Flavor modular symmetry

Tatsuo Kobayashi

1. Introduction
2. Modular symmetry
3. Magnetic flux compactification
4. Model building
5. Summary

1. Introduction
toward the unified theory

string open string closed string

Superstring theory : the promising candidate for
the unified theory of our world, that is,
all of interactions including gravity
and matter such as quarks and leptons, higgs,
and all of cosmological aspects.

Introduction

Theory of Everything: Superstring theory

素粒子標準模型を再現

素粒子や宇宙の謎を解明した

Introduction

Superstring theory ⇒ 我々の4次元時空
＋6次元(コンパクト)空間　予言

数えきれないコンパクト空間の候補

The compactification scale is expected to be much higher than the weak scale.

weak scale までたどり着く、様々なシナリオ
SUSY, GUT, etc.

top-down approach + bottom-up approach
相補的に

Introduction

compactification scale and weak scale

a huge energy scale gap.

top-down approach + bottom-up approach
　相補的に

Symmetry
　　架け橋となるのではないか　？

コンパクト空間上の超弦理論のもつ素粒子物理の意味
を考える

場の理論での模型構築で有効とされる対称性の起源
を考える

Flavor mystery

素粒子の世界はいまだに多くの謎をふくむ

その1つが flavor mystery:

the origin of the flavor structure

なぜ3世代　？

どうして、クォーク・レプトンの質量は階層的　？

混合角、CP　　？

still a challenging issue

Flavor mystery

なぜ3世代　？

どうして、クォーク･レプトンの質量は階層的　？

混合角、CP　　？

多くのアイデアが提案された。

その　1つ　　non-Abelian discrete flavor symmetries

レプトンの混合角は大きい

非可換対称性で説明できるのではなか

Non-Abelian flavor symmetry

離散フレーバー対称性を仮定

様々な対称性

S_N, A_N D_N, Q_N, $\Delta(N)$,.......

scalar 場　flavon　のVEVでこの対称性を破って、

現実的な質量行列を導出

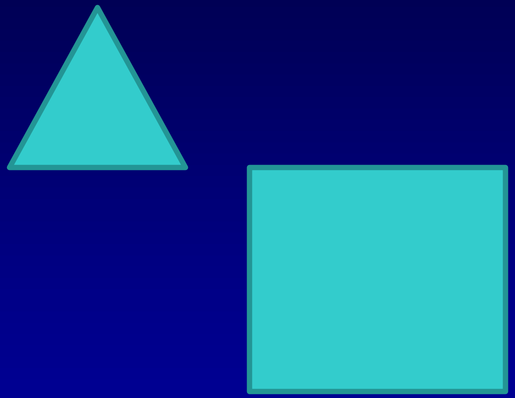
クォーク･レプトン

Geometrical symmetries

S3　　正三角形の対称性

D4　　正方形

A4　　正四面体



....

コンパクト空間は、離散対称性の起源　？？？

String theory on compact space

コンパクト空間の幾何学的対称性

＋

コンパクト空間上のstring の結合選択則

⇒ 大きな対称性へ

Heterotic string theory on orbifolds

intersecting/magnetized D-brane models

on torus

⇒ D4, Δ(27), Δ（54）, …..

