Symmetry and Geometry of Generalized Higgs Sectors

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in collaboration with
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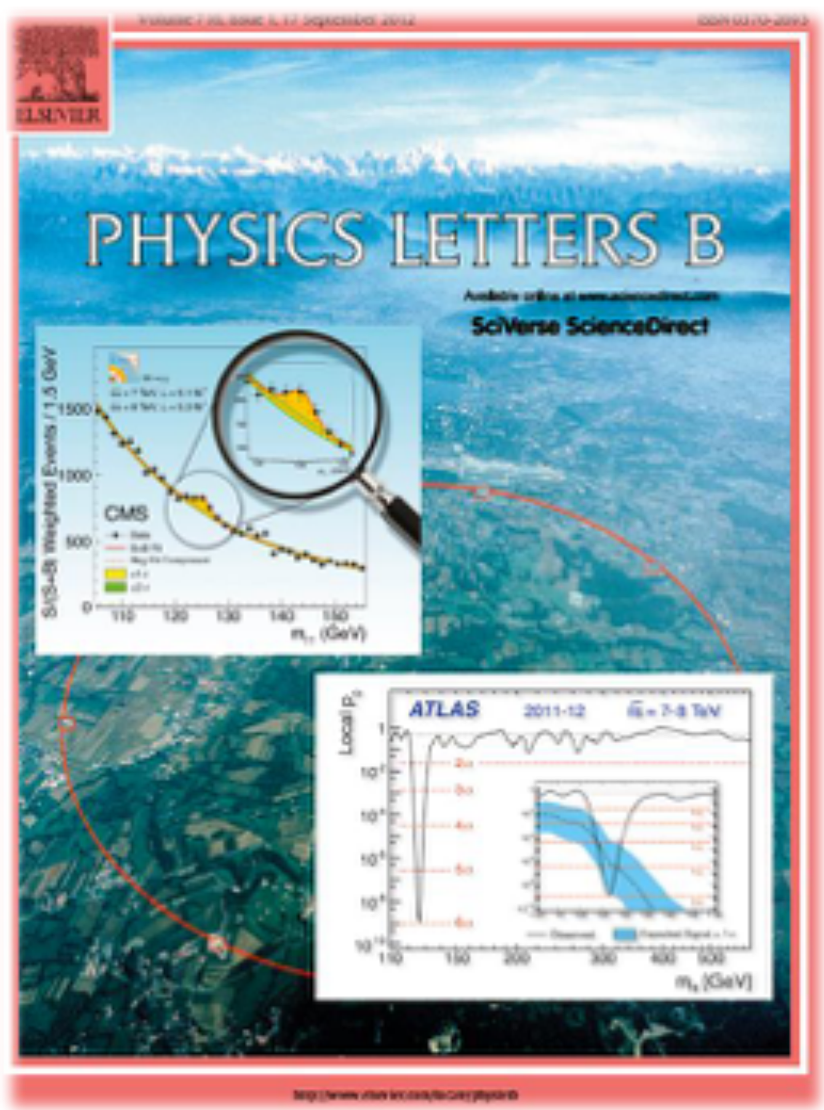
Outline

- Introduction
- Higgs Effective Field Theory
- Perturbative Unitarity vs. EWPTs
- Summary and Outlook

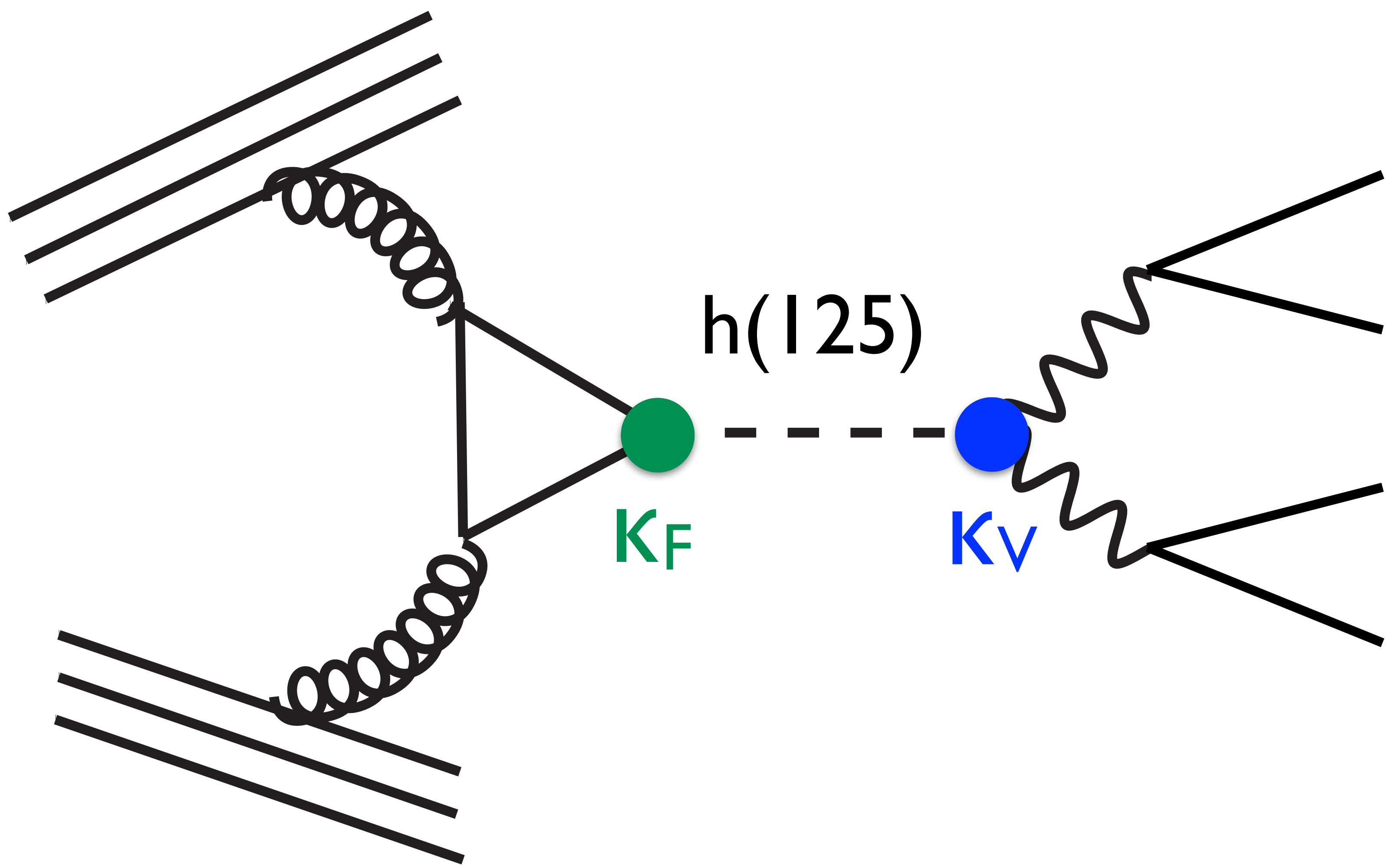
We found a h(125) !

- The 125GeV Higgs boson was discovered at the LHC.



h(125) = h_{SM} ?

