Exponentially Suppressed Cosmological Constant with Gauge Enhanced Symmetry in Heterotic Interpolating Models

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Introduction

When a top-down approach from string theory is considered, there are two choices depending on where SUSY breaking scale is ;

1. **SUSY is broken at low energy in supersymmetric EFT ;**
2. **SUSY is already broken at high energy like string/Planck scale.**

In this talk, the second one is focused on, and non-supersymmetric string models are considered.

In particular, **the $SO(16) \times SO(16)$ model** is a unique **tachyon-free** non-supersymmetric string model in ten-dimensions.

[Dixon, Hervey, (1986)]

Introduction

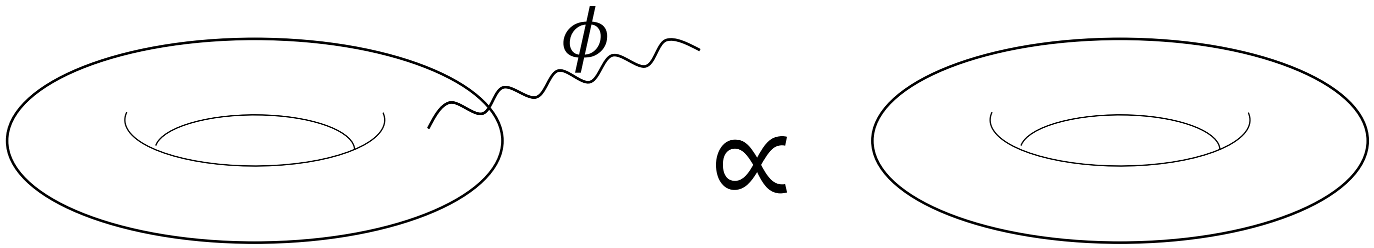
Considering non-supersymmetric string models, however, we face with the problem of vacuum instability arising from nonzero dilaton tadpoles;

$$V(\phi) \propto \Lambda$$

$V(\phi)$: dilaton tadpole

Λ: cosmological constant (vacuum energy)

At 1-loop level,



The desired model is a non-supersymmetric one whose **cosmolosical constant is vanishing or as small as possible.**

Interpolating models have the possibility of such properties.

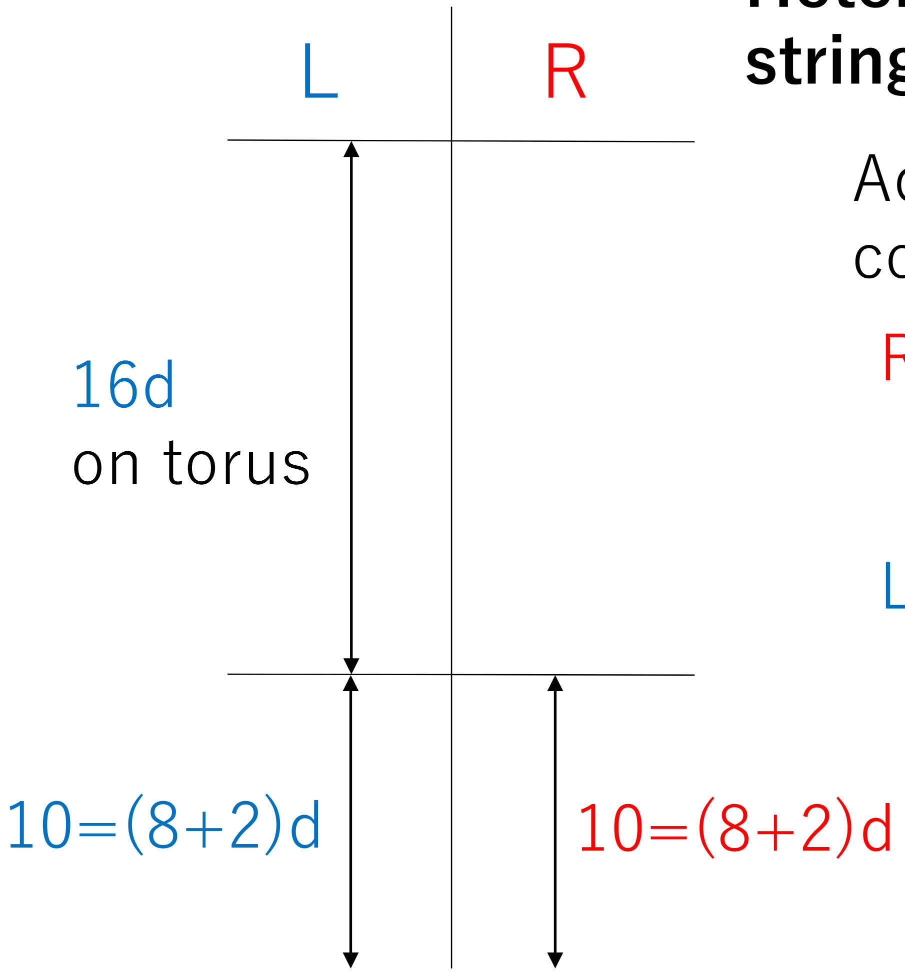
[Itoyama, Taylor, (1987)]