T.K. Raby, Zhang, '04

T.K. Nilles, Ploger, Raby, Ratz, '06

Abe, Choi, T.K. Ohki, '09

Flavor modular symmetry

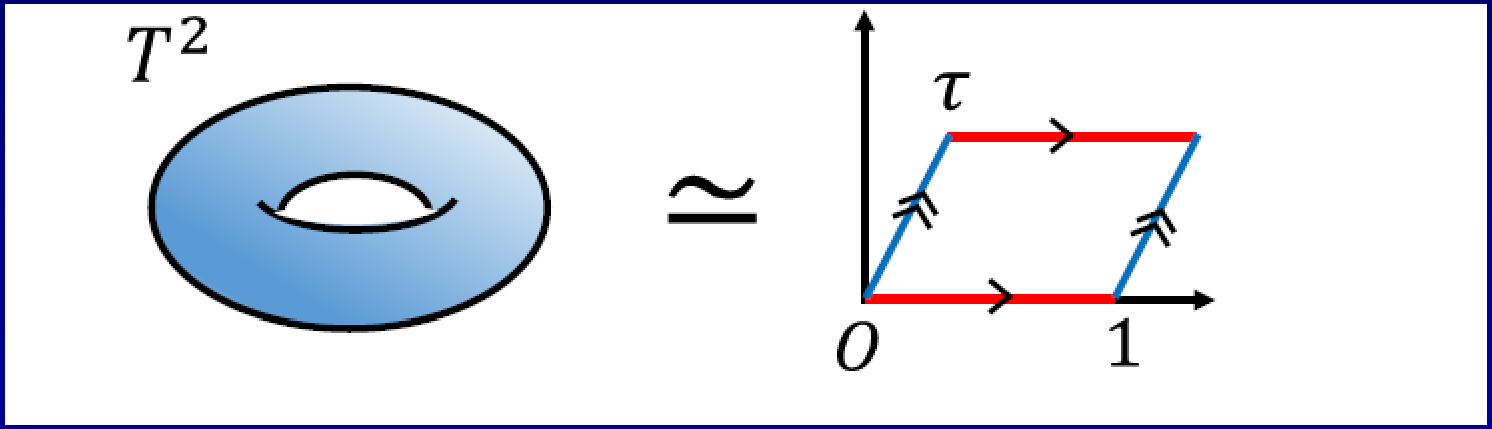
今回のものは、以前の対称性と
　　ちょっと違う

たとえば、　　Yukawa 結合も変換
　　　　　　　flavon を導入しなくてもよい

top-down approach
bottom-up approach　の両方を以下で、紹介
これらにはまだ隔たりがある

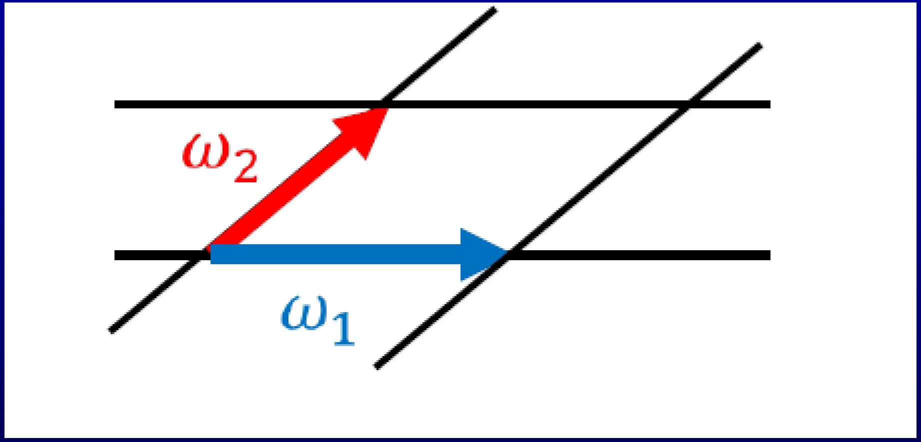
2. Modular symmetry: keyword

torus compactification



modulus

Lattice vectors



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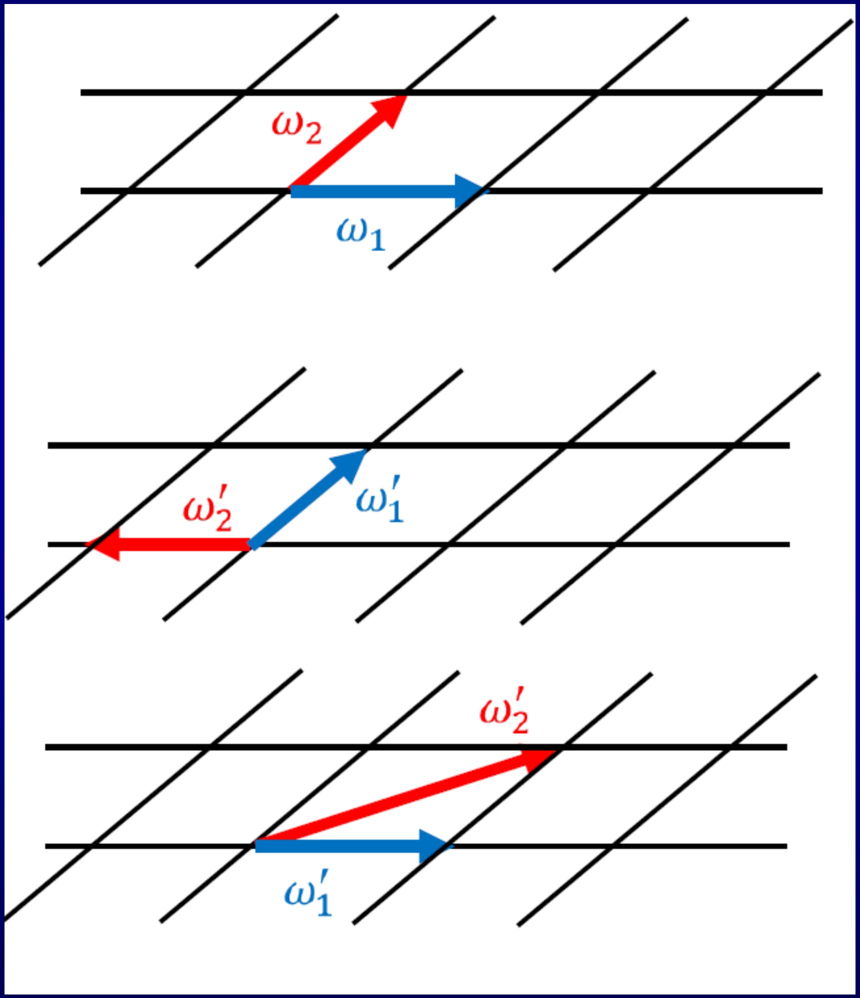
Modular symmetry

change of lattice

vectors



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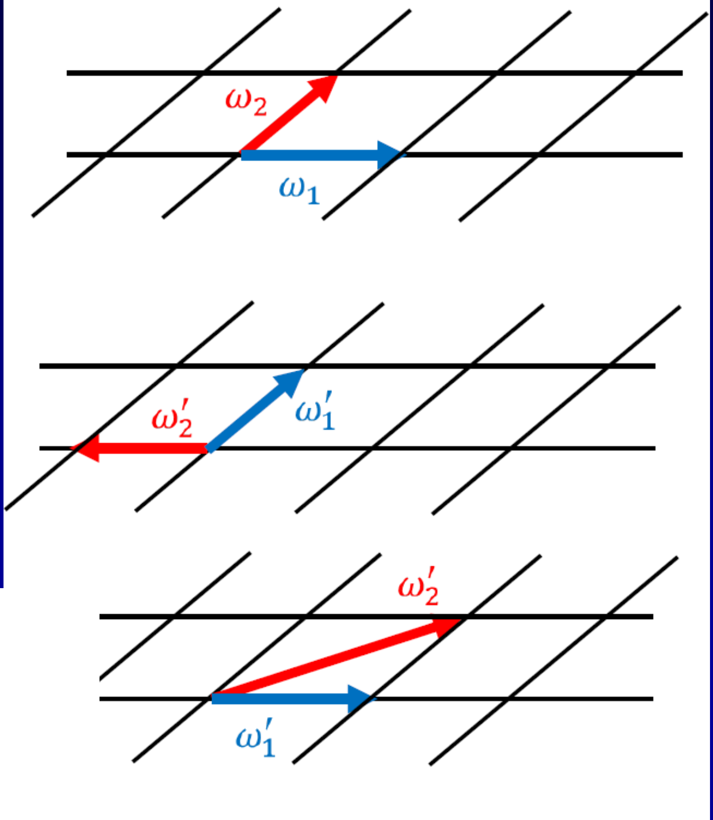
Modular symmetry

Change of lattice vectors



$$\begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \qquad \begin{array}{l} ad - bc = 1 \\ a, b, c, d \in \mathbb{Z} \end{array}$$

It is equivalent to

$$\tau' = \frac{a\tau + b}{c\tau + d}, \qquad \begin{array}{l} ad - bc = 1 \\ a, b, c, d \in \mathbb{Z} \end{array}$$

