有限温度QCD: 相転移、トポロジー、axion

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KEK

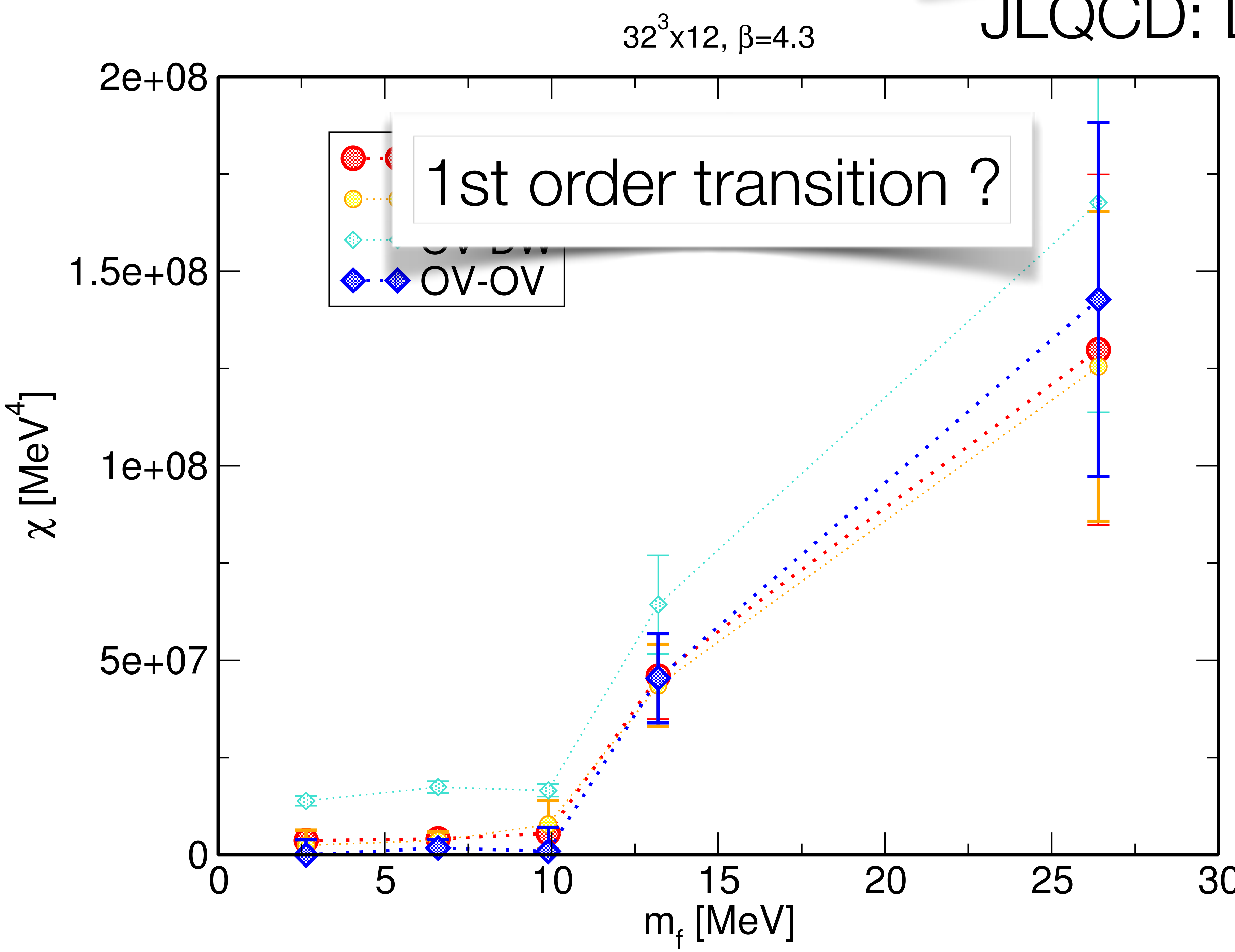
素粒子物理学の進展2018 @ 基研

Aug. 9, 2018

$\chi_t(m_f)$ for N_f=2 T=220 MeV

GL-DW	gluonic charge on DW
GL-OV	gluonic charge on OV
OV-	OV index on DW ensemble
OV-OV	OV index on OV ensemble

JLQCD: Lattice 2017



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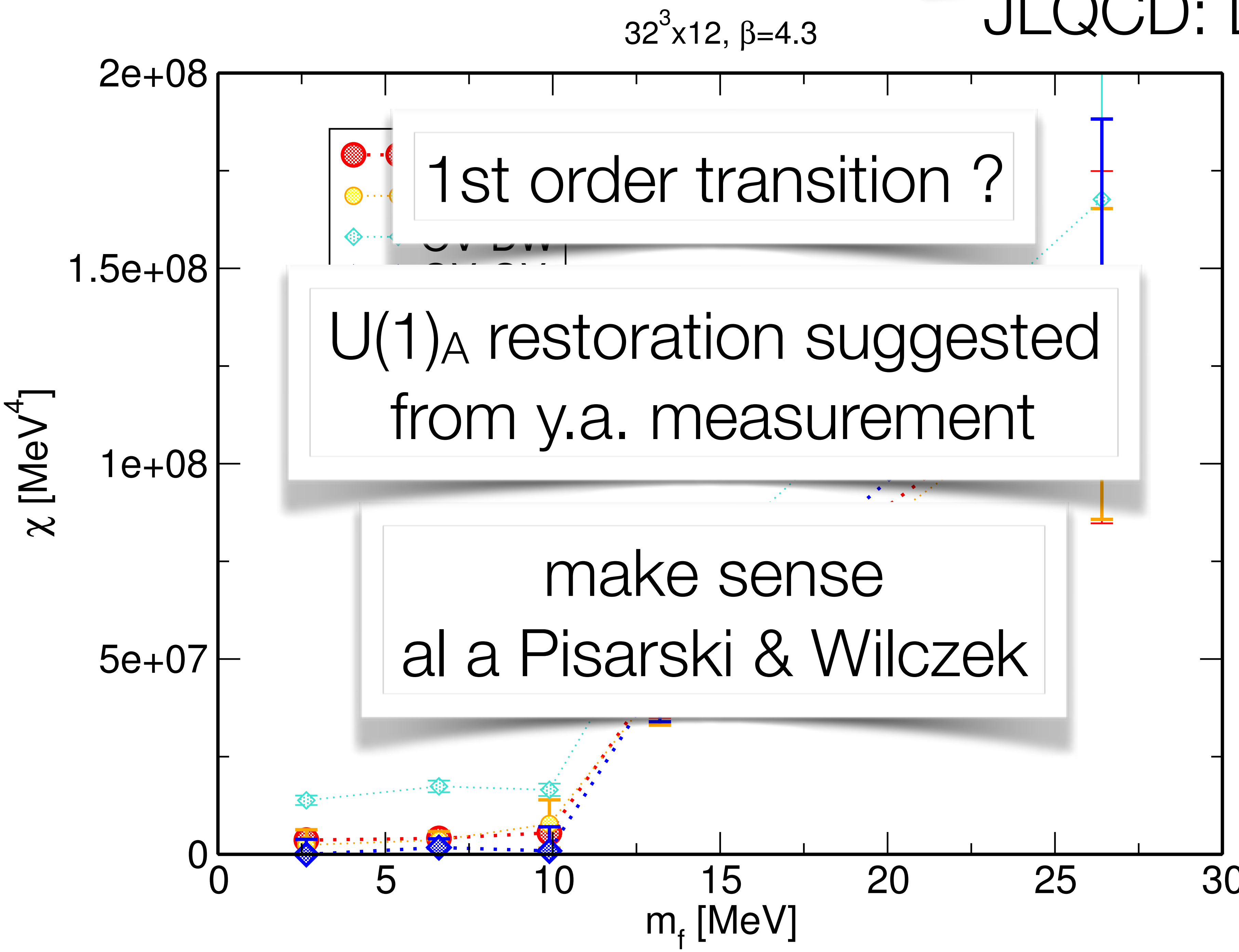

1st order transition ?

U(1)$_A$ restoration suggested from y.a. measurement

make sense al a Pisarski & Wilczek

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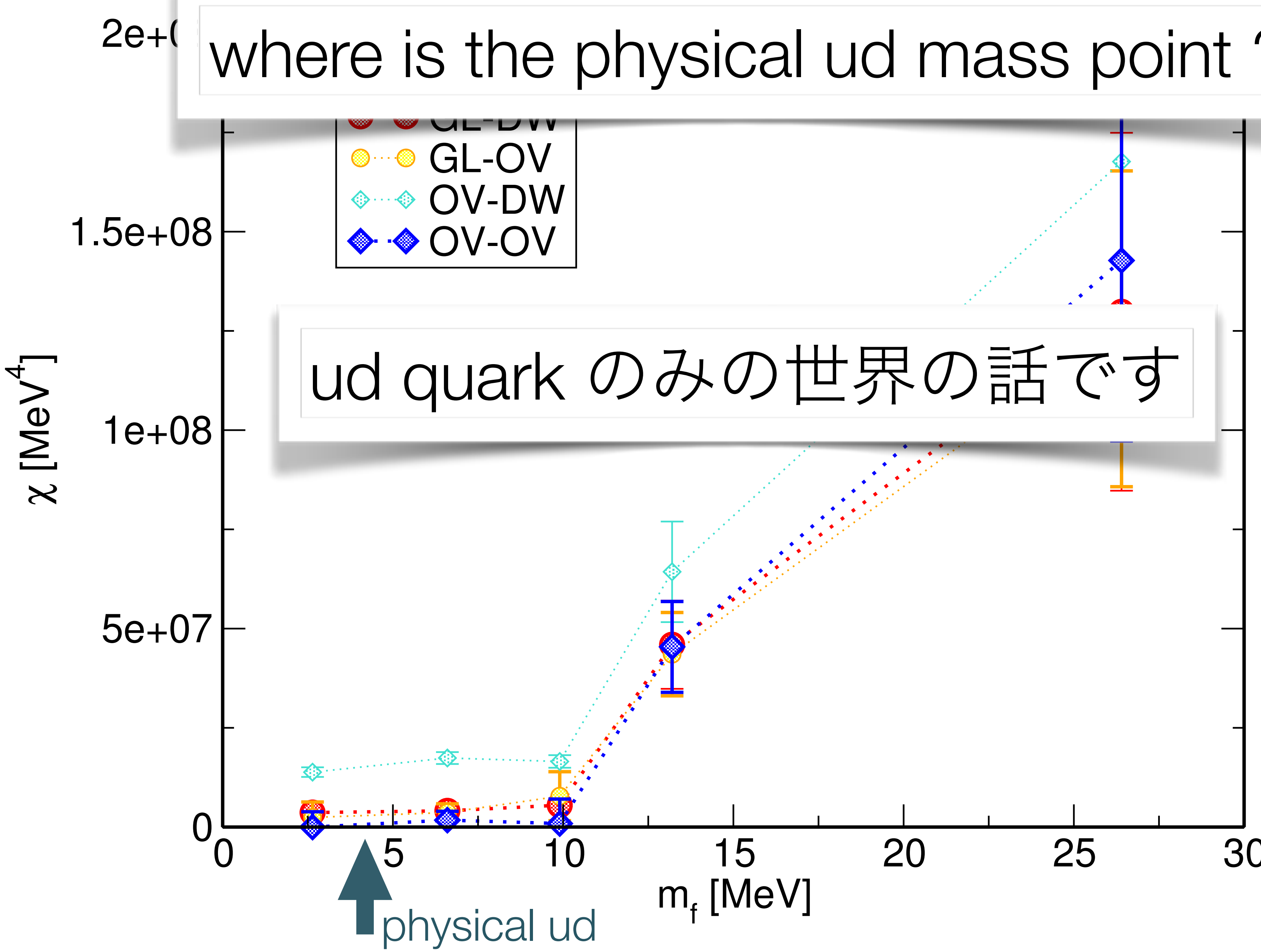
where is the physical ud mass point ?



ud quark のみの世界の話です

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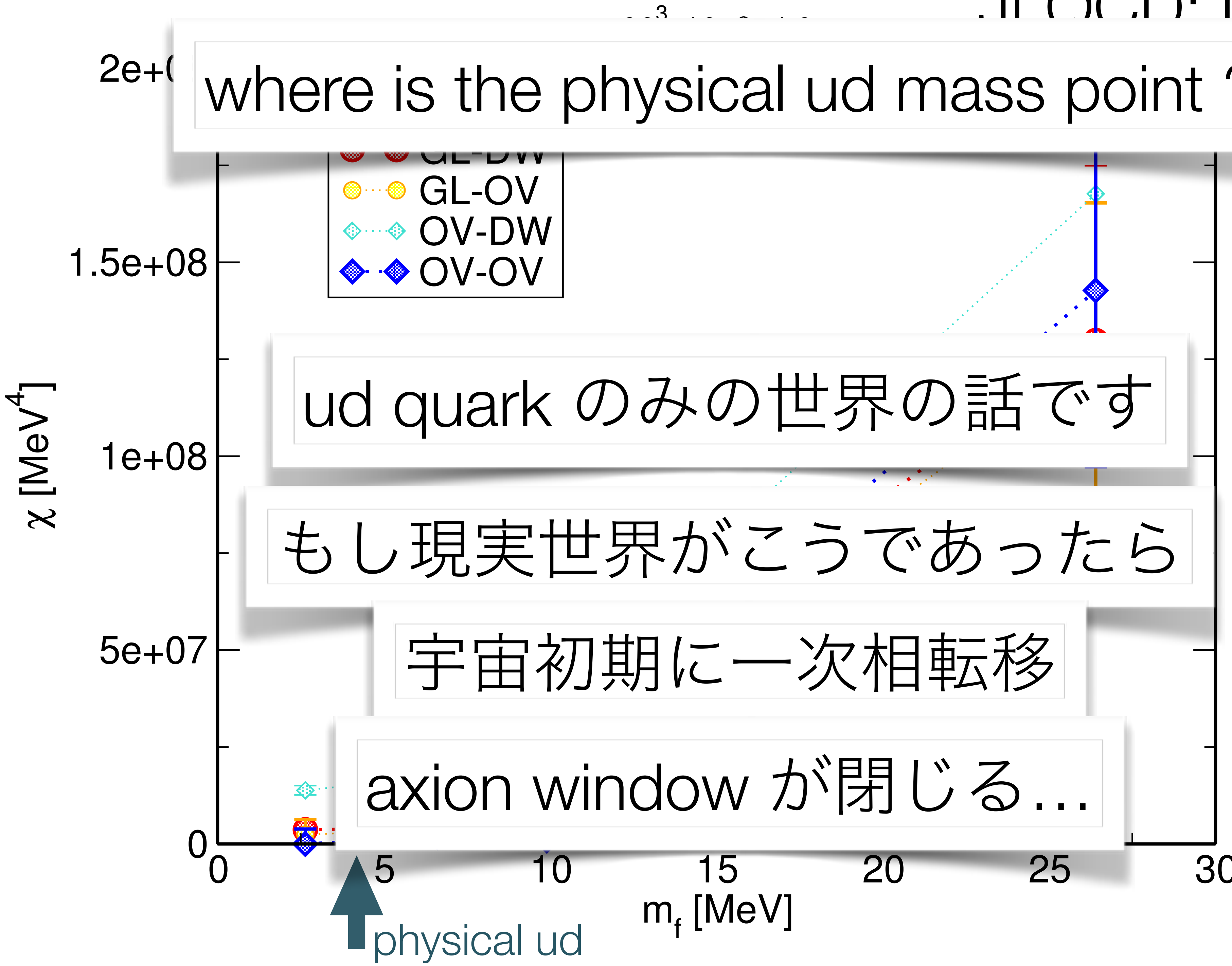


ud quark のみの世界の話です

もし現実世界がこうであったら

宇宙初期に一次相転移

axion window が閉じる…

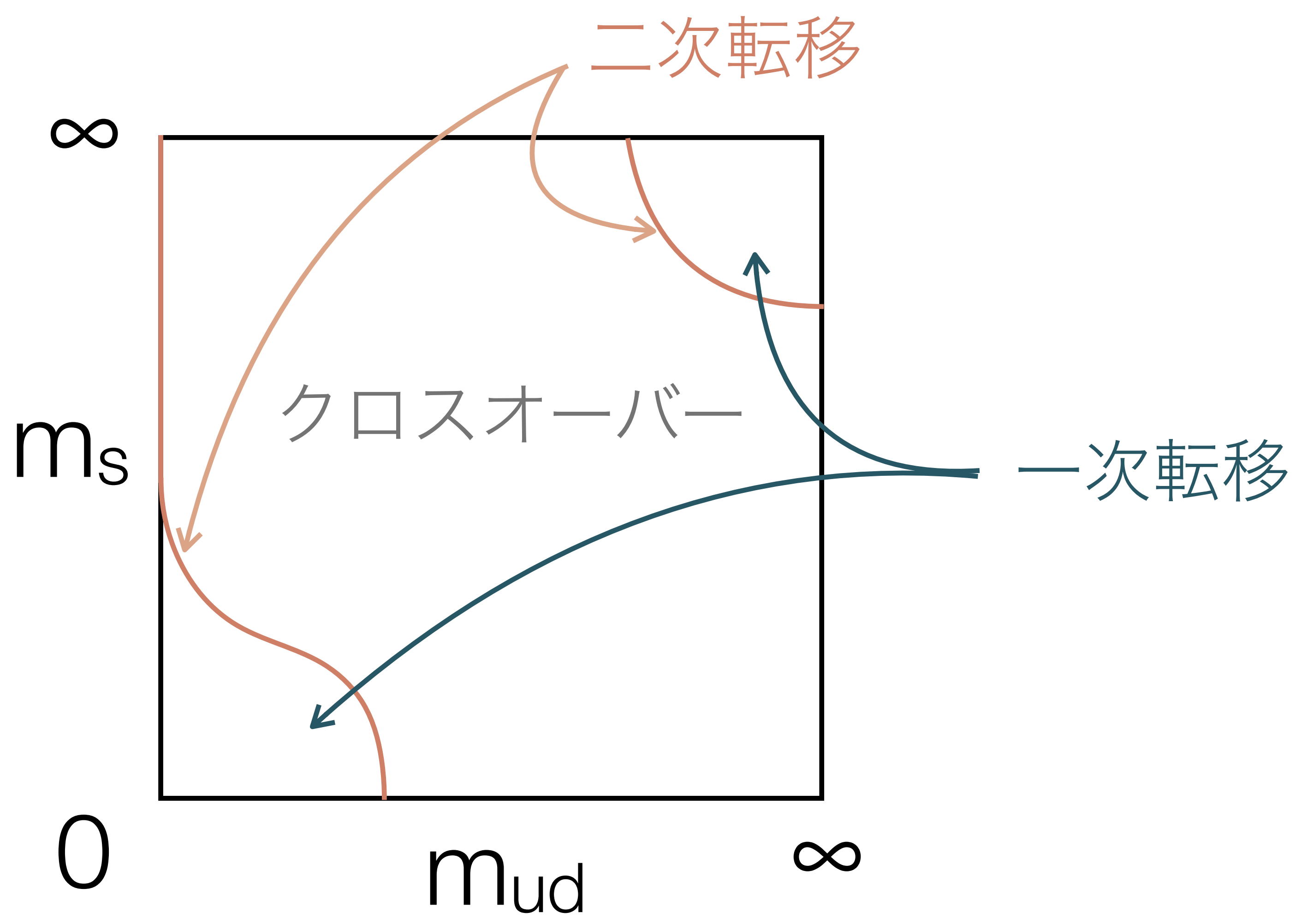
JLQCD members involved in recent finite temperature study

Sinya Aoki
YA
Guido Cossu
Hidenori Fukaya
Shoji Hashimoto
Takashi Kaneko
Kei Suzuki
…

もくじ

- QCD相図: 理解の現状　(μ=0: zero chemical potential)
  - 格子作用のいろいろ
  - axion との関係
- N_f=2　JLQCD の結果を中心に
  - topological susceptibility
  - fate of the $U_A(1)$ symmetry
- N_f=2+1
  - review of topological susceptibility

現在でも: Columbia Plot = 大方の人の理解 || 期待



[original Columbia plot: Brown et al 1990]

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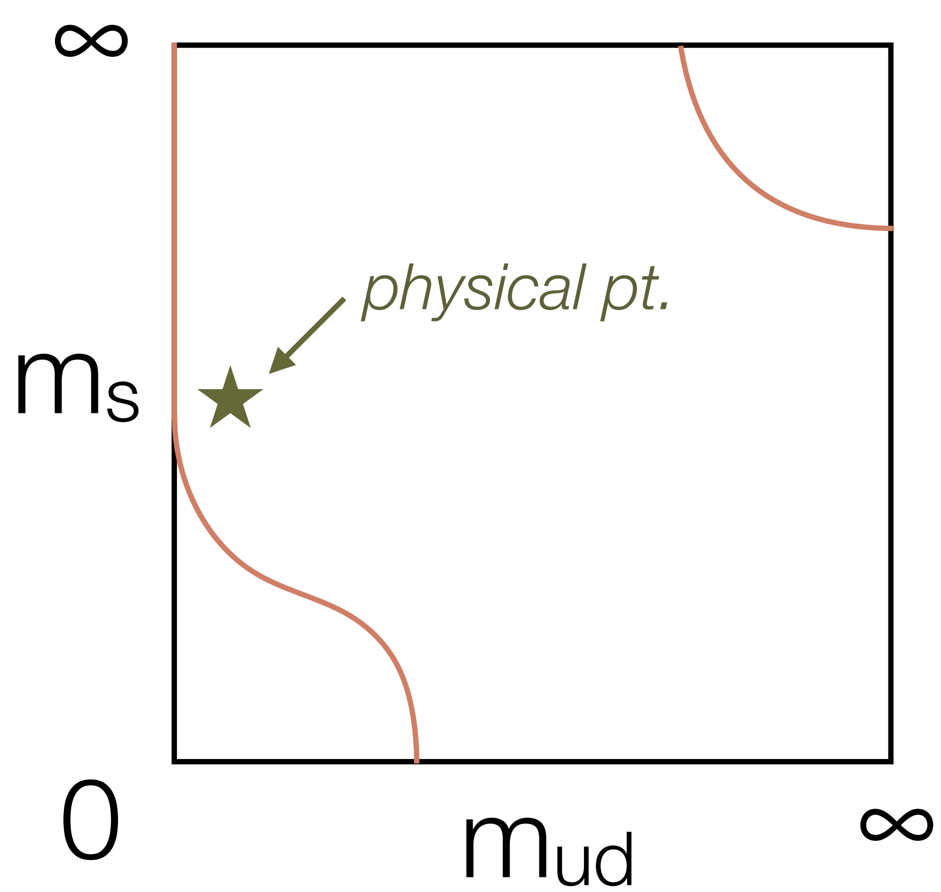
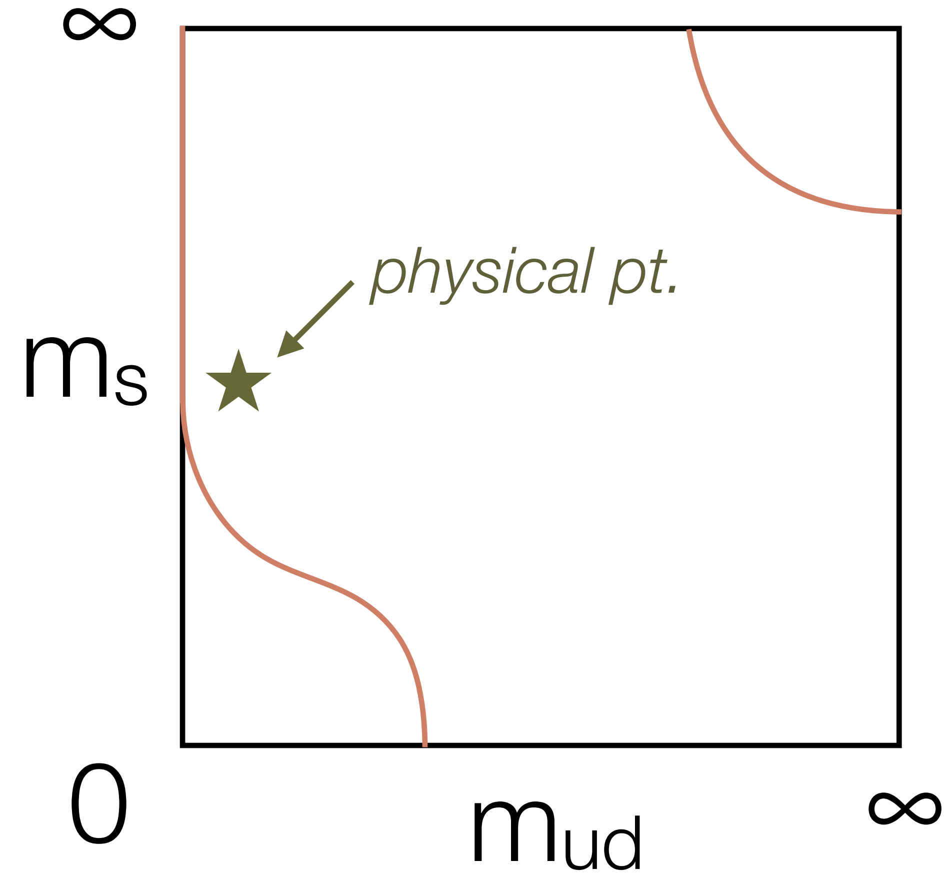


[original Columbia plot: Brown et al 1990]

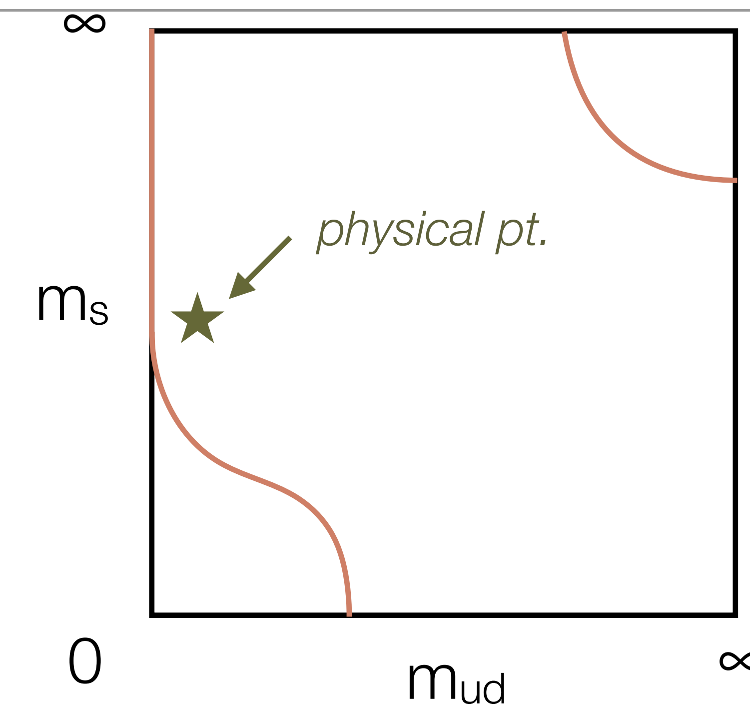
N_f=2+1相図

- 連続極限で分かっていること
  - N_f=0: 一次転移
    - 右上隅はよく分かっている
  - N_f=2+1 物理点: cross-over
    - staggered (YA, Endrodi, Fodor, Katz, Szabo: Nature 2006)
    - 他の正則化でも反証なし
    - 厳密なカイラル対称性を持つアプローチでは未踏
- その他の領域は未確定



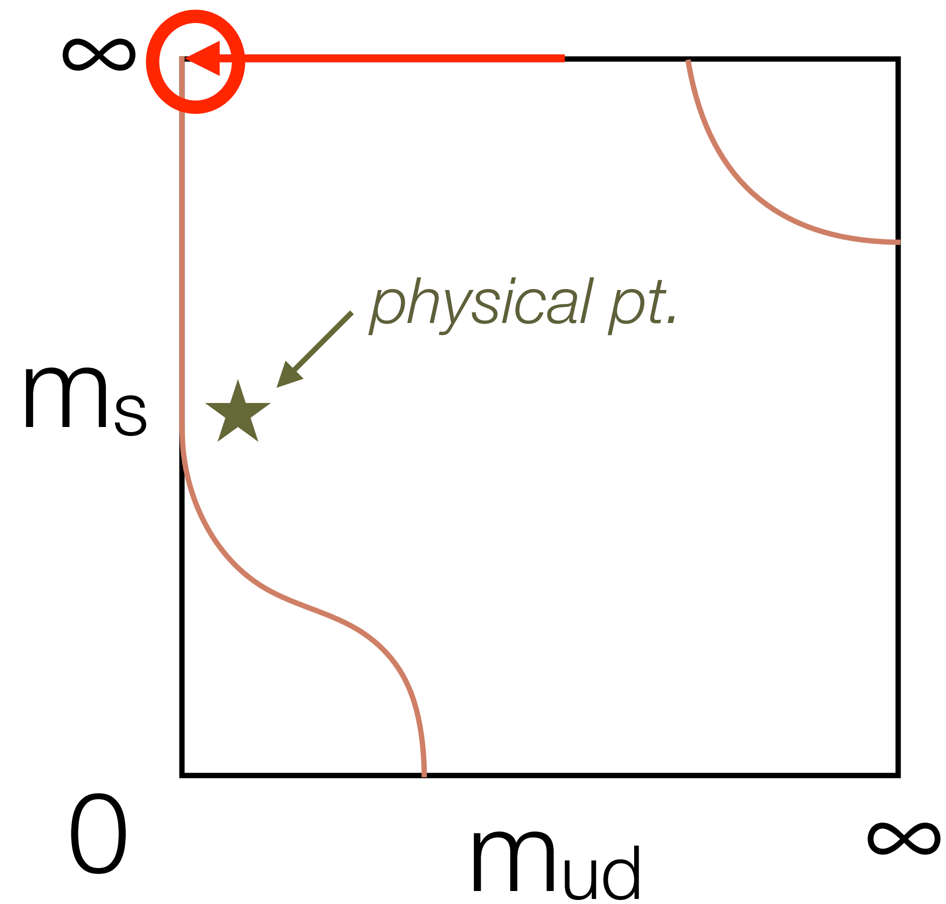
QCD 有限温度相転移の理論: N_f=2+1 Lattice

- Nf=2+1 相図が完成すれば
  - QCD の理解
  - 物理点の相転移の存在、次数が分かる。
    - 遠回りだが確実な方法
  - 相境界(μ=0)の μ>0 への伸び方を調べる→(T,μ)臨界終点の研究へつなげる
- 大変重要／有用である → ポスト京 重点課題9 のプロジェクトのひとつ



まずは N_f=2

- N_f=2+1 physical pt. から遠い？
  - m_s ~100 MeV → ∞
    - T=0 では s のあるなしは微細効果
  - boundary の情報としては有用



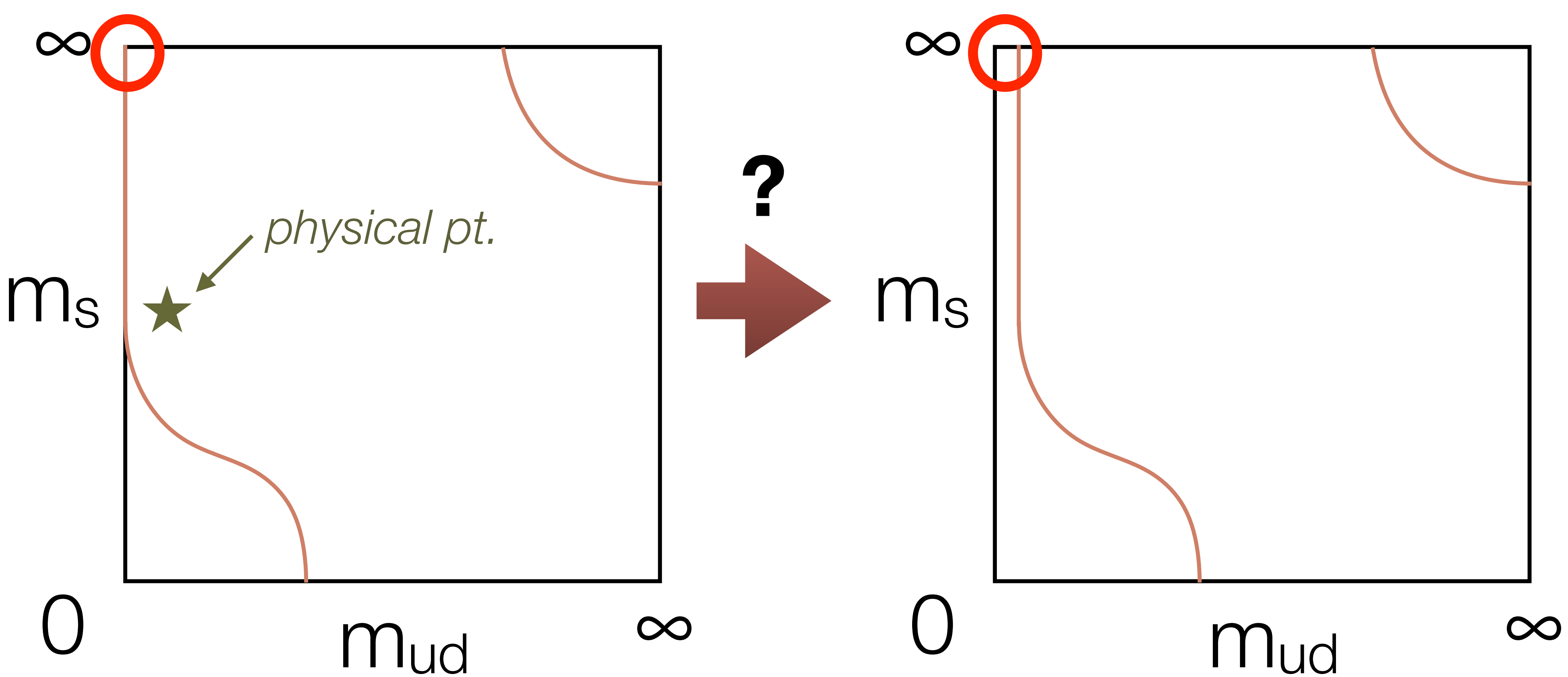
- N_f=2
  - Wilson, staggered: 未確定
  - 厳密な格子カイラル対称性

➡U(1)$_A$ 回復を示唆[JLQCD16]

