Resonance Chiral Lagrangian に基づくタウ崩壊分布の解析

梅枝 宏之 (広島大学)

共同研究者　両角 卓也　(広島大学)
木村 大自　(宇部高専)

(work in progress)

素粒子物理学の進展2016
9/8, 基礎物理学研究所

Introduction

- Light hadron の崩壊は精密に観測されている.

- 近年の実験データ(NA60, BES III, …)を使うことで、

QCD effective dynamicsのvalidityを調べることができる.

Introduction

- Light hadron の崩壊は精密に観測されている.
- 近年の実験データ(NA60, BES III, …)を使うことで、QCD effective dynamicsのvalidityを調べることができる.
- Resonance chiral Lagrangianは、light hadronのダイナミクスを扱える模型として使われている.
- ハドロン模型の予言値を、新物理探索に適用することができる.

Introduction

- Light hadron の崩壊は精密に観測されている

Plan

(1) ハドロン物理の実験データを使って、
模型のパラメーターを決める.

(2) タウの物理での新物理探索に適用する.

適用することができる.

Outline

(1) <u>模型</u>

-Resonance chiral Lagrangian-

(2) <u>ハドロン崩壊</u>

-Dalitz崩壊に関するダイレプトン質量分布-

(3) <u>タウレプトン崩壊</u>

-hadronic崩壊におけるタウの偏極度-

-新物理の効果-

Outline

(1) 模型

-Resonance chiral Lagrangian-

(2) ハドロン崩壊

-Dalitz崩壊に関するダイレプトン質量分布-

(3) タウレプトン崩壊

-hadronic崩壊におけるタウの偏極度-

-新物理の効果-

Chiral Lagrangian + ベクトル中間子

- QCDの有効理論パート.　$\pi, \eta, \cdots$
- 擬スカラー間にはたらく相互作用.

$$\mathcal{L}_{\mathrm{ChPT}} + \mathcal{L}_V$$

- 模型パート. Ecker *et. al.*, Phys. Lett. B 223, 425
  Resonance field of vectors
- $(\rho, \omega, \phi, K^{*+}, K^{*0})$　を扱う.

模型

(1-loop order counter terms)

$$\mathcal{L}_\chi = \mathcal{L}_{\rm ChPT} + \mathcal{L}_V + \mathcal{L}_c$$

$$U = \exp\left(\frac{2i\pi}{f}\right)$$

SU(3) **Octet + singlet**

η_0 : Singlet V_μ : Octet ϕ^0_μ : Singlet

$$\mathcal{L}_{\rm ChPT} = \frac{f^2}{4}{\rm Tr}(D_\mu U D^\mu U^\dagger) + B{\rm Tr}[M(U+U^\dagger)] + C{\rm Tr}QUQU^\dagger + \frac{1}{2}\partial_\mu\eta_0\partial^\mu\eta_0 - \frac{1}{2}M_{00}^2\eta_0^2 - ig_{2p}\eta_0{\rm Tr}[M(U-U^\dagger)]$$

$$\mathcal{L}_V = -\frac{1}{2}{\rm Tr}F_V^{\mu\nu}F_{V\mu\nu} + M_V^2{\rm Tr}\left(V_\mu - \frac{\alpha_\mu}{g}\right)^2 + g_{1V}\phi^0_\mu{\rm Tr}\left\{\left(V^\mu - \frac{\alpha^\mu}{g}\right)\left(\frac{\xi M\xi + \xi^\dagger M\xi^\dagger}{2}\right)\right\} - \frac{1}{4}F^0_{V\mu\nu}F_V^{0\mu\nu} + \frac{1}{2}M_{0V}^2\phi^0_\mu\phi^{0\mu}$$

固有パリティの破れ

"*Anomalous*" interactions

(a) SU(3) quantum anomaly

$$\mathcal{L}_{\mathrm{WZW}} \sim \epsilon^{\mu\nu\rho\sigma}\pi^0\partial_\mu A_\nu \partial_\rho A_\sigma$$

(b) Resonance field が入った相互作用項

$$\mathcal{L}_{\mathrm{IPV}} \sim \epsilon^{\mu\nu\rho\sigma} V_\mu \cdots$$

SU(3) invarianceを仮定する

固有パリティの破れ

$$\mathcal{L}_1 = i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\alpha_{L\mu}\alpha_{L\nu}\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)],$$

$$\mathcal{L}_2 = i\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\alpha_{L\mu}\alpha_{R\nu}\alpha_{L\rho}\alpha_{R\sigma}],$$

$$\mathcal{L}_3 = \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[gF_{V\mu\nu}\{\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)\}],$$

$$\mathcal{L}_4 = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[(\hat{F}_{L\mu\nu} + \hat{F}_{R\mu\nu})\{\alpha_{L\rho}, \alpha_{R\sigma}\}]$$

Fujiwara et. al.
PTP73(4)926-941

$$\alpha_\mu = \frac{1}{2i}(\xi^\dagger D_{L\mu}\xi + \xi D_{R\mu}\xi^\dagger),$$

$$\alpha_{L\mu} = \alpha_\mu + \alpha_{\perp\mu} - gV_\mu,$$

$$\alpha_{R\mu} = \alpha_\mu - \alpha_{\perp\mu} - gV_\mu,$$

$$\alpha_{\perp\mu} = \frac{1}{2i}(\xi^\dagger D_{L\mu}\xi - \xi D_{R\mu}\xi^\dagger)$$

$$\mathcal{L}_5 = \epsilon^{\mu\nu\rho\sigma}F^0_{V\mu\nu}\mathrm{Tr}[\alpha_{L\rho}\alpha_{R\sigma} - (R \leftrightarrow L)]$$

$$\mathcal{L}_6 = \frac{\eta_0}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{V\mu\nu}F_{V\rho\sigma}$$

$$\mathcal{L}_7 = \frac{\eta_0}{f}\epsilon^{\mu\nu\rho\sigma}F^0_{V\mu\nu}F^0_{V\rho\sigma}$$

$$\mathcal{L}_8 = \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(\hat{F}_{L\mu\nu} + \hat{F}_{R\mu\nu})\phi^0_\rho\frac{\alpha_{L\sigma} - \alpha_{R\sigma}}{2}$$

$$\mathcal{L}_9 = \frac{\eta_0}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(\hat{F}_{L\mu\nu} + \hat{F}_{R\mu\nu})F_{V\rho\sigma}$$

$$\mathcal{L}_{10} = \frac{\eta_0}{f}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(\hat{F}_{L\mu\nu} + \hat{F}_{R\mu\nu})(\hat{F}_{L\rho\sigma} + \hat{F}_{R\rho\sigma})$$

$\mathrm{SU}(3)$ singlets

η_0 ϕ^0

が入った相互作用.

$$\mathcal{L}_{\mathrm{model}} \sim \sum_{i=1}^{10} c_i^{\mathrm{IP}}\mathcal{L}_i$$

Outline

(1) 模型

-Resonance chiral Lagrangian-

(2) ハドロン崩壊

-ダイレプトン不変質量分布-

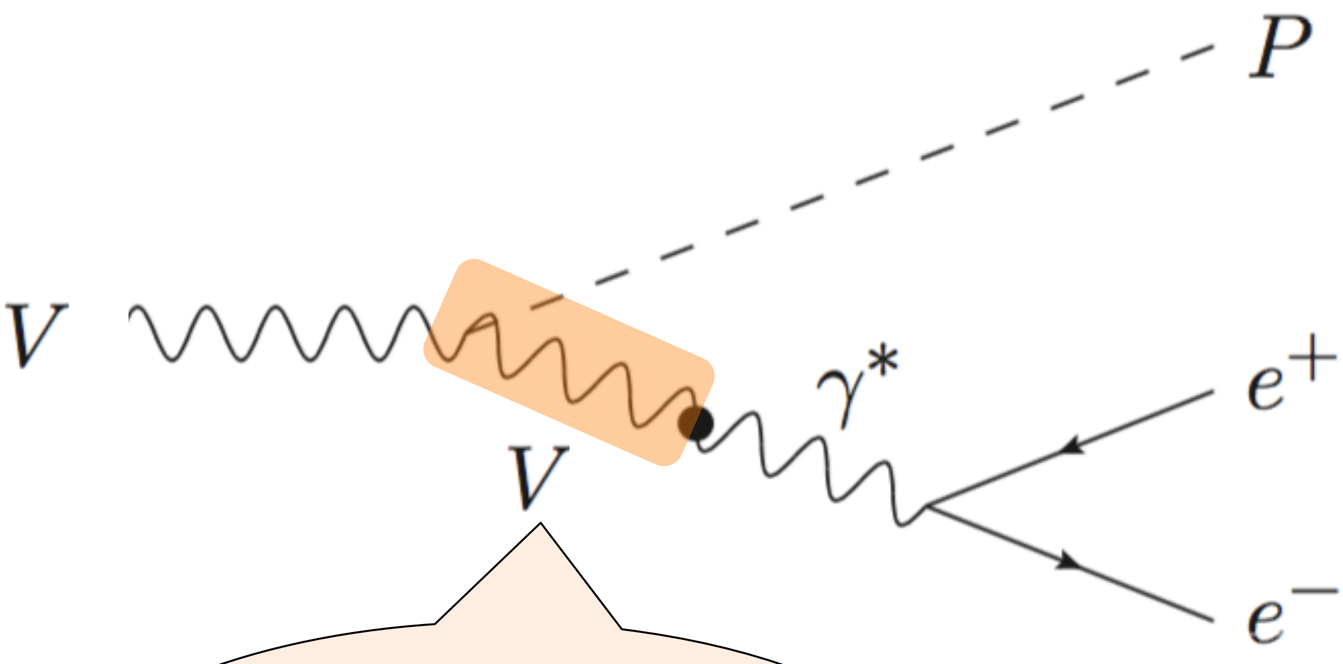
(3) タウレプトン崩壊

-hadronic崩壊におけるタウの偏極-

-新物理の効果-

Dalitz崩壊　$V \to Pl^+l^-$

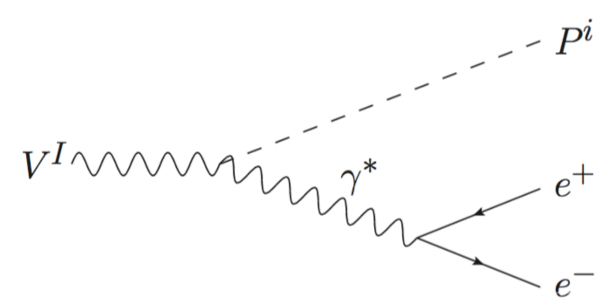
$V : 1^-$
$P : 0^-$



invariant mass

$$s = (p^+ + p^-)^2$$

Resonance of vector meson



$V \to Pl^+l^-$ の微分分岐比

$$\frac{\mathrm{d}\Gamma(V^I \to P^i l^+ l^-)}{\mathrm{d}s} = \frac{\alpha}{3\pi}\Gamma(V^I \to P^i\gamma)\frac{\beta_l}{s}\left(1+\frac{2m_l^2}{s}\right)$$
$$\times\left[\left(1+\frac{s}{M_{V^I}^2-M_{P^i}^2}\right)^2-\frac{4M_{V^I}^2 s}{(M_{V^I}^2-M_{P^i}^2)^2}\right]^{\frac{3}{2}}|F_{V^I P^i}(s)|^2$$

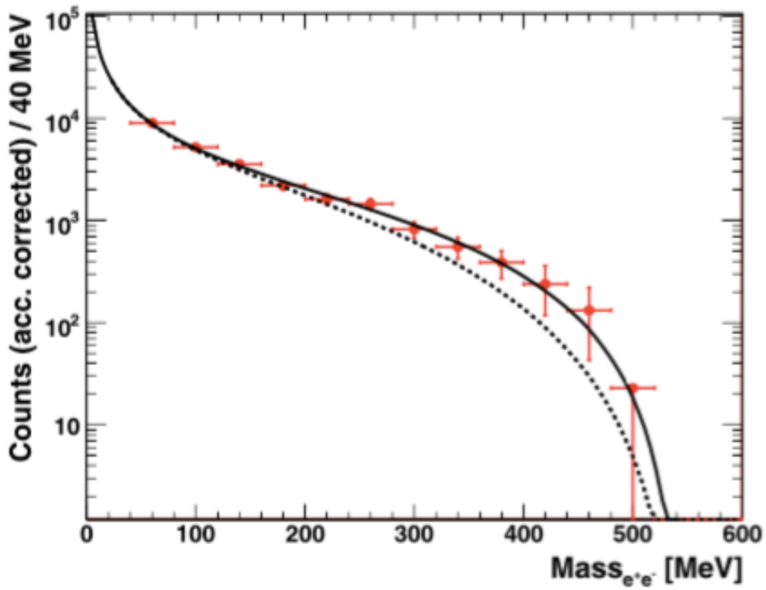
Transition form factor (TFF)

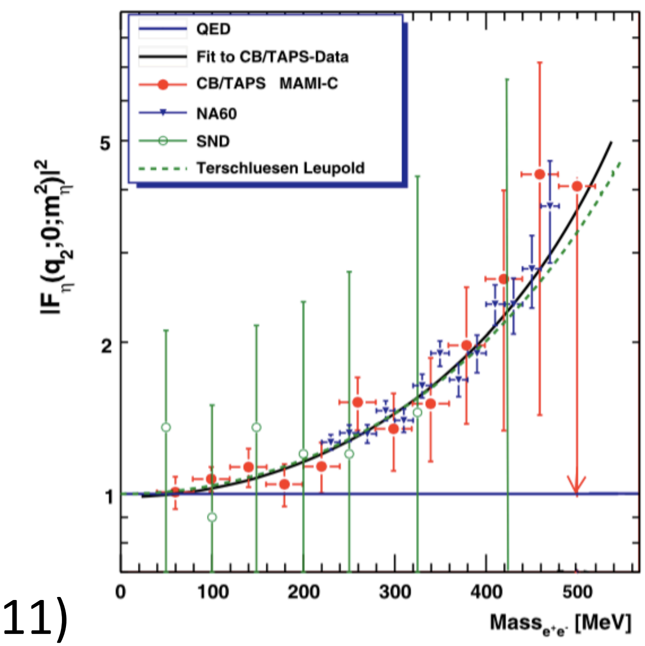
TFFs are observed in experiments:
NA60, KLOE-2, SND, Lepton-G, CMD-2.

