Instability of De Sitter Spacetime and Eternal Inflation

Hiroki Matsui

Department of Physics, Tohoku University, Sendai,
Miyagi 980-8578, Japan, *hiroki.matsui.c6@tohoku.ac.jp*

Based on: **H, Mastui** and **F, Takahashi**,....... arXiv:1806.10339, arXiv:1807.1193.......

August 11, 2018

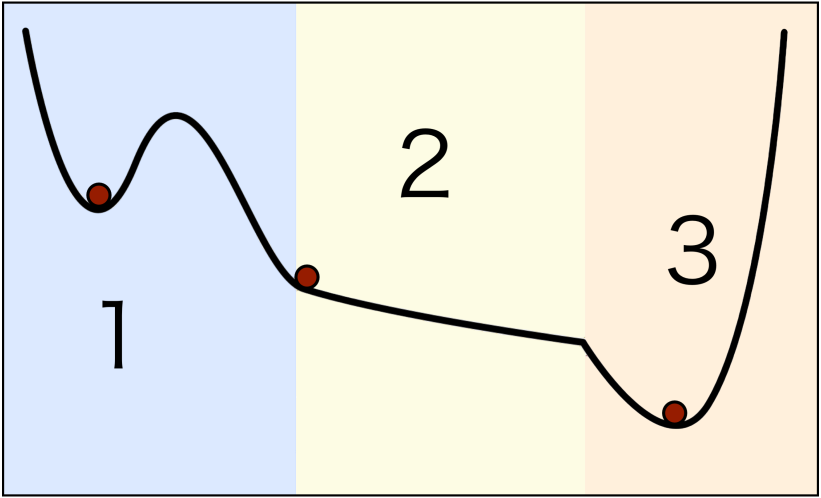
Cosmological Inflation

Inflation solves several problems in big bang cosmology: Horizon, flatness, magnetic-monopole problem. And, it precisely matches cosmological observations of CMB, etc.

But, we do not know the origin of the inflation and the shape of the inflaton potential. Additionally, most inflation models are thought to be eternal. Broadly speaking, there are three types for eternal inflation: old, new and chaotic inflation.

[Guth, J. Phys. A40, 6811 (2007), hep-th/0702178]

Eternal Inflation



Eternal Inflation

The vacuum decay rate in de Sitter spacetime is given by the Hawking-Moss instanton

$$\Gamma_{\rm decay} = A \exp(-B), \quad B \approx \frac{8\pi^2 V(\phi_{\rm max})}{3H^4}$$

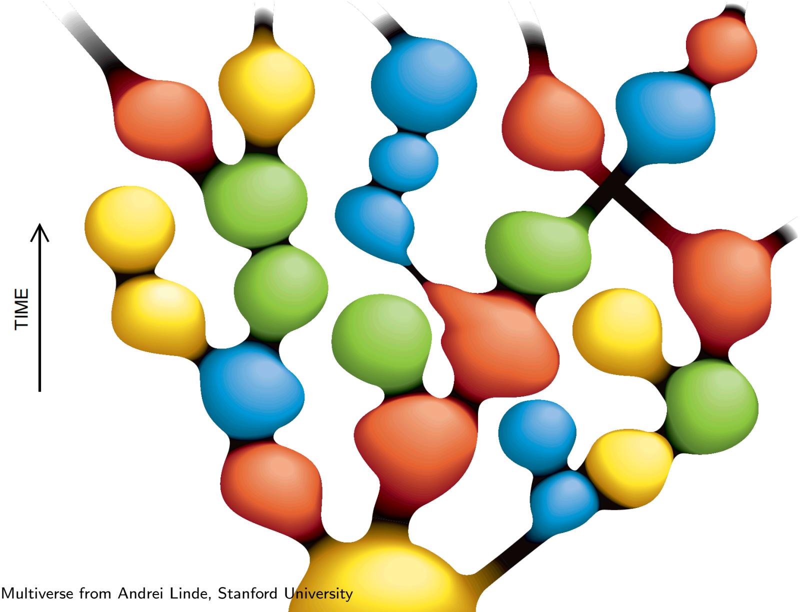
However, the inflation increase the number of the Hubble-horizon patches exponentially

$$\mathcal{N}_{\rm patch} \sim \exp\left(3Ht\right)$$

Thus, the number of patches continuing the inflation grows exponentially with Hubble time

