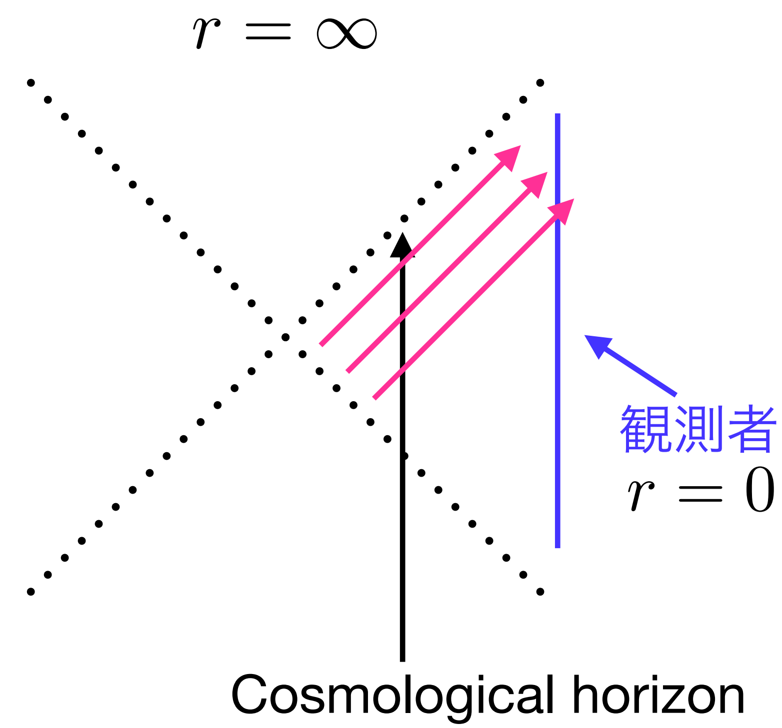
Unruh radiation $T = \frac{a}{2\pi}$

Black Hole



Hawking radiation $T = \frac{\kappa}{2\pi}$

de Sitter 宇宙



Gibbons-Hawking radiation $T = \frac{H}{2\pi}$

inflaton fluctuation $\delta\phi = \frac{H}{2\pi}$

curvature perturbation $\zeta = \delta N = H\delta t = \frac{H}{\dot{\phi}}\delta\phi$

Power spectra

$$\mathcal{P}_{\rm s}(k) = A_{\rm s}\left(\frac{k}{k_*}\right)^{n_{\rm s}-1}$$

$$\mathcal{P}_{\rm t}(k) = A_{\rm t}\left(\frac{k}{k_*}\right)^{n_{\rm t}}$$

$$n_{\rm s} - 1 = -6\epsilon + 2\eta$$

$$r \equiv \frac{A_{\rm t}}{A_{\rm s}} = 16\epsilon$$

Excellent fit by ΛCDM

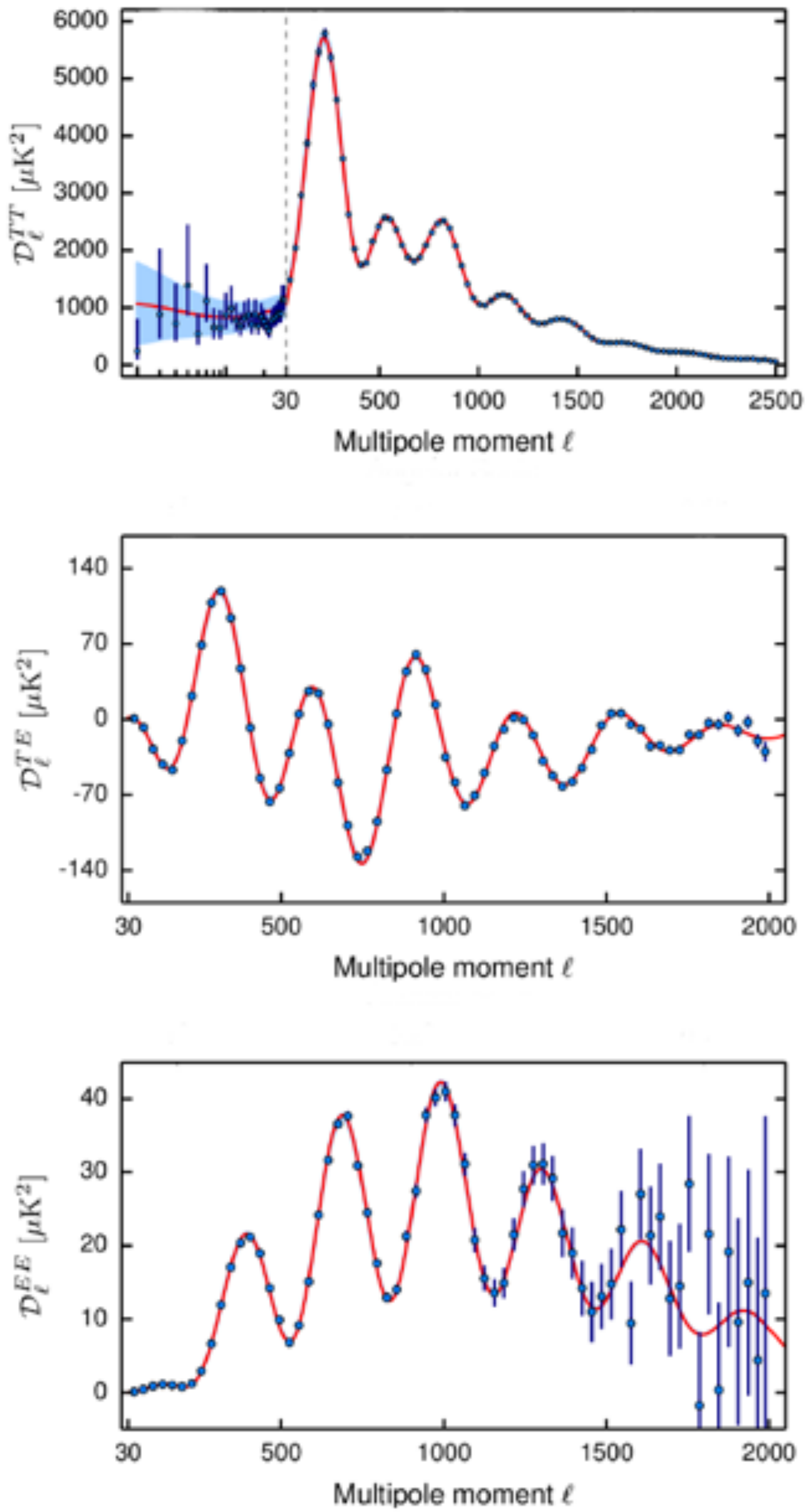


Fig. 2. *Planck TT* (top), high-ℓ *TE* (centre), and high-ℓ *EE* (bottom) angular power spectra. Here $\mathcal{D}_\ell \equiv \ell(\ell+1)C_\ell/(2\pi)$.

Scale invariant $(n_s \sim 1)$

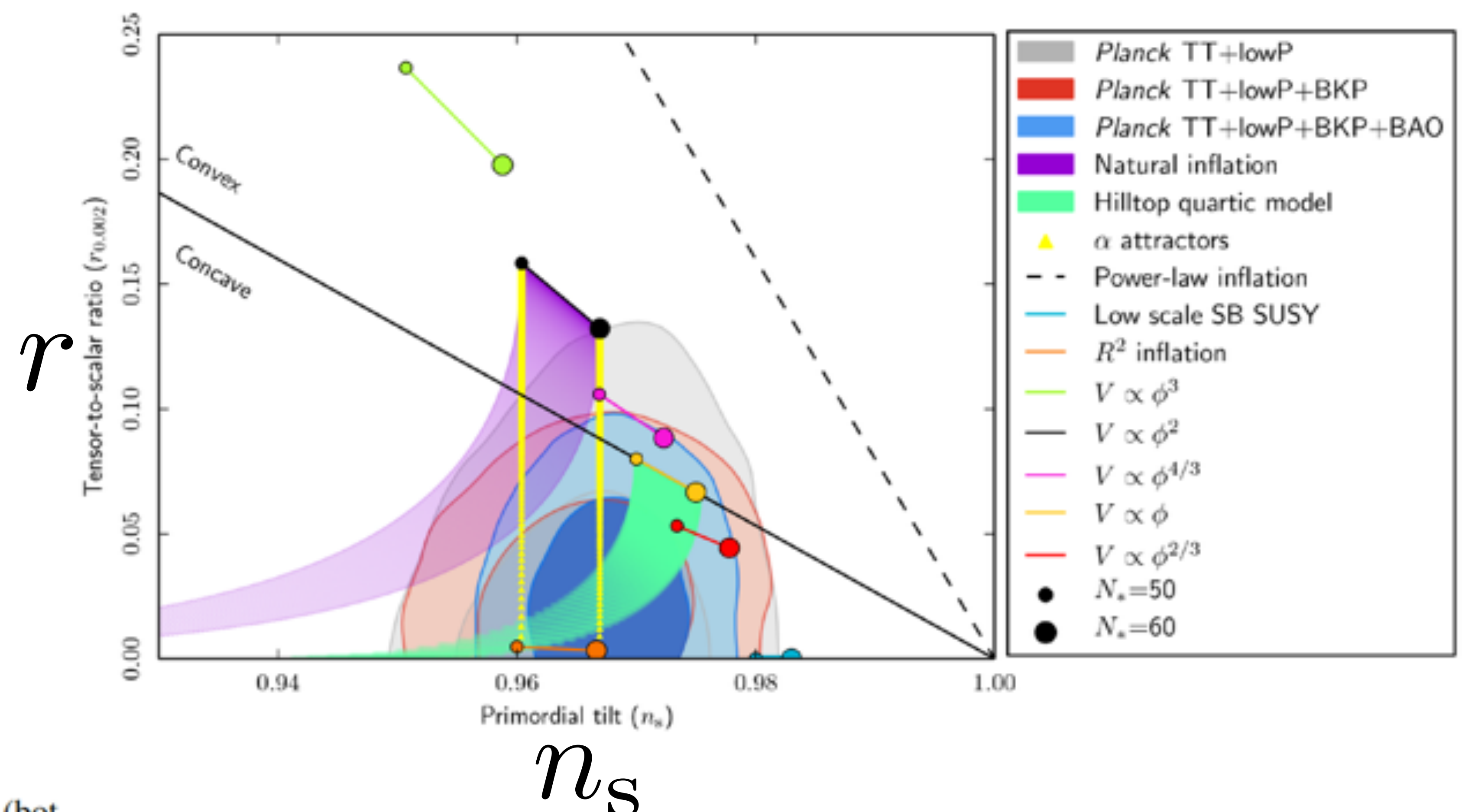
$$n_s = 0.9655 \pm 0.0062$$ (Planck TT+ low P)

