g-2の格子理論計算

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(RBC&UKQCD collaboration)

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RIKEN BNL Research Center

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Contents & References

- g-2 Hadronic Vacuum Polarization (HVP)
  Phys. Rev. Lett. 121 (2018) 022003
- g-2 Hadronic Light-by-Light (HLbL)
  Phys. Rev. D96 (2017) 034515
  Phys. Rev. Lett. 118 (2017) 022005
- Luchang Jin, Christoph Lehner, Aaron Meyer, talks at Lattice 2019
- Tom Blum, talk at Anomalies 2019

Collaborators / Machines

g-2 DWF
HVP & HLbL

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Peter Boyle (Edinburgh)
Norman Christ (Columbia)
Vera Guelpers (Southampton)
Masashi Hayakawa (Nagoya)
James Harrison (Southampton)
Taku Izubuchi (BNL/RBRC)
Christoph Lehner (BNL)
Kim Maltman (York)
Chulwoo Jung (BNL)
Andreas Jüttner (Southampton)
Luchang Jin (BNL)
Antonin Portelli (Edinburgh)

tau input for
g-2 HVP &
HVP GEVP

Mattia Bruno (CERN)
Aaron Meyer (BNL)
Christoph Lehner (BNL & Regensburg)
Taku Izubuchi (BNL & RBRC)

tau decay

Peter Boyle (Edinburgh)
Taku Izubuchi (BNL/RBRC)
Christoph Lehner (BNL)
Kim Maltman (York)
Antonin Portelli (Edinburgh)
Renwick James Hudspith (York)
Andreas Ju¨ttner(Southampton)
Randy Lewis (Southampton)
Hiroshi Ohki (RBRC/Nara Women)
Matthew Spraggs (Edinburgh)

Part of related calculation are done by resources from
USQCD (DOE), XSEDE, ANL BG/Q Mira (DOE, ALCC), Edinburgh BG/Q,
BNL BG/Q, RIKEN BG/Q and Cluster (RICC, HOKUSAI)

Support from RIKEN, JSPS, US DOE, and BNL

The RBC & UKQCD collaborations

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Yong-Chull Jang
Chulwoo Jung
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Stony Brook University

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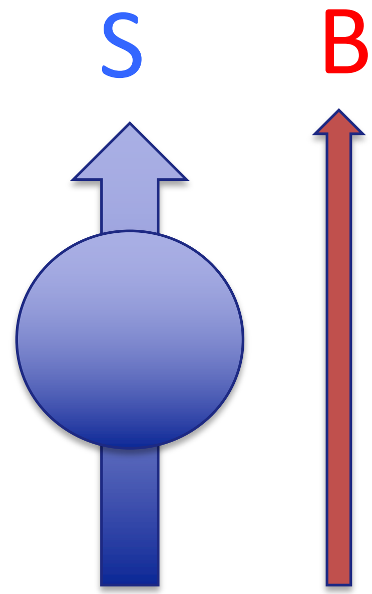
Sergey Syritsyn (RBRC)

Anomalous magnetic moment

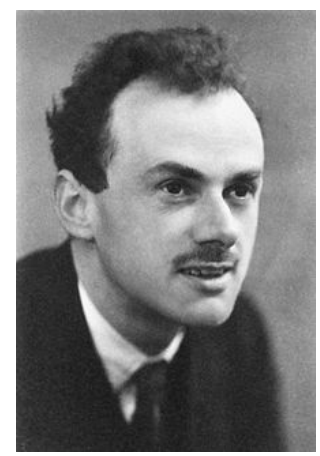


- Fermion's energy in the external magnetic field:

$$V(x) = -\vec{\mu}_l \cdot \vec{B} \qquad \vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l$$

- Magnetic moment Lande g-factor tree level value 2
- 1928 P.A.M. Dirac "Quantum Theory of Electron"
  Dirac equation (relativity, minimal gauge interaction)

$$i[\partial_\mu - ieA_\mu(x)]\gamma^\mu \psi(x) = m\psi(x)$$



- Non-relativistic and weak constant magnetic field limits of the Dirac equation :

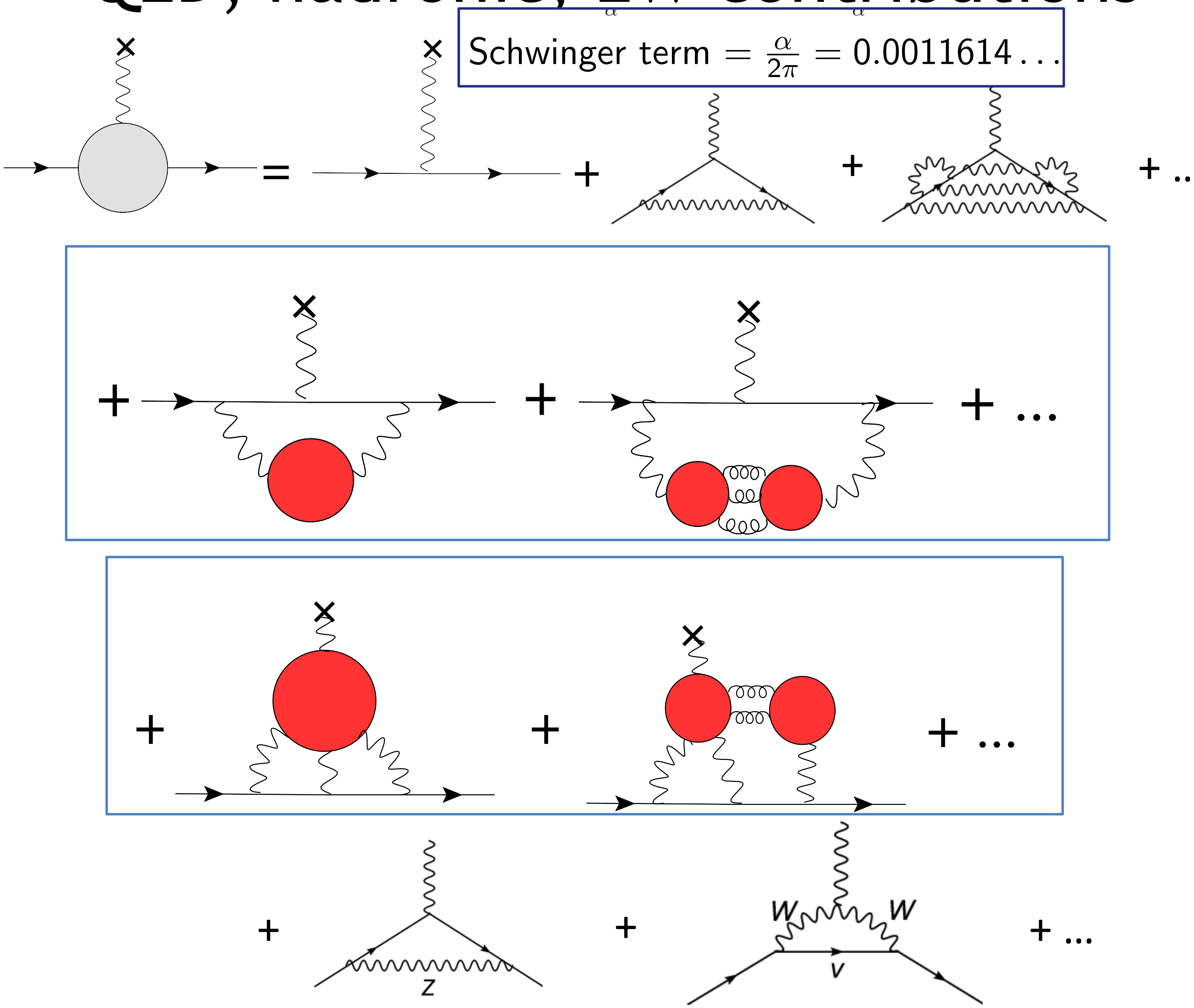
$$-i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{\nabla^2}{2m} + \frac{e}{2m} \left(\vec{L} + 2\vec{S} \right) \cdot \vec{B} \right] \psi$$

$$g_l = 2$$ (for Dirac Fermion l = e, μ, τ, ….)

SM Theory

$$\gamma^\mu \rightarrow \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2)\right)$$

- QED, hadronic, EW contributions



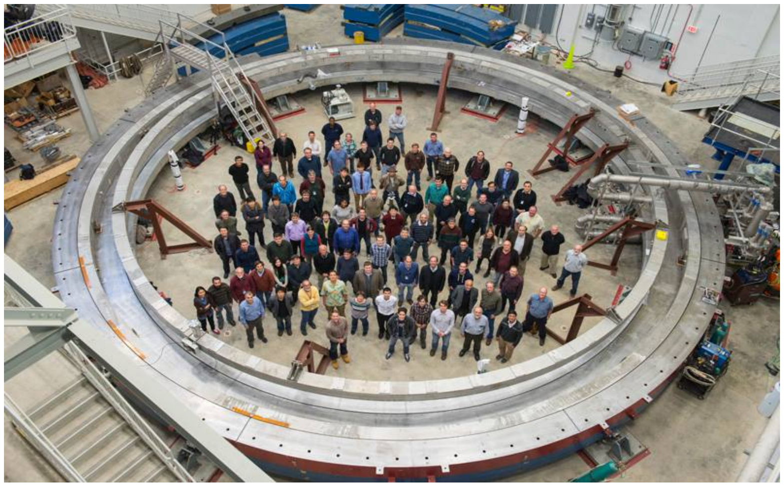
QED (5-loop)
Aoyama Hayakawa, Kinoshita, Nio
PRL109,111808 (2012)

Hadronic vacuum polarization (HVP)

Hadronic light-by-light (Hlbl)

Electroweak (EW)
Knecht et al 02
Czarnecki et al. 02
......

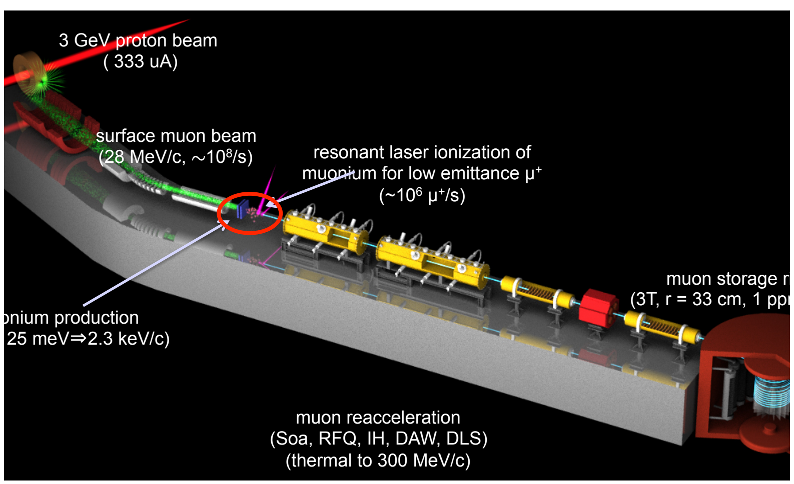
muon anomalous magnetic moment



BNL g-2 till 2004 : ~ 3.7 σ larger than SM prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\,\underbrace{(2.6)}_{\rm HLbL}\,\underbrace{(0.1)}_{\rm other}\,\underbrace{(6.3)}_{\rm EXP}\times 10^{-10}$$



FNAL E989 (began 2017-)

2019: BNL level error : (6.3) -> 4.5×10^{-10}

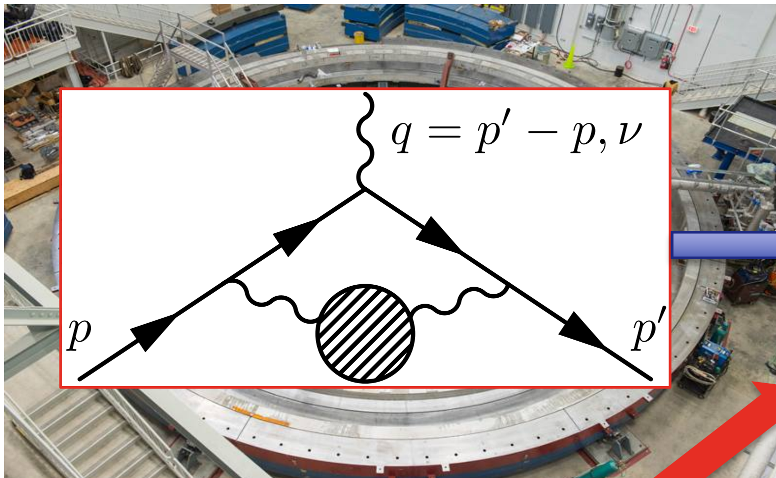
2022(?): $1.6\ 10^{-10}$ x4 precise 0.14ppm

J-PARC E34 (IMPORTANT different systematics !)

ultra-cold muon beam

0.37 ppm then 0.1 ppm, also EDM

muon anomalous magnetic moment



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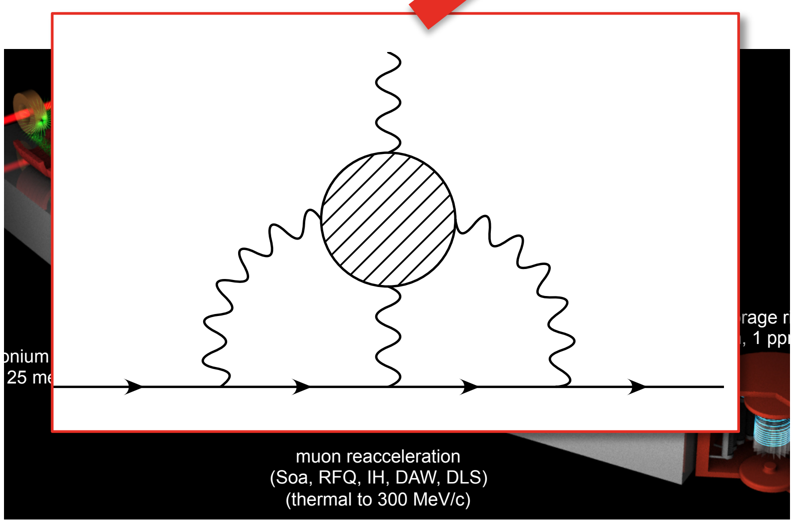

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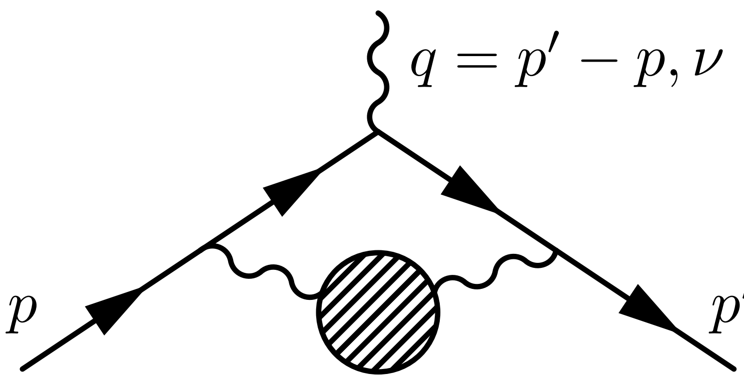
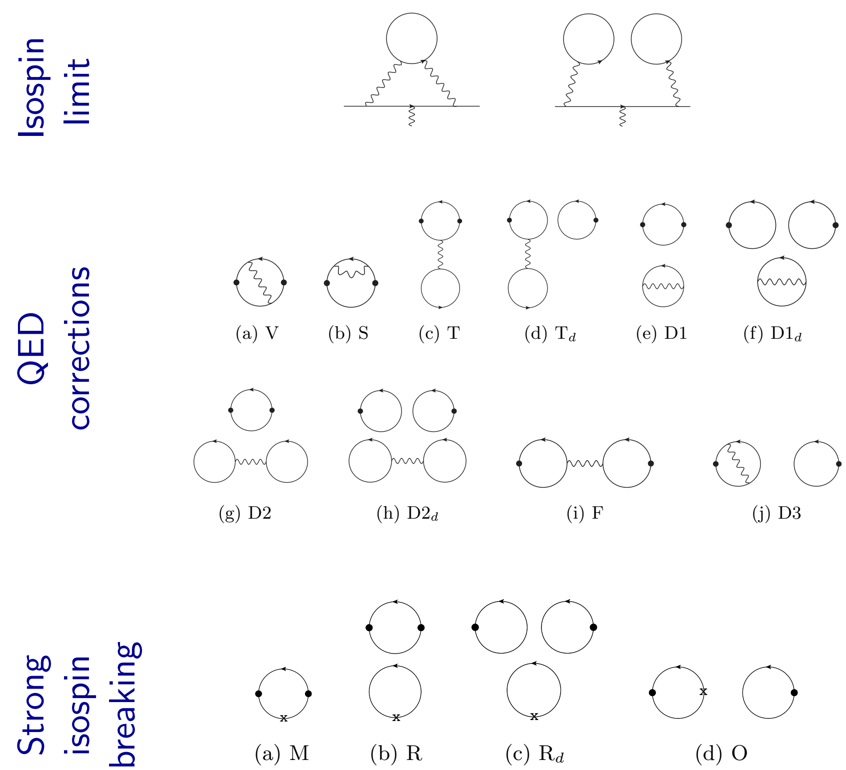
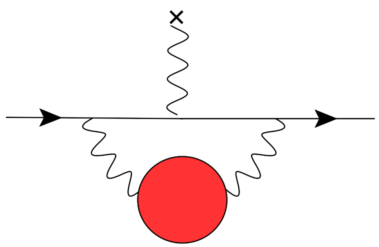
Hadronic Vacuum Polarization (HVP) contribution to g-2

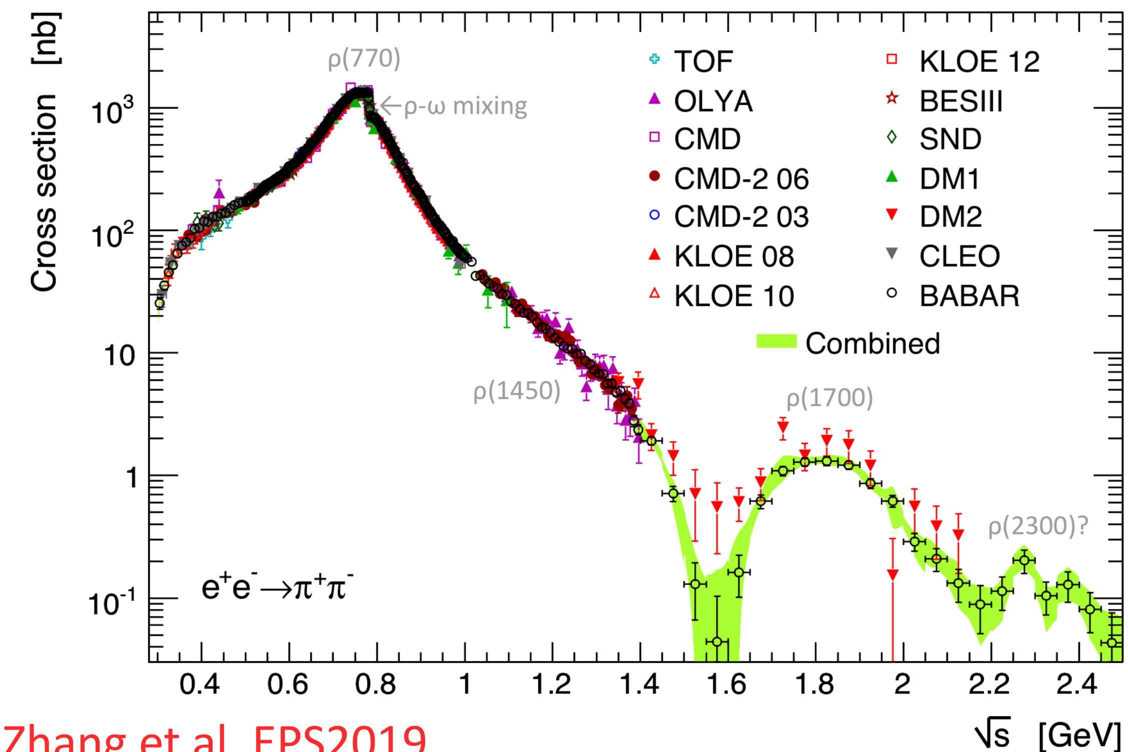




Quark & anti-quark contribution

g-2 from R-ratio

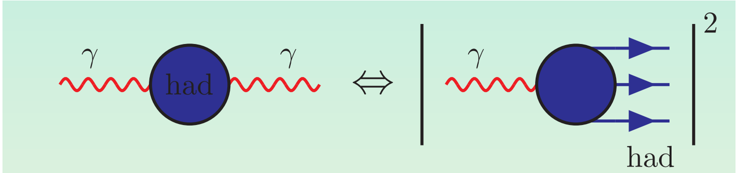




Zhang et al. EPS2019

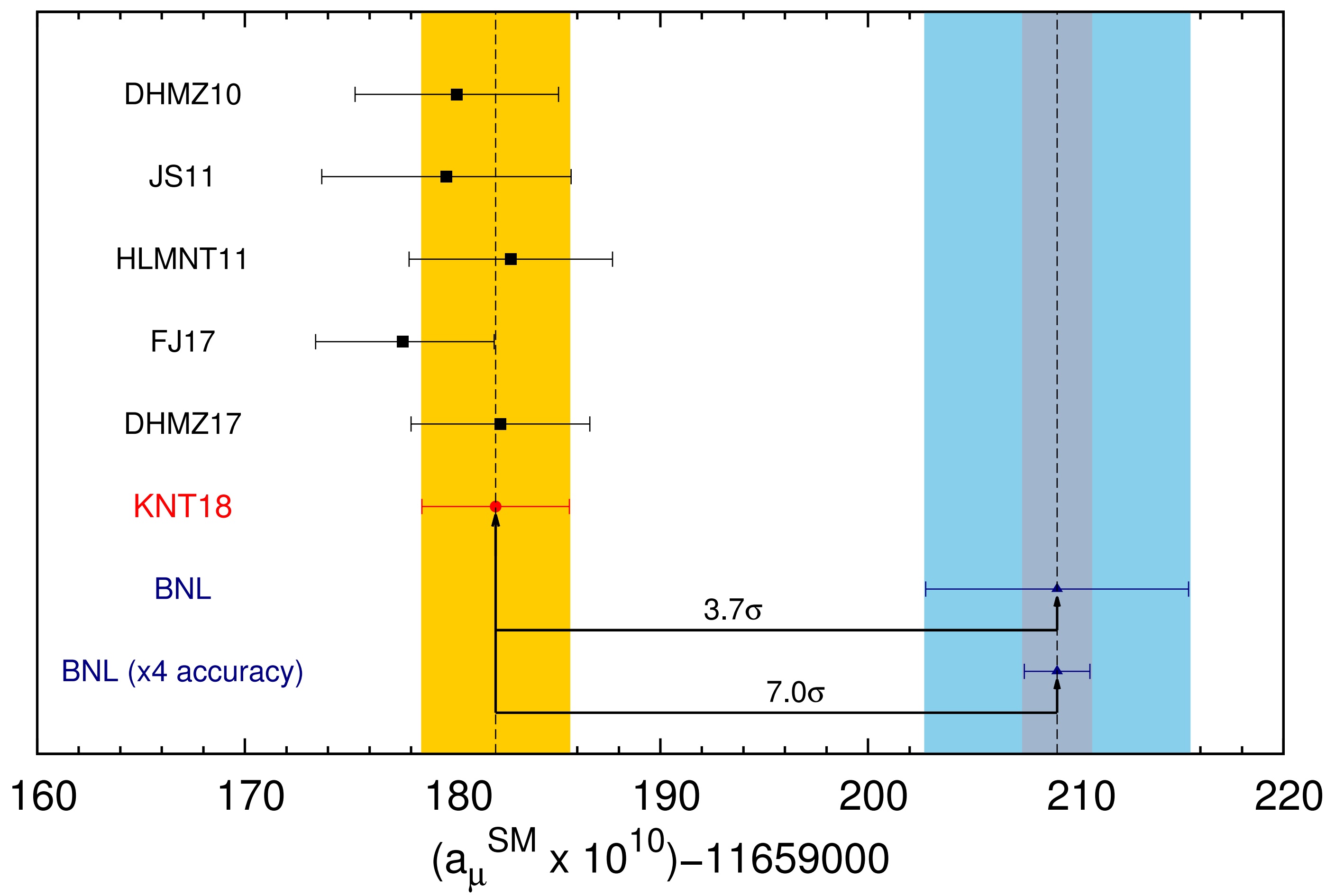
- From experimental e+ e- inclusive hadron decay cross section $\sigma_{total}(s)$ in time-like s = q^2 >0, and dispersion relation, optical theorem

$$a_\mu^{\rm HVP} = \frac{1}{4\pi^2}\int_{s_{th}}^{\infty} ds K(s)\sigma_{\rm total}(s)$$