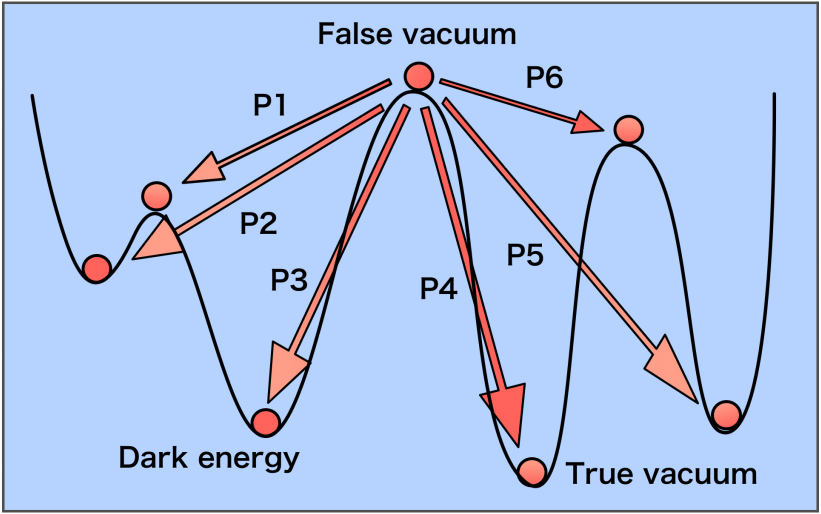
$$\mathcal{N}_{\rm inflation} \sim \mathcal{N}_{\rm patch} \cdot \left(1 - \Gamma_{\rm decay}\right)^{Ht} \sim e^{Ht \cdot \{3 + \ln\left(1 - \Gamma_{\rm decay}\right)\}} \gg \mathcal{O}(1)$$