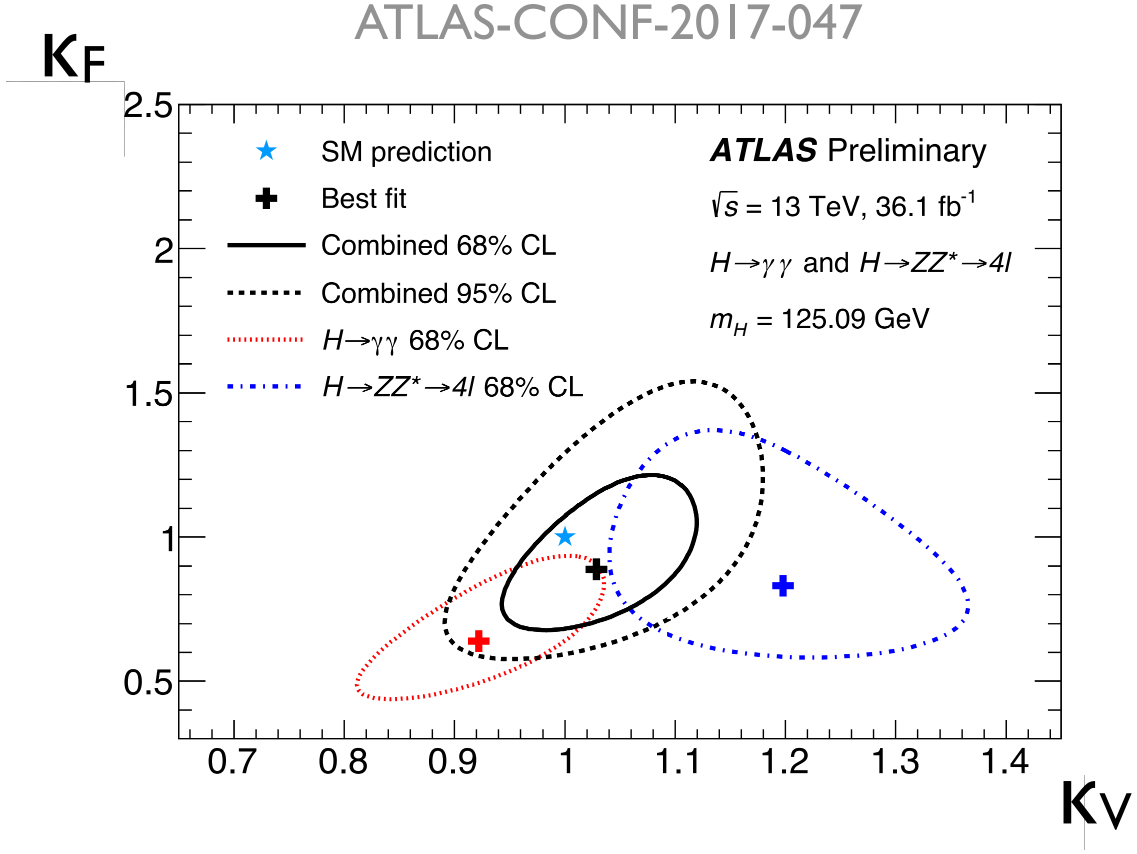
- Higgs coupling measurements



h(125) = h_{SM} ?

- Higgs coupling measurements

ATLAS-CONF-2017-047



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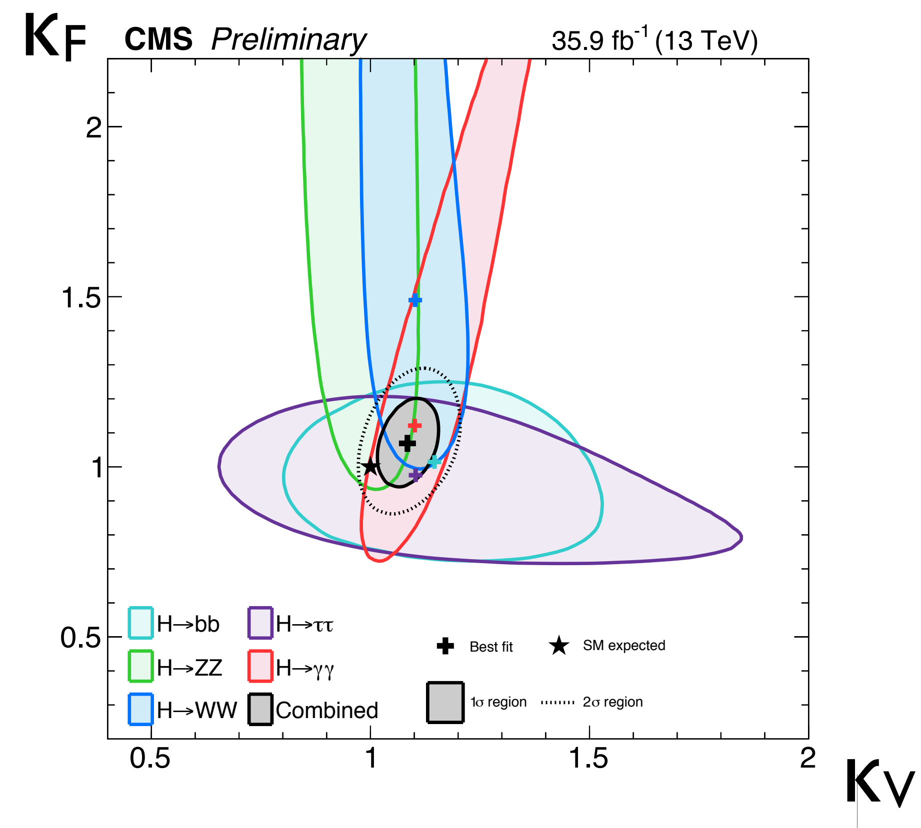
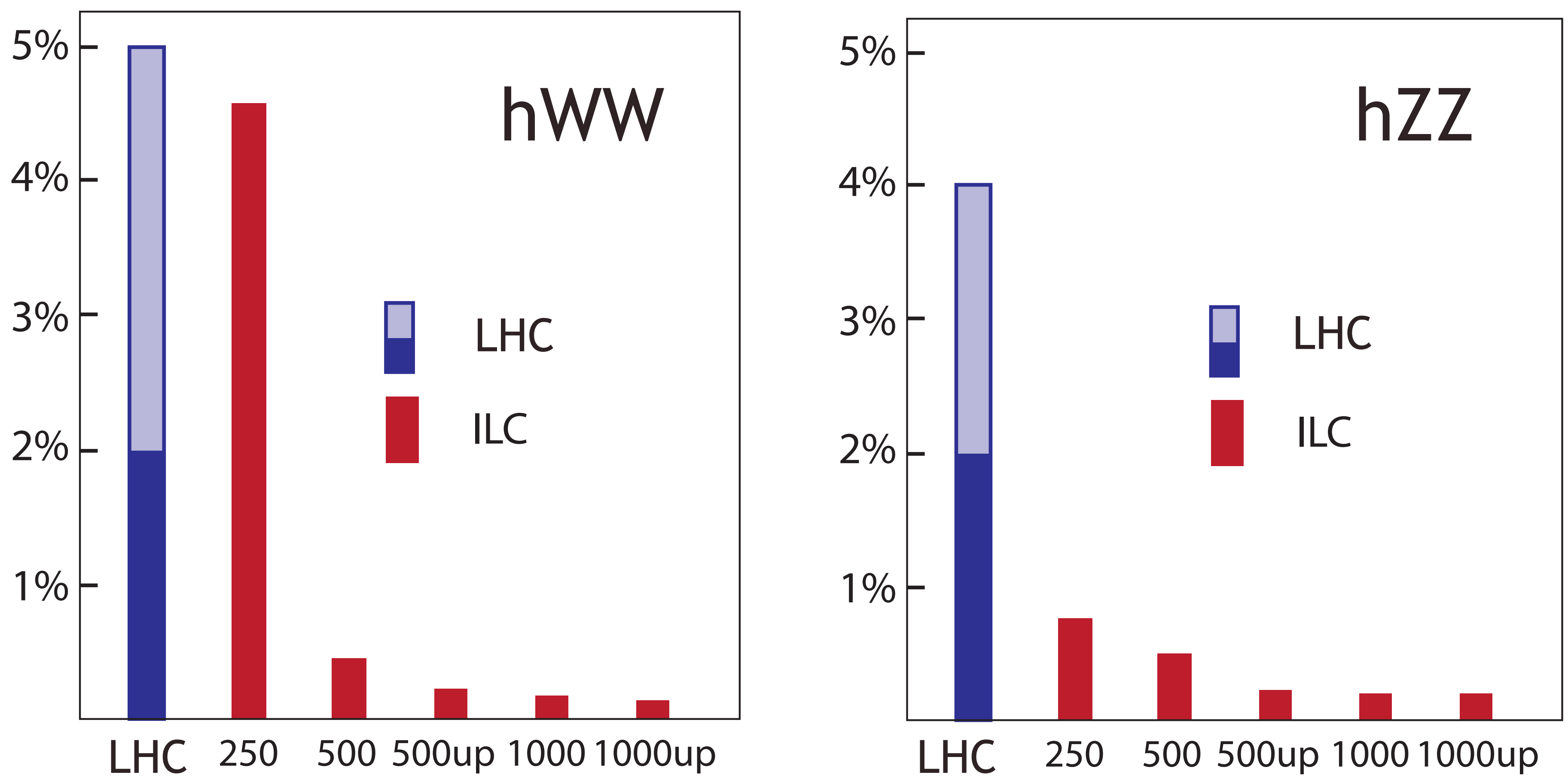


~10% deviation from $\kappa_{V,F} = 1$ is still allowed.