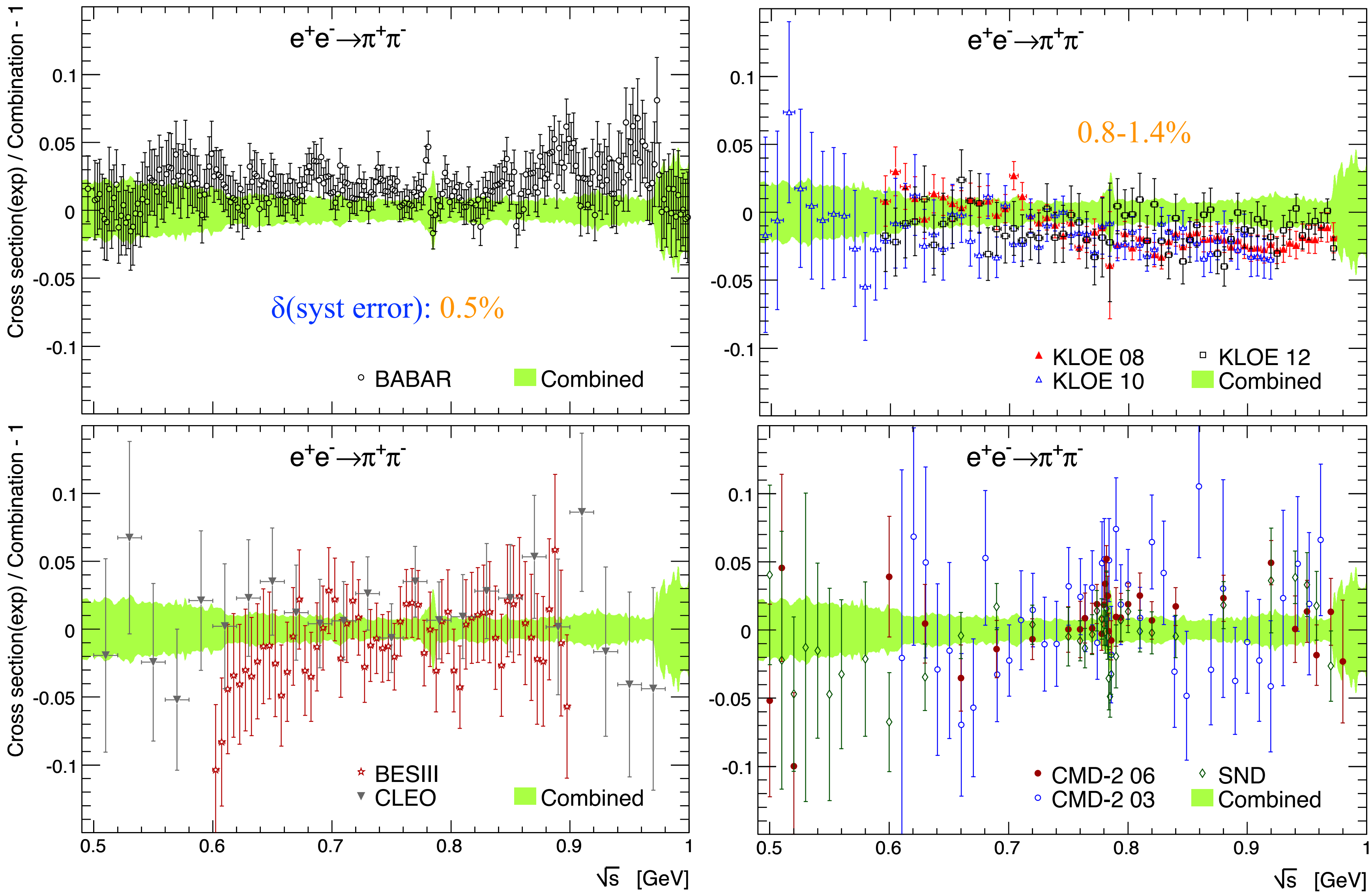


[Keshavarzi, Nomura, Teubner PRD96 (2018) 114025, arXiV 1802.02995]

KNT18 a_μ^{SM} update



The Dominant $\pi^+\pi^-$ Channel (2)



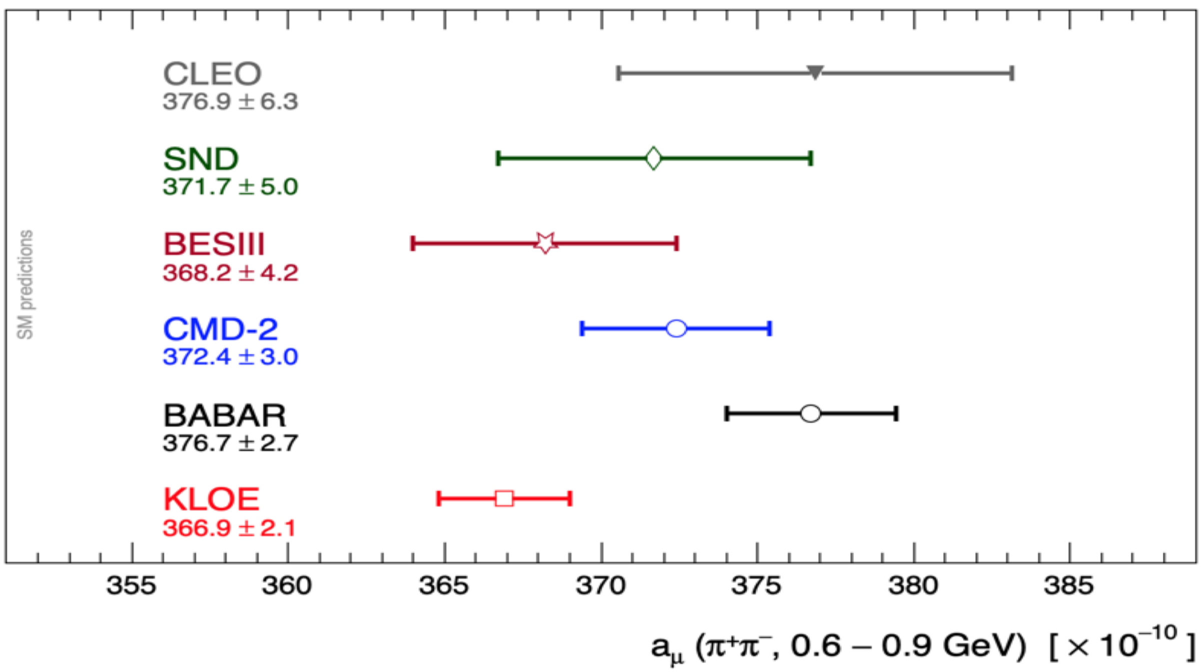
Zhang et al. EPS2019

BABAR & KLOE dominates 0.6-0.9 GeV ππ data,
Has a large discrepancy between BABAR & KLOE -> inflate error (dominant)

Combined Results Fit [<0.6 GeV] + Data [0.6-1.8 GeV]

√s range [GeV]	a_μ^{had} [10^{-10}] All data	a_μ^{had} [10^{-10}] All but BABAR	a_μ^{had} [10^{-10}] All but KLOE
threshold - 1.8	506.9 ± 1.9_{total}	505.0 ± 2.1_{total}	510.6 ± 2.2_{total}

⇒



ents?

BABAR or

⇒ ide to

- ➤ Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- ➤ Take the mean value "All but BABAR" and "All but KLOE" as our central value

Include other contributions in unit of 10^{-10}:

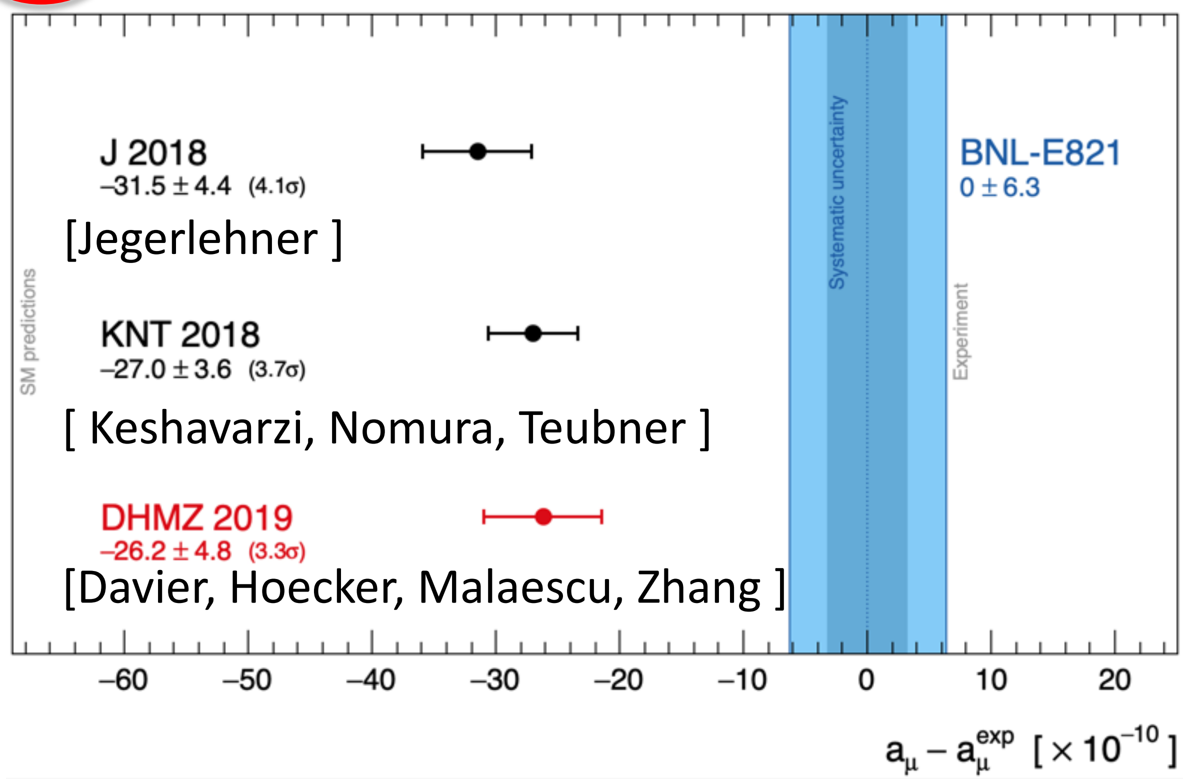
QCD NLO: -9.87 ± 0.07; NNLO: 1.24 ± 0.01; LBL: 10.5 ± 2.6

EW: 15.29 ± 0.10; QED: $11\,658\,471.895 \pm 0.008$

$\Rightarrow a_\mu = 11\,659\,182.9 \pm 4.8_{\text{total}}$

In comparison with the direct measurement:
$11\,659\,209.1 \pm 6.3_{\text{total}}$

$\Rightarrow 26.2 \pm 7.9\ (3.3\sigma)$

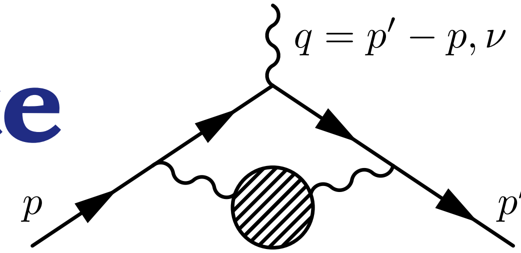


Zhang et al. EPS2019

DHMZ19 added half of discrepancy b/w BABAR and KLOE, 2.8×10^{-10}, as an additional uncertainty

→ Unless this discrepancy is understood, this limits the precision of dispersive analysis

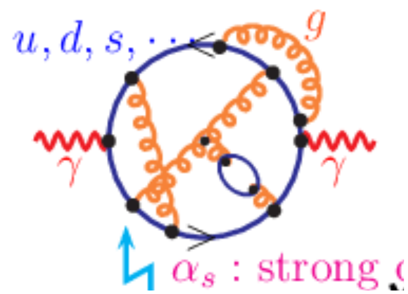
g-2 HVP from Lattice



[Bernecker Meyer 2011, Feng et al. 2013]

In Euclidean space-time, project verctor 2 pt to zero spacial momentum, $\vec{p} = 0$:

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$



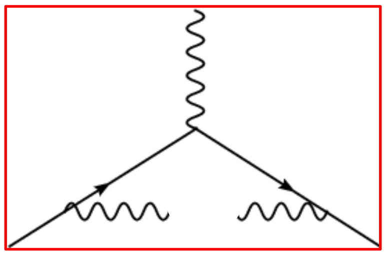
g-2 HVP contribution is

w(t) ~ t^4

$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

$$w(t) = 2 \int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right]$$

$f_{QED}(\omega^2)$



- Subtraction $\Pi(0)$ is performed.

  Noise/Signal $\sim e^{(E_{\pi\pi} - m_\pi)t}$, is improved [Lehner et al. 2015] .

Euclidean time correlation from e^+e^- $R(s)$ data

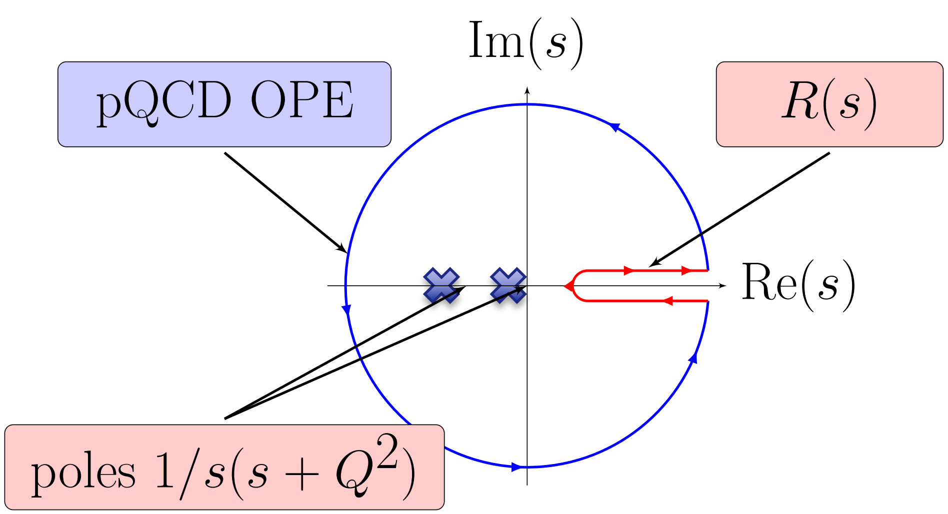
From e^+e^- $R(s)$ ratio, using disparsive relation, zero-spacial momentum projected Euclidean correlation function $C(t)$ is obtained

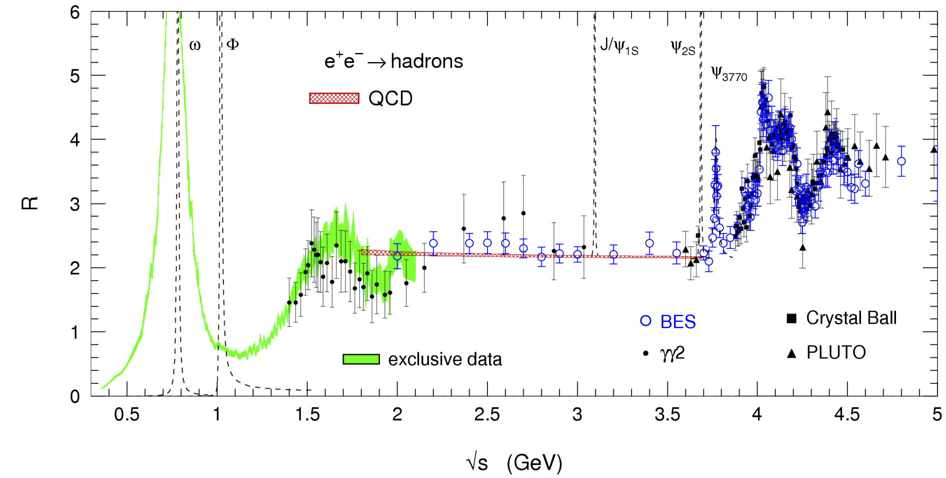
$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

$$C^{\text{R-ratio}}(t) = \frac{1}{12\pi^2} \int_0^\infty \frac{d\omega}{2\pi} \hat{\Pi}(\omega^2) e^{i\omega t} = \frac{1}{12\pi^2} \int_0^\infty ds \sqrt{s} R(s) e^{-\sqrt{s}t}$$

Lattice can compute Integral of
Inclusive cross sections accurately

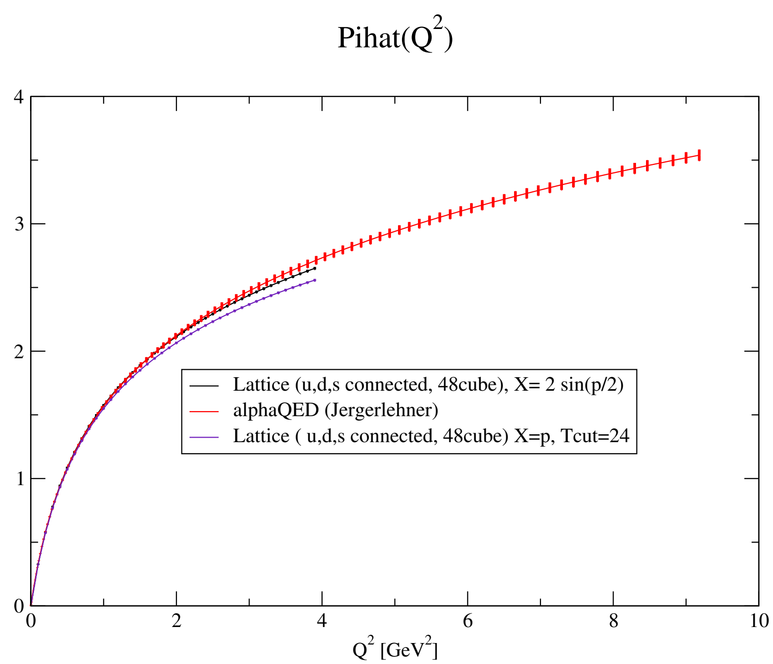
- $C(t)$ or $w(t)C(t)$ are directly comparable to Lattice results with the proper limits ($m_q \to m_q^{\text{phys}}, a \to 0, V \to \infty$, QED ...)
- Lattice: long distance has large statistical noise, (short distance: discretization error, removed by $a \to 0$ and/or pQCD)
- R-ratio : short distance has larger error