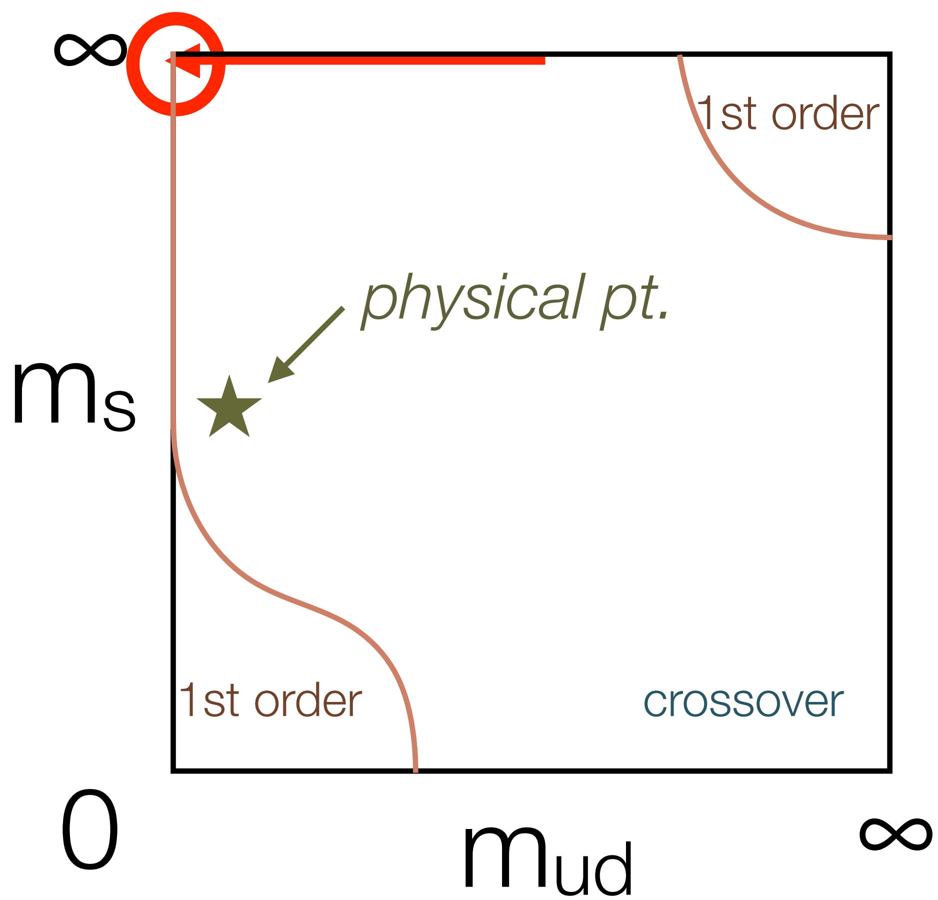
➡一次転移の可能性 → $\chi_t(m)$に飛び?
[Pisarski&Wilczek]

一次転移だとどうなるか？

- $0 \leq m_f < m_c$: 一次転移
- 一つの可能性として: 左下(N_f=3)の一次転移領域と繋がる
- 物理点への影響も考えられる
    - 現状では staggered → 連続極限の結果のみ



Columbia plot: direct search of PT / scaling



- 2nd order
  - improved Wilson
    - WHOT-QCD Lat2016 (O(4) scaling)
    - Ejiri et al PRD 2016 [heavy many flavor]
- 1st oder
  - imaginary μ → 0
    - staggered Bonati et al PRD 2014
    - Wilson Phillipsen et al PRD 2016

external parameter
→ phase boundary
→ point of interest
➡ detour the demanding region

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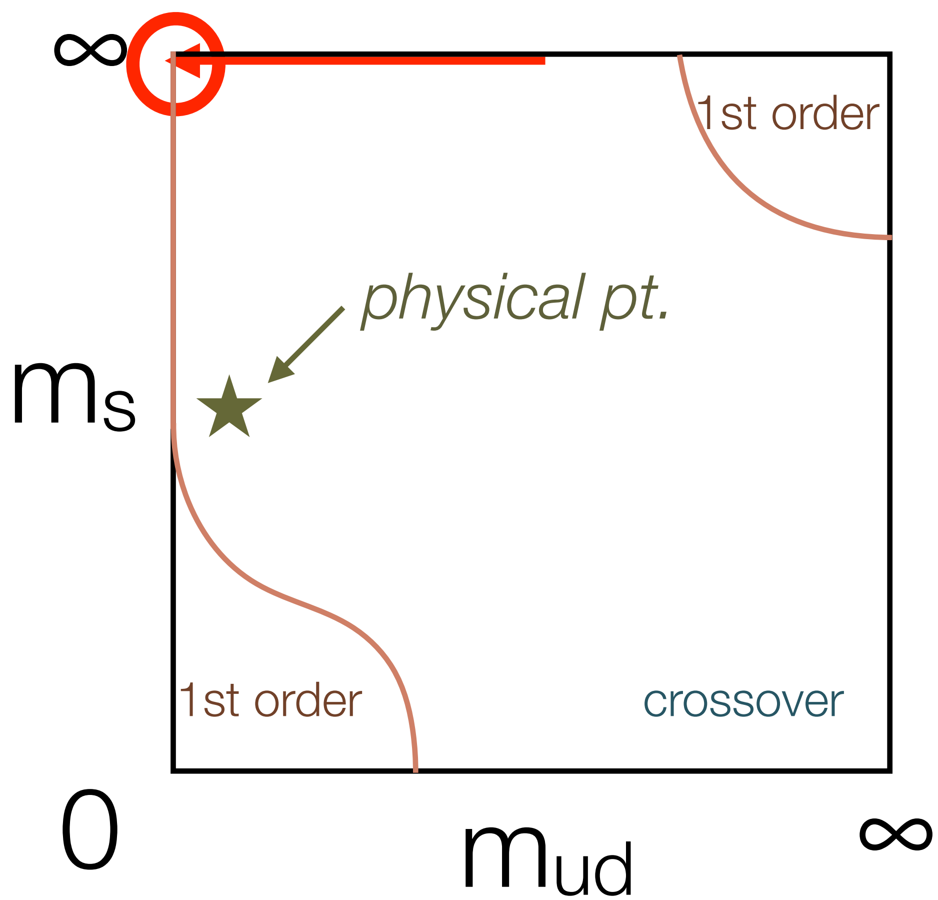

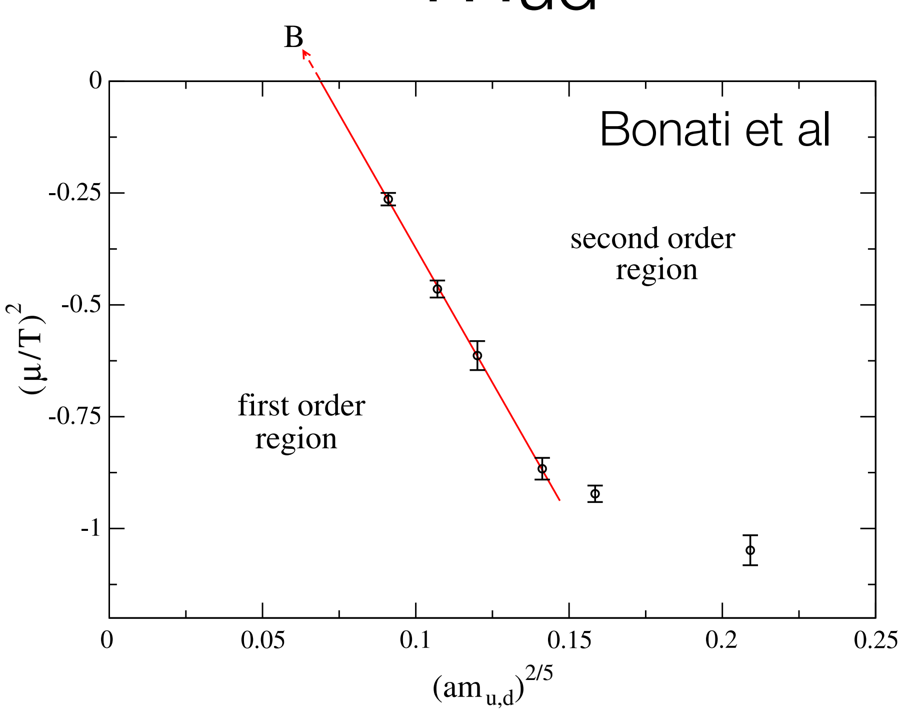

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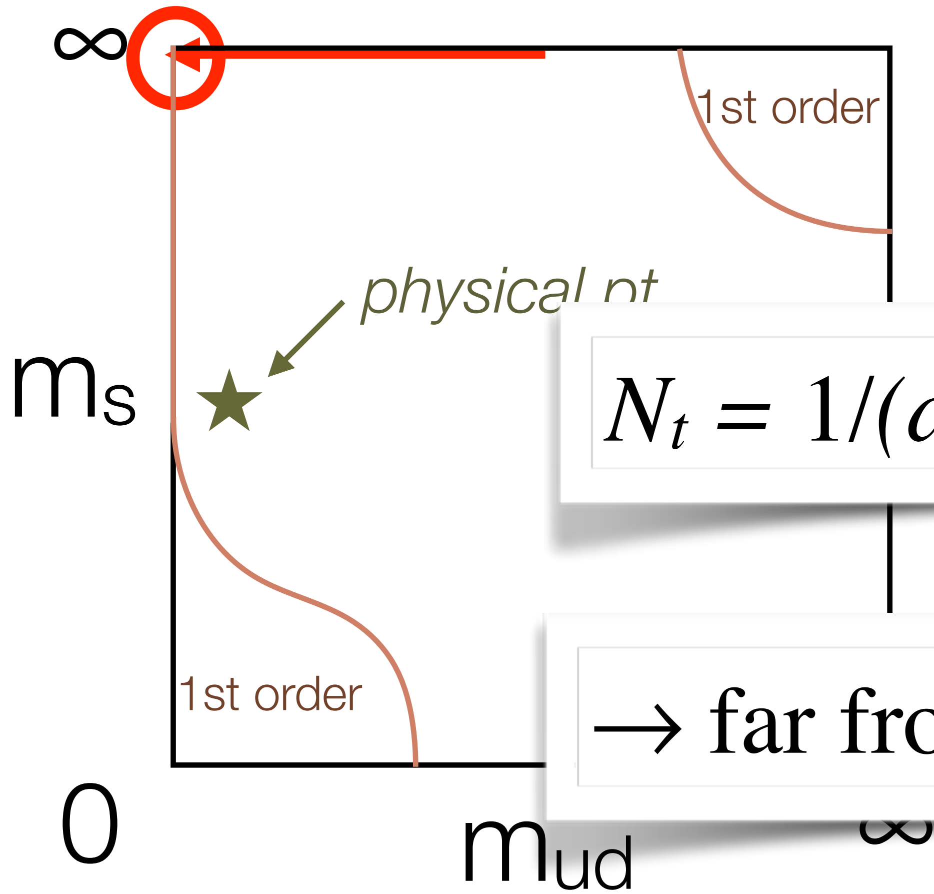

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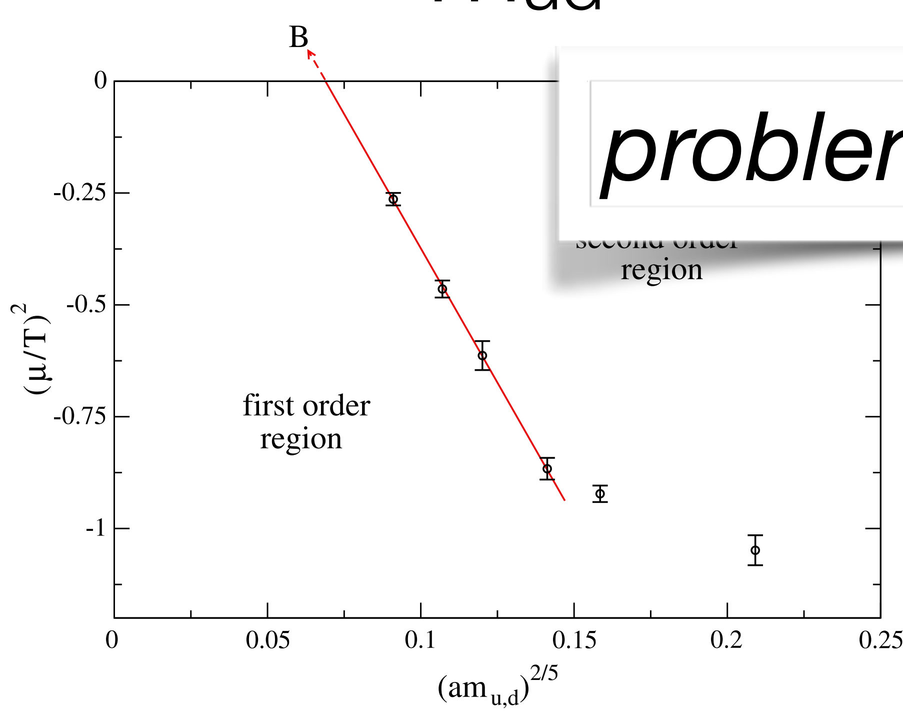
$N_t = 1/(aT)$ =4 or 6 so far

→ far from continuum limit

problem not settled yet



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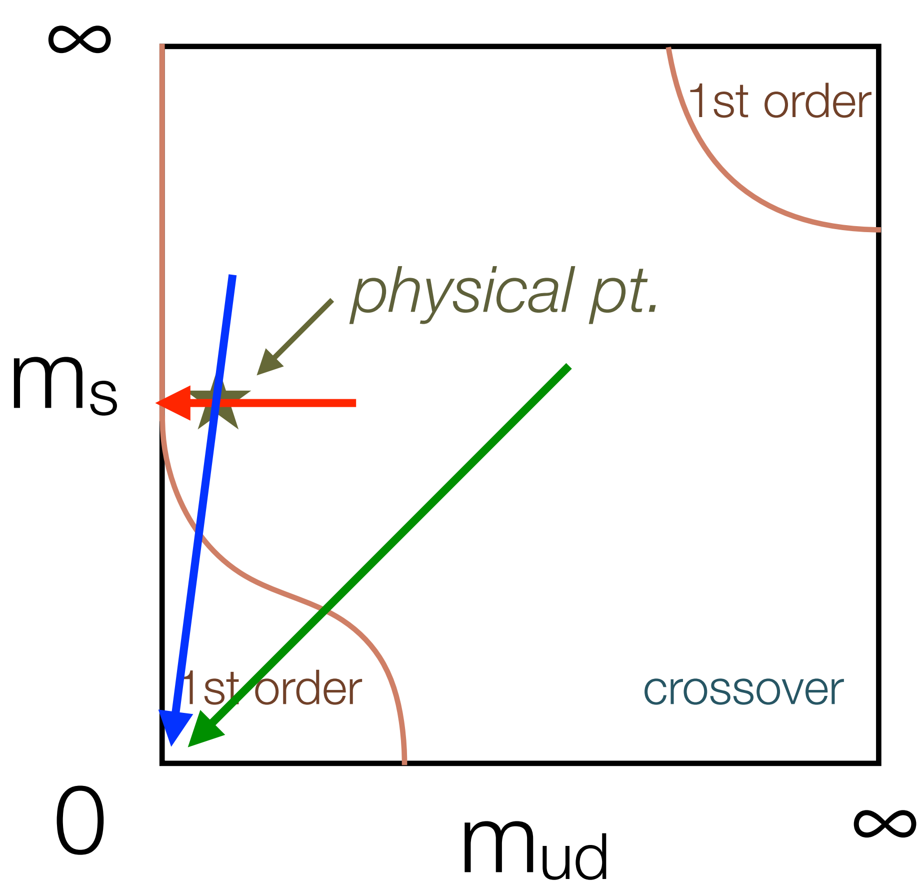
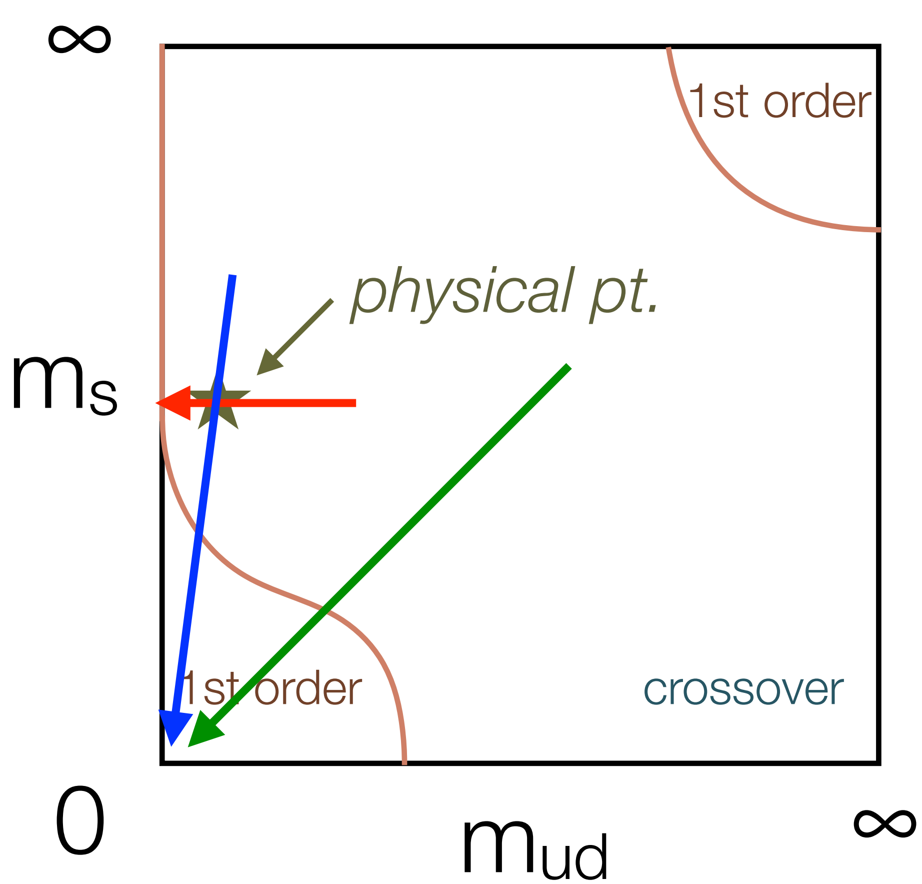


N_f=2+1 or 3

- either
  - no PT found
  - 1st order region
    - **shrinks** as $a \to 0$
      with both staggered and Wilson
    - or even disappear ?
- *for more information see eg*
  - Meyer Lattice 2015
  - Ding Lattice 2016
  - de Forcrand
    "Surprises in the Columbia plot"
    (Lapland talk 2018)

Columbia plot: direct search of PT / scaling



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Understanding of the diagram being changed a lot

- Ding Lattice 2016
- de Forcrand

  "Surprises in the Columbia plot"

  (Lapland talk 2018)

格子作用いろいろ

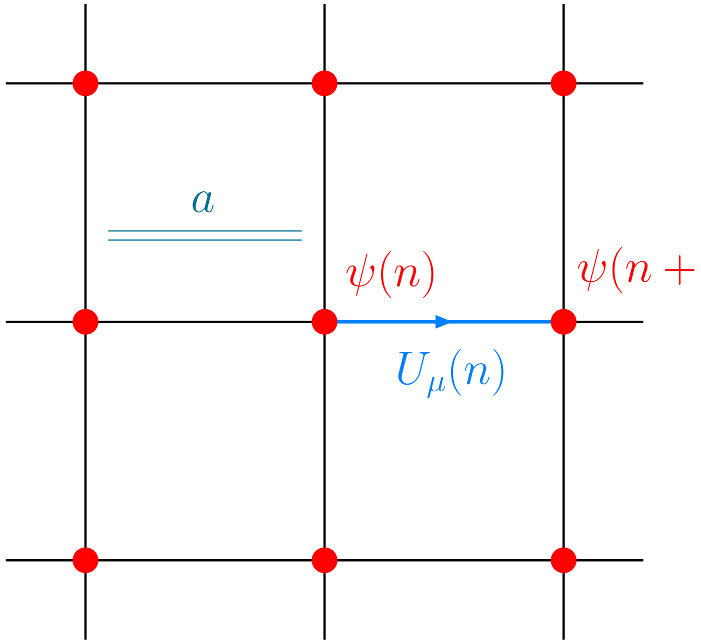
	$U(1)_B$	$SU(N_f)_V$	$SU(N_f)_A$	simulation cost
Wilson	✓	✓	×	moderate
staggered	✓	×	U(1)	cheep
domain wall	✓	✓	almost exact	expensive
overlap	✓	✓	✓	almost impossible

現状良く行われる改良

- Wilson → improved version
- staggered → improved version
- domain wall fermion → “reweighting” to overlap [JLQCD]

QCD and Lattice QCD

- Lattice QCD = QCD defined on discretized **Euclidian** space-time
  - discreteness: lattice spacing = a (~ 0.1 *fm* $\sim$ (2 *GeV*)$^{-1}$)
    - eventually continuum limit: $a \to 0$ needed
- put the system in finite 4d box : $V = L_S^3 \times L_t$
    - eventually: $V \to \infty$ needed
- able to put on the computer as a statistical system
  - $Z = \Sigma \exp(-S) \to$ Monte Carlo simulation
- some symmetry is lost
  - infinitesimal translation and rotation
  - chiral: partially or completely lost
 - expected to recover in the continuum lim. $a \to 0$
- exact symmetry
  - gauge !
  - "chiral" for special discretization
    - (close to) exact chiral symmetry crucial for some applications



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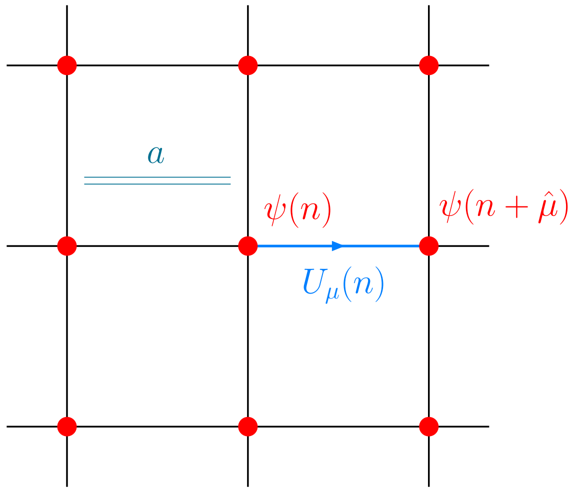


- Lattice QCD = QCD defined on discretized **Euclidian** space-time
  - discreteness: lattice spacing = a (~0.1 fm ~ (2 GeV)$^{-1}$)
  - continuum limit is needed: $a \to 0$
- **near the continuum limit**
  - lattice operators can be expanded in powers of a

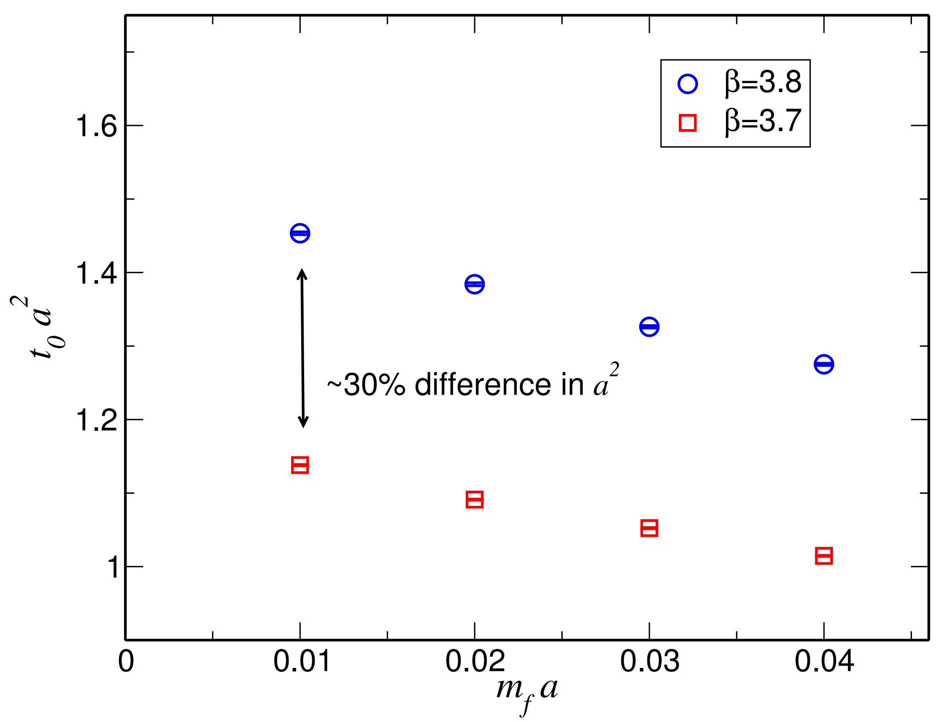
$$\mathcal{O}|_{LQCD} = \mathcal{O}|_{QCD} + ac_1\mathcal{O}_1 + a^2c_2\mathcal{O}_2 \ldots$$

- for some operators in some lattice discretizations
  - $c_1 = 0$ automatically → effectively close to cont. lim.
  - $c_1 = 0$ by engineering = "improvements"
- most of the lattice actions used now → $c_1 = 0$ or $c_1 \simeq 0$
- *However, the size of c_2 term wildly varies among different actions*

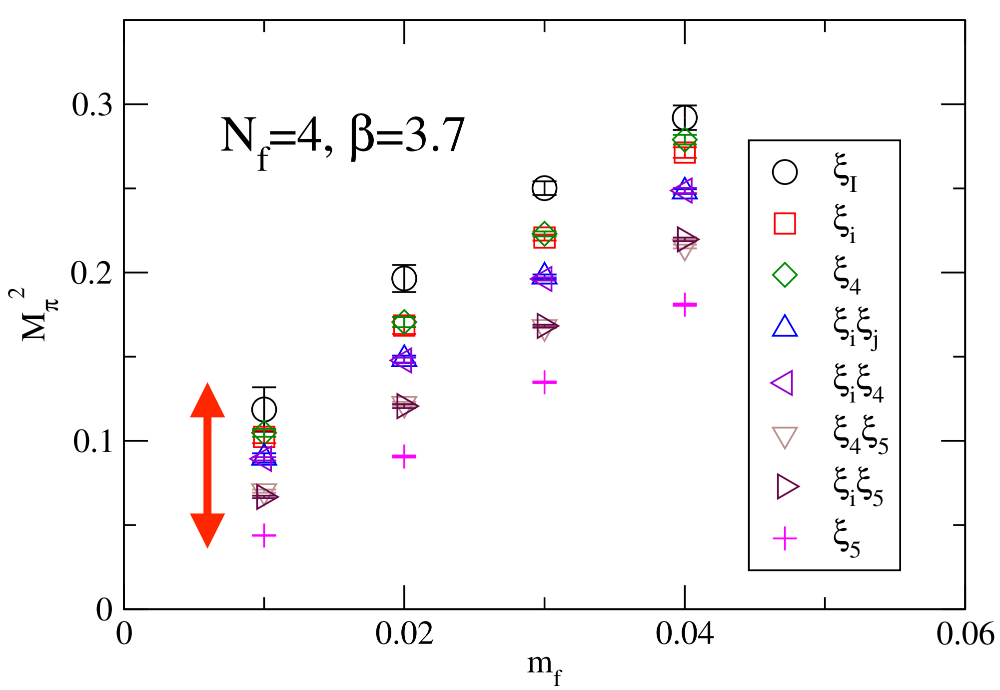
T=0

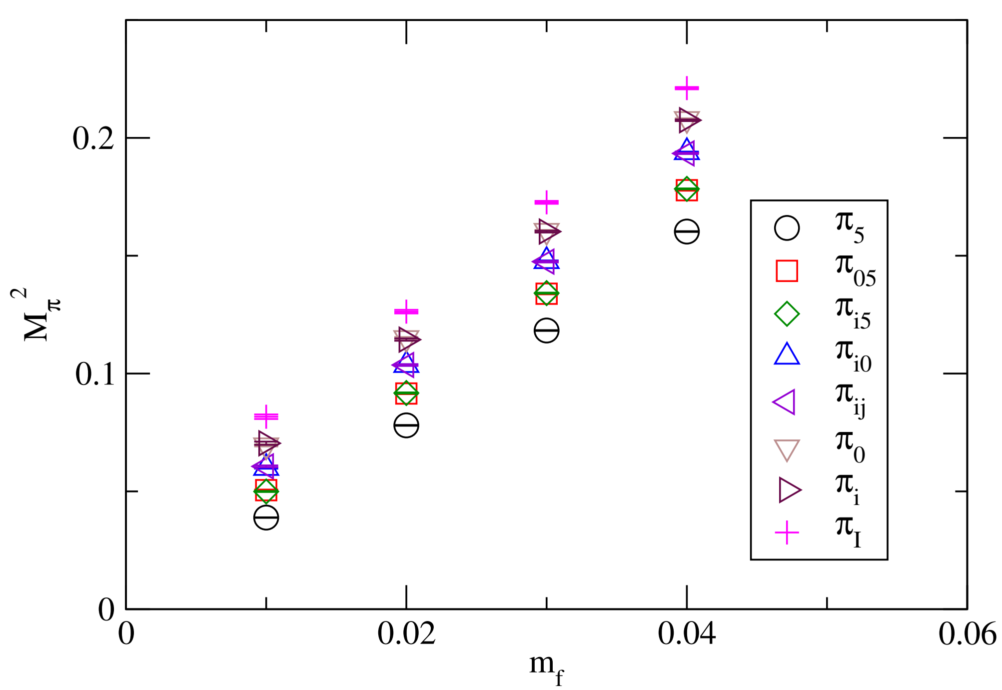
N_f=4: ~~stout~~ HISQ improved staggered [LatKMI collab.]

- t_0 from Symanzik flow:
  - $a^2(\beta=3.7)/a^2(\beta=3.8) \simeq 1.3$



- taste symmetry violation





改良した作用でも $SU(4)_V$ の破れが大きい

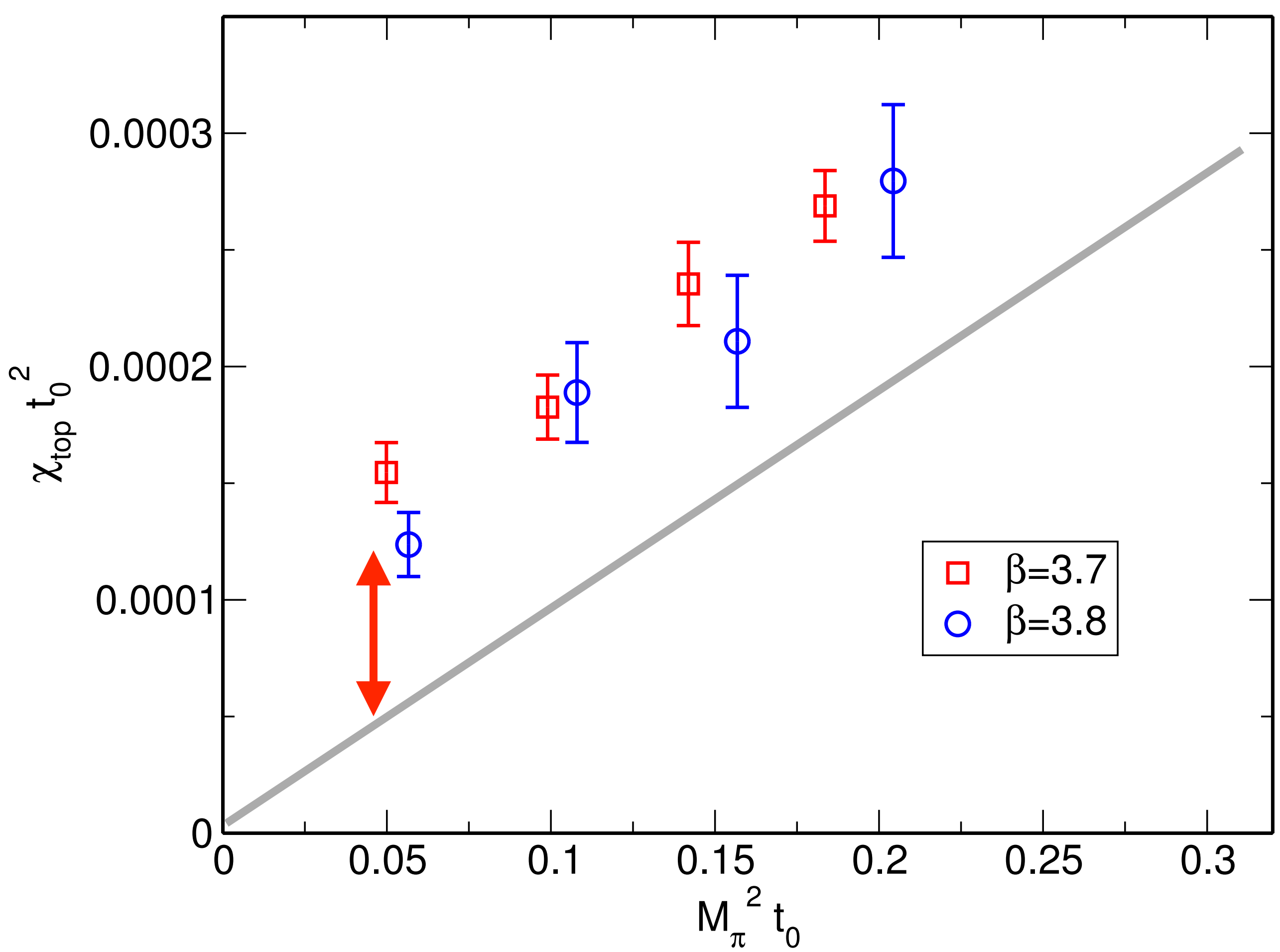
T=0

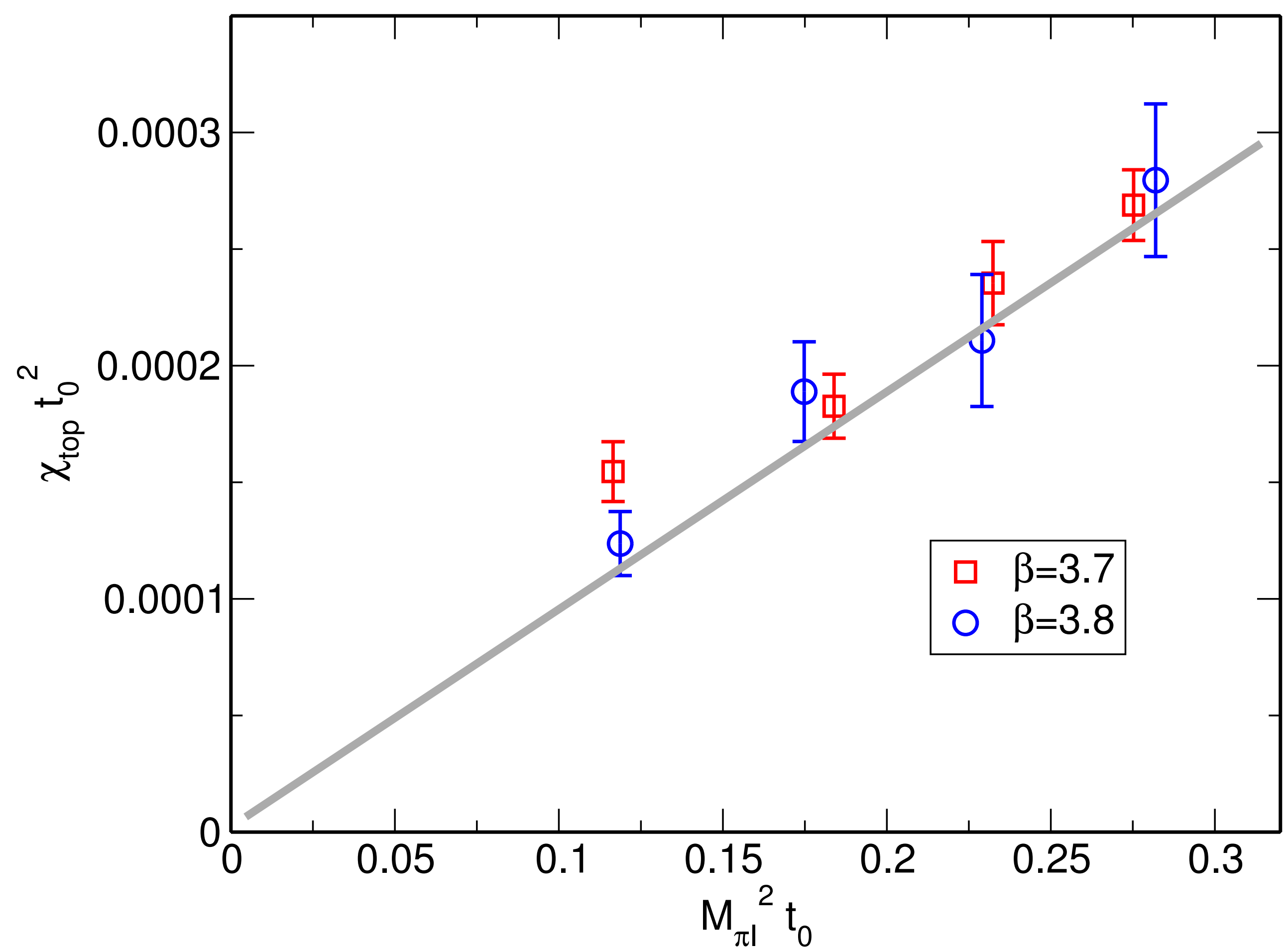
N_f=4 topological susceptibility [LatKMI collab.]