Adiabatic $(\beta_{\rm iso} \sim 0)$

$$\beta_{\rm iso}(0.002\,{\rm Mpc}^{-1}) < 4.1 \times 10^{-2}$$ (for CDM)

Gaussian $(f_{\rm NL} \sim 0)$

$$f_{\rm NL}^{\rm local} = 0.8 \pm 5.0$$ (Planck TT+ low P)



[Planck collaboration, 1502.02114, 1502.01592]

Universality classes of inflation

Analogy to Renormalization Group

The Hamilton-Jacobi formalism $\phi(t) \leftrightarrow t(\phi)$

"superpotential" $W(\phi) \equiv -H$

→ This implies: $W_\phi = \dot{\phi}/2$ $V = 3W^2 - 2W_\phi^2$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\ln a} = \beta(\phi)$$

cf.) $$\frac{\mathrm{d}g}{\mathrm{d}\ln \mu} = \beta(g)$$

where $\beta(\phi) = -2\frac{W_\phi}{W} = \frac{\dot{\phi}}{H} = \pm\sqrt{\frac{3(P+\rho)}{\rho}} = \pm\sqrt{2\epsilon}$

→ classified by the behavior near the fixed point (de Sitter).

Underlying connections?

[McFadden, Skenderis, 0907.5542, 1001.2007]

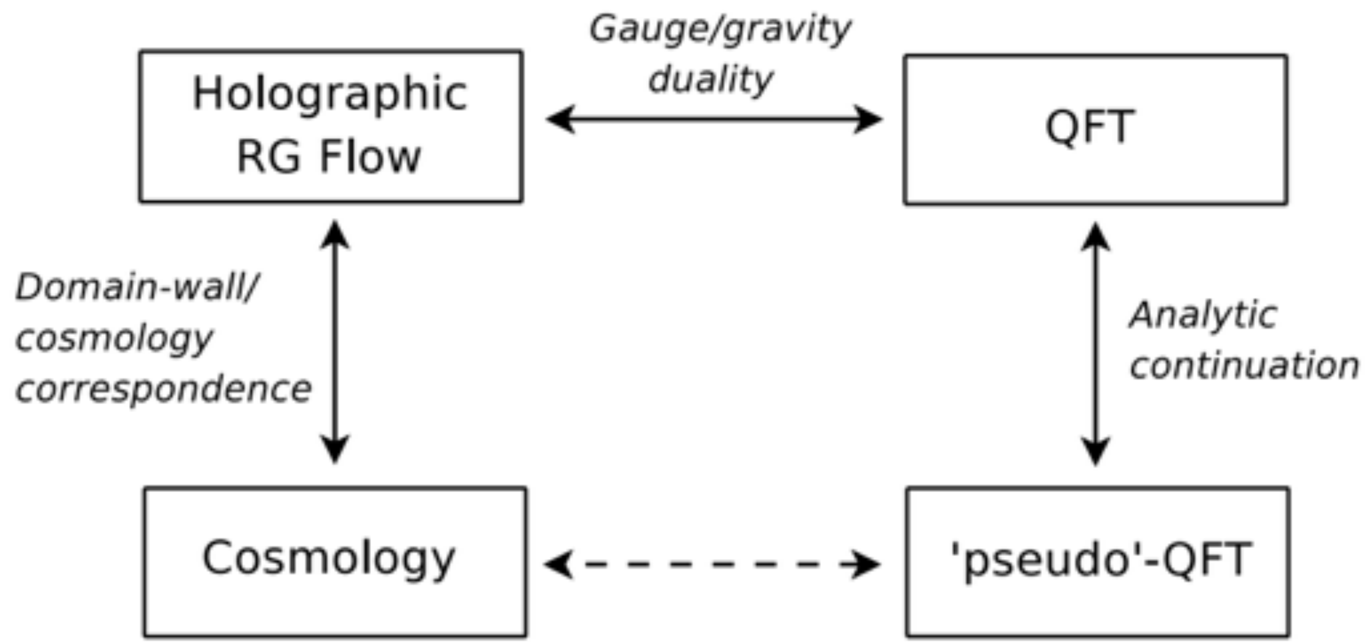


FIG. 1: The 'pseudo'-QFT dual to inflationary cosmology is operationally defined using the correspondence of cosmologies to domain-walls and standard gauge/gravity duality.

See also, dS/CFT and FRW/CFT.

dS/CFT:
[Strominger, hep-th/0106113]
[Witten hep-th/0106109]
[Larsen et al., hep-th/0202127]
[Halyo, hep-th/0203235]

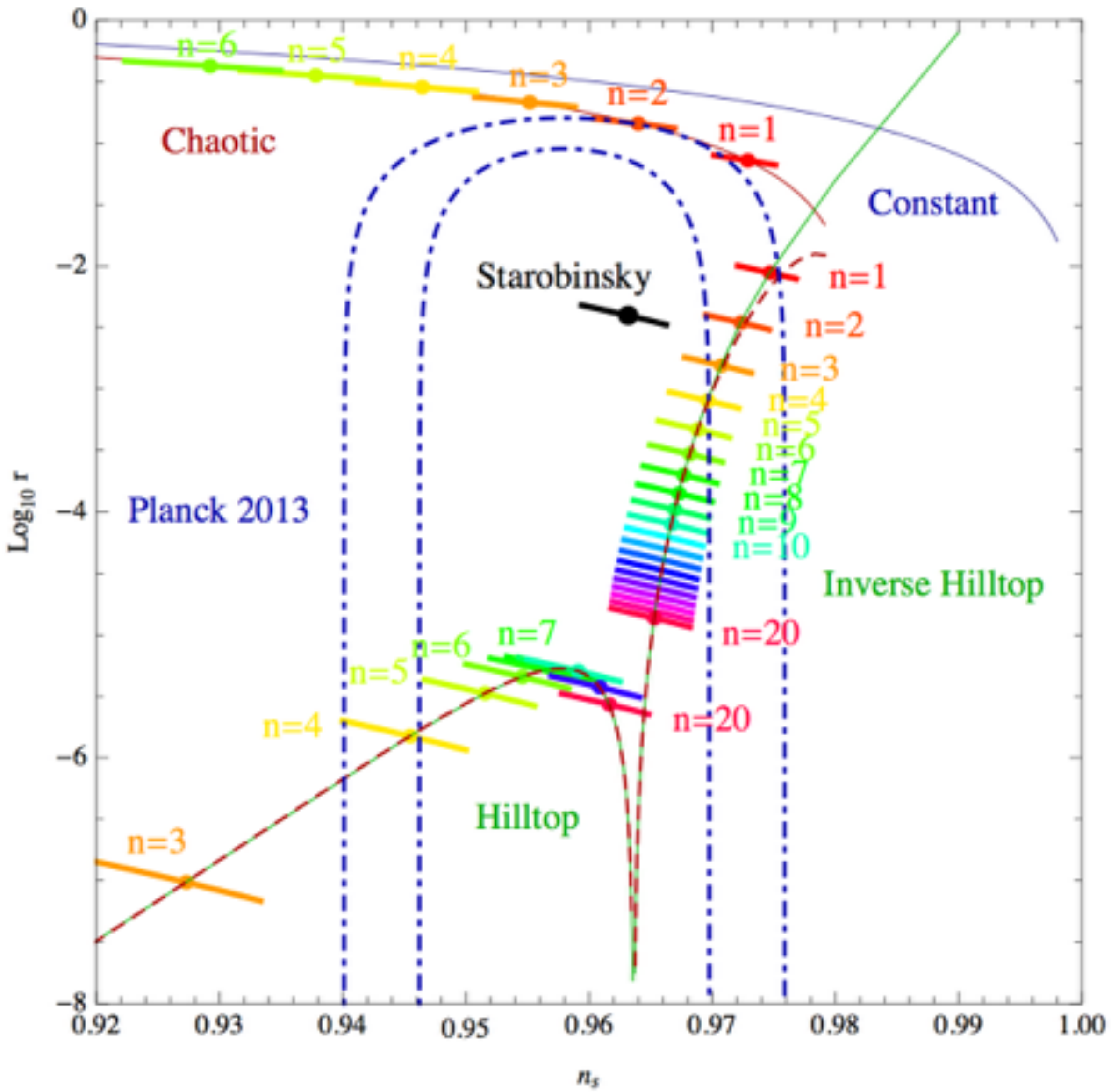
FRW/CFT:
[Freivogel et al., hep-th/0606204]
[Sekino et al., 0908.3844]