Modular transformation

Modular group Γ

Modular symmetry

Generator S and T

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

algebraic relations

$$S^2 = 1, \qquad (ST)^3 = 1.$$

infinite number of elements

Modular symmetry

Generator S and T

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

trivial な torus compact化

ゼロモード constant

non-trivial example

magnetic flux compactification

torus and orbifolds

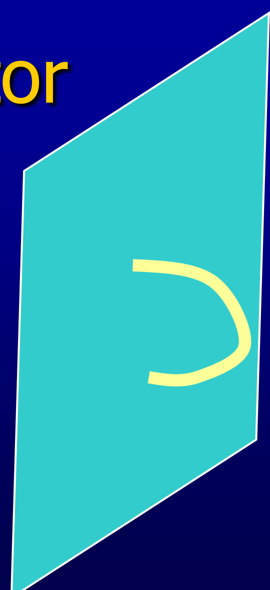
3. Magnetic flux compactification of D-brane models: top-down approach

gauge boson: open string, whose two end-points are on the same (set of) D-brane(s)

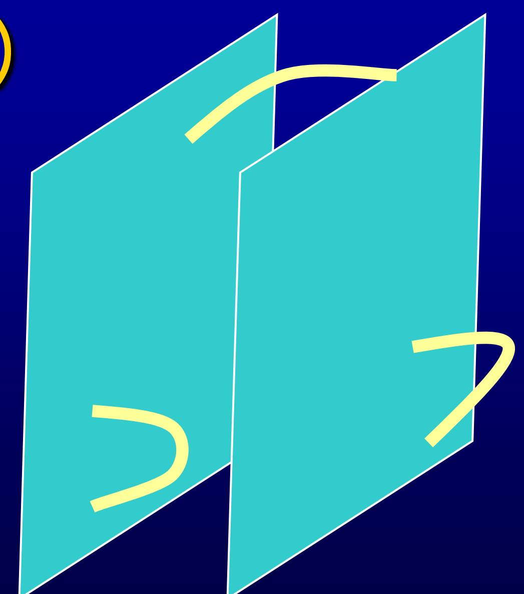
N parallel D-branes ⇒ U(N) gauge group

higher-dimensional SYM

U(1) vector multiplet

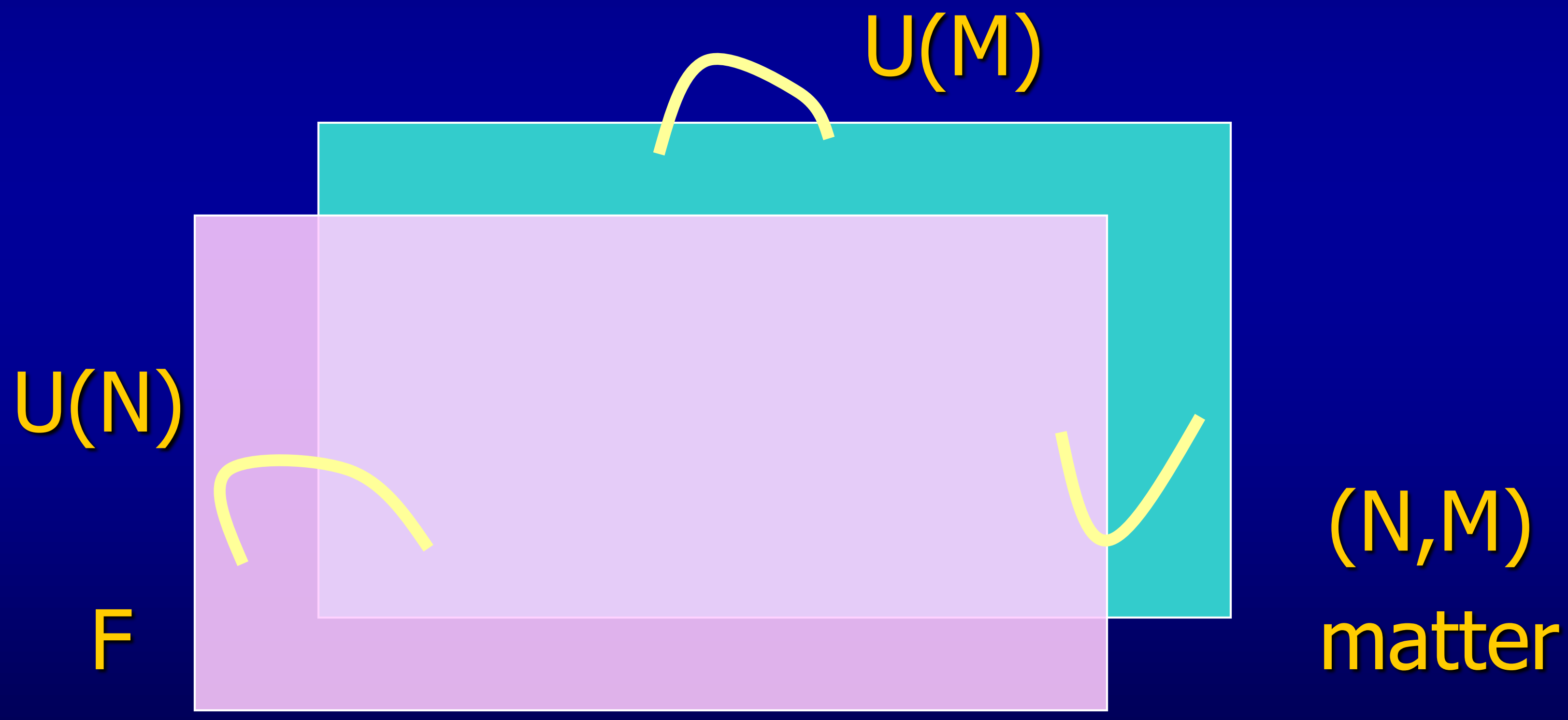


U(2)



Magnetized D-branes

We consider torus compactification with magnetic flux background, F_{45}, etc.



Super YM theory

effective field theory

6D super U(N) super YM theory

2D torus compact化

U(Na) とU（Nb）のトレース成分へ

magnetic flux　　トーラス方向に

10D super YM on 6D も同様

Fermions in bifundamentals $(N = N_a + N_b)$

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}.$$

Breaking the gauge group $U(N) \to U(N_a) \times U(N_b)$

(Abelian flux case $M_a, M_b \in Z$

The gaugino fields $\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$

⇨ λ^{aa} and λ^{bb} gaugino of unbroken gauge Adj N_a, Adj N_b.

λ^{ab} and λ^{ba} bi-fundamental matter fields $(N_a, \bar{N}_b), (\bar{N}_a, N_b)$.