- normalized with t_0

- x-axis: NG pion → taste singlet

  (a la sChPT: see
  Billeter, DeTar, Osborn, PRD2004)





does not vanish in $M_\pi \to 0$?

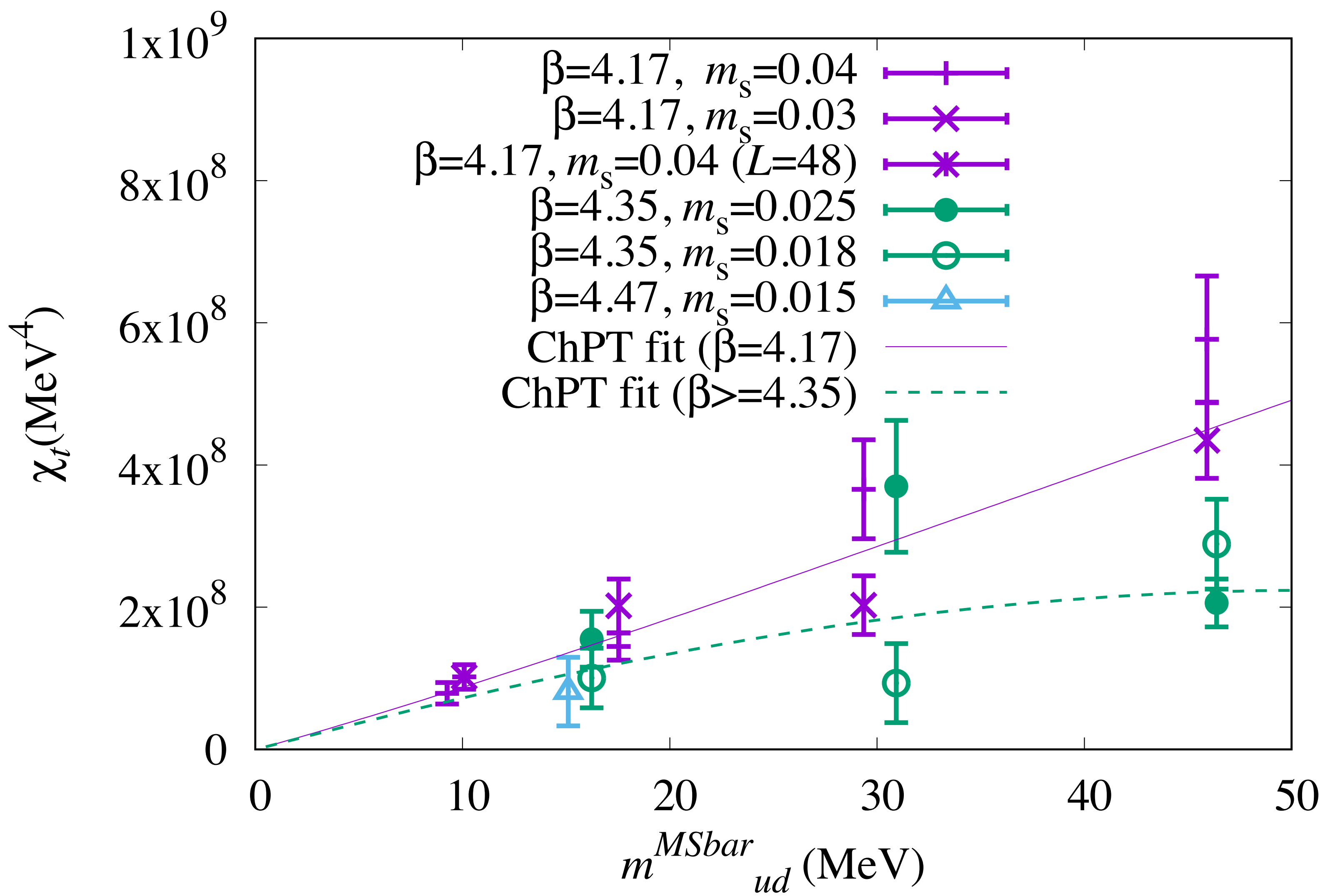
χ_t: $O(a^2)$が巨大

EFT (sChPT)に頼れば改善するが、

第一原理計算の意義は？

T=0

N_f=2+1 domain wall fermion



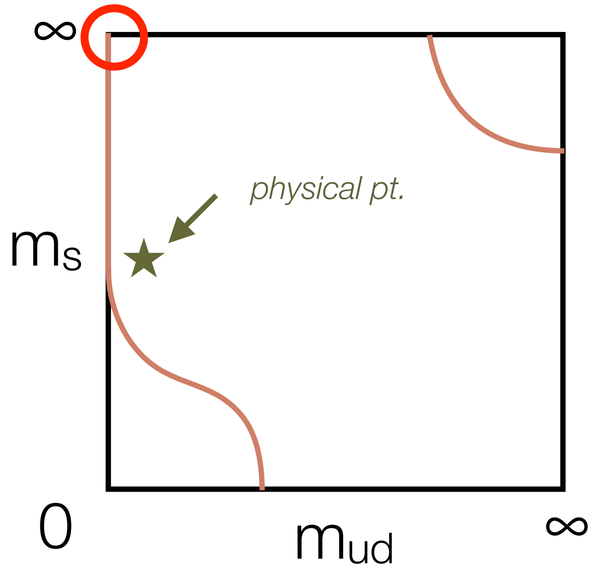
[JLQCD: S.Aoki et al 2017, N_f=2+1 DWF]

O(a^2) error 制御可

ChPT matching 良好

DWF を使うべき!

転移はともかく、U(1)$_A$ 回復すると…



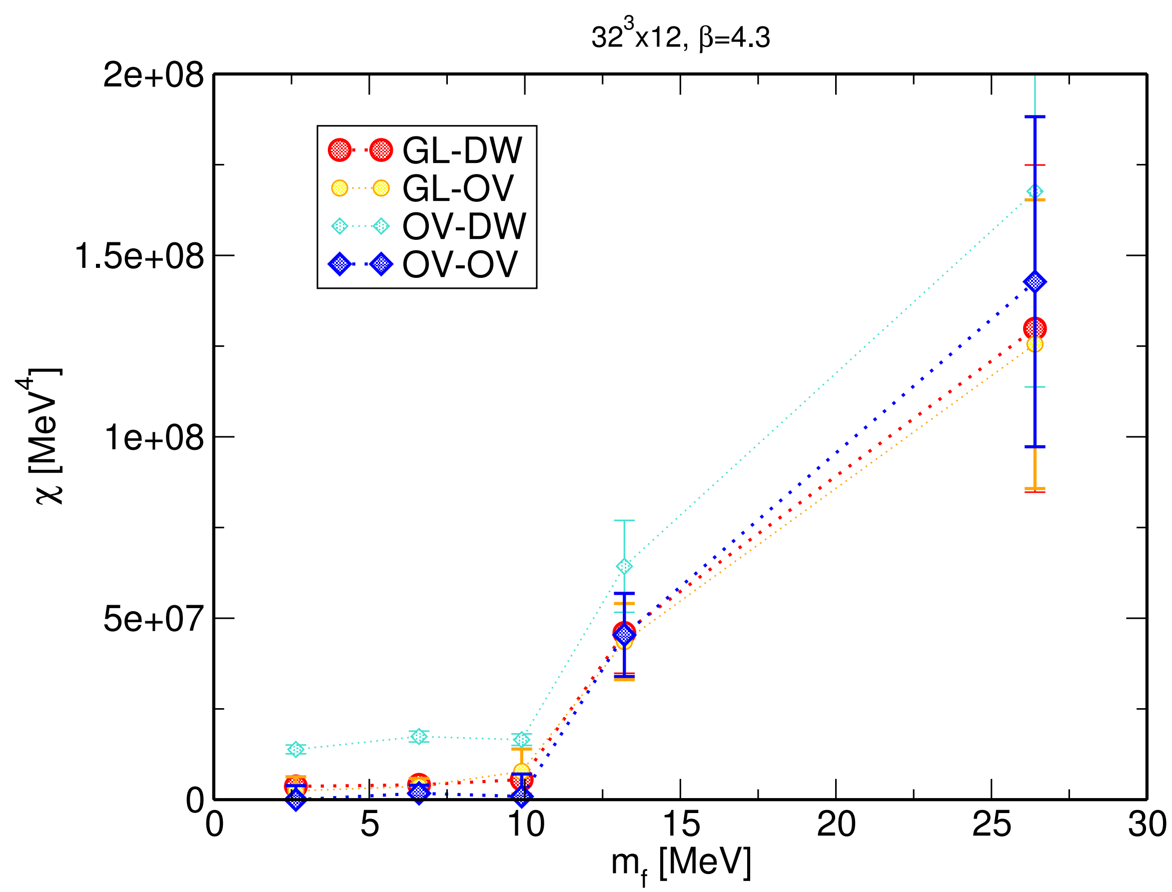
- (U(1)$_A$ broken case: $\chi_t(T) \propto m^2$: m=u,d quark mass)

➡ $\chi_t \sim O(m^4)$ Cohen

refined

➡ $\chi_t|_{m=0} = 0$ & $d^n\chi_t / dm^n|_{m=0} = 0$ Aoki-Fukaya-Tanigchi

➡ $\chi_t = 0$ for $0 \le m < m_c$ → 実現？ →



- physical u,d で **$\chi_t = 0$** の可能性

topological susceptibility and axion mass

- Peccei Quinn mechanism for a solution to strong CP problem
  - new complex pseudo scalar field to remedy the fine tuning problem of θ
  - U(1) symmetry is spontaneously broken → axion
  - effective potential tilted by chiral anomaly
    - → gets mass through χ = topological susceptibility at $\theta=0$
- Axion is a candidate of dark matter
  - axion mass as a function of temperature $m_A(T)$ is a crucial information
  - $\chi(T)$ of QCD @ $\theta=0$ is the target quantity!

U(1)$_A$ 回復すると…

- **axion cosmology scenario may fail for U(1)$_A$ restoration**

  due to vanishing / suppressed topological susceptivility

  - $\chi_t|_{m=0} = 0$ & $d^n\chi_t / dm^n|_{m=0} = 0$ Aoki-Fukaya-Tanigchi

    ➡ $\chi_t = 0$ for small non-zero m OR

    ➡ exponential decay for T>T$_c$

$$\chi_t(T) \sim \begin{cases} m_q \Lambda_{\rm QCD}^3, & T < T_c, \\ m_q^2 \Lambda_{\rm QCD}^2 e^{-2c(m_q)T^2/T_c^2}, & T > T_c, \end{cases}$$

$$c(m_q) \to \infty \text{ as } m_q \to 0,$$

$$\chi_t = m_a^2 f_a^2$$

  - axion mass and decay constant:

  ➡ axion window can possibly be closed

Kitano-Yamada JHEP [1506.00370]

topological susceptibility, $U(1)_A$:

- QCD相図の理解
- axion の可能性

に重要。

それらを順に見ていく。

得に断りの無いものは JLQCD による仕事 (最新結果はpreliminary) topological susceptibility

Method

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- Q_t measurements
  - global sum of the gluonic charge density (clover) after Wilson Flow ($t \simeq t_0$)
  - Overlap Index

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$ susceptibility

- reweighting: before / after and above 2 meas. yield 4 χ_t values
- current main focus: 1/a = 2.6 GeV *** **PRELIMINARY** ***

Method

- DWF ensemble → reweighted to overlap
  - Möbius D… V (β=4.3, L_s=16)
  - Overlap:
- Q_t measuren…
  - global su… …low ($t \simeq t_0$)
  - Overlap I…

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$\chi_t(m_f)$ for N_f=2 T=220 MeV, 32^3

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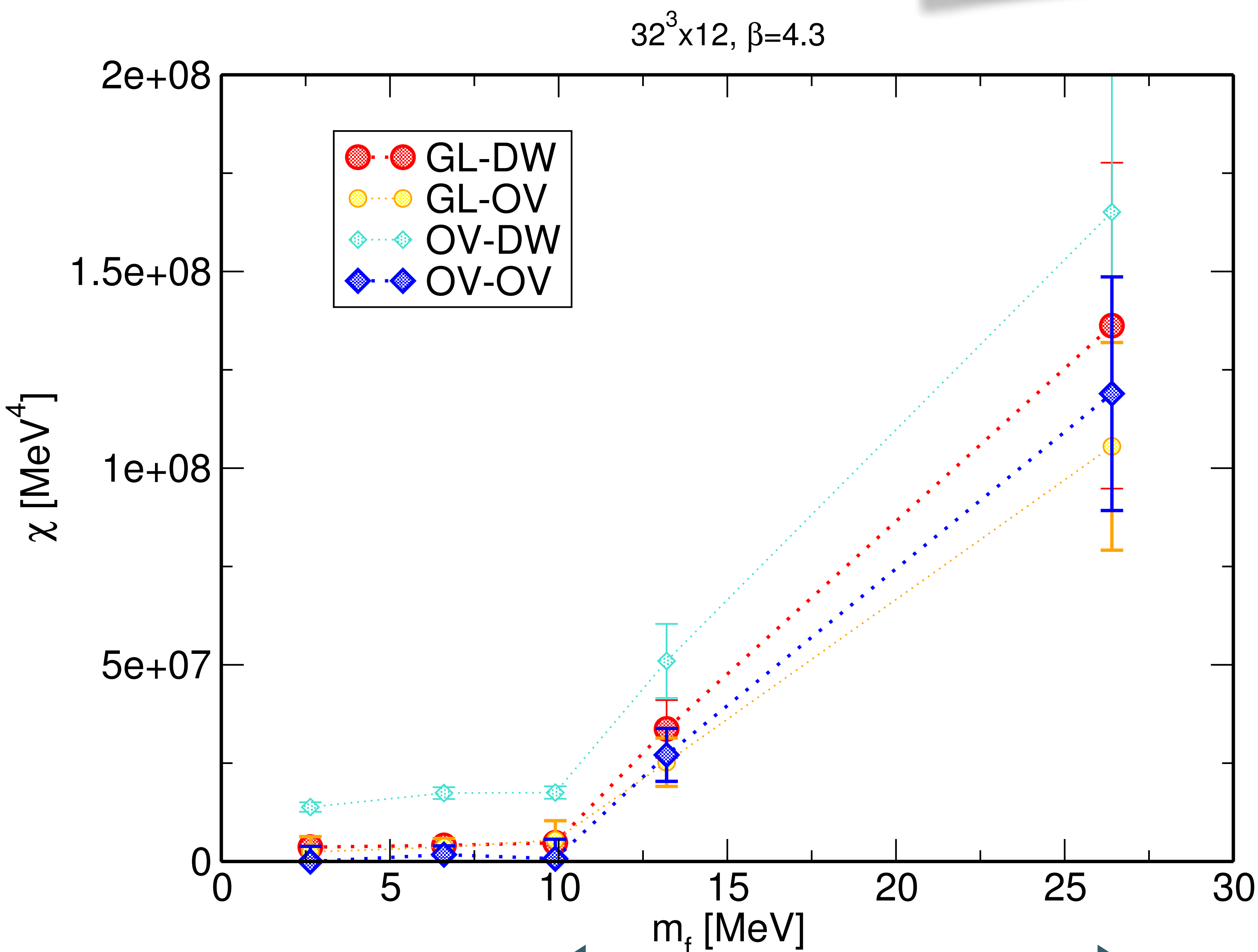
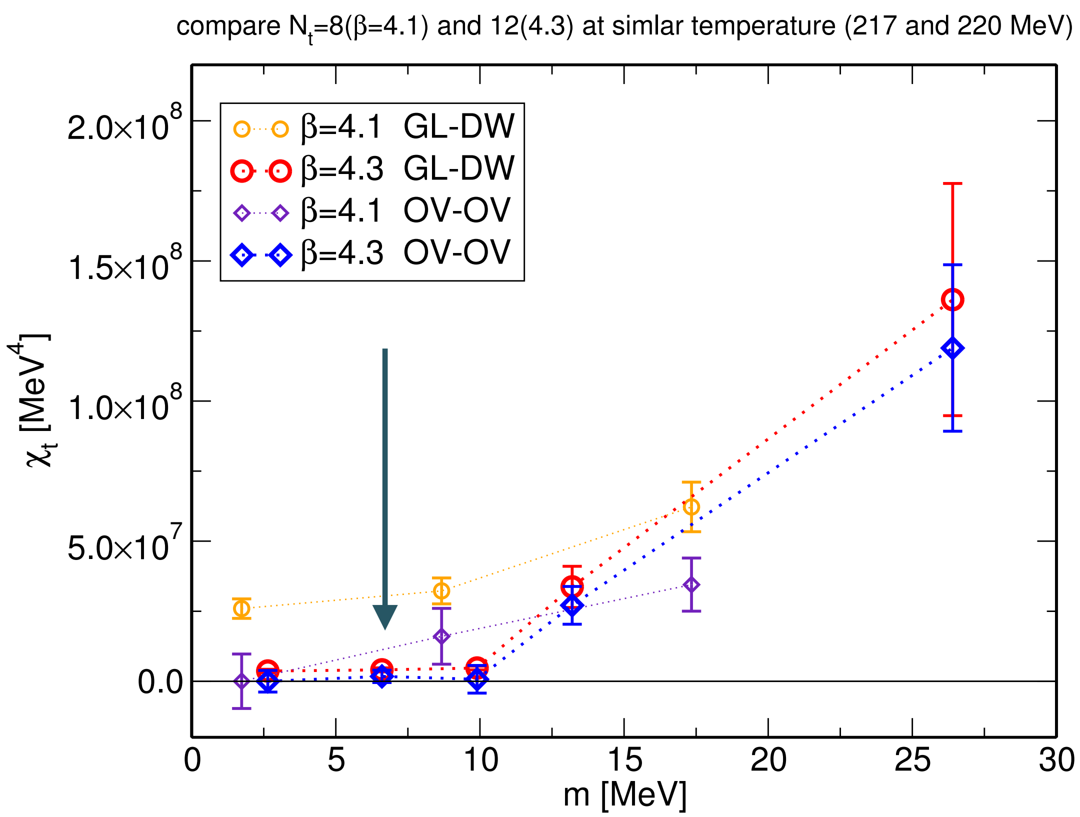
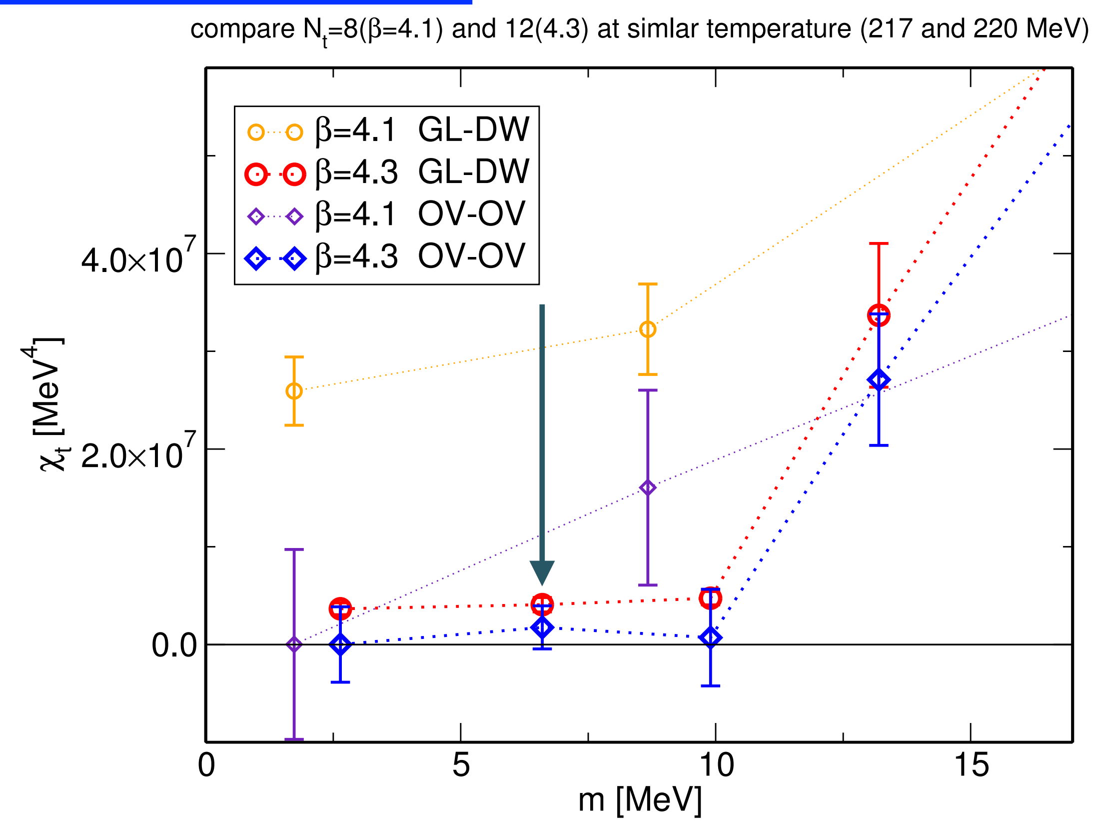


increased stat. from Lattice 2017 : ~30k traj.

$\chi_t(m)$ T=~220 MeV discretization effect

comparing 1/a=1.7 GeV and 1/a=2.6 GeV ($(3.6\text{fm})^3$ and $(2.4\text{fm})^3$)

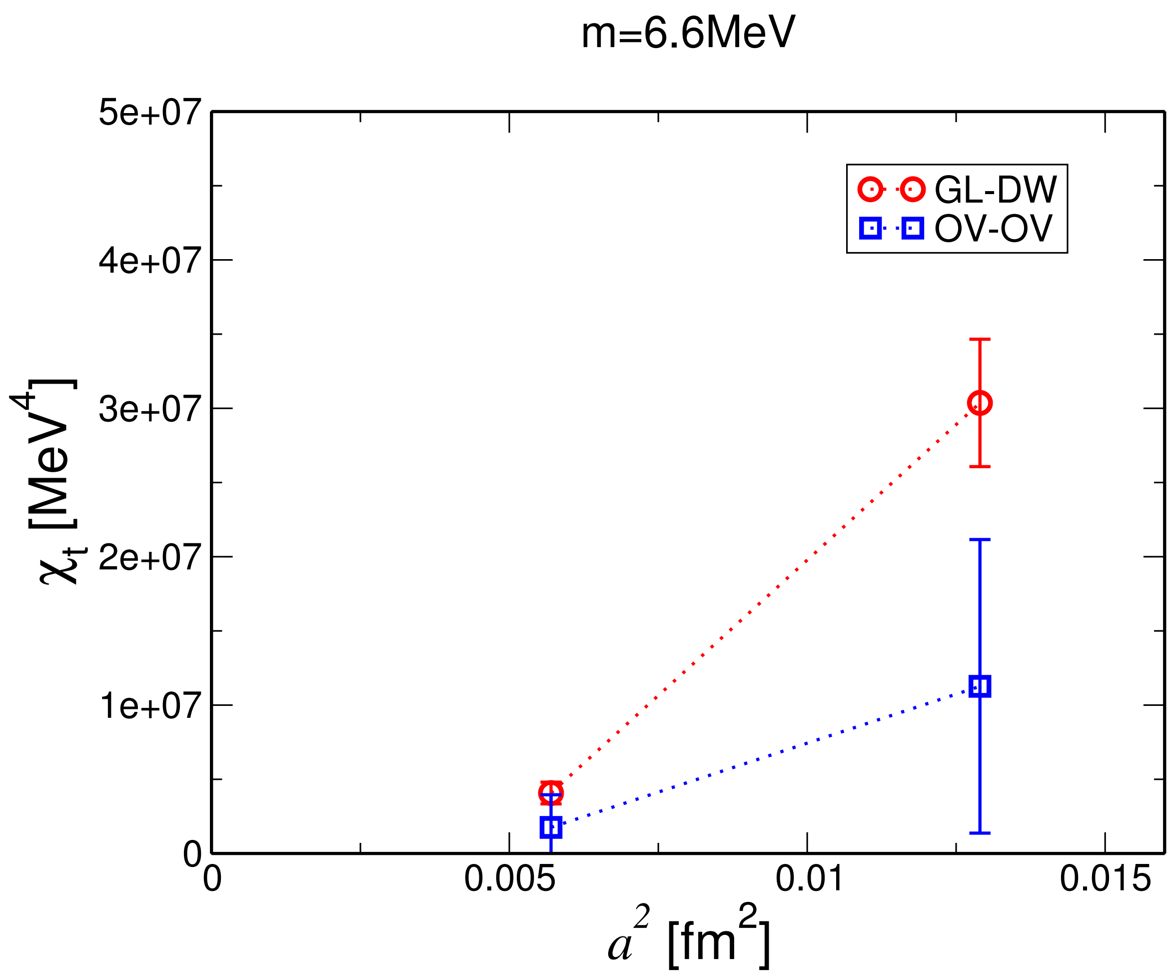




- OV-OV: better scaling
- GL-DW: large scaling violation for smaller m
- OV-OV: $\chi_t = 0$ (within error) for $0 \leq m \lesssim 10$ MeV
- GL-DW: $\chi_t > 0$, but, may well decrease as a

➡ *(consistent with OV-OV with large error of OV-OV)*

$\chi_t(m)$ T=220 MeV a^2 scaling: m=6.6 MeV



(V=(3.6fm)^3 and (2.4fm)^3)

continuum scaling in 1st region

- m=6.6 MeV
- vanishing towards continuum limit
- caveat: physical volume is different → needs further invest.

$\chi_t(m)$ T=~220 MeV, 32^3x12

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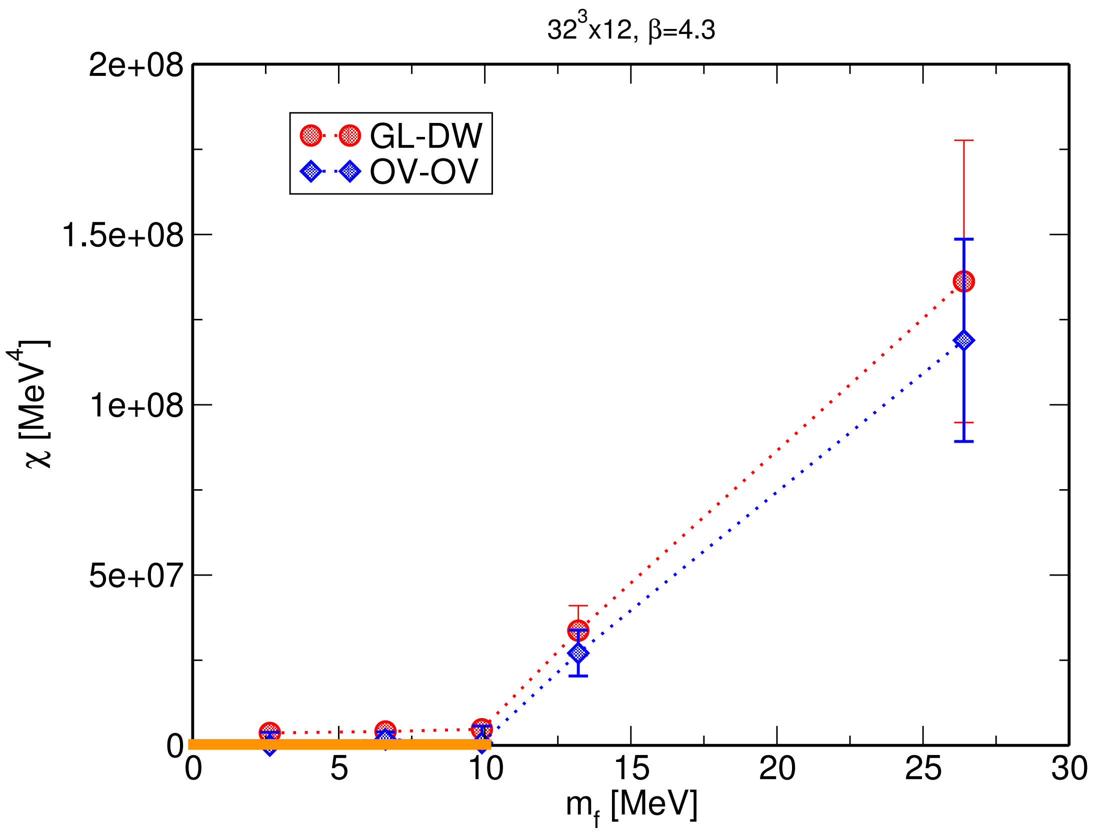
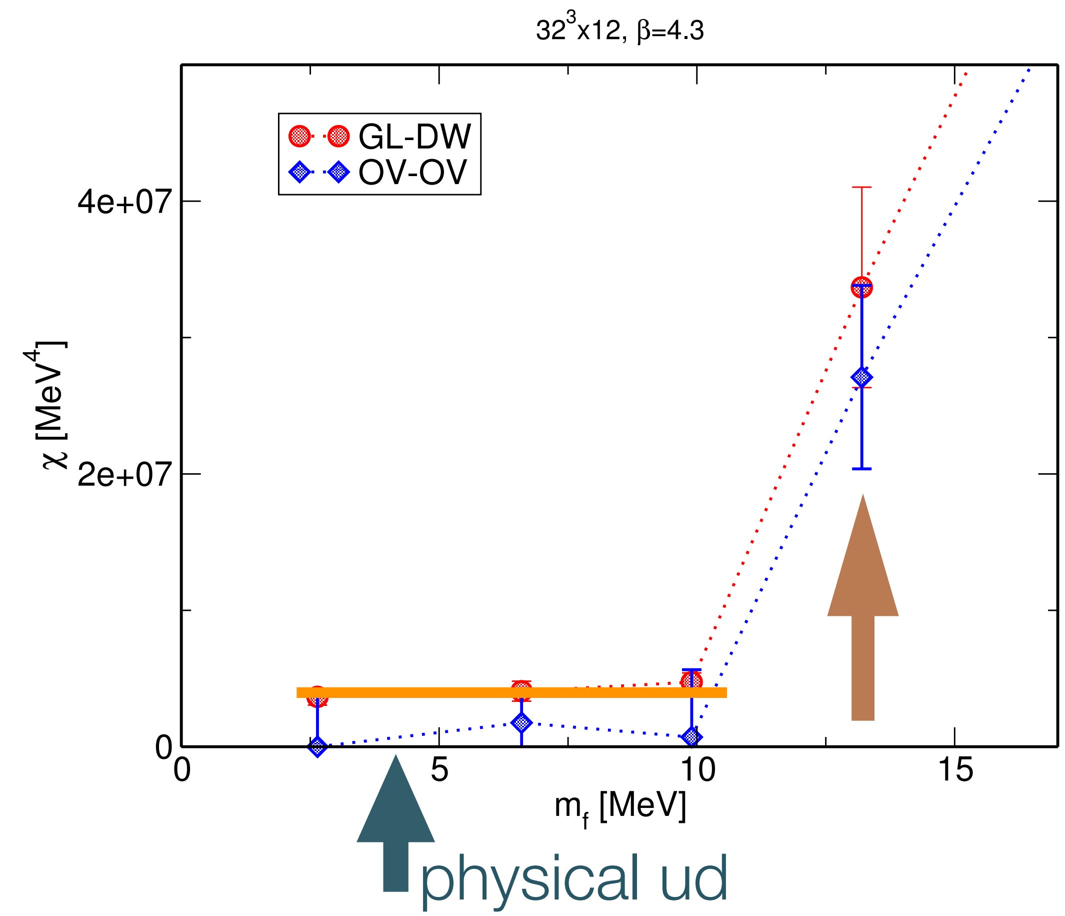
1/a=2.6 GeV





suggesting 2 regions

1: χ_t is very small (may vanish in $a \to 0$): $0 \leq m \lesssim 10$ MeV

(→ consistent w/ Aoki-Fukaya-Tanigchi for $U(1)_A$ symm.)

