Cosmological Constant Problem and Scale Invariance

Taichiro Kugo
Maskawa Institute, Physics Dept., Kyoto Sangyo University

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1 Cosmological Constant Problem

Dark Clouds hanging over the two well-established theories

$$\text{Quantum Field Theory} \Longleftrightarrow \text{Einstein Gravity Theory}$$

We know the recently observed Dark Energy Λ_0, which looks like a small Cosmological Constant (CC):

$$\text{Present observed CC} \quad 10^{-29}\text{gr/cm}^3 \sim 10^{-47}\text{GeV}^4 \equiv \Lambda_0 \tag{1}$$

We do not mind this tiny CC, which will be explained after our CC problem is solved. However, we use it as the scale unit Λ_0 of our discussion.

本当に問題なのは何か？

Essential point: multiple mass scales are involved!

There are several dynamical symmetry breakings and they are necessarily accompanied by vacuum condensation energy:

In particular, we are confident from the success of the Standard Model of the existence of at least two symmetry breakings:

$$\text{Higgs Condensation} \sim \quad (\,200\,\text{GeV}\,)^4 \;\sim 10^9\text{GeV}^4 \sim 10^{56}\Lambda_0$$

$$\text{QCD Chiral Condensation}\ \langle \bar{q}q\rangle^{4/3} \sim \quad (\,200\,\text{MeV}\,)^4 \sim 10^{-3}\text{GeV}^4 \sim 10^{44}\Lambda_0$$

Nevertheless, these seem not contributing to the Cosmological Constant!

It is a Super fine tuning problem:

$$c \;:\; \text{initially prepared CC } (>0)$$

$$c - 10^{56}\Lambda_0 \;:\; \text{should cancell, but leaving 1 part per } 10^{12}\text{; i.e., } \sim 10^{44}\Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \;:\; \text{should cancell, but leaving 1 part per } 10^{44}\text{; i.e., } \sim \Lambda_0$$

$$c - 10^{56}\Lambda_0 - 10^{44}\Lambda_0 \sim \Lambda_0 \;:\; \text{present Dark Energy}$$

$c =$ initially prepared CC

$$\underbrace{654321,098765}_{12\text{桁}}4321,0987654321,0987654321,0987654321,0987654321 \times \Lambda_0 \ \sim 10^{56}\Lambda_0$$

$c + V_{\rm Higgs}$

$$\underbrace{4321,0987654321,0987654321,0987654321,0987654321}_{44\text{桁}} \times \Lambda_0 \ \sim 10^{44}\Lambda_0$$

$c + V_{\rm Higgs} + V_{\rm QCD} =$ present Dark Energy

$$1 \times \Lambda_0 \ \sim \Lambda_0$$

Note that the vacuum energy is almost totally cancelled at each stage of spontaneous breaking as far as the the relevant energy scale order.

2　真空エネルギー ≃ 真空凝縮エネルギー

宇宙定数の「2つ」の起源

Vacuum Energy in QFT:

$$\sum_{\boldsymbol{k},s}\frac{1}{2}\hbar\omega_{\boldsymbol{k}}-\sum_{\boldsymbol{k},s}\hbar E_{\boldsymbol{k}} \tag{2}$$

Vacuum Condensation Energy:

$$V(\phi_c)\ \text{: potential} \tag{3}$$

They are separately stored in our (or my, at least) memory, but actually, almost the same object, as we see now.