Universality classes of inflation

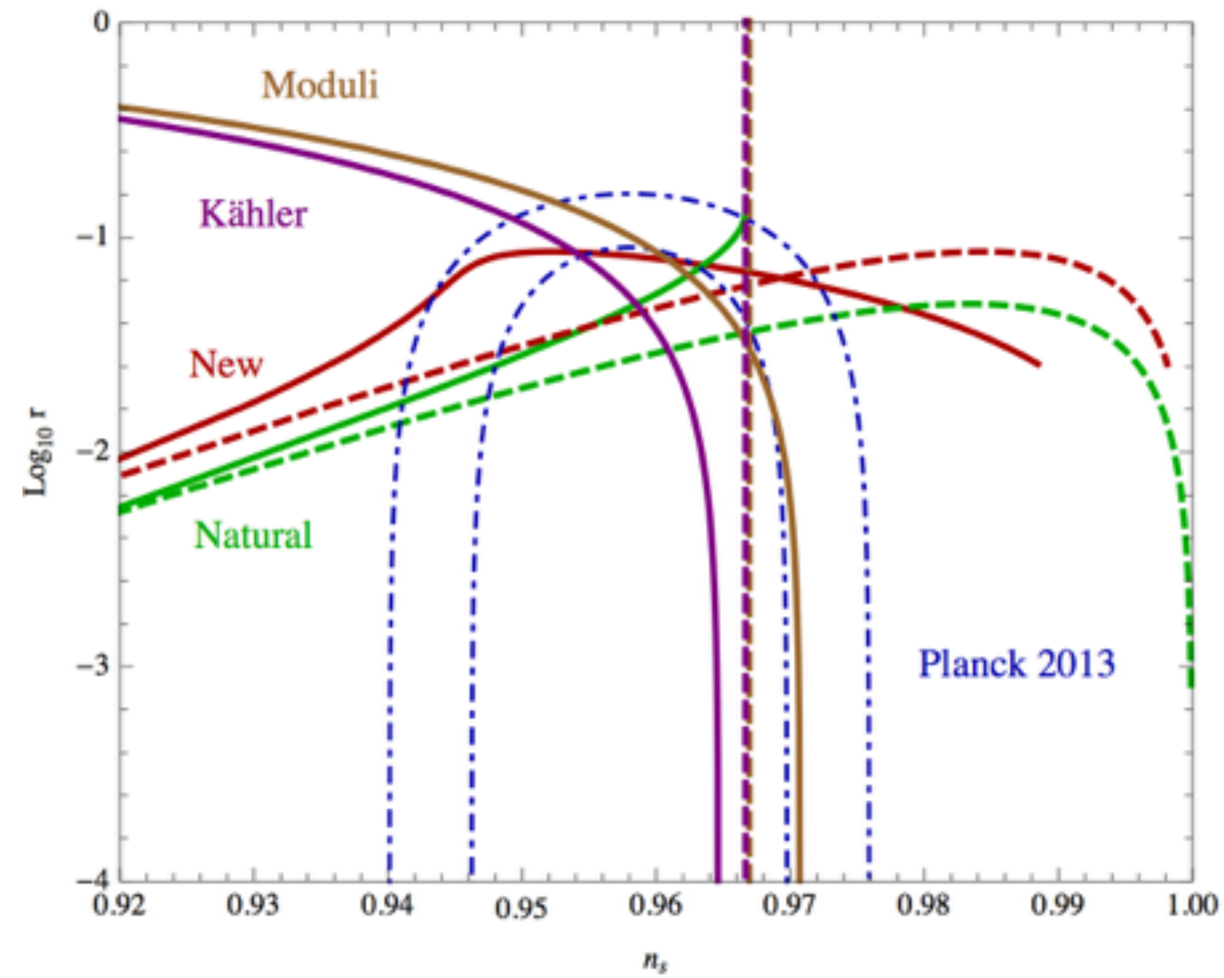
[Mukhanov, 1303.3925] [Roest, 1309.1285] [Garcia-Bellido et al., 1402.2059] [Binetruy et al., 1407.0820]

class	$\beta(\phi)$	$V(\phi)$	inflation model
SF(1)	$\beta_1\phi$	$1-\beta\phi^2$	(Natural)
SF(ℓ)	$\beta_\ell\phi^\ell$	$1-\beta_\ell\phi^{\ell+1}$	Hilltop
MF	$-\beta e^{-\gamma\phi}$	$1-\beta e^{-\gamma\phi}$	Starobinsky
LF(k)	$-\hat{\beta}_k/\phi^k$	$1-\hat{\beta}_k\phi^{-k+1}$	Inverse-Hilltop
LF(1)	$-\hat{\beta}_1/\phi$	$\phi^{\hat{\beta}_1}$	Chaotic
LF(0)	$-\hat{\beta}_0$	$e^{\hat{\beta}_0\phi}$	Power-law

Universality classes of inflation



Figures from [Garcia-Bellido, Roest, 1402.2059]



Universality classes of inflation

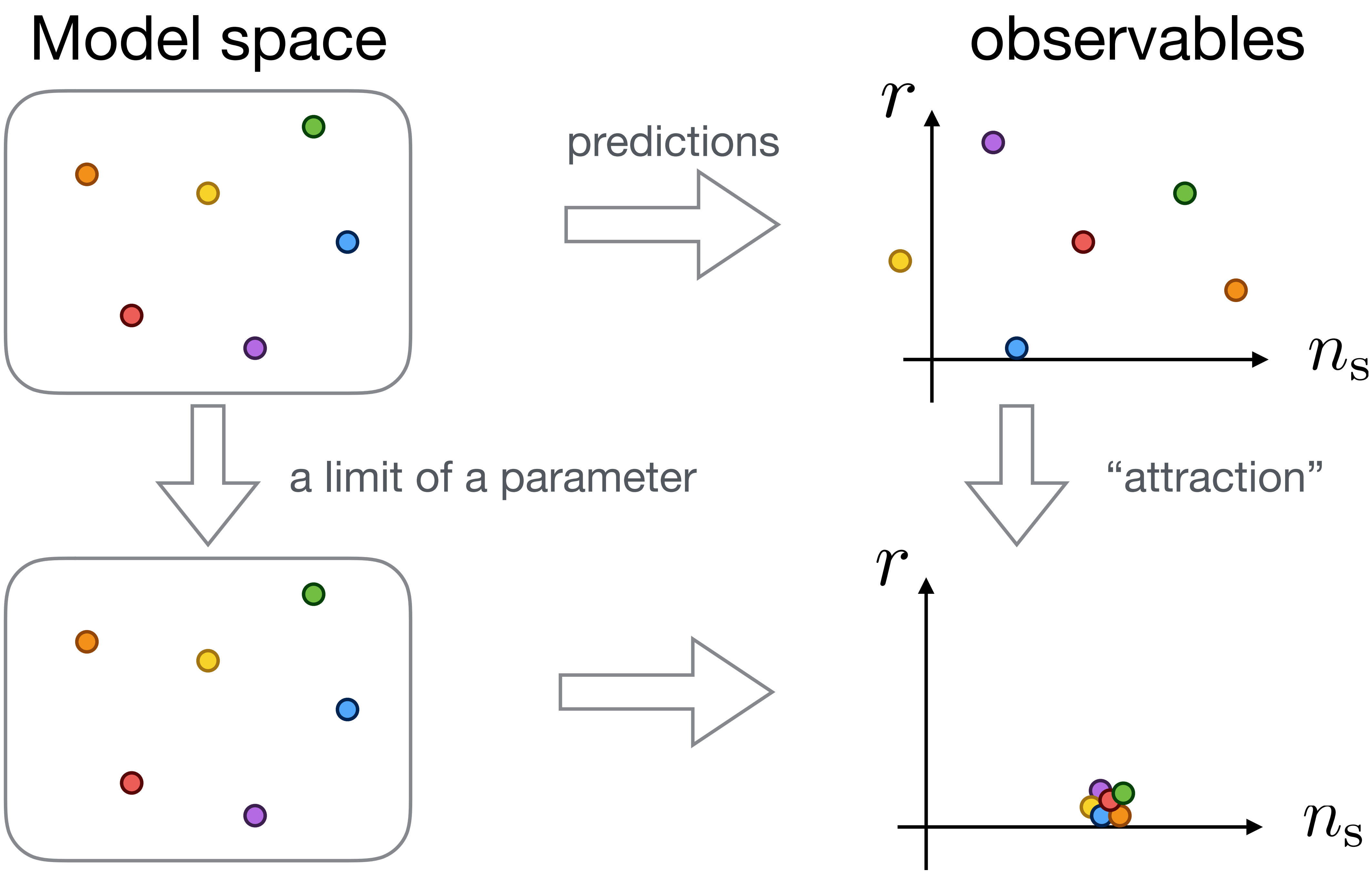
[Mukhanov, 1303.3925] [Roest, 1309.1285] [Garcia-Bellido et al., 1402.2059] [Binetruy et al., 1407.0820]

観測的ステータス

	class	$\beta(\phi)$	$V(\phi)$	inflation model
△	SF(1)	$\beta_1\phi$	$1-\beta\phi^2$	(Natural)
△	SF(ℓ)	$\beta_\ell\phi^\ell$	$1-\beta_\ell\phi^{\ell+1}$	Hilltop
◎	MF	$-\beta e^{-\gamma\phi}$	$1-\beta e^{-\gamma\phi}$	Starobinsky
○	LF(k)	$-\hat{\beta}_k/\phi^k$	$1-\hat{\beta}_k\phi^{-k+1}$	Inverse-Hilltop
△	LF(1)	$-\hat{\beta}_1/\phi$	$\phi^{\hat{\beta}_1}$	Chaotic
×	LF(0)	$-\hat{\beta}_0$	$e^{\hat{\beta}_0\phi}$	Power-law

Any underlying mechanism for the universality?