$h(125) = h_{SM}$?

- Higgs coupling measurements

M.Peskin (2014)



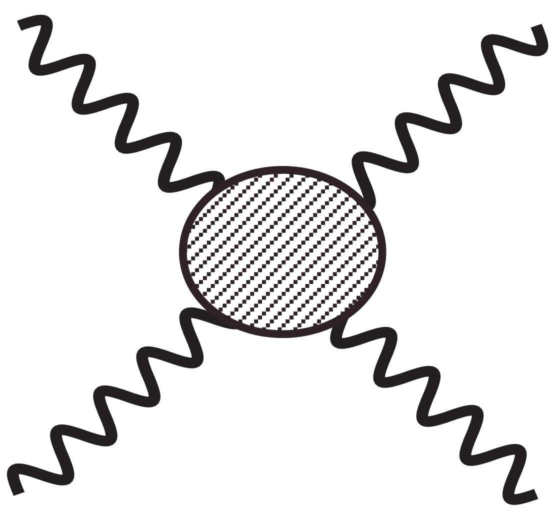
~1% precise measurement is expected to be realized.

What if $\kappa_V \neq 1$?

$W_L W_L \to W_L W_L$

Longitudinal

Longitudinal

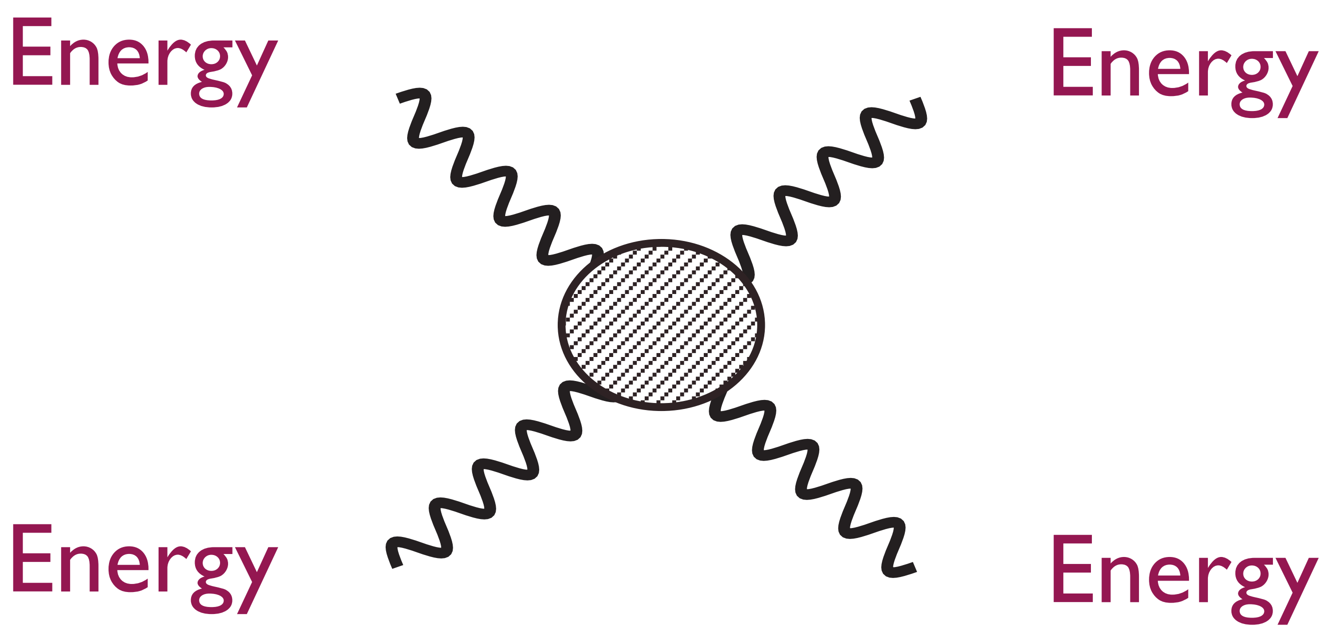


Longitudinal

Longitudinal

What if $\kappa_V \neq 1$?

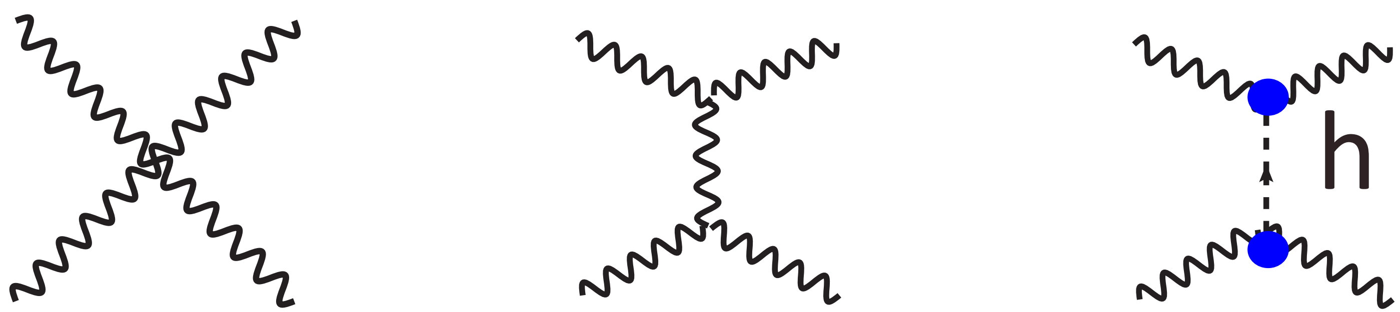
<u>$W_L W_L \to W_L W_L$</u>



Unitarity is violated ?

What if $\kappa_V \neq 1$?

$\underline{W_L W_L \to W_L W_L}$

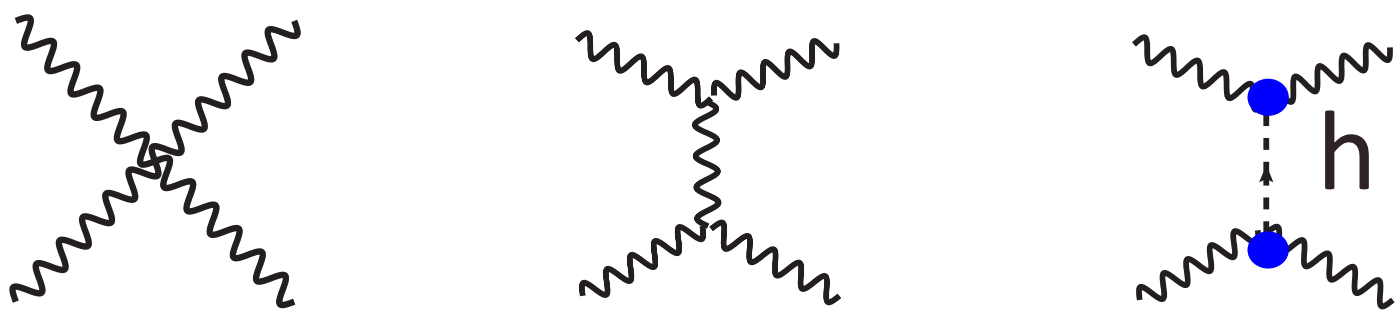


$$\mathcal{M}_{W_L W_L \to W_L W_L} \sim \frac{E^2}{M^2}(1 - \kappa_V^2)$$

Gauge boson   Higgs boson

What if $\kappa_V \neq 1$?