場の理論のテキストの最初に現れる「真空エネルギー」は無限大。しかし、massless の場合の無限大は、例えば Supersymmetry でボソン―フェルミオン間で相殺していると考える。

massless からのズレのエネルギー密度は、有限に計算され、それは結局は真空凝縮エネルギー $V(\phi_c)$ などに効いている。Consider the chiral quark condensation in QCD. For simplicity, consider NJL model as a parallel model for the realistic QCD:

$$\begin{aligned}\mathcal{L}_{\rm NJL} &= \bar{q}i\gamma^\mu\partial_\mu q+\frac{G}{4}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5 q)^2\right]\\ &\to \quad \bar{q}(i\gamma^\mu\partial_\mu-\sigma-i\gamma_5\pi)q-\frac{1}{G}(\sigma^2+\pi^2)\end{aligned}$$

The effective potential $V(\sigma, \pi)$ is a function of $\sigma^2 + \pi^2$ and can be computed at the $\pi = 0$ section $V(\sigma) = V(\sigma, \pi = 0)$:

$$V(\sigma) = \frac{1}{G}\sigma^2 - \int \frac{d^4p}{i(2\pi)^4} \ln\det(\not{p} - \sigma)$$

But the second term is nothing but the vacuum energy

$$-\int \frac{d^4p}{i(2\pi)^4} \ln\det(\not{p} - \sigma) = -\sum_{\boldsymbol{p},s} \hbar\sqrt{\boldsymbol{p}^2 + \sigma^2} + (\sigma\text{-independent const})$$

implying that

$$\langle \bar{q}q \rangle \text{ condensation energy } \simeq \text{ Dirac sea vacuum energy} \tag{4}$$

Moreover, in a Shwinger-Dyson approach to realistic QCD, the quark mass is calculated as a function $\Sigma(p)$ possessing the support only $\lesssim \Lambda_{\mathrm{QCD}}$, and the condensation energy is computed finite.

3 Quantum Gravity is irrelevant

CC problem is to be considered in Einstein Gravity theory.

$$\text{Einstein gravity is a unique Low Energy Effective Theory} \tag{5}$$

Just like Chiral Lagrangian

$$\mathcal{L} = f_\pi^2 \, \mathrm{tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$
$$U = \exp(i\pi/f_\pi), \qquad \pi = \pi^a(x)T^a$$

is a unique Effective Theory in the low energy region $E \lesssim f_\pi$, i.e., in the lowest (second) order in the derivative. We know that the fundamental theory describing the strong interaction is QCD. But, whatever the dynamical theory is beyond $E > f_\pi$, the sysytem is described by the the Nambu-Goldstone (NG) bosons $\boldsymbol{\pi}$ based on the coset $SU(3)_L \times SU(3)_R/SU(3)_V$, and the dynamics is uniquely described by this non-linear sigam model. The non-linearly realized chiral symmetry uniquely determines the dynamics of the NG bosons, self-coupling and coupling to other matters in the low energy regime. Moreover, even the quantum correction in this system can be computed by this Lagrangian in the sense of Weinberg.

In exactly the same manner, the general coordinate (GC) invariance uniquely determine the Lagrangian in the lowest (second) order in the derivative; that is, it is the Einstein-Hilbert action. In this analogy, it is worth noticing

Graviton is a NG tensor boson corresponding to GL(4) $\to$ SO(3,1)

Nakanishi-Ojima (1979)

So the Einstein-Hilbert action is exactly analogous to the chiral Lagrangian, and $M_{\rm Pl}$ is the counterpart of the pion decay constant f_π:

$$S_{\rm eff} = \int d^4x\,\sqrt{-g}\Big\{c_0 M_{\rm Pl}^4 + c_1 M_{\rm Pl}^2 R + c_2 R^2 + c_3 R_{\mu\nu}R^{\mu\nu} + \cdots\Big\}$$
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

The CC term (with no derivatives) is consistent with GC invariance and its natural scale is $O(M_{\rm Pl}^4)$.

Below the Planck energy scale $M_{\rm Pl}$, the dynamics is uniquely described by the E-H action plus interaction terms with matter fields. The quantum gravity is quite irrelevant to any problem in much lower energy region than Planck sacale, $E \ll M_{\rm Pl}$, in particular, to the CC problem associated with the spontaneous breaking of Electro-weak symmetry and chiral symmetry.

4 Scale Invariance solves the problem!

Our world is almost scale invariant: that is, the standard model Lagrangian is scale invariant except for the Higgs mass term!