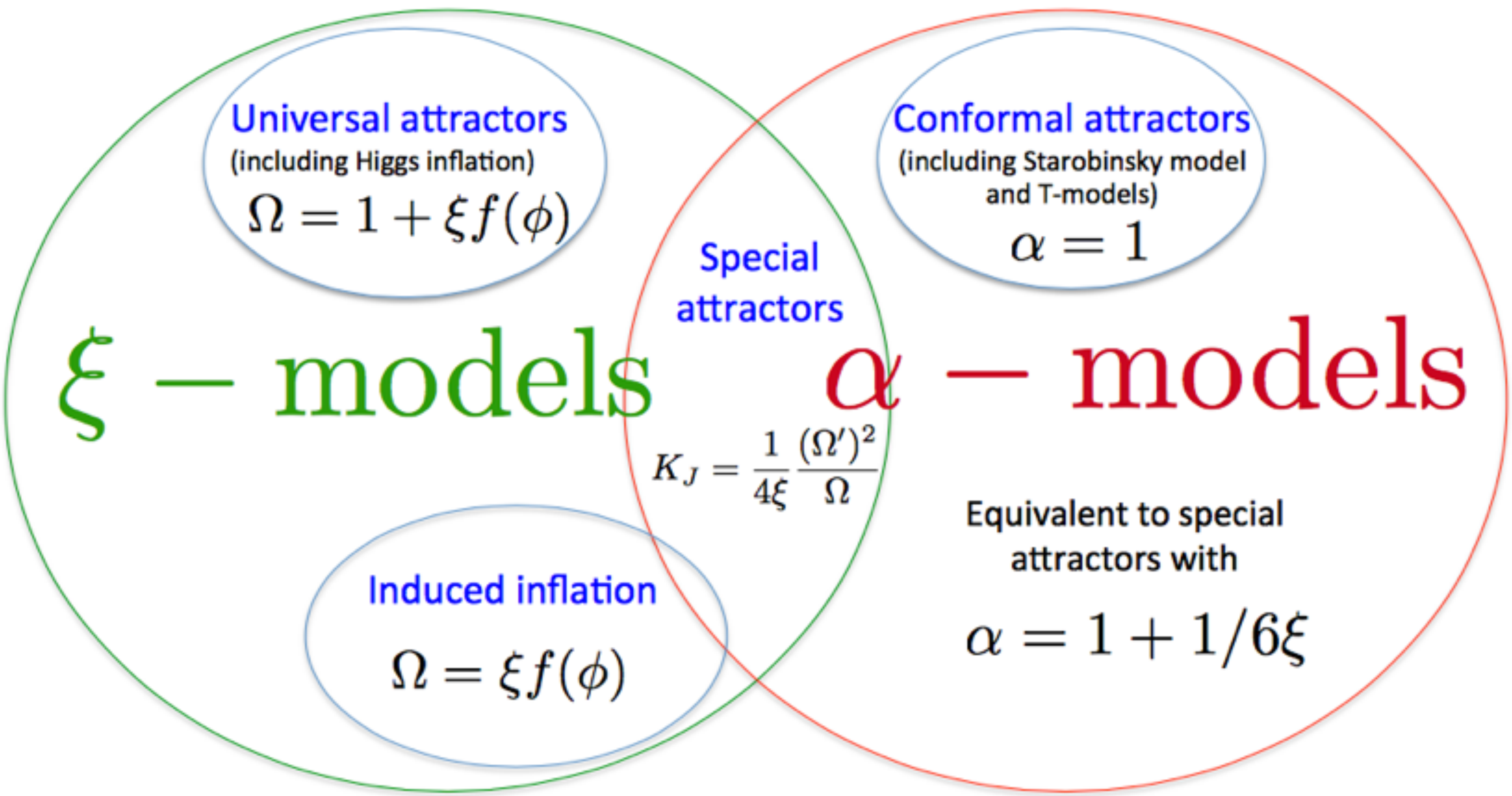
Inflationary Attractor Models



Unity of cosmological attractors

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}\Omega(\phi)R - \frac{1}{2}K_{\mathrm{J}}(\phi)(\partial_\mu\phi)^2 - V_{\mathrm{J}}(\phi)$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

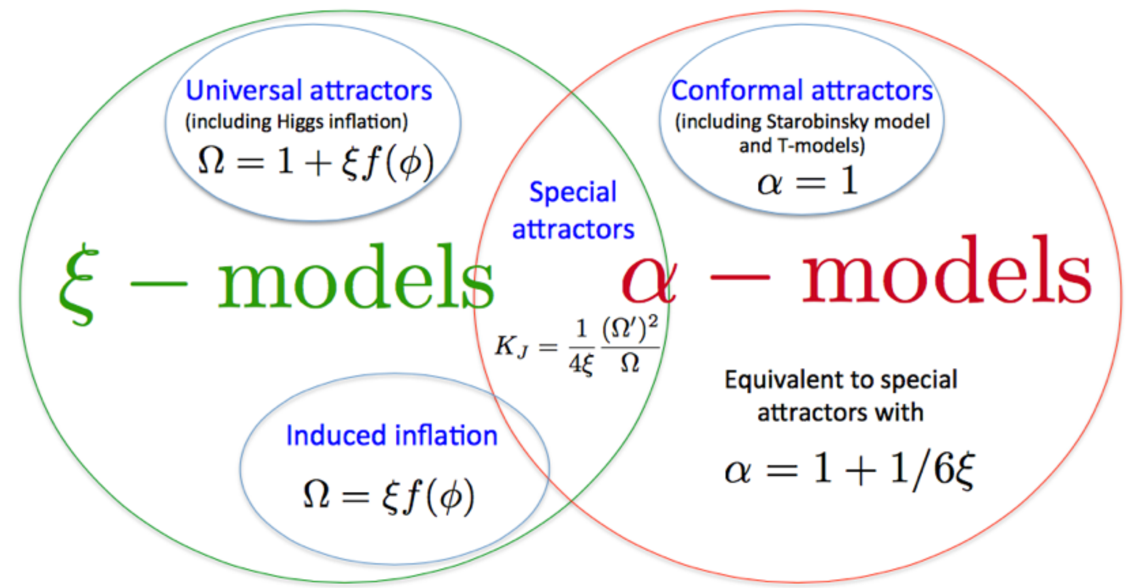


Unity of cosmological attractors

$$(\sqrt{-g})^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}K_{\mathrm{E}}(\varphi)(\partial_\mu\varphi)^2 - V_{\mathrm{E}}(\varphi)$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

$$K_{\mathrm{E}}(\phi) \simeq \frac{3\alpha/2}{(\phi-\phi_0)^2}$$



2nd order pole(s) in K_{E} !

Inflation occurs near the pole.

Canonical normalization makes the potential flat.

Pole inflation

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2\varphi^p}\partial^\mu\varphi\partial_\mu\varphi - V_0\left(1-\varphi+\mathcal{O}(\varphi^2)\right)$$

$$V = \begin{cases} V_0\left(1-\left(\frac{p-2}{2\sqrt{a_p}}\phi\right)^{-\frac{2}{p-2}}+\cdots\right) & (p\neq 2), \\ V_0\left(1-e^{-\phi/\sqrt{a_p}}+\cdots\right) & (p=2), \end{cases}$$

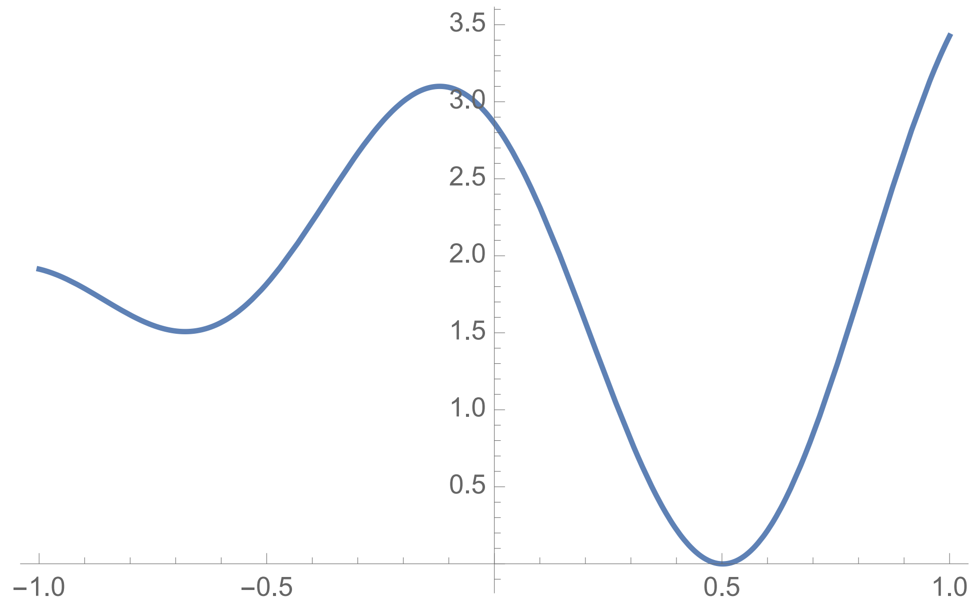
⇨ $$n_s = 1-\frac{p}{(p-1)N} \qquad r = \frac{8}{a_p}\left(\frac{a_p}{(p-1)N}\right)^{\frac{p}{p-1}}$$

[Galante, Kallosh, Linde, Roest, 1412.3797]

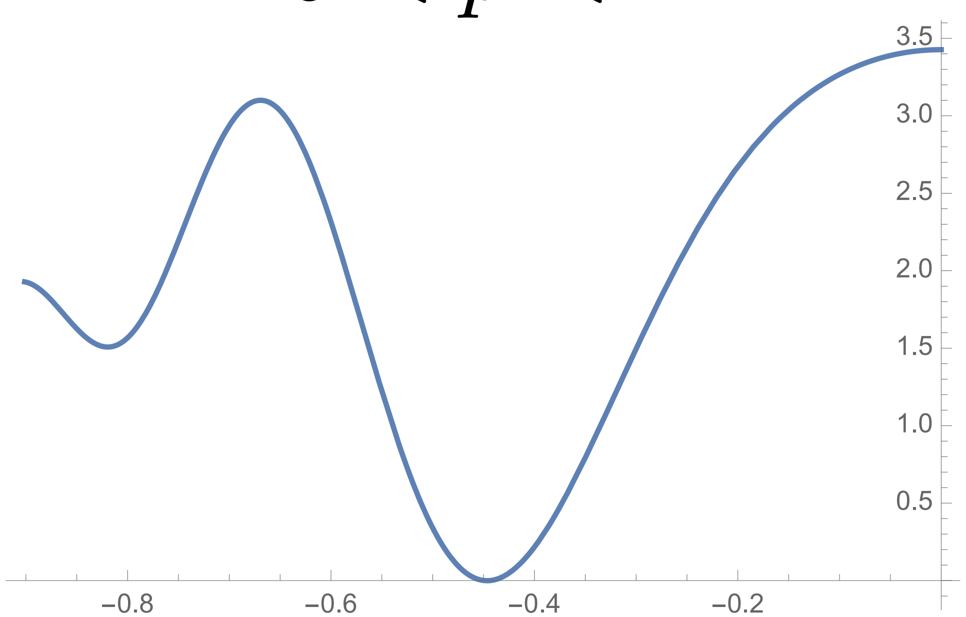
[Broy, Galante, Roest, Westphal, 1507.02277]