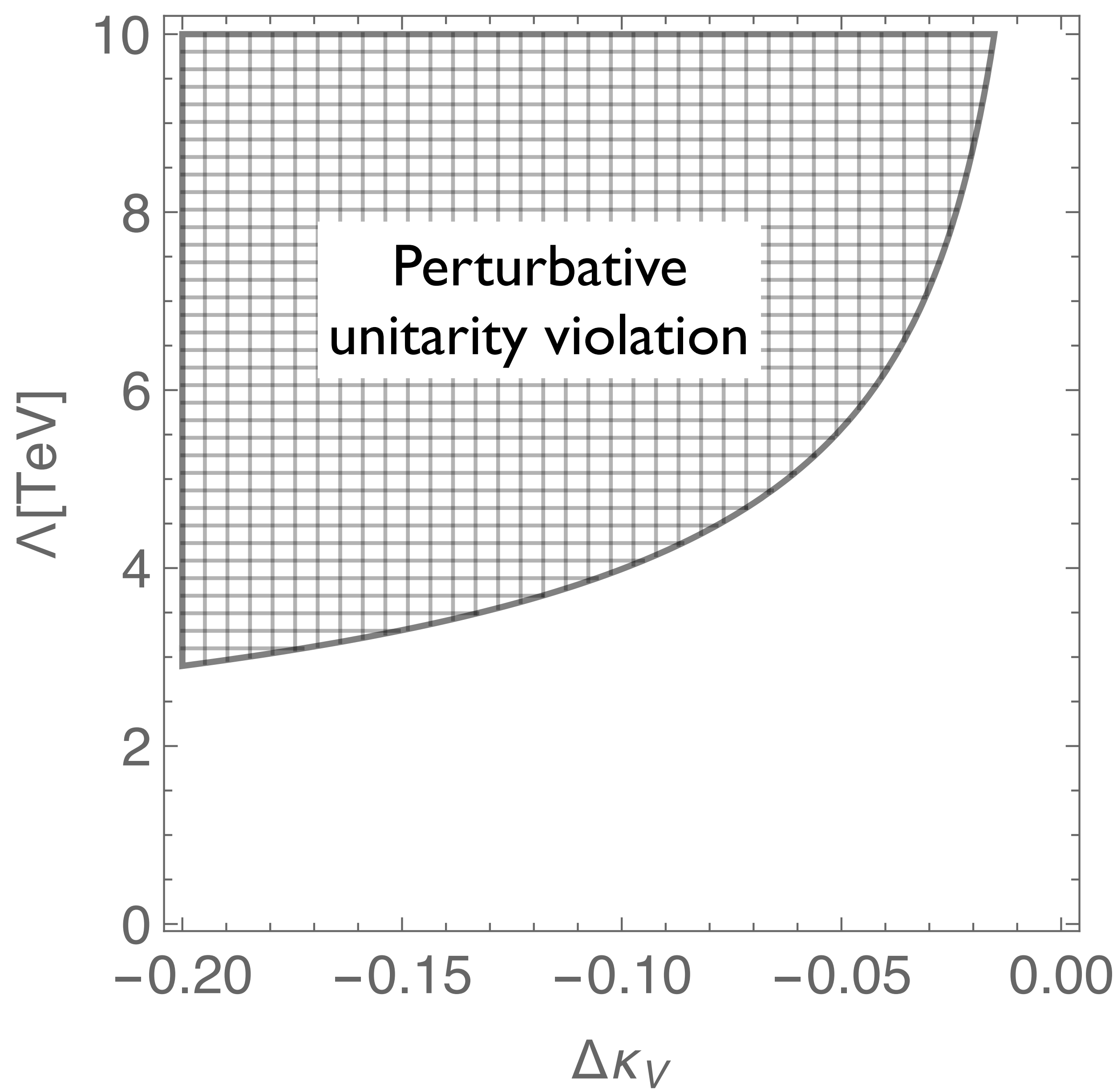
$W_L W_L \to W_L W_L$



$$\mathcal{M}_{W_L W_L \to W_L W_L} \sim \frac{E^2}{M^2}(1-\kappa_V^2)$$

- The SM higgs boson ($\kappa_V = 1$) keeps the theory perturbatively unitary.

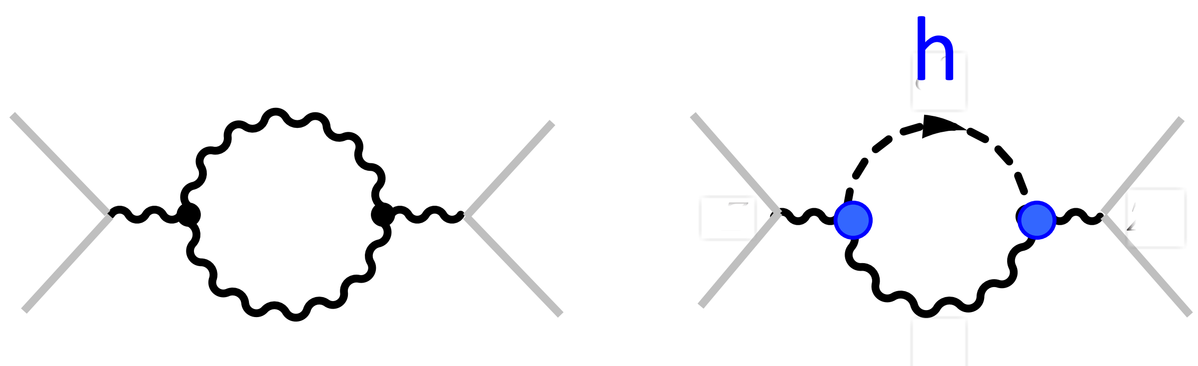
What if $\kappa_V \neq 1$?



- If $\kappa_V \neq 1$, perturbative **unitarity** in $W_L W_L$ scattering seems to be **violated** at certain energy scale.

What if $\kappa_V \neq 1$?