Eternal Inflation and Multiverse

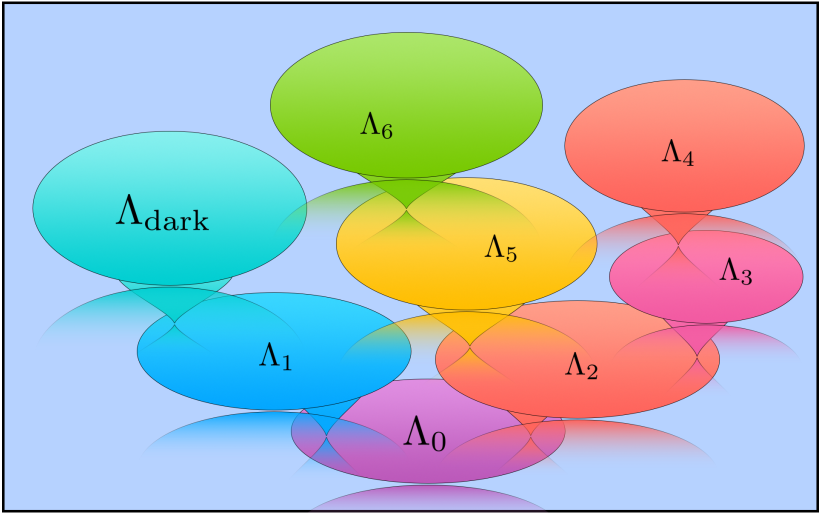


Multiverse from Andrei Linde, Stanford University

String Landscape

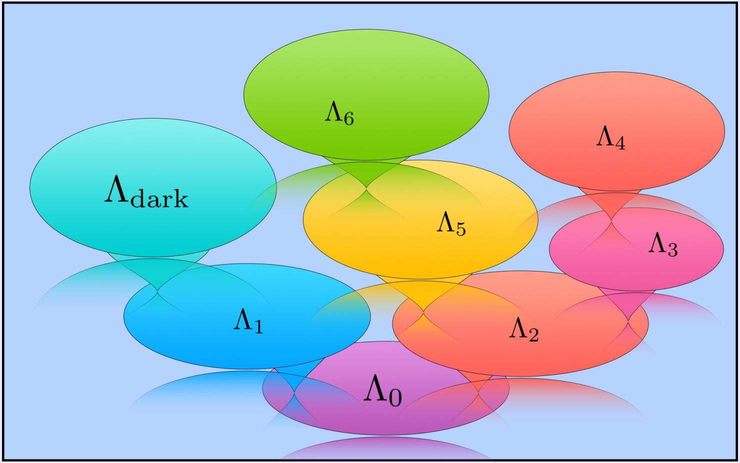


Eternal Inflation and Multiverse



Anthropic Principle

String Landscape + Eternal Inflation $\Longrightarrow$ Finetuning Problem



Swampland Conjectures

Swampland De Sitter Conjecture

$$M_{\mathrm{P}} \frac{|\nabla V|}{V} > c$$

[G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, 1806.08362]

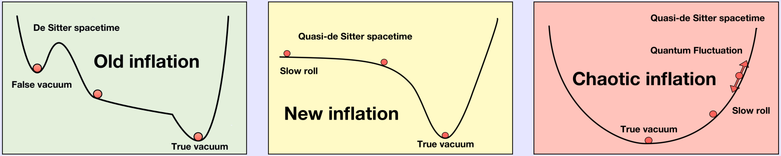
where c is a numerical constants of order unity, but its precise value depends on details of the compactification.

$$\text{Swampland De Sitter Conjecture} \Longrightarrow \text{No dS vacuua or minima}$$

The slow-roll inflation requires $c < \sqrt{2}$ and the CMB measurements show $\epsilon < 0.0045$ which leads to $c < 0.094$.

$$\epsilon \simeq \frac{M_{\mathrm{P}}^2}{2} \left(\frac{\nabla V}{V} \right)^2 \quad \Rightarrow \quad \epsilon^{1/2} > \frac{c}{\sqrt{2}}$$

Eternal Inflation vs Swampland Conjectures

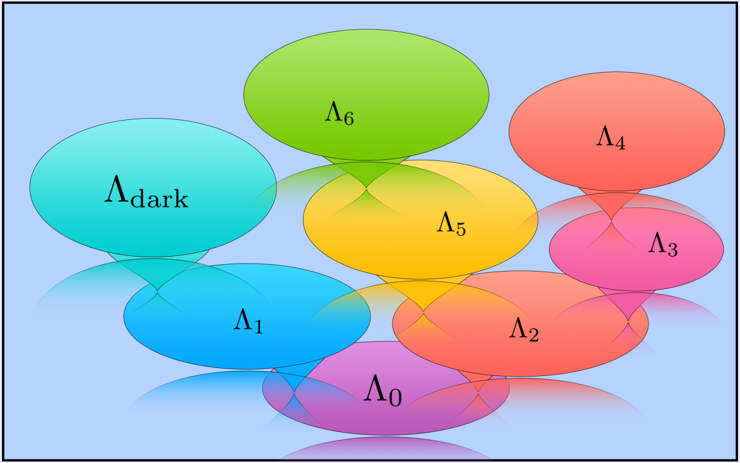


- Swampland De Sitter Conjecture forbids de Sitter vacua or minima.
- The eternal old/hilltop inflation is impossible for this criteria.
- The chaotic eternal inflation is only possible for $c \sim \mathcal{O}(0.01)$ and $1/\mathcal{D} \sim \mathcal{O}(0.01)$, and that the Hubble parameter $H_{\rm inf}$ during the eternal inflation is parametrically close to the Planck scale, and we get a new constraint $2\pi c \lesssim H_{\rm inf}/M_{\rm P} < 1/\sqrt{3}$.

[**H, Mastui** and **F, Takahashi**, arXiv:1807.1193]

Anthropic Principle ?