2: sudden growth of χ_t : 10 MeV $\lesssim$ m

- physical ud mass point: m≃4 MeV

$\chi_t(m)$ T=~220 MeV, 32^3x12

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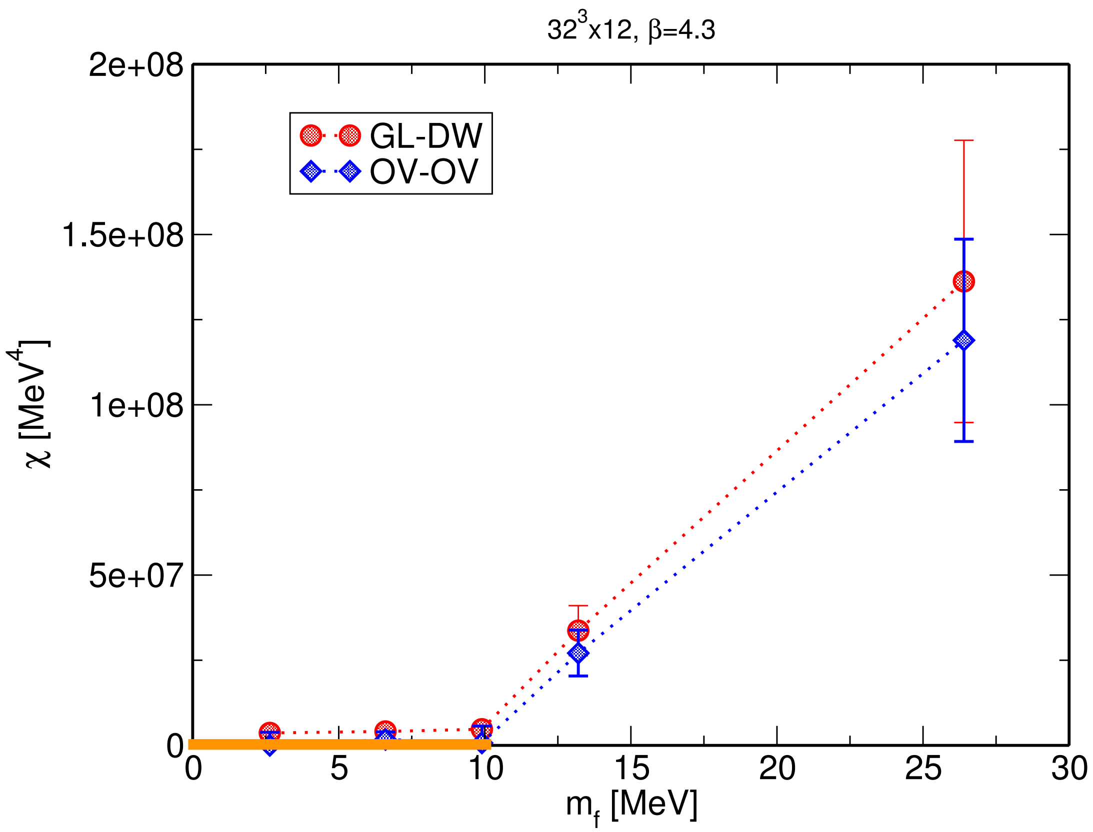
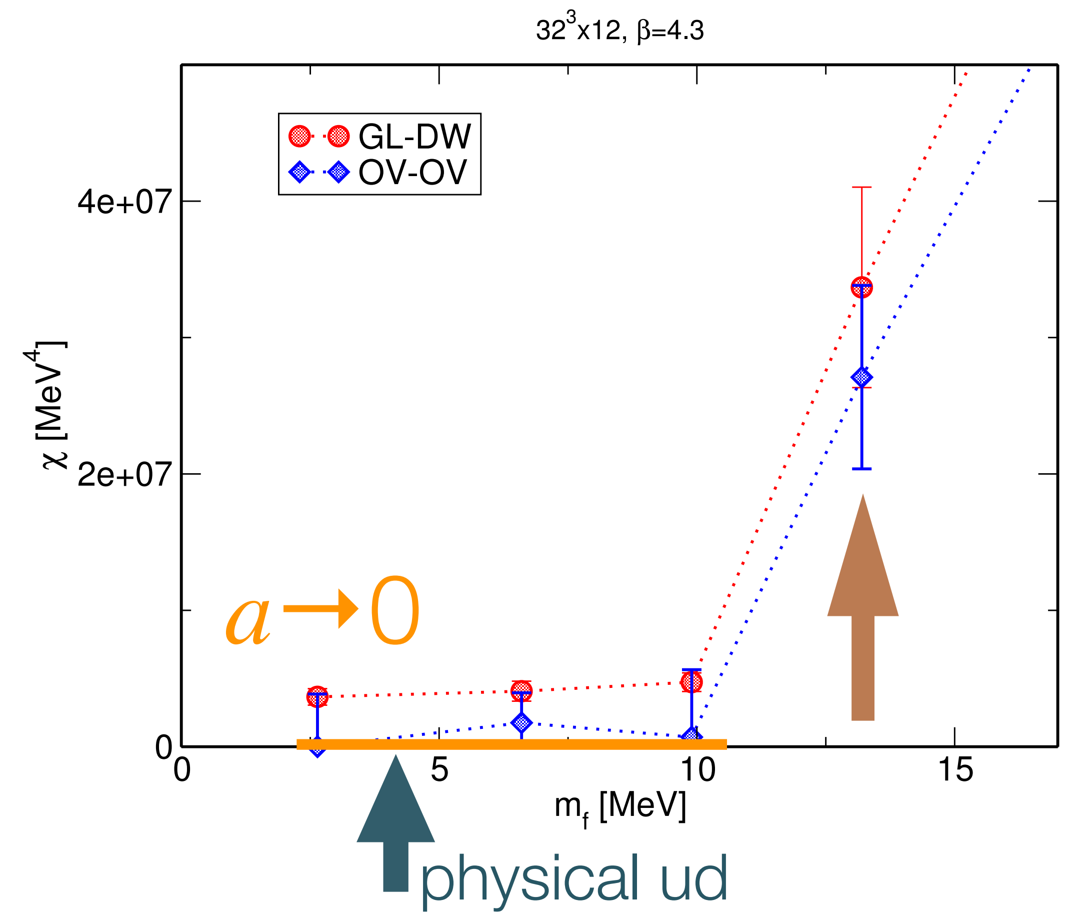
1/a=2.6 GeV





suggesting 2 regions

1: χ_t is very small (may vanish in $a\to 0$): $0 \leq m \lesssim 10$ MeV

(→ consistent w/ Aoki-Fukaya-Tanigchi for $U(1)_A$ symm.)

2: sudden growth of χ_t : 10 MeV $\lesssim$ m

- physical ud mass point: m≃4 MeV

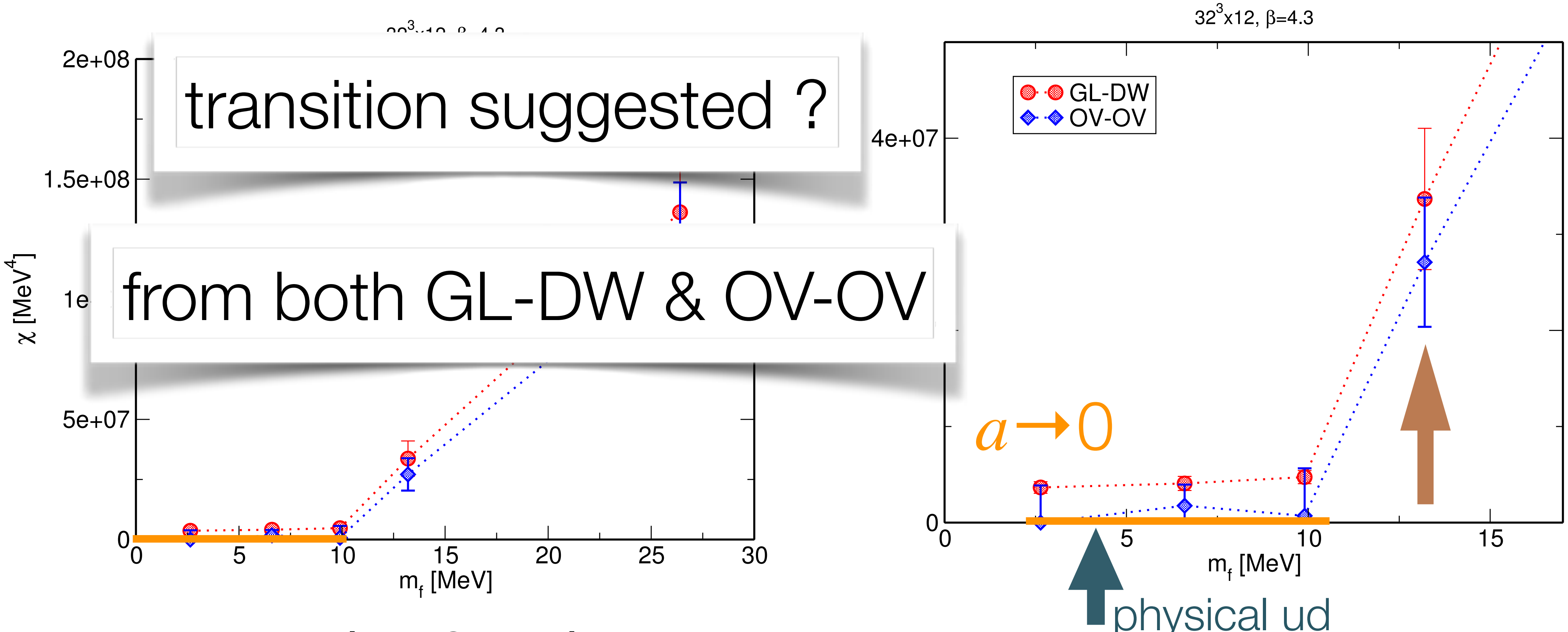
$\chi_t(m)$ T=~220 MeV, 32^3x12

GL-DW	gluonic charge on DW
GL-OV	gluonic charge on OV
OV-DW	OV index on DW ensemble
OV-OV	OV index on OV ensemble

1/a=2.6 GeV

transition suggested ?

from both GL-DW & OV-OV



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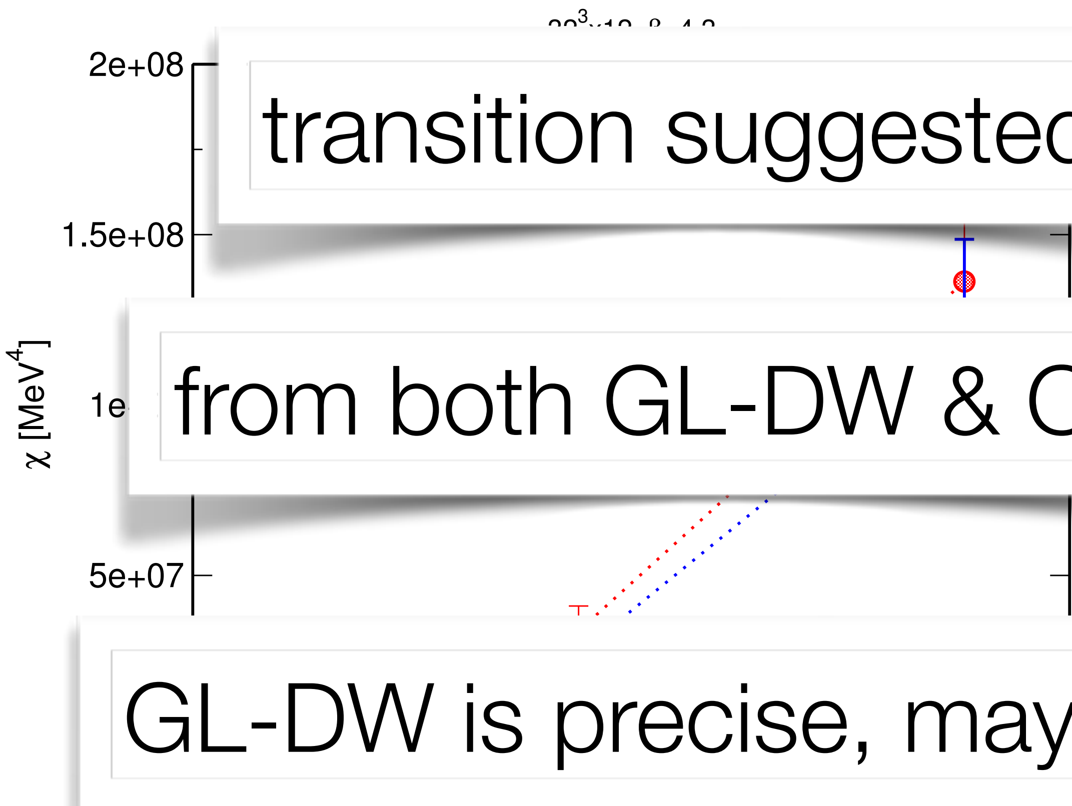

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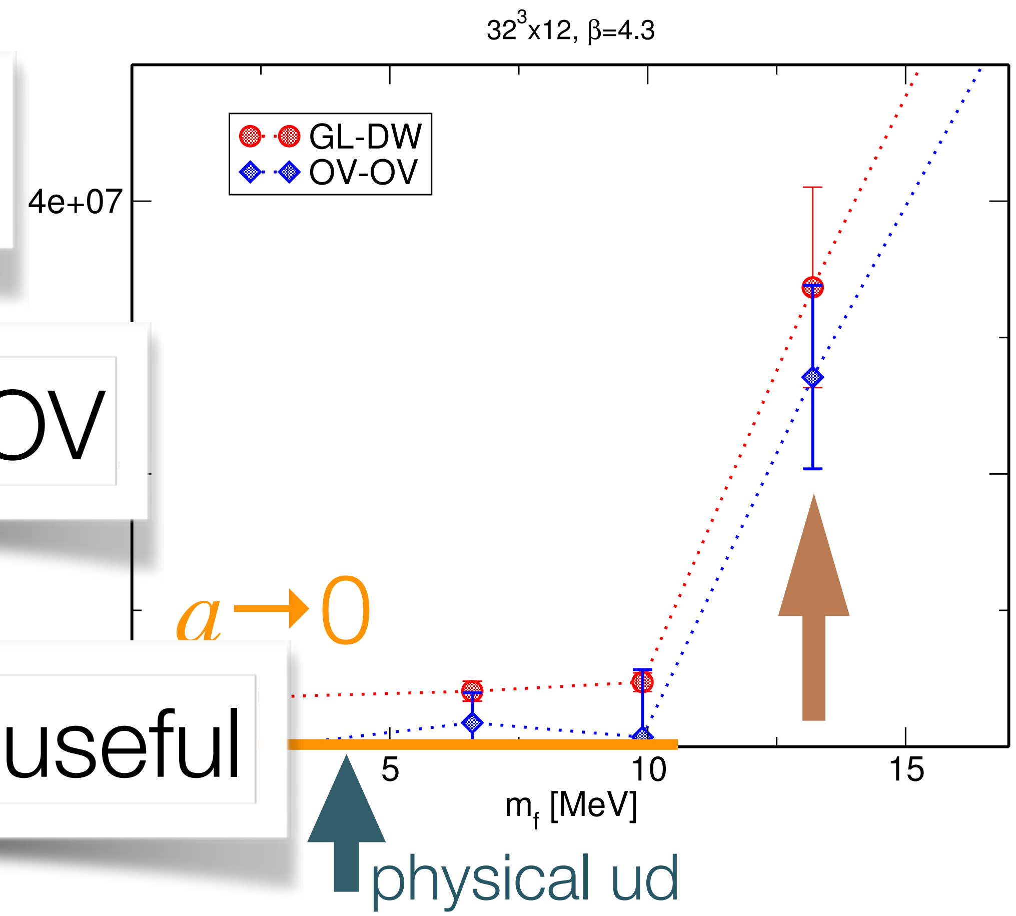


GL-DW is precise, maybe useful

suggestin

Next step: Volume Study

1: χ_t is v ... m ≲ 10 MeV

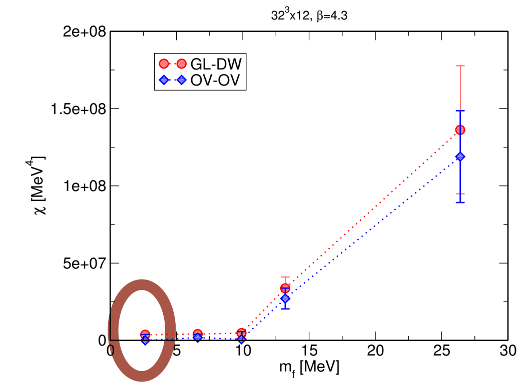
symm.)

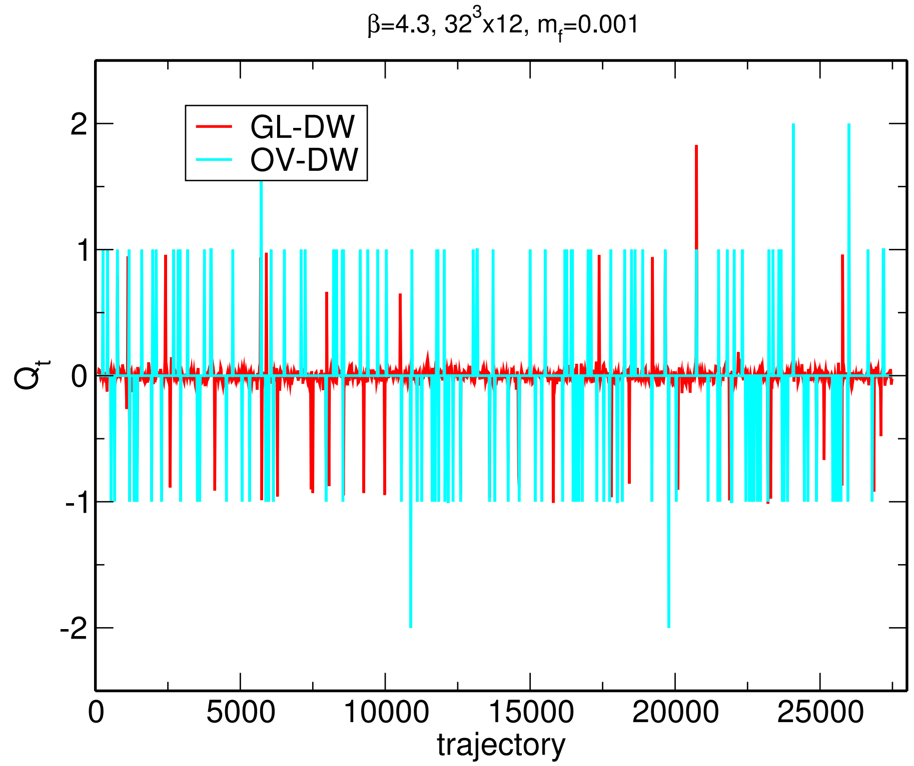
In addition to 32^3, **24^3** & **48^3** are studied

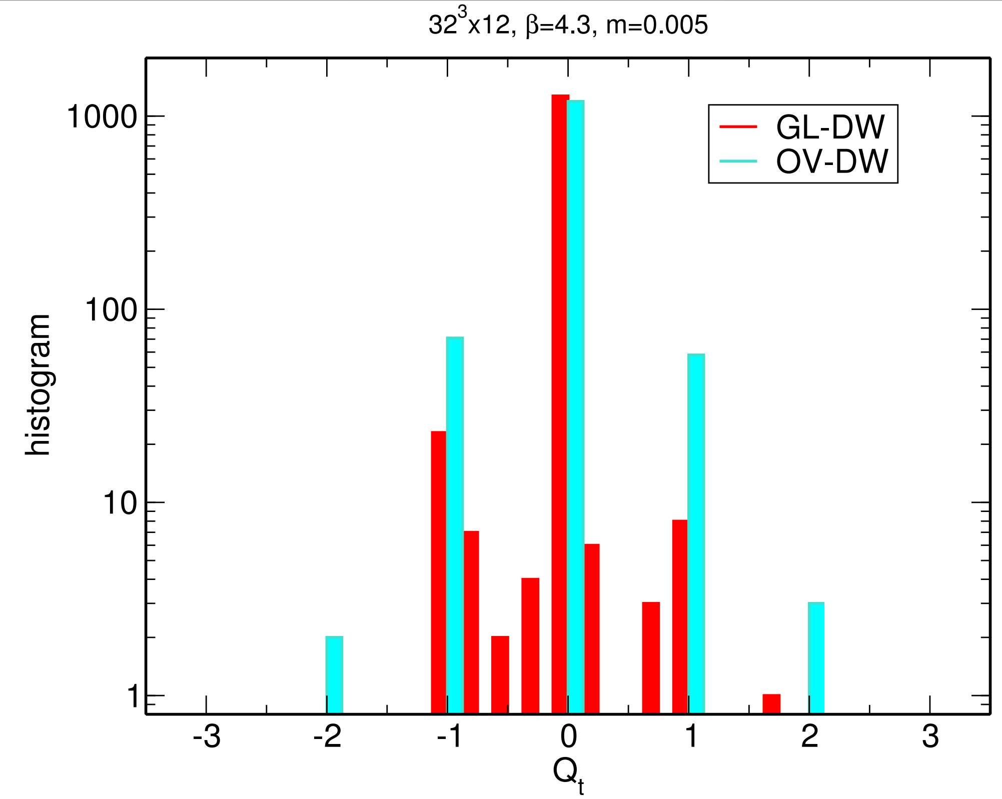
2:

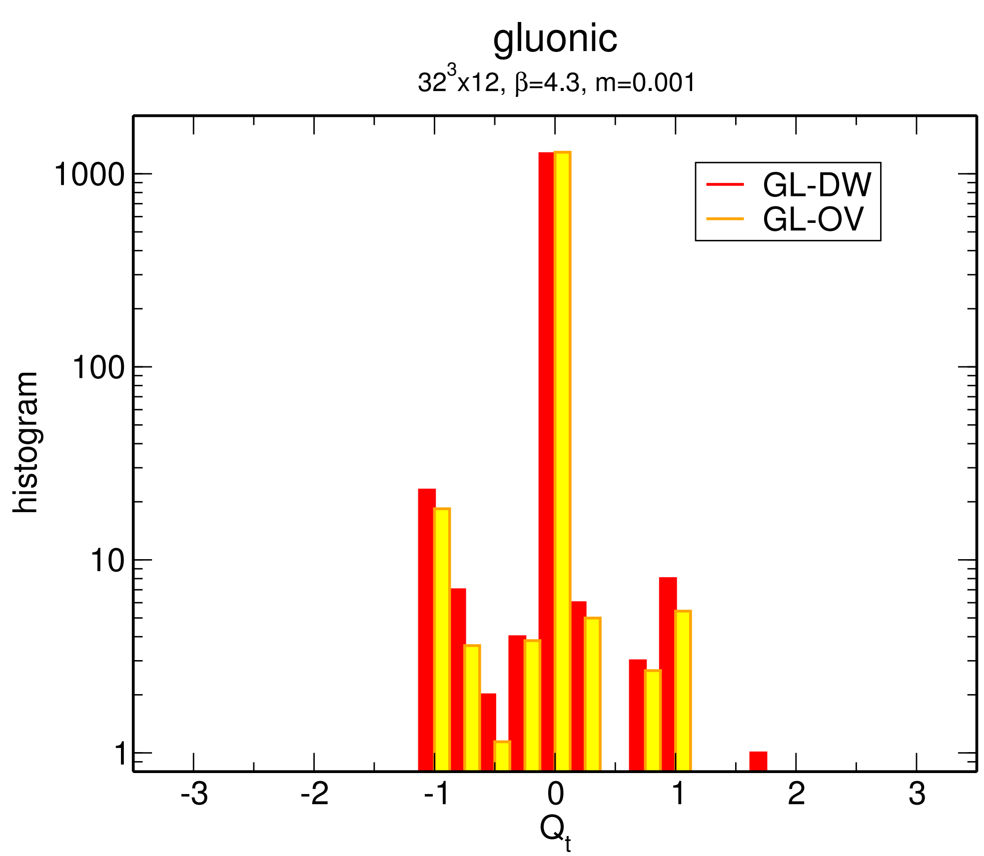
- physical ud mass point: m≃4 MeV

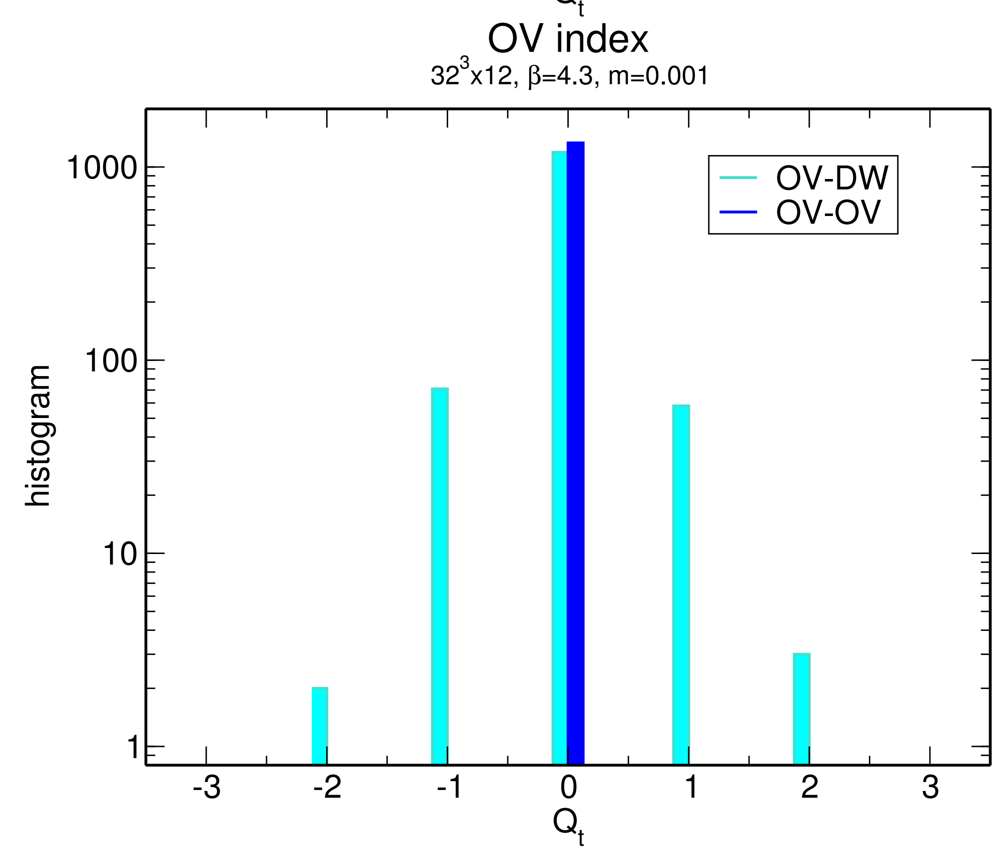
32^3 m=2.6 MeV history and histogram



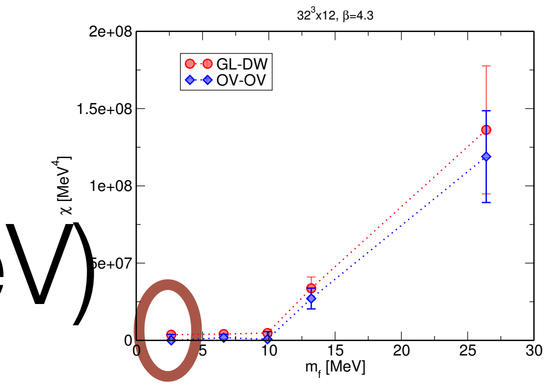


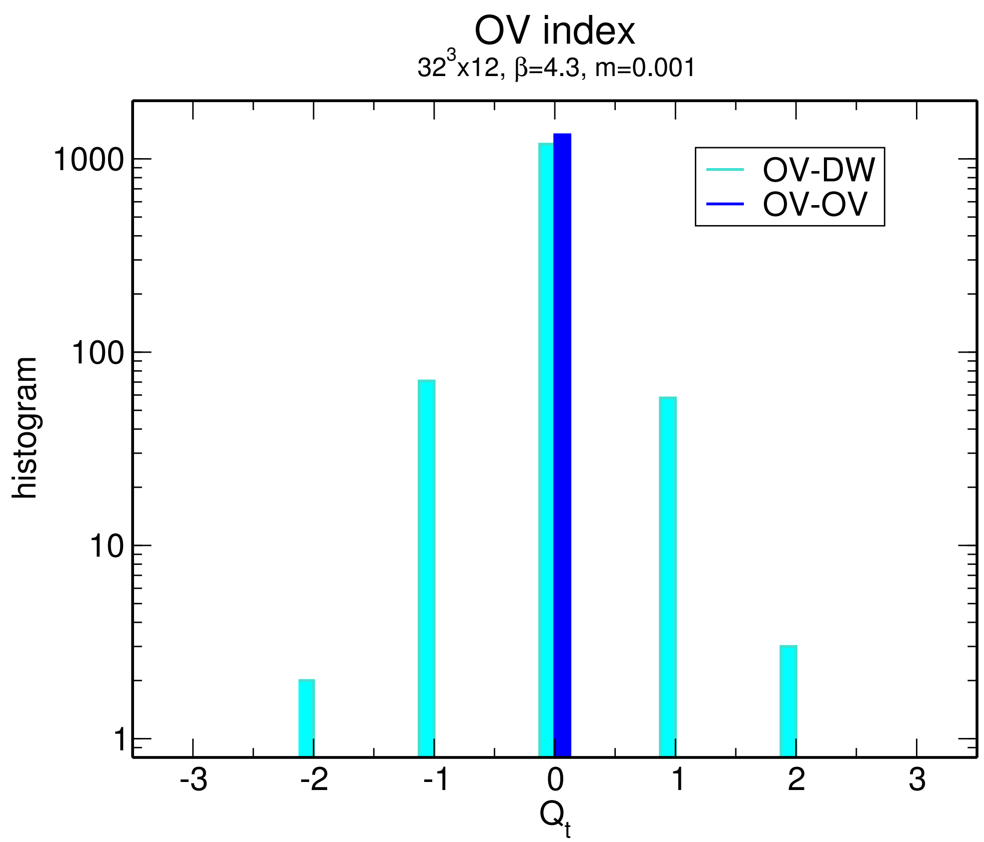






resolution of susceptibility (ex: m=2.6 MeV)





Effective number of statistics

- decreases with reweighting
- $N_{eff}=N_{conf} <R>/R_{max}$
- $N_{conf}=1326 \rightarrow N_{eff} = 32$

null measurement of topological excitation after reweighting

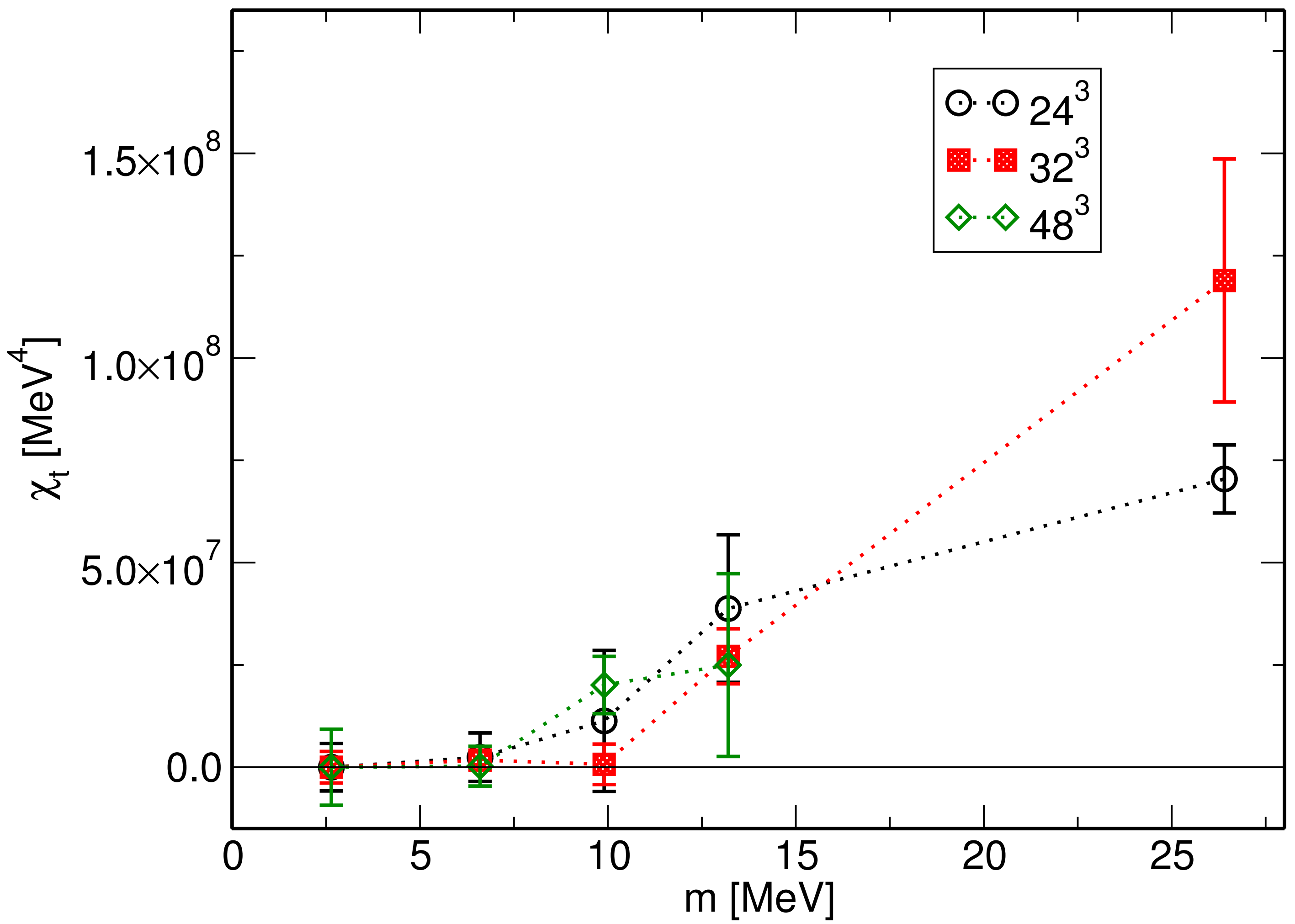
- does not readily mean $\chi_t=0$:

  (this case $<Q^2>=4(4) \times 10^{-6} \leftrightarrow 6(3) \times 10^{-3}$ @m=13MeV)

- there must be a resolution of χ_t under given statistics
  - [resolution of $<Q^2>$] = $1/N_{eff}$
- shall take the “statistical error” of $<Q^2> = \max(\Delta<Q^2>, 1/N_{eff})$

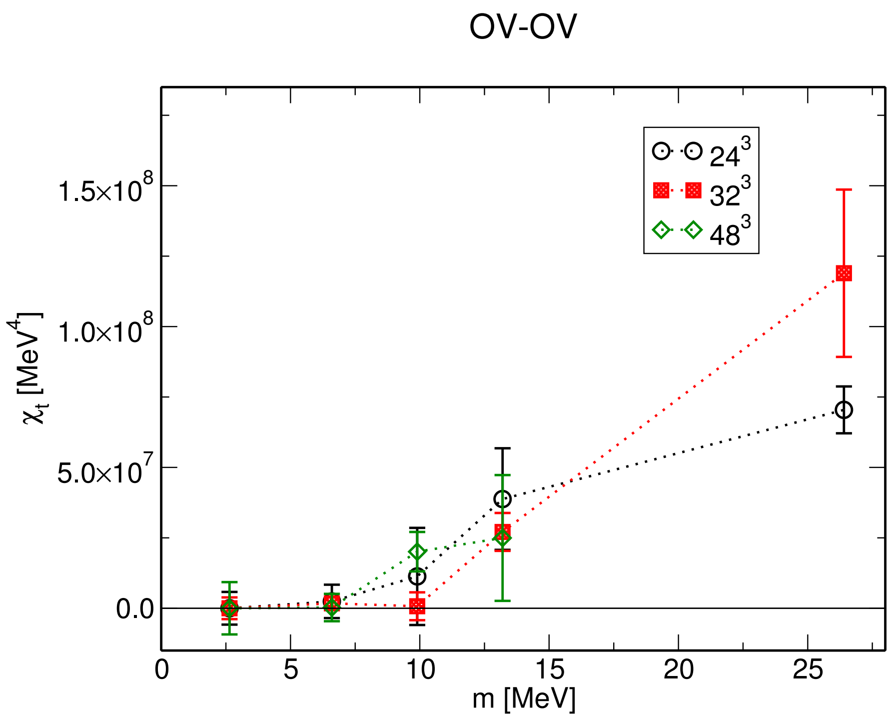
Results of $\chi_t(m)$ at T=220 MeV; multiple volume