Zero-mode Dirac equations

$$\begin{pmatrix} \bar{\partial}\psi_+^{aa} & \left[\bar{\partial} + 2\pi(M_a - M_b)y_4\right]\psi_+^{ab} \\ \left[\bar{\partial} + 2\pi(M_b - M_a)y_4\right]\psi_+^{ba} & \bar{\partial}\psi_+^{bb} \end{pmatrix} = 0.$$

$$\begin{pmatrix} \partial\psi_-^{aa} & \left[\partial - 2\pi(M_a - M_b)y_4\right]\psi_-^{ab} \\ \left[\partial - 2\pi(M_b - M_a)y_4\right]\psi_-^{ba} & \partial\psi_-^{bb} \end{pmatrix} = 0.$$

No effect due to magnetic flux for adjoint matter fields, λ^{aa} and λ^{bb}

Total number of zero-modes of λ^{ab} ⇨ $I_{ab} = |M_a - M_b|.$

$M_a - M_b > 0 \Rightarrow$

ψ_+^{ab}, ψ_-^{ba} : Zero-modes

ψ_-^{ab}, ψ_+^{ba} : No zero-mode

Torus with magnetic flux

We solve the zero-mode Dirac equation,

$$i\gamma^m D_m \psi = 0$$

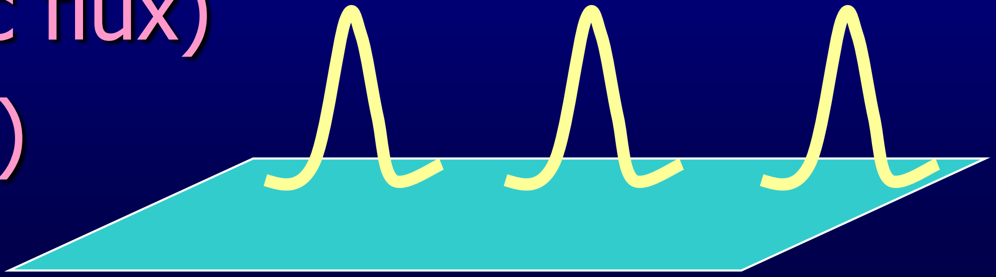
e.g. for U(1) charge q=1.

Torus background with magnetic flux leads to chiral spectra.

the number of zero-modes

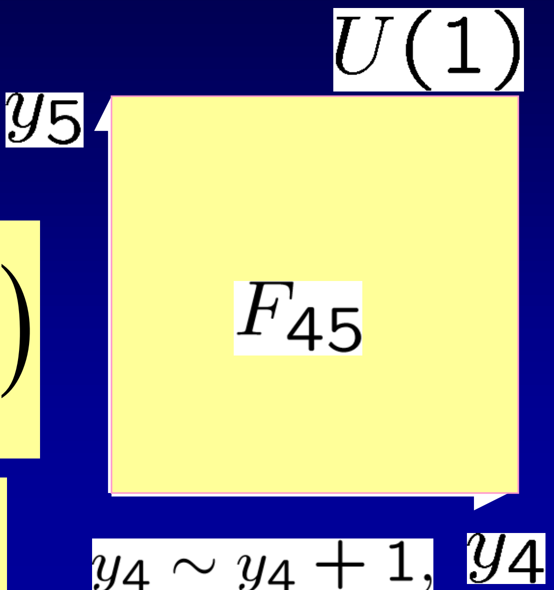
= M (magnetic flux)

x q (charge)



Quantum mechanics: Chap. 15 in Landau-Lifshitz book

particle in magnetic flux(Landau)



$$F_{45} = 2\pi M, \;\; A_4 = 0, \;\; A_5 = 2\pi M y_4$$

$$H = \frac{1}{2m}\left(P_4^2 + (P_5 - 2\pi M y_4)^2\right)$$

$$[H, P_5] = 0 \quad \Downarrow \quad P_5 = 2\pi k$$

$$y_4 \sim y_4 + 1, \quad y_5 \sim y_5 + 1$$

$$H = \frac{1}{2m}\left(P_4^2 + 4\pi^2 M^2 (y_4 - k/M)^2\right)$$

Harmonic oscillator at y=k/M **M=integer**

M degenerate ground states

k=0,1,2,…………,(M-1)

Dirac equation and chiral fermion

|M| independent zero mode solutions in Dirac equation.

$$\Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \cdot \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)$$

$$(j = 0, 1, \cdots, |M| - 1)$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) \equiv \sum_n e^{\pi i (n+a)^2 \tau} e^{2\pi (a+n)(\nu+b)}$$ (Theta function)

Properties of theta functions

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m, \tau) = e^{2\pi i m a} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m\tau, \tau) = e^{-\pi m^2 \tau - 2\pi i m(\nu+b)} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

chiral fermion

$M \gtrless 0 \Rightarrow$ $\psi_{+/-}$:Normalizable mode

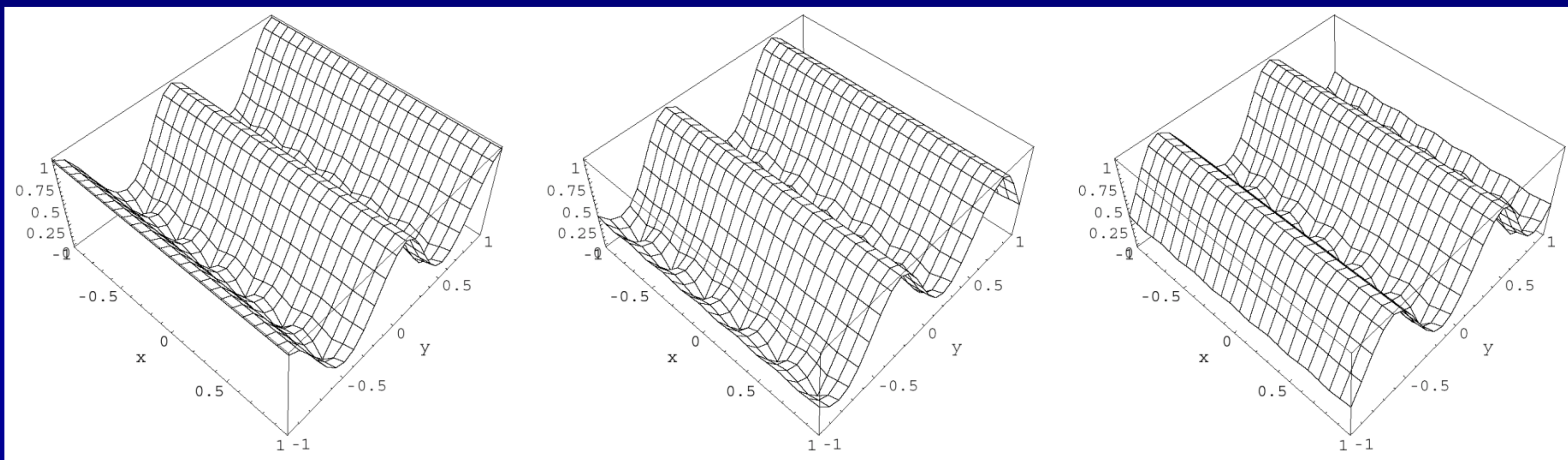
$\psi_{-/+}$:Non-normalizable mode

By introducing magnetic flux, we can obtain chiral theory.

Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Illustrating model: U(8) → Pati-Salam → SM

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & m_2 \mathrm{I}_{N_2} & \\ 0 & & m_3 \mathrm{I}_{N_3} \end{pmatrix}$$

$$N_1 = 4,\ N_2 = 2,\ N_3 = 2$$

Pati-Salam group $U(4) \times U(2)_L \times U(2)_R$

$$(m_1 - m_2) = (m_3 - m_1) = 3 \text{ for the first } T^2$$
$$(m_1 - m_2) = (m_3 - m_1) = 1 \text{ for the other tori}$$

WLs along a U(1) in U(4) and a U(1) in U(2)R

=> Standard gauge group up to U(1) factors

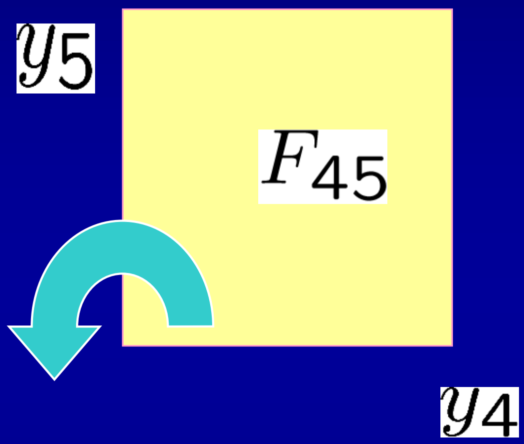
$$U(3)_C \times U(2)_L \times U(1)^3$$

U(1)Y is a linear combination.

Orbifold with magnetic flux

Abe, T.K., Ohki, '08

T2/Z2



We can also embed Z2 into the gauge space.

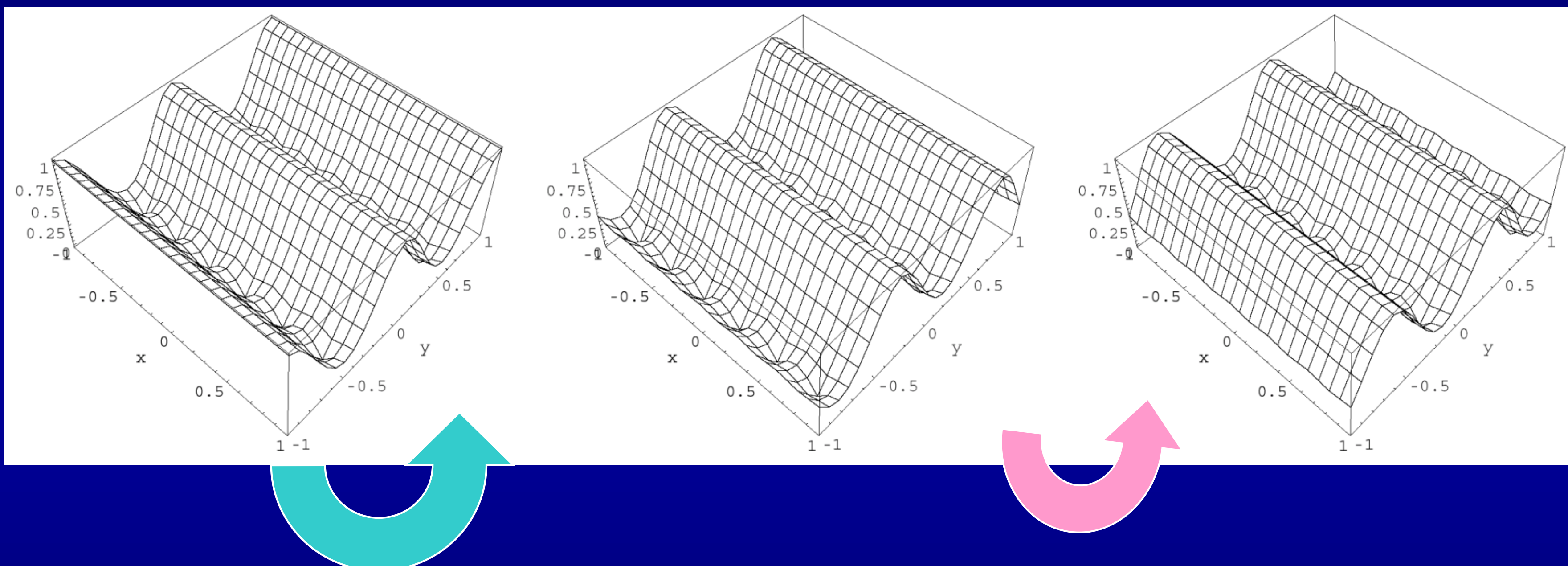
$$\mathrm{Z}_2 : \psi(\mathrm{y}_4, \mathrm{y}_5) \to \psi(-\mathrm{y}_4, -\mathrm{y}_5) = (-\mathrm{i})\Gamma^4\Gamma^5 \mathrm{P}\psi(-\mathrm{y}_4, -\mathrm{y}_5)$$

$(P^2 = 1)$

Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Z2 even and odd modes

$$\Theta^{j}(z) \qquad j = 0,1,\ldots,(M-1)$$

j is related to the peak position.