Outline

1. Introduction
2. Heterotic Strings
3. 9D Interpolating models
4. 9D Interpolating models with Wilson line
5. Summary

Outline

Idea of Heterotic Strings



Heterotic strings are hybrid closed strings of bosonic string in 26D and superstrings in 10D.

Adopting the lightcone coordinates, the worldsheet contents are

Right mover: 10d superstring

$$X_R^i(\tau-\sigma),\ \tilde{\psi}^i(\tau-\sigma)$$

Left mover: 26d bosonic string out of which

internal 16d realize rank 16 current algebra

$$X_L^i(\tau+\sigma),\ X_L^I(\tau+\sigma)$$

$$(i=1,2,\cdots,8 \quad I=1,2,\cdots,16)$$

[Gross, Hervey, Martinec, Rohm, (1985)]

The one-loop partition func. & State Counting

- The one-loop partition function is the trace over string Fock space:

$$Z(\tau) = \mathrm{Tr}(-1)^F q^{L_0} \bar{q}^{\tilde{L}_0}$$

$$\left(\begin{array}{l} q = e^{2\pi i \tau} \\ F\text{: the spacetime fermion number} \end{array} \right)$$

- $Z(\tau)$ counts #(states) at each mass level as coeff. in q $(\bar{q})$ expansion.

$$Z(\tau) = \tau_2^{-\frac{D-2}{2}} \sum_{m,n} a_{mn} \bar{q}^m q^n$$

a_{mn} denotes #(bosons) minus #(fermions) at mass levels (m, n)

In the string model with spacetime SUSY, $a_{mn} = 0$ for all (m, n) because of fermion-boson degeneracy.

$\Longrightarrow$ $Z(\tau) = 0$ for supersymmetric string models.

- In order for the string model to be consistent, $Z(\tau)$ has to be invariant under modular transformation:

$$Z(-1/\tau) = Z(\tau + 1) = Z(\tau)$$

Characters

- $Z(\tau)$ is written in terms of $SO(2n)$ characters $O_{2n}, V_{2n}, S_{2n}, C_{2n}$ and the Dedekind eta function $\eta(\tau)$, e.g,

$SO(32)$ hetero:

$$Z_{SO(32)}(\tau) = Z_B^{(8)} \left(\bar{V}_8 - \bar{S}_8\right) \left(O_{16}O_{16} + V_{16}V_{16} + S_{16}S_{16} + C_{16}C_{16}\right)$$

$E_8 \times E_8$ hetero:

$$Z_{E_8\times E_8}(\tau) = Z_B^{(8)} \left(\bar{V}_8 - \bar{S}_8\right) \left(O_{16} + S_{16}\right) \left(O_{16} + S_{16}\right)$$

$SO(16) \times SO(16)$ hetero:

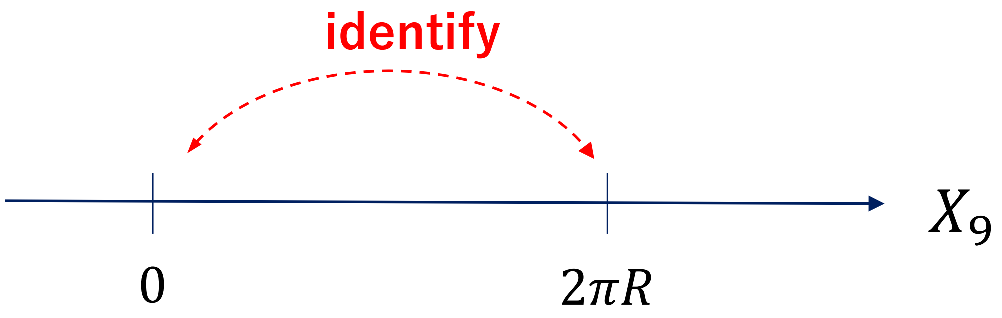
$$\begin{aligned} Z_{SO(16)\times SO(16)} = Z_B^{(8)} \{ & \bar{O}_8 \left(V_{16}C_{16} + C_{16}V_{16}\right) + \bar{V}_8 \left(O_{16}O_{16} + S_{16}S_{16}\right) \\ & - \bar{S}_8 \left(V_{16}V_{16} + C_{16}C_{16}\right) - \bar{C}_8 \left(O_{16}S_{16} + S_{16}O_{16}\right) \} \end{aligned}$$