OV-OV



- Statistics in trajectory
  ~30k, 30k, 10k

Results of $\chi_t(m)$ at T=220 MeV; multiple volume



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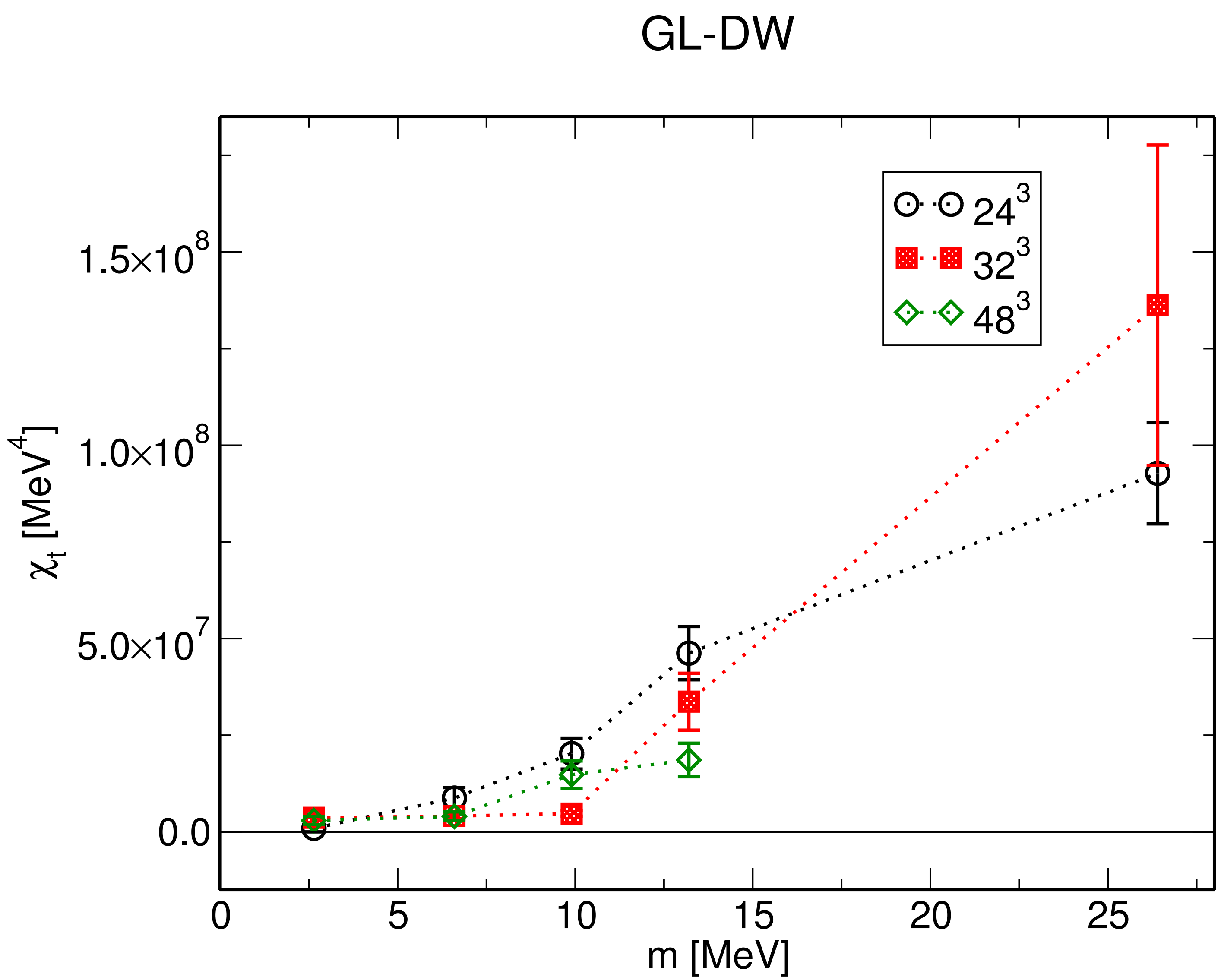

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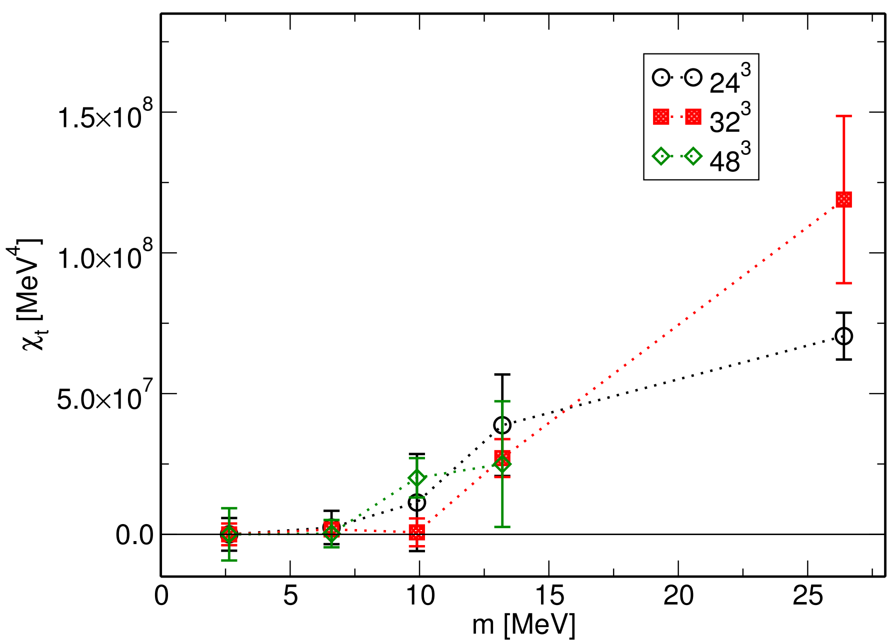
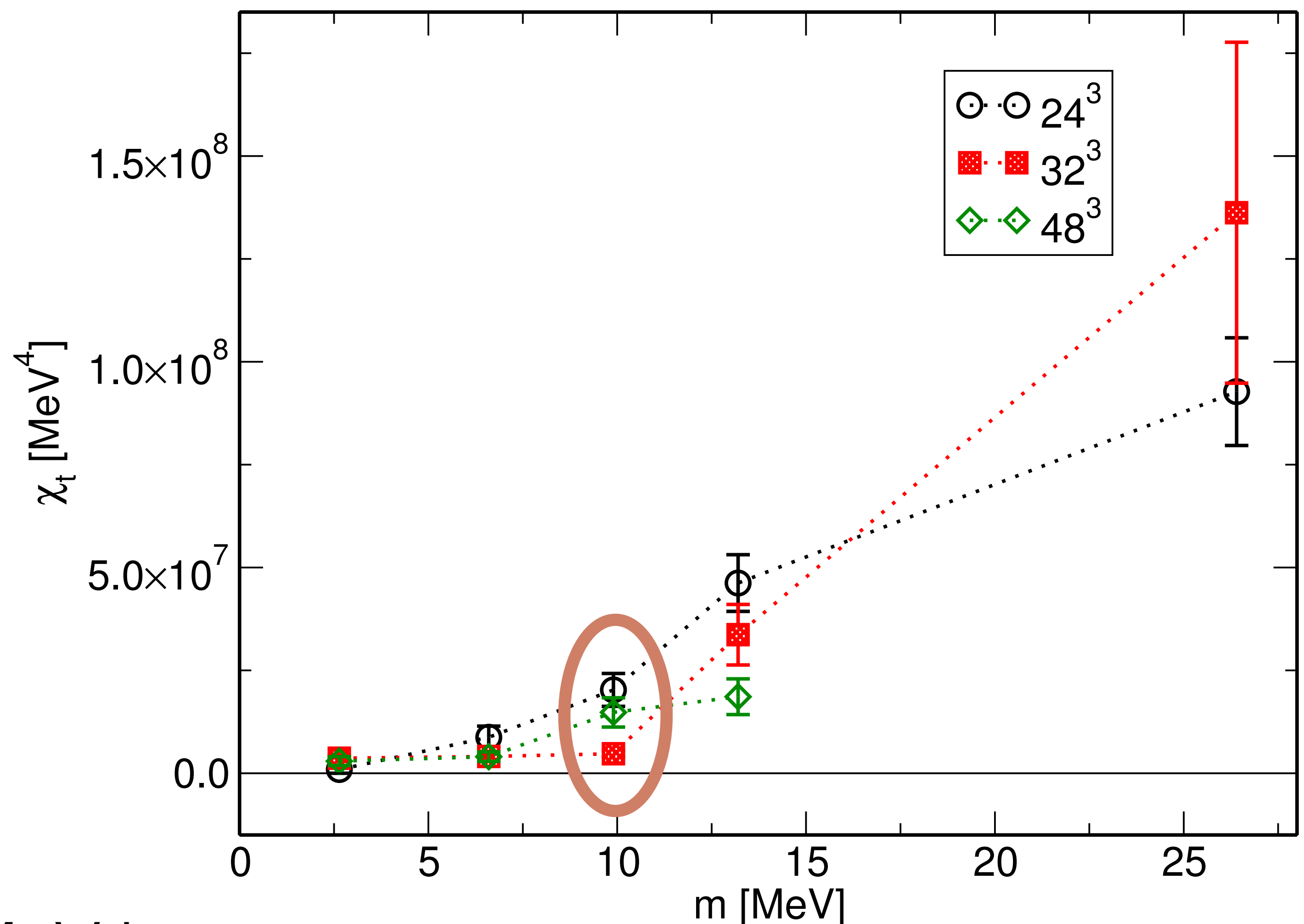


GL-DW

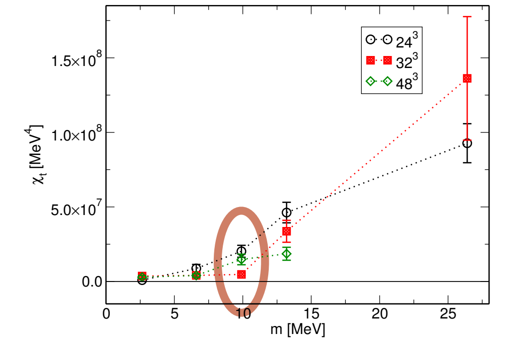


Statistics in trajectory

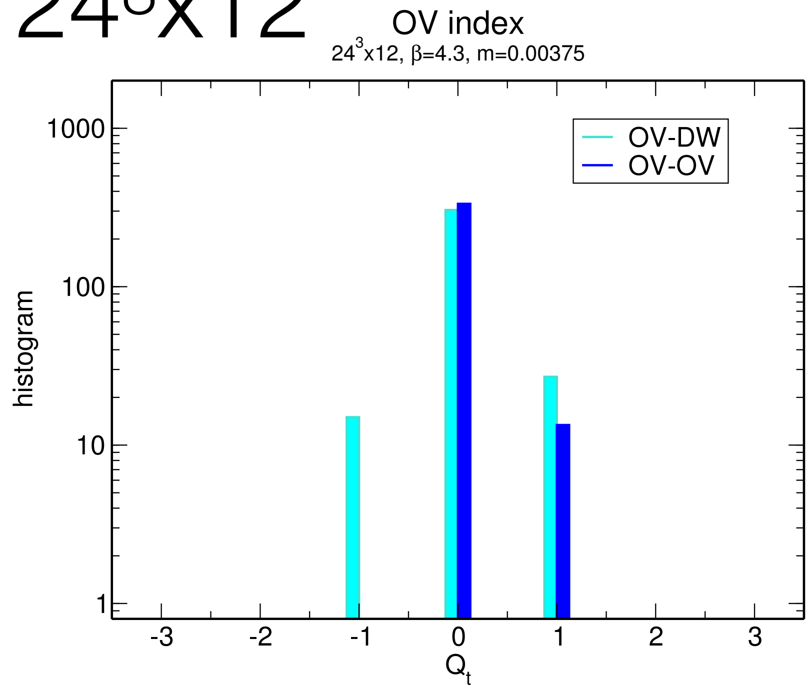
~30k, 30k, 10k

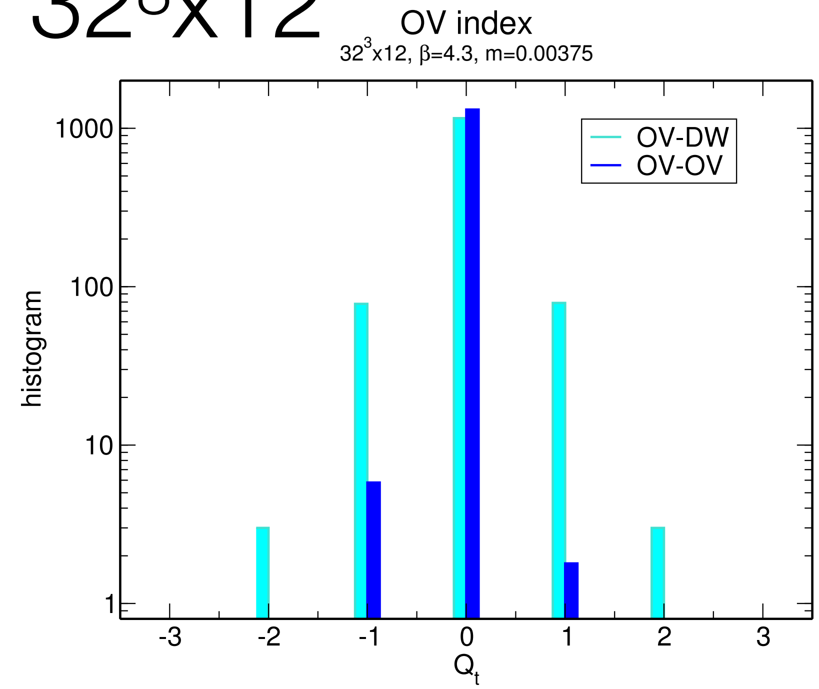
- V dependence at m=10 MeV is strange
  - non-monotonic: cannot take thermodynamic limit
  - important region, where a phase boundary was suggested w/ 32^3
- Let's look at the histogram of Q

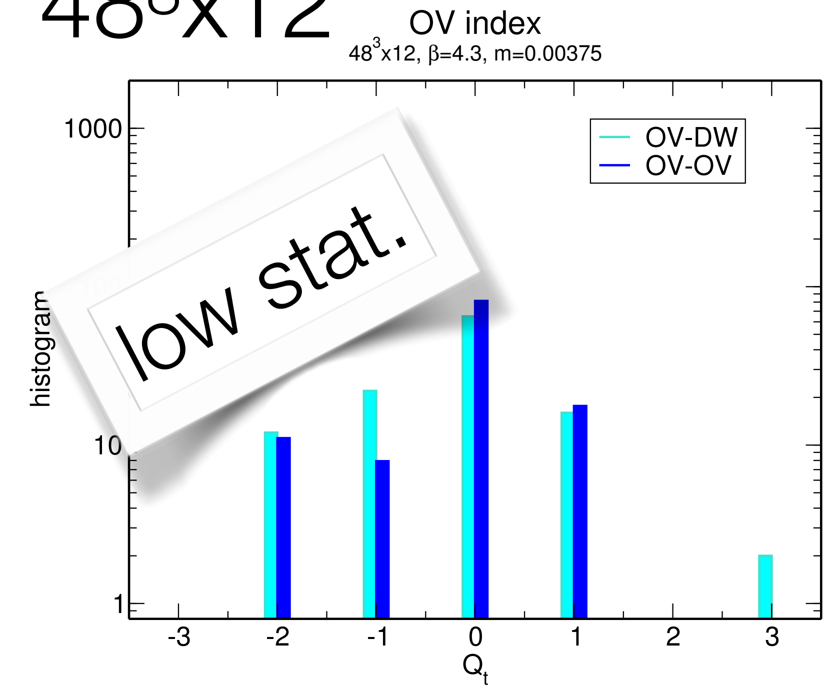
summary of histogram: T=220 MeV, m=10 MeV

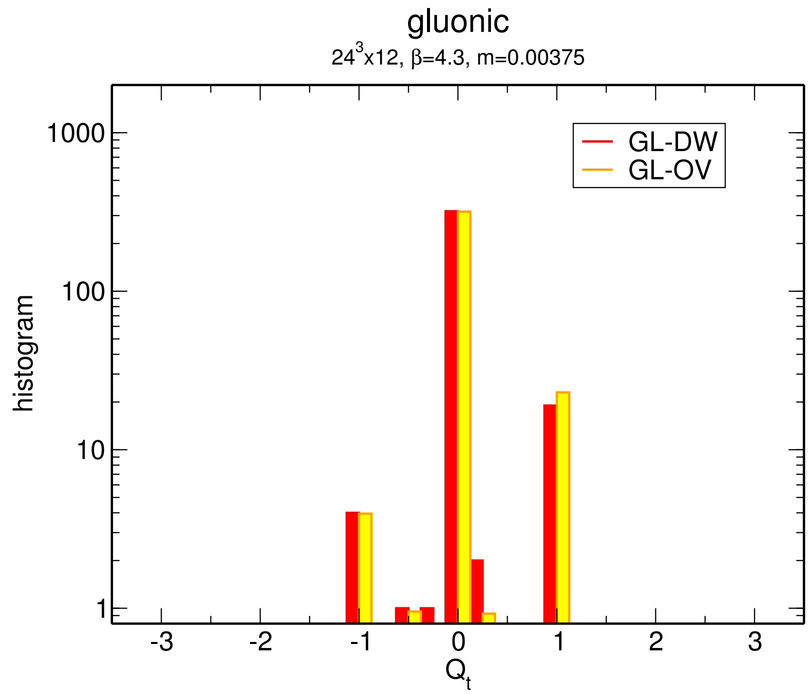


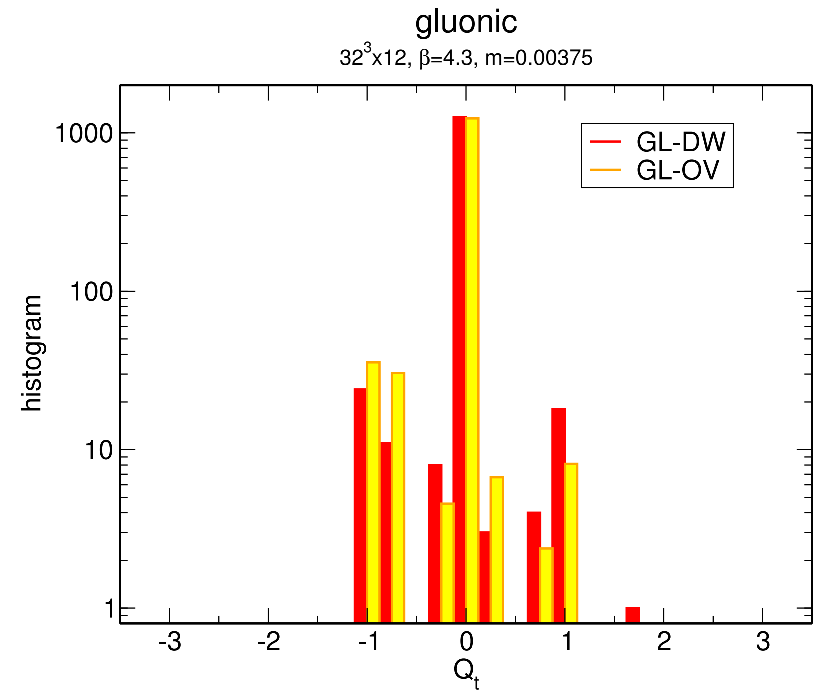
24^3x12	32^3x12	48^3x12

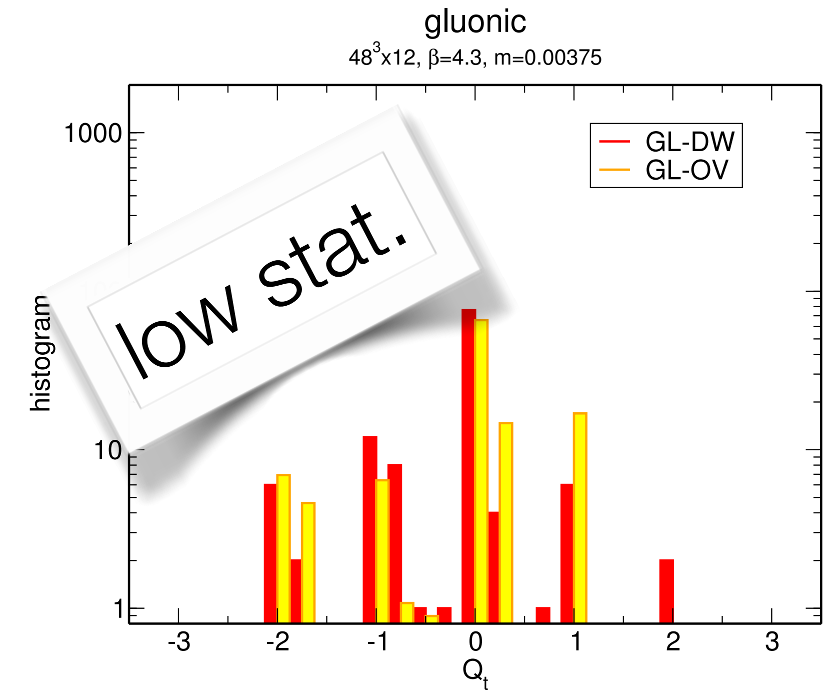






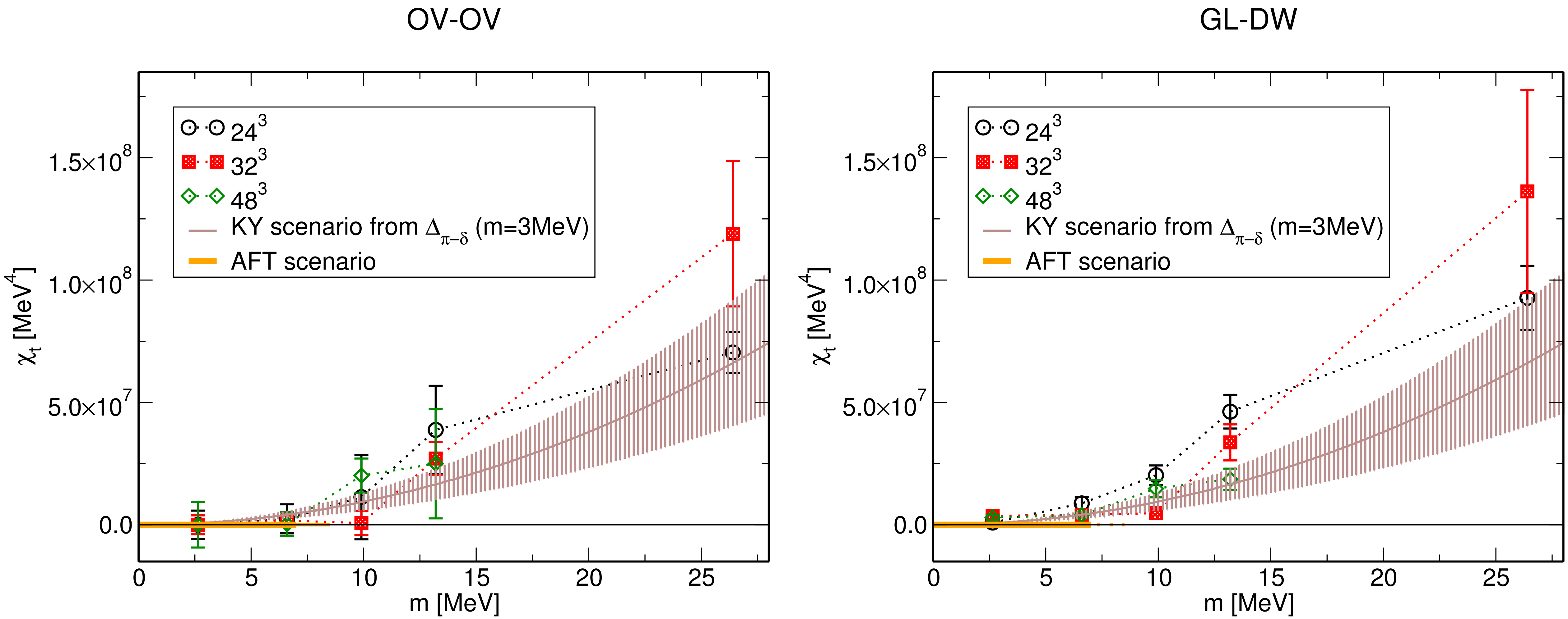






	24^3x12	32^3x12	48^3x12
# trajectory:	~30k	~30k	~10k
sample rate:	100	20	100

competing scenarios with multiple volumes for χ_t given $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=220 MeV



- AFK scenario: $\chi_t = 0$ for $0<m<m_c$
- KY scenario: $\chi_t = 2 f_A m^2$
- There are no strong tensions
- Neither scenario is excluded

Kanazawa-Yamamoto

- assume $f_A \neq 0$ (breaking param)
- expansing free energy in m

- discussing
  - finite m and V effect
  - in each topological sector

$U(1)_A$

U(1) axial

$$\partial_\mu J_5^\mu = \frac{N_f}{32\pi^2} F\tilde{F}$$

- violated by quantum anomaly

$$\langle \partial_\mu J_5^\mu(x) \cdot O(0) \rangle = \frac{N_f}{32\pi^2} \langle F\tilde{F}(x) \cdot O(0) \rangle$$

up to contact terms

- at T=0, responsible for η' mass
    - non-trivial topology of gauge field

- at high T, this Ward-Takahashi identity is still valid

- however, if configurations that contribute to RHS is suppressed………

➡ the symmetry effectively recovers

◉ here N_f=2 (including N_f=2+1 with "2" driven to chiral limit)

Why bother ?

- **Because it is unsettled problem !**
- fate of $U(1)_A$ - analytic
  - Gross-Pisarski-Yaffe (1981) restores in high temperature limit
    - Dilute instanton gas
  - Cohen (1996)
    - measure zero instanton effect → restores
  - Lee-Hatsuda (1996)
    - zero mode does contributes → broken
  - Aoki-Fukaya-Tanigchi (2012)
    - QCD analysis (overlap) → restores w/ assumption (lattice)
  - Kanazawa-Yamamoto (2015)
    - EFT case study how restore / break
  - Azcoiti (2017)
    - case study how restore / break

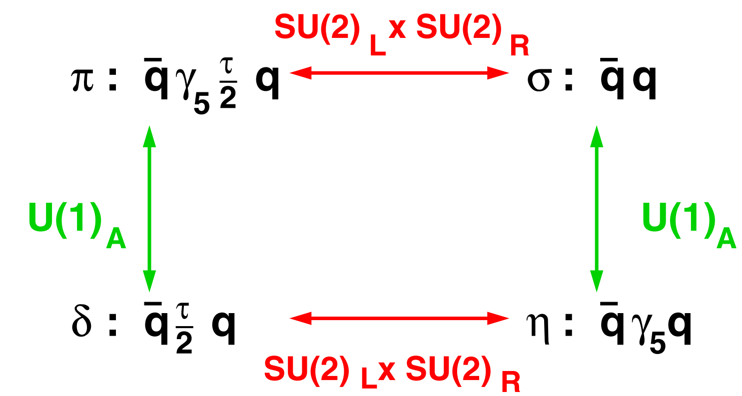
Why bother ?

- **Because it is unsettled problem !**
- fate of $U(1)_A$ lattice
  - HotQCD (DW, 2012) broken
  - JLQCD (topology fixed overlap, 2013) restores
  - TWQCD (optimal DW, 2013) restores ?
  - LLNL/RBC (DW, 2014) broken
  - HotQCD (DW, 2014) broken
  - Dick et al. (overlap on HISQ, 2015) broken
  - Brandt et al. (O(a) improved Wilson 2016) restores
  - JLQCD (reweighted overlap from DW, 2016) restores
  - JLQCD (current: see Suzuki et al Lattice 2017) restores
  - Ishikawa et al (Wilson, 2017) at least Z_4 restores

$U(1)_A$ restoration or not

- need to make sure if not comparing apples and oranges…
- key points
  - systematics effects of lattice discretization under control ?
    - finite V, a, m…
  - ud chiral limit of
    - N_f=2 QCD or
    - N_f=2+1 QCD → strange quark mass effect !
  - discussing m_{ud}→0 or just around physical ud mass
  - discussing $X = 0$? or $X \simeq 0$?

a U(1)A order parameter



- symmetry in switching flavor non-singlet pseudoscalar and scalar

- order parameter:

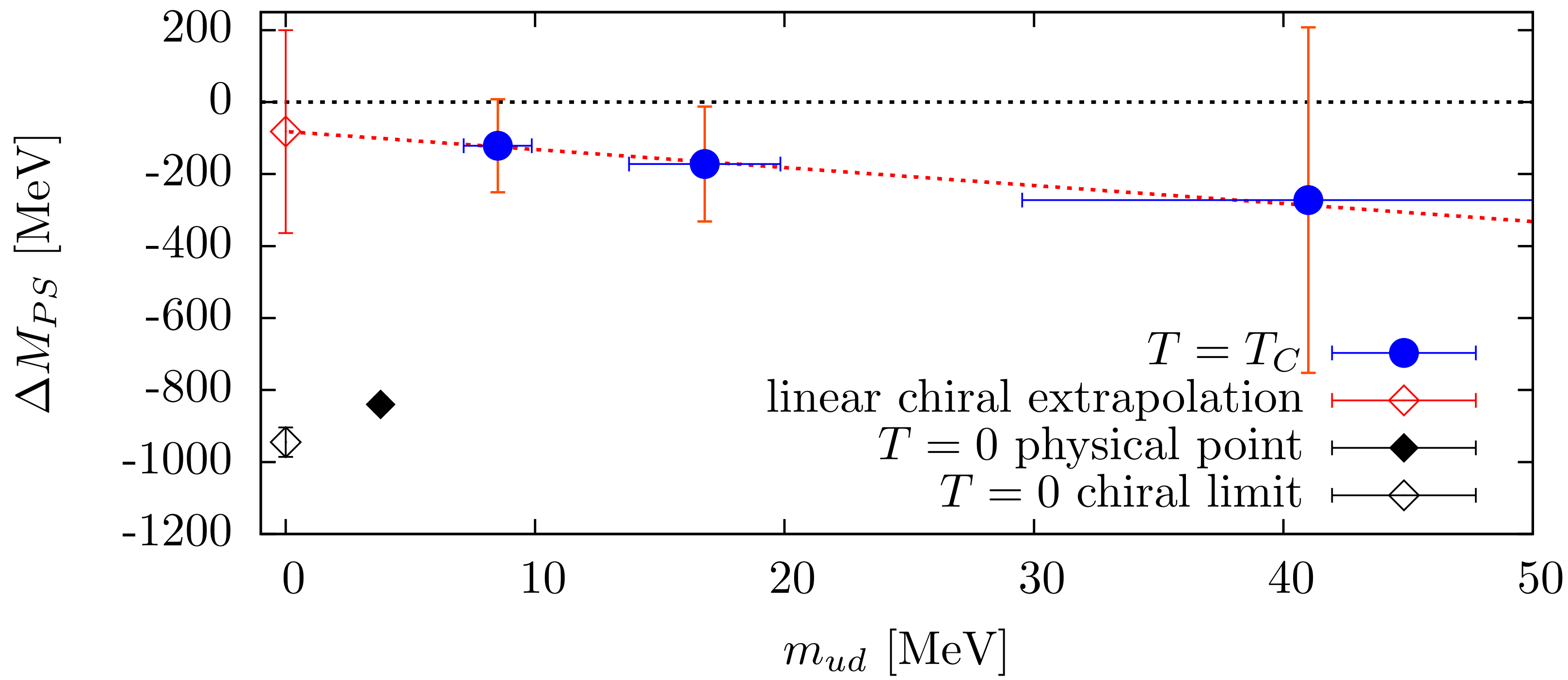
$$\Delta_{\pi-\delta} = \int d^4x [\langle \pi^a(x)\pi^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle],$$

➡ 0 for U(1)A restoration

- as a result, screening masses for these channel will degenerate
  - not a sufficient condition for U(1)A restoration

screening mass from O(a) improved Wilson f N_f=2

- mass difference between π and δ



Brandt et al JHEP [1608.06882]

- $N_t = 1/(aT) = 16$ - quite fine lattice
- $T=T_c$ - **on top of transition temperature**

only one existing study for N_f=2

- $\Delta M_{PS} = 0$ (with a sizable error) → consistent with $U(1)_A$ restoration

relation with Dirac eigenmode spectrum ρ(λ)

- chiral condensate : order parameter of $SU(2)_A$: Banks-Casher rel.

$$-\langle \bar{q} q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

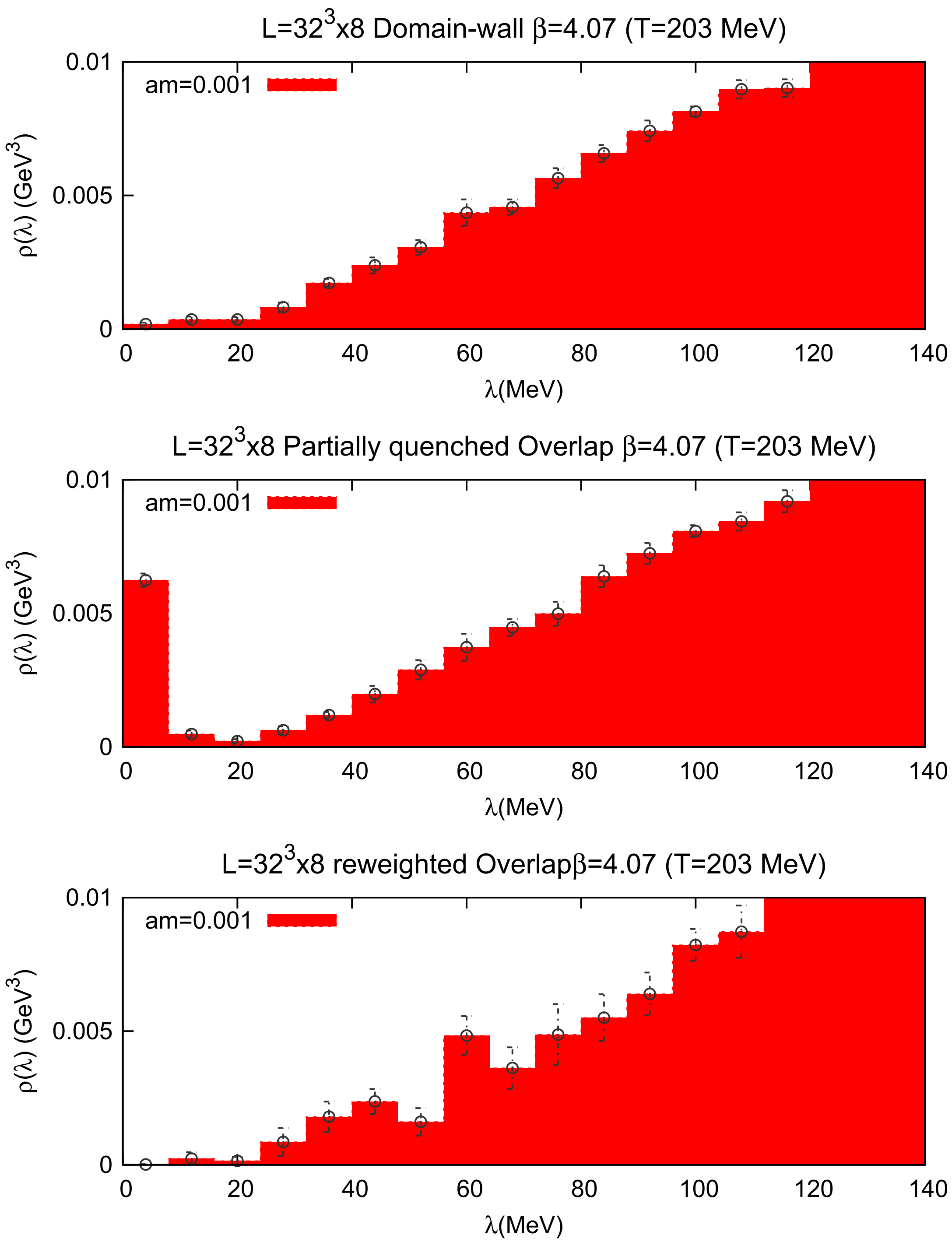
- $U(1)_A$:

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \to\sim \rho'(0)$$

very roughly speaking

  - very sensitive to the spectrum near λ=0
  - overlap fermion, able to distinguish zero/nonzero modes, is ideal