Change of potential shape

The original potential

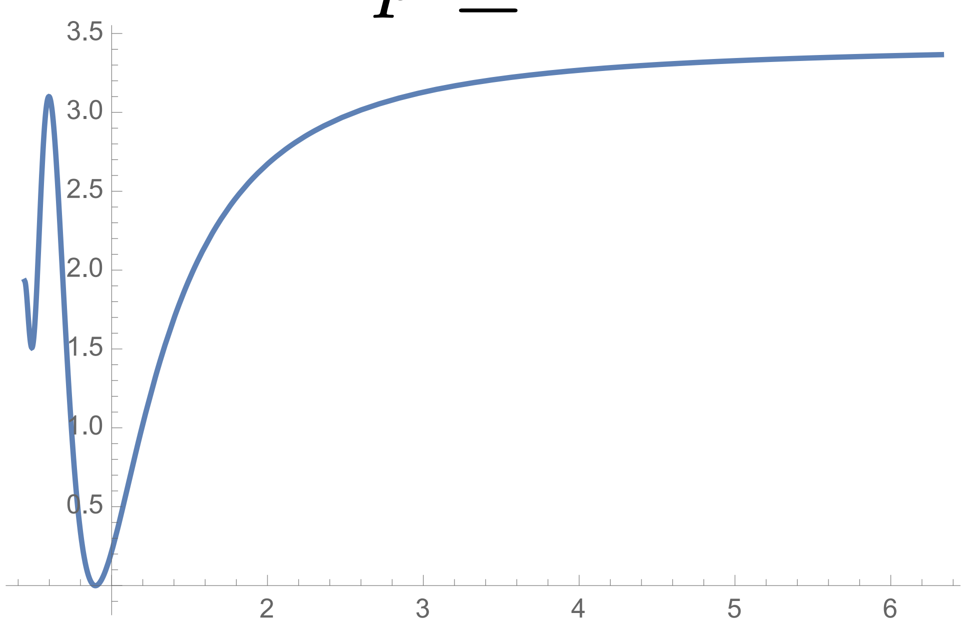


$0 < p < 2$



“hilltop”

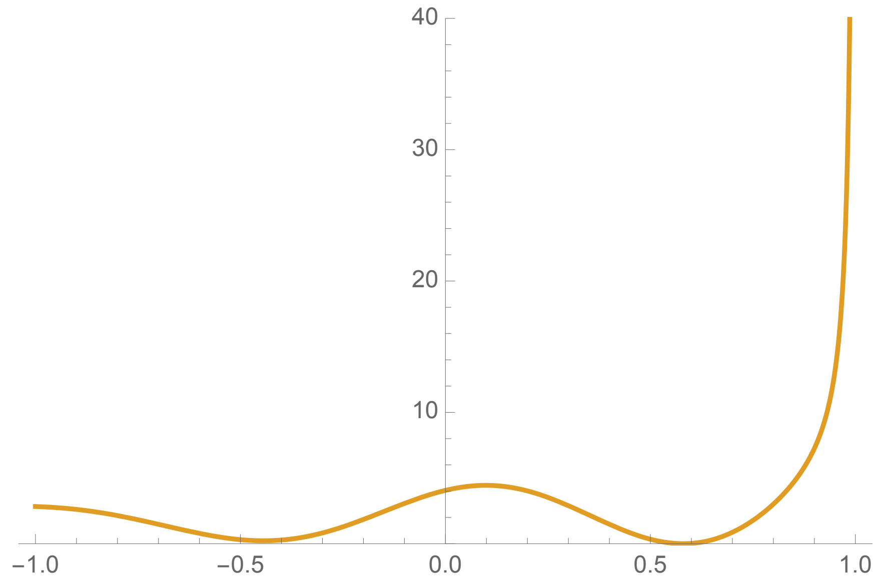
$p \geq 2$



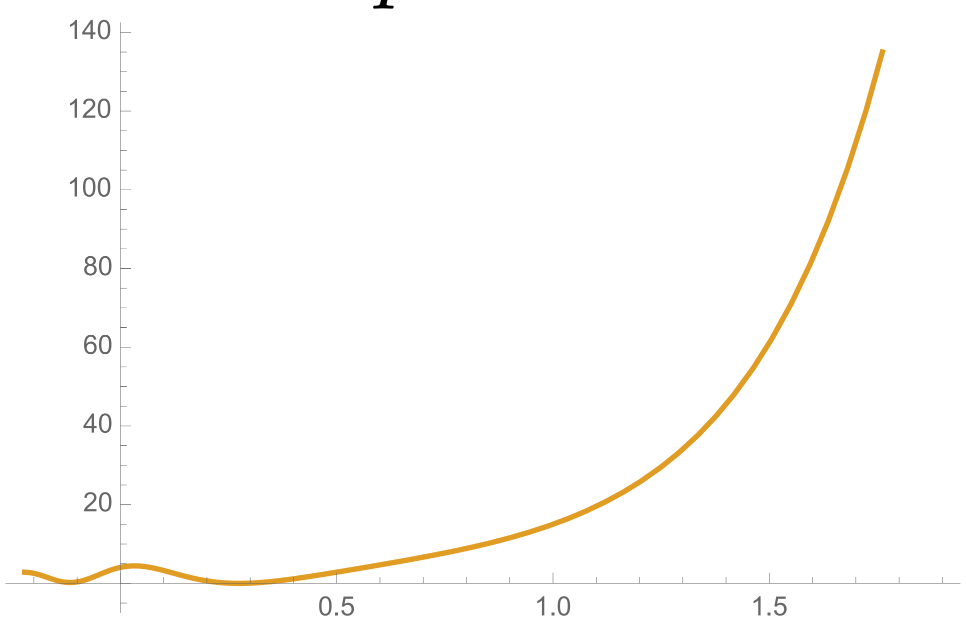
“inverse-hilltop”

Inflation with a singular potential

The original *diverging* potential

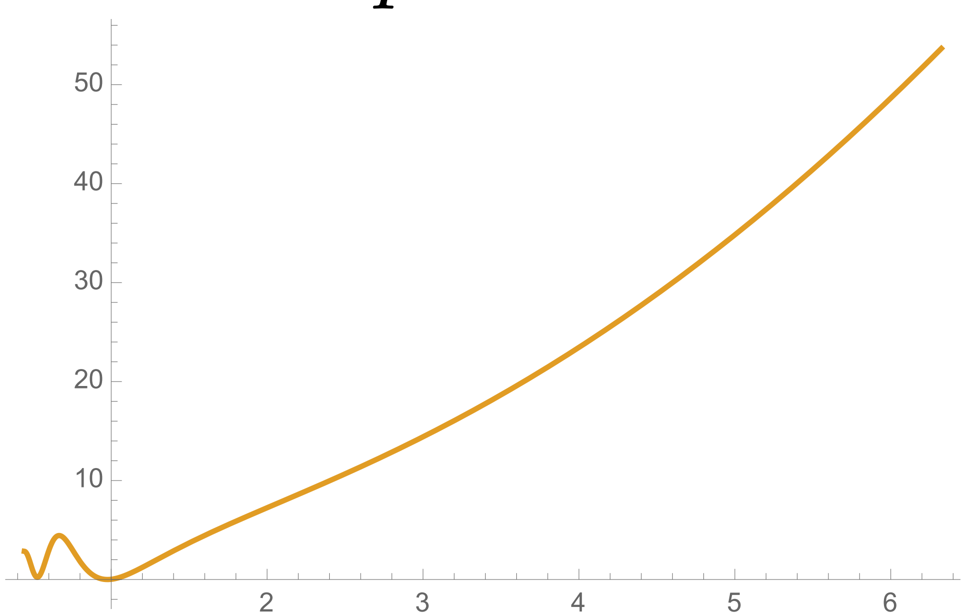


$p = 2$



"power-law"

$p > 2$



"chaotic"

Inflation with a singular potential

[Rinaldi, L. Vanzo, S. Zerbini, and G. Venturi, 1505.03386]

[TT, 1602.07867]

$$\left(\sqrt{-g}\right)^{-1}\mathcal{L} = -\frac{a_p}{2\varphi^p}\partial^\mu\varphi\partial_\mu\varphi - \frac{C}{\varphi^s}\left(1+\mathcal{O}(\varphi)\right)$$

$$V = \begin{cases} C\left(\frac{p-2}{2\sqrt{a_p}}\phi\right)^{\frac{2s}{p-2}} + \cdots & (p \neq 2), \\ Ce^{s\phi/\sqrt{a_p}} + \cdots & (p = 2). \end{cases}$$

Potentials for monomial **chaotic** and **power-law** inflation

$$n_{\mathrm{s}} = 1 - \frac{p+s-2}{(p-2)N} \qquad r = \frac{8s}{(p-2)N}$$

Summary of general pole inflation

For more details, see [TT, arXiv:1602.07867].

	$p=1$	$1<p<2$	$p=2$	$2<p$
non-singular potential	**2nd order hilltop** **generalization of natural inflation**	**hilltop**	**alpha-attractor** **xi-attractor** **Starobinsky model** **Higgs inflation**	**inverse-hilltop**
singular potential	**run-away**	**run-away**	**power-law inflation (exponential potential)**	**monomial chaotic**