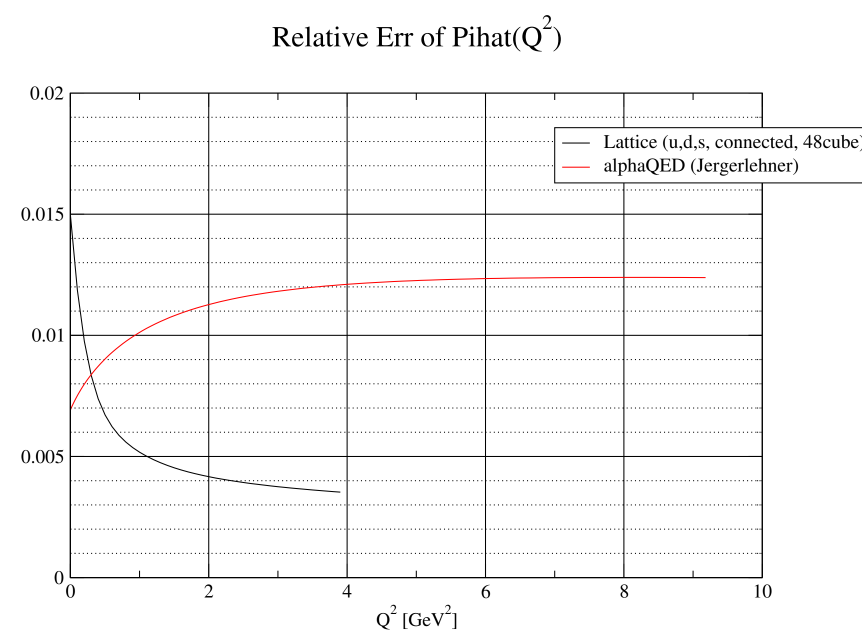




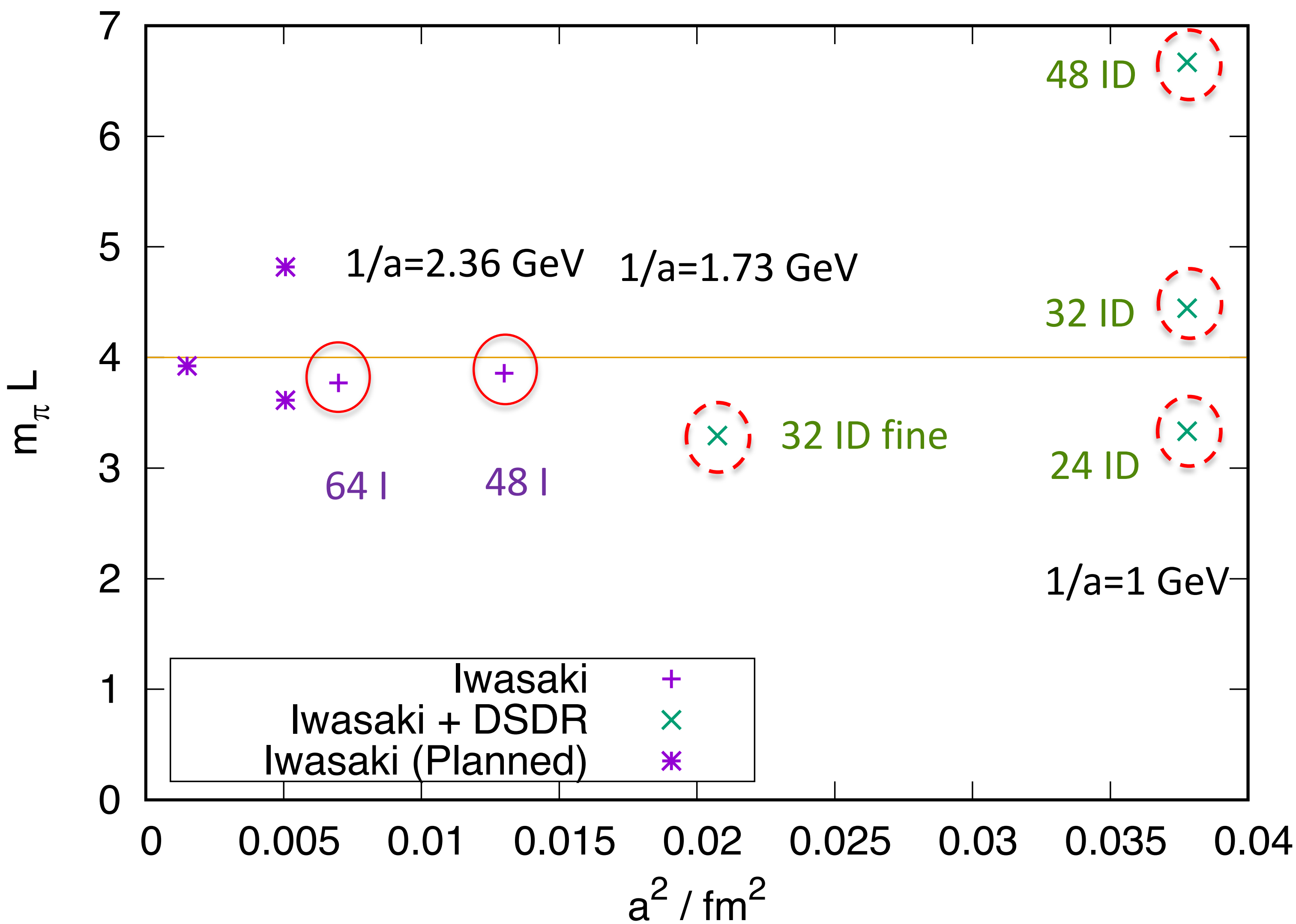
$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

$(1/a = 1.78$ **GeV,** **Relative statistical error)**





Nf=2+1 DWF QCD ensemble at physical quark mass

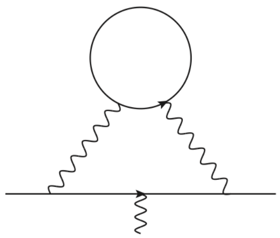


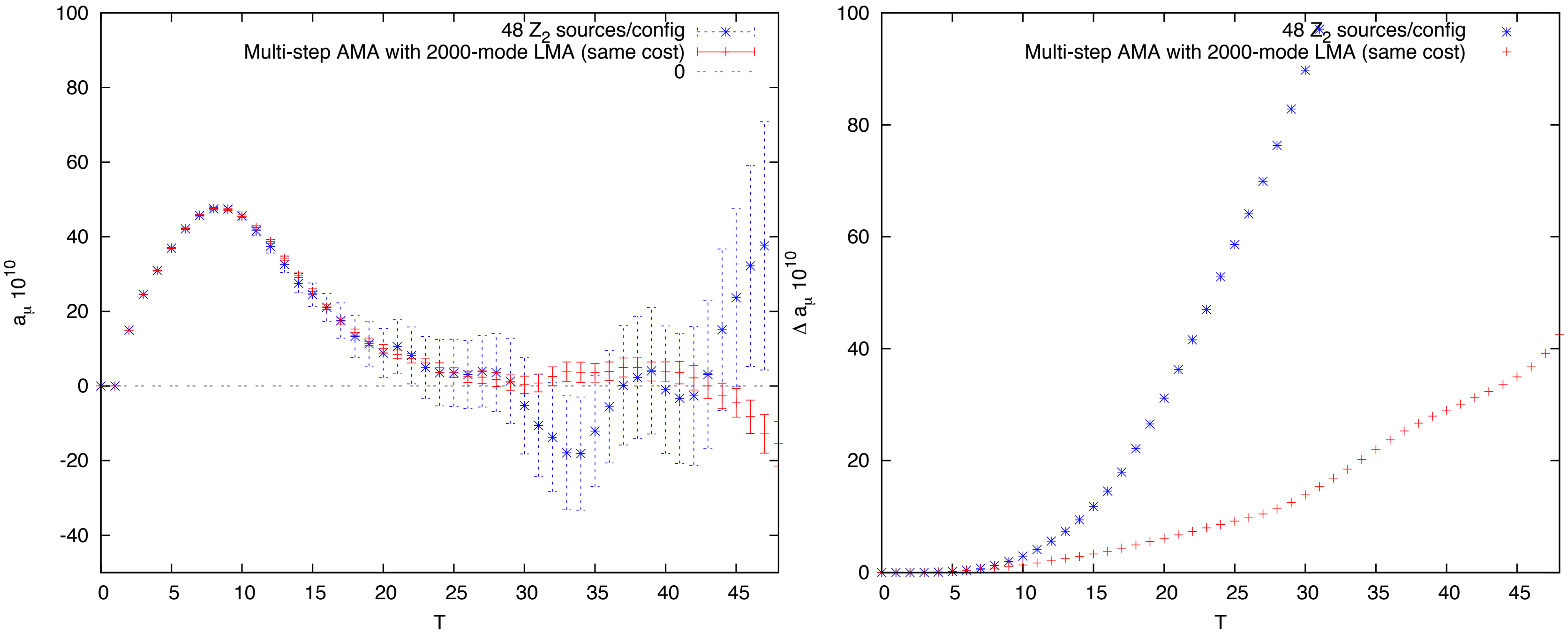
New Data since 2018

RBC/UKQCD 2019 (data for light quarks, changes from 2018):

- A2A data for connected isospin symmetric: 48I (127 conf $\rightarrow$ 400 conf), 64I (160 conf $\rightarrow$ 250 conf), 24ID (new 130 conf, multi mass), 32ID (new 88 conf, multi mass)
- A2A data (tadpole fields) for disconnected: 48I (33 conf), 24ID (new 260 conf, multi mass), 32IDf (new 103 conf)
- QED and SIB corrections to meson and Ω masses, Z_V: 48I (30 conf) and 64I (new 30 conf)
- QED and SIB from HLbL point sources on 48I, 24ID, 32ID, 32IDf (on order of 20 conf each, 2000 points per config)
- Distillation data on 48I (33 conf), 64I (in progr.), 24ID (33 conf), 32ID (11 conf, multi-mass)
- New Ω mass operators (excited states control): 48I (130 conf)

DWF light HVP
[2016 Christoph Lehner]



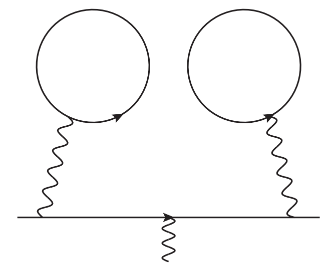


120 conf (a=0.11fm), 80 conf (a=0.086fm)   physical point Nf=2+1 Mobius DWF
4D full volume LMA with 2,000 eigen vector (of e/o preconditioned zMobius $D^{+}D$)
EV compression (1/10 memory) using local coherence [C. Lehner Lat2017 Poster]
In addition, 50 sloppy / conf via multi-level AMA
more than x 1,000 speed up compared to simple CG

disconnected quark loop contribution

- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit,

  Qu+Qd+Qs = 0



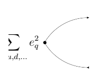
- Use low mode of quark propagator, treat it exactly

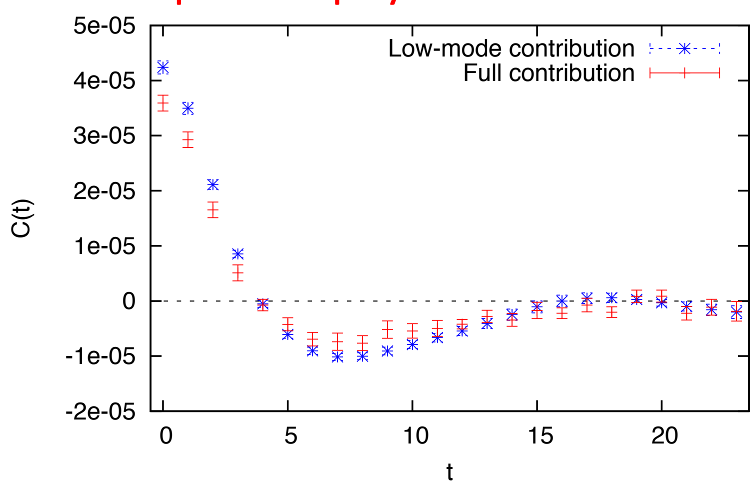
  (all-to-all propagator with sparse random source)
- First non-zero signal

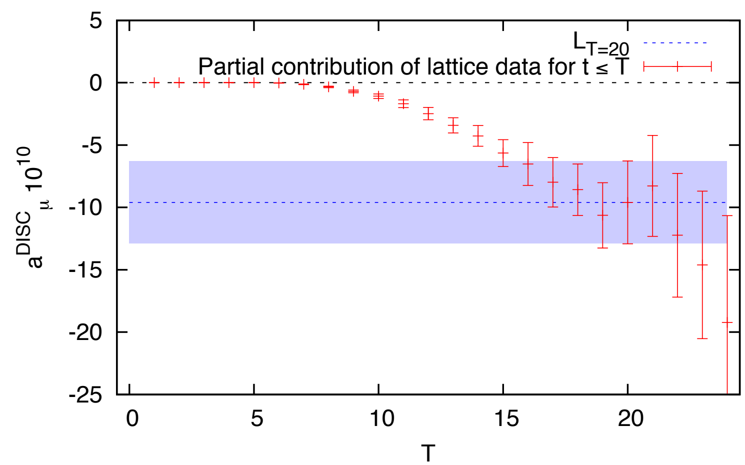
$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

Sensitive to m_π

crucial to compute at physical mass

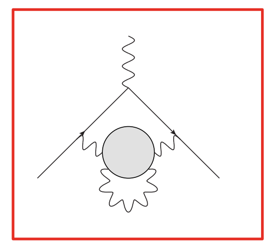


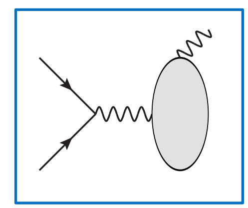


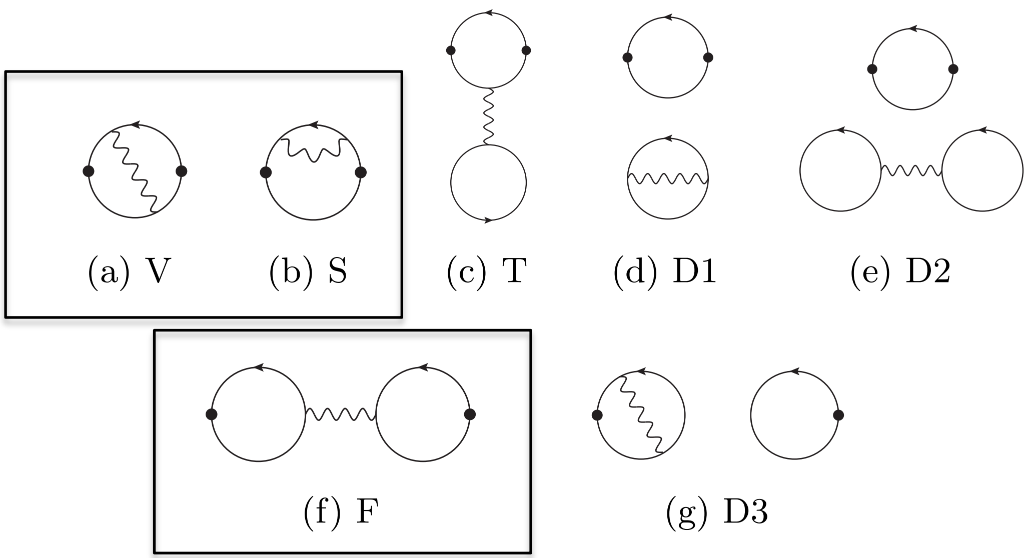


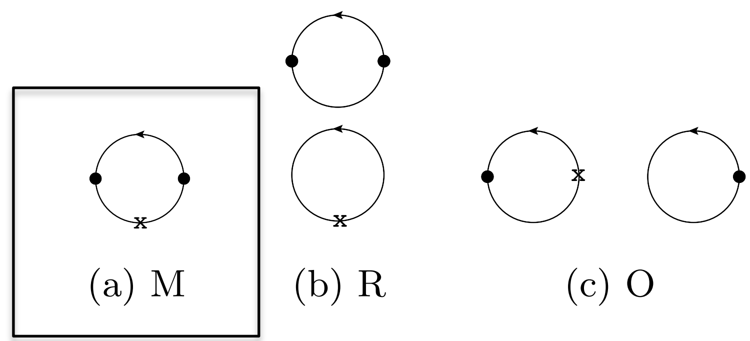
HVP QED+ strong IB corrections

- HVP is computed so far at Iso-symmetric quark mass, needs to compute isospin breaking corrections : Qu, Qd, mu-md ≠0
- u,d,s quark mass and lattice spacing are re-tuned using {charge,neutral} x{pion,kaon} and (Omega baryon masses)
- For now, V, S, F, M are computed : assumes EM and IB of sea quark and also shift to lattice spacing is small (correction to disconnected diagram)
- Point-source method : stochastically sample pair of 2 EM vertices a la important sampling with exact photon



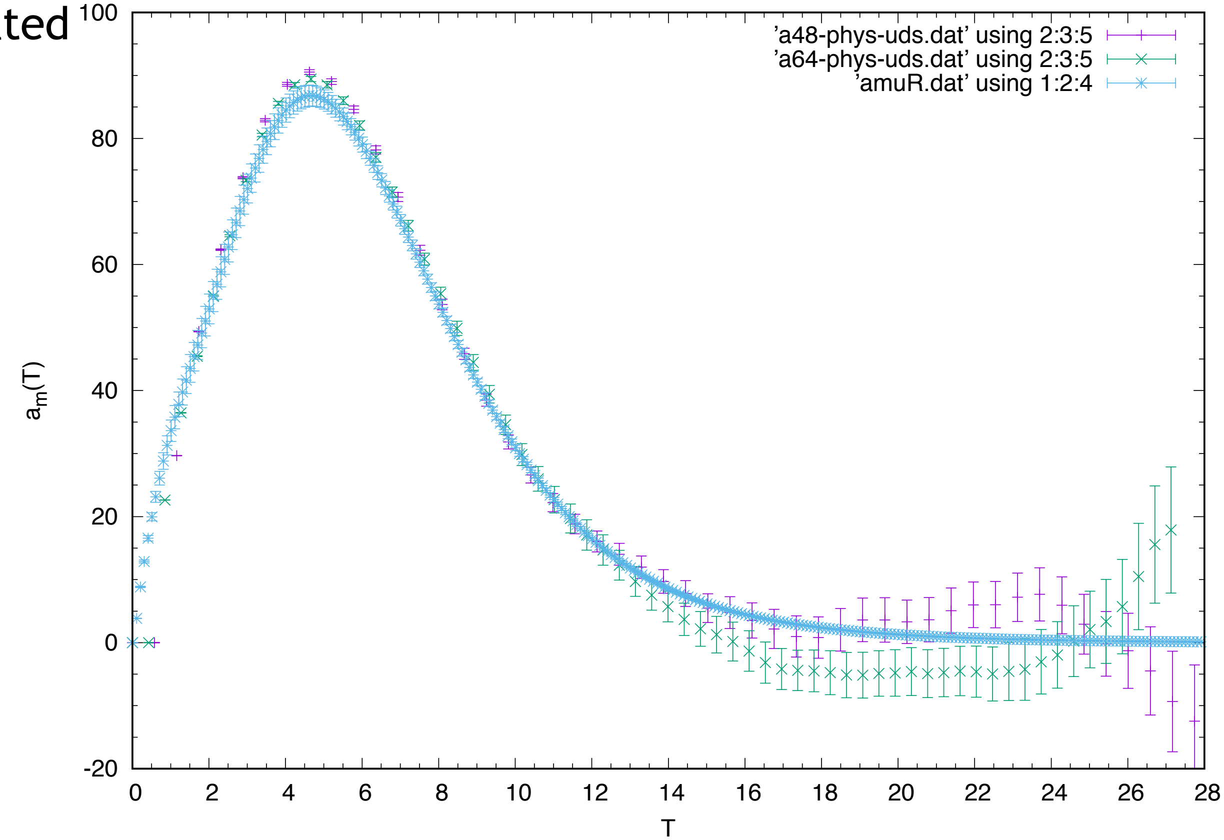






Comparison of R-ratio and Lattice
[F. Jegerlehner alphaQED 2016]

- Covariance matrix among energy bin in R-ratio is not available, assumes 100% correlated



$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

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$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$

Combine R-ratio and Lattice
[Christoph Lehner et al PRL18]

- Use short and long distance from R-ratio using smearing function, and mid-distance from lattice

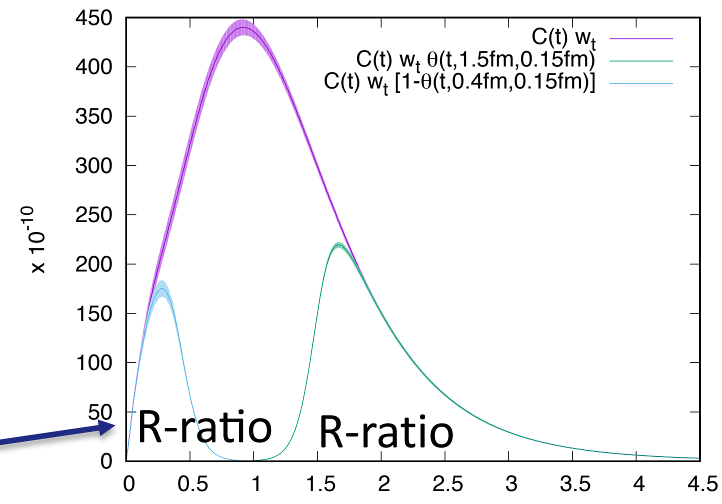
$$\Theta(t,\mu,\sigma) \equiv \left[1 + \tanh\left[(t-\mu)/\sigma\right]\right]/2$$

$$a_\mu = \sum_t w_t C(t) \equiv a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

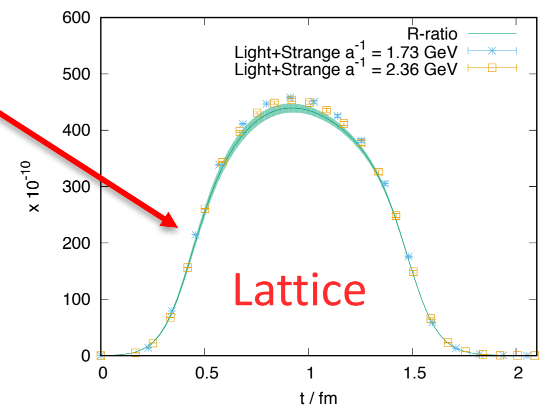
$$a_\mu^{\mathrm{SD}} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)],$$

$$a_\mu^{\mathrm{W}} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)],$$

$$a_\mu^{\mathrm{LD}} = \sum_t C(t) w_t \Theta(t, t_1, \Delta)$$

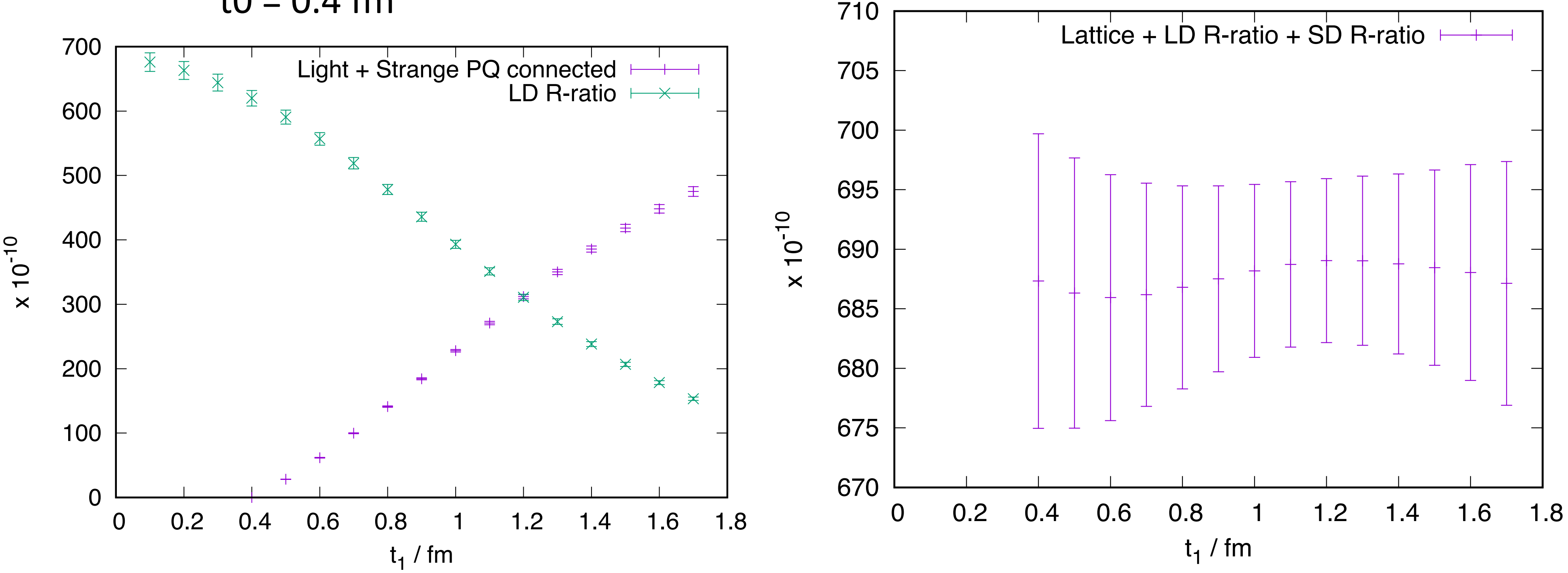


+



R-ratio + Lattice

t0 = 0.4 fm



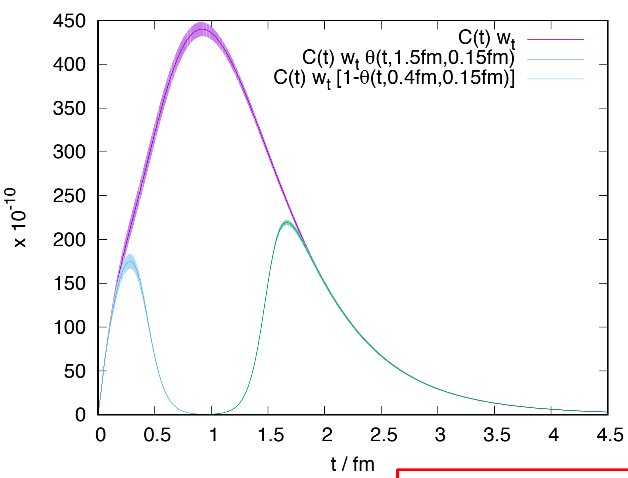
t1 dependence is flat => a consistency between R-ratio and Lattice

t1 = 1.2 fm, R-ratio : Lattice = 50:50

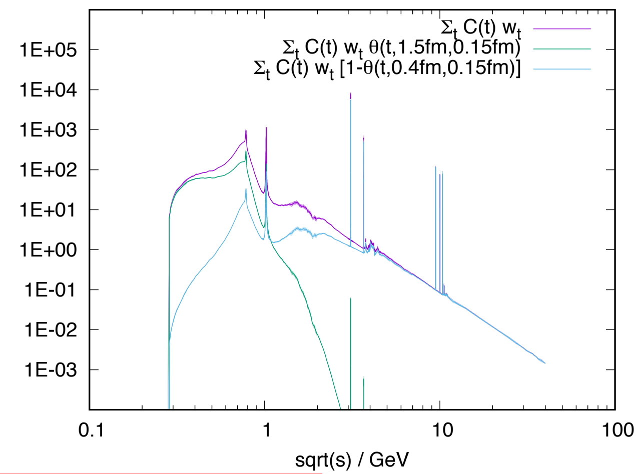
t1=1.2 fm current error (note 100% correlation in R-ratio) is minimum

How does this translate to the time-like region?