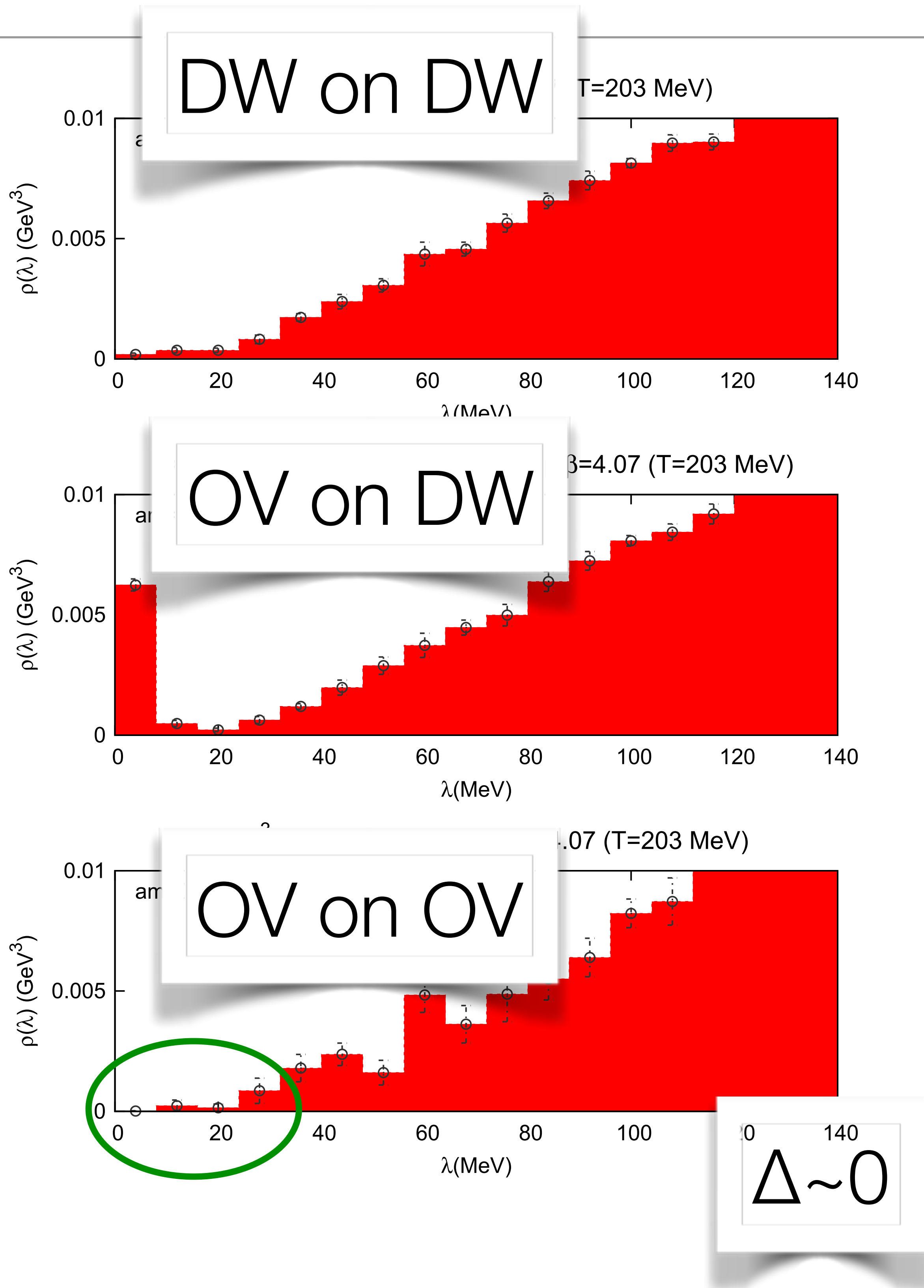
JLQCD 16: H_{OV}, H_{DW} spectrum: above T_c N_f=2



- DW: Domain wall fermion sea
- OV: Overlap valence
    - exact "chiral symmetry"
- reweighting to OV

[JLQCD 2016 Tomiya et al]

JLQCD 16: H_{OV}, H_{DW} spectrum: above T_c N_f=2



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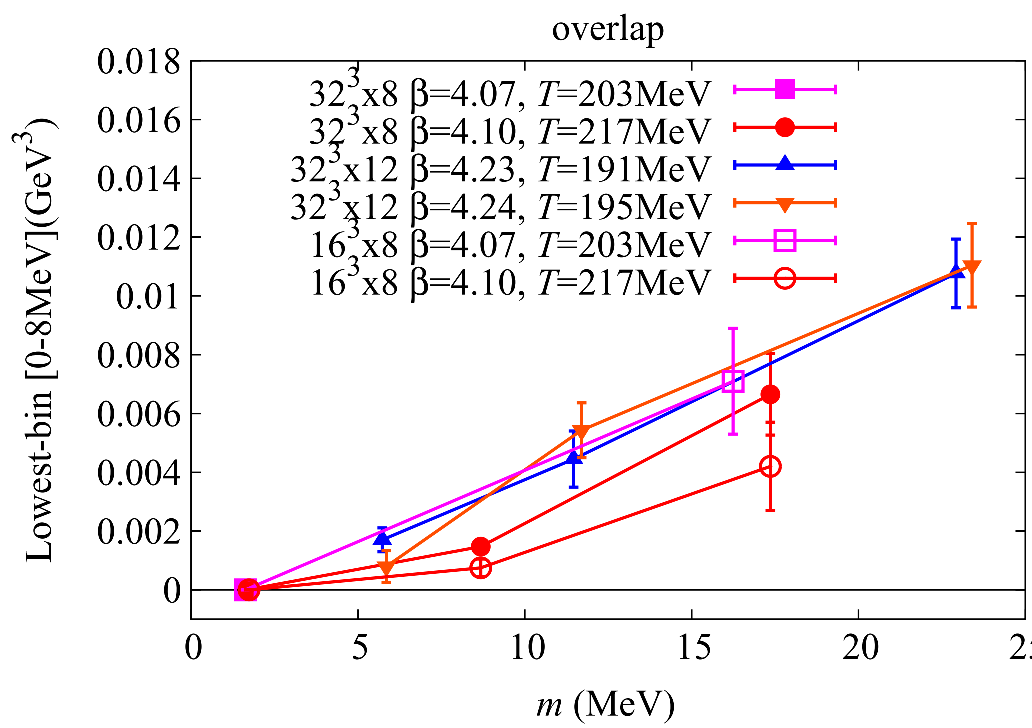
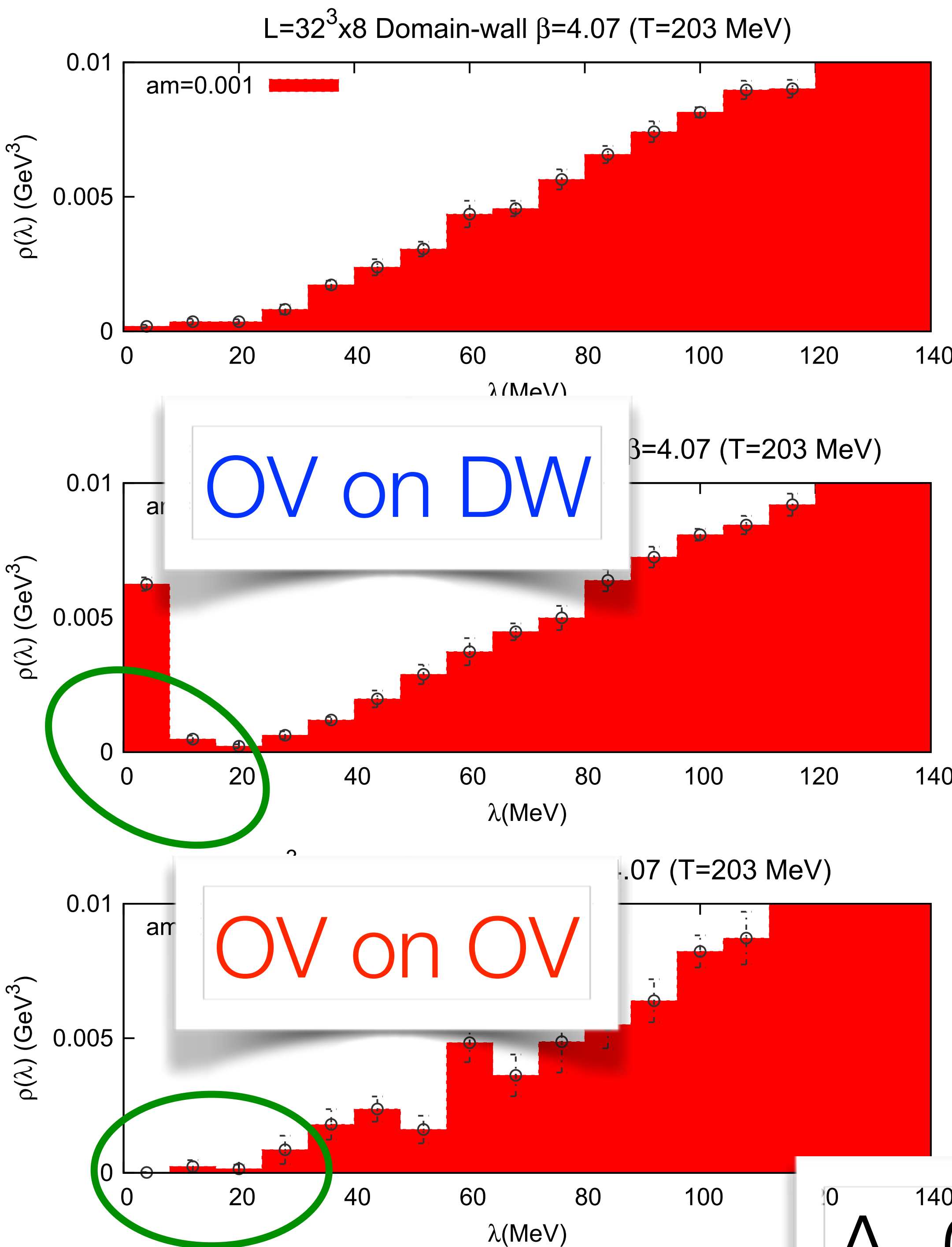


Lowest bin→0

consistent with $SU_A(2)$ restoration

[JLQCD 2016 Tomiya et al]

Comparison: unitary <-> partially quench



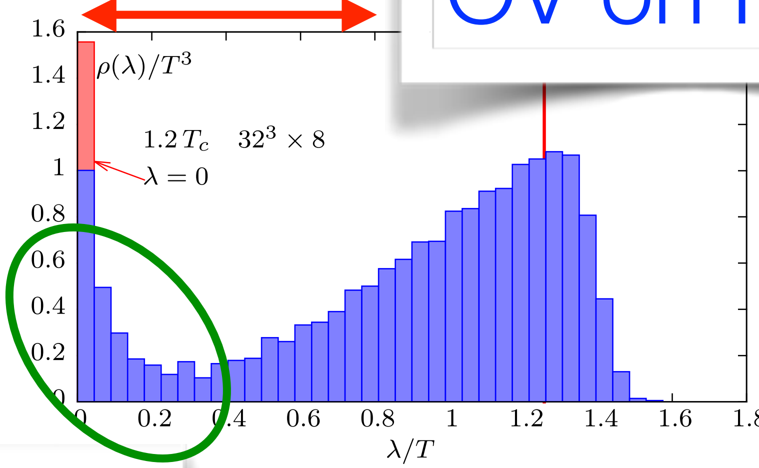
OV on DW

OV on OV

Δ~0

range of JLQCD

OV on HISQ

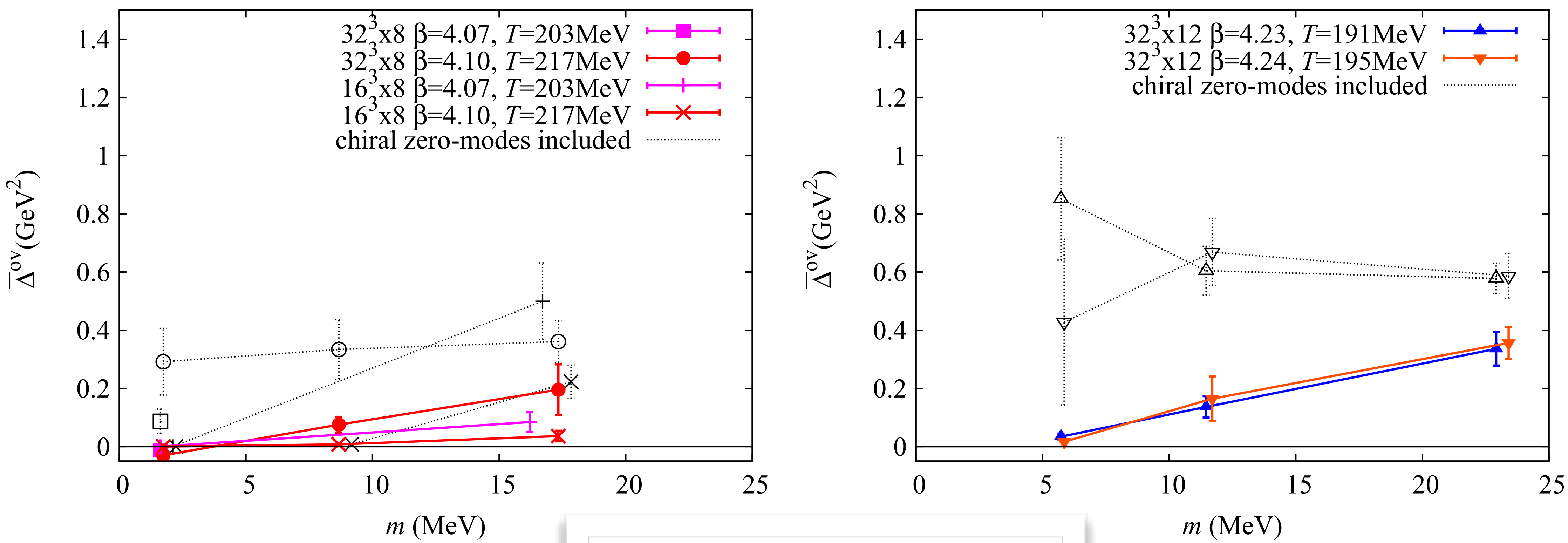


Δ>0

Dick et al PRD [1502.06190]

Partially quench effect needs to be investigated

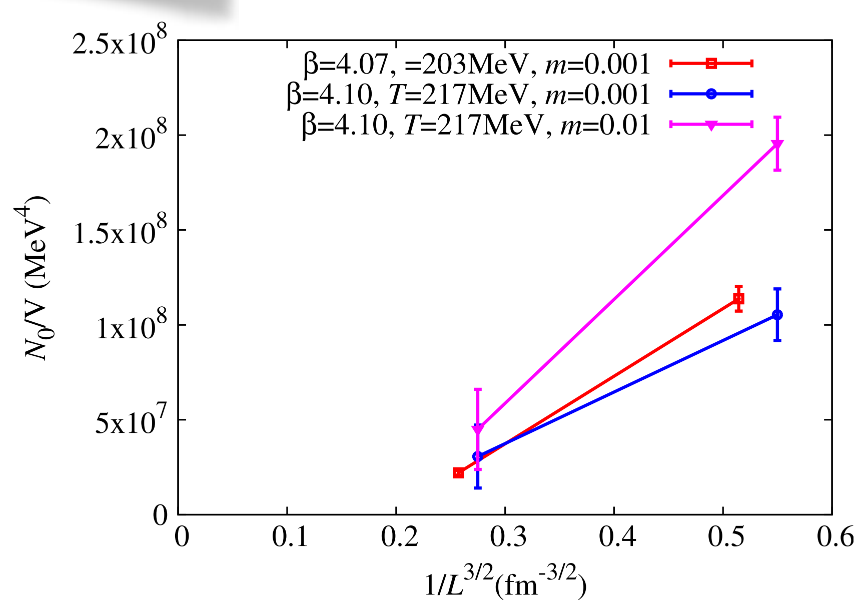
JLQCD 16: $U_A(1)$ susceptibility: T=190-220 MeV



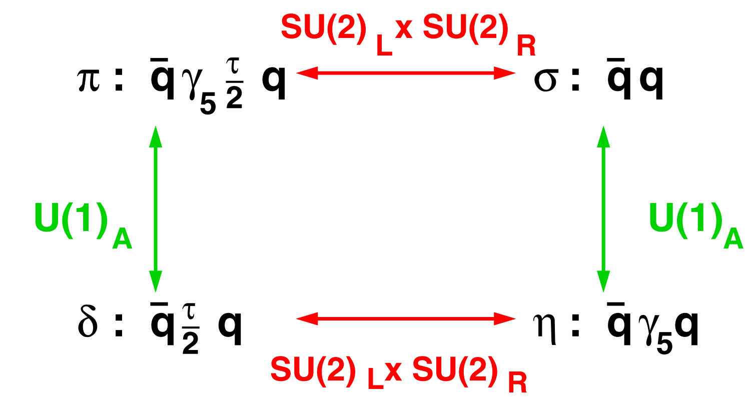
seemingly Δ→0

$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2}.$$

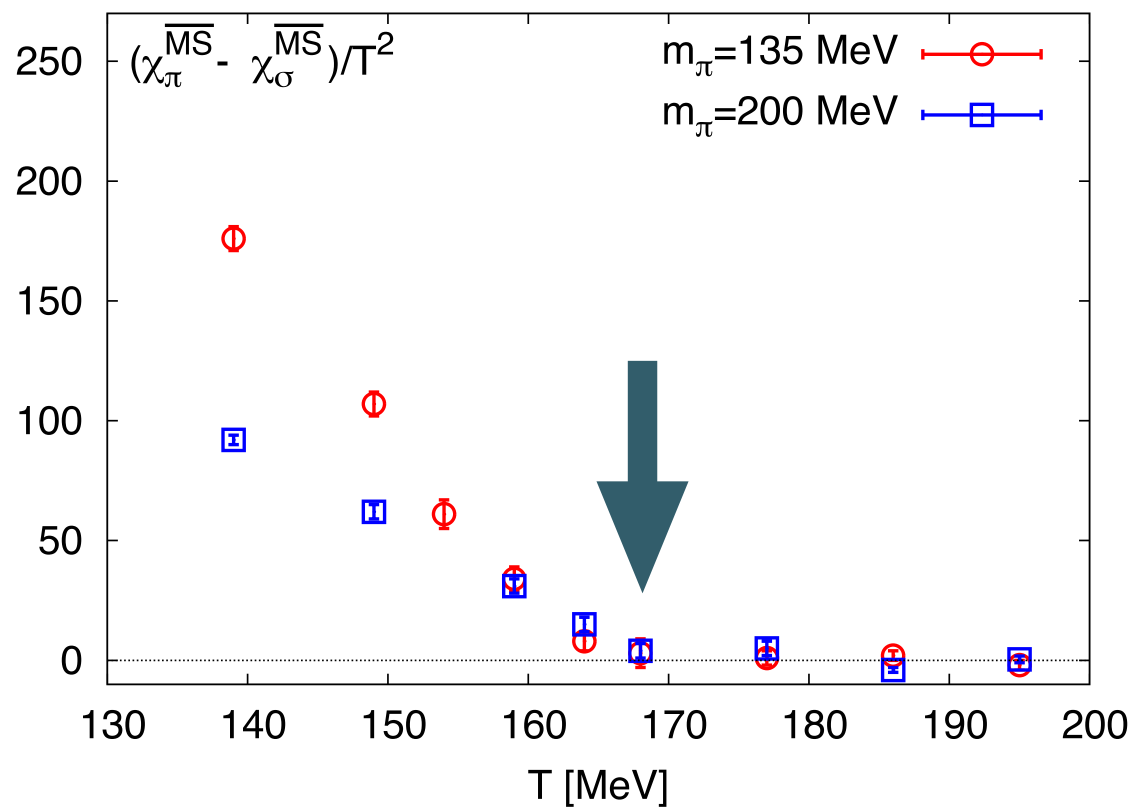
zero mode effect



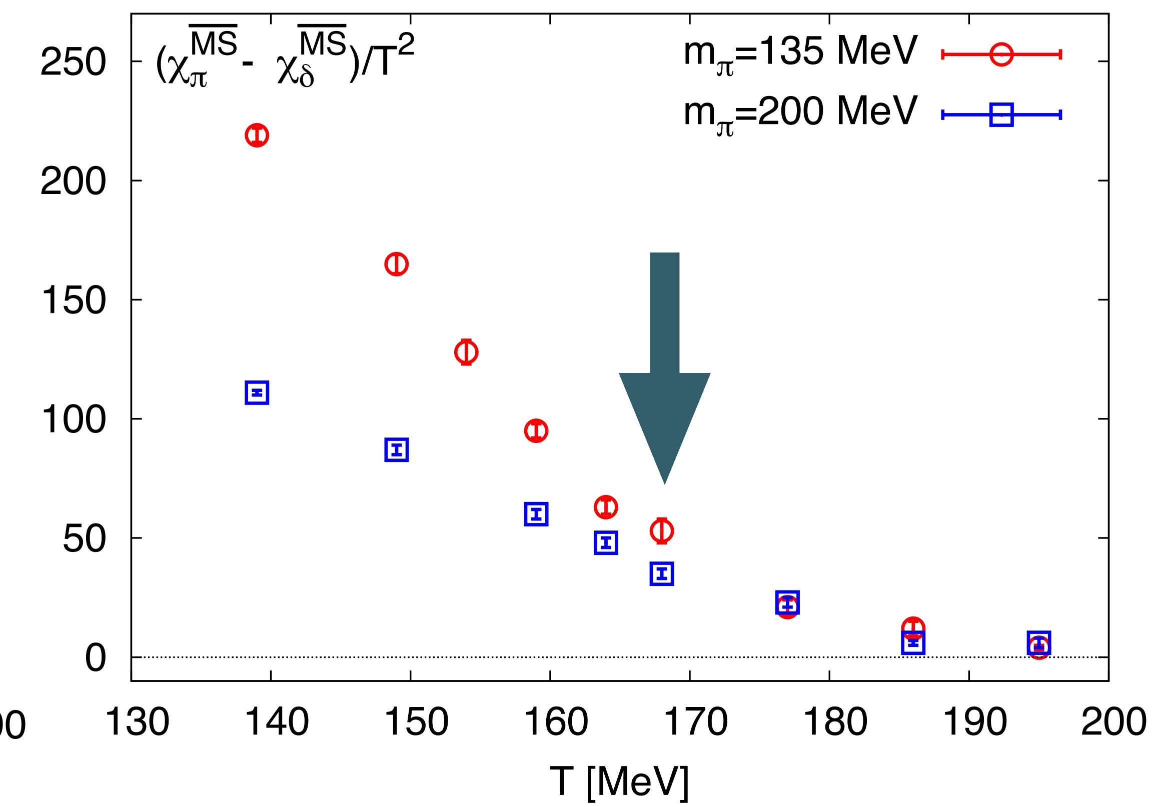
HotQCD 2014: DWF N_f=2+1



SU(2)A



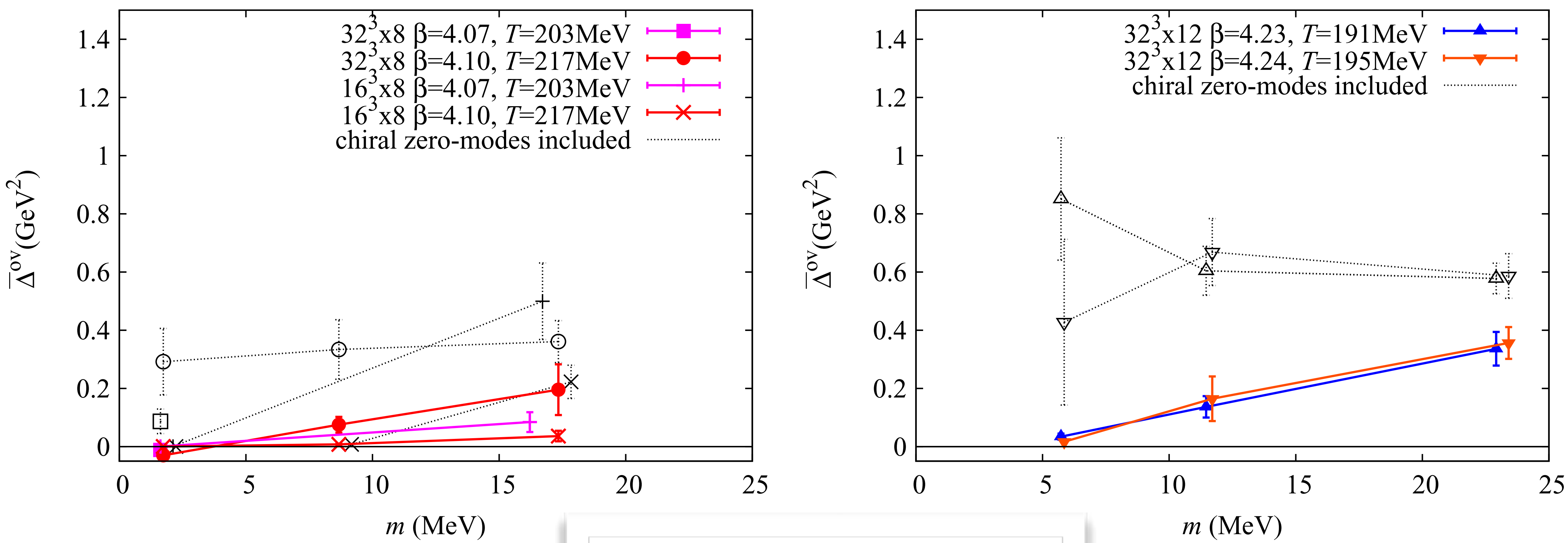
U(1)A



体積研究は？

[figures from Ding Lattice 2016]

JLQCD 16: $U_A(1)$ susceptibility



seemingly $\Delta \to 0$

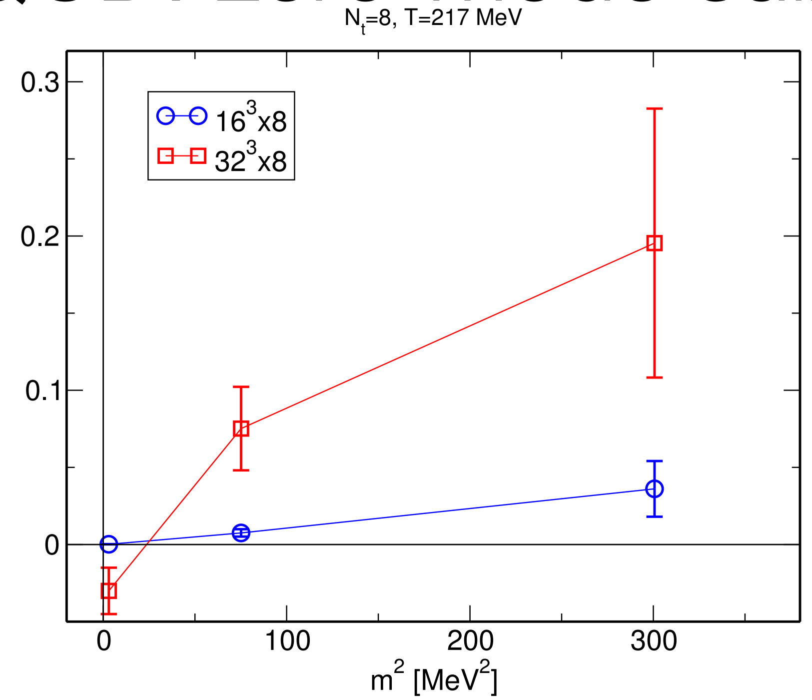
is this showing really, exactly $\Delta \to 0$?

update available

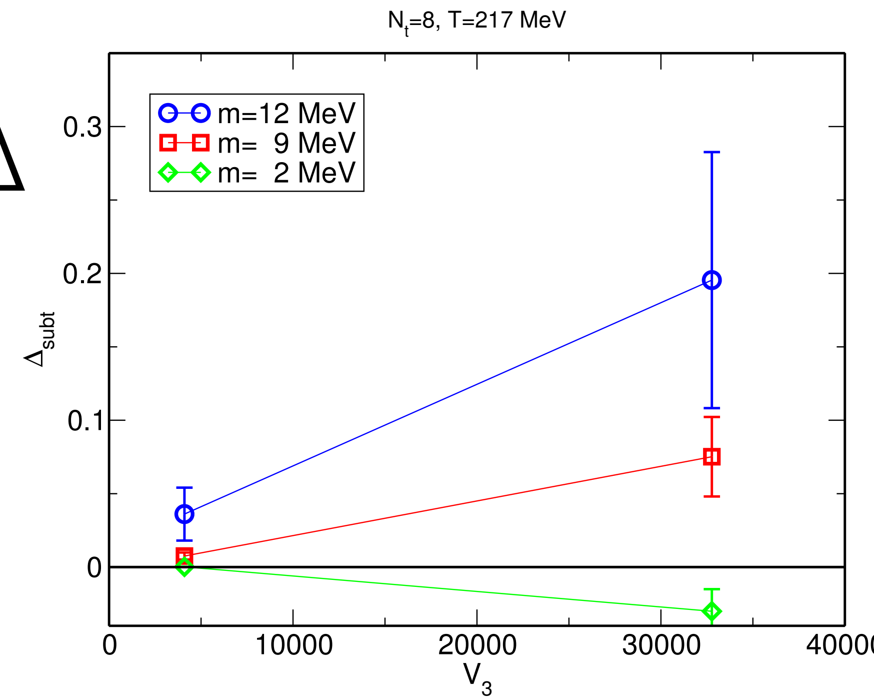
closer to continuum limit

チェック事項: V→∞ 極限を取れるか

JLQCD: zero-mode subtracted



Δ

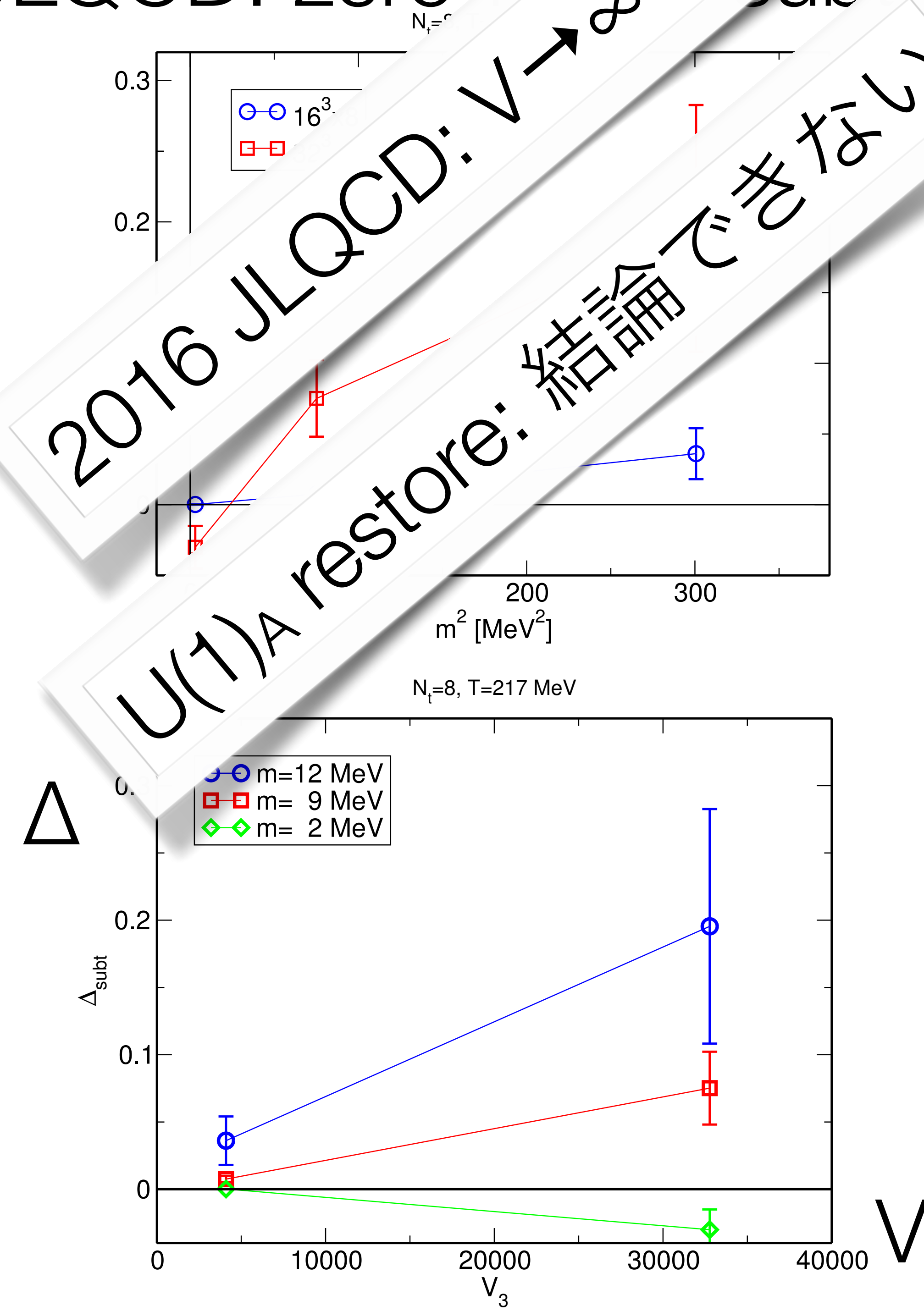


V

[JLQCD 2016 Tomiya et al]

チェック事項: V→∞ 極限を取れるか

JLQCD: zero-m… subtracted

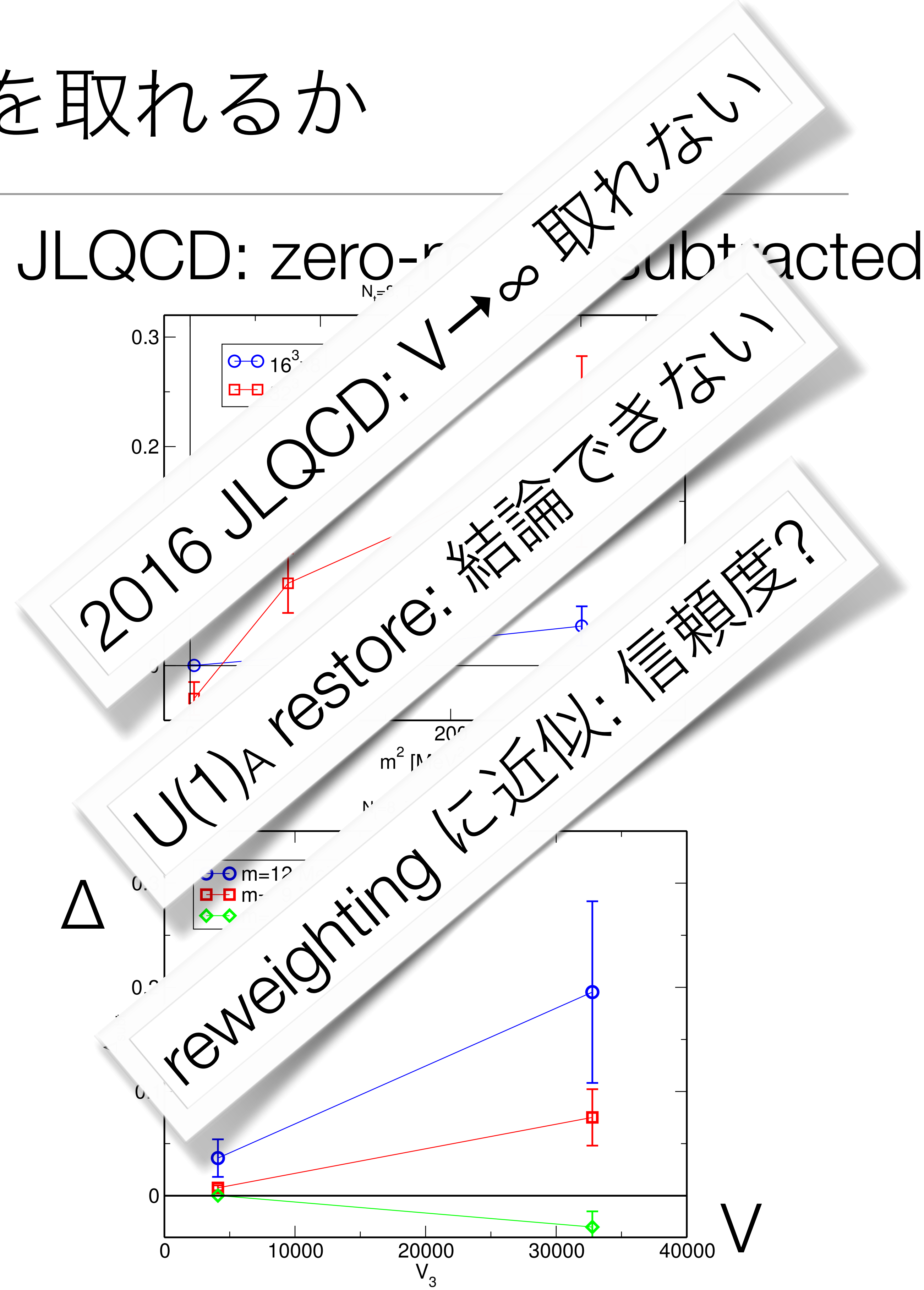


2016 JLQCD: V→∞ 取れない

U(1)$_A$ restore: 結論できない

[JLQCD 2016 Tomiya et al]