<u>ff → ff</u>

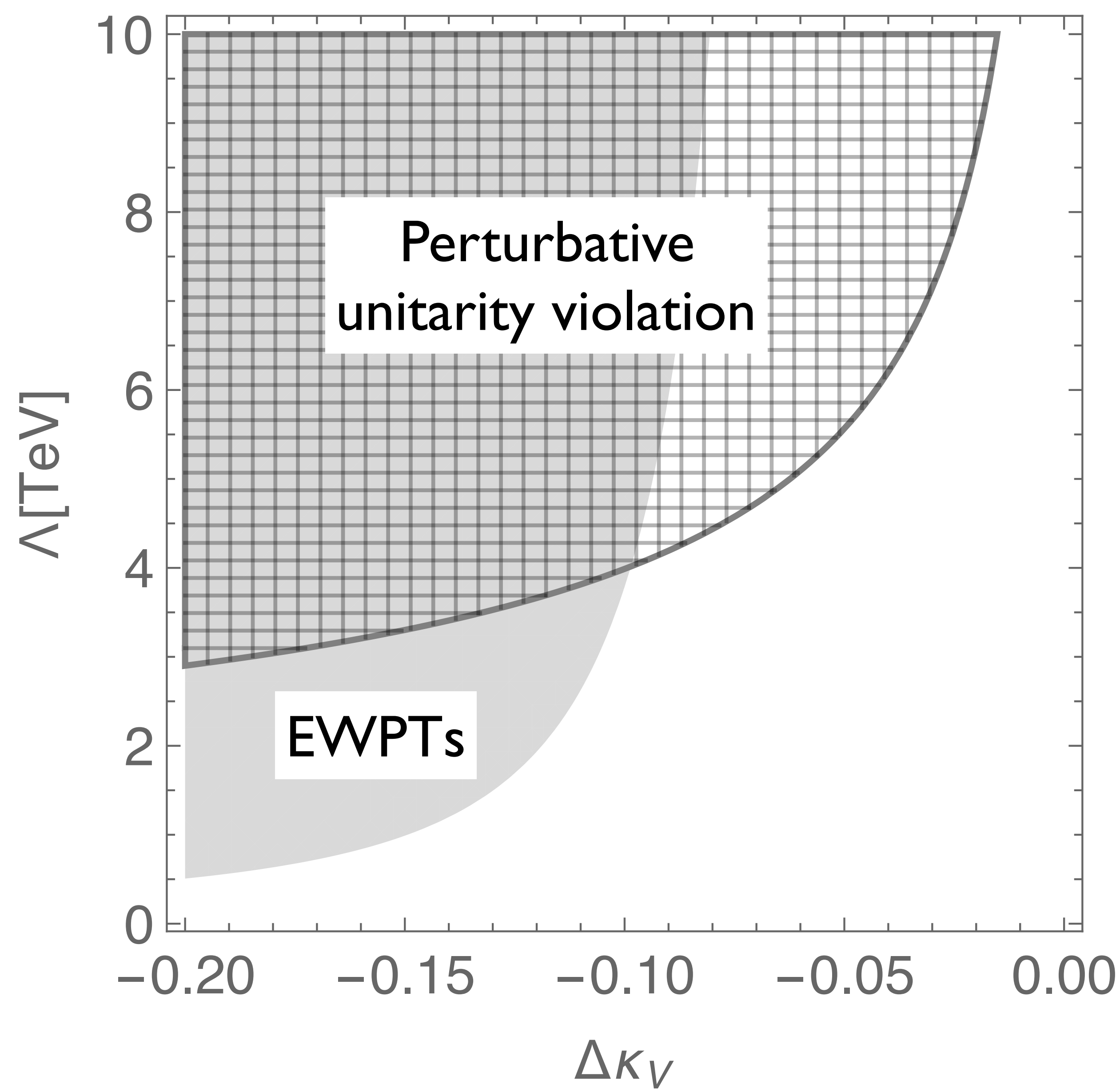


$$S \simeq \frac{1}{12}\left(1 - \kappa_V^2\right) \ln \frac{\Lambda^2}{M_h^2}$$

- Electroweak Precision Tests (EWPTs) at LEP :

$S = 0.03 \pm 0.10$ Global fitting by Gfitter Group

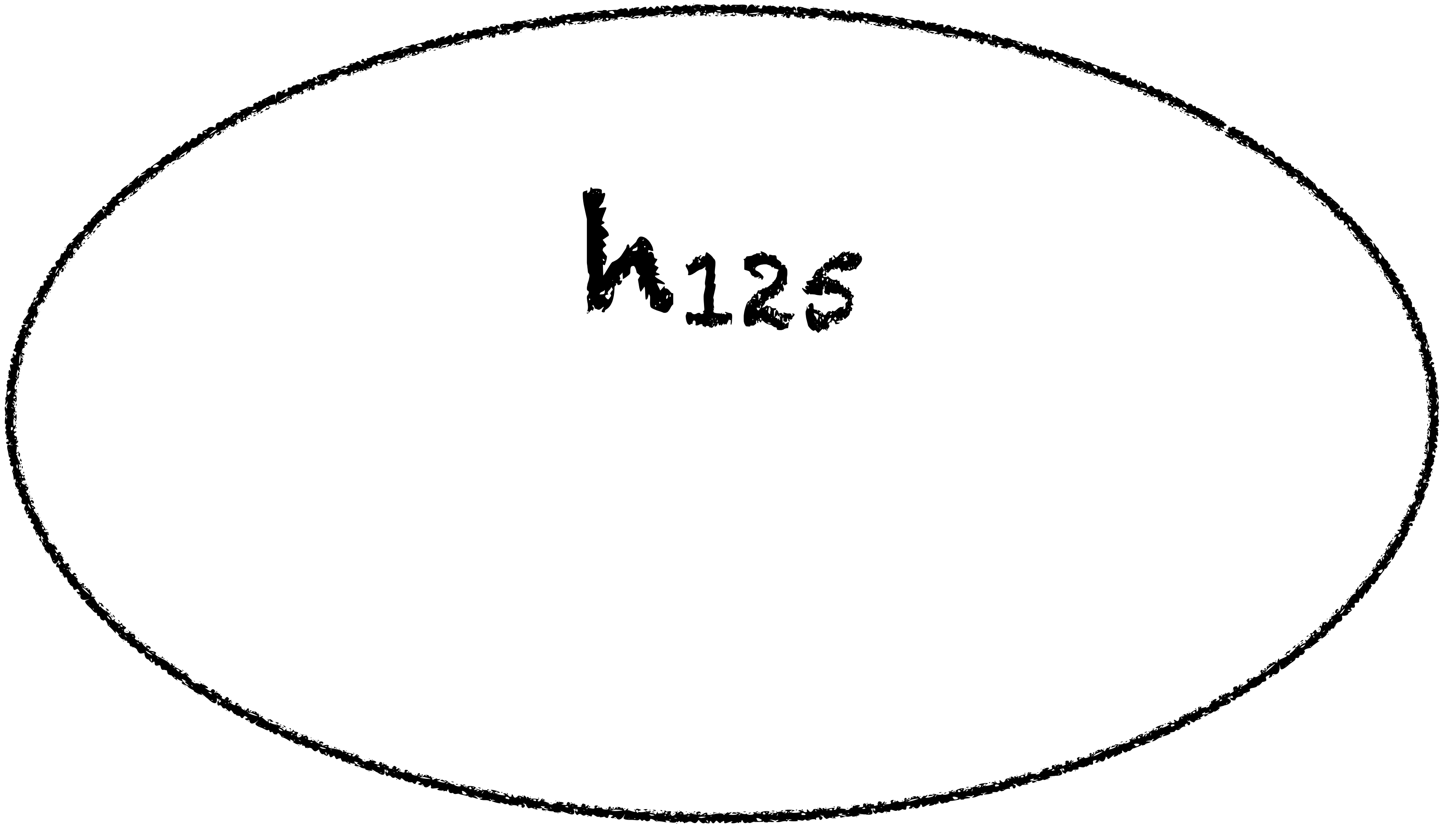
What if $\kappa_V \neq 1$?



- $\kappa_V \neq 1$ causes not only **perturbative unitarity violation** but also **inconsistency with EWPTs**.

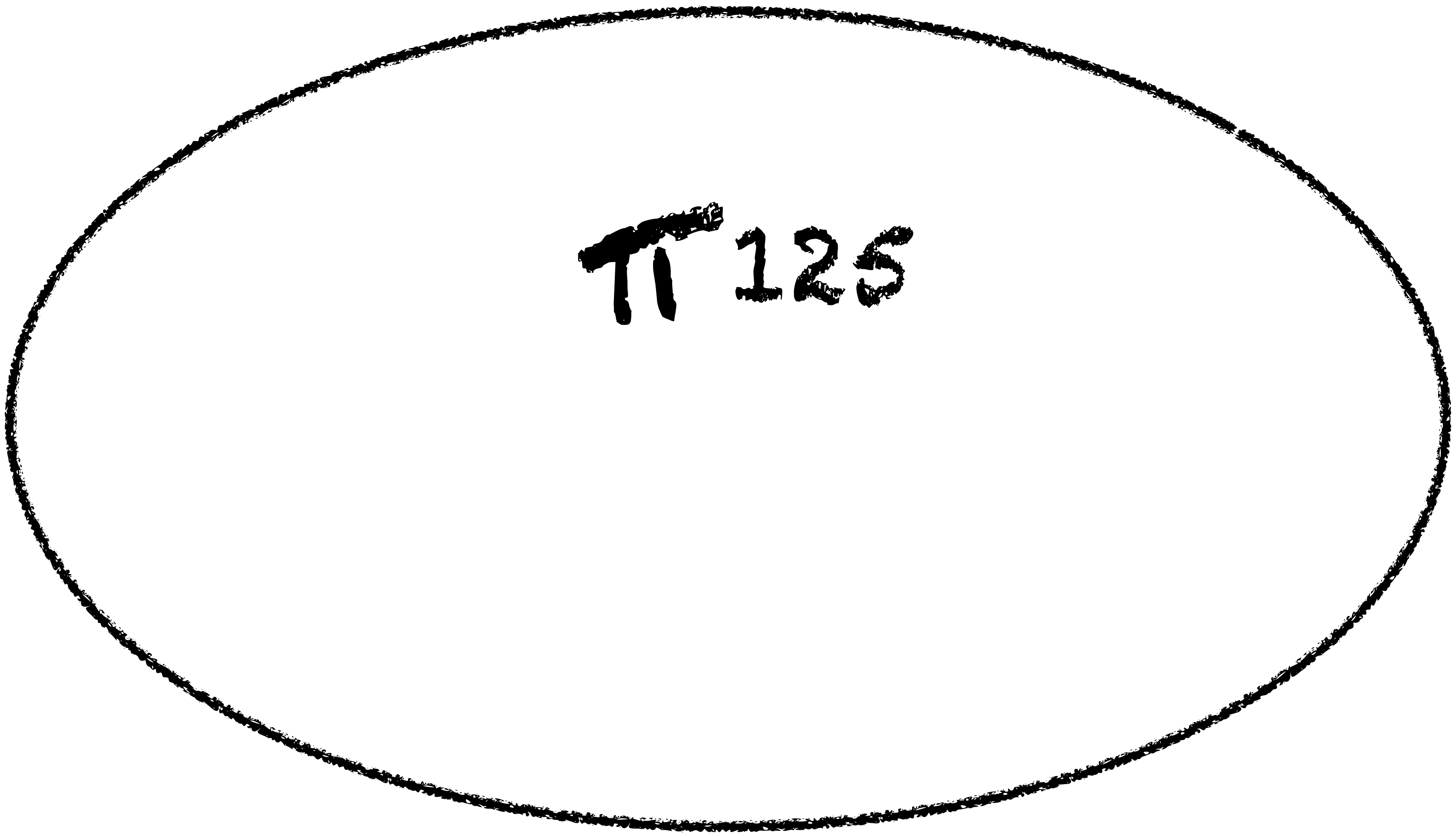
What if $\kappa_V \neq 1$?