Euclidean from Lattice

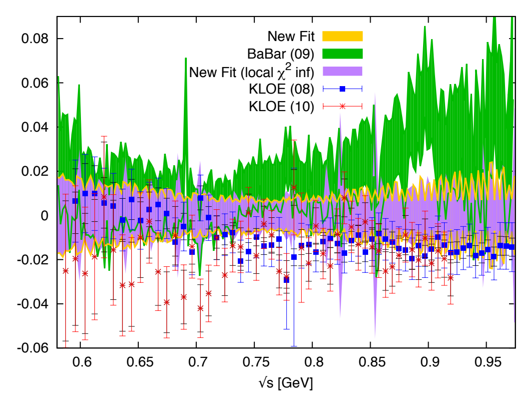


Time-like from e+e-



Blue : low-pass window
Green: high-pass window
Purple : total

Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.



Current status & Improvements

[Christoph Lehner Lattice2019]

The pure lattice calculation of RBC/UKQCD 2018:

[RBC/UKQCD, PRL 121 (2018) 022003]

$$10^{10} \times a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7)$$
$$= 715.4(16.3)_{\mathrm{S}}(7.8)_{\mathrm{V}}(3.0)_{\mathrm{C}}(1.9)_{\mathrm{A}}(3.2)_{\mathrm{other}}$$

(S) statistics, (V) finite-volume errors, (C) the continuum limit extrapolation, (A) scale setting uncertainty;

other $\supset$ neglected diagrams for QED and SIB, estimate of bottom quark contribution

Statistical noise mostly from isospin symmetric light quark connected (14.2) and disconnected (3.3), QED (5.7), SIB (4.3)

RBC/UKQCD 2019 update (in preparation):

- Improved methodology
- A lot of new data

[Aaron Meyer LATTICE2019]

Reconstruction of HVP from multi-channel Greens function

- Correlation function among N operators O_n, n=0,1,..., N-1

- Point (or smeared) vector $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x),\ \mu \in \{1,2,3\}$
- 2 π operator $\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2$
- 4 π operator $\mathcal{O}_{4\pi} = \left|\sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2 \left|\sum_{xy} \bar{\psi}(x) f(x-y)\gamma_5\psi(y)\right|^2$
- NxN correlation function < O_i(t) O_j(0) > (using distillation)

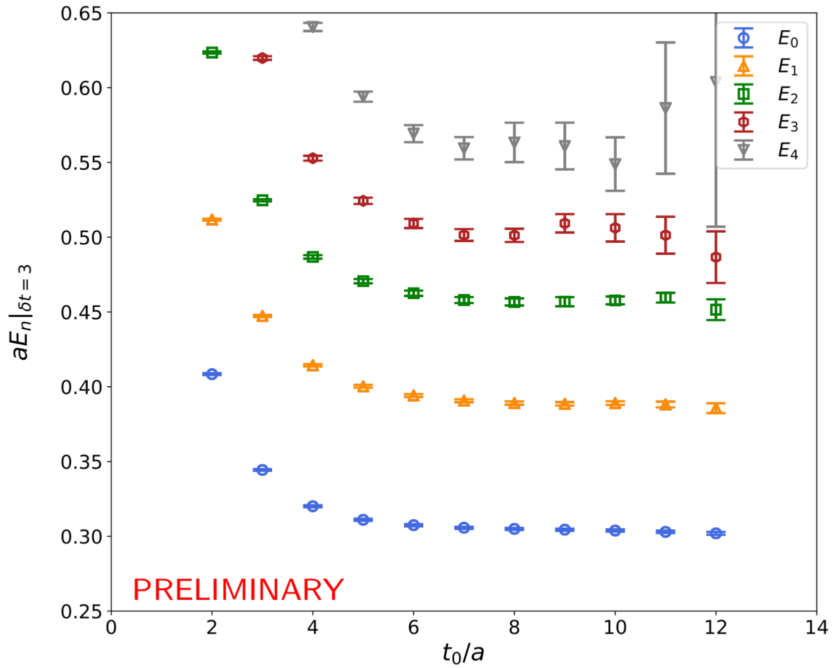
- Solve NxN spectrum E_n of eigenstates |E_n> and Overwrap factors <E_n|O_0|0> (GEVP)

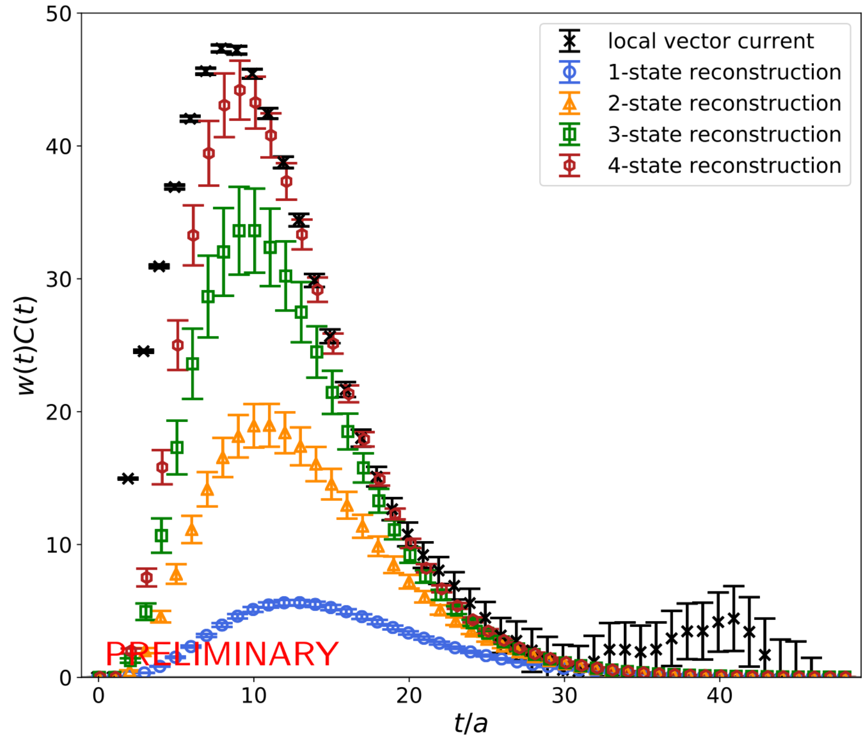
- Reconstruct V-V correlator, and bound contribution from the (N+1)-th states and above

$$\langle O_0(t) O_0^\dagger(0)\rangle = \sum_{n=0}^{N-1} |\langle 0|O_0|n\rangle|^2 e^{-E_n t}$$
$$+ \text{(contributions from } n \geq N \text{ states}$$

[Aaron Meyer LATTICE2019]

GEVP & Reconstruct I=1 VV





6-operator basis on 48I ensemble: local+smeared vector, $4\times(2\pi)$

$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad V_{im} \propto \langle\Omega|\,\mathcal{O}_i\,|m\rangle$$

Bounds for a_μ

- Upper & lower bounds from unitarity

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max}) e^{-E(t - t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

Lower bound: $E = \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

- Also bounds for the n in [N+1, ∞] states contribution

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

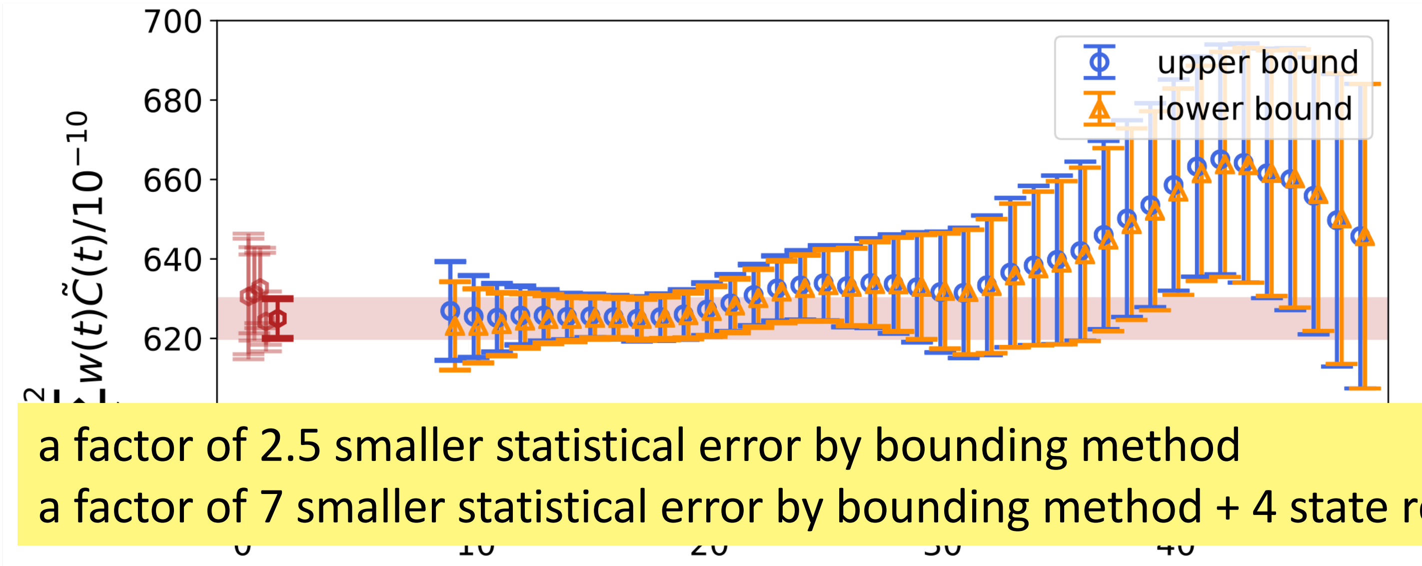
$\Longrightarrow$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\Longrightarrow$ Smaller overall contribution from neglected states

test of GEVP+Bounding method

[A. Meyer]

Bounding Method Results - 48I



a factor of 2.5 smaller statistical error by bounding method
a factor of 7 smaller statistical error by bounding method + 4 state reconstruction

No bounding method:	$a_\mu^{HVP} = 646(38)$
Bounding method $t_{\max} = 3.3\ \mathrm{fm}$, no reconstruction:	$a_\mu^{HVP} = 631(16)$
Bounding method $t_{\max} = 3.0\ \mathrm{fm}$, 1 state reconstruction:	$a_\mu^{HVP} = 631(12)$
Bounding method $t_{\max} = 2.9\ \mathrm{fm}$, 2 state reconstruction:	$a_\mu^{HVP} = 633(10)$
Bounding method $t_{\max} = 2.2\ \mathrm{fm}$, 3 state reconstruction:	$a_\mu^{HVP} = 624.3(7.5)$
Bounding method $t_{\max} = 1.8\ \mathrm{fm}$, 4 state reconstruction:	$a_\mu^{HVP} = 625.0(5.4)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

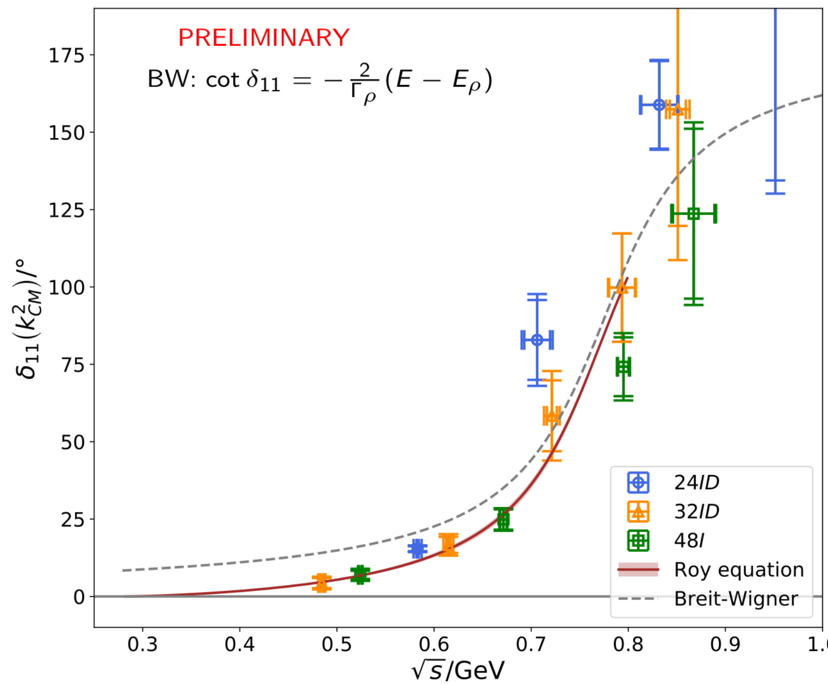
Finite Volume correction estimates

- L=6.22 fm (24cube) bax vs L=4.66 fm (32cube)
- scalar QED
- Using pion form factor (Gounaris-Sakurai parametrization) & Lellouche Luscher's FV formula

$$a_\mu^{HVP}(L=6.22\ \mathrm{fm}) - a_\mu^{HVP}(L=4.66\ \mathrm{fm})$$

$$= \begin{cases} 12.2 \times 10^{-10} & \text{sQED} \\ 21.6(6.3) \times 10^{-10} & \text{LQCD} \\ 20(3) \times 10^{-10} & \text{GSL} \end{cases}$$

-

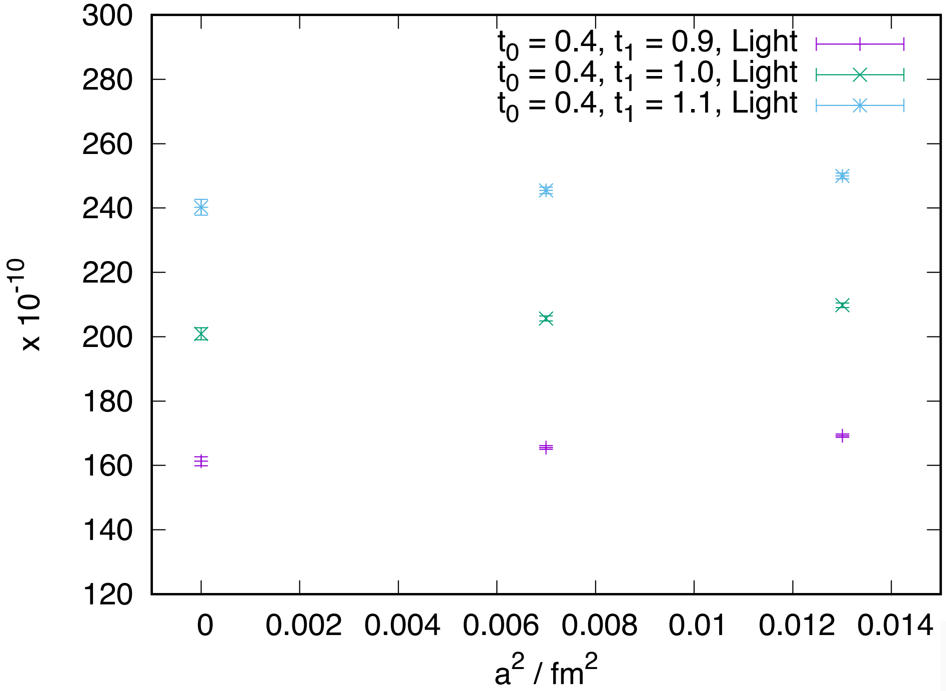


- Revised FV estimation :

$$a_\mu^{HVP}(L=\infty) - a_\mu^{HVP}(L=5.47\ \mathrm{fm}) = 22(1) \times 10^{-10}$$

Continuum limit of a^W

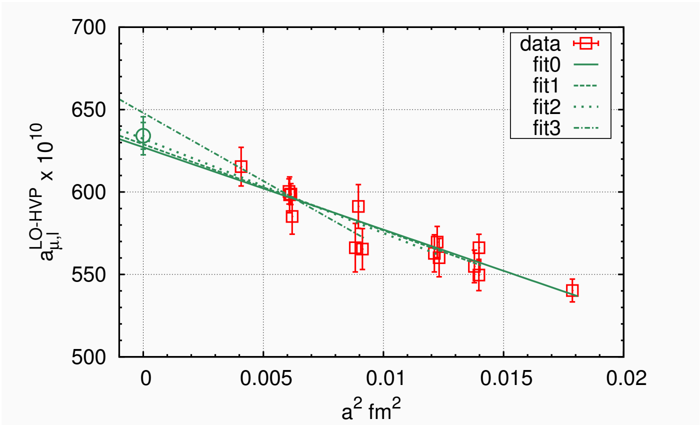
Continuum limit of a_μ^{W} from our lattice data; below $t_0 = 0.4$ fm and $\Delta = 0.15$ fm



RBC/UKQCD [C. Lehner Lat17]

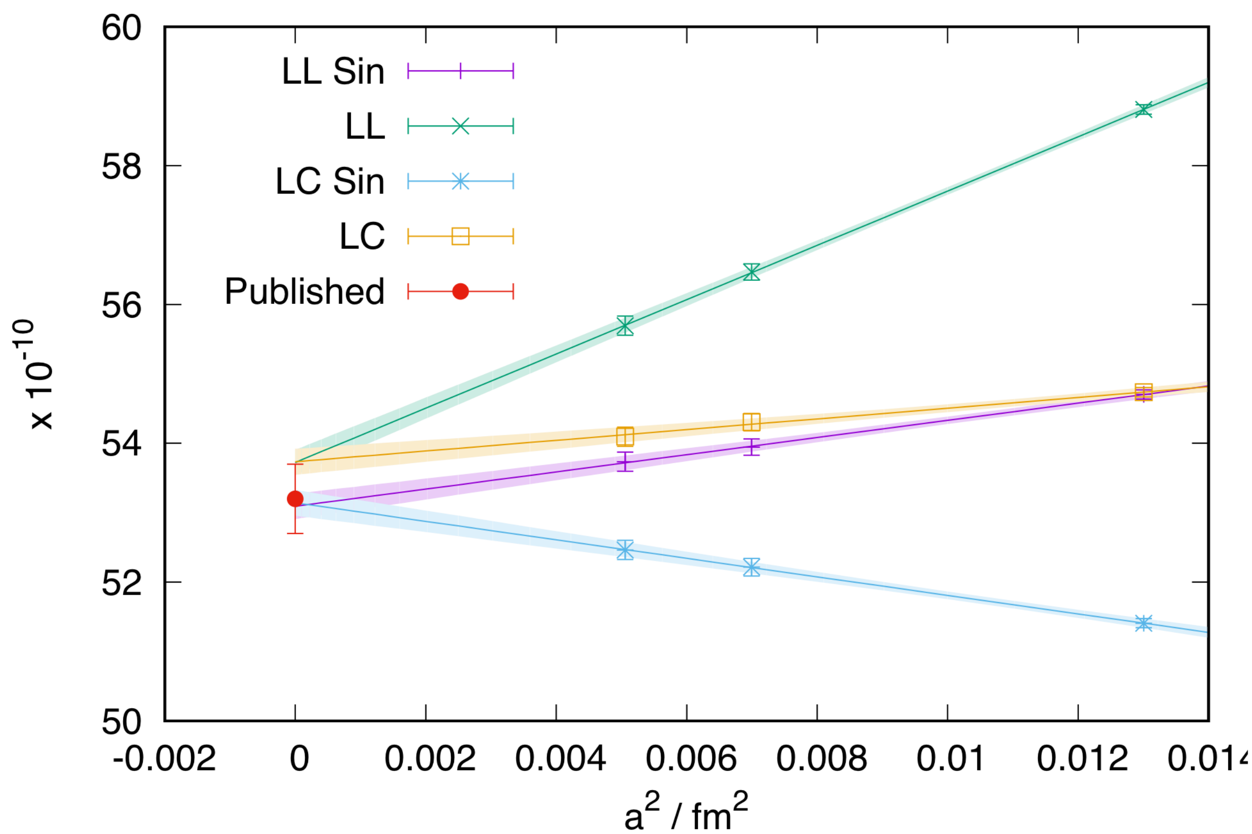
Continuum extrapolation is mild

c.f BMWc [K. Miura Lat17]