$$Z_B^{(n)} = \tau_2^{-n/2} \left(\bar{\eta}\eta\right)^{-n}, \quad \begin{pmatrix} O_{2n} \\ V_{2n} \end{pmatrix} = \frac{1}{2\eta^n} \left(\vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^n \pm \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^n \right), \quad \begin{pmatrix} S_{2n} \\ C_{2n} \end{pmatrix} = \frac{1}{2\eta^n} \left(\vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^n \pm \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^n \right)$$

the Jacobi's abstruse identity: $V_8 - S_8 = 0$

SUSY breaking by Compactification

● Compactification on a circle

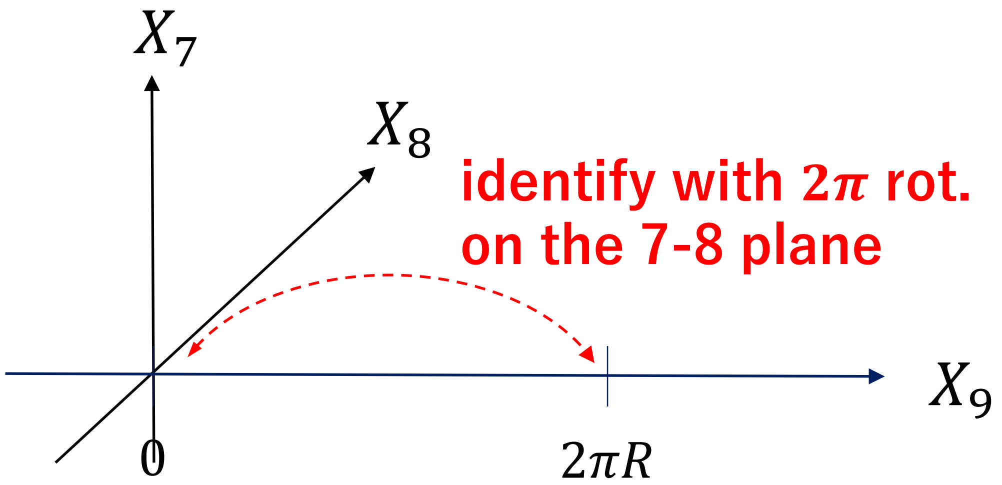


The translation operator for X_9 satifies

$$e^{2\pi i P_9 R} = 1 \quad \therefore P_9 = \frac{n}{R} \quad (n \in \boldsymbol{Z})$$

This comp. affects bosonic and fermionic states **in the same way**. ⟶ **SUSY is NOT broken**.

● Compactification on a twisted circle



The translation operator for X_9 satifies

$$e^{2\pi i P_9 R} = e^{2\pi i J_{78}} \quad \therefore P_9 = \frac{n + \frac{F}{2}}{R} \quad (n \in \boldsymbol{Z})$$

$\left(F\text{: the spacetime fermion number} \right)$

This comp. affects bosonic and fermionic states **in the different way**. It induces the mass splitting between bosonic and fermionic states.

➡ **SUSY is broken** [Rohm, (1984)]