Higgs Sector



What if $\kappa_V \neq 1$?

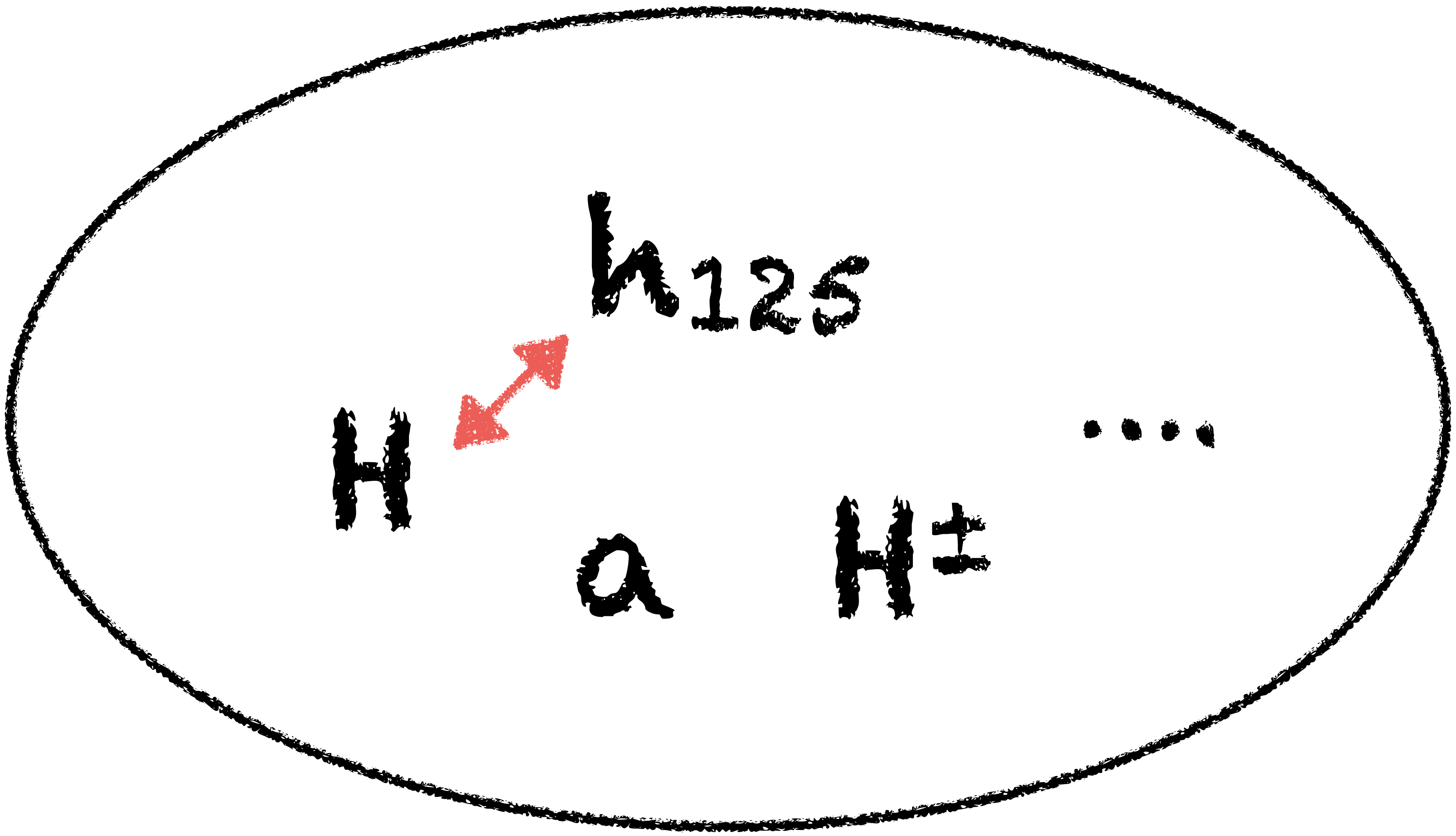
Higgs Sector



Composite ?

What if $\kappa_V \neq 1$?

Higgs Sector



Elementary. But mixed state ?

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Higgs EFT

Q. How can we formulate the Higgs sector ?

- $SU(2)_W$ doublet filed : 4 = 3(NGBs) + 1(higgs)

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i(\phi^1 + i\phi^2) \\ \phi^0 - i\phi^3 \end{pmatrix}$$

Higgs EFT

Q. How can we formulate the Higgs sector ?

- $SU(2)_W$ doublet filed : 4 = 3(NGBs) + 1(higgs)

$$\Phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v}\pi^a\tau^a\right)\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

NGBs (π^a) 125GeV higgs (h)

Higgs EFT

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- $SU(2)_W$ doublet filed : 4 = 3(NGBs) + 1(higgs)

$$\Phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v}\pi^a \tau^a\right) \left(\begin{array}{c} 0 \\ v + h \end{array} \right)$$

- Examples :

$$\mathcal{L}_{\mathrm{SM}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \cdots$$

$$\mathcal{L}_{\mathrm{SMEFT}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{C}{\Lambda^2} \Phi^\dagger \Phi \, (D_\mu \Phi)^\dagger (D^\mu \Phi) + \cdots$$

- What if $h(125) \not\in \Phi$? e.g. Composite Higgs models

CCWZ method

- Systematic way for constructing model with G → H

Coleman-Wess-Zumino (1969)
Coleman-Callen-Wess-Zumino (1969)

- General Lagrangian for NGBs arising through EWSB :

$$\mathcal{L}_\pi = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]$$

* We here assume "custodial symmetry" for simplicity.