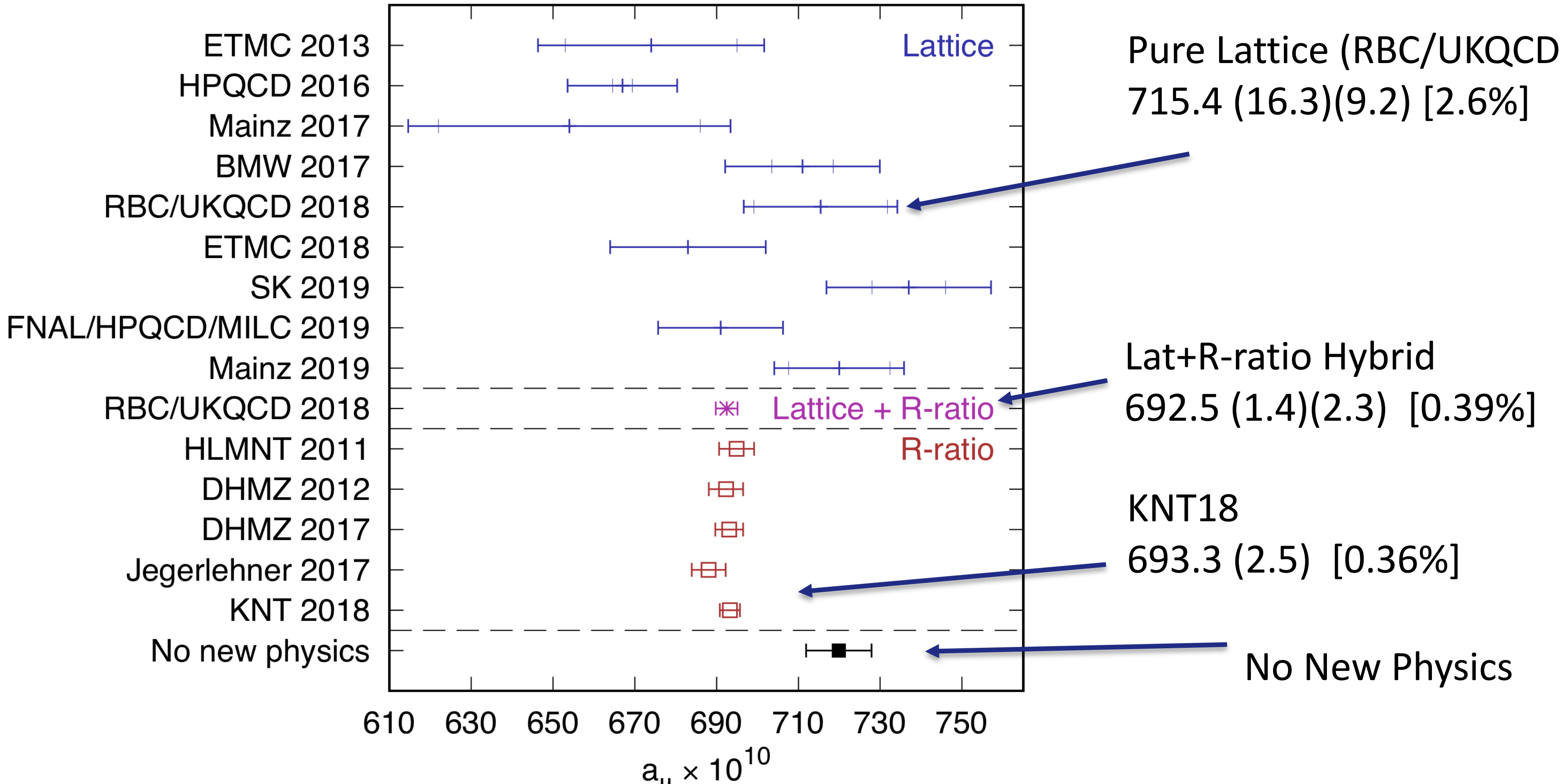
Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



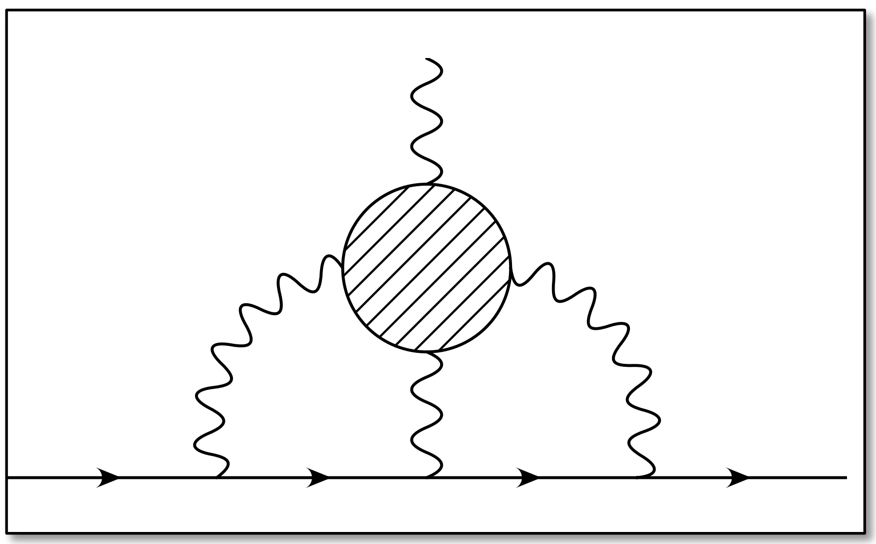
- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

HVP results
[Christoph Lehner Lat19]



- Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
- Pure Lattice HVP $5\text{x}10^{-10}$ this year, $1\text{x}10^{-10}$ for long term
- Check BABAR-KLOE tention by windown method, consolidate error at $3\text{x}10^{-10}$

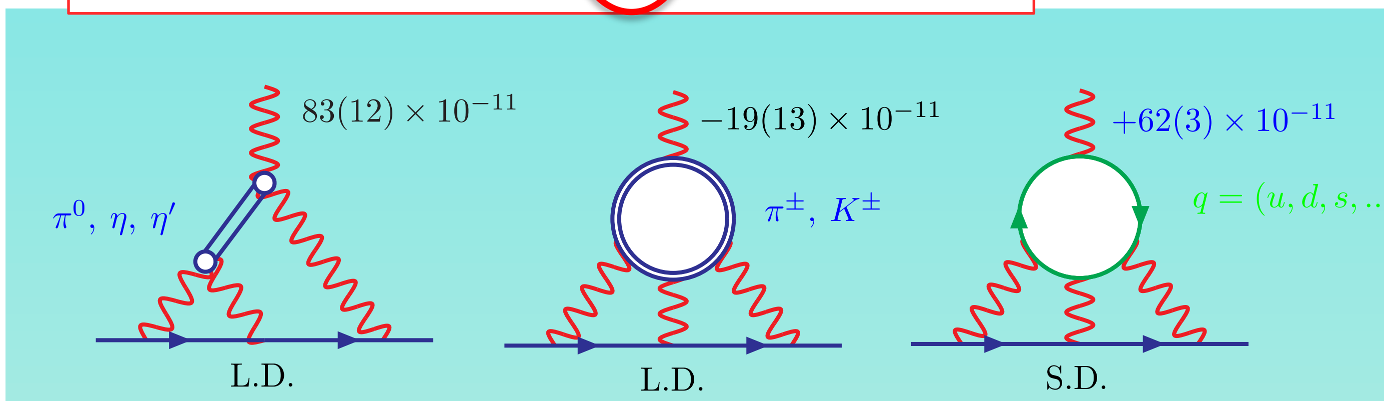
Hadronic Light-by-Light (HLbL) contributions



HLbL from Models

- Model estimate with non-perturbative constraints at the chiral / low energy limits using anomaly : $(9\text{—}12) \times 10^{-10}$ with 25-40% uncertainty

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\underbrace{(2.6)}_{\rm HLbL}\underbrace{(0.1)}_{\rm other}\underbrace{(6.3)}_{\rm EXP} \times 10^{-10}$$

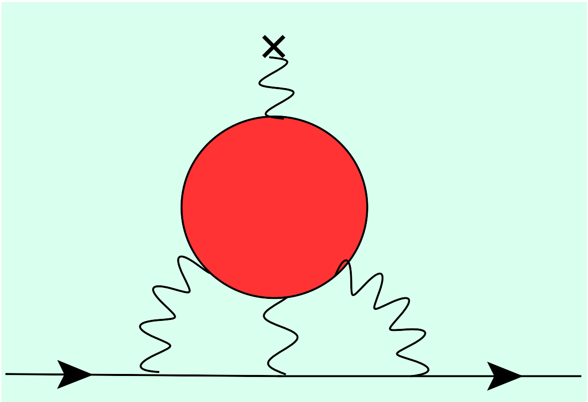


F. Jegerlehner , x 10^{11}

Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	15±10	22± 5
scalars	−6.8±2.0	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

Hadronic Light-by-Light

- 4pt function of EM currents
- No direct experimental data available
- Dispersive approach



$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times \gamma_\nu S^{(\mu)}(\not{p}_2+\not{k}_2)\gamma_\rho S^{(\mu)}(\not{p}_1+\not{k}_1)\gamma_\sigma$$

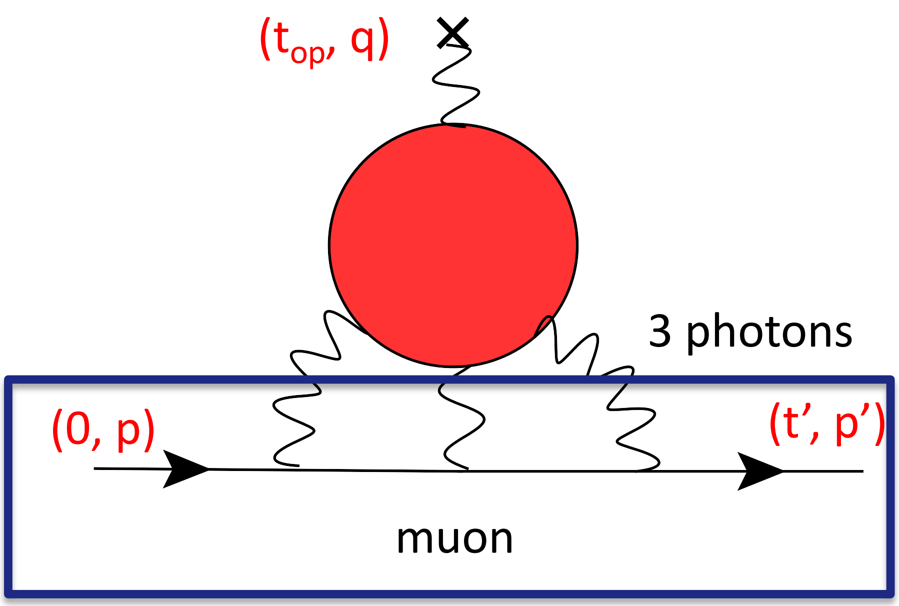
$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\,d^4x_2\,d^4x_3\,\exp[-i(k_1\cdot x_1+k_2\cdot x_2+k_3\cdot x_3)] \times\langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle$$

$$\text{Form factor}: \ \Gamma_\mu(q) = \gamma_\mu\,F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2\,m_l}\,F_2(q^2)$$

Our Basic strategy : Lattice QCD+QED system

- 4pt function has too much information to parameterize (?)
- Do Monte Carlo integration for QED two-loop with 4 pt function $\pi^{(4)}$ which is sampled in lattice QCD with chiral quark (Domain-Wall fermion)
- Photon & lepton part of diagram is derived either in lattice QED+QCD [Blum et al 2014] (stat noise from QED), or exactly derive for loop momenta / location at currents [L. Jin et al 2015] (no noise from QED+lepton).

$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_2,k_3)$$
$$\times\, [S(p_2)\gamma_\nu S(p_2+k_2)\gamma_\rho S(p_1+k_1)\gamma_\sigma S(p_1) + (\mathrm{perm.})]$$

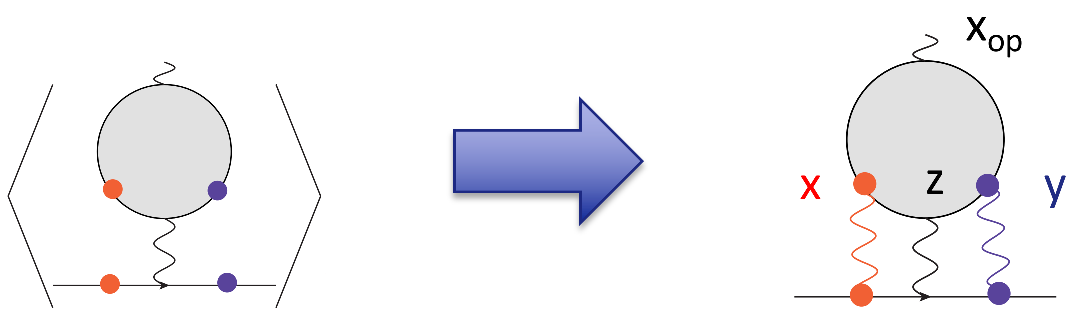


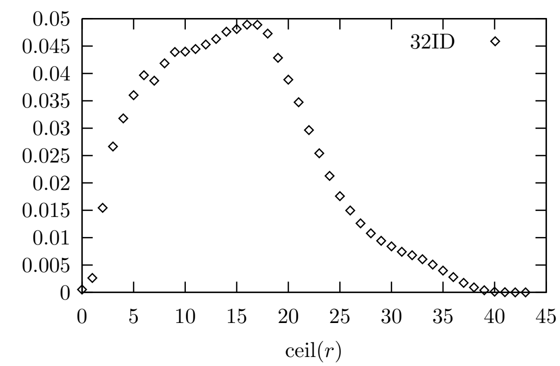
- set spacial momentum for
  - external EM vertex q
  - in- and out- muon p, p'
  q = p-p'
- set time slice of muon source(t=0), sink(t') and operator (t_{op})
- take large time separation for ground state matrix element

Coordinate space Point photon method

[Luchang Jin et al. , PRD93, 014503 (2016)]

- Treat all 3 photon propagators exactly (3 analytical photons) , which makes the quark loop and the lepton line connected :
  disconnected problem in Lattice QED+QCD -> connected problem with analytic photon
- QED 2-loop in coordinate space. Stochastically sample, two of quark-photon vertex location x,y, z and x_{op} is summed over space-time exactly

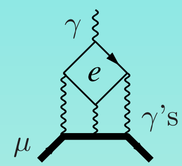




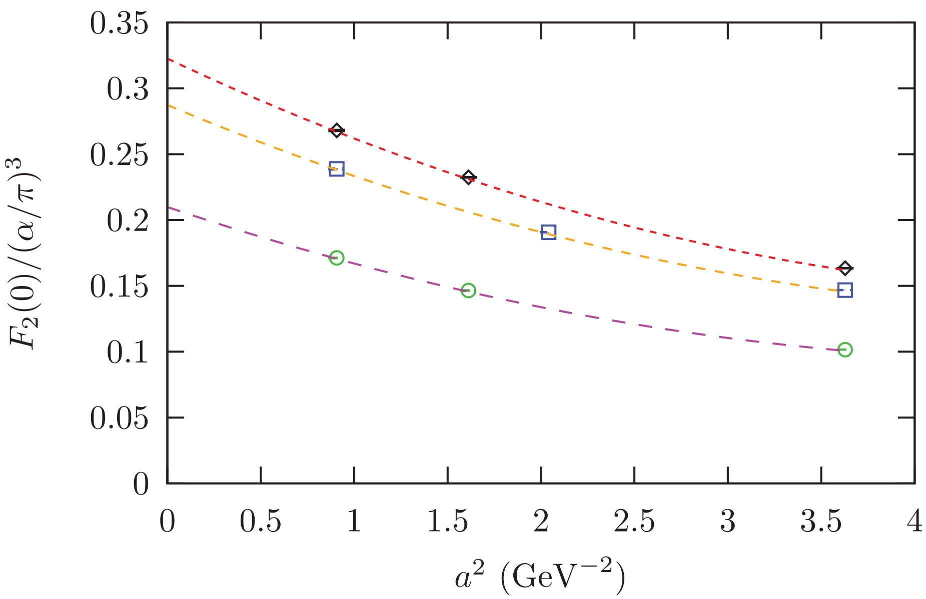
- Short separations, Min[|x-z|,|y-z|,|x-y|] < R ~ O(0.5) fm, which has a large contribution due to confinement, are summed for all pairs
- longer separations, Min[|x-z|,|y-z|,|x-y|] >= R, are done stochastically with a probability shown above (Adaptive Monte Carlo sampling)

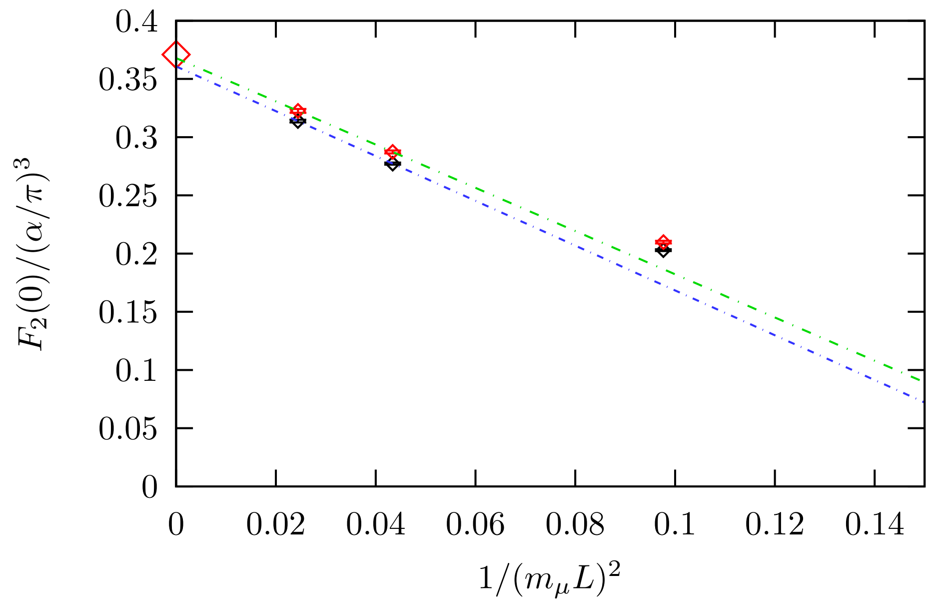
Systematic effects in QED only study

- muon loop, muon line
- a = a m_μ / (106 MeV)
- L= 11.9, 8.9, 5.9 fm
- known result : F2 = 0.371 (diamond) correctly reproduced (good check)



$$a_\mu^{(6)}(\text{lbl}, e) = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \frac{59}{270}\pi^4 - 3\,\zeta(3) - \frac{10}{3}\pi^2 + \frac{2}{3} + O\left(\frac{m_e}{m_\mu}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^3$$



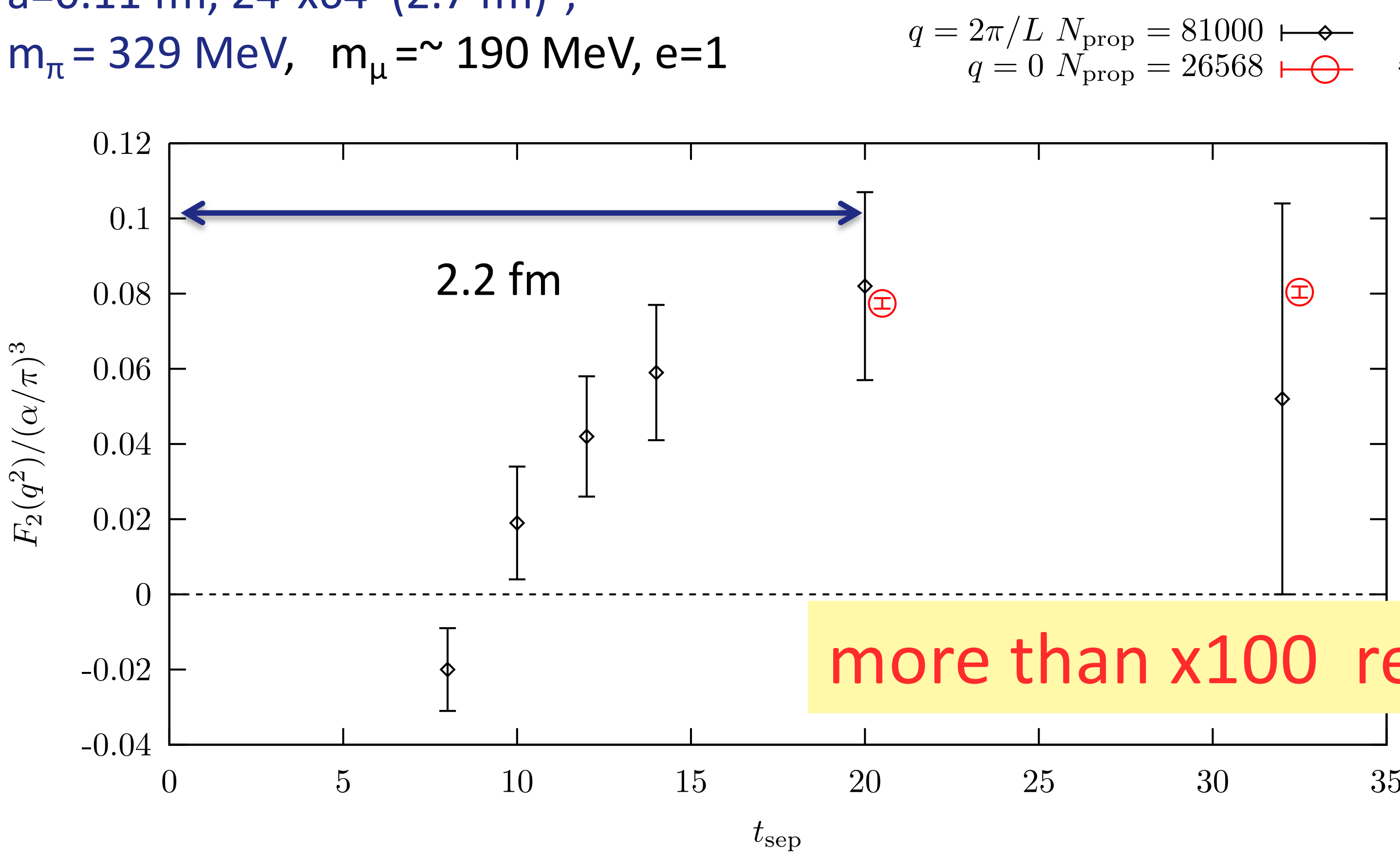


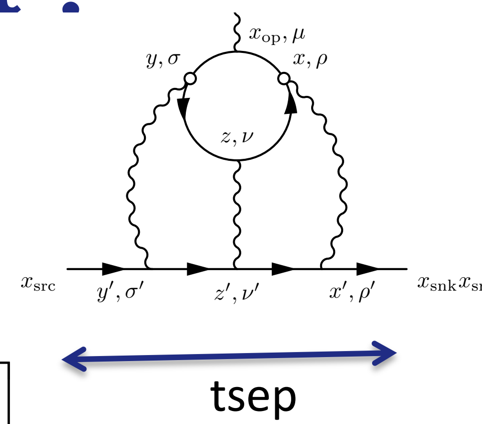
FV and discretization error could be as large as 20-30 %,
similar discretization error seen from QCD+QED study

Dramatic Improvement !