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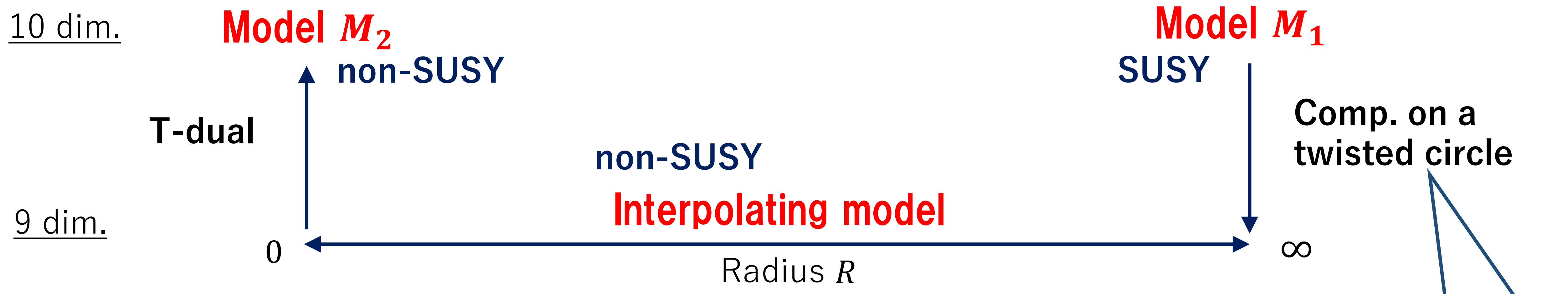
Interpolation between SUSY and non-SUSY models

An interpolating model is a lower dimensional string model relating two different higher dimensional string models continuously.



In the large R (small a) region, the cosmological constant is

$$\Lambda_9 \simeq (n_F - n_B)\, a^{-9}\xi + \mathcal{O}\left(e^{-a^{-2}}\right)$$

$\left(a = \sqrt{\alpha'}/R, \quad \xi > 0, \quad n_F, \ n_B \right.$: # of massless fermions, bosons $\left. \right)$

If $n_F = n_B$, the cosmological constant is exponentially suppressed.

[Itoyama, Taylor, (1987)]

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \{ & \Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right] \\
& +\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right] \\
& +\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right] \\
& +\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right] \}
\end{aligned}
$$

$$
\Lambda_{\alpha,\beta} = (\bar{\eta}\eta)^{-1}\sum_{n,w}\bar{q}^{\alpha' p_R^2/2} q^{\alpha' p_L^2/2} = (\bar{\eta}\eta)^{-1}\sum_{n,w}\exp\left[2\pi i n w \tau_1 - \pi\tau_2\left(n^2 a^2 + w^2/a^2\right)\right]
$$

where the sum is taken over $n \in 2(\boldsymbol{Z}+\alpha),\ \ w\in \boldsymbol{Z}+\beta$

- $R\to\infty$: contribution from the zero winding # only $\Rightarrow$ $\Lambda_{\alpha,0}\to(2a)^{-1}Z_B^{(1)},\quad \Lambda_{\alpha,1/2}\to 0$

- $R\to 0$: contribution from the zero momentum only $\Rightarrow$ $\Lambda_{0,\beta}\to aZ_B^{(1)},\quad \Lambda_{1/2,\beta}\to 0$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **The limiting case: $R\to\infty$** $\quad\Lambda_{\alpha,0}\to(2a)^{-1}Z_B^{(1)},\quad\Lambda_{\alpha,1/2}\to 0$

$$\begin{aligned}Z_{\rm int}^{(9)}=Z_B^{(7)}\{&\Lambda_{0,0}\left[\bar V_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar S_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right]\\&+\Lambda_{1/2,0}\left[\bar V_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar S_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right]\\&\cancel{+\Lambda_{0,1/2}\left[\bar O_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar C_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right]}\\&\cancel{+\Lambda_{1/2,1/2}\left[\bar O_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar C_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right]\}}\end{aligned}$$

➡ the one-loop partition function of SUSY $SO(32)$ heterotic model, which is vanishing

SUSY is restored in $R\to\infty$ $(a\to 0)$

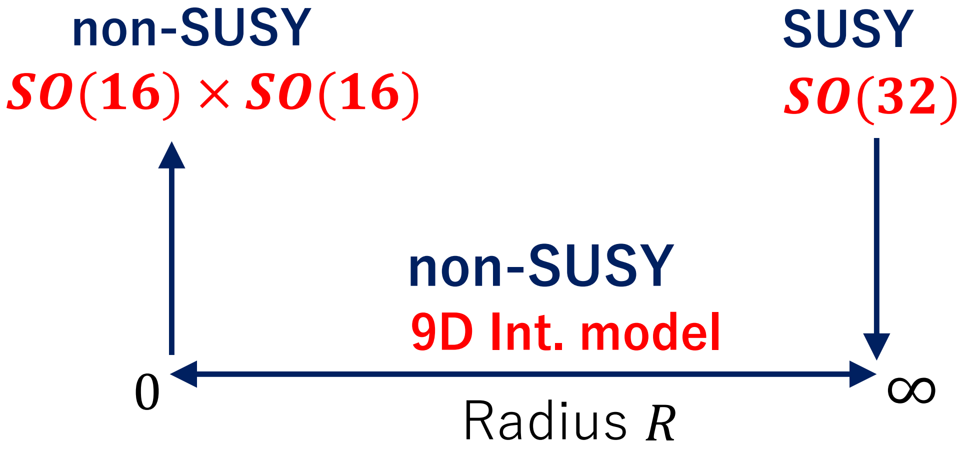
Interpolation between $SO(32)$ and $SO(16)\times SO(16)$