Z2 transformation

$$\Theta^{j}(-z) = \Theta^{M-j}(z)$$

Z2 even modes

$$(\Theta^{j}(z) + \Theta^{M-j}(z))/\sqrt{2}$$

Z2 odd modes

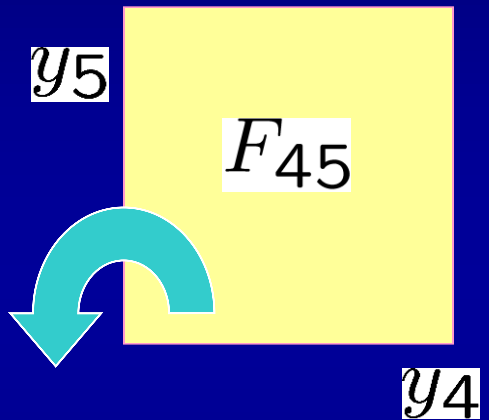
$$(\Theta^{j}(z) - \Theta^{M-j}(z))/\sqrt{2}$$

Orbifold with magnetic flux

Abe, T.K., Ohki, '08

The number of even and odd zero-modes

$M = I^{ab}$	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4



We can also embed Z2 into the gauge space.

$$Z_2 : \psi(y_4, y_5) \rightarrow \psi(-y_4, -y_5) = (-i)\Gamma^4\Gamma^5 P \psi(-y_4, -y_5)$$

$$(P^2 = 1)$$

Orbifolding projects out adjoint matter fields.

Modular symmetry

Generator　S and T

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

trivial な torus compact化

　　ゼロモード　constant

non-trivial example

magnetic flux compactification

torus　and orbifolds

Modular symmetry

T.K., Nagamoto, 1709.09784,

T.K.,Nagamoto, Takada, Tamba, Tatsuishi,1804.06644

Zero-modes

S

$$\psi^{j,M} \to \frac{1}{\sqrt{M}} \sum_k e^{2\pi i jk/M} \psi^{k,M}.$$

T

$$\psi^{j,M} \to e^{\pi i j^2/M} \psi^{j,M},$$

zero-modes transform each other

flavor symmetry

Modular symmetry

T.K.,Nagamoto, Takada, Tamba, Tatsuishi,1804.06644

Example M=2

$\psi^{0,2}, \psi^{1,2}$. The S-transformation acts on these zero-modes as

$$\begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix} \longrightarrow S_{(2)} \begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}, \qquad S_{(2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

The T-transformation acts as

$$\begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix} \longrightarrow T_{(2)} \begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}, \qquad T_{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

They satisfy the following algebraic relations,

$$S_{(2)}^2 = \mathbb{I}, \qquad T_{(2)}^4 = \mathbb{I}, \qquad (S_{(2)}T_{(2)})^3 = e^{\pi i/4}\mathbb{I}.$$

flavor symmetry

$(Z_8 \times Z_4) \rtimes S_3.$

Modular symmetry

T.K.,Nagamoto, Takada, Tamba, Tatsuishi,1804.06644

Example M=2

$$(Z_8 \times Z_4) \rtimes S_3.$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = (T_{(2)})^2. \quad (55)$$

In addition, the permutation Z_2^C element in D_4 corresponds to $S_{(2)}T_{(2)}T_{(2)}S_{(2)}$, i.e.

$$C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = S_{(2)}T_{(2)}T_{(2)}S_{(2)}. \quad (56)$$

Thus, the D_4 group, which includes the eight elements (A8), is subgroup of $G_{(2)} \simeq (Z_8 \times Z_4) \rtimes S_3$.

D4 is a subgroup.

The previous D4 is included in modular symmetry.

Modular symmetry

T.K.,Nagamoto, Takada, Tamba, Tatsuishi,1804.06644

Example M=4

4 zero-modes decompose into 3 + 1

in orbifold basis

$$S_{(4)+} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}, \qquad T_{(4)+} = \begin{pmatrix} 1 & & \\ & e^{\pi i/4} & \\ & & -1 \end{pmatrix}.$$

flavor symmetry

semi-direct product of (Z8xZ8) and A4

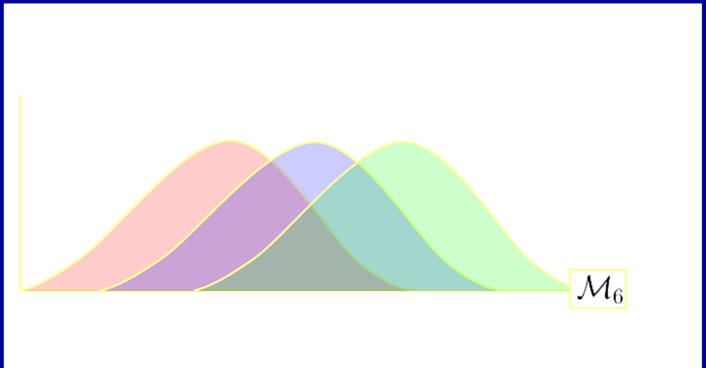
3-point couplings

Cremades, Ibanez, Marchesano, '04

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.'s.

$$Y_{ijk} = \int d^2 z\, \psi^i_M(z) \psi^j_N(z) \left(\psi^k_{M+N}(z)\right)^*$$

$$\int d^2 z \;\; \psi^i_M(z) \left(\psi^k_M(z)\right)^* = \delta^{ik}$$



If they are localized far away each other, the Yukawa coupling is suppressed.

Gaussian suppression

Yukawa couplings

yukawa coupling

$$Y_{ijk} = g \int d^2 z \psi^{i,M} \psi^{j,N} (\psi^{k,M'})^*$$

$$= g \left(\frac{2 \mathrm{Im}\, \tau}{\mathcal{A}^2} \right)^{1/4} \sum_{m \in Z_{M'}} \delta_{k,i+j+Mm} \cdot \vartheta \begin{bmatrix} \frac{Ni - Mj + MNm}{MNM'} \\ 0 \end{bmatrix} (0, MNM'\tau) ,$$

Yukawa couplings as well as other couplings depend on modulus

They transform non-trivially under Modular symmetry.

Other models

Blow-up orbifolds

T.K., Otsuka, Uchida, 1904.02867

Twisted and shifted orbifolds

Kariyazono, et. al. 1908.XXXXX

These also lead to non-trivial (finite) modular symmetry.

4. Model building with flavor symm. = modular symm.: bottom-up

modular group

Generator S and T

$$S^2 = 1, \qquad (ST)^3 = 1.$$

algebra infinite

subgroup

$$T^N = 1,$$

S3, A4, S4, A5 for N=2,3,4,5

Γ_N = SL(2,Z)/Γ(N) Γ(N):合同部分群

Δ(96), Δ(384) も

Field-theoretical flavor models

$$\sin^2\theta_{12} \approx 0.3, \qquad \sin^2\theta_{23} \approx 0.5, \qquad \sin^2\theta_{13} \approx 0.02$$

Assuming S3, A4, S4,A5 flavor symmetries,

One can explain mass matrices

Assign quarks and leptons as certain representations.