$$\frac{\mathrm{d}\Gamma(V^I \to P^i l^+ l^-)}{\mathrm{d}s} = \frac{\alpha}{3\pi}\Gamma(V^I \to P^i\gamma)\frac{\beta_l}{s}\left(1+\frac{2m_l^2}{s}\right) \times \left[\left(1+\frac{s}{M_{V^I}^2 - M_{P^i}^2}\right)^2 - \frac{4M_{V^I}^2 s}{(M_{V^I}^2 - M_{P^i}^2)^2}\right]^{\frac{3}{2}} |F_{V^I P^i}(s)|^2$$

Differential width $\mathrm{d}\Gamma/\mathrm{d}s$

TFF





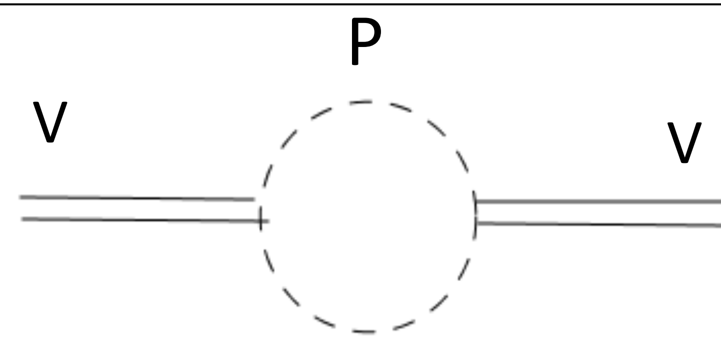
CB/TAPS, PLB701(2011)

TFFs in the model

$$|F_{\phi\pi^0}(s)|^2 = \left|1 + \frac{s}{\bar{\chi}_{\phi\pi^0}} \sum_{J}^{\rho,\omega,\phi} \bar{\theta}^J_{\phi\pi^0} \eta_J \delta B_{VJJ} D_J(s)\right|^2$$

Vector meson propagator

ベクトル中間子のSelf-energy



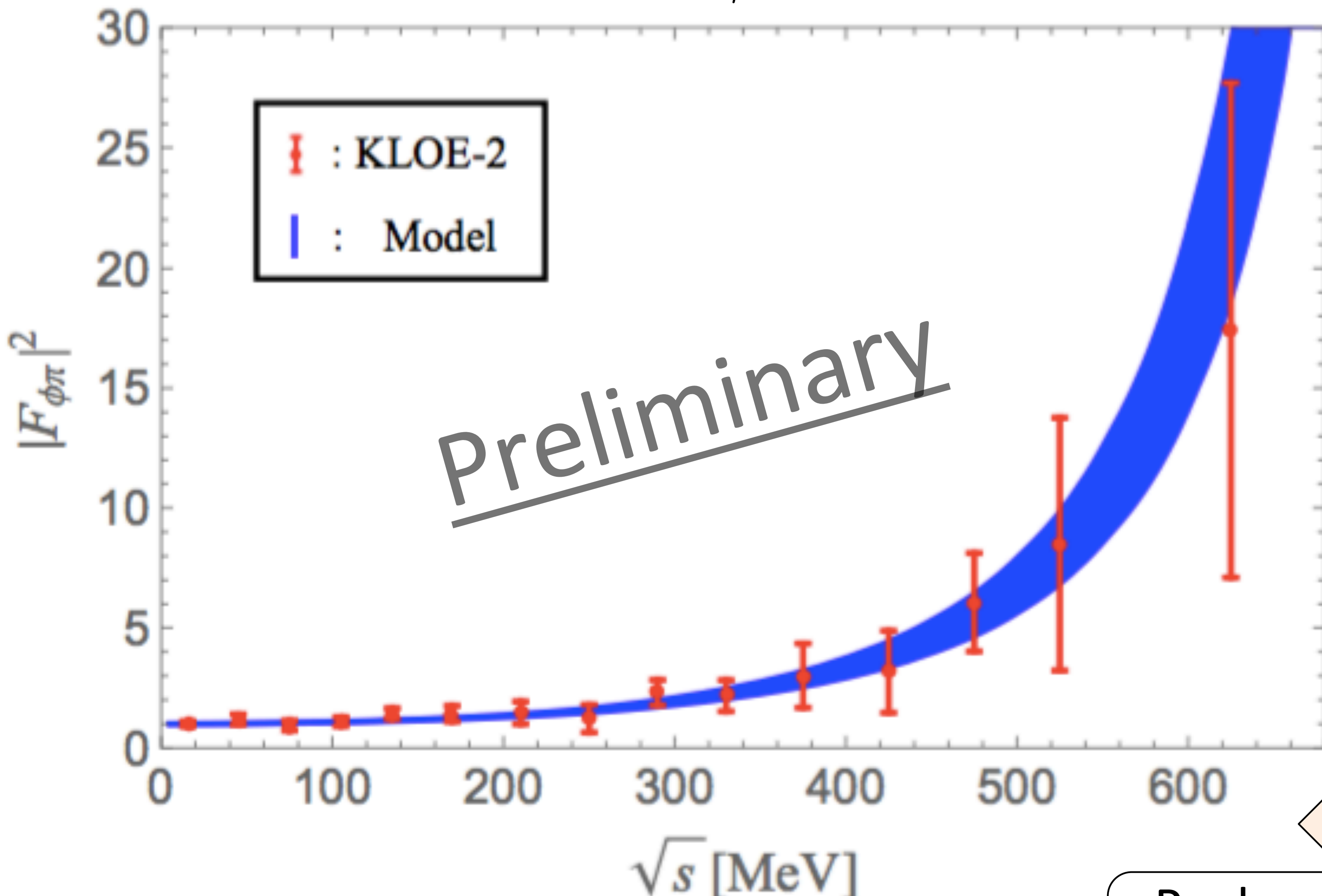
$\bar{\theta}_{VVP}$: VVP coupling

η_V : $V - \gamma$ conversion vertex

$\bar{\chi}_{VP}$: effective vertex for $VP\gamma$ with on-shell photon

$g_{\rho\pi\pi} = 5.83$

TFF of $\phi \to \pi^0 e^+ e^-$



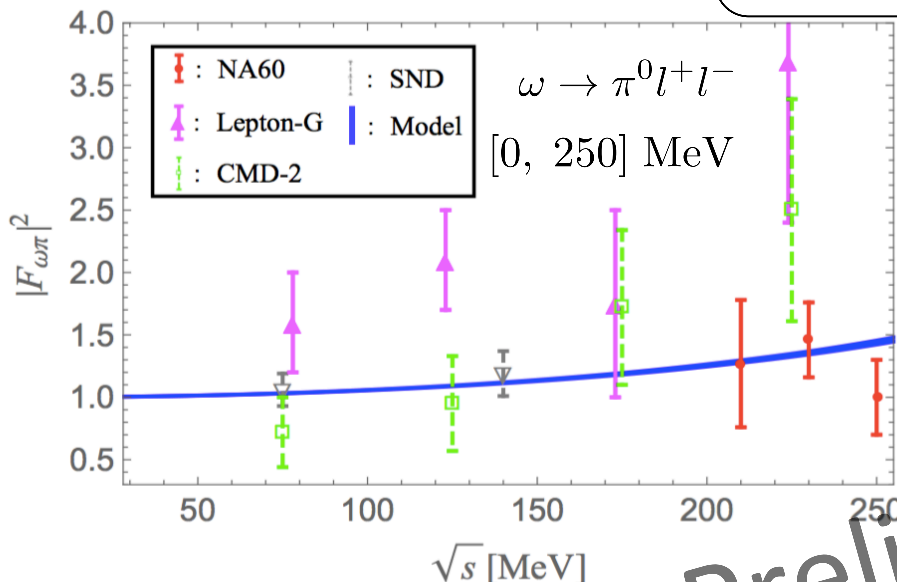
Peak region
$\rho(770), \omega(782)$

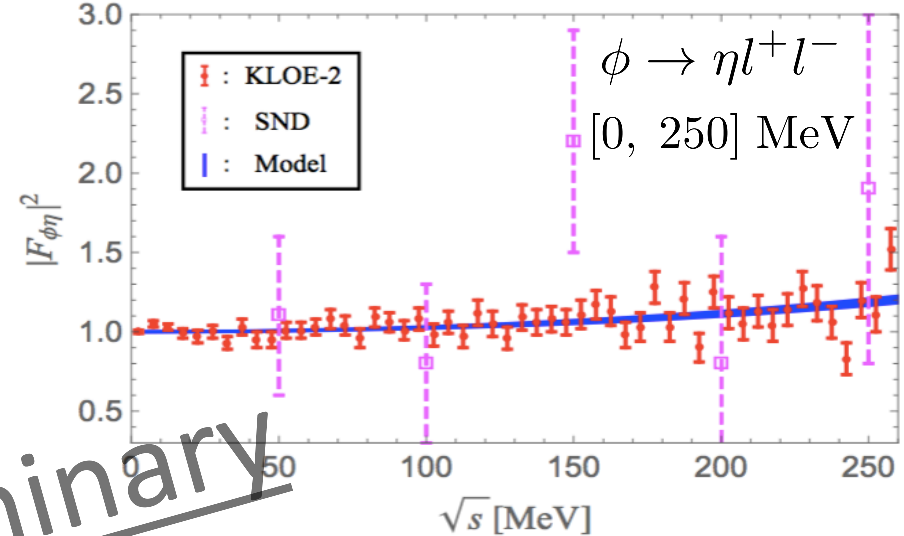
TFFのフィットの結果

Fit in total for $V \to Pl^+l^-$

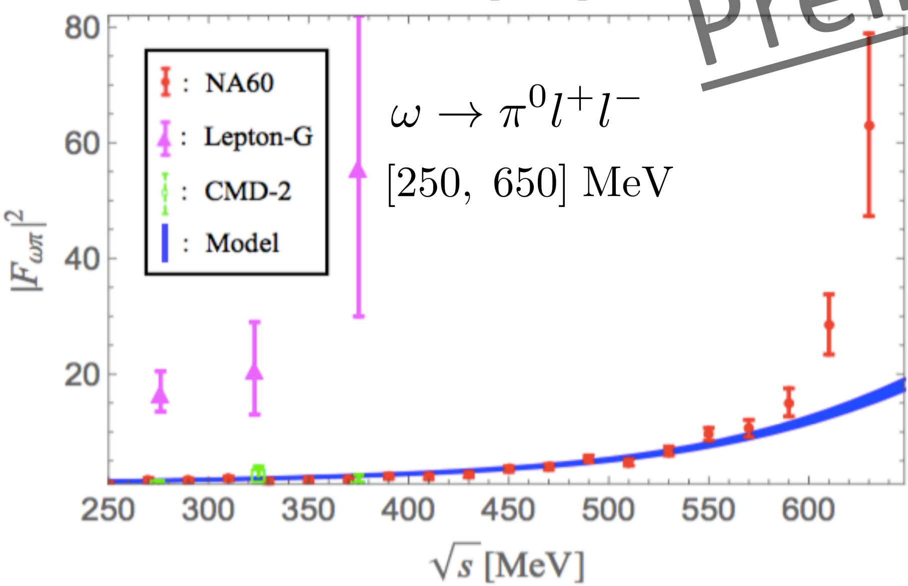
$\chi^2/\mathrm{d.o.f} = 190.6/151$

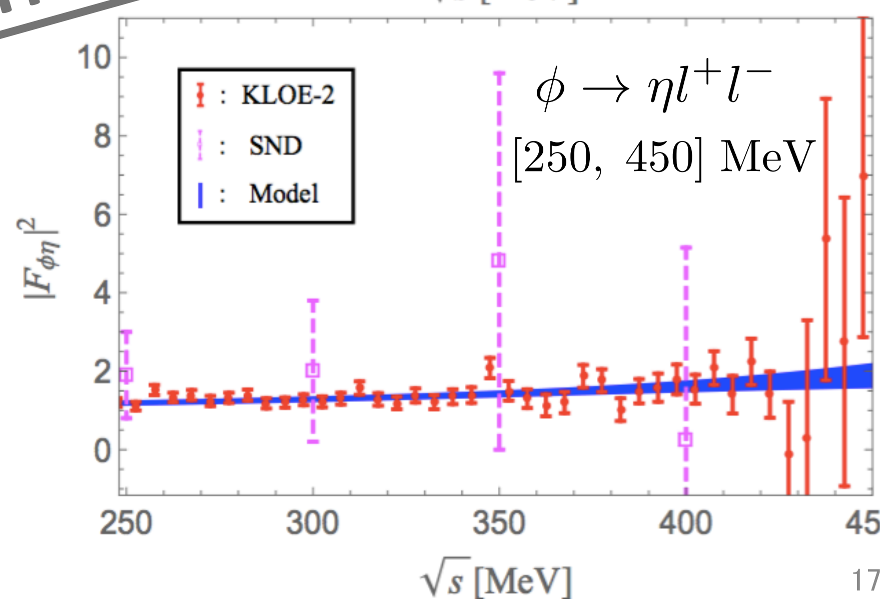
Blue bands: Model in 68% C.L.