- **The limiting case: $R\to 0$** $\qquad \Lambda_{0,\beta}\to aZ_B^{(1)},\quad \Lambda_{1/2,\beta}\to 0$

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)}\Big\{&\Lambda_{0,0}\underline{\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right]}\\
&\cancel{+\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right]}\\
&+\Lambda_{0,1/2}\underline{\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right]}\\
&\cancel{+\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right]\Big\}}
\end{aligned}
$$

⇒ the one-loop partition function of $SO(16)\times SO(16)$ heterotic model

$Z_{\text{int}}^{(9)}$ realizes $\begin{cases} \text{SUSY } SO(32) \text{ model in } R\to\infty \\ SO(16)\times SO(16) \text{ model in } R\to 0 \end{cases}$



Interpolation between $SO(32)$ and $SO(16) \times SO(16)$

- **Massless spectrum**

at generic *R*, massless states come from *n=w=0* part

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \left\{ \underline{\Lambda_{0,0} \left[\bar{V}_8 \left(O_{16} O_{16} + S_{16} S_{16} \right) - \bar{S}_8 \left(V_{16} V_{16} + C_{16} C_{16} \right) \right]} \right.$$
$$+ \Lambda_{1/2,0} \left[\bar{V}_8 \left(V_{16} V_{16} + C_{16} C_{16} \right) - \bar{S}_8 \left(O_{16} O_{16} + S_{16} S_{16} \right) \right]$$
$$+ \Lambda_{0,1/2} \left[\bar{O}_8 \left(V_{16} C_{16} + C_{16} V_{16} \right) - \bar{C}_8 \left(O_{16} S_{16} + S_{16} O_{16} \right) \right]$$
$$\left. + \Lambda_{1/2,1/2} \left[\bar{O}_8 \left(O_{16} S_{16} + S_{16} O_{16} \right) - \bar{C}_8 \left(V_{16} C_{16} + C_{16} V_{16} \right) \right] \right\}$$

Massless bosons

- **9-dim. graviton, anti-symmetric tensor, dilaton:** $g_{\mu\nu}, \quad B_{\mu\nu}, \quad \phi$
- **Gauge bosons in adj rep of** $SO(16) \times SO(16) \times \underline{U_{G,B}^2(1)}$

$g_{9\mu}, B_{9\mu}$

Massless fermions

- $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{16})$

$$n_F - n_B = 64$$

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Boost on momentum lattice

- Considering d-dimensional compactification, **the boost in the momentum lattice corresponds to putting massless constant backgrounds**, that is, adding the following term to the worldsheet action

$$C_{Aa}\int d^2z\partial X_L^A\bar{\partial}X_R^a \qquad \begin{pmatrix} a = 10-d,\cdots,9 \\ A=(a,I)=10-d,\cdots,26 \end{pmatrix}$$

C_{ba}: metric and antisymmetric tensor, C_{Ia}: $U(1)^{16}$ gauge fields (WL)

[Narain, Sarmadi, Witten, (1986)]

- The d-dimensional compactifications are classified by the transformation

$$\frac{SO(16+d,d)}{SO(16+d)\times SO(d)},$$

whose DOF agree with that of C_{Aa}.

- In this work, we will consider one-dimensional compactification and put a single WL background $A=C_{I=1,a=9}$ for simplicity.