Luchang Jin

a=0.11 fm, 24^3x64 $(2.7\ \text{fm})^3$,
m_π = 329 MeV, m_μ =~ 190 MeV, e=1



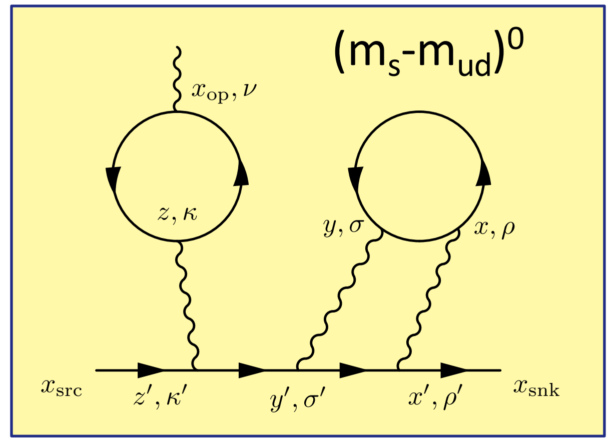


more than x100 reduced cost !

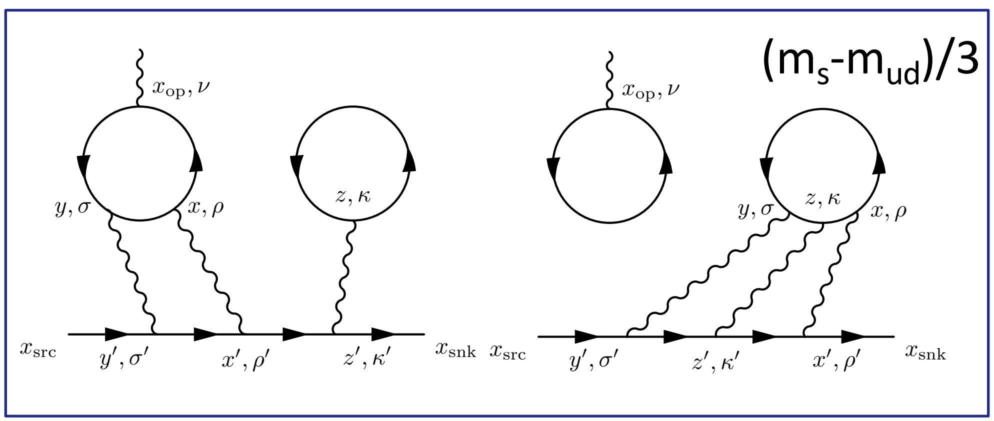
Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$
Conserved	0.0825(32)	12	$(118 + 128) \times 2 \times 7$	0.65
Mom.	0.0804(15)	18	$(118 + 128) \times 2 \times 3$	0.24

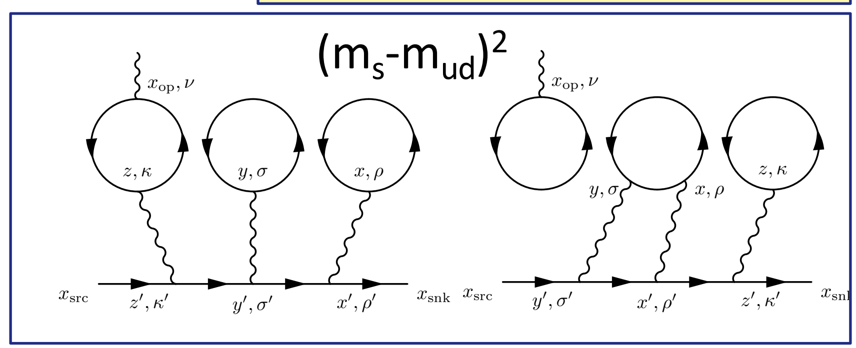
SU(3) hierarchies for d-HLbL

- At m_s=m_{ud} limit, following type of disconnected HLbL diagrams survive $Q_u + Q_d + Q_s = 0$
- Physical point run using similar techniques to c-HLbL.



- other diagrams suppressed by
  $O(m_s\text{-}m_{ud})$ /3 and $O(\ (m_s\text{-}m_{ud})^2\ )$

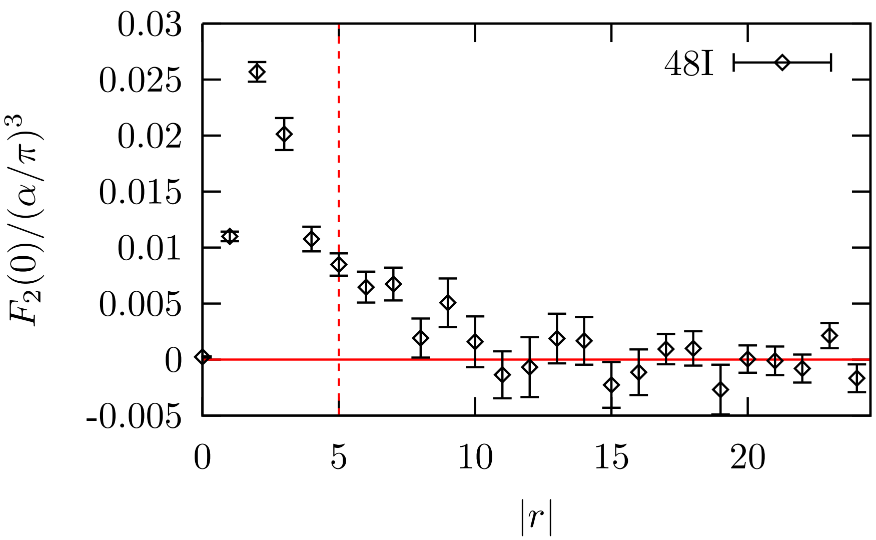


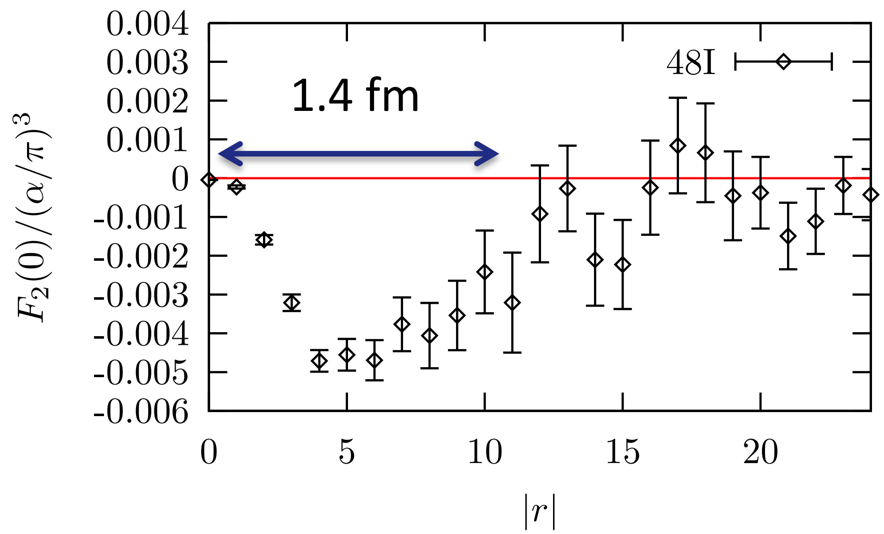


140 MeV Pion, connected and disconnected LbL results

[Luchang Jin et al. , Phys.Rev.Lett. 118 (2017) 022005]

- left: connected, right : leading disconnected

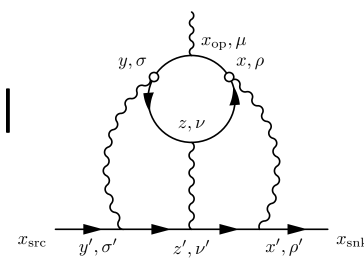




- Using AMA with 2,000 zMobius low modes, AMA

(statistical error only)

r = |x-y|



$$\left.\frac{g_\mu - 2}{2}\right|_{\rm cHLbL} = (0.0926 \pm 0.0077) \times \left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10}$$

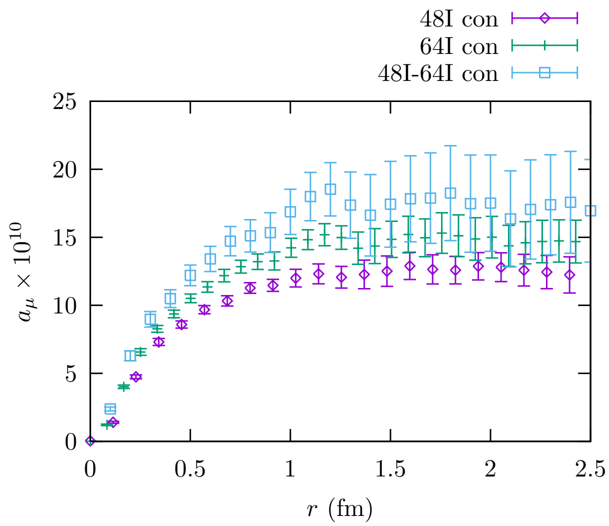
$$\left.\frac{g_\mu - 2}{2}\right|_{\rm dHLbL} = (-0.0498 \pm 0.0064) \times \left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10}$$

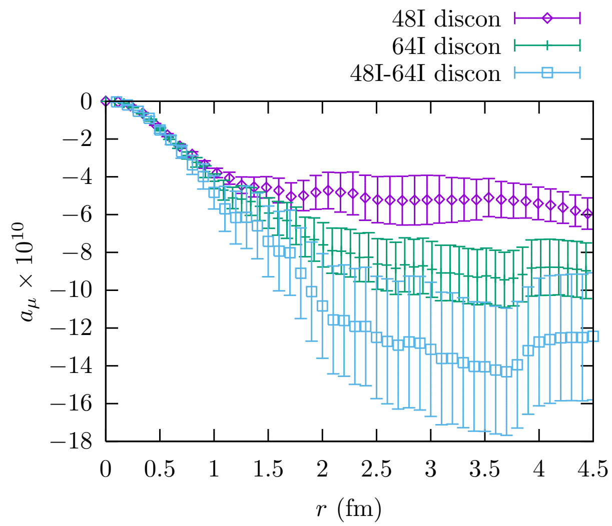
$$\left.\frac{g_\mu - 2}{2}\right|_{\rm HLbL} = (0.0427 \pm 0.0108) \times \left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10}$$

Continuum / infinite volume extrapolation

- Discretization error   1/a = 2.7, 1.4 GeV at physical quark mass

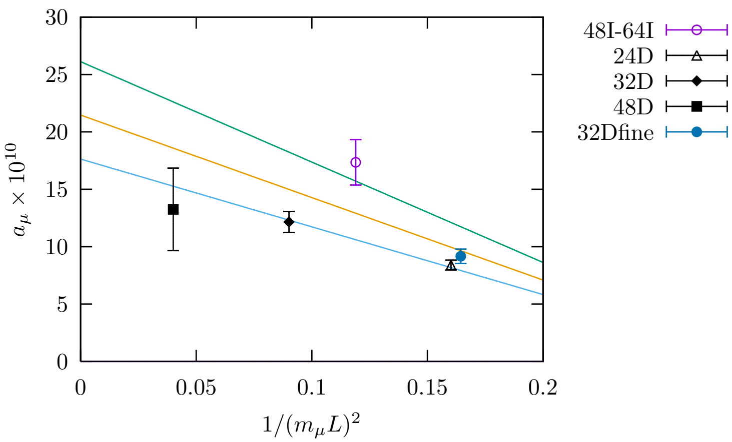
connected



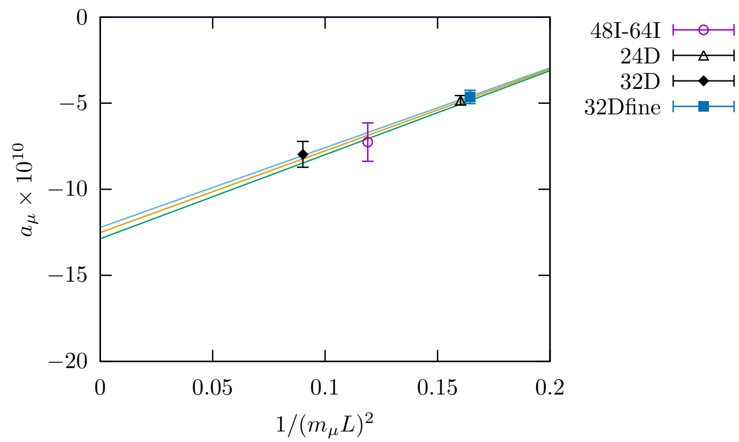


disconnected

- Finite volume   L = 4.8, 6.4, 9.6 fm at 1/a=1 GeV at physical mass



Connected diagrams

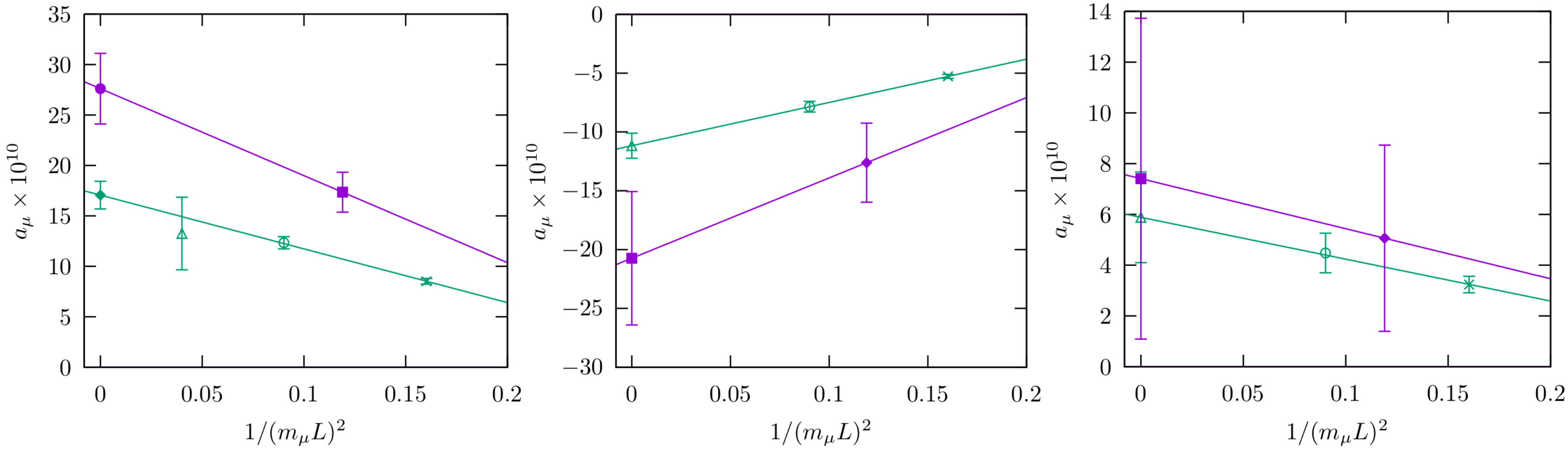


Disconnected diagrams

[Blum et al.   Moriond2019]

Using QED_L
[Hayakawa Uno PTP 2008]

QED_L continuum and infinite volume extrapolation [Blum et al., 2019] (preliminary)



- Iwasaki ensembles: $a \to 0$ ($c_2 = 0$, conn. extrap.: up to 1 fm, 48^3 for $r > 1$ fm)
- I-DSDR ensembles: $L \to \infty$ ($b_2 = 0$)

$$
\begin{aligned}
a_\mu^{cHLbL} &= (27.61 \pm 3.51_{\mathrm{stat}} \pm 0.32_{\mathrm{sys},a^2}) \times 10^{-10} \\
a_\mu^{dHLbL} &= -20.20 \pm 5.65_{\mathrm{stat}} \times 10^{-10} \\
a_\mu^{HLbL} &= 7.41 \pm 6.32_{\mathrm{stat}} \pm 0.32_{\mathrm{sys},a^2} \times 10^{-10}
\end{aligned}
$$

$$
F_2(a, L) = F_2\left(1 - \frac{c_1}{(m_\mu L)^2}\right)(1 - c_2\, a^2)
$$

Infinite Volume Photon and Lepton QED$_\infty$

[Feynman, Schwinger, Tomonaga]

- Instead of, or, in addition to, larger QED box, one could use infinite volume QED to compute $\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z)$.

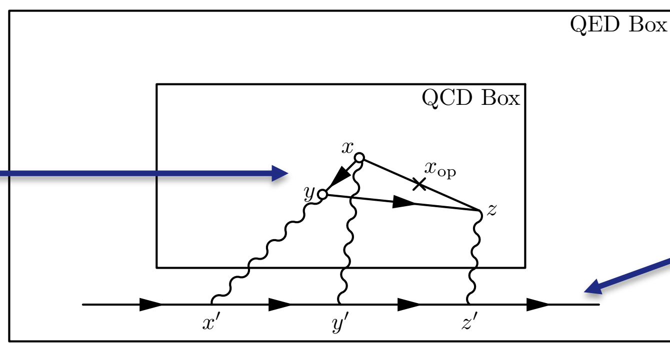
- Hadron part $\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op})$ has following features due to the mass gap :
  - ▷ *For large distance separation, the 4pt Green function is exponentially suppressed:* $\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op}) \sim \exp[-m_\pi \times \text{dist}(x,y,z,x_{\rm op})]$
  - ▷ *For fixed* $(x,y,z,x_{\rm op})$, *FV error (wraparound effect etc.) is exponentially suppressed:* $\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}|_V - \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}|_\infty \sim \exp[-m_\pi \times L]$

- By using QED$_\infty$ weight function $\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z)$, which is not exponentially growing, asymptotic FV correction is exponentially suppressed

$$\Delta_V \left[\sum_{x,y,z,x_{\rm op}} \mathcal{G}_{\rho,\sigma,\kappa}(x,y,z)\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op})) \right] \sim \exp[-m_\pi L]$$

($x_{\rm ref} = (x+y)/2$ is at middle of QCD box using transnational invariance)

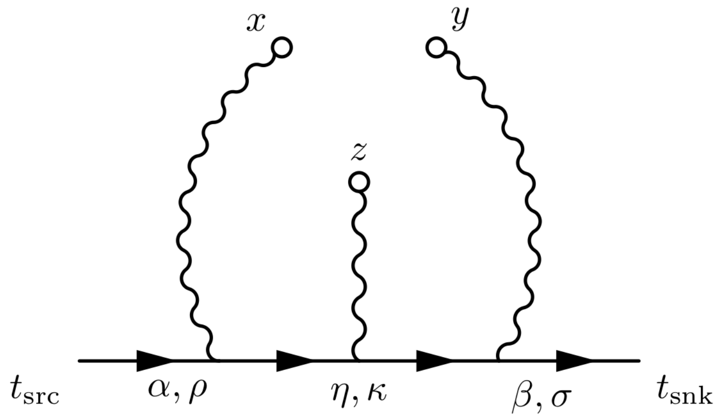
$\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op})$



$\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z)$

Infinite volume QED_∞ [Green et al., 2015, Asmussen et al., 2016, Lehner and Izubuchi, 2015, Jin et al., 2015, Blum et al., 2017b]

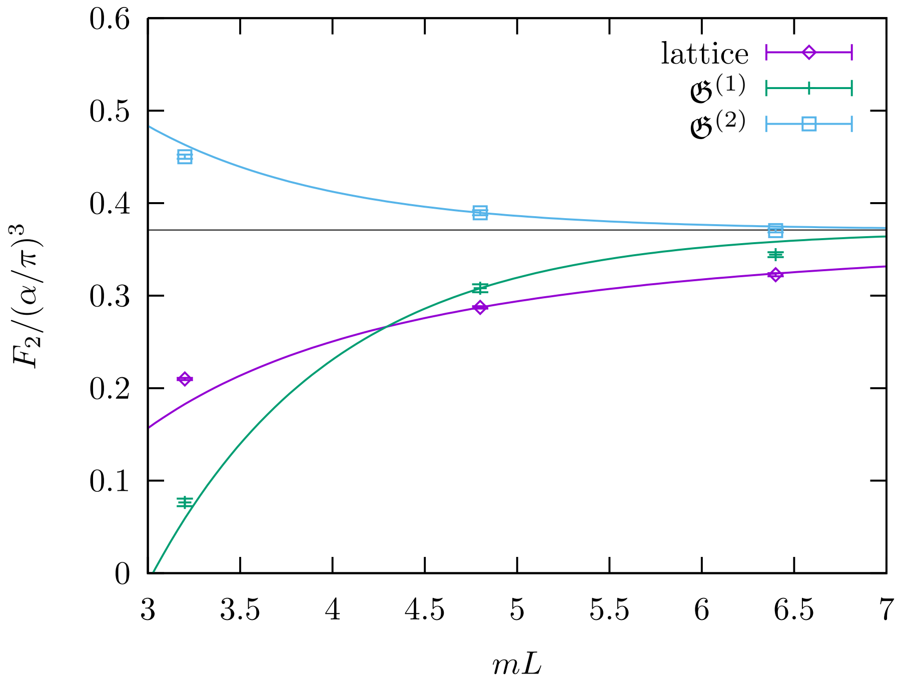
QCD in finite volume, QED in ∞ volume



$= \mathcal{Q}(x, y, z)$

- Mainz group first proposed QED_∞ method
- QED_∞: muon, photons computed in infinite volume, continuum (*c.f.* HVP)
- QED "weight" function $\mathcal{Q}(x, y, z)$ pre-computed
- subtract terms that vanish as $a \to 0$, $L \to \infty$ to reduce $O(a^2)$ errors
- Leading FV error is exponentially suppressed (*c.f.* HVP) instead of $O(1/L^2)$
  - QCD mass gap: $\mathcal{H}(x, y, z, x_{\rm op}) \sim \exp - m_\pi \times \mathrm{dist}(x, y, z, x_{\rm op})$
  - QED weight function does not grow exponentially