Correspondence to universality classes of inflation

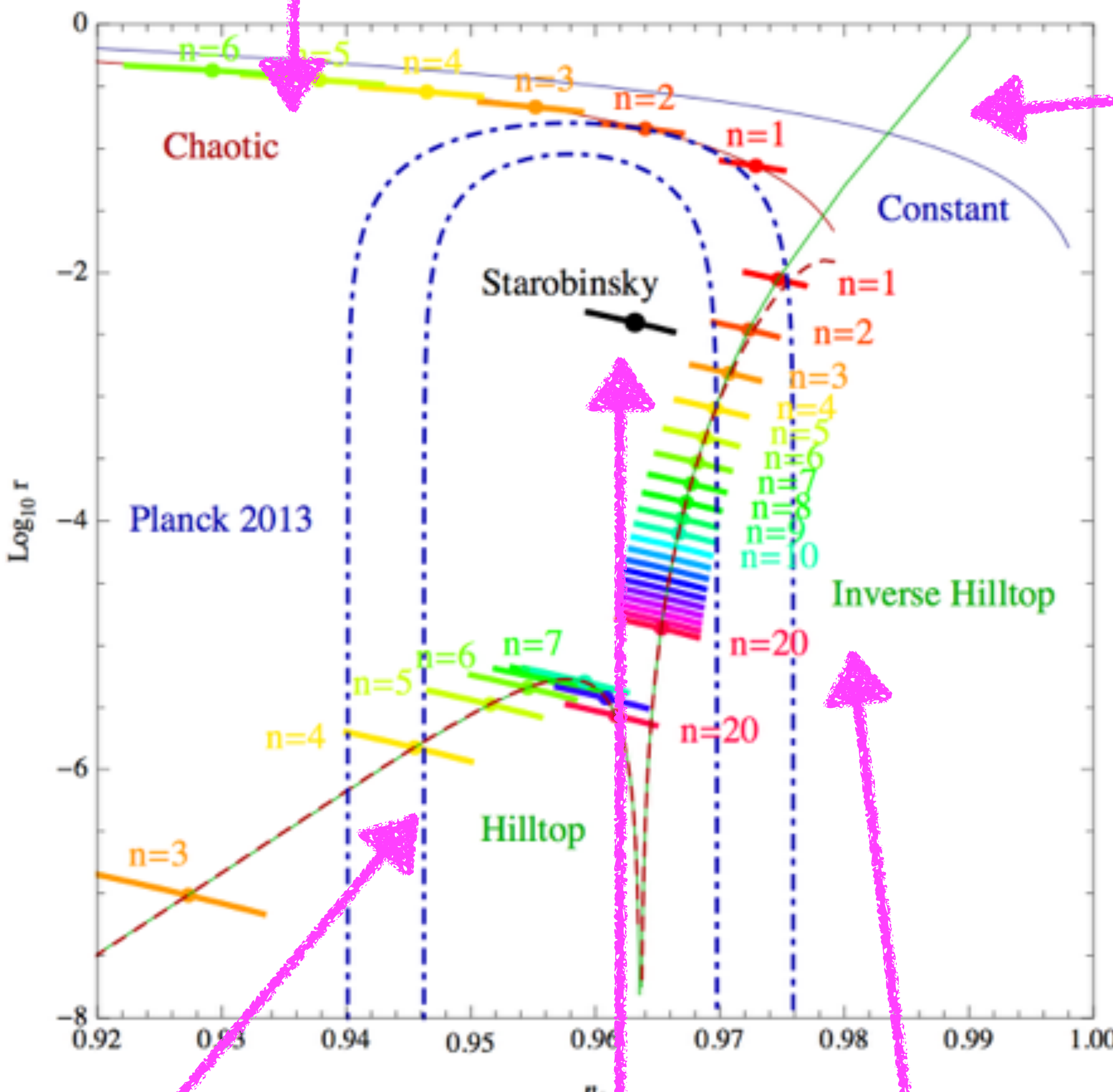
class	$\beta(\phi)$	$V(\phi)$	inflation model	corresponding pole
SF(1)	$\beta_1\phi$	$1-\beta\phi^2$	(Natural)	$p=1$
SF(ℓ)	$\beta_\ell\phi^\ell$	$1-\beta_\ell\phi^{\ell+1}$	Hilltop	$1<p<2, \quad \ell+1=-2/(p-2)$
MF	$-\beta e^{-\gamma\phi}$	$1-\beta e^{-\gamma\phi}$	Starobinsky	$p=2$
LF(k)	$-\hat{\beta}_k/\phi^k$	$1-\hat{\beta}_k\phi^{-k+1}$	Inverse-Hilltop	$p>2, \quad -k+1=-2/(p-2)$
LF(1)	$-\hat{\beta}_1/\phi$	$\phi^{\hat{\beta}_1}$	Chaotic	$p>2$ w/ a singular potential
LF(0)	$-\hat{\beta}_0$	$e^{\hat{\beta}_0\phi}$	Power-law	$p=2$ w/ a singular potential

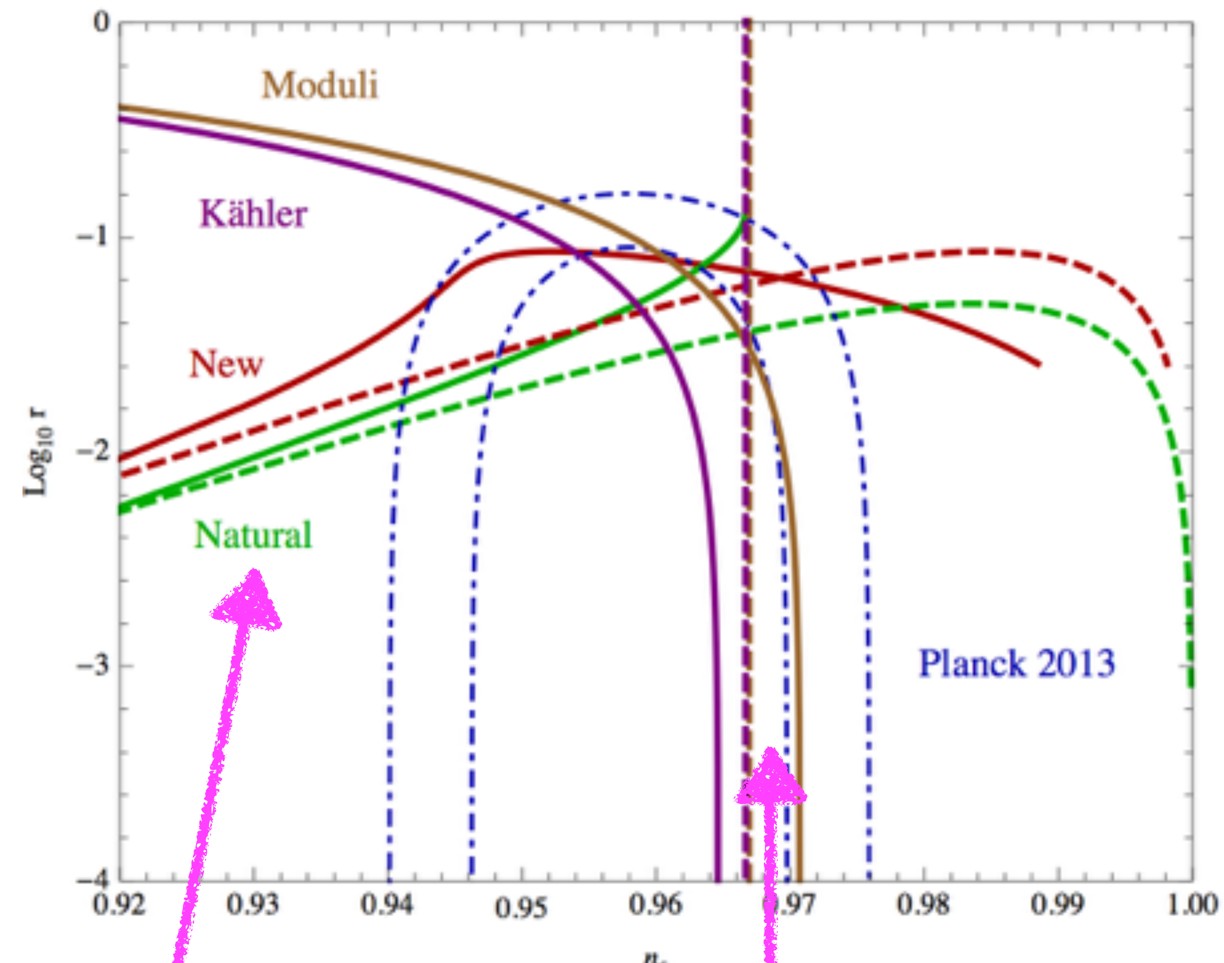
General Pole Inflation
As a realization of Universality Classes

Figures from [Garcia-Bellido, Roest, 1402.2059]

$p > 2$ w/ sing. pot.

$p = 2$ w/ sing. pot.





$p < 2$ $p = 2$ $p > 2$ $p = 1$ $p = 2$ w/ log. corr.

ここまでのまとめ

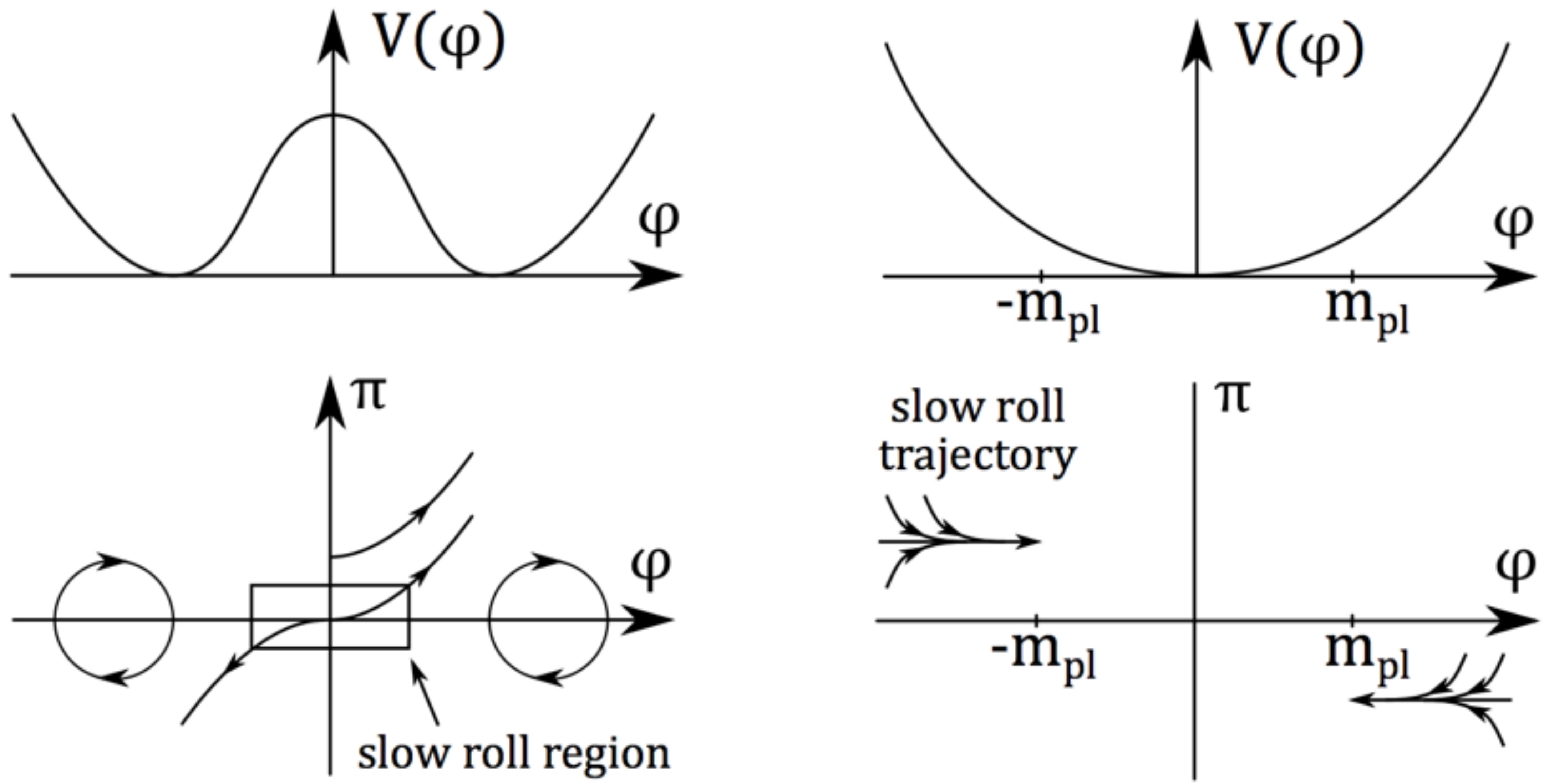
- インフレーション模型は Universality class に分類できる。
- インフラトン作用の極と次数による分類は、
  それを実現する具体例となっている。
- さて、どのクラスが現実的でしょう？

Initial condition problems:
Small-field or Large-field?