Preliminary





$c_3^{\mathrm{IP}} = (1.80 \pm 0.08) \times 10^{-2}, c_4^{\mathrm{IP}} = -(4.0 \pm 0.2) \times 10^{-2}, c_6^{\mathrm{IP}} = -0.78 \pm 0.97$ and $c_7^{\mathrm{IP}} = -0.6 \pm 1.8.$

Decay constant of π^+ and K^+

$$\frac{f_{K^+}}{f_{\pi^+}} = \sqrt{\frac{Z_{\pi^+}}{Z_{K^+}}} = 1.197 \pm 0.009$$

実験値

$$\begin{cases} \langle \pi^+(p)|\bar{u}\gamma_\mu\gamma_5 d|0\rangle\,|_{1-\text{loop order}} = i\sqrt{2} f_{\pi^+} p_\mu, \\ \langle K^+(p)|\bar{u}\gamma_\mu\gamma_5 s|0\rangle\,|_{1-\text{loop order}} = i\sqrt{2} f_{K^+} p_\mu. \end{cases}$$

$$\begin{cases} Z_{\pi^+(1)} \sim -8\left(\frac{M_{\pi^+}^2 + 2\bar{M}_K^2}{f^2} L_4^r + \frac{M_{\pi^+}^2}{f^2} L_5^r\right) + 2c(2\mu_{\pi^+} + \mu_{\bar{K}}), \\ Z_{K^+(1)} \simeq Z_{K^0(1)} \sim -8\left(\frac{M_{\pi^+}^2 + 2\bar{M}_K^2}{f^2} L_4^r + \frac{\bar{M}_K^2}{f^2} L_5^r\right) + c\left(\frac{3}{2}\mu_{\pi^+} + \frac{3}{2}\mu_{88} + 3\mu_{\bar{K}}\right) \end{cases}$$

$$c = 1 - \frac{M_V^2}{g^2 f^2}$$

フィットの結果

	Model (MeV)	PDG (MeV)
$\Gamma[\pi^0 \to 2\gamma]$	7.6×10^{-6}	$(7.6 \pm 0.2) \times 10^{-6}$
$\Gamma[\eta \to 2\gamma]$	5.2×10^{-4}	$(5.2 \pm 0.2) \times 10^{-4}$
$\Gamma[\eta' \to 2\gamma]$	4.5×10^{-3}	$(4.4 \pm 0.2) \times 10^{-3}$
$\Gamma[\rho^0 \to \pi^0\gamma]$	4.3×10^{-2}	$(8.9 \pm 1.2) \times 10^{-2}$
$\Gamma[\omega \to \pi^0\gamma]$	0.73	0.70 ± 0.03
$\Gamma[\phi \to \pi^0\gamma]$	7.1×10^{-3}	$(5.4 \pm 0.3) \times 10^{-3}$
$\Gamma[\rho \to \eta\gamma]$	3.5×10^{-2}	$(4.4 \pm 0.3) \times 10^{-2}$
$\Gamma[\omega \to \eta\gamma]$	5.3×10^{-3}	3.6×10^{-3}
$\Gamma[\phi \to \eta\gamma]$	1.2×10^{-2}	$(5.5 \pm 0.1) \times 10^{-2}$
$\Gamma[\phi \to \eta'\gamma]$	3.4×10^{-4}	$(2.67 \pm 0.09) \times 10^{-4}$
$\Gamma[\eta' \to \omega\gamma]$	4.1×10^{-3}	$(5.4 \pm 0.5) \times 10^{-3}$
f_{K^-}/f_{π^-}	1.172	1.197 ± 0.009

The minimum of χ^2 is realized with,

$$c_{34}^+ = c_3^{\mathrm{IP}} + c_4^{\mathrm{IP}}$$

$$g = 4.80, \quad L_4^r = 7.4 \times 10^{-4}, \quad L_5^r = 3.0 \times 10^{-3}, \quad c_{6-9-10} = 7.5 \times 10^{-3},$$
$$\frac{c_{69}}{g^2}\frac{1}{c_{34}^+} = -0.50, \quad \frac{c_8^{\mathrm{IP}}}{g c_{34}^+} = 0.78, \quad \theta_1 = 0.36, \quad \theta_2 = 5.5 \times 10^{-2}, \quad \theta_3 = 5.6 \times 10^{-2}$$

フィットの結果

擬スカラーのmixing

$$O = \begin{pmatrix} \cos\theta_2\cos\theta_3 - \cos\theta_1\sin\theta_2\sin\theta_3 & \cos\theta_2\sin\theta_3 + \cos\theta_1\sin\theta_2\cos\theta_3 & \sin\theta_1\sin\theta_2 \\ -\sin\theta_2\cos\theta_3 - \cos\theta_1\cos\theta_2\sin\theta_3 & -\sin\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_2\cos\theta_3 & \sin\theta_1\cos\theta_2 \\ \sin\theta_1\sin\theta_3 & -\sin\theta_1\cos\theta_3 & \cos\theta_1 \end{pmatrix}$$

If IPV parameters are zero,

$$\Gamma[\eta' \to 2\gamma] = 0.08 \times 10^{-3}\mathrm{MeV}$$

	Model (MeV)	PDG (MeV)
$\Gamma[\eta' \to 2\gamma]$	4.5×10^{-3}	$(4.4 \pm 0.2) \times 10^{-3}$

1-loop order counter term in ChPT $\begin{cases} L_4^r \\ L_5^r \end{cases}$

$\eta_0 - \eta_8$ Mixing angle: 20.9°

$$c_{34}^+ = c_3^{\mathrm{IP}} + c_4^{\mathrm{IP}}$$

$$g = 4.80, \quad L_4^r = 7.4 \times 10^{-4}, \quad L_5^r = 3.0 \times 10^{-3}, \quad c_{6-9-10} = 7.5 \times 10^{-3},$$

$$\frac{c_{69}}{g^2}\frac{1}{c_{34}^+} = -0.50, \quad \frac{c_8^{\mathrm{IP}}}{g c_{34}^+} = 0.78, \quad \theta_1 = 0.36, \quad \theta_2 = 5.5 \times 10^{-2}, \quad \theta_3 = 5.6 \times 10^{-2}$$

フィットの結果

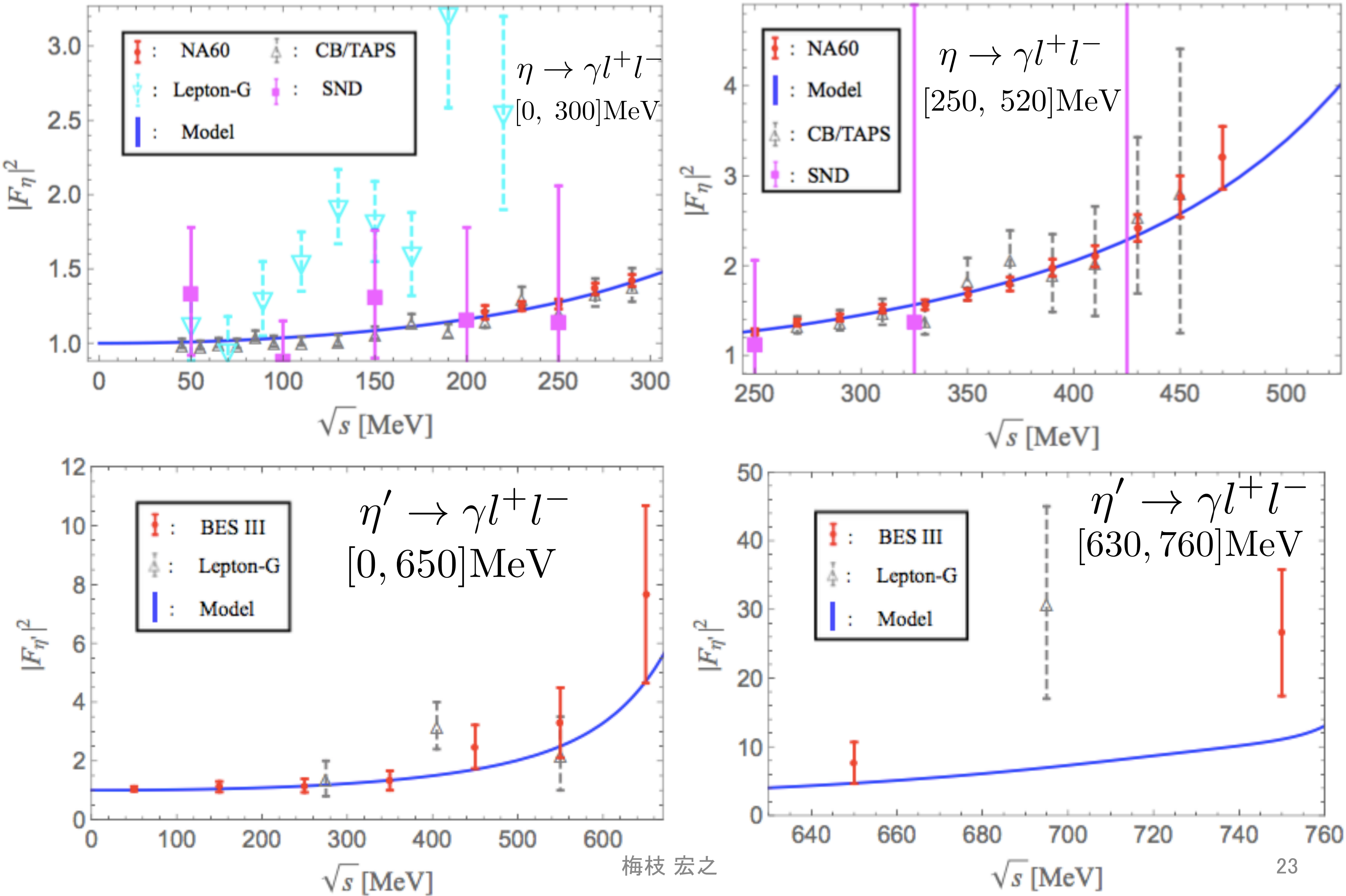
Decay mode	Model (MeV)	PDG (MeV)
$\Gamma[\rho^+ \to \pi^+\gamma]$	$(8.0 \pm 0.6) \times 10^{-2}$	$(6.7 \pm 0.7) \times 10^{-2}$
$\Gamma[K^{*+} \to K^+\gamma]$	$(3.1 \pm 0.2) \times 10^{-2}$	$(5.0 \pm 0.5) \times 10^{-2}$
$\Gamma[K^{*0} \to K^0\gamma]$	0.121 ± 0.009	0.12 ± 0.01

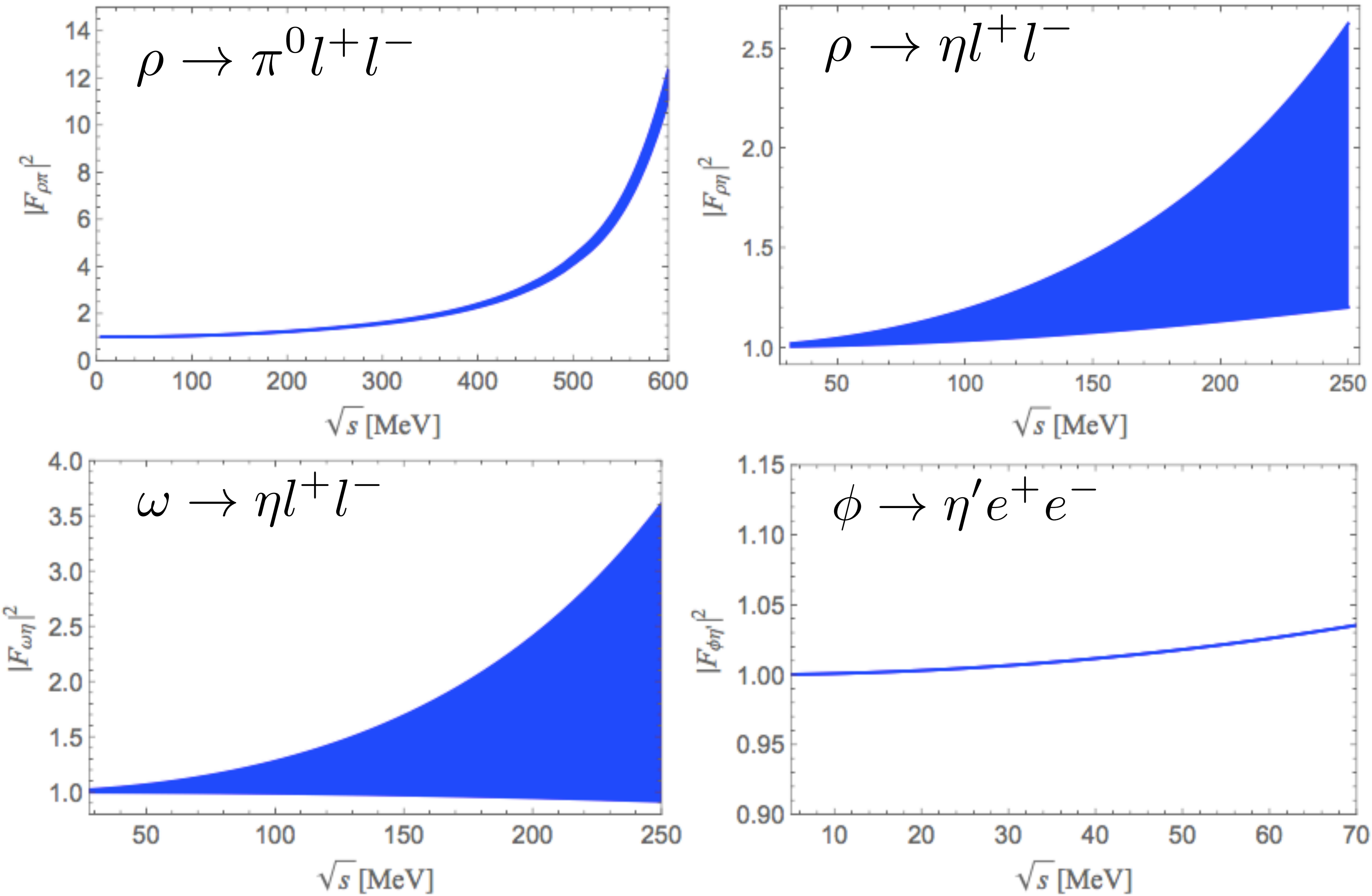
$$g|c_3^{\mathrm{IP}} + c_4^{\mathrm{IP}}| = 0.107 \pm 0.04$$