- NGB fields : $U = e^{\frac{i}{v}\pi^a\tau^a}$
- Gauge bosons : $D_\mu U = \partial_\mu U - igU\mathbf{W}_\mu + ig_Y\mathbf{B}_\mu U$

CCWZ method

- General Lagrangian for NGBs and singlet scalar(s)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\sum_h \kappa_{WW}^h \frac{h}{v} + \sum_{h,h'} \kappa_{WW}^{hh'} \frac{hh'}{v^2} + \cdots\right)$$

Nagai-Tanabashi-Tsumura (2016)

- Examples:

$$\mathcal{L}_{\mathrm{SM}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\frac{h}{v} + \frac{h^2}{v^2}\right)$$

$$\mathcal{L}_{\mathrm{SM+S}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2c\frac{h}{v} + 2s\frac{H}{v} + c^2\frac{h^2}{v^2} + s^2\frac{H^2}{v^2} + 2cs\frac{hH}{v^2}\right)$$

$$\mathcal{L}_{\mathrm{MCSM}} = \frac{v^2}{4}\mathrm{Tr}[D_\mu U^\dagger D^\mu U]\left(1 + 2\left(1 - \frac{v^2}{8f^2}\right)\frac{h}{v} + \left(1 - \frac{v^2}{f^2}\right)\frac{h^2}{v^2} + \cdots\right)$$

CCWZ method

- General Lagrangian for NGBs and singlet scalar(s)

$$\mathcal{L} = \frac{v^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial^\mu U]\left(1 + 2\sum_h \kappa_{WW}^h \frac{h}{v} + \sum_{h,h'} \kappa_{WW}^{hh'} \frac{hh'}{v^2} + \cdots\right)$$

$$= \frac{1}{2}\,\partial_\mu(\pi^a \; h \; h' \; \cdots)\begin{pmatrix} \left(1 + 2\sum \kappa_{WW}^h \frac{h}{v} + \sum \kappa_{WW}^{hh'} \frac{hh'}{v^2}\right)(\delta_{ab} + \mathcal{O}(\pi^2)) & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \partial^\mu \begin{pmatrix} \pi^a \\ h \\ h' \\ \vdots \end{pmatrix}$$

$$= \frac{1}{2} g_{ij}(\phi)\,\partial_\mu \phi^i\,\partial^\mu \phi^j$$

- NGBs and Higgs field(s) define coordinates on a scalar manifold.

Generalized Higgs sector

- The EW sector in general extended Higgs scenarios can be expressed by the following form:

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j$$

$$\phi^i = (\underbrace{\pi^a}_{\text{NGBs}}, \underbrace{h, h', \cdots}_{\text{Higgses}})$$

- The interaction between π (~ W_L/Z_L) and Higgs bosons is determined by the structure of g_{ij} (Φ)

Alonso-Jenkins-Manohar (2016)

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Perturbative Unitarity

- The EW sector in general extended Higgs scenarios can be expressed by the following form:

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j$$

$$\phi^i = (\underbrace{\pi^a}_{\text{NGBs}}, \underbrace{h, h', \cdots}_{\text{Higgses}})$$

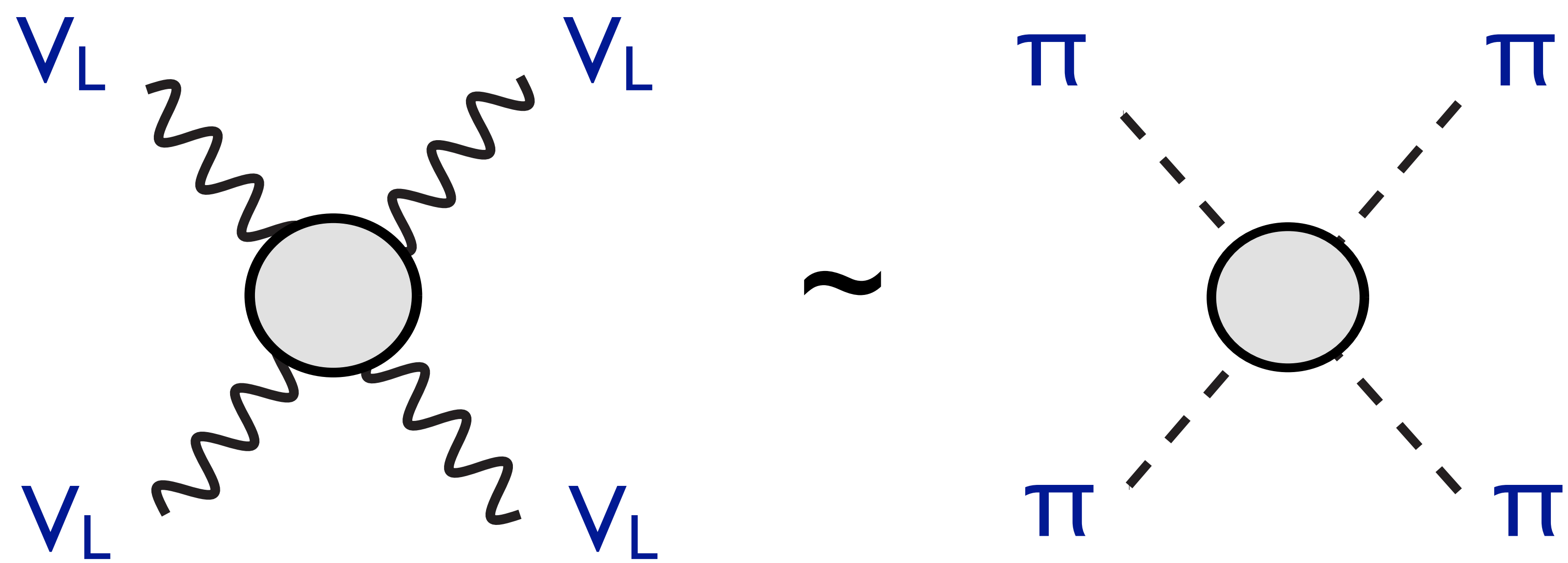
- The interaction between π (~ W_L/Z_L) and Higgs bosons is determined by the structure of $g_{ij}(\Phi)$

Alonso-Jenkins-Manohar (2016)

- **Perturbative Unitarity** / EWPTs → $g_{ij}(\Phi)$??

Perturbative Unitarity

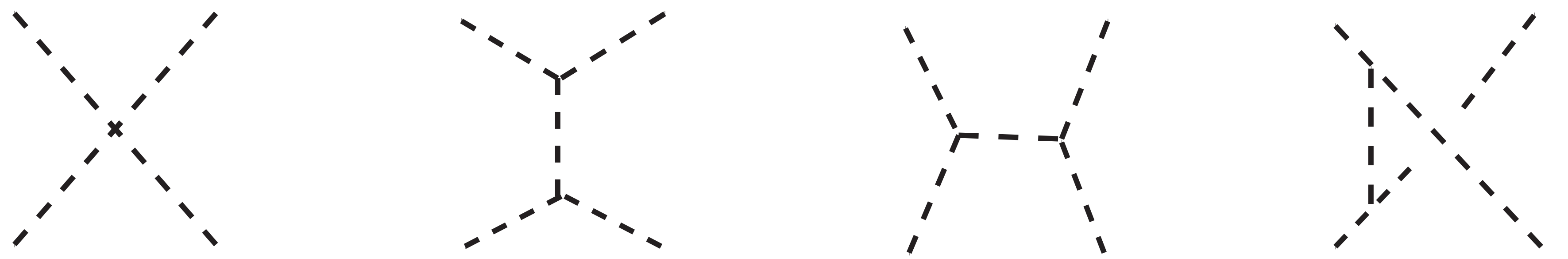
- High-energy behavior of W_L/Z_L ~ π. (Equivalence Theorem).



- NGBs/Higgses scattering amplitudes can be expressed in terms of geometry of the scalar manifold.