String Landscape + ~~Eternal Inflation~~ $\Longrightarrow$ Finetuning Problem



De Sitter Spacetime (FLRW)

Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetimes

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j$$

Einstein's field equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}.$$

with no matter and lead to de Sitter solution

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \Longleftrightarrow H^2 = \frac{\Lambda}{3} \Longleftrightarrow \dot{H} = 0 \Longleftrightarrow a(t) = e^{H\cdot t}$$

The most famous examples of the de Sitter spacetime are cosmic inflation and dark energy, $\Lambda \sim V(\phi)$.

De Sitter Spacetime Instability

The quantum fluctuations on de Sitter spacetime

$$\delta\phi \approx H/2\pi.$$

The Einstein's equation

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G_N \left\langle T_{\mu\nu} \right\rangle .$$

The de Sitter instability from quantum backreaction

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} \simeq \frac{H^4}{M_{\mathrm{P}}^2} \Longrightarrow \text{dS spacetime may be destabilized}$$

[Mottola '85 '86, Tsamis, Woodard '93, Abramo, Brandenberger, Mukhanov '97, Goheer, Kleban, Sussking '03, Polyakov '07, Anderson, Mottola '14, Dvali, Gomez, Zell '17, Markkanen '16 '17]

Evaporation of DS Spacetime

The de Sitter entropy

$$S_{\rm dS} = \frac{A}{4G}, \quad A = 4\pi H^{-2}$$

The de Sitter thermodynamics like black hole

$$dU = TdS - PdV \quad \Longrightarrow \quad 2\dot{H} M_P^2 = \frac{\dot{\rho}_{\rm vac}}{3H},$$
$$dU = -d\left(\frac{4\pi\rho_{\rm vac}}{3H^3}\right), \quad TdS = HM_P^2 d\left(\frac{4\pi}{H^2}\right), \quad PdV = -p_{\rm vac} d\left(\frac{4\pi}{3H^3}\right)$$

The de Sitter thermodynamics

$$2\dot{H} M_P^2 = \frac{\dot{\rho}_{\rm vac}}{3H}, \quad \dot{\rho}_{\rm vac} \simeq \mathcal{O}(H^5) \quad \Longrightarrow \quad 2\dot{H} M_P^2 = \mathcal{O}(H^4)$$

which shows the time-dependent cosmological constant.

[Spradlin, Strominger, Volovich '01, Padmanabhan '03, Markkanen '17]

De Sitter Instability from Quantum Conformal Anomaly

We focus on quantum backreaction from conformal massless fields and trace of EMT is classically zero.

$$T^{\mu}{}_{\mu} = 0.$$

But, the vacuum expectation values of EMT is non-zero

$$\left\langle T^{\mu}{}_{\mu} \right\rangle \neq 0 \Longrightarrow \text{conformal anomaly}$$

We persist in the semiclassical approach of the gravity

$$\frac{1}{8\pi G_N} G_{\mu\nu} + \rho_\Lambda g_{\mu\nu} + a_1 H^{(1)}_{\mu\nu} + a_2 H^{(2)}_{\mu\nu} + a_3 H^{(3)}_{\mu\nu} = \langle T_{\mu\nu} \rangle$$

The semiclassical gravity has no unitary problem about the gravitational S-matrix since the gravity is not quantized.

Quantum Conformal Anomaly

Quantum trace of energy momentum tensor

$$
\begin{aligned}
\langle T^{\mu}{}_{\mu} \rangle = \frac{m^2}{4\pi^2 C(\eta)} \int dk k^2 \bigg[& \frac{1}{\omega_k} + \frac{C m^2}{8\omega_k^5}\left(D' + D^2\right) - \frac{5 C^2 m^4 D^2}{32\omega_k^7} \\
& - \frac{C m^2}{32\omega_k^7}\left(D''' + 4D''D + 3D'^2 + 6D'D^2 + D^4\right) \\
& + \frac{C^2 m^4}{128\omega_k^9}\left(28D''D + 21D'^2 + 126D'D^2 + 49D^4\right) \\
& - \frac{231 C^3 m^6}{256\omega_k^{11}}\left(D'D^2 + D^4\right) + \frac{1155 C^4 m^8 D^4}{2048\omega_k^{13}} \bigg] \\
& = -\frac{m^4}{32\pi^2}\left[\frac{1}{\epsilon} + 1 - \gamma + \ln 4\pi + \ln\frac{\mu^2}{m^2}\right] \\
& + \frac{m^2 D^2}{192\pi^2 C}\left(2D' + D^2\right) - \frac{1}{960\pi^2 C^2}\left(D''' - D'D^2\right)
\end{aligned}
$$