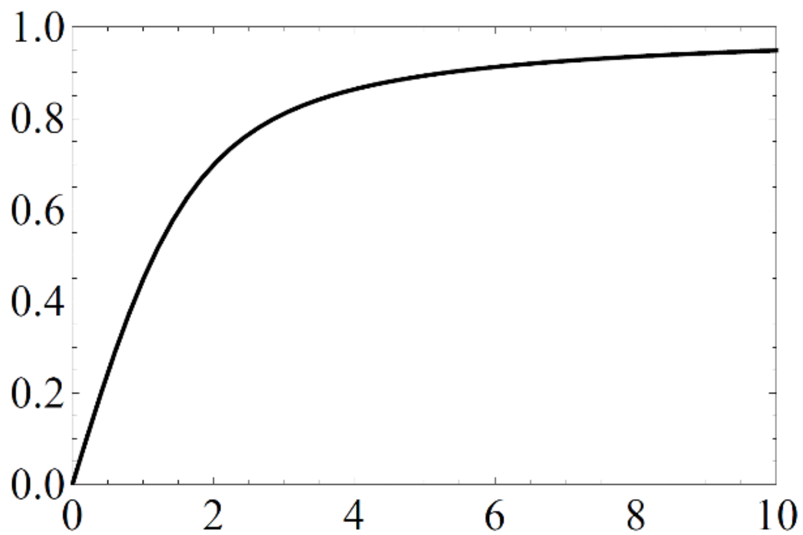
チェック事項: V→∞ 極限を取れるか



[JLQCD 2016 Tomiya et al]

チェック事項: V→∞ 極限を取れるか

KY: without |Q|>0 sector



$$x = 2V_4 f_A m^2$$

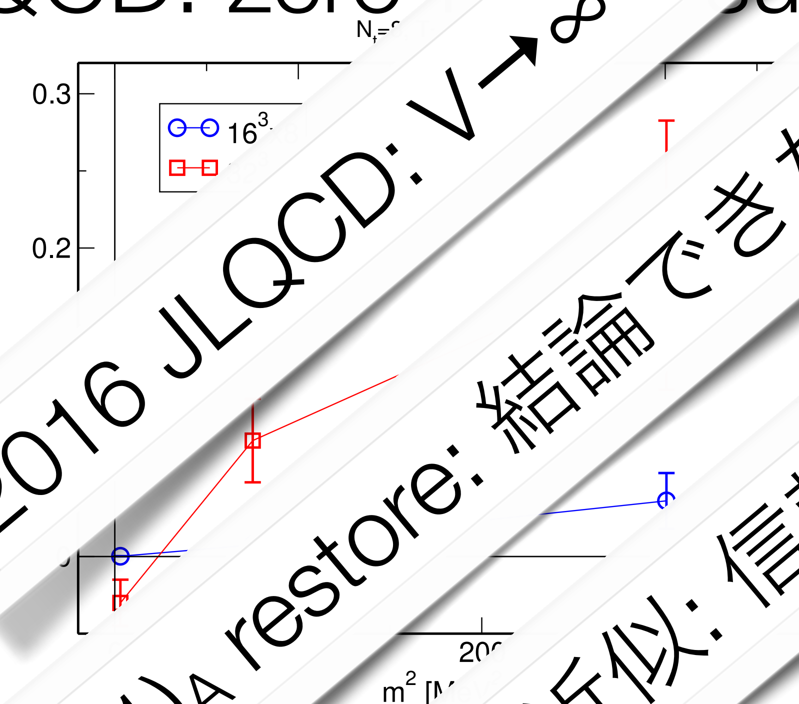
fix V: Δ→0 as m² for m→0
even for U(1)$_A$ br. case

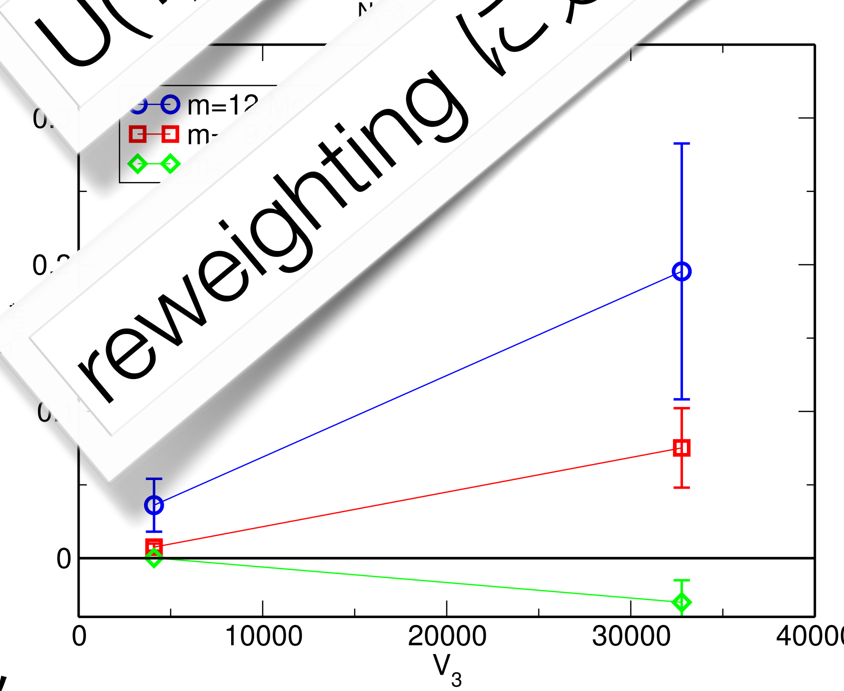
fix m : Δ∝V

必ずしも一致する必要ない

しかし x→0 の極限では似たような物

JLQCD: zero-mode subtracted





2016 JLQCD: V→∞ 取れない

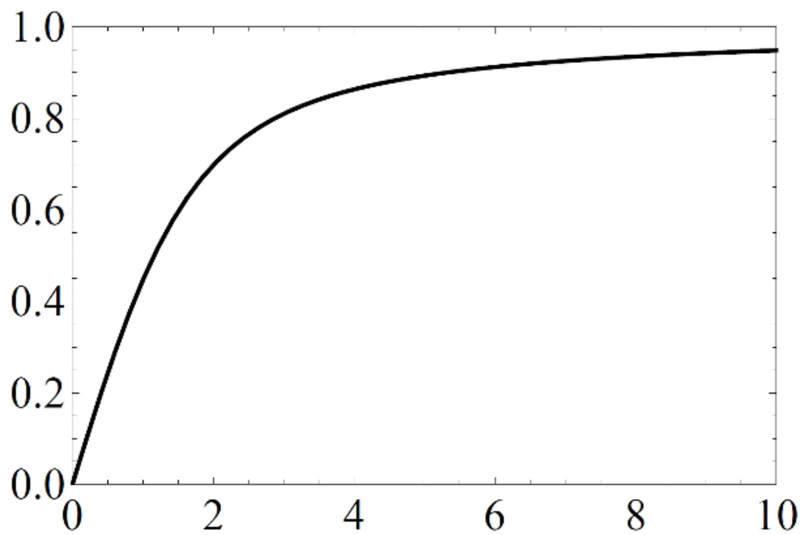
U(1)A restore: 結論できない

reweighting に近似: 信頼度?

[JLQCD 2016 Tomiya et al]

チェック事項: V→∞ 極限を取れるか

KY: without |Q|>0 sector

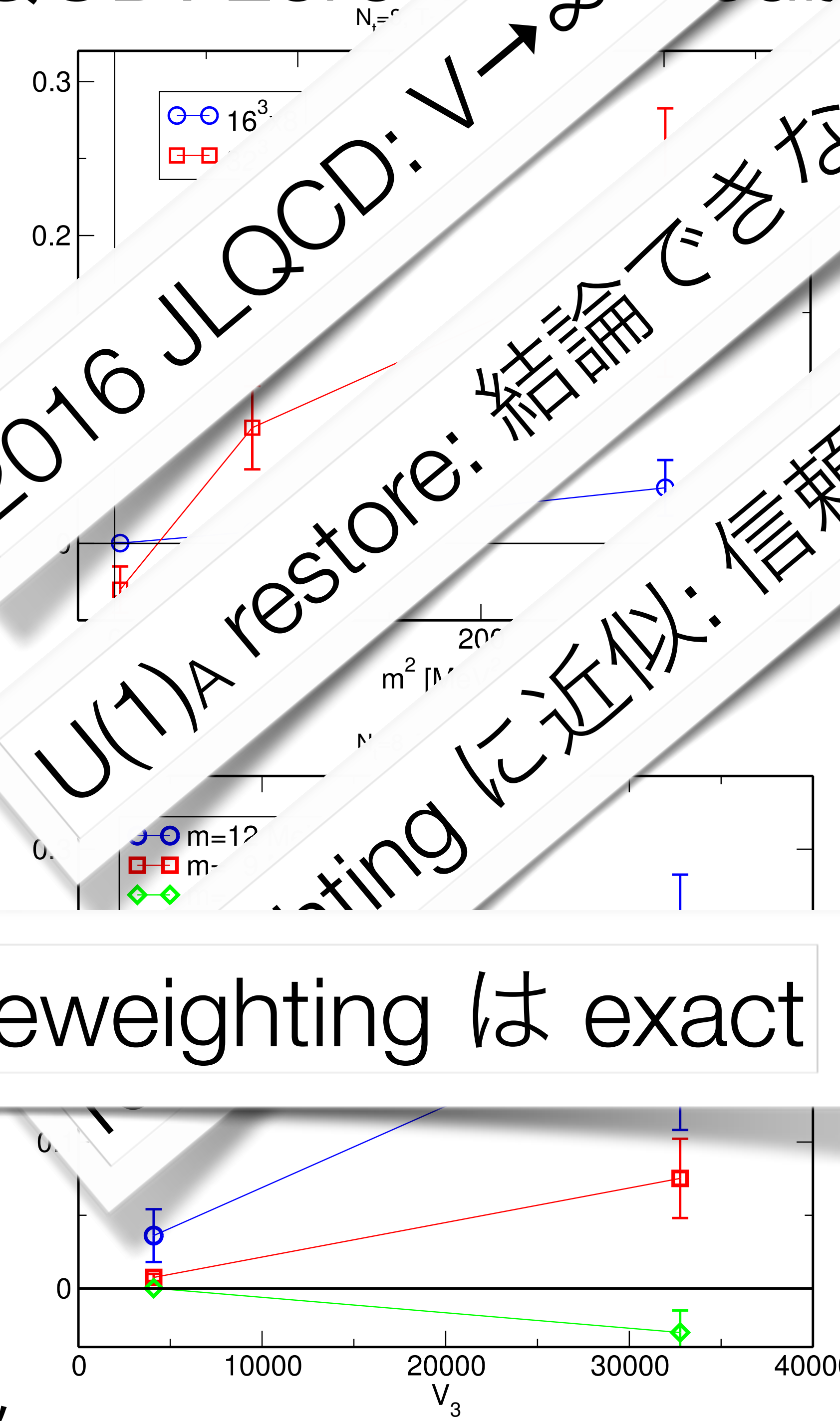


$$x = 2V_4 f_A m^2$$

fix V: Δ→0 as m² for m→0
even f

fix m : Δ∝V

JLQCD: zero-



2016 JLQCD: V→∞ 取れない

U(1)A restore: 結論できない

ting に近似: 信頼度?

最新のJLQCD では reweighting は exact

必ずしも一致する必要ない

しかし x→0 の極限では似たような物

[JLQCD 2016 Tomiya et al]

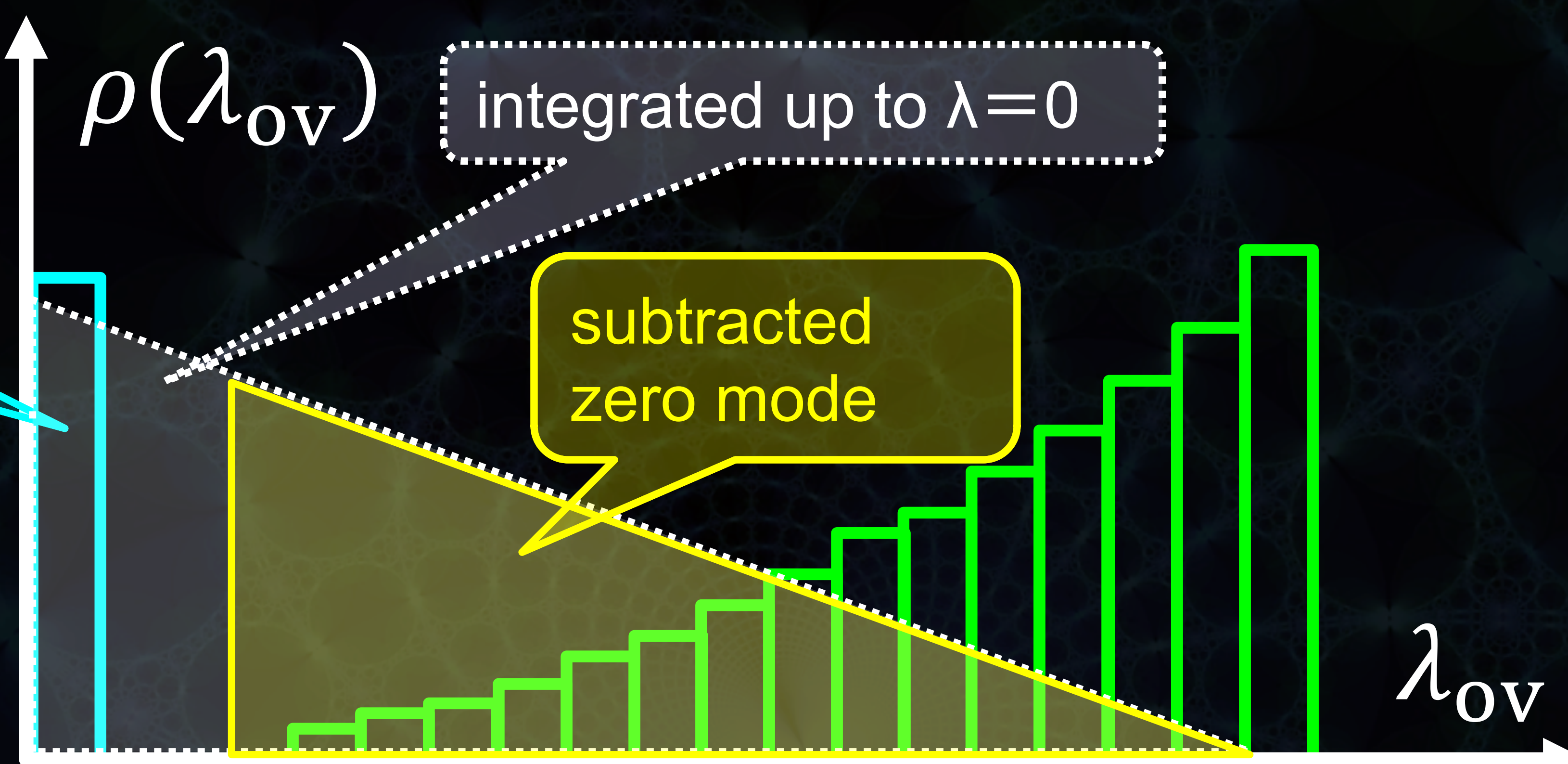
S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note 1 :

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2} \Rightarrow \Delta^{\rm ov}_{\pi-\delta} \equiv \frac{1}{V(1-m^2)^2}\sum_i \frac{2m^2(1-\lambda_{\rm ov}^{(i)2})^2}{\lambda_{\rm ov}^{(i)4}}$$

The factor of $1/\lambda^4$ enhances zero-mode contribution?



In $V \to \infty$ limit, we know zero-mode contribution is suppressed:

$$\Delta^{\rm ov}_{0-mode} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$$

New order parameter: we subtract zero mode

$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2}$$

Note 2:

U(1)$_A$ susc. = Physics + ~~Ultraviolet divergence~~?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$\rho(\lambda) \sim \lambda^3$; $\sim 1/\lambda^4$

$$\Delta^{\mathrm{ov}}_{\pi-\delta} \propto m^2 \ln\Lambda + \cdots$$

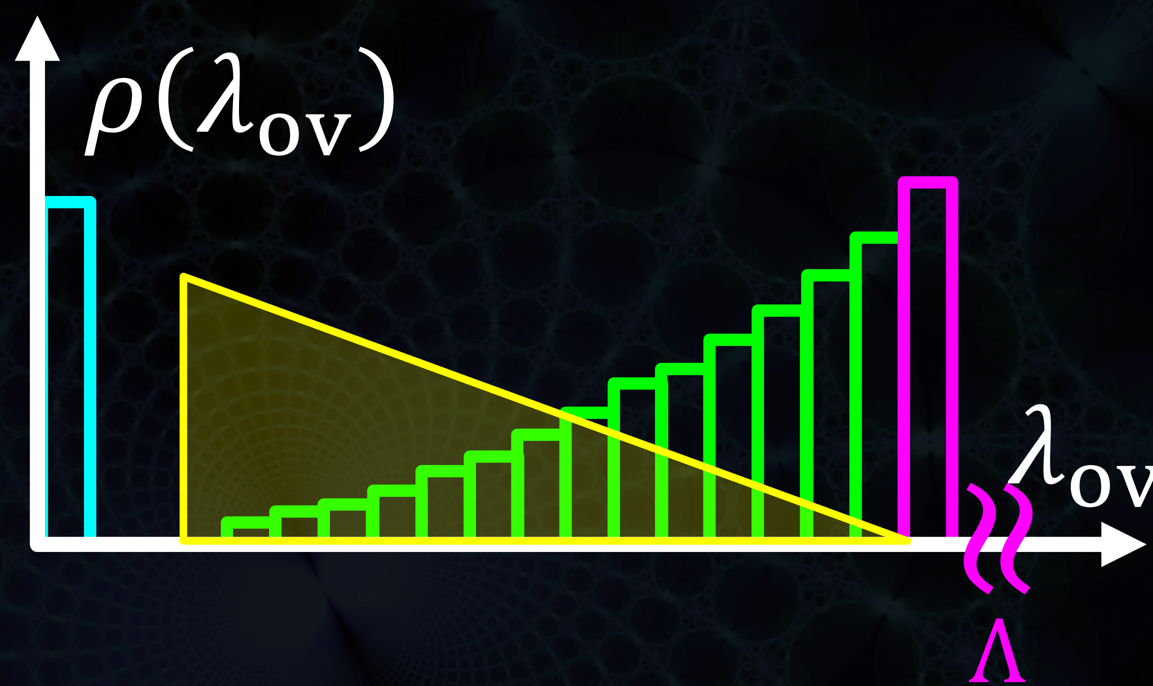
The term depends on cutoff Λ and valence quark mass m

We assume valence quark mass dependence of $\Delta_{\pi-\delta}$ (for small m):

$$\Delta_{\pi-\delta}(m) = \frac{a}{m^2} + b + cm^2 + O(m^4)$$

Zero-mode (disappears in $V \to \infty$)

$m^2 \ln\Lambda$ (disappears in $m \to 0$)

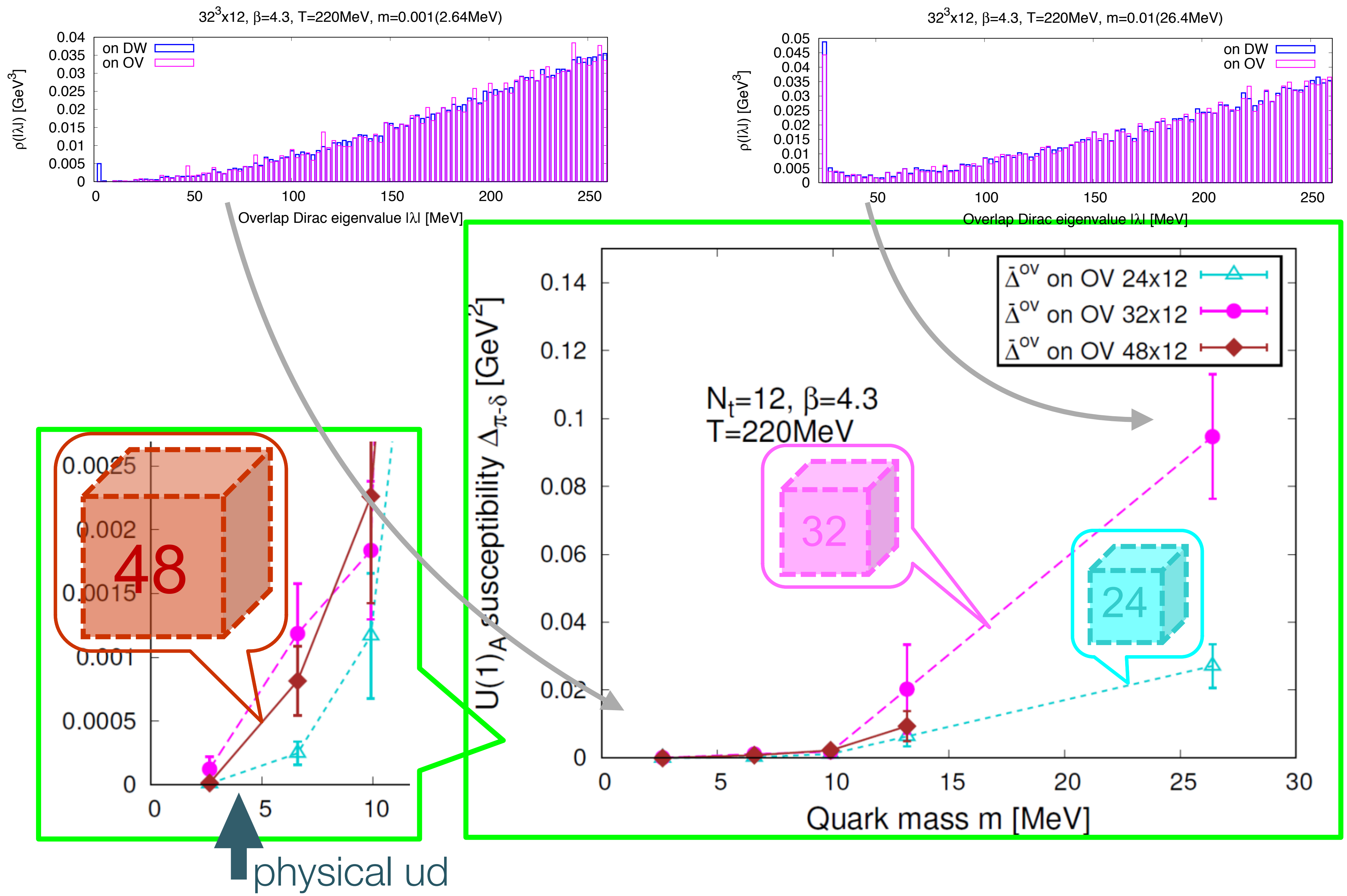


$\Rightarrow$ From 3 eqs. for $\Delta_{\pi-\delta}(m_1)$, $\Delta_{\pi-\delta}(m_2)$, $\Delta_{\pi-\delta}(m_3)$, a and c are eliminated

$\Rightarrow$ $\Delta_{\pi-\delta} \sim b + O(m^4)$ (, that depends on sea quark mass)

$U(1)_A$ susceptibility $N_f=2$

[JLQCD preliminary]

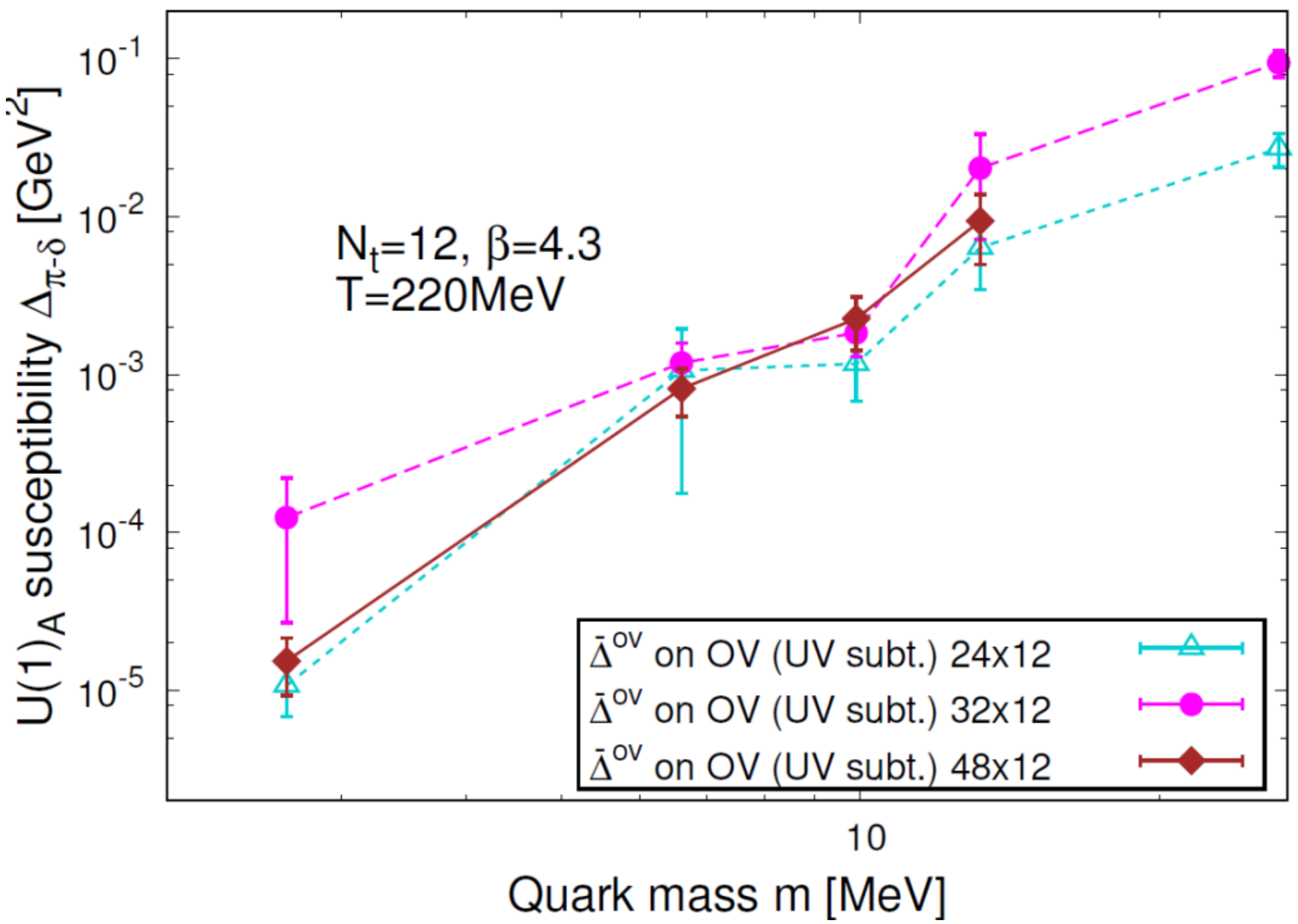


seemingly vanishing as m→0

Suzuki, Lattice 2018

$U(1)_A$ susceptibility N_f=2

[JLQCD preliminary]

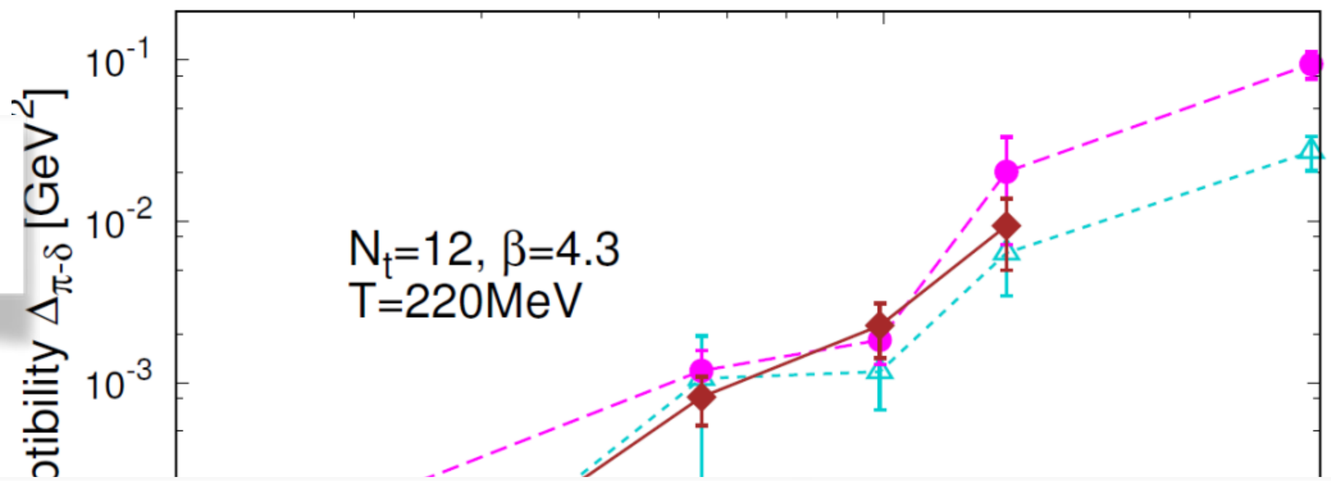


seemingly vanishing as m→0,
more evident in log-log prot

U(1)$_A$ susceptibility　N_f=2

[JLQCD preliminary]

ただし



UV subtraction が finite V effect も引いていないかは
精査する必要あり

その精査を通過したら
この結果は U(1)$_A$ 回復を示している

to be continued …

もう一つの見方？

- **Because it is unsettled problem !**
- fate of $U(1)_A$ lattice

		N_f
HotQCD (DW, 2012)	broken	2+1
JLQCD (topology fixed overlap, 2013)	restores	2
TWQCD (optimal DW, 2013)	restores ?	2
LLNL/RBC (DW, 2014)	broken	2+1
HotQCD (DW, 2014)	broken	2+1
Dick et al. (overlap on HISQ, 2015)	broken	2+1
Brandt et al. (O(a) improved Wilson 2016)	restores	2
JLQCD (reweighted overlap from DW, 2016)	restores	2
JLQCD (current: see Suzuki et al Lattice 2018)	restores ?	2
Ishikawa et al (Wilson, 2017)	at least Z_4 restores	2

ここまでのまとめ

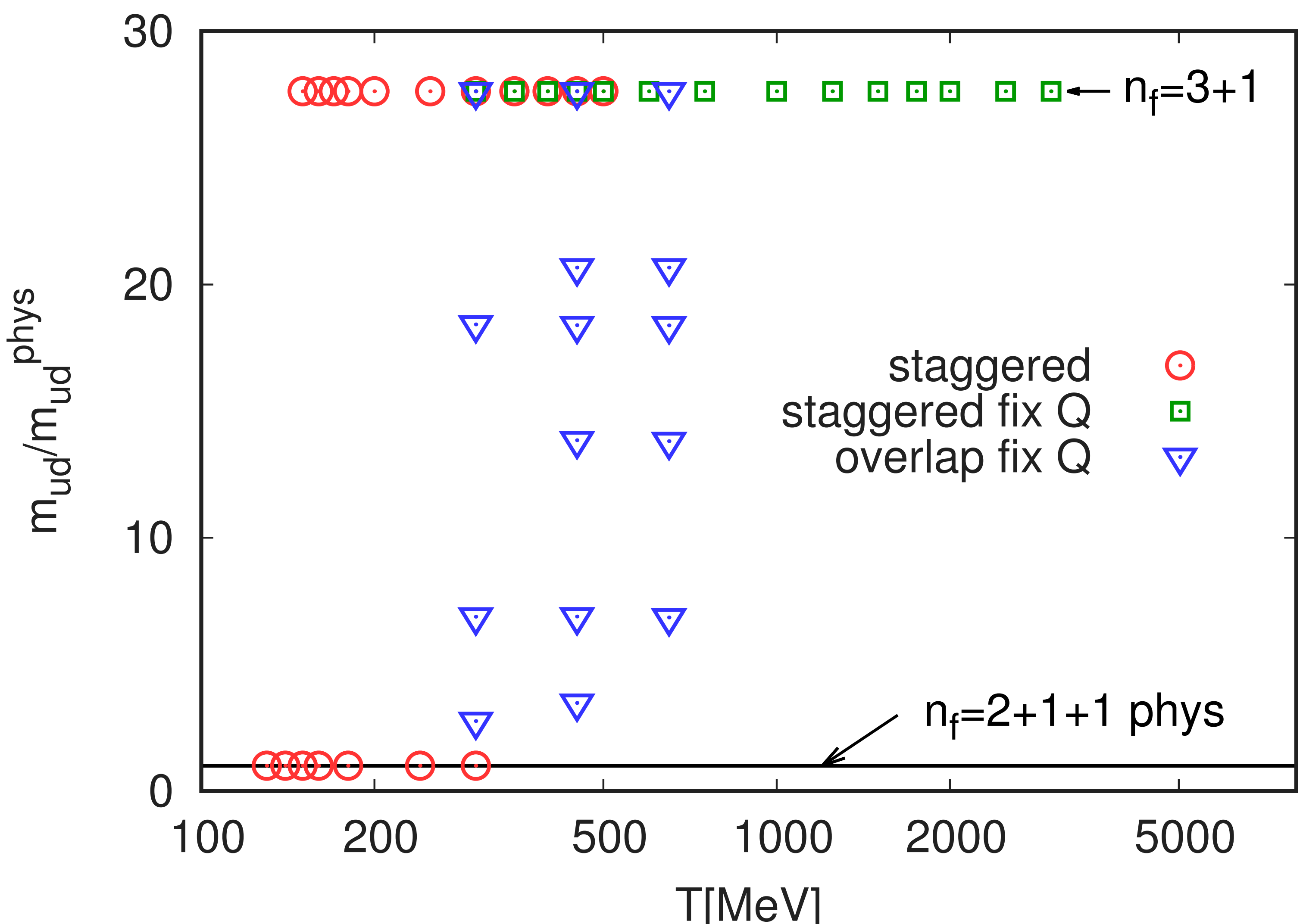
- topological susceptibility
    - $T>T_C$ でゼロの可能性:　結論出ず
    - 相転移の有無:　結論出ず
- fate of $U(1)_A$
    - $T>T_C$ で回復するか:　結論出ず
- しかし、より連続極限に近い格子で、より精密な手法を開発
- 更なる研究が必要:　そもそも簡単な問題ではない
- 今後
    - 現状の統計で 様々な解析手法を使い調査継続
    - subtraction の理解 (得に個人的)
    - parameter の変更により、より見やすい所を追跡: T_C 近傍など
        - T=220 MeV → 180 MeV (> T_C chiral transition)

$U(1)_A$ @ N_f=2+1 (+1) その他のグループ

references

- topological susceptibility for axion mass
  - 1606.07494, S. Borsanyi et al, (Budapest-Wuppertal), Nature
    - "Calculation of the axion mass based on high-temperature lattice quantum chromodynamics"
  - 1606.07175, J.Frison, R.Kitano, H.Matsufuru, S.Mori, N.Yamada
    - "Topological susceptibility at high temperature on the lattice"
    - crucial technique of above

simulation parameters and integral path



- starting from 3+1 at T=300 MeV

$$\langle O \rangle_{Q-0} = \langle O \rangle_Q - \langle O \rangle_0$$

↓ use Q=1

- T direction: integrate

$$-b_Q \equiv \frac{d \log Z_Q/Z_0}{d \log T} = \frac{d\beta}{d \log a} \langle S_g \rangle_{Q-0} + \sum_f \frac{d \log m_f}{d \log a} m_f \langle \overline{\psi} \psi_f \rangle_{Q-0}.$$

- m-direction:

$$\left.\frac{Z_1}{Z_0}\right|_{2+1+1} = \exp\left(\int_{m_{ud}^{phys}}^{m_s^{phys}} d \log m_{ud} \; m_{ud} \langle \overline{\psi} \psi_{ud} \rangle_{1-0} \right) \cdot \left.\frac{Z_1}{Z_0}\right|_{3+1}$$

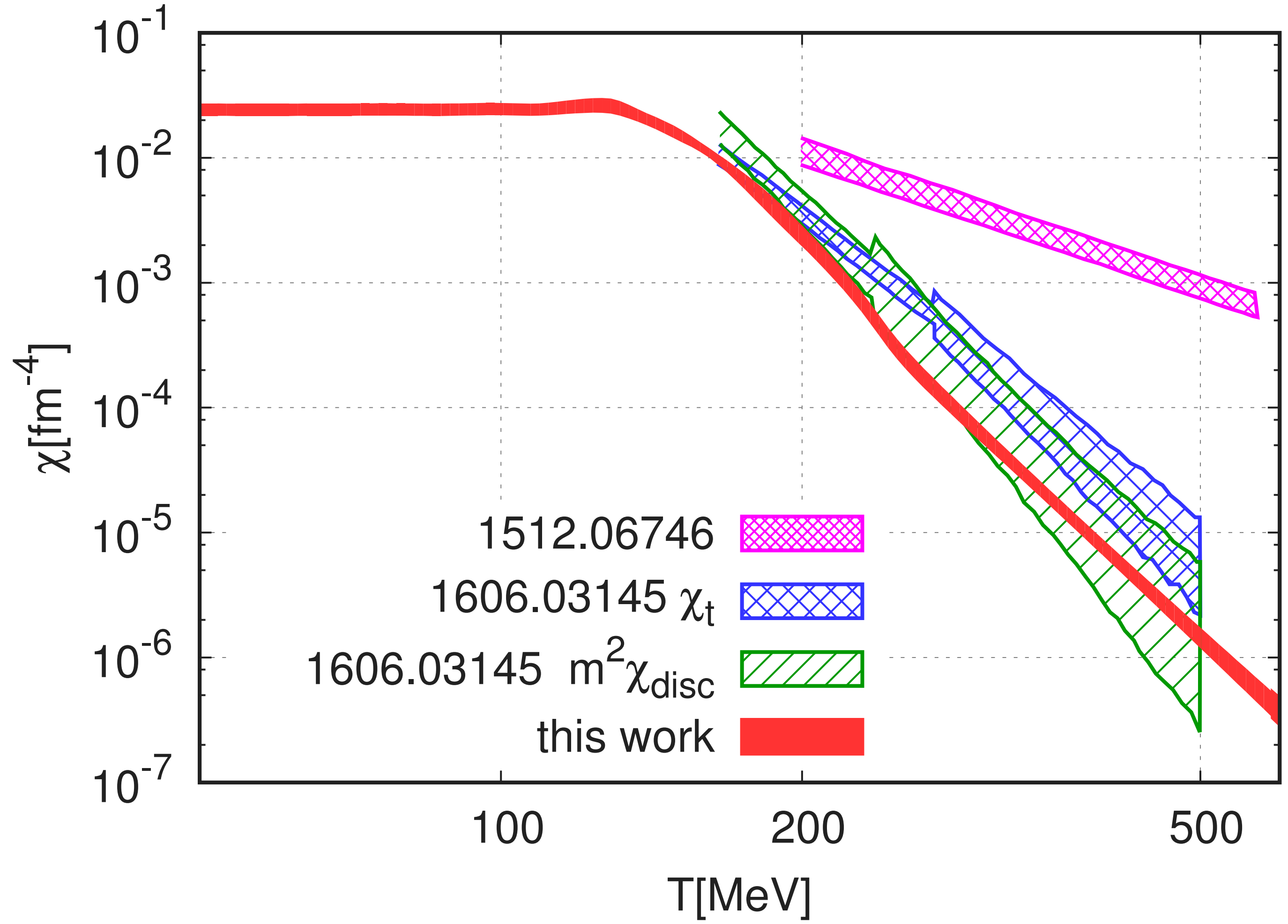
other key methodologies

- “reweighting” of staggered simulations to better ones with small $O(a^2)$
  - lowest modes engineering :
    - would induce non-local term in the action → similar to 4th root ?