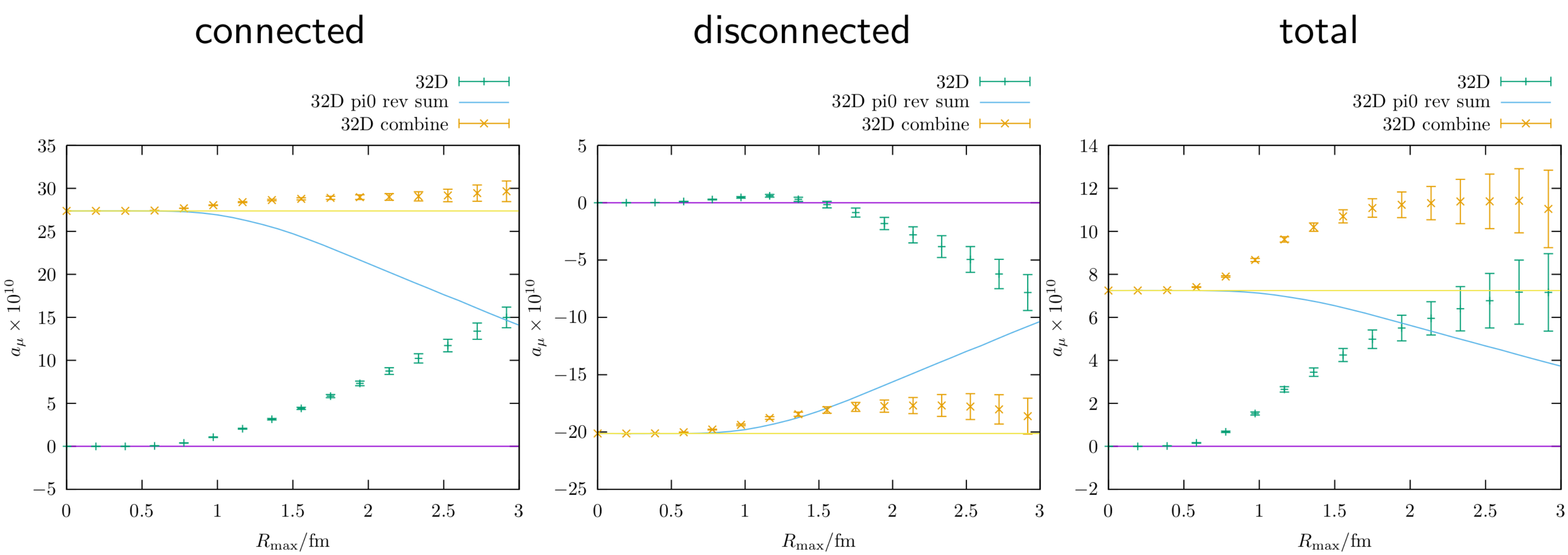
Results, QED case, Finite Volume Error



- QED weight : QED_L (purple diamond), QED_∞ without subtraction (green plus), with subtraction (blue square)
- Curves correspond to expected finite volume scaling $(0.371 + k/L^2)$ and infinite volume scaling $(0.371 + ke^{-mL})$, where the coefficient k is chosen to match the data at $mL = 4.8$.
- The right most point for the finite volume weighting function lies a bit off its scaling curve because the discretization error has not been completely removed, and the coefficient k does not contain any possible volume dependence.

[Tom Blum et al. Anomalies 2019]

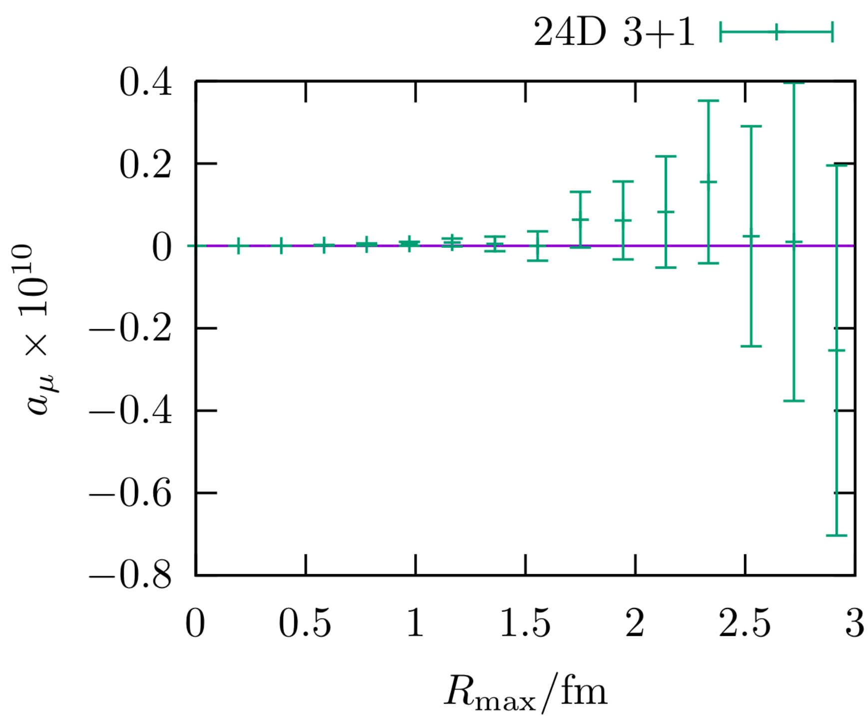
QED$_\infty$, 139 MeV pion, $a = 0.2$ fm, $L = 6.4$ fm (preliminary)



Combine full lattice result, up to R_{max}, with π^0 contribution from model or lattice from R_{max} to ∞ for most precise result ($c.f.$, QED$_L$ result)

[Tom Blum et al. Anomalies 2019]

dHLbL, QED_∞ (non-leading diagram), $m_\pi = 139$ MeV, $a = 0.2$ fm (preliminary)



negligible contribution compared to error on leading contributions

Summary & Perspectives

- **HVP**
  - R-ratio has 2.5-4.0 x10^{-10} [0.35-0.58 %], BABAR-KLOE discrepancy, New data coming (e.g. Belle-II)
  - Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
  - Pure Lattice HVP 5x10^{-10} [0.7%] this year possible, ~1x10^{-10} [0.14%] for long term
  - consolidate error of R-ratio + Lattice at 3x10^{-10} [0.4%]
  - Check BABAR-KLOE tention by windown method, AND different lattice group results !

- **HLbL**
  - computing connected and leading disconnected diagrams, take continue & infinite volume limits
  - QED_L and QED_∞ Long distance from neutral pion pole, cross-check each other, Mainz group
  - pi0 pole contribution & higher order disconnected diagrams are in progress
  - preliminary result not very different from the model results (Glasgow consensus)
  - Unlikely explain the exp-theory 3+ σ discrepancy

- Discrepancy between SM and Exp ~ 3.3 – 3.7 σ , New physics, or …. ?

- Also τ lepton hadronic decay (Belle-2, x50 statistics) , for CKM physics (V_{us}) [H. Ohki et al Phys.Rev.Lett. 121 (2018) 202003] , new physics and g-2 inputs [M. Bruno, PoS Lattice 2018 (2018) 135]

Subtraction using current conservation

- From current conservation, $\partial_\rho V_\rho(x) = 0$, and mass gap, $\langle x V_\rho(x) \mathcal{O}(0)\rangle \sim |x|^n \exp(-m_\pi |x|)$

$$\sum_x \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\text{op}}) = \sum_x \langle V_\rho(x) V_\sigma(y) V_\kappa(z) V_\nu(x_{\textsf{op}})\rangle = 0$$

$$\sum_z \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\text{op}}) = 0$$

at $V \to \infty$ and $a \to 0$ limit (we use local currents).

- We could further change QED weight

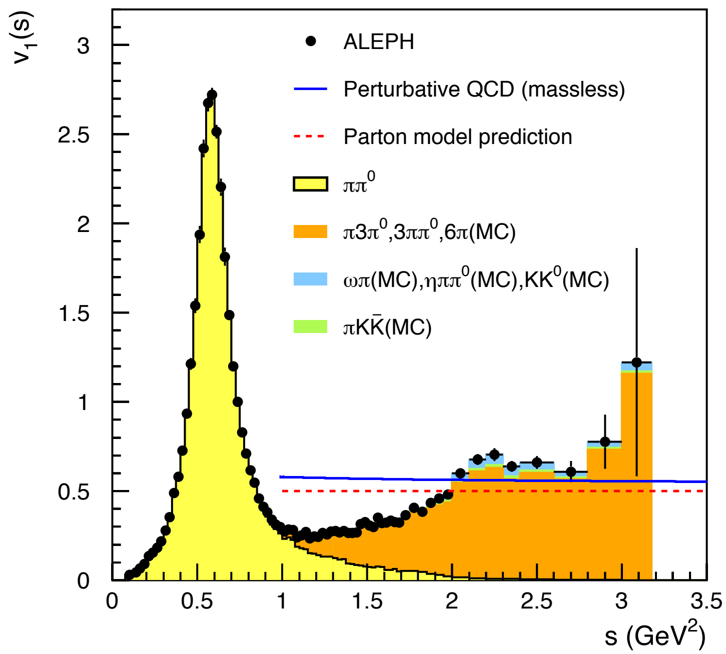
$$\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,y) + \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,y)$$

without changing sum $\sum_{x,y,z} \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\text{op}})$.

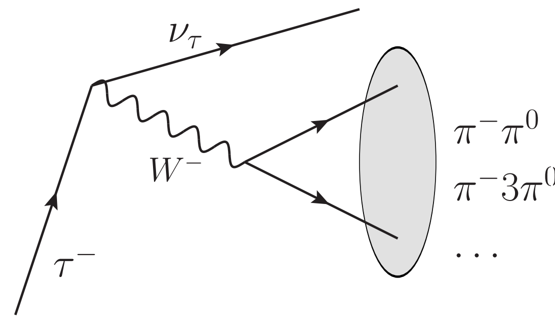
- Subtraction changes discretization error and finite volume error.
- Similar subtraction is used for HVP case in TMR kernel, which makes FV error smaller.
- Also now $\mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(z,z,x) = \mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,z) = 0$, so short distance $\mathcal{O}(a^2)$ is suppressed.
- The 4 dimensional integral is calculated numerically with the CUBA library cubature rules. (x,y,z) is represented by 5 parameters, compute on N^5 grid points and interpolates. ($|x-y| < 11$ fm).

[M. Bruno's slide]

Tau input for g-2 HVP

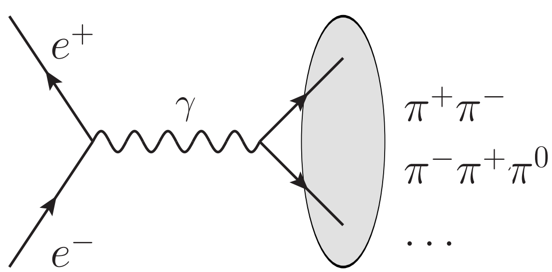


[M. Bruno et al, arXiv:1811.00508]



$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\rightarrow 72\%$ of total Hadronic LO

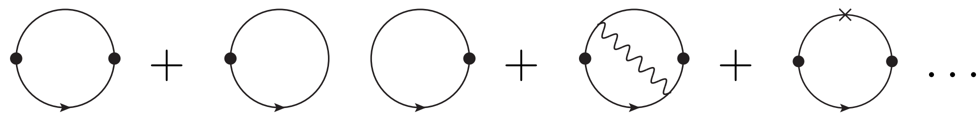
or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

amu & isospin components

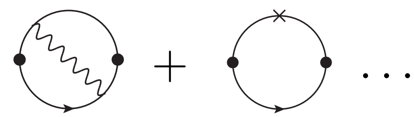
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$



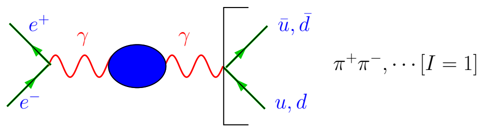
$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$

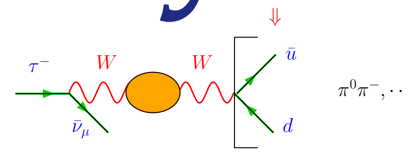


$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

difference b/w tau decay and e+e-



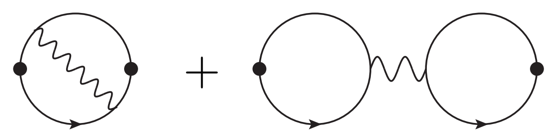


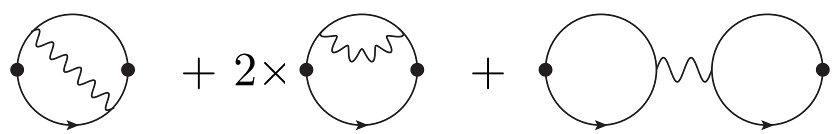
$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

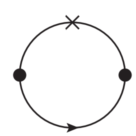
Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left[G_{01}^\gamma(t) + G_{11}^\gamma(t) - G_{11}^W(t)\right]$$

$$\delta G^{(1,1)} \equiv G_{11}^\gamma - G_{11}^W$$

$$= Z_V^4 (4\pi\alpha) \frac{(Q_\mathrm{u} - Q_\mathrm{d})^4}{4} \Big[$$  $$\Big]$$

$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Big[$$  $$+ \ldots \Big]$$

$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Big[2\times$$  $$+ \ldots \Big]$$

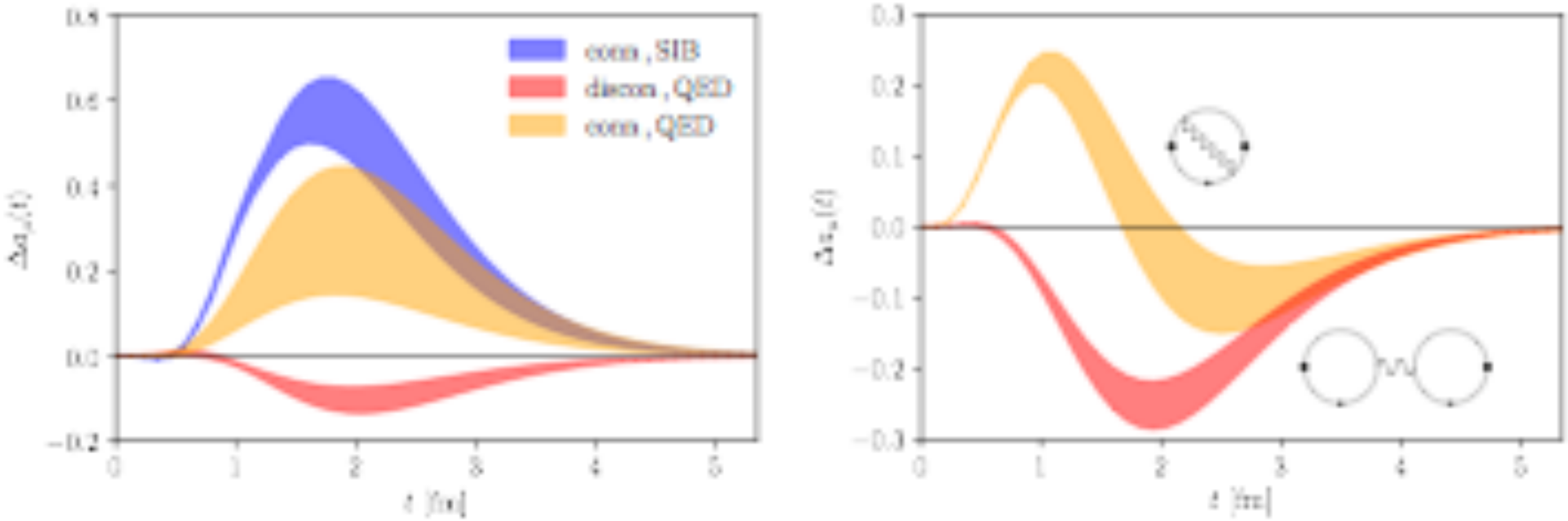
$\ldots$ = subleading diagrams currently not included

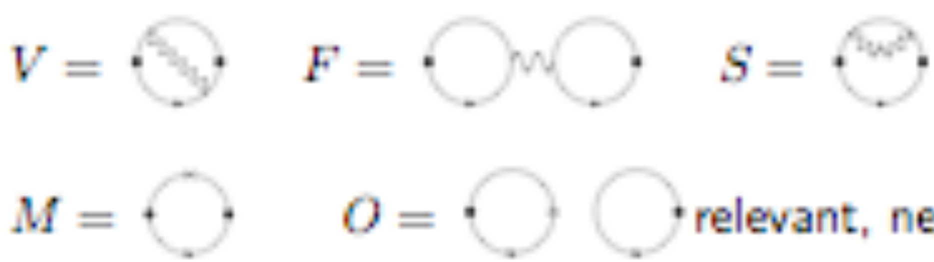
[M. Bruno's slide]

Δa_μ (Preliminary)

Δa_μ from G^{γ}_{01} (QED and SIB):

Pure $I = 1$ only $O(\alpha)$ terms:



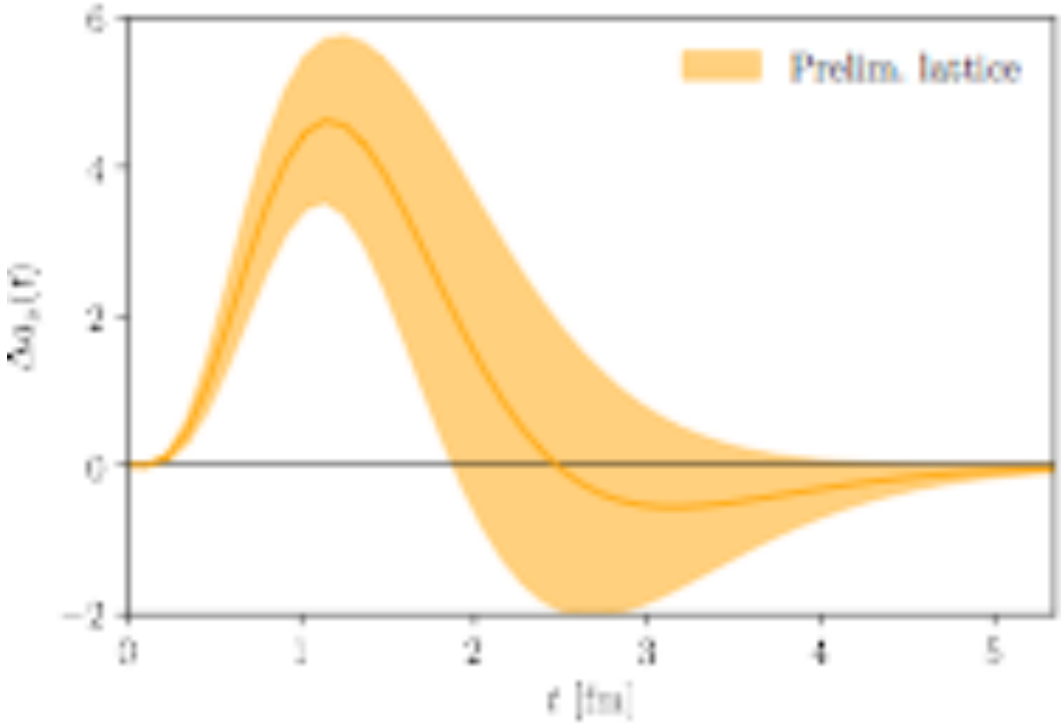


relevant, negative, neglected

Tau spectral function (vector, Strange=0) is very welcome !

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left[G_{01}^{\gamma}(t) + G_{11}^{\gamma}(t) - G_{11}^{W}(t)\right]$$

Preliminary lattice (full) calculation: $G_{01}^{\gamma} + \delta G$



Not included:

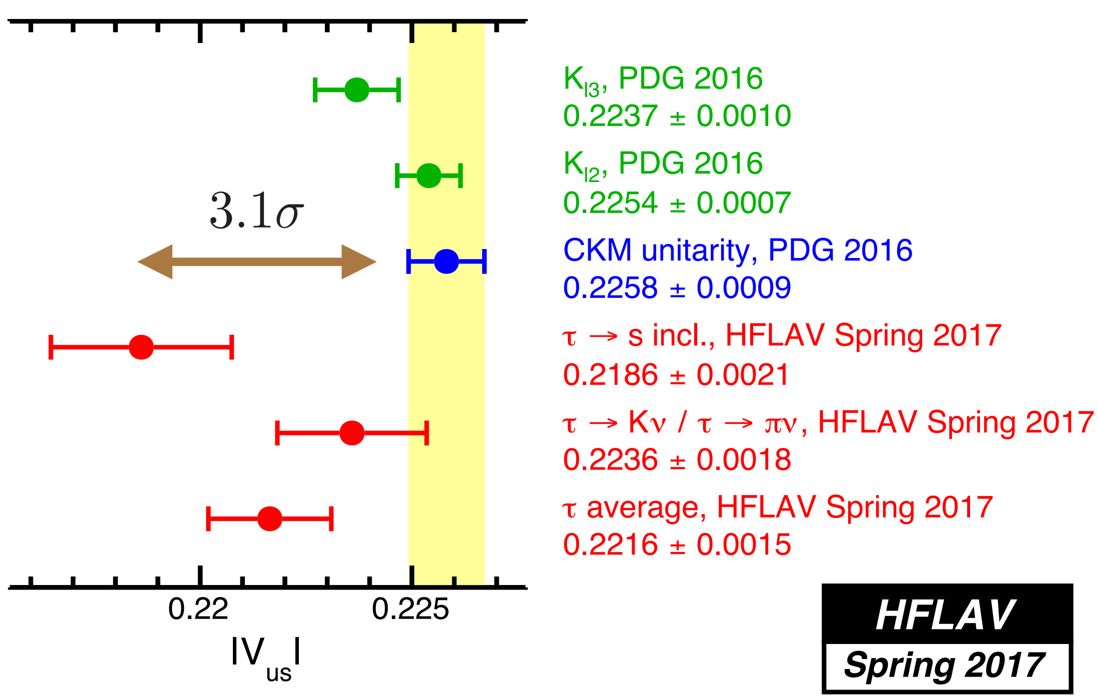
1.  relevant
2. sub-leading $1/N_c$, $1/N_f$
3. finite-volume errors
4. discretization errors

CKM V_{us} from Inclusive tau decay

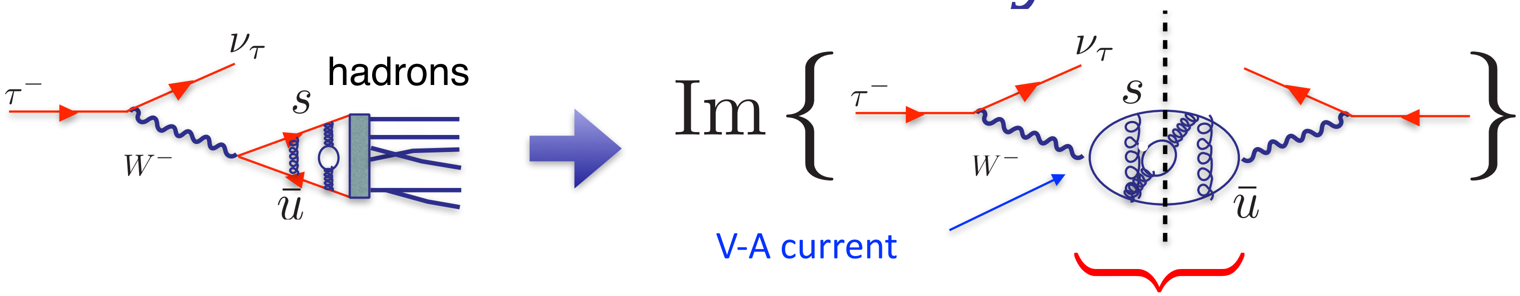
Yet another by-product of muon g-2 HVP

Phys.Rev.Lett. 121 (2018) 202003

[Hiroshi Ohki et al.]