How likely the slow-roll is?

Small field

Large field



Figures from a review [Brandenberger, 1601.01918]

Inhomogeneous initial conditions

[East, Kleban, Linde, and Senatore, 1511.05143]

スカラー場の揺らぎがスローロールの領域を越えない限り
非一様性が大きくてもインフレーションが起こる事を示した。

$$\langle \nabla\phi \cdot \nabla\phi \rangle = 10^3 \Lambda$$

3＋1次元数値計算で

結論

small field: チューニングが必要

large field: robust

Universality classes of inflation

[Mukhanov, 1303.3925] [Roest, 1309.1285] [Garcia-Bellido et al., 1402.2059] [Binetruy et al., 1407.0820]

観測的ステータス	class	$\beta(\phi)$	$V(\phi)$	inflation model	初期条件
△	SF(1)	$\beta_1\phi$	$1-\beta\phi^2$	(Natural)	
△	SF(ℓ)	$\beta_\ell\phi^\ell$	$1-\beta_\ell\phi^{\ell+1}$	Hilltop	×
◎	MF	$-\beta e^{-\gamma\phi}$	$1-\beta e^{-\gamma\phi}$	Starobinsky	◎
○	LF(k)	$-\hat{\beta}_k/\phi^k$	$1-\hat{\beta}_k\phi^{-k+1}$	Inverse-Hilltop	◎
△	LF(1)	$-\hat{\beta}_1/\phi$	$\phi^{\hat{\beta}_1}$	Chaotic	◎
×	LF(0)	$-\hat{\beta}_0$	$e^{\hat{\beta}_0\phi}$	Power-law	◎

Any underlying mechanism for the universality?

Shift symmetry and its origin

Planck-suppressed terms are NOT suppressed enough!

$$V \sim m^2\phi^2 + \lambda_3\phi^3 + \lambda_4\phi^4 + \sum_{n>4} \lambda_{4+n}\phi^4 \left(\frac{\phi}{M_{\rm P}}\right)^n$$

In particular,

$$V\frac{\phi^2}{M_{\rm P}^2} \quad \Longrightarrow \quad \eta = \mathcal{O}(1)$$

Also a naturalness question:

Why $m \ll M_{\rm P}$ $(\text{or } \Lambda)$?

See e.g. a good review [Westphal, 1409.5350].

Shift symmetry

$$\phi \to \phi + c$$

with an explicit, soft breaking $V(\phi) \ll 1$

Perturbative quantum gravity corrections:

$$\delta V \sim V \left(a \frac{V''}{M_{\rm P}^2} + b \frac{V}{M_{\rm P}^4} \right)$$

[Smolin, PLB 93, 95 (1980)]

[Linde, PLB 202 (1988) 194]

[Kaloper, Lawrence, Sorbo, 1101.0026]

Technically natural. ['t Hooft, NATO Sci.Ser.B 59 (1980) 135]

Shift symmetry in SUGRA

SUSY breaking effects $\Rightarrow$ $m_{\rm soft} \sim \mathcal{O}(H)$

Why $m \ll H$ ($\eta \ll 1$) ?

SUGRA scalar potential

$$V = e^{K}\left(K^{\bar{j}i} D_i W \bar{D}_{\bar{j}} \overline{W} - 3|W|^2\right) + \frac{1}{2} f_{AB} D^A D^B$$

where $D_i W = W_i + K_i W$.

shift symmetry [Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

$$\Phi \to \Phi + c \qquad K(\Phi, \bar{\Phi}) = K(i(\Phi - \bar{\Phi}))$$

Origin of the shift symmetry?

shift symmetry … non-linearly realized symmetry … any linear realization?

[Freese, Frieman, Olinto, PRL 65 (1990) 3233]

U(1)

Axion

pNG boson of U(1) or
Wilson line of extra dimensions

R

Dilaton

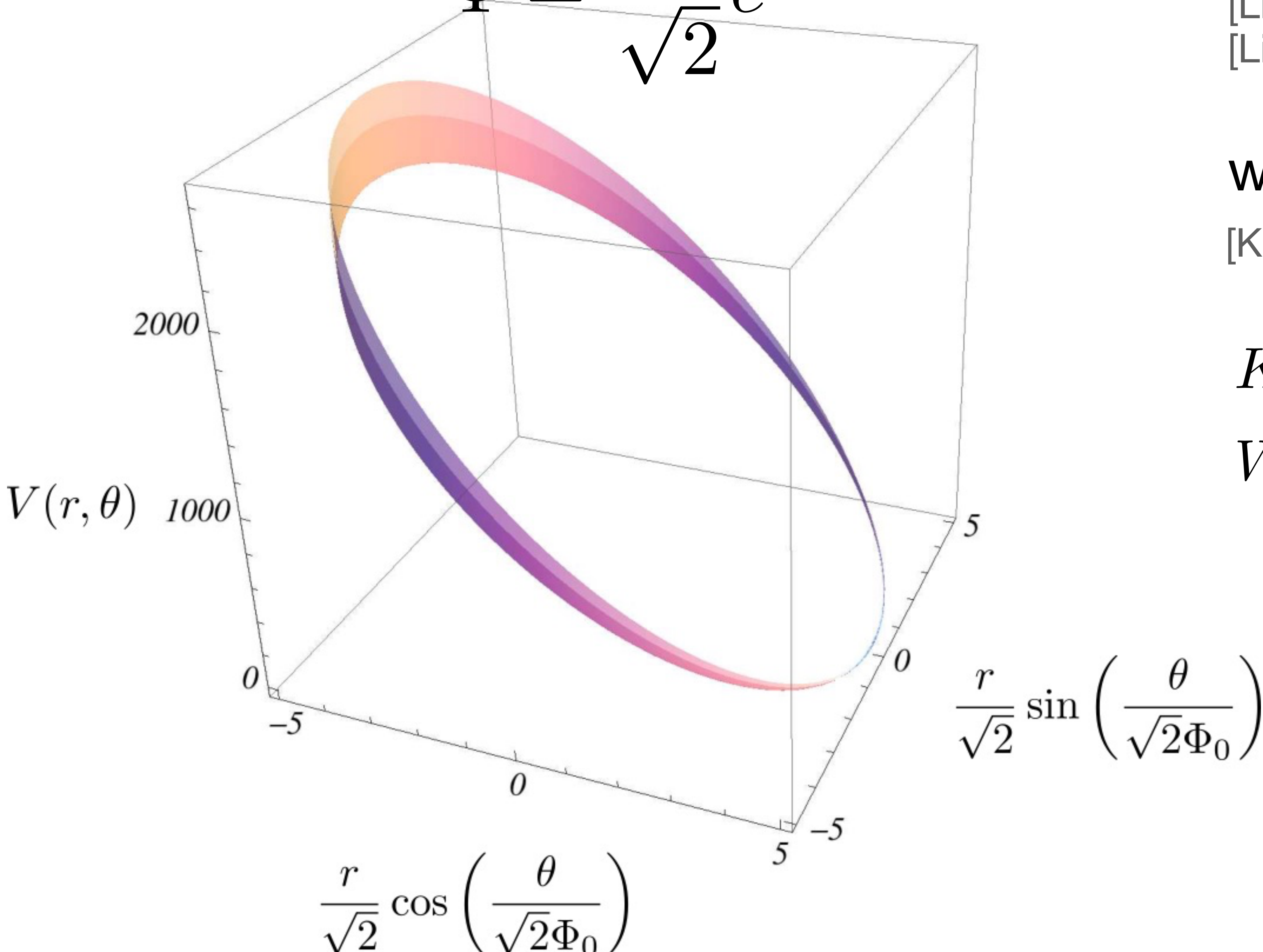
pNG boson of scale invariance

Inverse-hilltop class と chaotic class は、このどちらかに帰着するべき？

U(1)-sym. inflation in SUGRA

“Helical phase inflation”

$$\Phi = \frac{r}{\sqrt{2}} e^{i\theta/\sqrt{2}\Phi_0}$$



with the stabilizer field

[Li, Li, Nanopoulos, 1409.3267]
[Li, Li, Nanopoulos, 1412.5093]
[Li, Li, Nanopoulos, 1502.05005]
[Li, Li, Nanopoulos, 1507.04687]