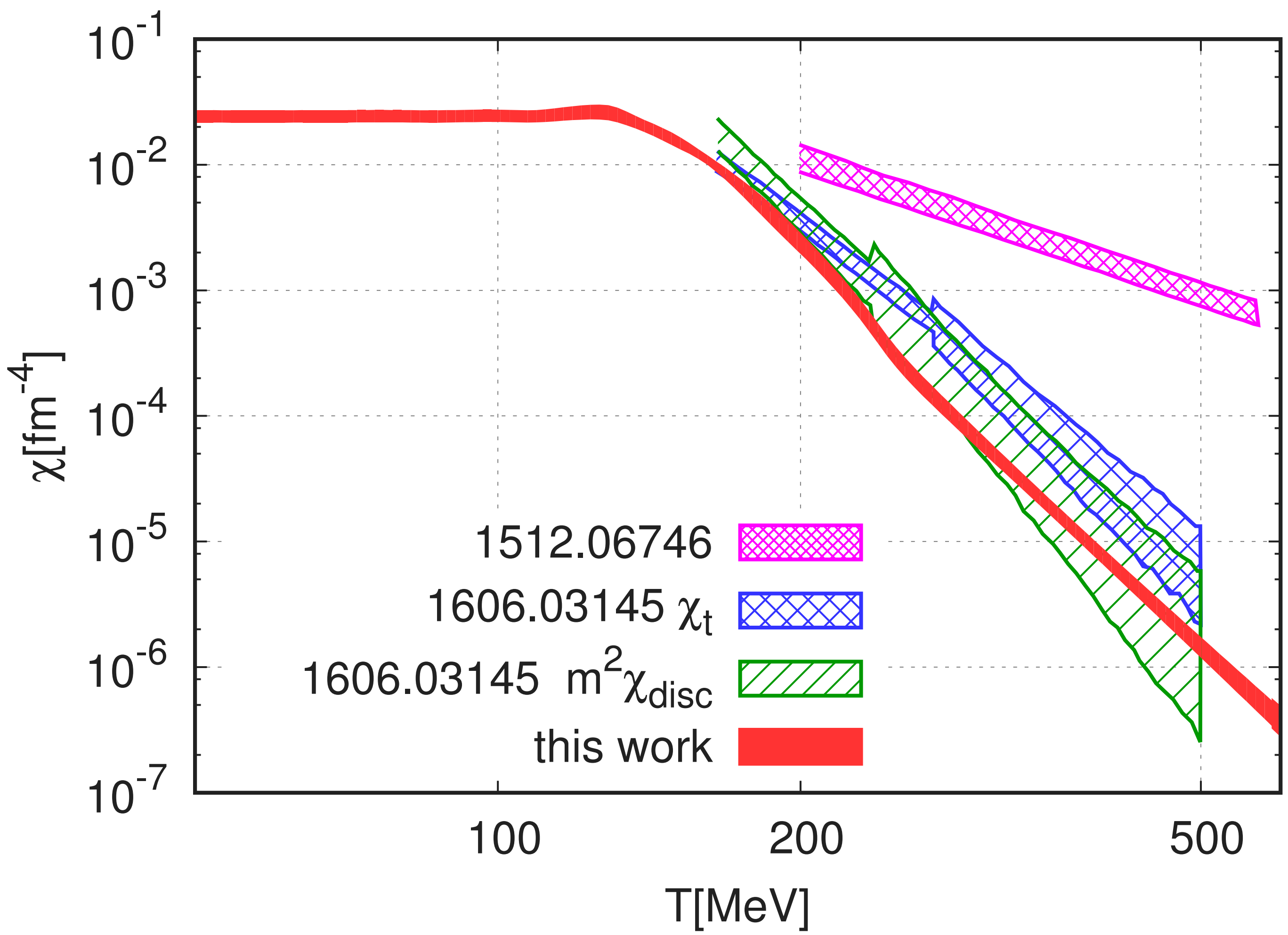
↖この操作の正当性が大きな問題

- isospin breaking effect
- finite volume effect
- charm quark effect
- line of constant physics: $m_{ud}(\beta)$, $m_s(\beta)$, $m_c(\beta)$, $a(\beta)$; $\beta=6/g^2$
- systematic error associated with the ambiguous definition of Q from Gluonic
- …

the result and comparison with other works



the result and comparison with other works

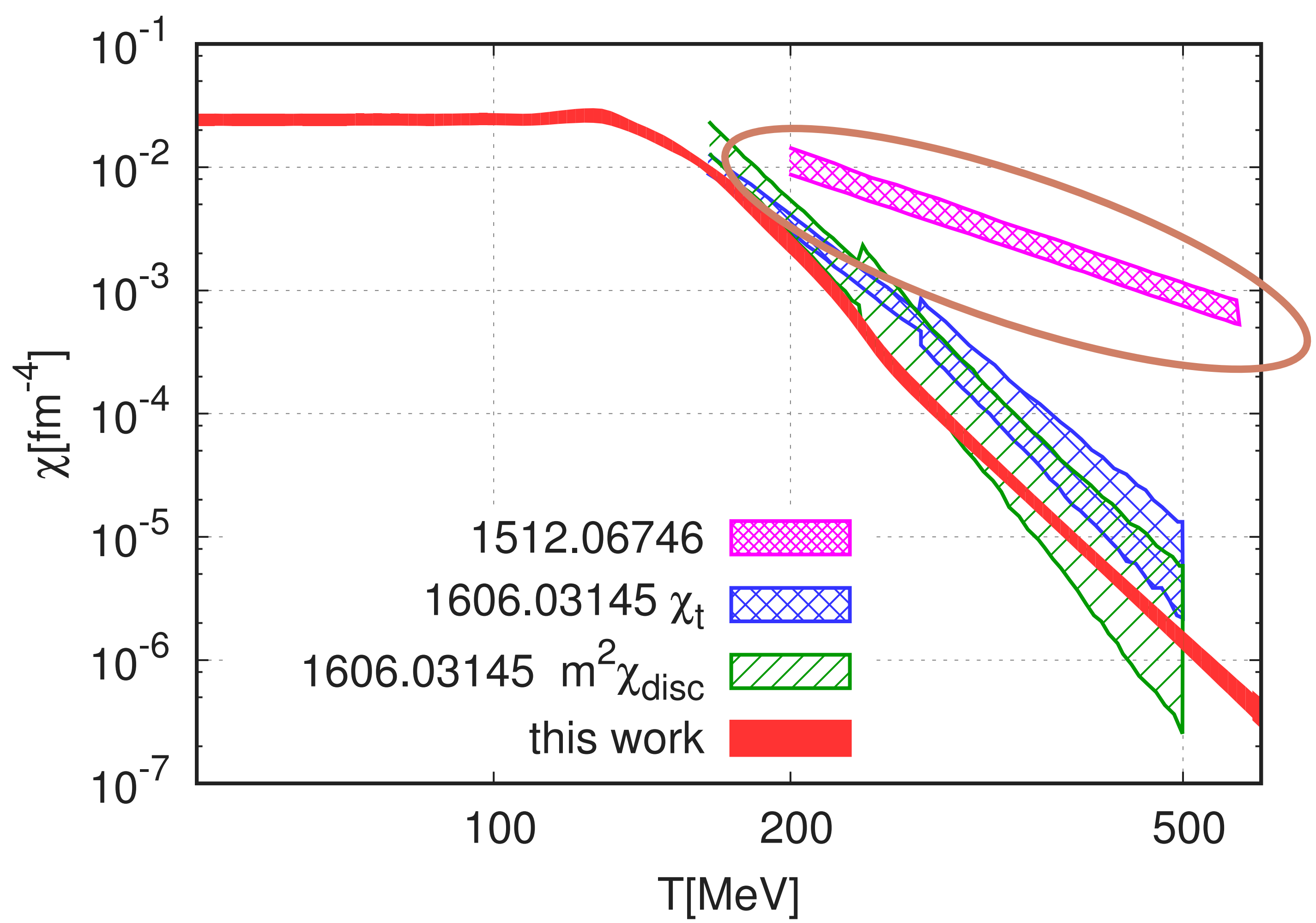


he direct determination of $\chi(T)$ all the way up to 3 GeV means that we do not have to rely on the dilute instanton gas approximation (DIGA).

Note that *a posteriori* the exponent predicted by DIGA turned out to be compatible with our finding, but its prefactor is off by an order of magnitude, similar to the quenched case.

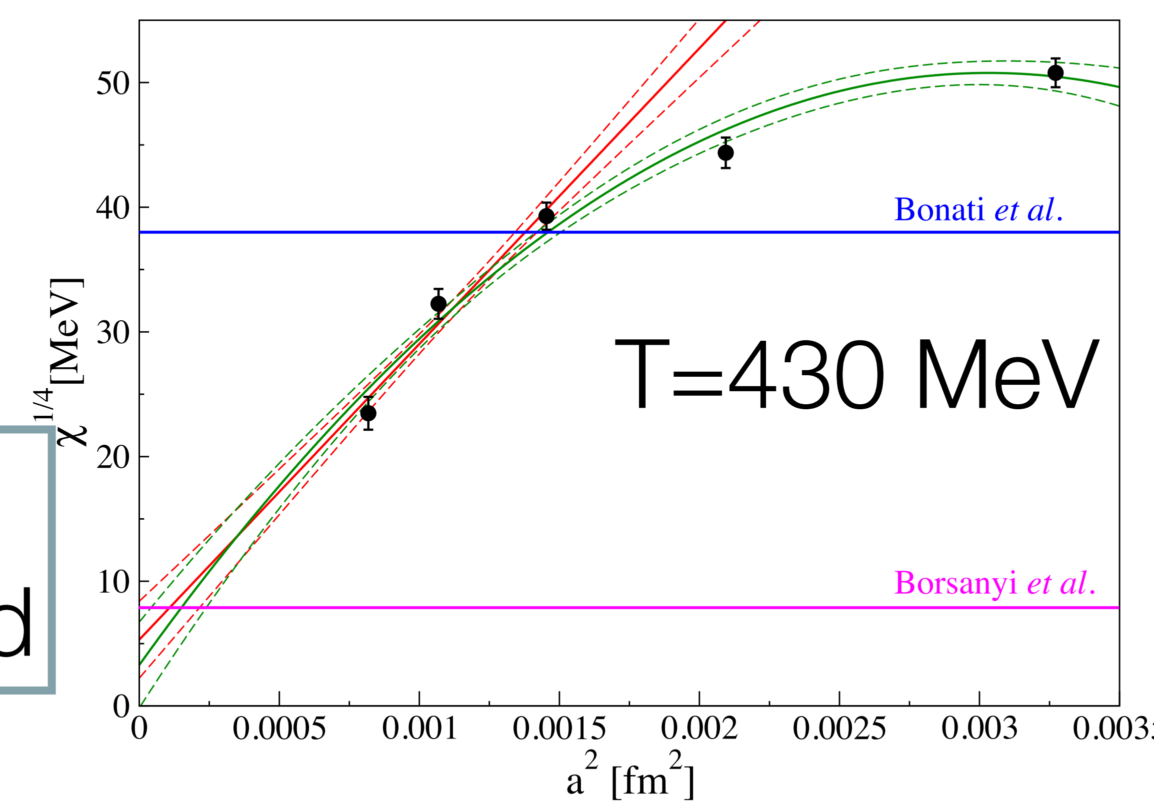
Though some of our simulations (see Supplementary Fig. 18) are already carried out with chiral (overlap) fermions, where large cut-off effects are *a priori* absent, it is an important task for the future to crosscheck these results with a calculation using chiral fermions only.

the result and comparison with other works



Bonati et al 2015: outlier ?

Bonati et al 2018 new



Note:
these are all based on staggered

DW or overlap での検証必要

now, consistent

Thank you very much for your attention !

Lattice framework

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- DW→OV reweighting

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$$R \equiv \frac{\det[H_{\rm ov}(m)]^2}{\det[H_{\rm DW}^{\rm 4D}(m)]^2} \times \frac{\det[H_{\rm DW}^{\rm 4D}(1/4a)]^2}{\det[H_{\rm ov}(1/4a)]^2}.$$

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$$D_{ov} = \underbrace{\frac{1}{2} \sum_{\lambda_i < \lambda_{th}} (1 + \gamma_5 \mathrm{sgn}\lambda_i) |\lambda_i\rangle\langle\lambda_i|}_{\text{Exact low modes}} + \underbrace{D_{DW}^{4D} \left(1 - \sum_{\lambda_i < \lambda_{th}} |\lambda_i\rangle\langle\lambda_i| \right)}_{\text{High modes}},$$

Lattice framework

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λ for $H_M = \gamma_5 \frac{\alpha D_W}{2 + D_W}$

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$$R \equiv \frac{\det[H_{\text{ov}}(m)]^2}{\det[H_{\text{DW}}^{\text{4D}}(m)]^2} \times \frac{\det[H_{\text{DW}}^{\text{4D}}(1/4a)]^2}{\det[H_{\text{ov}}(1/4a)]^2}.$$

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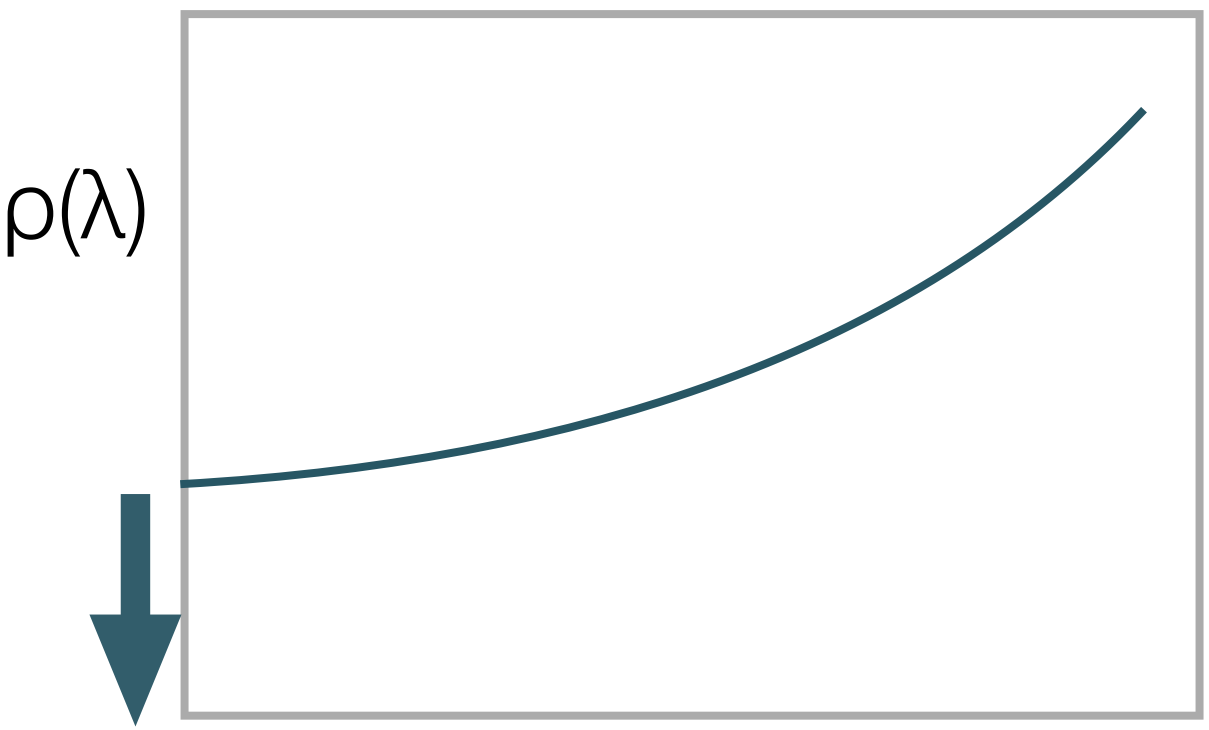
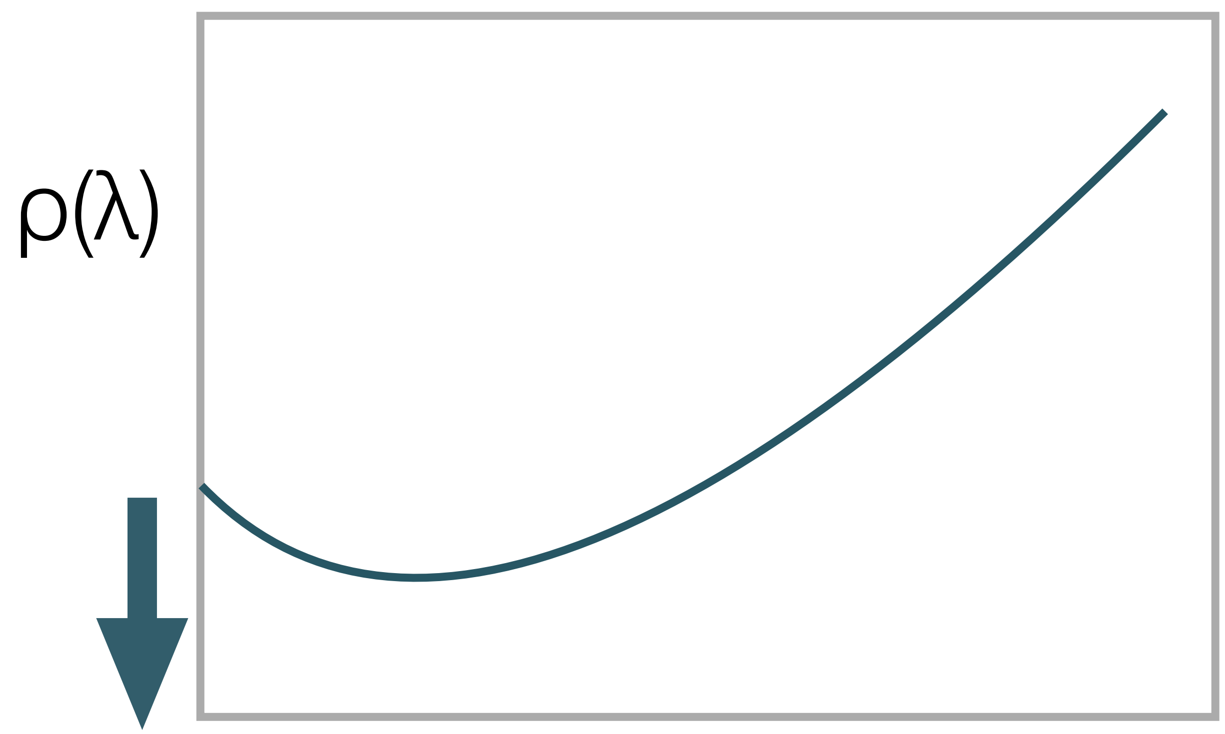
simply speaking,   in the $m \to 0$ limit

- $U(1)_A$ restores if



with $\rho(0) \to 0$ and $\rho'(0) \to 0$

- and not if



with $\rho(0) \to 0$ and $\rho'(0) \neq 0$

- non-analyticity at $\lambda \to 0$ required

Analytic works

- Aoki-Fukaya-Taniguchi
  - QCD with OV regulator
  - assuming analyticity of $\rho(0)$
- $f_A \to 0$: $U(1)_A$ br. parameter
- $\chi_{top} = 0$ for $0<m<m_c$

- Kanazawa-Yamamoto
  - assuming $f_A \neq 0$
  - expansing free energy in m
- discussing
  - finite m and V effect
  - contributions of topological sectors

Kanazawa - Yamamoto

- assuming $f_A \neq 0$
- expansing free energy in m

$$Z(T,V_3,M) = \exp\left[-\frac{V_3}{T} f(T,V_3,M)\right],$$

$$f(T,V_3,M) = f_0 - f_2 \,\mathrm{tr}\, M^\dagger M - \underline{f_A(\det M + \det M^\dagger)} + \mathcal{O}(M^4),$$

$$M \to e^{-2i\theta_A} V_L M V_R^\dagger \qquad \det M \to \mathrm{e}^{4i\theta_A} \det M$$ breaks $U(1)_A$

other terms are invariant under $U(1)_A$
all invariant under SU(2)LxR

- to study topological sectors

$$M \to M\,\mathrm{e}^{i\theta/N_f} \qquad Z_Q(T,V_3,M) \equiv \oint \frac{\mathrm{d}\theta}{2\pi}\,\mathrm{e}^{-iQ\theta}\, Z(T,V_3,M\mathrm{e}^{i\theta/2}).$$

$$= \mathrm{e}^{-V_4[f_0 - f_2(m_u^2+m_d^2)]} \oint \frac{\mathrm{d}\theta}{2\pi}\,\mathrm{e}^{-iQ\theta}\,\mathrm{e}^{2V_4 f_A m_u m_d \cos\theta}$$

$$= \mathrm{e}^{-V_4[f_0 - f_2(m_u^2+m_d^2)]}\, I_Q(2V_4 f_A m_u m_d),$$

$$\Delta_{\pi-\delta} = \sum_{Q=-\infty}^{\infty} \frac{Z_Q}{Z} P_Q \qquad P_Q = 8 f_A \frac{I_Q'(2V_4 f_A m^2)}{I_Q(2V_4 f_A m^2)}$$

Kanazawa - Yamamoto: U(1)$_A$ br. scenario

- to study topological sectors

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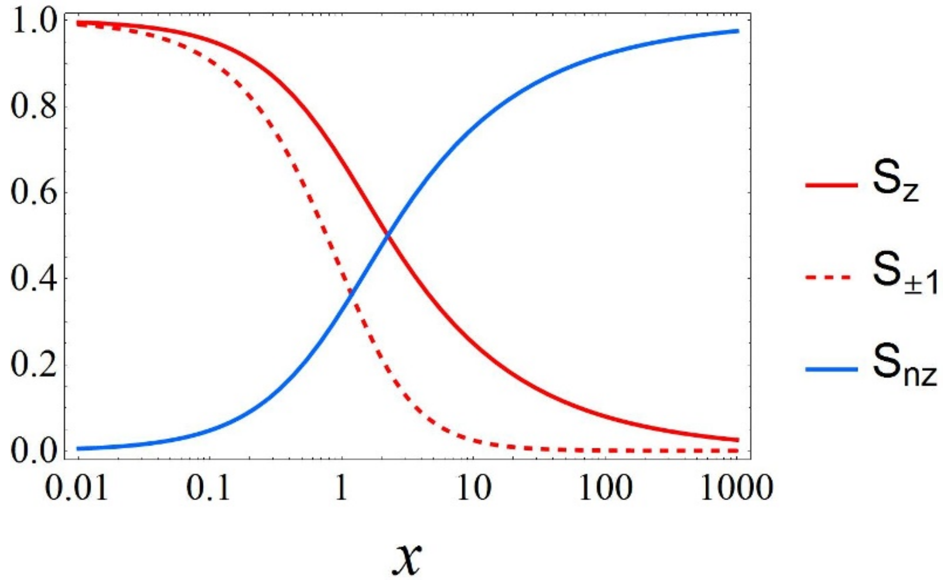
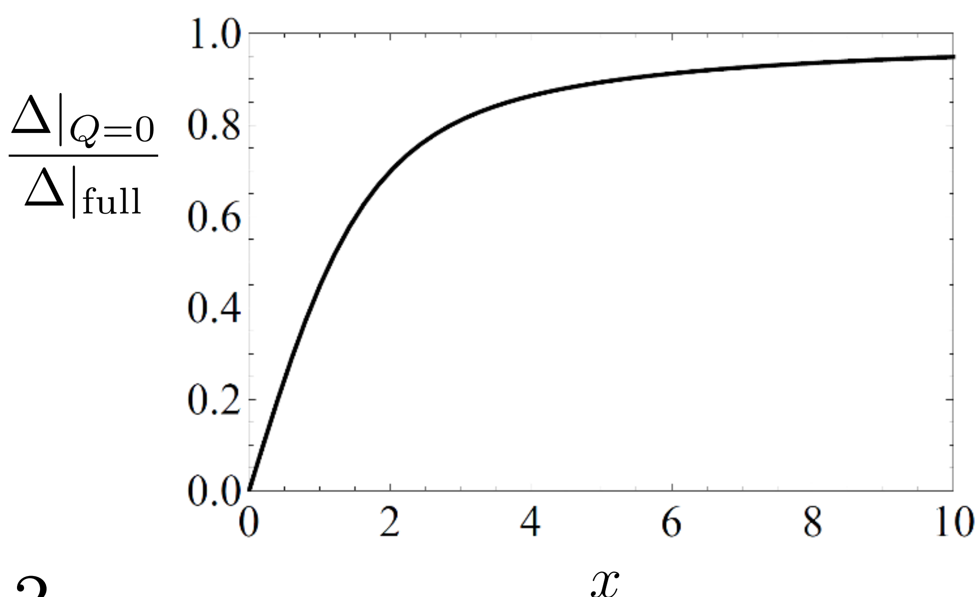
relative contribution of modes





$$x = 2V_4 f_A m^2$$

Kanazawa - Yamamoto: $U(1)_A$ br. scenario

KY tells

- fixed topology gives wrong result at small V
- adding all Q sector or large enough volume necessary

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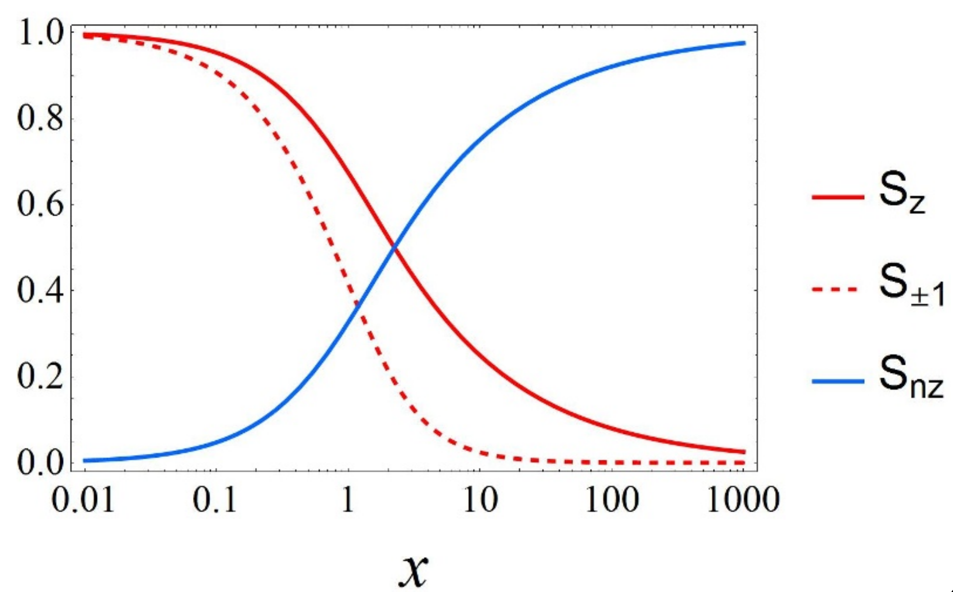

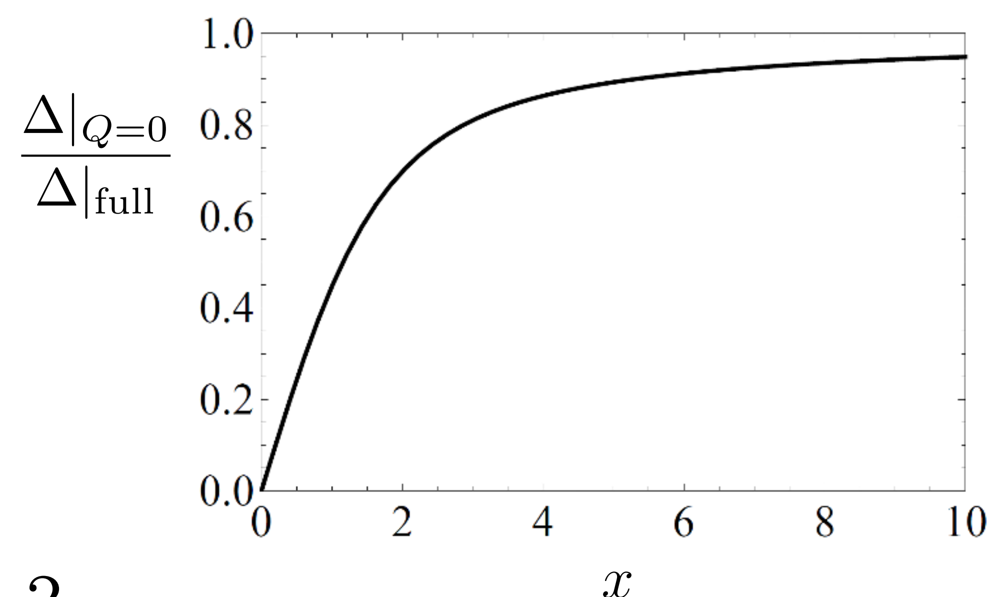

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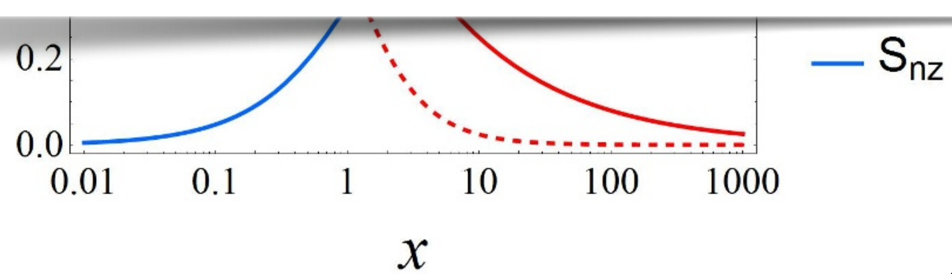
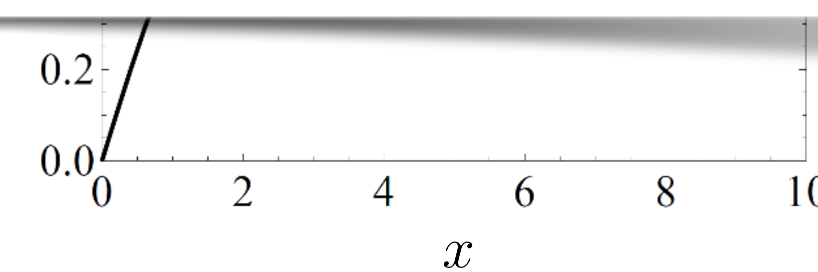
JLQCD

- does not fix topology (DW)
- zero-mode subtraction may have similar effect to fix Q=0
  - for smallest m: actually effectively fixed to Q=0





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