Prediction of the model

TFF: $P \to \gamma l^+ l^-$

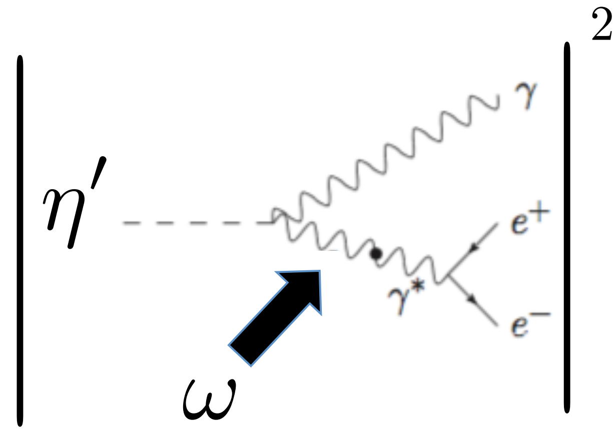
Blue curves: Model prediction

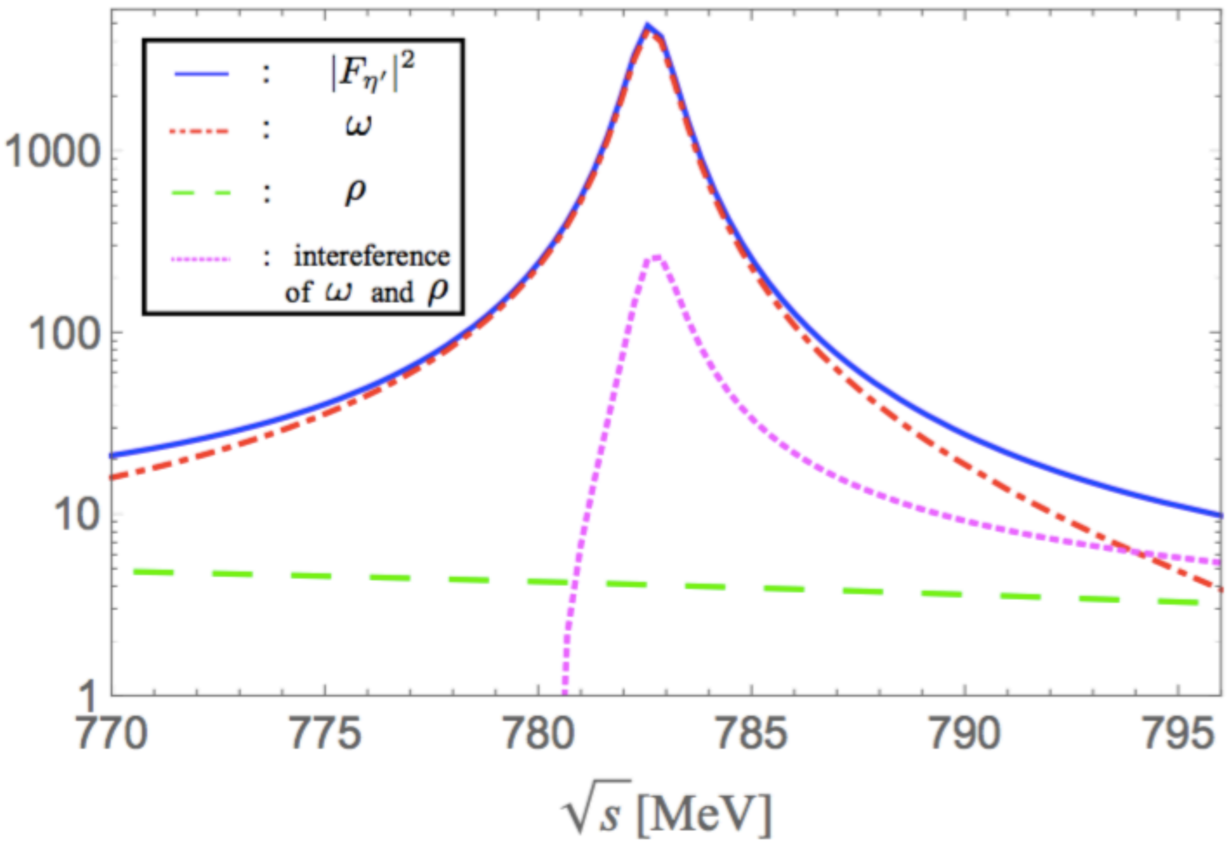




$\rho \to \pi^0 l^+ l^-$
$|F_{\rho\pi}|^2$
0 2 4 6 8 10 12 14
0 100 200 300 400 500 600
$\sqrt{s}$ [MeV]
$\rho \to \eta l^+ l^-$
$|F_{\rho\eta}|^2$
1.0 1.5 2.0 2.5
50 100 150 200 250
$\sqrt{s}$ [MeV]
$\omega \to \eta l^+ l^-$
$|F_{\omega\eta}|^2$
1.0 1.5 2.0 2.5 3.0 3.5 4.0
50 100 150 200 250
$\sqrt{s}$ [MeV]
$\phi \to \eta' e^+ e^-$
$|F_{\phi\eta'}|^2$
0.90 0.95 1.00 1.05 1.10 1.15
10 20 30 40 50 60 70
$\sqrt{s}$ [MeV]

ピーク領域: $\eta' \to \gamma e^+ e^-$

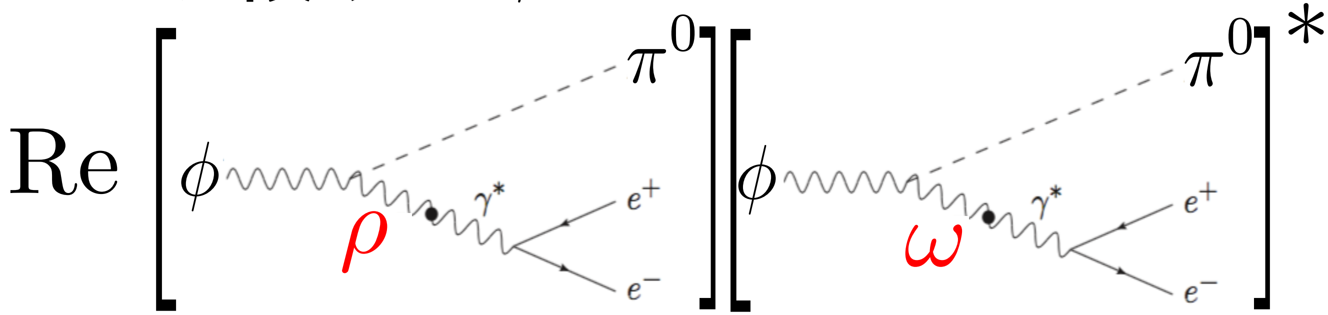


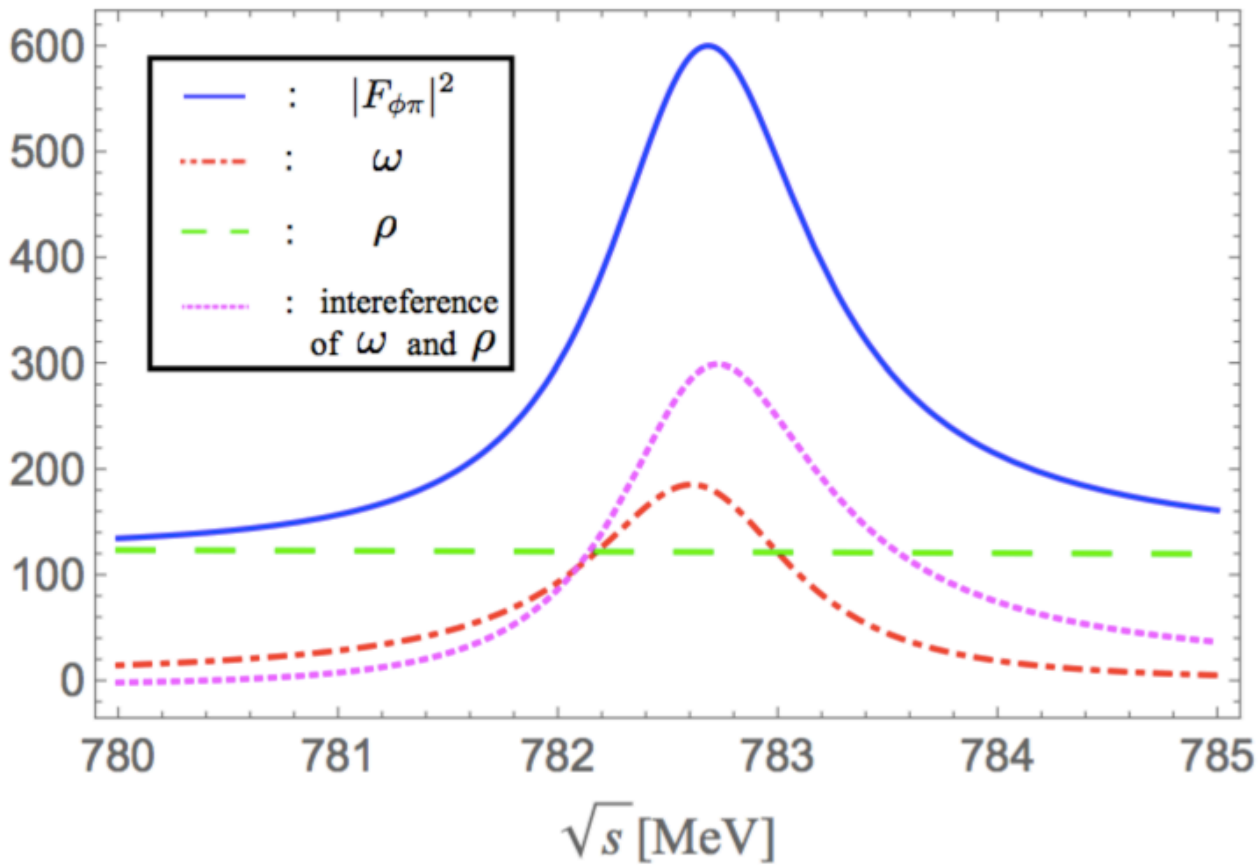


Omega pole is dominant.

ピーク領域: $\phi \to \pi^0 e^+ e^-$

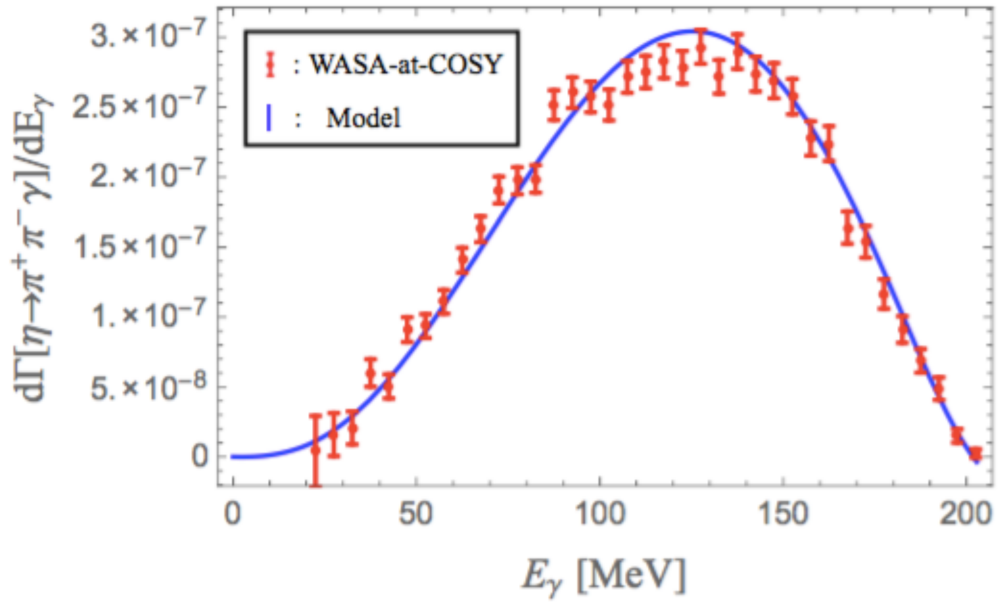
A typical contribution is,

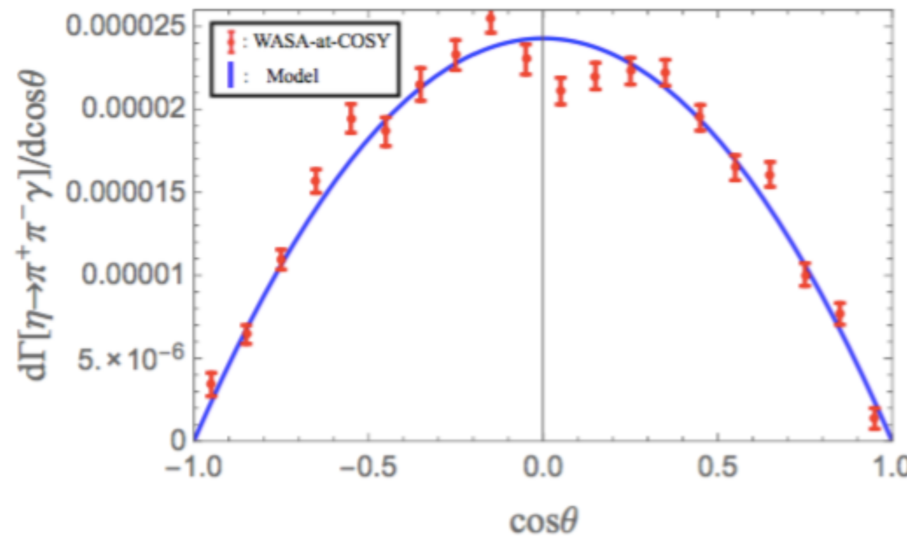




Interference is sizable.

$\eta \to \gamma \pi^+ \pi^-$ の微分分岐比





E_γ : photon energy
in the rest frame of η .