Symmetry breaking

flavon VEVs -> realistic mass matrices

Couplings are singlets.

(too) many models

flavon 模型を複雑化

New approach: flavor models

Feruglio, 1706.08749

$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

$$\mathcal{L}(\tau) \longrightarrow \mathcal{L}(\tau')$$

subgroups of modular group A4

レプトン A4のnon-trivial 表現

Yukawa couplings も A4 modular 群の

non-trivial 表現 Y(τ)

Modular forms

Yukawa couplings も A4 modular 群の

non-trivial 表現 Y(τ) modulus の特別な関数

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

w modular weight

rho(S), rho(T)

A4 modular 群の表現

$$f(S\tau)_i = \tau^w \rho(S)_{ij} f_j(\tau)$$
$$f(T\tau)_i = \rho(T)_{ij} f(\tau)$$

Γ(3) のmodular form

New approach:flavor models

Feruglio, 1706.08749

$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

$$\mathcal{L}(\tau) \longrightarrow \mathcal{L}(\tau')$$

subgroups of modular group A4

レプトン A4のnon-trivial 表現

weight もアサイン

Yukawa couplings も A4 modular 群の

non-trivial 表現 Y(τ) Γ(3) modular form

New approach:flavor models

Feruglio, 1706.08749

Yukawa couplings も A4 modular 群の

non-trivial 表現 Y(τ) Γ(3) modular form

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right]\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \qquad (28)\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

where $\eta(\tau)$ is the Dedekind eta-function, defined in the upper complex plane:

$$\eta(\tau)=q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad q\equiv e^{i2\pi\tau} \quad . \qquad (29)$$

They transform in the three-dimensional representation of A_4. In a vector notation where $Y^T=(Y_1,Y_2,Y_3)$ we have

$$Y(-1/\tau)=\tau^2\ \rho(S)Y(\tau) \quad , \qquad Y(\tau+1)=\rho(T)Y(\tau) \quad ,$$

with unitary matrices $\rho(S)$ and $\rho(T)$

$$\rho(S)=\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix} \quad , \qquad \rho(T)=\begin{pmatrix}1&0&0\\0&\omega&0\\0&0&\omega^2\end{pmatrix} \quad , \qquad \omega=-\frac{1}{2}+\frac{\sqrt{3}}{2}i \quad .$$

New approach:flavor models

T.K., Omoto, Shimizu, Takagi, Tanimoto, Tatsuishi,
1808.03012

レプトン　A4のnon-trivial　表現　weight もアサイン

Yukawa couplings もA4 modular 群のnon-trivial 表現

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	1, 1″, 1′	3	1	1	3
$-k_I$	−1 (1)	−1 (−3)	−1	0	0	$k=2$

$$w_e = \alpha e_R H_d(LY) + \beta \mu_R H_d(LY) + \gamma \tau_R H_d(LY) ,$$

$$w_D = g(\nu_R H_u L Y)_1 ,$$

$$w_N = \Lambda(\nu_R \nu_R Y)_1 ,$$

New approach:flavor models

T.K., Omoto, Shimizu, Takagi, Tanimoto, Tatsuishi,

1808.03012

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$$w_D = g(\nu_R H_u LY)_1\ ,$$

$$w_N = \Lambda(\nu_R \nu_R Y)_1\ ,$$

この模型は、flavon を含まない

moduli VEV → A4 が破れる

New approach:flavor models

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1808.03012

moduli VEV → A4 が破れる　　実験を再現

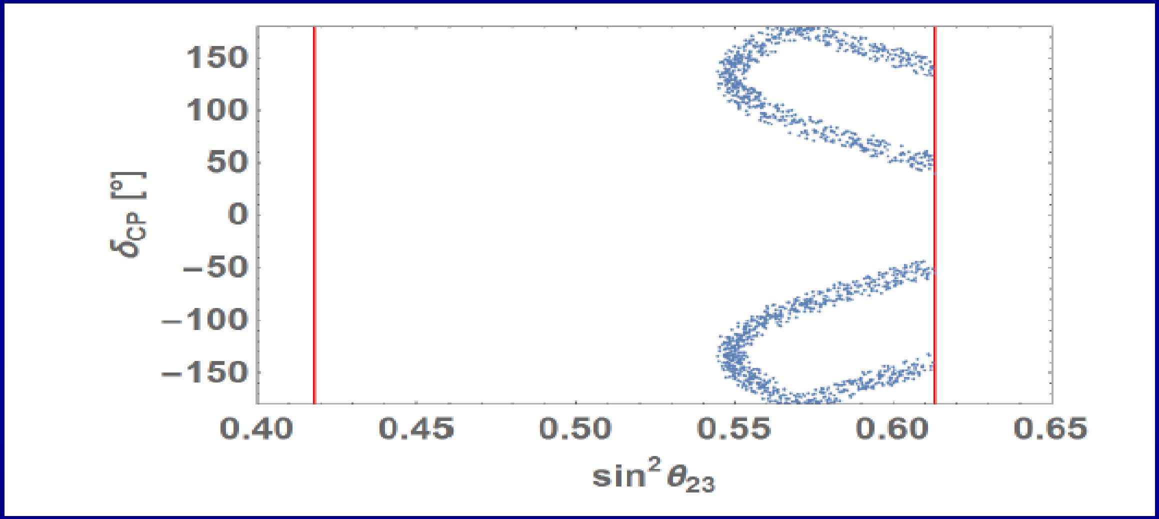
observable	$3\,\sigma$ range for NH	$3\,\sigma$ range for IH
$\Delta m^2_{\rm atm}$	$(2.399$ - $2.593) \times 10^{-3}{\rm eV}^2$	$(-2.562$ - $-2.369) \times 10^{-3}{\rm eV}^2$
$\Delta m^2_{\rm sol}$	$(6.80$ - $8.02) \times 10^{-5}{\rm eV}^2$	$(6.80$ - $8.02) \times 10^{-5}{\rm eV}^2$
$\sin^2\theta_{23}$	0.418 - 0.613	0.435 - 0.616
$\sin^2\theta_{12}$	0.272 - 0.346	0.272 - 0.346
$\sin^2\theta_{13}$	0.01981 - 0.02436	0.02006 - 0.02452

Table 3: The $3\,\sigma$ ranges of neutrino oscillation parameters from NuFIT 3.2 for NH and IH [35].