Tau decay



(Hadronic) vacuum polarization function $\Pi(Q^2)$

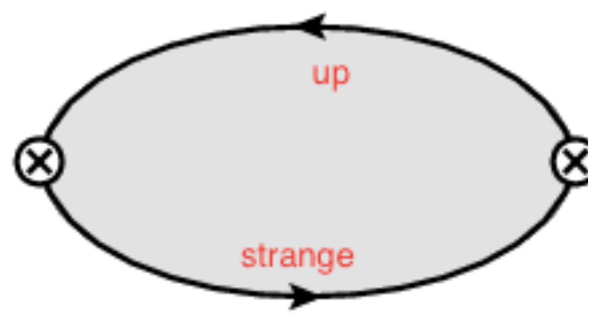
- Experiment side : $\tau \to \nu + had$ through V-A vertex. EW correction S_{EW}

$$R_{ij} = \frac{\Gamma(\tau^- \to \text{hadrons}_{ij}\ \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$$

$$= \frac{12\pi |V_{ij}|^2 S_{EW}}{m_\tau^2} \int_0^{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \underbrace{\left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]}_{\equiv\ \mathrm{Im}\,\Pi(s)}$$

- Lattice side : The Spin=0 and 1, vacuum polarization, Vector(V) or Axial (A) current-current two point

$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i \int d^4x e^{iqx} \left\langle 0 | T J^{\mu}_{ij;V/A}(x) J^{\dagger\mu}_{ij;V/A}(0) | 0 \right\rangle$$
$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$



Finite Energy Sum Rule (FESR)

[Shifman, Vainstein, Zakharov 1979]

- FESR = Optical theorem (Unitarity) + Dispersion relation (Analyticity)
- Optical theorem relate S=-1 spectral function $\rho^{0/1}_{V/A,ij}(s)$ and HVP $\Pi^{0/1}_{V/A,ij}(s)$ for given quantum number: flavor (us or ud), spin (0 or 1), parity (V or A)

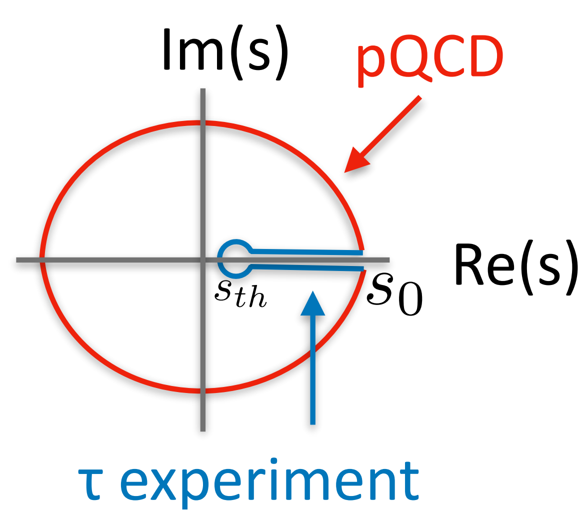
$$\frac{1}{\pi}\mathrm{Im}\Pi(s) = \rho(s)$$

- Do *finite* radius contour integral for arbitrary regular weight function $w(s)$

$$\int_{s_{th}}^{s_0} ds\,\rho(s)w(s) = +\frac{i}{2\pi}\oint_{|s|=s_0} ds\,\Pi(s)w(s)$$

- Real axis integral is extracted from experimental decay energy distribution dR_τ/ds

$$\frac{dR_{ij;V/A}}{ds} = \frac{12\pi^2|V_{ij}|^2 S_{EW}}{m_\tau^2}\omega_\tau(s)\rho(s)$$



$|V_{us}|$ determination from FESR

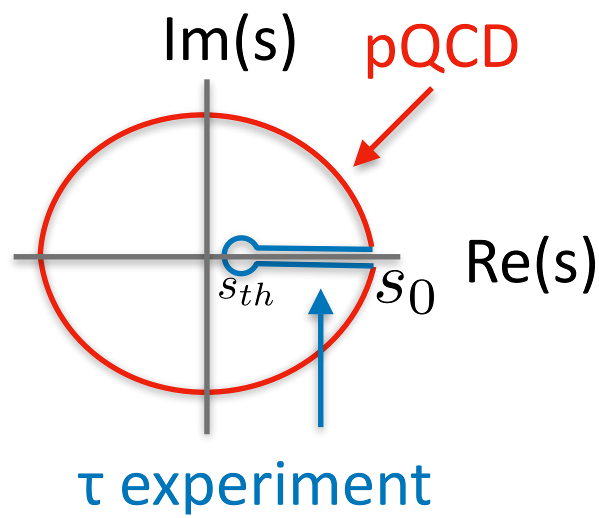
[E. Gamiz, *et al.*, 2003, 2005, Maltman et al 2006]

- Inclusive differencial τ decay rate with wieght $w(s)$

$$R^{\omega}{}_{ij}(s_0) \equiv \int_{s_{th}}^{s_0} ds \frac{dR_{ij}}{ds} \frac{\omega(s/s_0)}{\omega_\tau(s/m_\tau^2)}$$

- Take difference between up-down and up-strange channel

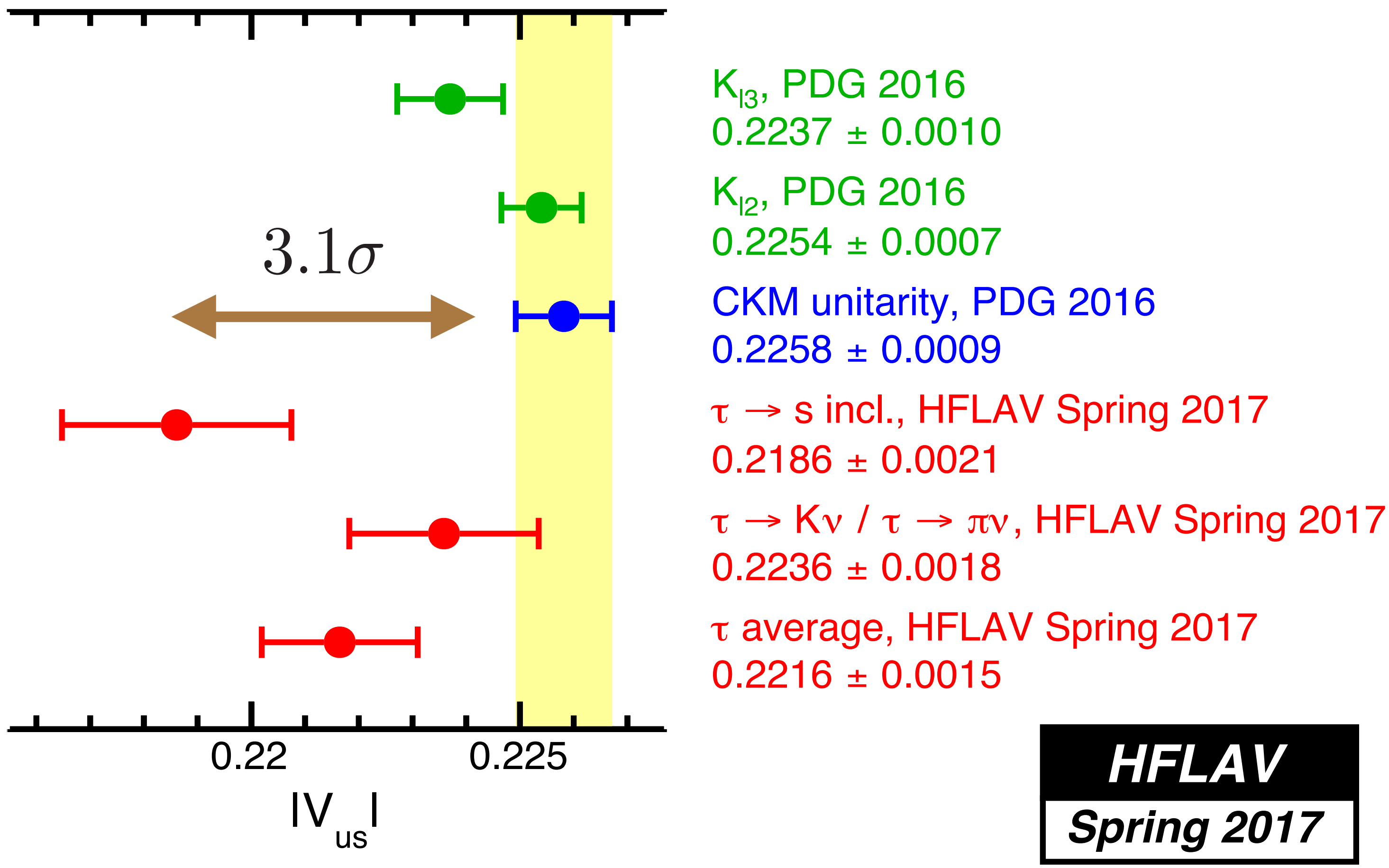
$$\Delta R^{\omega} = \frac{R^{\omega}_{ud}}{|V_{ud}|^2} - \frac{R^{\omega}_{us}}{|V_{us}|^2}$$



- $|V_{ud}|$ and m_s as input, selecting $s_0 = m_\tau^2, \omega = \omega_\tau(s/s_0)$

$$|V_{us}| = \sqrt{\frac{R^{\omega}_{us}(s_0)}{\frac{R^{\omega}_{ud}(s_0)}{|V_{ud}|^2} - [\Delta R^{\omega}(s_0)]^{\text{pQCD}}}}$$

- For $s > s_0$, fixed-order or contour-improved pQCD is used. OPE condensations at dim=4,6 ... are input/assumed. (a source of unaccounted uncertainties)



- τ result v.s. non-τ result : more than 3 σ deviation : |Vus| puzzle
- new physics effect?
- incl. analysis uses Finite energy sum rule (FESR)
- pQCD and higher order OPE for FESR:

  underestimation of truncation error and/or non-perturbative effects ?

  (c.f. alternative FESR approach, R. Hudspith et. al arXiv:1702.01767)

Our new method : Combining FESR and Lattice

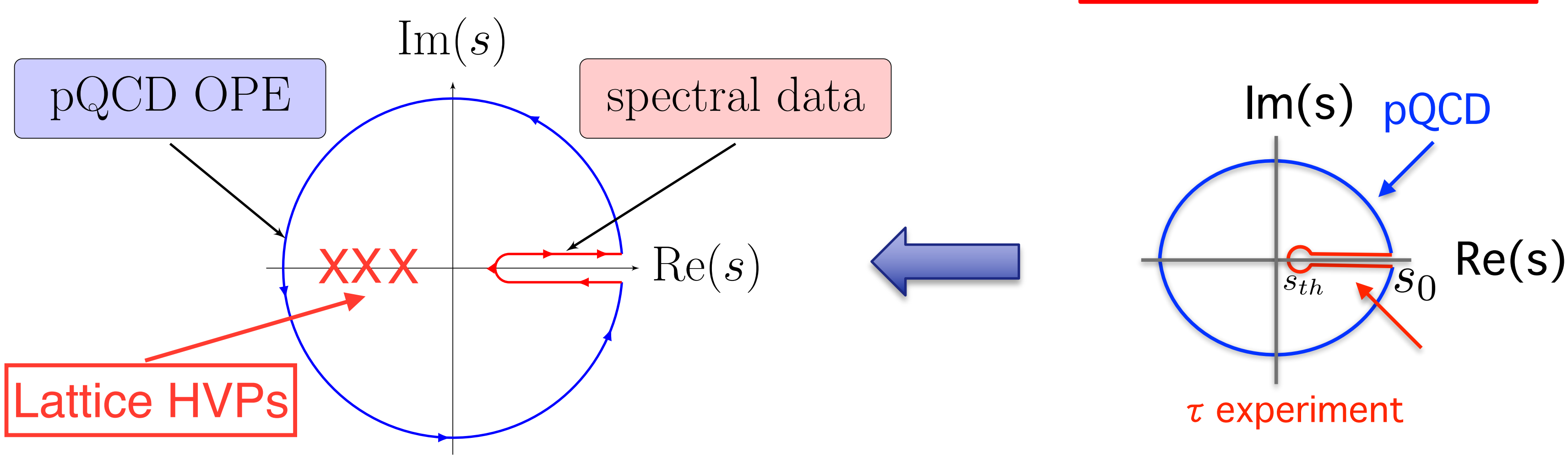
- If we have a reliable estimate for $\Pi(s)$ in Euclidean (space-like) points, $s = -Q_k^2 < 0$, we could extend the FESR with weight function $w(s)$ to have poles there,

$$\int_{s_{th}}^{\infty} w(s)\mathrm{Im}\Pi(s) = \pi \sum_{k}^{N_p} \mathrm{Res}_k[w(s)\Pi(s)]_{s=-Q_k^2}$$

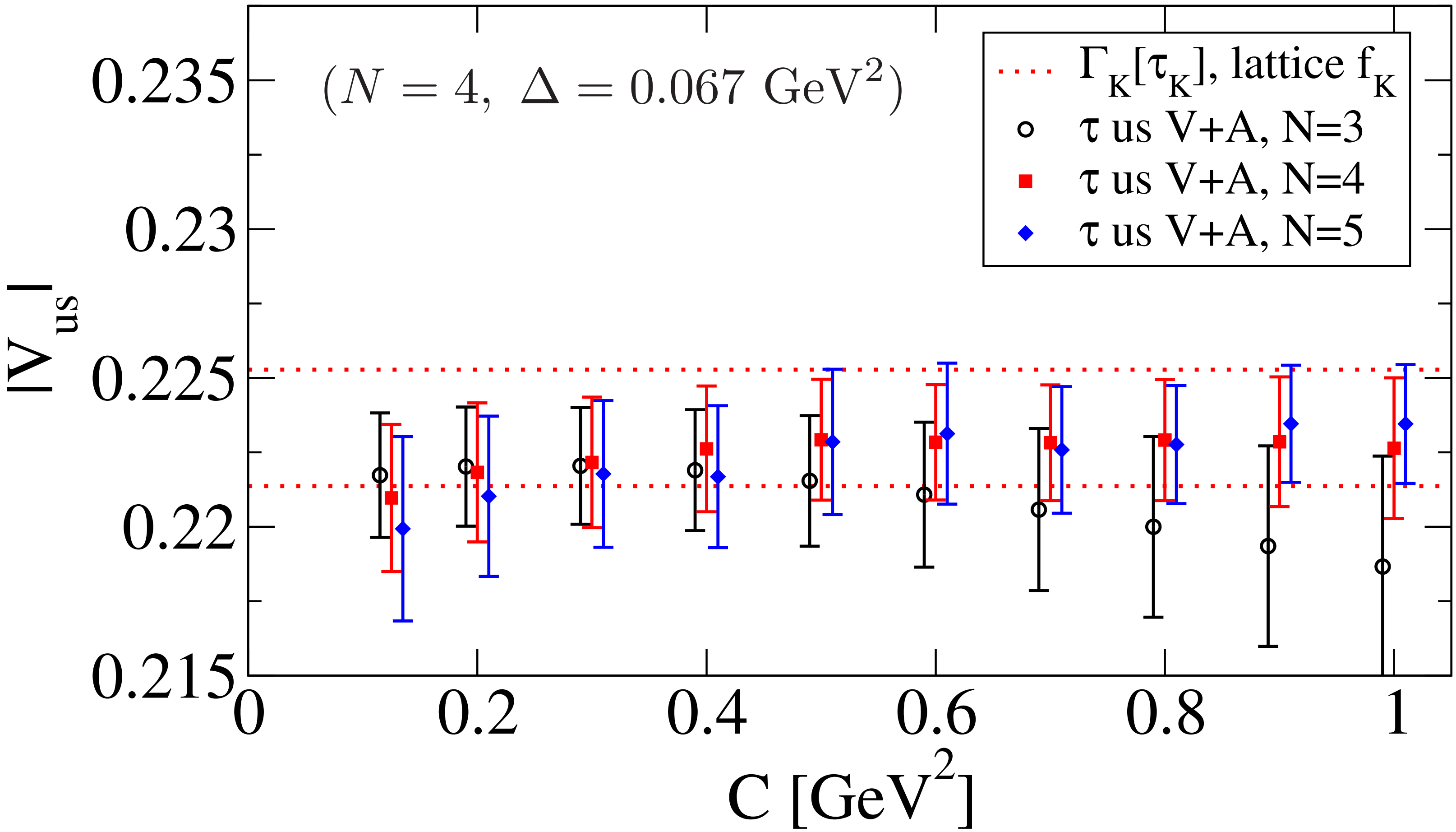
$$\Pi(s) = \left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s) \propto s \quad (|s| \to \infty)$$

- For $N_p \geq 3$, the $|s| \to \infty$ circle integral vanishes.

$$w(s) = \prod_{k}^{N_p} \frac{1}{(s + Q_k^2)}$$



Lattice Inclusive $|V_{us}|$ determinations

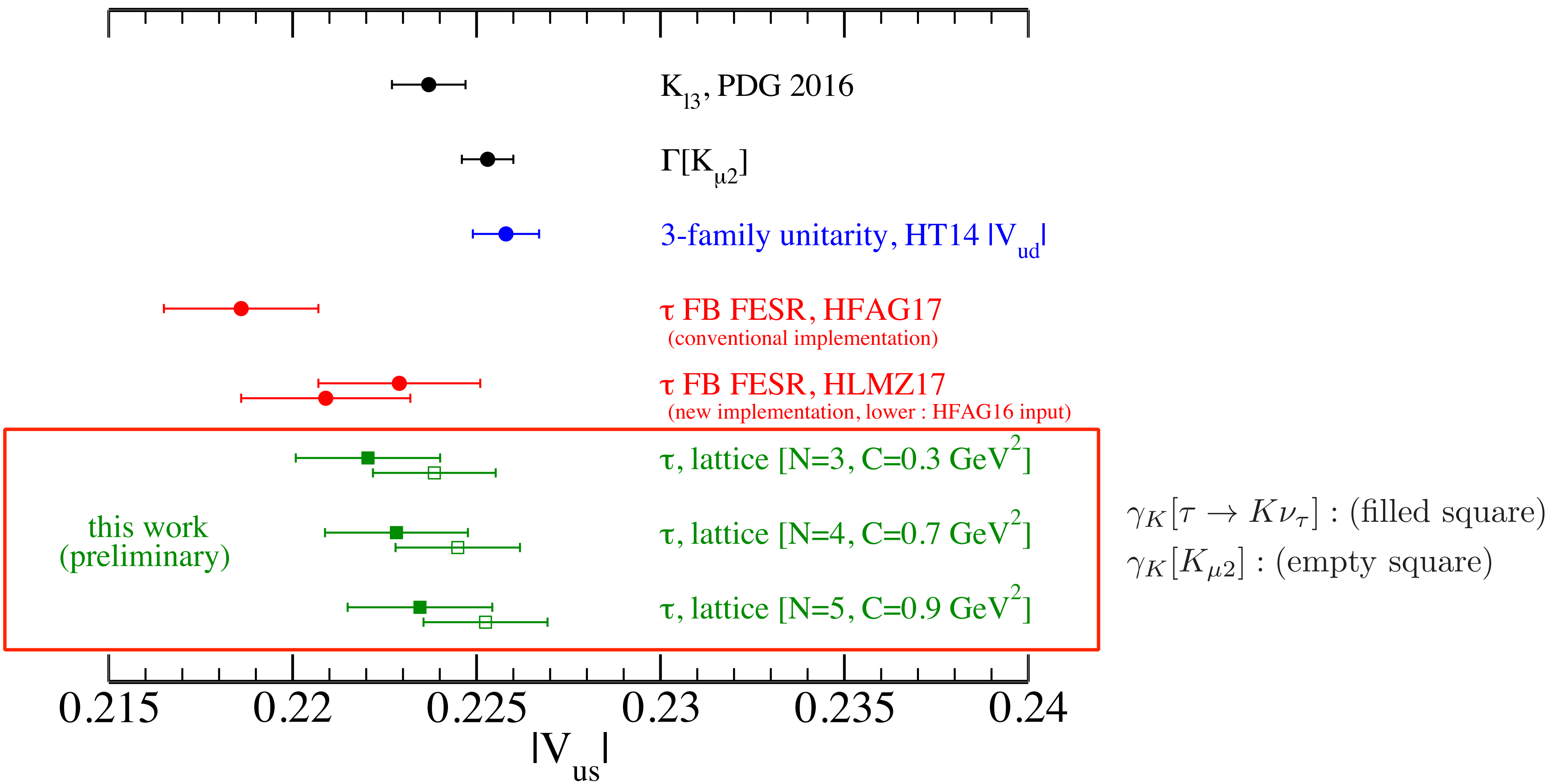


Theory and experimental errors are included.

The result is stable against changes of C and N.

$N = 4, C = 0.7[\mathrm{GeV}^2] : |V_{us}| = 0.2228(15)_{exp}(13)_{th}$ (0.87% total error)

Comparison to $|V_{us}|$ from others



Tau spectral function (vector/axial, Strange=-1) is very welcome !

[Luchang Jin's analogy]

Precession of Mercury and GR

discrepancy recognized since 1859

Amount (arc-sec/century)	Cause
5025.6	Coordinate (due to precession of equinoxes)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the sun (quadrupole moment)
42.98±0.04	General relativity
5600.0	Total
5599.7	Observed

Known physics

1915 by-then New physics
GR revolution

http://worldnpa.org/abstracts/abstracts_6066.pdf

precession of perihelion