θ : angle between γ and π^+
in the rest frame of $\pi^+\pi^-$.

$$\frac{d^2\Gamma[P^i \to \pi^+\pi^-\gamma]}{ds\, d\cos\theta} = \frac{1}{8192\pi^3 M_{P^i}^3} |Y_i^\gamma|^2 \sin^2\theta s^4 \beta_{\pi^+}^3 \left(1 - \frac{M_{P^i}^2}{s}\right)^3$$

$$\beta_{\pi^+} = \sqrt{1 - 4m_{\pi^+}^2/s}$$

We multiplied WASA-at-COSY data
(including 1σ error range) by

$$\begin{cases} 4.06 \times 10^{-11} & \text{(Left panel)} \\ 2.01 \times 10^{-9} & \text{(Right panel)} \end{cases}$$

Prediction for widths

$$V \to P l^+ l^-, V \to 3P$$

Decay mode	Model (MeV)	Exp. (MeV)
$\Gamma[\rho^0 \to \pi^0 e^+ e^-]$	$(8.33^{+0.05}_{-0.04}) \times 10^{-4}$	$< 6.0 \times 10^{-3}$
$\Gamma[\rho^0 \to \pi^0 \mu^+ \mu^-]$	$(9.7^{+0.4}_{-0.3}) \times 10^{-5}$	–
$\Gamma[\rho^0 \to \eta e^+ e^-]$	$(3.28 \pm 0.06) \times 10^{-4}$	–
$\Gamma[\rho^0 \to \eta \mu^+ \mu^-]$	$(4.5^{+1.4}_{-1.2}) \times 10^{-8}$	–
$\Gamma[\omega \to \pi^0 e^+ e^-]$	$(6.70 \pm 0.04) \times 10^{-3}$	$(6.5 \pm 0.5) \times 10^{-3}$
$\Gamma[\omega \to \pi^0 \mu^+ \mu^-]$	$(0.89 \pm 0.03) \times 10^{-3}$	$(1.1 \pm 0.3) \times 10^{-3}$
$\Gamma[\omega \to \eta e^+ e^-]$	$(2.9 \pm 0.1) \times 10^{-5}$	–
$\Gamma[\omega \to \eta \mu^+ \mu^-]$	$(1.2^{+0.6}_{-0.4}) \times 10^{-8}$	–
$\Gamma[\phi \to \pi^0 e^+ e^-]$	$(7.2^{+0.9}_{-0.7}) \times 10^{-5}$	$(4.8 \pm 1.2) \times 10^{-5}$
$\Gamma[\phi \to \pi^0 \mu^+ \mu^-]$	$(3.4^{+0.8}_{-0.7}) \times 10^{-5}$	–
$\Gamma[\phi \to \eta e^+ e^-]$	$(4.40 \pm 0.01) \times 10^{-4}$	$(4.9 \pm 0.4) \times 10^{-4}$
$\Gamma[\phi \to \eta \mu^+ \mu^-]$	$(2.43 \pm 0.05) \times 10^{-5}$	$< 4.0 \times 10^{-5}$
$\Gamma[\phi \to \eta' e^+ e^-]$	$(1.3800 \pm 0.0004) \times 10^{-6}$	–
$\Gamma[\rho \to \pi^0 \pi^+ \pi^-]$	$(7.5 \pm 1.3) \times 10^{-3}$	$(1.5 \pm 1.3) \times 10^{-2}$
$\Gamma[\omega \to \pi^0 \pi^+ \pi^-]$	$0.97^{+0.13}_{-0.12}$	7.57 ± 0.09
$\Gamma[\phi \to \pi^0 \pi^+ \pi^-]$	$0.95^{+0.28}_{-0.19}$	0.65 ± 0.02

$$P \to l^+ l^- \gamma$$

Decay mode	Model (MeV)	Exp. (MeV)
$\Gamma[\pi^0 \to e^+ e^- \gamma]$	9.1×10^{-8}	$(9.1 \pm 0.3) \times 10^{-8}$
$\Gamma[\eta \to e^+ e^- \gamma]$	8.6×10^{-6}	$(9.0 \pm 0.6) \times 10^{-6}$
$\Gamma[\eta \to \mu^+ \mu^- \gamma]$	4.2×10^{-7}	$(4.1 \pm 0.6) \times 10^{-7}$
$\Gamma[\eta' \to \mu^+ \mu^- \gamma]$	1.9×10^{-5}	$(2.1 \pm 0.6) \times 10^{-5}$
$\Gamma[\eta' \to e^+ e^- \gamma]$	9.32×10^{-5}	$(9.28 \pm 0.95) \times 10^{-5}$
$\Gamma[\eta \to \pi^+ \pi^- \gamma]$	3.2×10^{-5}	$(5.5 \pm 0.2) \times 10^{-5}$
$\Gamma[\eta' \to \pi^+ \pi^- \gamma]$	2.2×10^{-2}	$(5.8 \pm 0.3) \times 10^{-2}$
$\Gamma[\phi \to \omega \pi^0]$	$(8^{+30}_{-7}) \times 10^{-4}$	$(2.0 \pm 0.2) \times 10^{-4}$

Outline

(1) 模型

-Resonance chiral Lagrangian-

(2) ハドロン崩壊

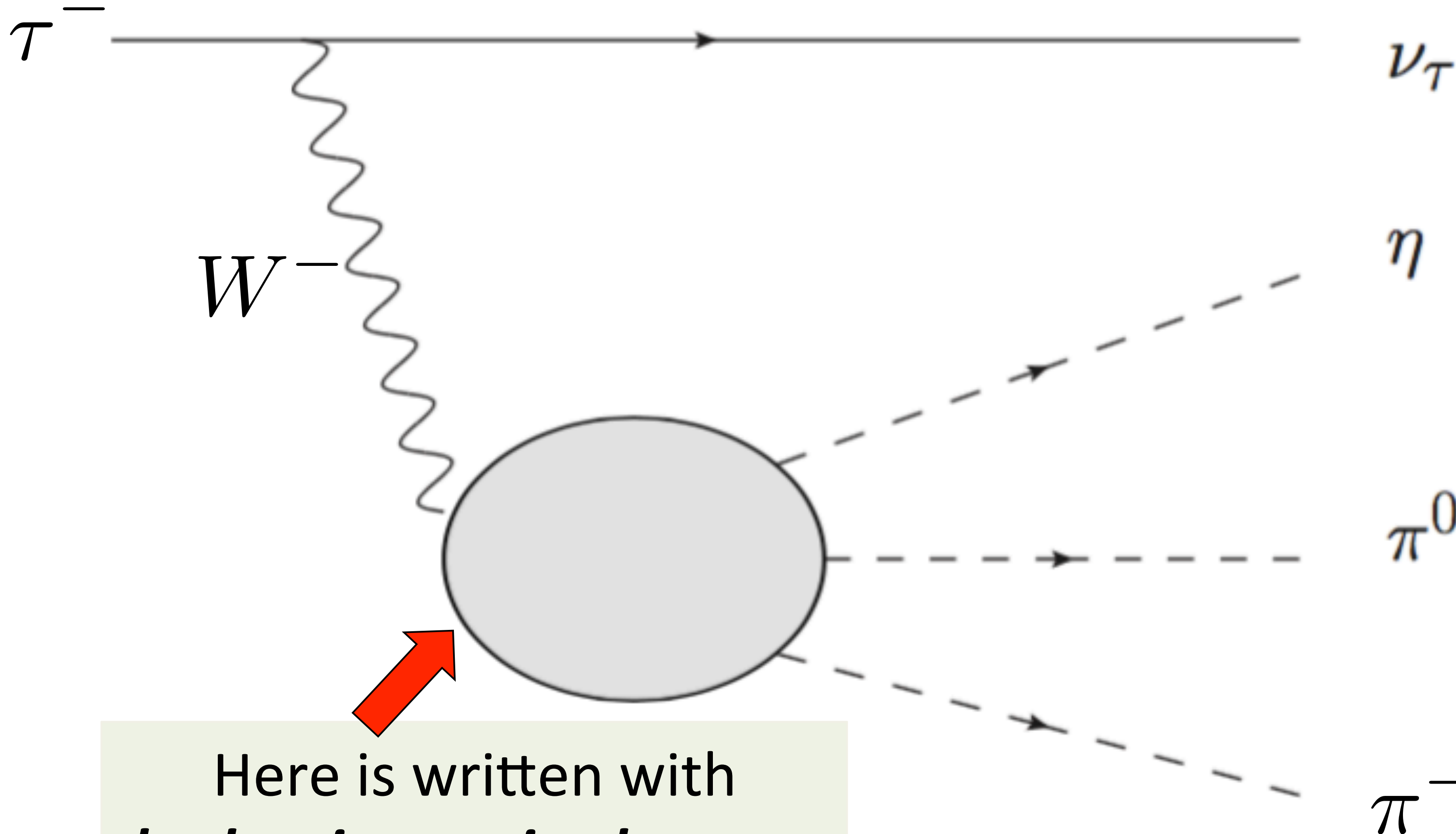
-Dalitz崩壊に関するダイレプトン質量分布-

(3) タウレプトン崩壊

-新物理の効果-

Decays of τ

3-body ハドロニック崩壊



Here is written with
hadronic matrix element.

τ Hadronic decay

$$\mathcal{M} \sim \frac{G_F}{\sqrt{2}} H_\mu L^\mu$$

H_μ : Hadronic current

$$H_\mu =< \eta\pi^0\pi^- | J_\mu^V(0) - J_\mu^A(0) | 0 >$$

L_μ : Leptonic current

$$L_\mu = \bar{u}\gamma_\mu(g_V - g_A\gamma_5)u$$

the SM: V-A current $\begin{cases} g_V = 1 \\ g_A = 1 \end{cases}$

Kinematics and form factors

Kuhn, Mirkes, Z.Phys.C56, 1992
Z.Phys.C67, 1995

$$J^\mu(q_1,q_2,q_3) = \langle h_1(q_1)h_2(q_2)h_3(q_3)|J^\mu_A(0)+J^\mu_V(0)|0\rangle = V_1^\mu F_1 + V_2^\mu F_2 + i V_3^\mu F_3 + V_4^\mu F_4,$$

$$\begin{cases} W_A = (x_1^2+x_3^2)|F_1|^2 + (x_2^2+x_3^2)|F_2|^2 + 2(x_1x_2 - x_3^2)\mathrm{Re}(F_1F_2^*), \\ W_B = x_4^2|F_3|^2, \\ W_{SA} = Q^2|F_4|^2, \end{cases}$$

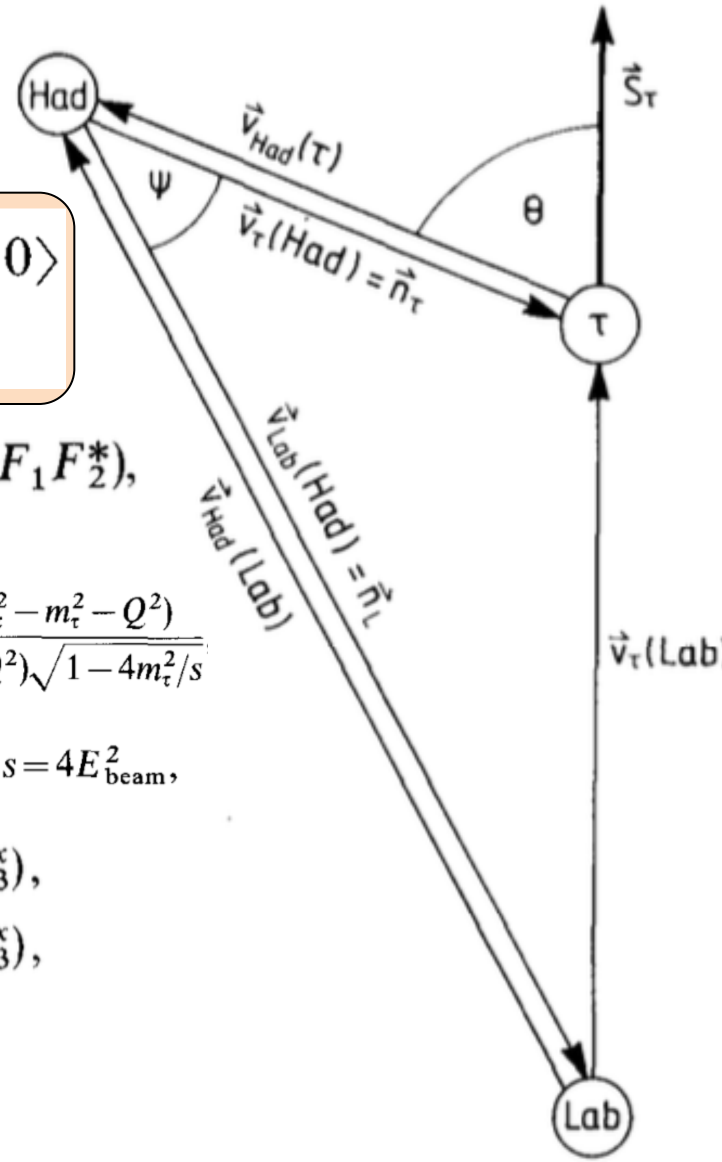
$$\begin{cases} q_3^\mu = (E_3, q_3^x, 0, 0), \\ q_2^\mu = (E_2, q_2^x, q_2^y, 0), \\ q_1^\mu = (E_1, q_1^x, q_1^y, 0), \end{cases}$$

$$\begin{cases} x_1 = V_1^x = q_1^x - q_3^x, \\ x_2 = V_2^x = q_2^x - q_3^x, \\ x_3 = V_1^y = q_1^y \\ x_4 = V_3^z = \sqrt{Q^2}\, x_3 q_3^x, \end{cases}$$

$$\begin{cases} E_i = \dfrac{Q^2 - s_i + m_i^2}{2\sqrt{Q^2}}, \\ q_3^x = \sqrt{E_3^2 - m_3^2}, \\ q_2^x = (2E_2E_3 - s_1 + m_2^2 + m_3^2)/(2q_3^x), \\ q_1^x = (2E_1E_3 - s_2 + m_1^2 + m_3^2)/(2q_3^x), \\ q_2^y = -\sqrt{E_2^2 - (q_2^x)^2 - m_2^2}, \\ q_1^y = \sqrt{E_1^2 - (q_1^x)^2 - m_1^2} = -q_2^y. \end{cases}$$

$$\cos\theta = \frac{(2xm_\tau^2 - m_\tau^2 - Q^2)}{(m_\tau^2 - Q^2)\sqrt{1-4m_\tau^2/s}}$$

$$x = 2\frac{E_h}{\sqrt{s}}, \quad s = 4E_{\mathrm{beam}}^2,$$



角度分布

$$\left(\frac{\Gamma[\tau^- \to \nu_\tau 3h]}{2}\right)^{-1} \frac{d\Gamma[\tau^- \to \nu_\tau 3h]}{d\cos\theta} - 1 = (2\overline{F_{AB}} - 1)\gamma_{VA}\, P\cos\theta$$

$$\gamma_{VA} = \frac{2g_V g_A}{g_V^2 + g_A^2}$$

P : タウの偏極度

$$P = \frac{-v_\tau a_\tau}{v_\tau^2 + a_\tau^2} \sim -0.16$$

at LEP, Kuhn, Mirkes

$\bar{F}_{AB}$: Form Factor の比をphase space 積分したもの

$$\overline{F_{AB}} = \frac{\int (m_\tau^2 - Q^2)^2 \left(\frac{1}{3}\frac{m_\tau^2}{Q^2}(W_A + W_B) + \frac{m_\tau^2}{Q^2} W_{SA}\right) \frac{dQ^2}{Q^2} ds_2 ds_3}{\int (m_\tau^2 - Q^2)^2 \left(\frac{1}{3}\left(2 + \frac{m_\tau^2}{Q^2}\right)(W_A + W_B) + \frac{m_\tau^2}{Q^2} W_{SA}\right) \frac{dQ^2}{Q^2} ds_2 ds_3}.$$

$$0 < \bar{F}_{AB} < 1$$

角度分布

$$\gamma_{VA} = \frac{2g_V g_A}{g_V^2 + g_A^2}$$

$$\left(\frac{\Gamma[\tau^- \to \nu_\tau 3h]}{2}\right)^{-1} \frac{d\Gamma[\tau^- \to \nu_\tau 3h]}{d\cos\theta} - 1 = \underline{(2\overline{F_{AB}} - 1)}\,\gamma_{VA}\, P\cos\theta$$