If the Higgs mass term comes from the spontaneous breaking of scale invariance at higher energy scale physics, the total system can be really be scale invariant.

4.1 Classical Scale Invariance

Suppose that our world has no dimensionful parameters.

Suppose that the effective potential V of the total system looks like

$$\begin{array}{ccccccc} V(\phi) = & V_0(\Phi) & + & V_1(\Phi, h) & + & V_2(\Phi, h, \varphi) \\ & \downarrow & & \downarrow & & \downarrow \\ & M & \gg & \mu & \gg & m \end{array}$$

and it is scale invariant. Then, classically, it satisfies the scale invariance relation :

$$\sum_i \phi^i \frac{\partial}{\partial \phi^i} V(\phi) = 4V(\phi), \tag{6}$$

so that the vacuum energy vanishes at any stationary point $\langle \phi^i \rangle = \phi^i_0$:

$$V(\phi_0) = 0.$$

Important point is that this holds at every stages of spontaneous symmetry breaking.

In the above potential V, we can retain only $V_0(\Phi)$ when discussing the physics at scale M, since φ and ϕ are expected to get VEVs of order μ or lower. Then the scale invariance guarantees $V_0(\Phi_0) = 0$.

If we discuss the next stage spontaneous breaking at energy scale μ, we should take $V_0(\Phi) + V_1(\Phi, h)$, and can conclude $V_0(\Phi_0') + V_1(\Phi_0', h_0) = 0$.

Similarly, at scale m, we have the potential $V_0(\Phi) + V_1(\Phi, h) + V_2(\Phi, h, \varphi)$, and can conclude $V_0(\Phi_0'') + V_1(\Phi_0'', h_0') + V_2(\Phi_0'', h_0', \phi\varphi_0) = 0$.

This miracle is realized since the scale invariance holds at each energy scale of spontaneous symmetry breaking.

For the help of understanding, we now write a toy model of potentials.

$$V_0(\Phi) = \frac{1}{2}\lambda_0(\Phi_1^2 - \varepsilon_0\Phi_0^2)^2,$$

in terms of two real scalars Φ_0, Φ_1, to realize a VEV

$$\langle\Phi_0\rangle = M \quad \text{and} \quad \langle\Phi_1\rangle = \sqrt{\varepsilon_0}M. \tag{7}$$

This M is totally spontaneous and we suppose it be Planck mass giving the Newton coupling constant via the scale invariant Einstein-Hilbert term

$$S_{\rm eff} = \int d^4x \sqrt{-g}\Big\{c_1\Phi_0^2\, R + c_2R^2 + c_3R_{\mu\nu}R^{\mu\nu} + \cdots\Big\}$$
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\rm Pl}$$

If GUT stage exists, ε_0 may be a constant as small as 10^{-4} and then Φ_1 gives the scalar field breaking GUT symmetry.

$V_1(\Phi, h)$ causes the electroweak breaking:

$$V_1(\Phi, h) = \frac{1}{2}\lambda_1\left(h^\dagger h - \varepsilon_1\Phi_1^2\right)^2,$$

with very small parameter $\varepsilon_1 \simeq 10^{-24}$. This reproduces the Higgs potential when h is the Higgs doublet field and $\varepsilon_1\Phi_1^2$ term is replaced by the VEV $\varepsilon_1\varepsilon_0 M^2 = \mu^2/\lambda_1$.