Quantum Conformal Anomaly

Quantum trace of energy momentum tensor

$$
\begin{aligned}
\left\langle T^{\mu}{}_{\mu}\right\rangle_{\text{anomaly}} &= \lim_{m\to 0}\left\langle T^{\mu}{}_{\mu}\right\rangle_{\text{ren}} \\
&= -\frac{1}{960\pi^2 C^2}\left(D''' - D' D^2\right) \\
&= -\frac{1}{2880\pi^2}\left[\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) + \Box R\right] \\
&= \frac{1}{360(4\pi)^2}\left(E - \frac{2}{3}\Box R\right) + \frac{-1}{270(4\pi)^2}\Box R \\
&= \frac{1}{360(4\pi)^2}E - \frac{1}{180(4\pi)^2}\Box R
\end{aligned}
$$

where $m \to 0$ for conformal massless fields

Quantum Conformal Anomaly

The general form of conformal anomaly for four dimensions

$$\begin{aligned} \langle T^{\mu}{}_{\mu} \rangle &= bF + b' \left(E - \frac{2}{3} \Box R \right) + b'' \Box R \\ &= bF + b'E + c\,\Box R \end{aligned}$$

E is Gauss-Bonnet invariant term and F is the square of the Weyl tensor.

$$\begin{aligned} E &\equiv {}^{*}R_{\mu\nu\kappa\lambda}{}^{*}R^{\mu\nu\kappa\lambda} = R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \\ F &\equiv C_{\mu\nu\kappa\lambda}C^{\mu\nu\kappa\lambda} = R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} - 2R_{\mu\nu}R^{\mu\nu} + R^2/3, \end{aligned}$$

[Capper, Duff '74, Deser, Duff, Isham '76, Duff '77]

Quantum Conformal Anomaly

The dimensionless parameters b, b' and c are given by:

$$b = -\frac{1}{120(4\pi)^2}\left(N_S + 6N_F + 12N_G\right)$$
$$b' = \frac{1}{360(4\pi)^2}\left(N_S + \frac{11}{2}N_F + 62N_G\right)$$
$$c = -\frac{1}{180(4\pi)^2}\left(N_S + 6N_F - 18N_G\right),$$

where we consider N_S scalars (spin-0), N_F Dirac fermions (spin-1/2) and N_G abelian gauge (spin-1) fields.

$$\begin{aligned} \mathrm{MSSM}: N_S = 104,\ N_F = 32,\ N_G = 12 \\ \mathrm{SM}: N_S = 4,\ N_F = 24,\ N_G = 12 \\ \mathrm{Curent\ Universe}: N_S = 0,\ N_F = 0,\ N_G = 1 \end{aligned}$$

Quantum Backreaction

The differential equation derived from Einstein equations

$$\frac{2\,\dot{a}\,\dddot{a}}{a^2}-\frac{\ddot{a}^2}{a^2}+\frac{2\,\ddot{a}\,\dot{a}^2}{a^3}-\left(3+\frac{2b'}{c}\right)\frac{\dot{a}^4}{a^4}-\frac{1}{8\pi c\,G}\frac{\Lambda}{3}+\frac{1}{8\pi c\,G}\frac{\dot{a}^2}{a^2}=0\,.$$

The differential equation with respect to Hubble parameter,

$$6H^2\dot{H}+2H\ddot{H}-\dot{H}^2-\frac{2b'}{c}H^4-\frac{1}{8\pi c\,G}\frac{\Lambda}{3}+\frac{1}{8\pi c\,G}H^2=0$$

For the relatively small cosmological constant $8b'\Lambda/3 \ll M_{\mathrm{P}}$,

$$H_{\mathrm{C}}\simeq\sqrt{\frac{\Lambda}{3}},\quad H_{\mathrm{Q}}\simeq\sqrt{\frac{1}{16\pi b' G}}$$