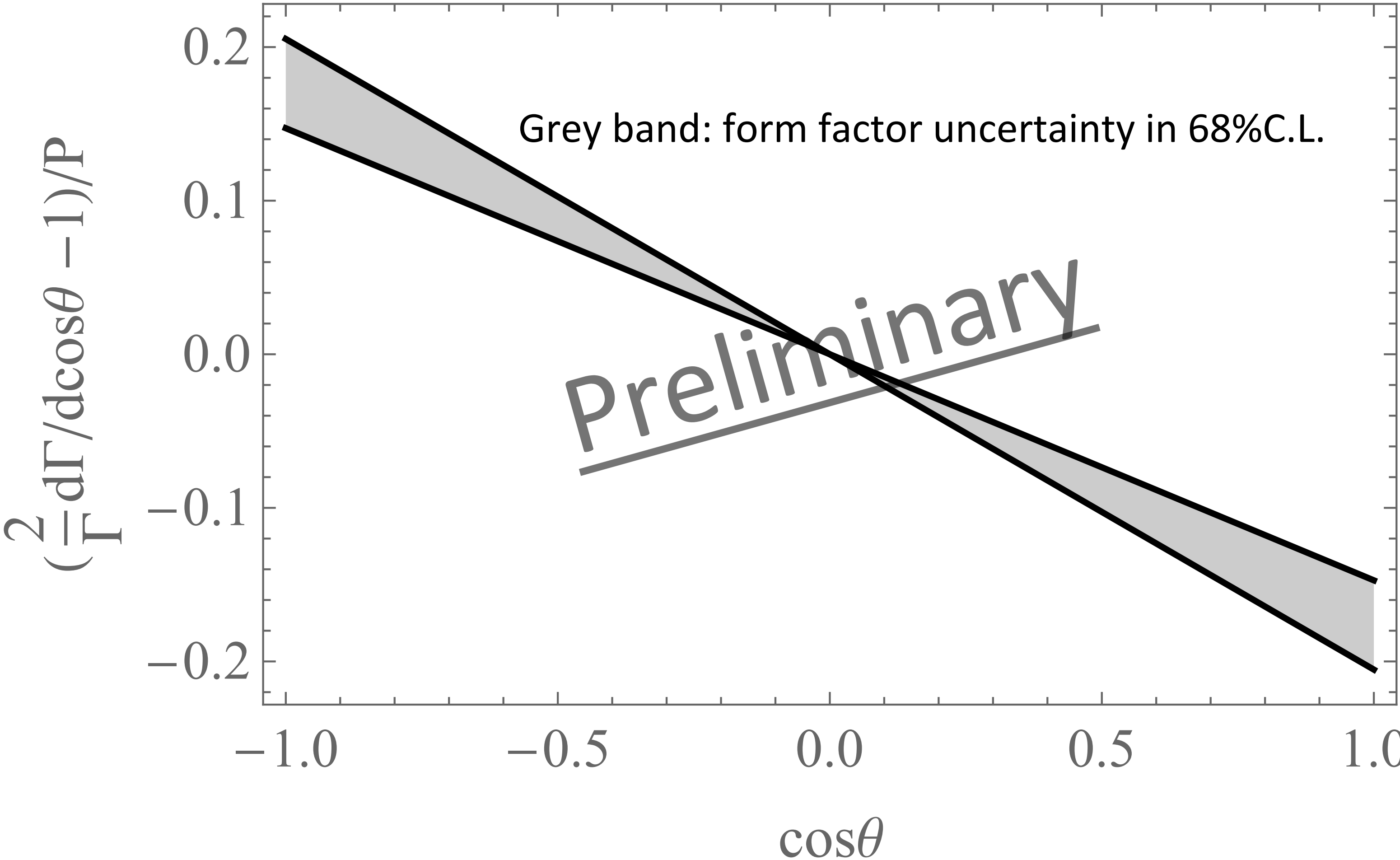
(1) (2) (3)

(1) ハドロン物理の不定性のfactorがある.

(2) $\gamma_{VA} \neq 1$ は, Leptonic currentの中の新物理の効果を示す.

(3) タウの偏極度に比例する観測量である.

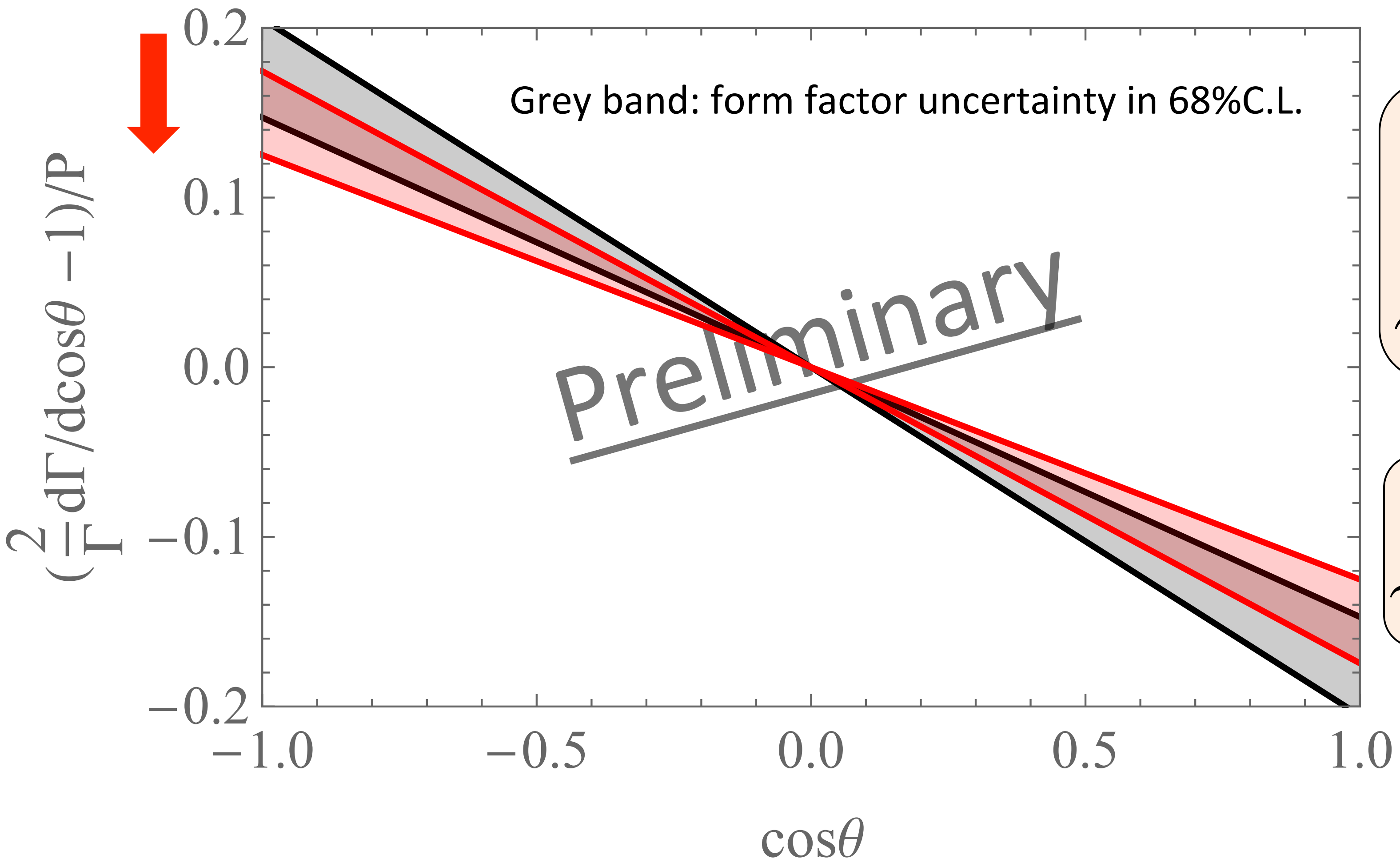
角度分布 $\tau^- \to \nu_\tau \eta \pi^0 \pi^-$



Grey:
The SM
$\gamma_{VA} = 1$

$$\left(\frac{\Gamma[\tau^- \to \nu_\tau 3h]}{2}\right)^{-1} \frac{d\Gamma[\tau^- \to \nu_\tau 3h]}{\mathrm{d}\cos\theta} - 1 = (2\overline{F_{\mathrm{AB}}} - 1)\gamma_{\mathrm{VA}}\, P\cos\theta$$

角度分布 $\tau^- \to \nu_\tau \eta \pi^0 \pi^-$

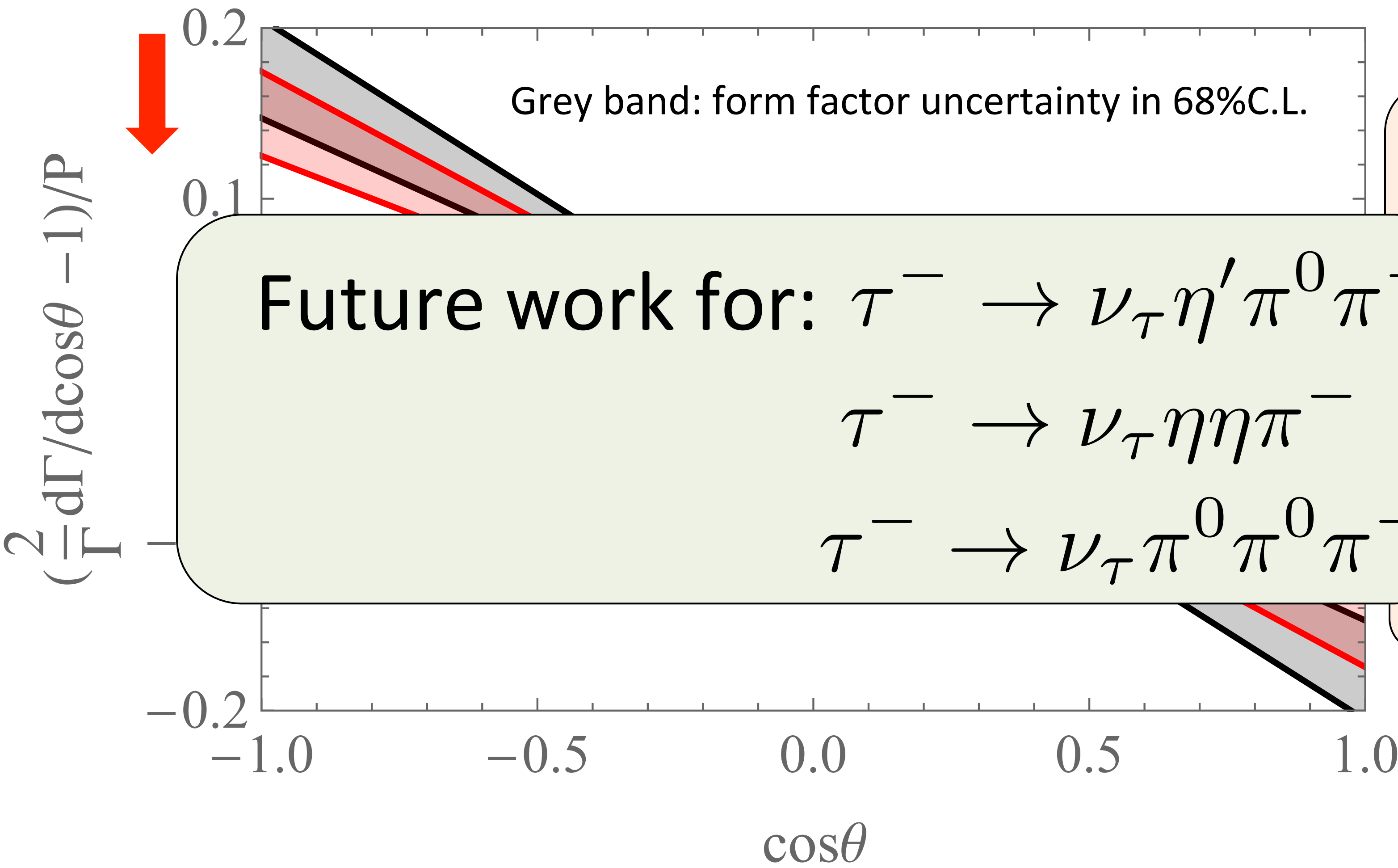


Grey:
The SM
$\gamma_{VA} = 1$

Red:
$\gamma_{VA} = 0.85$

$$\left(\frac{\Gamma[\tau^- \to \nu_\tau 3h]}{2}\right)^{-1} \frac{d\Gamma[\tau^- \to \nu_\tau 3h]}{\mathrm{d}\cos\theta} - 1 = (2\overline{F_{\mathrm{AB}}} - 1)\gamma_{\mathrm{VA}}\, P\cos\theta$$

角度分布 $\tau^- \to \nu_\tau \eta \pi^0 \pi^-$



Grey: The SM $\gamma_{VA} = 1$

Red: $\gamma_{VA} = 0.85$

Future work for: $\tau^- \to \nu_\tau \eta' \pi^0 \pi^-$

$\tau^- \to \nu_\tau \eta \eta \pi^-$

$\tau^- \to \nu_\tau \pi^0 \pi^0 \pi^-$

$$\left(\frac{\Gamma[\tau^- \to \nu_\tau 3h]}{2}\right)^{-1} \frac{d\Gamma[\tau^- \to \nu_\tau 3h]}{d\cos\theta} - 1 = (2\overline{F_{AB}} - 1)\gamma_{VA} P \cos\theta$$

Summary

- SU(3) のoctetとsingletを含むmodelを扱った.
- Intrinsic parity violation の効果を、singletを含んだformalismで取り入れた.
- Dalitz崩壊 $V \to Pl^+l^-$ の実験データに対してフィッティングを行った.
- $\rho^0 \to \pi^0 l^+ l^-, \rho^0 \to \eta l^+ l^-, \omega \to \eta l^+ l^-$ and $\phi \to \eta' l^+ l^-$ のTFFに対して予言を与えた. 特に, $\eta' \to \gamma e^+ e^-$ と $\phi \to \pi^0 e^+ e^-$ のピーク領域に対してomegaとrhoのcontributionを解析した.