$V_2(\Phi, h, \varphi)$ causes the chiral symmetry breaking, $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R} \to \mathrm{SU(2)_V}$. Using the 2×2 matrix scalar field $\varphi = \sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}$, we may similarly write the potential

$$V_2(\Phi, h, \varphi) = \tfrac{1}{4}\lambda_2\left(\mathrm{tr}(\varphi^\dagger\varphi) - \varepsilon_2\Phi_1^2\right)^2 + V_{\rm break}(\Phi, h, \varphi)$$

with another small parameter ε_2. The first term reproduces the linear σ-model potential invariant under the chiral $\mathrm{SU(2)_L}\times\mathrm{SU(2)_R}$ transformation $\varphi \to g_{\rm L}\varphi g_{\rm R}$ when $\varepsilon_2\Phi_1^2$ is replaced by the VEV $\varepsilon_2\varepsilon_0 M^2 = m^2/\lambda_2$. The last term $V_{\rm break}$ stands for the chiral symmetry breaking term which is caused by the explicit quark mass terms appearing as the result of tiny Yukawa couplings of u, d quarks, y_u, y_d, to the Higgs doublet h; e.g.,

$$V_{\rm break}(\Phi, h, \varphi) = \frac{1}{2}\varepsilon_2 \Phi_1^2 \, {\rm tr}\left(\varphi^\dagger \left(y_u \epsilon h^* \;\; y_d h\right) + {\rm h.c.}\right)$$

4.2 Quantum Mechanically

However, we have neglected the scale invariance anomaly in quantum field theory. Actually, if we take account of the renormalization point μ, we have the RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a} + \sum_i \gamma_i(\lambda)\phi_i\frac{\partial}{\partial\phi_i}\right) V(\phi) = 0$$

and the dimension counting identity

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_i \phi_i\frac{\partial}{\partial\phi_i}\right) V(\phi) = 4V(\phi)$$

From these we obtain

$$\left(\sum_i (1-\gamma_i(\lambda))\phi_i\frac{\partial}{\partial\phi_i} - \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\right) V(\phi) = 4V(\phi)$$

This is the correct equation in place of the above naive one:

$$\sum_i \phi_i\frac{\partial}{\partial\phi_i} \, V(\phi) = 4V(\phi)$$

This shows the anomalous dimension $\gamma_i(\lambda)$ is not the problem.
$\beta_a(\lambda)$ terms may be problematic:

$$\longrightarrow \qquad 4\,V(\phi_0) = -\sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\,V(\phi_0)$$

So, an obvious possibility is that all the coupling constants go to the Infrared Fixed Points: $\beta_a(\lambda_{\rm IR}) = 0$. But,

What does this equation really imply?

We argue that the potential value $V(\phi_0)$ at the stationary point $\phi = \phi_0$, $\left.\dfrac{dV}{d\phi}\right|_{\phi=\phi_0} = 0$, is zero at any μ, even before reaching the IR limit $\mu = 0$.

The potential value $V(\phi_0) = V_0(\lambda;\mu^2)$ at stationary points satisfies the RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a(\lambda)\frac{\partial}{\partial\lambda_a}\right) V_0(\lambda;\mu^2) = 0$$

(The first term $\mu\partial/\partial\mu$ may be replaced by 4 since $V_0 = \mu^4 v(\lambda)$.)

The solution is given by

$$V_0(\lambda;\mu^2) = V(\bar{\lambda}(t);\mu^2 e^{4t}),$$

where $t = \ln\mu$,

$$\frac{d\bar{\lambda}_a(t)}{dt} = \beta_a(\bar{\lambda}(t)) \quad \text{with} \quad \bar{\lambda}_a(t=0) = \lambda_a.$$

Or, writing $V_0(\lambda;\mu^2) = \mu^4 v(\lambda)$, we must have

$$v(\lambda) = e^{4t} v(\bar{\lambda}(t)) \quad \to \quad v(\bar{\lambda}(t)) = e^{-4t} v(\lambda)$$

Taking the IR limit $t \to -\infty$ $(\mu \to 0)$ gives

$$v(\lambda_{\rm IR}) = e^{+\infty} v(\lambda)$$

If $v(\lambda_{\rm IR})$ is finite, then we have

$$v(\lambda) = 0 \quad \to \quad V_0(\lambda;\mu^2) = 0$$

That is, provided that IR fixed point $\lambda_{\rm IR}$, as well as the theory on top of that point, exist, then, the vanishing property of the potential value at stationary point is not injured by the anomaly!