Boost on momentum lattice

After turning on WL, the momenta of $X_L^{I=1}$, $X_L^{a=9}$ and $X_R^{a=9}$ are changed as

$$\begin{cases} l_L = \frac{1}{\sqrt{\alpha'}} m \\ p_L = \frac{1}{\sqrt{2\alpha'}} \left(an + \frac{w}{a} \right) \\ p_R = \frac{1}{\sqrt{2\alpha'}} \left(an - \frac{w}{a} \right) \end{cases} \xrightarrow{\text{boost and rotation}} \begin{cases} l'_L = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2}m - 2\frac{A}{\sqrt{1+A^2}} \frac{w}{a} \right) \\ p'_L = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2}Am + \sqrt{1+A^2}an - \frac{1-A^2}{\sqrt{1+A^2}} \frac{w}{a} \right) \\ p'_R = \frac{1}{\sqrt{2\alpha'}} \left(\sqrt{2}Am + \sqrt{1+A^2}an - \sqrt{1+A^2} \frac{w}{a} \right) \end{cases}$$

l_L is the left-moving momentum of $X_L^{I=1}$

The effective change in the 1-loop partition function is

$$\Lambda_{\alpha,\beta}(a) \left(\frac{\vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix}}{\eta} \right)^8 \xrightarrow{\text{introduction of WL}} \Lambda^{(\alpha,\beta)}_{(\gamma,\delta)}(a, A) \left(\frac{\vartheta \begin{bmatrix} \gamma \\ \delta \end{bmatrix}}{\eta} \right)^7$$

$$\Lambda^{(\alpha,\beta)}_{(\gamma,\delta)} = (\bar{\eta}\eta)^{-1} \eta^{-1} \sum_{n,w,m} (-1)^{2\delta m} q^{\frac{\alpha'}{2}(l'^2_L + p'^2_L)} \bar{q}^{\frac{\alpha'}{2} p'^2_R} \qquad n \in 2(\boldsymbol{Z} + \alpha), \ \ w \in \boldsymbol{Z} + \beta, \ \ m \in \boldsymbol{Z} + \gamma$$

The fundamental region of moduli space

Do all the points in moduli space correspond to different models? $\longrightarrow$ **NO!**

It is convenient to introduce a modular parameter $\tilde{\tau}$ as

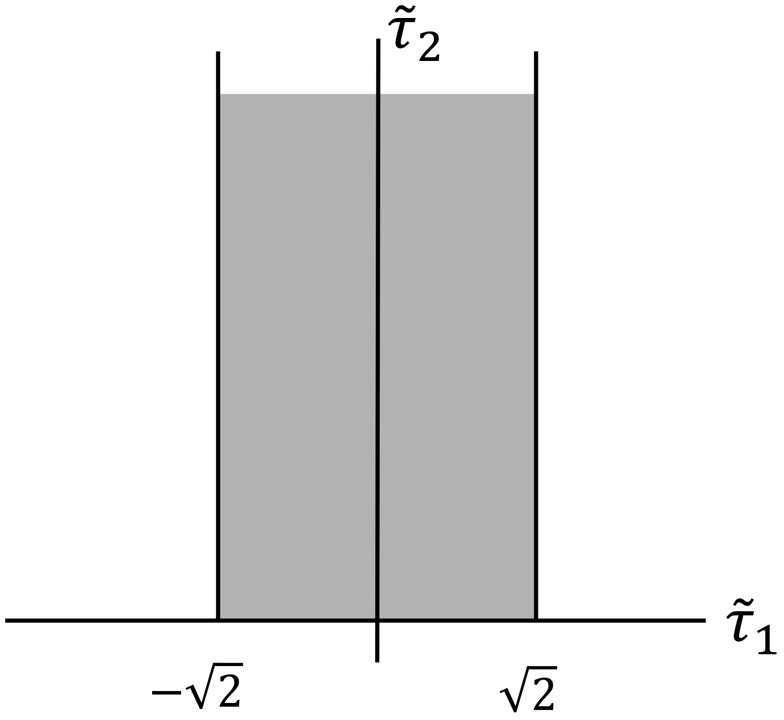
$$\tilde{\tau} = \tilde{\tau}_1 + i\tilde{\tau}_2 = \frac{A}{\sqrt{1+A^2}}a^{-1} + i\frac{1}{\sqrt{1+A^2}}a^{-1}$$

Momentum lattice $\Lambda^{(\alpha,\beta)}_{(\gamma,\delta)}$ is invariant under the shift

$$\tilde{\tau} \to \tilde{\tau} + 2\sqrt{2}$$

The fundamental region of moduli space is

$$-\sqrt{2} \le \tilde{\tau}_1 \le \sqrt{2}$$



Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **The one-loop partition function**