Summary (cont'd)

- $\tau^- \to \nu_\tau 3h$ に対して、以下の量にsensitiveな角度分布を定義した.

(1) Initial tauの偏極度.

(2) Leptonic current における新物理の効果.

タウのハドロニック崩壊(LFVでない)での新物理探索.

- QCD effective dynamics のチェックになる.
- ハドロン実験のデータとクロスチェックが重要(と思う).

Back up

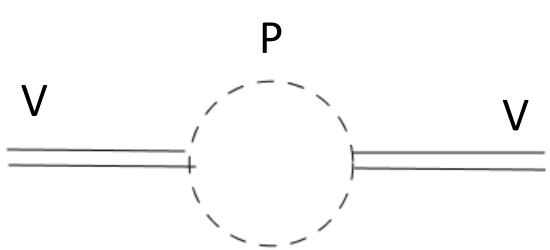
Vector self-energy

Decay mode	Theory (MeV)	PDG (MeV)
$\Gamma[\rho \to \pi\pi]$	152.0 ± 0.7	149.1 ± 0.8
$\Gamma[K^{*\pm} \to (K\pi)^{\pm}]$	42.5 ± 0.2	46.2 ± 1.3
$\Gamma[K^{*0} \to (K\pi)^{0}]$	42.2 ± 0.2	47.4 ± 0.6

$g_{\rho\pi\pi} = 5.83$

$\omega - \phi$ mixing : $49.3°$

Mass	Theory(MeV)	PDG (MeV)
m_{ρ^0}	774.61	775.26 ± 0.25
m_ω	782.65	782.65 ± 0.12
m_ϕ	1019.46	1019.461 ± 0.019



1-loop order counter terms

Back ground field methodでの計算結果.
(octet pseudoscalarのchiral loopの効果)

$$v_\mu = \frac{M_V^2}{2gf^2}\left(V_\mu - \frac{\alpha_\mu}{g}\right)$$

$$\begin{aligned}
\mathcal{L}_c = \; & L_1\left(\mathrm{Tr}(D_\mu U(D^\mu U)^\dagger)\right)^2 + L_2\mathrm{Tr}(D_\mu U(D_\nu U)^\dagger)\mathrm{Tr}(D^\mu U(D^\nu U)^\dagger) \\
& + L_3\mathrm{Tr}\{D^\mu U(D_\mu U)^\dagger D^\nu U(D_\nu U)^\dagger\} \\
& + \frac{4B}{f^2}L_4\mathrm{Tr}\{D_\mu U(D^\mu U)^\dagger\}\mathrm{Tr}\{M(U+U^\dagger)\} \\
& + \frac{4B}{f^2}L_5\mathrm{Tr}\{D_\mu U(D^\mu U)^\dagger(UM+MU^\dagger)\} \\
& + \frac{16B^2}{f^4}L_6\{\mathrm{Tr}(M(U+U^\dagger))\}^2 \\
& + \frac{16B^2}{f^4}L_7\{\mathrm{Tr}(M(U-U^\dagger))\}^2 \\
& + \frac{16B^2}{f^4}L_8\mathrm{Tr}(MUMU+MU^\dagger MU^\dagger) \\
& + iL_9\mathrm{Tr}\{F_{L\mu\nu}(D^\mu U)(D^\nu U)^\dagger + F_{R\mu\nu}(D^\mu U)^\dagger D^\nu U\} \\
& + L_{10}\mathrm{Tr}(F_{L\mu\nu}UF_R{}^{\mu\nu}U^\dagger) \\
& + H_1\mathrm{Tr}(F_{L\mu\nu}F_L{}^{\mu\nu} + F_{R\mu\nu}F_R{}^{\mu\nu}) \\
& + H_2\left(\frac{4B}{f^2}\right)^2\mathrm{Tr}(M^2)
\end{aligned}$$

$$\begin{aligned}
& + i\frac{K_1}{2}\mathrm{Tr}(\xi^\dagger D^\mu U(D^\nu U)^\dagger\xi)(D_\mu v_\nu - D_\nu v_\mu + i[v_\mu, v_\nu]) \\
& - \frac{1}{2}\left(K_2\mathrm{Tr}(\xi^\dagger F_{L\mu\nu}\xi + \xi F_{R\mu\nu}\xi^\dagger)(D^\mu v^\nu - D^\nu v^\mu + i[v^\mu, v^\nu]\right) \\
& + K_3\mathrm{Tr}(D_\mu v_\nu - D_\nu v_\mu + i[v_\mu, v_\nu])(D^\mu v^\nu - D^\nu v^\mu + i[v^\mu, v^\nu])) \\
& + \frac{4B}{f^2}\left(K_4\mathrm{Tr}\{(\xi M\xi + \xi^\dagger M\xi^\dagger)v^2\} + K_5\mathrm{Tr}\{M(U+U^\dagger)\}\mathrm{Tr}(v^2)\right) \\
& + K_6\mathrm{Tr}(v_\rho\alpha_\perp^\mu)\mathrm{Tr}(v^\rho\alpha_{\perp\mu}) + K_7\mathrm{Tr}(v^2\alpha_{\perp\mu}\alpha_\perp^\mu) + K_8\mathrm{Tr}(\alpha_\perp^2)\mathrm{Tr}(v^2) \\
& + K_9\{\mathrm{Tr}(v^2)\}^2 + K_{10}\mathrm{Tr}(v^4) \\
& + i\frac{g_{2p}}{f^2}T_1\eta_0\mathrm{Tr}\{(\xi M\xi - \xi^\dagger M\xi^\dagger)v^2\} \\
& + i\frac{g_{2p}}{f^2}T_2\eta_0\mathrm{Tr}\{M(U-U^\dagger)\}\mathrm{Tr}(v^2) \\
& + T_3 i\frac{g_{2p}}{f^2}\frac{4B}{f^2}\eta_0\mathrm{Tr}M(U+U^\dagger)\mathrm{Tr}M(U-U^\dagger) \\
& + T_4\left(\frac{g_{2p}}{f^2}\right)^2\eta_0{}^2\left(\mathrm{Tr}M(U-U^\dagger)\right)^2 + iT_5\frac{4B}{f^2}\frac{g_{2p}}{f^2}\eta_0\mathrm{Tr}(MUMU - MU^\dagger MU^\dagger) \\
& + T_6\left(\frac{g_{2p}}{f^2}\right)^2\eta_0^2\mathrm{Tr}(MUMU + MU^\dagger MU^\dagger - 2M^2) \\
& + i\frac{g_{2p}}{f^2}\eta_0\left[T_7\mathrm{Tr}\{M(D_\mu U(D^\mu U)^\dagger U - U^\dagger D_\mu U(D^\mu U)^\dagger)\}\right. \\
& \left. + T_8\mathrm{Tr}(M(U-U^\dagger))\mathrm{Tr}(D_\mu U(D^\mu U)^\dagger)\right]
\end{aligned}$$

Problems

Decay mode	Theory (MeV)	PDG (MeV)
$\Gamma[\omega \to \pi\pi]$	4.53	0.130 ± 0.016
$\Gamma[\phi \to K^+K^-]$	2.867	2.086 ± 0.026
$\Gamma[\phi \to K^0\bar{K}^0]$	1.86887	1.459 ± 0.020

Decay mode	Theory (MeV)	PDG (MeV)
$\Gamma[\rho^+ \to \pi^+\gamma]$	$(8.0 \pm 0.6) \times 10^{-2}$	$(6.7 \pm 0.7) \times 10^{-2}$
$\Gamma[\rho^0 \to \pi^0\gamma]$	4.3×10^{-2}	$(8.9 \pm 1.2) \times 10^{-2}$
$\Gamma[\omega \to \pi^0\pi^+\pi^-]$	$0.97^{+0.13}_{-0.12}$	7.57 ± 0.09
$\Gamma[\phi \to \pi^0\pi^+\pi^-]$	$0.95^{+0.28}_{-0.19}$	0.65 ± 0.02

} SU(3) breaking in IPV interactions ?

Effective coupling ratios

Ratio	PDG	Theory	Model in the isospin limit
$\left\|\frac{X_{11}}{X_{\rho^+}}\right\|$	1.15 ± 0.10	0.73	1
$\left\|\frac{X_{12}}{X_{\rho^+}}\right\|$	3.2 ± 0.2	3.0	$\sqrt{3}\|\cos\theta_V^{08} - \frac{3c_8^{\rm IP}}{gc_{34}^+}\sin\theta_V^{08}\|$
$\left\|\frac{X_{13}}{X_{\rho^+}}\right\|$	0.18 ± 0.01	0.19	$\sqrt{3}\|\sin\theta_V^{08} + \frac{3c_8^{\rm IP}}{gc_{34}^+}\cos\theta_V^{08}\|$
$\left\|\frac{X_{21}}{X_{\rho^+}}\right\|$	2.2 ± 0.1	1.8	$\sqrt{3}\|\cos\theta_1 - (\frac{c_{69}}{g^2 c_{34}})\tan\theta_1\|$
$\left\|\frac{X_{22}}{X_{\rho^+}}\right\|$	0.62 ± 0.05	0.66	$\|\cos\theta_V^{08}\|\|\cos\theta_1 + \sqrt{3}(\frac{c_{69}}{g^2 c_{34}})\sin\theta_1 - \frac{\sqrt{3}c_8^{\rm IP}}{gc_{34}^+}\cos\theta_1\tan\theta_V^{08}\|$
$\left\|\frac{X_{23}}{X_{\rho^+}}\right\|$	0.96 ± 0.06	0.41	$\|\sin\theta_V^{08}\|\|\cos\theta_1 + \sqrt{3}(\frac{c_{69}}{g^2 c_{34}})\sin\theta_1 + \frac{\sqrt{3}c_8^{\rm IP}}{gc_{34}^+}\cos\theta_1\cot\theta_V^{08}\|$
$\left\|\frac{X_{33}}{X_{\rho^+}}\right\|$	0.99 ± 0.06	1.02	$\|\sin\theta_V^{08}\|\|\sin\theta_1 - \sqrt{3}(\frac{c_{69}}{g^2 c_{34}})\cos\theta_1 + \frac{\sqrt{3}c_8^{\rm IP}}{gc_{34}^+}\sin\theta_1\cot\theta_V^{08}\|$
$\left\|\frac{X_{32}}{X_{\rho^+}}\right\|$	0.59 ± 0.04	0.47	$\|\cos\theta_V^{08}\|\|\sin\theta_1 - \sqrt{3}(\frac{c_{69}}{g^2 c_{34}})\cos\theta_1 - \frac{\sqrt{3}c_8^{\rm IP}}{gc_{34}^+}\sin\theta_1\cot\theta_V^{08}\|$

$$X_{11} : g_{\rho^0\pi^0\gamma} \qquad X_{\rho^+} : g_{\rho^+\pi^+\gamma}$$

SU(3) breaking in IPV

M. Hashimoto, Phys. Rev.D54(9) with Hidden Local Symmetry

$$\Delta\mathcal{L}_1=\mathrm{tr}[\hat{\alpha}_L^3(\hat{\alpha}_R\cdot\hat{\epsilon}^{(1)}+\hat{\epsilon}^{(1)}\cdot\hat{\alpha}_R)-\hat{\alpha}_R^3(\hat{\alpha}_L\cdot\hat{\epsilon}^{(1)}+\hat{\epsilon}^{(1)}\cdot\hat{\alpha}_L)],$$

$$\Delta\mathcal{L}_1'=\mathrm{tr}(\hat{\alpha}_L\hat{\epsilon}^{(1')}\hat{\alpha}_L^2\hat{\alpha}_R-\hat{\alpha}_R\hat{\epsilon}^{(1')}\hat{\alpha}_R^2\hat{\alpha}_L+\hat{\alpha}_L^2\hat{\epsilon}^{(1')}\hat{\alpha}_L\hat{\alpha}_R-\hat{\alpha}_R^2\hat{\epsilon}^{(1')}\hat{\alpha}_R\hat{\alpha}_L),$$

$$\Delta\mathcal{L}_2=\mathrm{tr}(\hat{\epsilon}^{(2)}\cdot\hat{\alpha}_L+\hat{\alpha}_L\cdot\hat{\epsilon}^{(2)})\hat{\alpha}_R\hat{\alpha}_L\hat{\alpha}_R,$$

$$\Delta\mathcal{L}_3=i\ \mathrm{tr}(F_V\cdot\hat{\epsilon}^{(3)}+\hat{\epsilon}^{(3)}\cdot F_V)\cdot(\hat{\alpha}_L\hat{\alpha}_R-\hat{\alpha}_R\hat{\alpha}_L),$$

$$\Delta\mathcal{L}_3'=i\ \mathrm{tr}F_V(\hat{\alpha}_L\hat{\epsilon}^{(3')}\hat{\alpha}_R-\hat{\alpha}_R\hat{\epsilon}^{(3')}\hat{\alpha}_L),$$

$$\Delta\mathcal{L}_4=i\ \mathrm{tr}[\{(\hat{F}_L+\hat{F}_R)\cdot\hat{\epsilon}^{(4)}+\hat{\epsilon}^{(4)}\cdot(\hat{F}_L+\hat{F}_R)\}\cdot(\hat{\alpha}_L\hat{\alpha}_R-\hat{\alpha}_R\hat{\alpha}_L)],$$

$$\Delta\mathcal{L}_4'=i\ \mathrm{tr}(\hat{F}_L+\hat{F}_R)\cdot(\hat{\alpha}_L\hat{\epsilon}^{(4')}\hat{\alpha}_R-\hat{\alpha}_R\hat{\epsilon}^{(4')}\hat{\alpha}_L),$$

$$\Delta\mathcal{L}_5=\mathrm{tr}\hat{\epsilon}^{(5)}(\hat{\alpha}_L^2\hat{\alpha}_R^2-\hat{\alpha}_R^2\hat{\alpha}_L^2),$$

$$\Delta\mathcal{L}_6=i\ \mathrm{tr}(\hat{\epsilon}^{(6)}F_V-F_V\hat{\epsilon}^{(6)})\cdot(\hat{\alpha}_L^2-\hat{\alpha}_R^2),$$

$$\Delta\mathcal{L}_7=i\ \mathrm{tr}[(\hat{\epsilon}^{(7)}\hat{F}_L-\hat{F}_L\hat{\epsilon}^{(7)})\hat{\alpha}_R^2-(\hat{\epsilon}^{(7)}\hat{F}_R-\hat{F}_R\hat{\epsilon}^{(7)})\hat{\alpha}_L^2],$$

$$\Delta\mathcal{L}_8=i\ \mathrm{tr}[(\hat{\epsilon}^{(8)}\hat{F}_L-\hat{F}_L\hat{\epsilon}^{(8)})\hat{\alpha}_L^2-(\hat{\epsilon}^{(8)}\hat{F}_R-\hat{F}_R\hat{\epsilon}^{(8)})\hat{\alpha}_R^2].$$