New approach:flavor models

T.K., Omoto, Shimizu, Takagi, Tanimoto, Tatsuishi, 1808.03012

moduli VEV → A4 が破れる　　実験を再現



Im[τ]	Re[τ]	g	ϕ_g	α/γ	β/γ
$0.66-0.73$ $1.17-1.32$	$\pm(0.25-0.31)$, $\pm(0.46-0.54)$, $\pm(0.66-0.75)$, $\pm(1.25-1.31)$, $\pm(1.46-1.50)$	$1.20-1.22$	$\pm(87-88)^\circ$ $\pm(92-93)^\circ$	$202-203$	$3286-3306$

Modular forms

A4 以外の modular forms （modulus の特別な関数）

A4 Feruglio, 1706.08749

S3 T.K., Tanaka, Tatsuishi, 1803.10391

S4 Penedo, Petcov, 1806.11040

A5 Novichkov, Penedo, Petcov, Titov, 1812.02158

Δ(96), Δ(384) T.K., Tamba, 1811.11384

double covering modular group

T′ Liu, Ding, 1907.01488

New approach:flavor models

S3, A4, S4, A5, T' を使った模型構築

レプトンセクター

クォークセクター

ＳＵ（５）GUT

generalized CP

New approach:flavor models

T.K., Shimizu, Takagi, Tanimoto, Tatsuishi, Uchida,
1812.11072

今回　T2 x T2 x T2

レプトンは、first T2　クォークは、 second T2

レプトンとクォークの対称性が違っていてもよいじゃいか。

レプトン　A4　　T.K, et. al. 1808.03012

クォーク　S3

$$M_{u,d} \simeq \begin{pmatrix} 0 & 2c^{u,d} Y_1(\tau')Y_2(\tau') & c_{13}^{u,d} Y_2(\tau') \\ 2c^{u,d} Y_1(\tau')Y_2(\tau') & -2c^{u,d} Y_1(\tau')^2 & -c_{13}^{u,d} Y_1(\tau') \\ c_{31}^{u,d} Y_2(\tau') & -c_{31}^{u,d} Y_1(\tau') & c_{33}^{u,d} \end{pmatrix}.$$

$$|Y_2(\tau')/Y_1(\tau')| = \lambda \simeq 0.2.$$

New approach:flavor models

T.K., Shimizu, Takagi, Tanimoto, Tatsuishi, Uchida,
1812.11072

今回　T2 x T2 x T2

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レプトン　A4　　T.K, et. al. 1808.03012

クォーク　S3

Supersymmtric SM　R-parity の代用になる

New approach:flavor models

T.K., Shimizu, Takagi, Tanimoto, Tatsuishi,

1907.09141

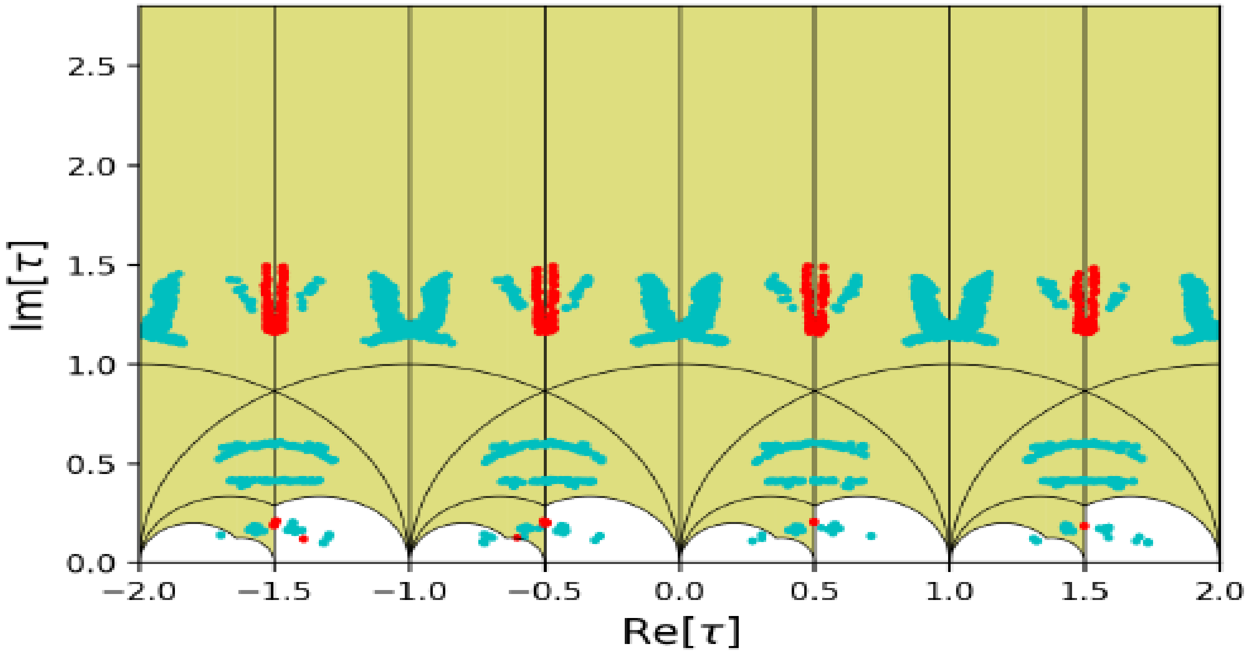

Figure 5: Allowed region on the Re[τ]–Im[τ] plane. The fundamental domain of Γ(4) are shown by olive-green. Cyan-points and red-points denote cases of NH and IH, respectively.

Summary

Modular symmetry は、ゼロモードの世代を変える

“フレーバー対称性”

湯川結合の変換

Modular symmetry は、

S3, A4, S4, A5, Δ(96), Δ(384)を含む

これらのことを使った現象論的模型構築

どのような対称性がよいか

クォーク･レプトンのアサインも含めて

⇒ どのようなコンパクト空間がよいか

Moduli stabilization 課題

Summary

高エネルギーのフレーバー模型が楽しいが、、、、
どうやって検証するか
（混合角など精密測定、CP, 0νββdecay,質量和
　模型の取捨選択はできるけど、直接的検証　？？）
Axion が軽いとか、、、、、、

どのような対称性がよいか
　　クォーク・レプトンのアサインも含めて
　　　⇒ どのようなコンパクト空間がよいか

そのコンパクト空間上の超弦理論の様々な
性質を寄せ集める

Summary

It is still a challenging issues how to derive realistic quark/lepton masses and mixing angles.