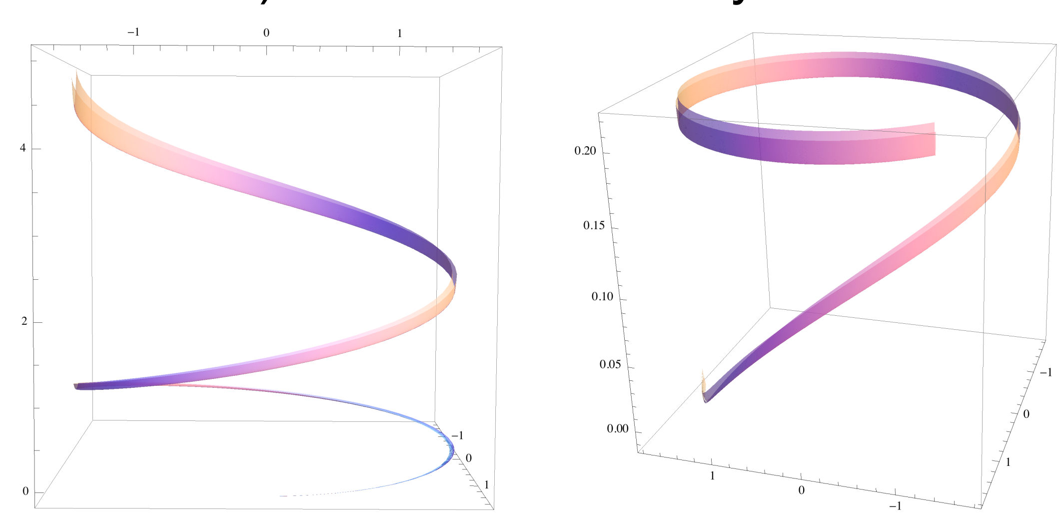
without the stabilizer field

[Ketov, TT, 1509.00953]

$$K = \left(\bar{\Phi}\Phi - \Phi_0^2\right) - \frac{\zeta}{4}\left(\bar{\Phi}\Phi - \Phi_0^2\right)^4 .$$

$$W = m\,(c + \Phi)$$

cf.) with a monodromy



Extranatural inflation

[Arkani-Hamed et al., hep-th/0301218] [Kaplan et al., hep-ph/0302014]

Inflaton as an extra-dim. component of a gauge field

$$e^{i\phi/f} = e^{i\oint A_5 \mathrm{d}x^5}$$

Discrete (gauged) shift symmetry: $\phi \to \phi + 2\pi n f$

NOTE: The Wilson line is a ***non-local*** effect.
Any local effects (including Quantum Gravity) cannot break the gauge symmetry explicitly.

$$V \sim \sum_n c_n e^{-nS} \cos\frac{n\phi}{f}$$

Weak Gravity Conjecture

"Gravity is the weakest force." [Arkani-Hamed et al., hep-th/0601001]

Statement of the conjecture:

For U(1) gauge theory with gravity, there must exist a particle satisfying

$$m < qM_{\mathrm{P}}$$

where m and q are the mass and U(1) charge of the particle.

Application to a magnetic monopole:

The cut-off scale of the effective theory must satisfy

$$\Lambda \lesssim gM_{\mathrm{P}}$$

where g is the gauge coupling (elementary charge).

WGC: reason

In $g \to 0$ limit, infinitely many charged black holes can exist.

The production probability will diverge
even if each probability is exponentially suppressed.

(same as the argument against black hole remnants)

[Susskind, hep-th/9501106]

However, there are objections to the arguments.
See a recent review [Chen, Ong, Yeom, 1412.8366]

It also violates the covariant entropy bound conjecture [Bousso, hep-th/9905177].

For the black holes to be able to decay, we need light enough particles.

$$M_{\rm BH} \geq Q_{\rm BH} M_{\rm P}$$

(Extremal black holes saturate the bound.)

WGC: generalization

[Arkani-Hamed et al., hep-th/0601001]

For *p*-form gauge field, *p-1*-dim object satisfies

$$T \lesssim g M_{\mathrm{P}}$$

where *T* is the tension, and *g* is the charge density.

For 0-form (axion),

$$S \lesssim \frac{M_{\mathrm{P}}}{f}$$

where *S* is the instanton action, and *f* is the decay constant.

(Extra)Natural inflation のまとめ

U(1)

- discrete gauged shift symmetry に基づく魅力的なアイディア
- WGC により decay constant は厳しく制限される。（ループホールに注意。） See e.g. [Saraswat, 1608.06951] and references therein.
- WGC を回避できたとしても、既に Planck 1 σ 領域の外…。

Scale-invariant models of inflation

$$g_{\mu\nu} \to g_{\mu\nu} e^{2\sigma}$$
$$\sqrt{-g} \to \sqrt{-g} e^{4\sigma}$$

Starobinsky model [Starobinsky, PLB 91 (1980) 99] $R \to R e^{-2\sigma}$

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{R}{2} + \frac{R^2}{12M^2} \right) \sim \int \mathrm{d}^4 x \sqrt{-g} \frac{R^2}{12M^2}$$

Higgs inflation [Bezrukov, Shaposhnikov, 0710.3755] $\phi \to \phi e^{-\sigma}$

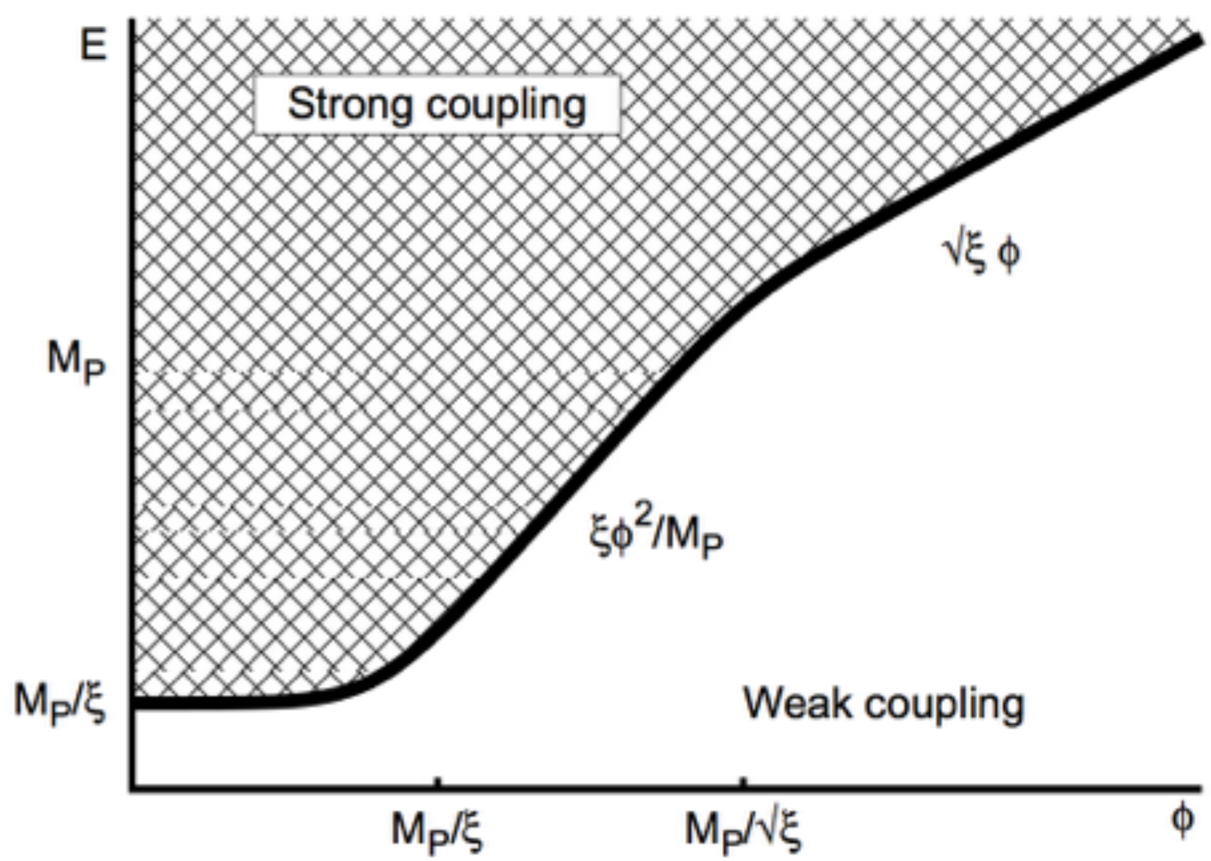
$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{R}{2} + \xi \phi^2 R - \frac{1}{2} (\partial_\mu \phi)^2 - \lambda \phi^4 \right) \sim \int \mathrm{d}^4 x \sqrt{-g} \left(\zeta \phi^2 R - \lambda \phi^4 \right)$$

From a low-energy EFT point of view, see also [Csáki et al., 1406.5192].

Unitarity violation or not

[Bezrukov, Magnin, Shaposhnikov, Sibiryakov, 1008.5157]

Taking into account the field dependence of the cut-off,
the tree-level unitarity is preserved throughout the history of the universe.



<u>Provided that</u> **UV physics** respects the asymptotic scale (shift) symmetry,
the form of the Lagrangian is preserved by quantum corrections.

Scale invariance in Swampland?

Scale invariance はグローバル対称性なので、
量子重力効果で破れると期待される。

[Hawking, PRD 14 (1976) 2460]

Any non-trivial constraints on EFT, like WGC?

[Vafa, hep-th/0509212] [Ooguri, Vafa, hep-th/0605264]

Scale-invariant inflation のまとめ

R

- 観測と非常によく合っている。
- Global 対称性なので量子重力補正をコントロールできない。
- Swampland からの非自明な制限はあるか？
  （特に、摂動的ユニタリティーは量子重力的要求と両立するか？）

Summary & Conclusion

現実的なインフレーション模型は何か

WGC を回避できれば現実的

観測的ステータス	class	$\beta(\phi)$	$V(\phi)$	inflation model	初期条件
△	SF(1)	$\beta_1\phi$	$1-\beta\phi^2$	(Natural)	
△	SF(ℓ)	$\beta_\ell\phi^\ell$	$1-\beta_\ell\phi^{\ell+1}$	Hilltop	×
◎	MF	$-\beta e^{-\gamma\phi}$	$1-\beta e^{-\gamma\phi}$	Starobinsky	◎
○	LF(k)	$-\beta_k/\phi^k$	$1-\beta_k\phi^{-k+1}$	Inverse-Hilltop	◎
△	LF(1)	$-\hat{\beta}_1/\phi$	$\phi^{\hat{\beta}_1}$	Chaotic	◎
×	LF(0)	$-\hat{\beta}_0$	$e^{\hat{\beta}_0\phi}$	Power-law	◎

シフト対称性の起源が不明瞭？

…人間原理でチューニングしてフラットにする？

→何故まだ重力波が見つからない程フラットにできる？

データと合うが、

どう対称性を制御するか。

更なる研究が必要！

Part II: Recent progress toward
UV (SUGRA) embedding of inflation models

セミナーに呼んでください！