Back up: Estimated values of the parameters

$$g = 4.80, \quad L_4^r = 7.4 \times 10^{-4}, \quad L_5^r = 3.0 \times 10^{-3}, \quad c_{6-9-10} = 7.5 \times 10^{-3},$$

$$\frac{c_{69}}{g^2}\frac{1}{c_{34}^+} = -0.50, \quad \frac{c_8^{\mathrm{IP}}}{g c_{34}^+} = 0.78, \quad \theta_1 = 0.36, \quad \theta_2 = 5.5 \times 10^{-2}, \quad \theta_3 = 5.6 \times 10^{-2}$$

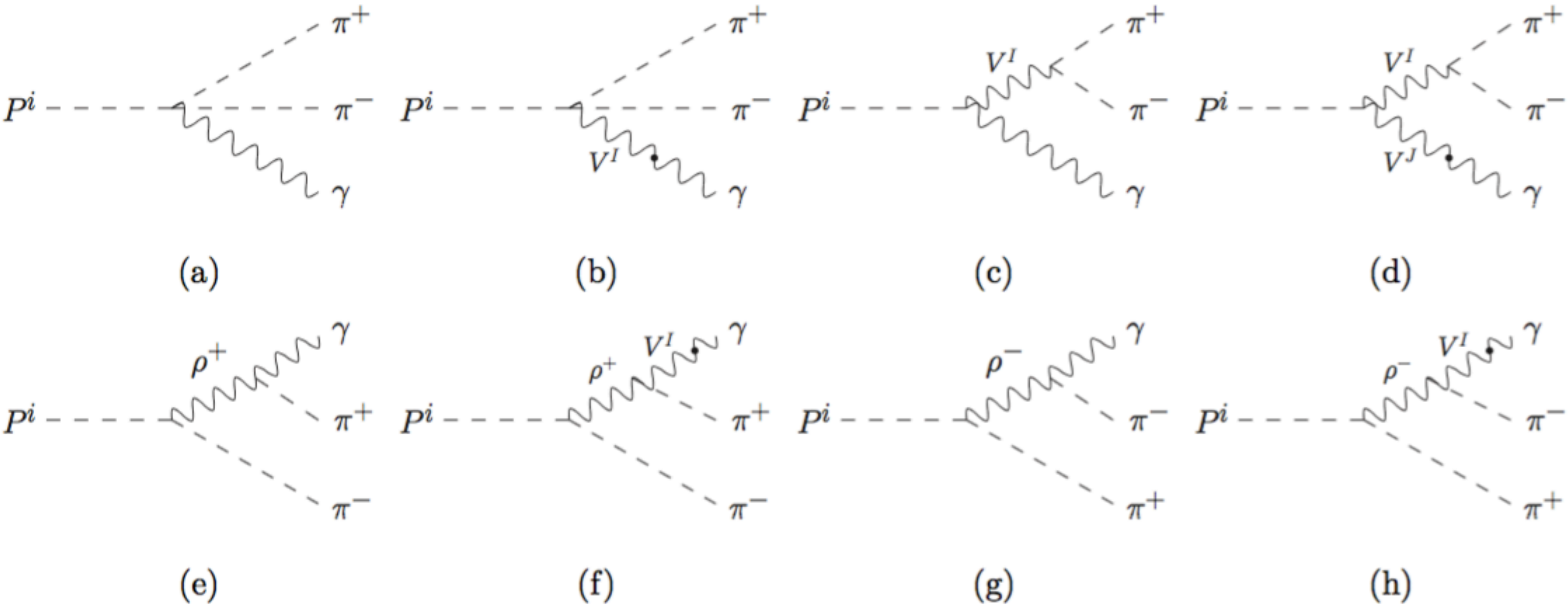
M_V	690MeV	c_3^{IP}	$(1.80 \pm 0.08) \times 10^{-2}$	c_7^{IP}	-0.6 ± 1.8
g	4.8	c_4^{IP}	$(-4.0 \pm 0.2) \times 10^{-2}$	c_8^{IP}	$(-8.4 \pm 0.3) \times 10^{-2}$
L_4^r	7.4×10^{-4}	c_5^{IP}	$(7.0 \pm 0.2) \times 10^{-2}$	c_9^{IP}	0.27 ± 0.40
L_5^r	3.0×10^{-3}	c_6^{IP}	-0.78 ± 0.97	c_{10}^{IP}	$(-5.8 \pm 9.8) \times 10^{-2}$

Back up: TFFs of $P \to \gamma l^+ l^-$

$$\frac{\mathrm{d}\Gamma(P^i \to \gamma l^+ l^-)}{\mathrm{d}s} = \frac{2\alpha}{3\pi}\Gamma(P^i \to 2\gamma)\frac{\beta_l}{s}\left(1 + \frac{2m_l^2}{s}\right)\left(1 - \frac{s}{M_{P^i}^2}\right)^3 |F_{P^i}(s)|^2$$

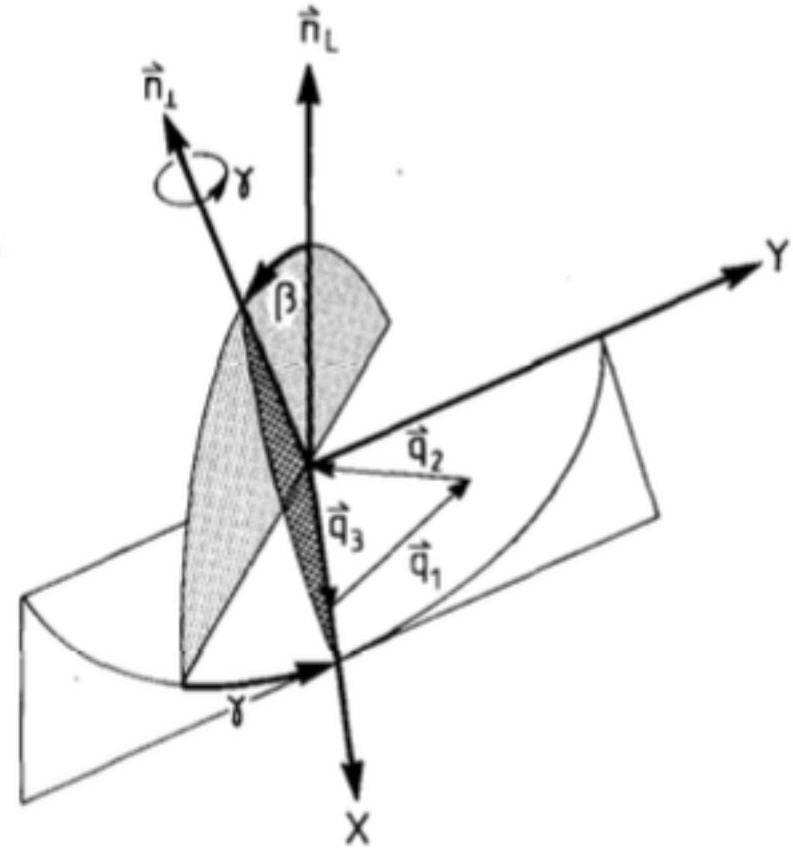
$$|F_{P^i}(s)|^2 = \left|1 + \frac{e^2 s}{g f_{\pi} R_i}\sum_{I=1}^{3}\bar{\chi}_{iI}\eta_I \delta B_{VII} D_I(s)\right|^2$$

Diagrams of $P \to \gamma\pi^+\pi^-$



$$\frac{d^2\Gamma[P^i \to \pi^+\pi^-\gamma]}{ds\, d\cos\theta} = \frac{1}{8192\pi^3 M_{P^i}^3} |Y_i^\gamma|^2 \sin^2\theta\, s^4 \beta_{\pi^+}^3 \left(1 - \frac{M_{P^i}^2}{s}\right)^3$$

$$\mathrm{d}\Gamma(\tau\to 3h)=\frac{G^2}{4m_\tau}(g_V^2+g_A^2)\cos^2\theta_c\left\{\sum_X L_X W_X\right\}$$
$$\times\frac{1}{(2\pi)^5}\frac{1}{64}\frac{(m_\tau^2-Q^2)^2}{m_\tau^2}\frac{\mathrm{d}Q^2}{Q^2}\,\mathrm{d}s_1\,\mathrm{d}s_2$$
$$\times\frac{\mathrm{d}\alpha}{2\pi}\frac{\mathrm{d}\gamma}{2\pi}\frac{\mathrm{d}\cos\beta}{2}\frac{\mathrm{d}\cos\theta}{2}.$$



α, β, γ
積分

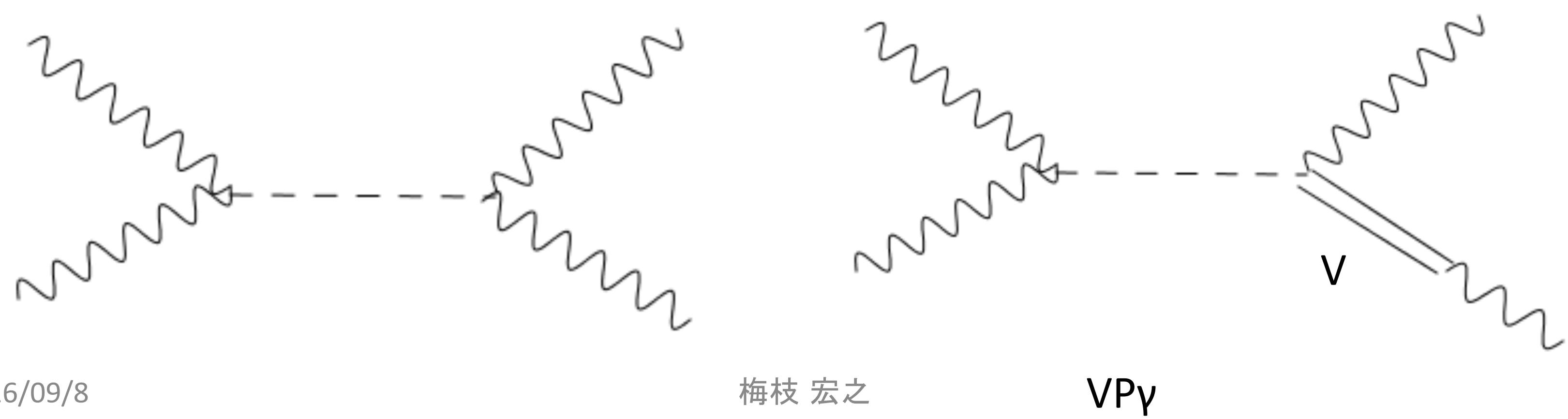
$$\left(\frac{\Gamma[\tau^-\to\nu_\tau 3h]}{2}\right)^{-1}\frac{d\Gamma[\tau^-\to\nu_\tau 3h]}{\mathrm{d}\cos\theta}-1 \;=\; (2\overline{F_{\mathrm{AB}}}-1)\gamma_{\mathrm{VA}}\,P\cos\theta$$

Muon g-2

$$a_\mu^{LbyL,\pi^0} = (65.8 \pm 1.2) \cdot 10^{-11}$$

By, T. Kadavý, K. Kampf, J. Novotný, 2016

$\pi^0\gamma\gamma$ Off-shell form factor



$$V^I V^J P^i: \ \theta_{iIJ} = -\frac{2gc_3^{\rm IP}}{\sqrt{3}} \left[\left(2O_{1i}O_{V1I} - \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i}O_{V2I} \right) O_{V2J} + \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i}O_{V1I}O_{V1J} \right]$$

$$-4c_5^{\rm IP} \left(O_{1i}O_{V3I}O_{V1J} + \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i}O_{V3I}O_{V2J} \right)$$

$$-\frac{2c_6^{\rm IP}}{g} \sqrt{\frac{1}{Z_1^\pi}} O_{3i}(O_{V1I}O_{V1J} + O_{V2I}O_{V2J}) - \frac{4c_7^{\rm IP}}{g} \sqrt{\frac{1}{Z_1^\pi}} O_{3i}O_{V3I}O_{V3J}$$

$$V^I P^i \gamma: \ \chi_{iI} = -\frac{2g}{3} c_{34}^- \left[\left(O_{1i} + \sqrt{3} \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i} \right) O_{V1I} + \left(\sqrt{3} O_{1i} - \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i} \right) O_{V2I} \right]$$

$$-4c_5^{\rm IP} \left(O_{1i} + \frac{1}{\sqrt{3}} \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i} \right) O_{V3I} + 2c_8^{\rm IP} \left(O_{1i} + \frac{1}{\sqrt{3}} \sqrt{\frac{Z_2^\pi}{Z_1^\pi}} O_{2i} \right) O_{V3I}$$

$$+2c_9^{\rm IP} \sqrt{\frac{1}{Z_1^\pi}} O_{3i} \left(O_{V1I} + \frac{1}{\sqrt{3}} O_{V2I} \right),$$

$$\mathcal{L}_\chi|_{V\gamma} = -\frac{eM_I^2}{g} \eta_I V_\mu^I A^\mu,$$

$$\eta_I = O_{V1I} + \frac{1}{\sqrt{3}} O_{V2I}.$$