Quantum Backreaction

The first-order differential equation from Einstein equations

$$\frac{dy}{dx} = \frac{b'(x - x^{-1/3} + \frac{2b'\Lambda}{3M_{\mathrm{P}}^2}x^{-5/3})}{6cy} - 1$$

where:

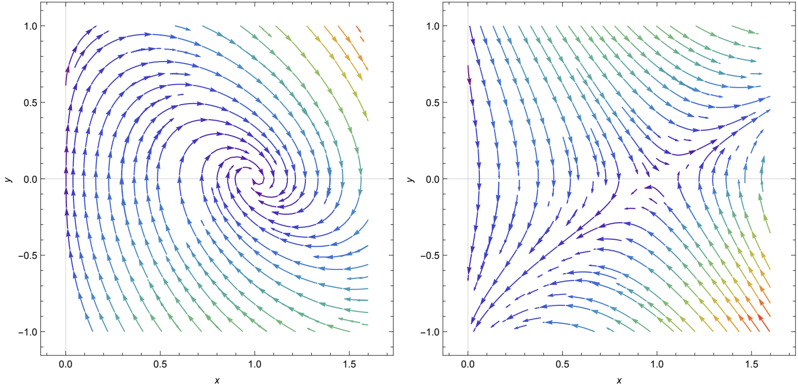
$$x = \left(\frac{H}{H_{\mathrm{Q}}}\right)^{3/2}, \quad y = \frac{\dot{H}}{2H_{\mathrm{Q}}^{3/2}}\,H^{-1/2}, \quad dt = \frac{dx}{3H_{\mathrm{Q}}x^{2/3}y},$$

We consider the following two differential equations

$$\frac{dx}{d\tau} = 3x^{2/3}y, \quad \frac{dy}{d\tau} = \frac{b'\left(x^{5/3} - x^{1/3} + \frac{2b'\Lambda}{3M_{\mathrm{P}}^2}x^{-1}\right)}{2c} - 3\,x^{2/3}y.$$

Hubble Diagram with $\Lambda = 0$

The critical point $(1,0)$ corresponds to $H_{\mathrm{Q}} = M_{\mathrm{P}}/\sqrt{2b'}$.

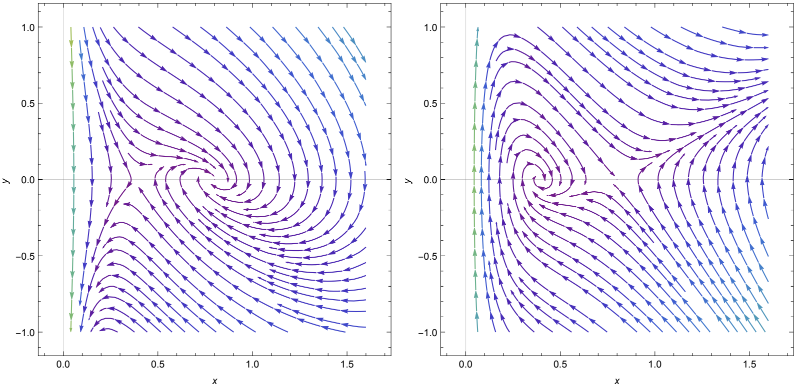


$$\mathrm{MSSM}: b' = 8/45\pi^2,\ c = -1/36\pi^2, \quad \mathrm{SM}: b' = 11/72\pi^2,\ c = 17/720\pi^2$$

[Starobinsky '80, Hawking, Hertog, Reall '01, Pelinson, Shapiro, Takakura '03]

Hubble Diagram with $\Lambda \neq 0$

De Sitter solution with $H_{\mathrm{C}} \simeq \sqrt{\Lambda/3}$, $H_{\mathrm{Q}} \simeq M_{\mathrm{P}}/\sqrt{2b'}$