5 Example Calculation in $\lambda\phi^4$ theory

One-loop RGE-improved tree potential:

$$V(\phi,\lambda;\mu^2) = \frac{\lambda}{4!}\frac{1}{1-\dfrac{3\lambda}{32\pi^2}\ln\dfrac{\frac{1}{2}\lambda\phi^2}{\mu^2}}\,\phi^4$$

Or, denoting $4\pi\phi = \varphi,\ \frac{1}{2}\lambda\phi^2 = \alpha\varphi^2,\ \frac{\lambda}{32\pi^2} = \alpha,$

$$192\pi^2 V = \frac{\alpha\varphi^4}{1-3\alpha\ln(\alpha\varphi^2/\mu^2)}$$

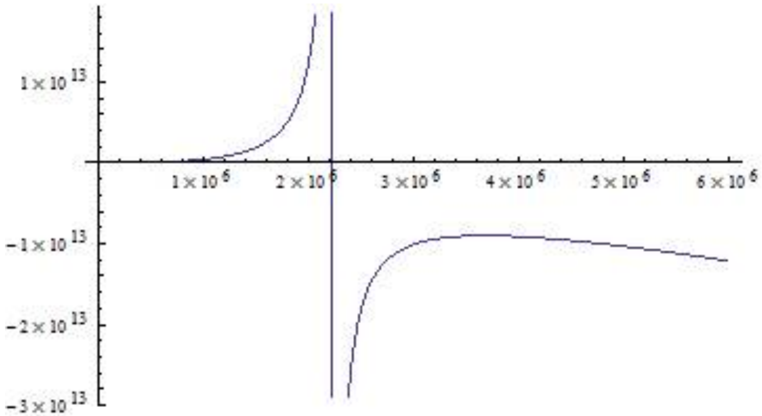

図 1: $y = \alpha x^2/(1-3\alpha\ln(\alpha x)),\ \alpha = 3/100,\ \mu = 1$

停留点 φ_0^2 ：

$$\frac{\partial V}{\partial \varphi^2} = \frac{\alpha\varphi^2}{(1-3\alpha\ln(\cdot))^2}(2(1-3\alpha\ln(\cdot))+3\alpha) = 0$$

$$\to \varphi_0^2 = 0 \quad \text{or} \quad \ln(\alpha\varphi_0^2/\mu^2) = \frac{1}{2}+\frac{1}{3\alpha}$$

$$\to \varphi_0^2 = 0 \quad \text{or} \quad \varphi_0^2 = \frac{\mu^2}{\alpha}\exp\left(\frac{1}{2}+\frac{1}{3\alpha}\right) \to \ \infty e^{\infty} \qquad \text{as } \alpha \to 0+ \tag{8}$$

停留値：

$$V_0(\varphi_0) = 0 \quad \text{or} \quad V_0(\varphi_0) = \frac{\alpha\varphi_0^4}{-(3\alpha/2)} = -\frac{2}{3}\varphi_0^4 \ \to\ -\infty^2 e^{\infty} \qquad \text{as } \alpha \to 0+$$

In this case of $\beta(\lambda) = b_1\lambda^2$, the RGE $\dfrac{d\lambda}{dt} = \beta(\lambda)$ leads to

$$t = \frac{1}{b_1}\left(\frac{1}{\bar\lambda(t)} - \frac{1}{\lambda}\right)$$

$$v(\bar\lambda(t)) = v(\lambda)e^{-4t} = v(\lambda)\exp\left[\frac{4}{b_1}\left(\frac{1}{\bar\lambda(t)}-\frac{1}{\lambda}\right)\right]$$

But $t \to -\infty$ で $\bar\lambda(t) \to \lambda_{\mathrm{IR}} = 0$ だが、$v(0)$ は free theory だから、それが発散するのはおかしい。ゆえに、$v(\lambda) = 0$.