$$
\begin{aligned}
Z^{(9)}_{\rm int} = Z^{(7)}_B \Big\{ &\bar{V}_8\left(O^{(0,0)}_{16} O_{16} + S^{(0,0)}_{16} S_{16}\right) - \bar{S}_8\left(V^{(0,0)}_{16} V_{16} + C^{(0,0)}_{16} C_{16}\right) \\
&+\bar{V}_8\left(V^{(1/2,0)}_{16} V_{16} + C^{(1/2,0)}_{16} C_{16}\right) - \bar{S}_8\left(O^{(1/2,0)}_{16} O_{16} + S^{(1/2,0)}_{16} S_{16}\right) \\
&+\bar{O}_8\left(V^{(0,1/2)}_{16} C_{16} + C^{(0,1/2)}_{16} V_{16}\right) - \bar{C}_8\left(O^{(0,1/2)}_{16} S_{16} + S^{(0,1/2)}_{16} O_{16}\right) \\
&+\bar{O}_8\left(O^{(1/2,1/2)}_{16} S_{16} + S^{(1/2,1/2)}_{16} O_{16}\right) - \bar{C}_8\left(V^{(1/2,1/2)}_{16} C_{16} + C^{(1/2,1/2)}_{16} V_{16}\right)\Big\}
\end{aligned}
$$

$$
\begin{pmatrix} O^{(\alpha,\beta)}_{16} \\ V^{(\alpha,\beta)}_{16} \end{pmatrix} = \frac{1}{2\eta^7}\left(\Lambda^{(\alpha,\beta)}_{(0,0)}\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 \pm \Lambda^{(\alpha,\beta)}_{(0,1/2)}\vartheta\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7\right) \qquad \begin{pmatrix} S^{(\alpha,\beta)}_{16} \\ C^{(\alpha,\beta)}_{16} \end{pmatrix} = \frac{1}{2\eta^7}\left(\Lambda^{(\alpha,\beta)}_{(1/2,0)}\vartheta\begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 \pm \Lambda^{(\alpha,\beta)}_{(1/2,1/2)}\vartheta\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7\right)
$$

- $R \to \infty$: $\left(O^{(\alpha,\beta)}_{16}, V^{(\alpha,\beta)}_{16}, S^{(\alpha,\beta)}_{16}, C^{(\alpha,\beta)}_{16}\right) \longrightarrow \dfrac{\sqrt{\alpha'}}{2r_\infty} Z^{(1)}_B \left(O_{16}, V_{16}, S_{16}, C_{16}\right)\delta_{\beta,0}$

- $R \to 0$: $\left(O^{(\alpha,\beta)}_{16}, V^{(\alpha,\beta)}_{16}, S^{(\alpha,\beta)}_{16}, C^{(\alpha,\beta)}_{16}\right) \longrightarrow \sqrt{\alpha'} r_0 Z^{(1)}_B \left(O_{16}, V_{16}, S_{16}, C_{16}\right)\delta_{\alpha,0}$

$$
r_\infty = \frac{R}{\sqrt{1+A^2}}, \qquad r_0 = \sqrt{1+A^2}\,R
$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