Perturbative Unitarity

<u>$\phi_i \phi_j \to \phi_k \phi_l$</u>



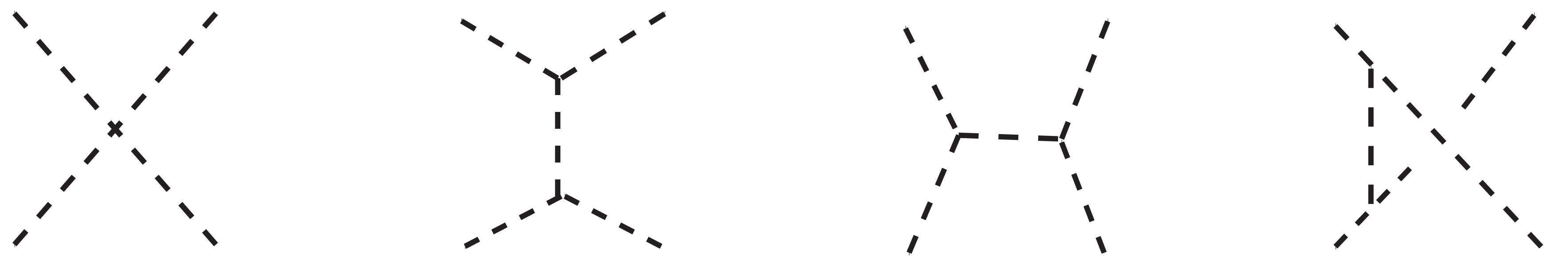
- Tree-level scattering amplitude:

$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}\left(\overline{R}_{iklj}+\overline{R}_{ilkj}\right)+\frac{t}{3}\left(\overline{R}_{ijlk}+\overline{R}_{iljk}\right)+\frac{u}{3}\left(\overline{R}_{ijkl}+\overline{R}_{ikjl}\right)$$

Riemann curvature tensor

Perturbative Unitarity

$\Phi_i \Phi_j \rightarrow \Phi_k \Phi_l$



- Tree-level scattering amplitude:

$$\mathcal{M}_{\phi_i\phi_j\to\phi_k\phi_l} \sim \frac{s}{3}\left(\overline{R}_{iklj}+\overline{R}_{ilkj}\right)+\frac{t}{3}\left(\overline{R}_{ijlk}+\overline{R}_{iljk}\right)+\frac{u}{3}\left(\overline{R}_{ijkl}+\overline{R}_{ikjl}\right)$$

- Perturbative unitarity condition:

$$\overline{R}_{ijkl}=0$$

EWPTs

- The EW sector in general extended Higgs scenarios can be expressed by the following form:

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j$$

$$\phi^i = (\underline{\pi^a}, \underline{h, h', \cdots})$$

NGBs ↗ ↖ Higgses

- The interaction between π (~ W_L/Z_L) and Higgs bosons is determined by the structure of g_{ij} (Φ)

Alonso-Jenkins-Manohar (2016)

- Perturbative Unitarity / **EWPTs** → g_{ij} (Φ) ??

EW corrections

- EW oblique corrections depend on not only geometry but also symmetry of the scalar manifold.
- The scalar manifold respects $SU(2)_W \times U(1)_Y$ gauge sym.

4 Killing vectors

w_a^i (a=1,2,3) $\quad$ y^i

$SU(2)_W$ symmetry $\quad$ $U(1)_Y$ symmetry

For examples:

$$w_a^b = \frac{v}{2}\delta_{ab} - \frac{1}{2}\epsilon_{abc}\pi^c + \mathcal{O}(\pi^2)$$

$$y^b = -\frac{v}{2}\delta_{3b} - \frac{1}{2}\epsilon_{3bc}\pi^c + \mathcal{O}(\pi^2)$$

EW corrections

S-parameter at one-loop

- UV divergence:

$$S_{\log} \propto (\bar{w}_3^j)_{;i}(\bar{y}^i)_{;j}$$

Alonso-Jenkins-Manohar (2016)

$$(w_a^i)_{;j} = \frac{\partial}{\partial \phi^j} w_a^i + \Gamma^i_{kj} w_a^k$$

$$(y^i)_{;j} = \frac{\partial}{\partial \phi^j} y^i + \Gamma^i_{kj} y^k$$

EW corrections

S-parameter at one-loop

- UV divergence:

$$S_{\log} \propto (\bar{w}_3^j)_{;i} (\bar{y}^i)_{;j}$$

Alonso-Jenkins-Manohar (2016)

SU(2)$_W$×U(1)$_Y$ sym.

$$[w_a, w_b] = -\epsilon_{abc} w_c$$

$$[w_a, y] = 0$$

$$v_{i;j;k} = R^l{}_{kji} v_l \quad (v = w_a, y)$$

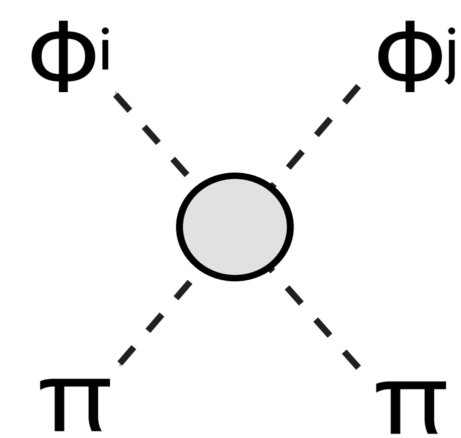
$$S_{\log} \propto \epsilon_{3bc} \overline{R}^i{}_{jc3} (\bar{w}_b^j)_{;i} + \epsilon_{3bc} \overline{R}^i{}_{jbc} (\bar{y}^j)_{;i}$$

EW corrections

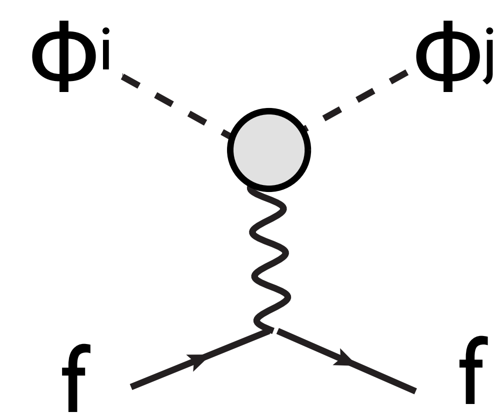
S-parameter at one-loop

- UV divergence:

$\pi\pi \to \phi^i\phi^j$



$f\bar{f} \to \phi^i\phi^j$



$$S_{\log} \propto \epsilon_{3bc}\overline{R}^i{}_{jc3}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}\overline{R}^i{}_{jbc}(\bar{y}^j)_{;i}$$

EW corrections

U-parameter at one-loop

- UV divergence:

$$U_{\log} \propto (\bar{w}_1^j)_{;i}(\bar{w}_1^i)_{;j} - (\bar{w}_3^j)_{;i}(\bar{w}_3^i)_{;j}$$

SU(2)$_W$×U(1)$_Y$ sym.

$$[w_a, w_b] = -\epsilon_{abc} w_c$$

$$[w_a, y] = 0$$

$$v_{i;j;k} = R^l{}_{kji} v_l \quad (v = w_a, y)$$

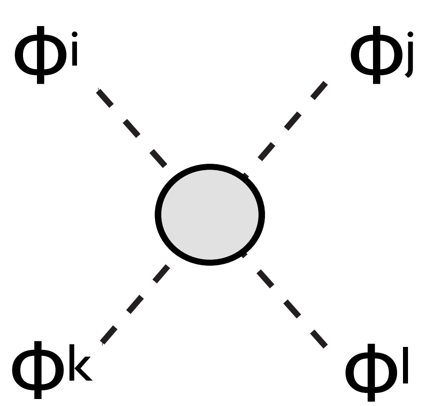
$$U_{\log} \propto \epsilon_{1bc} \overline{R}^i{}_{jbc} (\bar{w}_1^j)_{;i} - \epsilon_{3bc} \overline{R}^i{}_{jbc} (\bar{w}_3^j)_{;i}$$

PU vs. EWPTs

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j \qquad \phi^i = (\pi^a, h, h', \cdots)$$

Perturbative Unitarity

* $\mathcal{M}_{\phi^i\phi^j \to \phi^k \phi^l}\Big|_{E^2} \sim \overline{R}_{ijkl}$



EW oblique corrections

* $S_{\log} \sim \epsilon_{3bc} \overline{R}^{i}{}_{jc3} (\bar{w}^j_b)_{;i} + \epsilon_{3bc} \overline{R}^{i}{}_{jbc} (\bar{y}^j)_{;i}$

* $U_{\log} \sim \epsilon_{1bc} \overline{R}^{i}{}_{jbc} (\bar{w}^j_1)_{;i} - \epsilon_{3bc} \overline{R}^{i}{}_{jbc} (\bar{w}^j_3)_{;i}$

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Summary

- We have discussed extended Higgs scenarios from the view point of perturbative unitarity and consistency with EWPTs.

- Physics in extended Higgs scenarios can be understood in terms of geometry and symmetry of "the scalar manifold".

Flat	→	Perturbative
Curved	→	Non-Perturbative

- Consistency with EWPTs does not imply the complete flatness of the scalar manifold.

- Precise measurements of the 125GeV Higgs couplings reveal the structure of the scalar manifold.