In case of non-trivial IR fixed point, $\beta(\lambda) \simeq b(\lambda - \lambda_{\rm IR})$ with $b > 0$,

$$t = \frac{1}{b} \ln \left(\frac{\lambda_{\rm IR} - \bar{\lambda}(t)}{\lambda_{\rm IR} - \lambda} \right)$$
$$\rightarrow \quad v(\bar{\lambda}(t)) = v(\lambda) e^{-4t} = v(\lambda) \left(\frac{\lambda_{\rm IR} - \lambda}{\lambda_{\rm IR} - \bar{\lambda}(t)} \right)^{4/b} \tag{9}$$

$t \to -\infty$ で $\bar{\lambda}(t) \to \lambda_{\rm IR}-$

6 Gauge Hierarchy

上では

$$V_2 = \frac{1}{2}\lambda_2 \left(h^\dagger h - \varepsilon_1 \Phi_1^2\right)^2 \quad や \quad V_2 \supset \tfrac{1}{4}\lambda_2 \left(\mathrm{tr}(\varphi^\dagger\varphi) - \varepsilon_2\Phi_1^2\right)^2$$

で、単に小さな parameter $\varepsilon_1 \simeq 10^{-24}$, $\varepsilon_2 \simeq 10^{-30}$ を仮定することによって gauge hierarchy を「実現」した。

しかし、例えば chiral symmetry breaking scale ε_2 の値は、GUT を仮定すれば、SU(3) gauge coupling g_3 が、$\varepsilon_0 M$ からくり込み群発展で running coupling $\bar{g}_3(\mu)$ が $O(1)$ になるスケール $\mu \simeq \Lambda_{\mathrm{QCD}}$ で chiral symmetry が起こる、$\varepsilon_1\varepsilon_0 M \simeq \Lambda_{\mathrm{QCD}}$、ということで決まっている。Planck/GUT scale M と QCD scale Λ_{QCD} との関係は、スケール M でのゲージ結合定数 g_3 がどういう値をとっていたかによって決まる。

Electroweak breaking scale を決める ε_1 も、例えば Technicolor 等の下のレベルのゲージ相互作用があれば、同様に決まるだろう。

Bardeen が昔から指摘しているように、Higgs scalar の mass term は、Higgs が elementary の場合でも、sacale invariance があれば 4 次の相互作用項が起源なので、2 次発散ではなく log 発散であり、ε_1 がゼロであれば Φ_1 との相互作用も切れるので、ε_1 のくりこみは対数補正でかつ ε_1 に比例している。同様な指摘は、Shaposhnikov-Zenhausern によってもなされている。

W. A. Bardeen, "On naturalness in the standard model," FERMILAB-CONF-95-391-T.

M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671** (2009) 162

宇宙定数問題の解決に scale invariance が本質的な役割を果たすというアイデアは、
E. Rabinovici, B. Saering and W. A. Bardeen, Phys. Rev. D **36** (1987) 562.
M. Shaposhnikov and D. Zenhausern,ibid
C. Wetterich, Nucl. Phys. B **302** (1988) 668.
にあるが、前 2 者は quantum にも exact scale invariance を必要としている。どれも、multi-step spontaneous breaking との関連は言っていない。

また後者 2 者は、共に gravity の asymptotic safety を要求しているが、宇宙項問題の本質には Planck scale 以上の UV の振る舞いは無関係のはず。

7　これから

1. ∃Dilaton は何か？ $\rightarrow$ Higgs ?
2. Dilaton 質量は？ $\rightarrow$ anomaly が効く!
3. 現在の宇宙定数 Λ_0 の値はどう説明？
4. インフレーションはどう起こるのか？
5. Running cosmological constant? : Loop effects of matter and graviton fields below the Planck scale.
6. Thermal effects.
7. Scale invariant Beyond Standard Model の構成。
8. Scale Invariant Einstein Gravity: Local scale invariance will be meaningless.