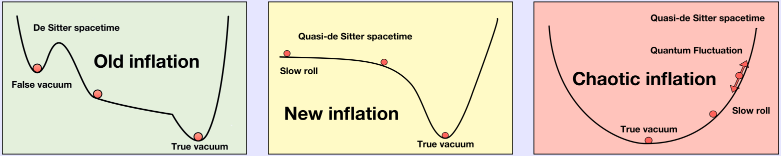


$$2b'\Lambda/3M_{\mathrm{P}}^2 = 10^{-0.7}$$

Results and Summary

- For $c < 0$ and $H(t_0) \lesssim \Lambda$, de Sitter solutions are generally destabilized and the expansion of spacetime terminates: $H(t) \to 0$.
- For $c < 0$ and $H(t_0) \gtrsim \Lambda$, de Sitter solutions approach the stable critical point corresponds to the quantum de Sitter attractor: $H(t) \to M_{\mathrm{P}}/\sqrt{2b'}$.
- For $c > 0$ and $H(t_0) \ll \Lambda$, de Sitter solutions go towards the infinity and de Sitter expansion of spacetime increases continuously: $H(t) \to \infty$.
- For $c > 0$ and $\Lambda \lesssim H(t_0) \lesssim M_{\mathrm{P}}/\sqrt{2b'}$, the de Sitter solutions approach the stable critical point corresponds to the classic de Sitter attractor: $H(t) \to \sqrt{\Lambda/3}$.
- For $c < 0$ and $M_{\mathrm{P}}/\sqrt{2b'} \lesssim H(t_0)$, de Sitter solutions go towards the infinity and the de Sitter expansion of spacetime increases continuously: $H(t) \to \infty$.

Eternal Inflation vs De Sitter Instability from Conformal Anomaly

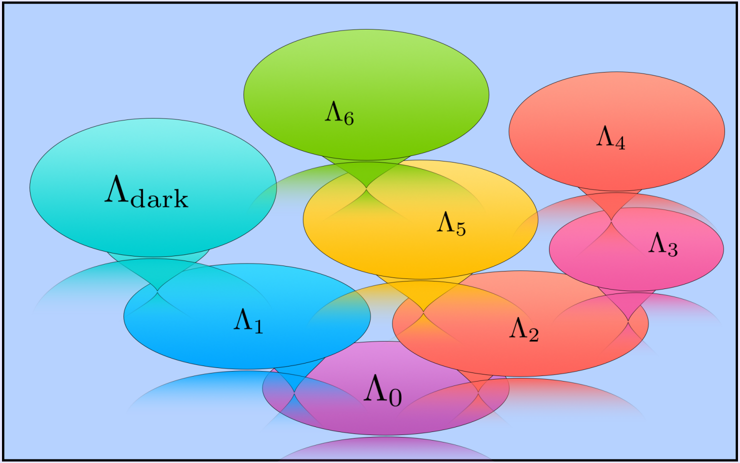


- The quantum backreaction from conformal anomaly generally destabilizes de Sitter spacetime.
- Unless the fine-tuning of the conformal anomaly and the higher derivative terms, the inflation finally becomes the Planckian inflation with the Hubble scale $H \approx M_{\mathrm{P}}$ or terminates $H(t) \longrightarrow 0$.
- The eternal inflation would be impossible for the later situation.

[**H, Mastui**, arXiv:1806.10339]

Anthropic Principle ?

String Landscape + ~~Eternal Inflation~~ $\Longrightarrow$ Finetuning Problem



Conclusion and Summary

- Until quite recently, most inflation models are thought to be eternal. If there are populating various vacua like string landscape and the eternal inflation takes place, the fine-tuning of the parameters like cosmological constant can be avoided by the anthropic argument.
- However, de Sitter instability provides negative evidences for eternal inflation.
- The de Sitter instability from conformal anomaly strongly depend on the initial conditions and the particle contexts. However, unless the fine-tuning of the conformal anomaly and the higher derivative terms which corresponds to the specific choice of QG or the fine-tuning of the initial conditions, the inflation finally becomes the Planckian inflation $H \approx M_{\mathrm{P}}$ or terminates $H(t) \to 0$.
- Furthermore, recently proposed Swampland De Sitter Conjecture and de Sitter instability from quantum backreaction strongly restrict eternal inflation scenarios. Both cases excludes eternal old/hilltop inflations. The chaotic inflation is also restricted, but the possibility would not be excluded.