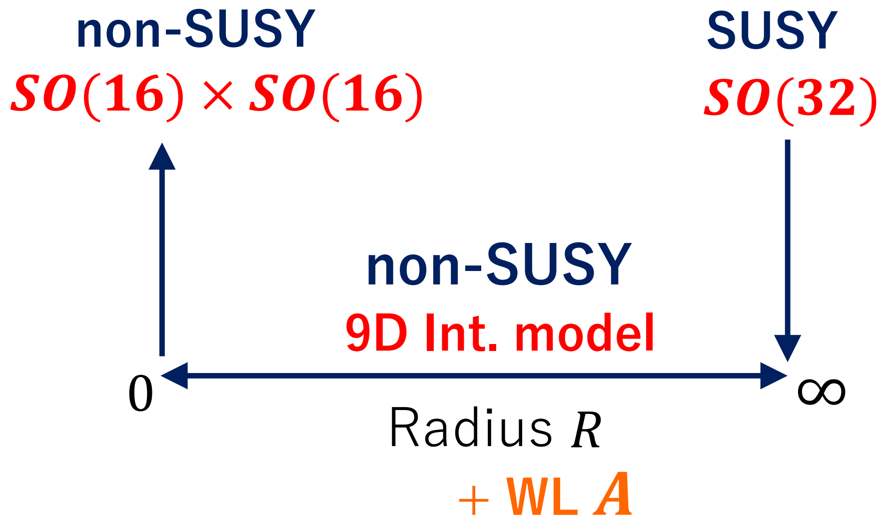
- **The one-loop partition function**

$$
\begin{aligned}
Z^{(9)}_{\text{int}} = Z^{(7)}_B \Big\{ &\bar{V}_8\left(O^{(0,0)}_{16}O_{16} + S^{(0,0)}_{16}S_{16}\right) - \bar{S}_8\left(V^{(0,0)}_{16}V_{16} + C^{(0,0)}_{16}C_{16}\right) \\
&+\bar{V}_8\left(V^{(1/2,0)}_{16}V_{16} + C^{(1/2,0)}_{16}C_{16}\right) - \bar{S}_8\left(O^{(1/2,0)}_{16}O_{16} + S^{(1/2,0)}_{16}S_{16}\right) \\
&+\bar{O}_8\left(V^{(0,1/2)}_{16}C_{16} + C^{(0,1/2)}_{16}V_{16}\right) - \bar{C}_8\left(O^{(0,1/2)}_{16}S_{16} + S^{(0,1/2)}_{16}O_{16}\right) \\
&+\bar{O}_8\left(O^{(1/2,1/2)}_{16}S_{16} + S^{(1/2,1/2)}_{16}O_{16}\right) - \bar{C}_8\left(V^{(1/2,1/2)}_{16}C_{16} + C^{(1/2,1/2)}_{16}V_{16}\right)\Big\}
\end{aligned}
$$

- **The limiting cases**
  - $R\to\infty$: the 1st and 2nd lines survive
  - $R\to 0$: the 1st and 3rd lines survive

For any WL A, $Z^{(9)}_{\text{int}}$ realizes

$$
\begin{cases}
\text{SUSY } SO(32) \text{ model in } R\to\infty \\
SO(16)\times SO(16) \text{ model in } R\to 0
\end{cases}
$$



Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- **Massless spectrum** at generic *R*, massless states come from *n=w=0* part

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ & \underline{\bar{V}_8\left(O_{16}^{(0,0)}O_{16}+S_{16}^{(0,0)}S_{16}\right)-\bar{S}_8\left(V_{16}^{(0,0)}V_{16}+C_{16}^{(0,0)}C_{16}\right)} \\
& +\bar{V}_8\left(V_{16}^{(1/2,0)}V_{16}+C_{16}^{(1/2,0)}C_{16}\right)-\bar{S}_8\left(O_{16}^{(1/2,0)}O_{16}+S_{16}^{(1/2,0)}S_{16}\right) \\
& +\bar{O}_8\left(V_{16}^{(0,1/2)}C_{16}+C_{16}^{(0,1/2)}V_{16}\right)-\bar{C}_8\left(O_{16}^{(0,1/2)}S_{16}+S_{16}^{(0,1/2)}O_{16}\right) \\
& +\bar{O}_8\left(O_{16}^{(1/2,1/2)}S_{16}+S_{16}^{(1/2,1/2)}O_{16}\right)-\bar{C}_8\left(V_{16}^{(1/2,1/2)}C_{16}+C_{16}^{(1/2,1/2)}V_{16}\right)\Big\}
\end{aligned}
$$

Massless bosons

- 9-dim. graviton, anti-symmetric tensor, dilaton: $g_{\mu\nu},\ \ B_{\mu\nu},\ \ \phi$
- Gauge bosons in adj rep of $SO(16)\times SO(14)\times U(1)\times U^2_{G,B}(1)$

Massless fermions

- $\mathbf{8}_S\otimes(\mathbf{16},\mathbf{14})$

$$n_F - n_B = 32$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- Massless spectrum　∃ a few conditions under which the additional massless states appear

$$Z_{\rm int}^{(9)} = Z_B^{(7)}\Big\{\bar V_8\left(O_{16}^{(0,0)}O_{16}+S_{16}^{(0,0)}S_{16}\right)-\bar S_8\left(V_{16}^{(0,0)}V_{16}+C_{16}^{(0,0)}C_{16}\right)$$
$$+\bar V_8\left(V_{16}^{(1/2,0)}V_{16}+C_{16}^{(1/2,0)}C_{16}\right)-\bar S_8\left(O_{16}^{(1/2,0)}O_{16}+S_{16}^{(1/2,0)}S_{16}\right)$$
$$+\bar O_8\left(V_{16}^{(0,1/2)}C_{16}+C_{16}^{(0,1/2)}V_{16}\right)-\bar C_8\left(O_{16}^{(0,1/2)}S_{16}+S_{16}^{(0,1/2)}O_{16}\right)$$
$$+\bar O_8\left(O_{16}^{(1/2,1/2)}S_{16}+S_{16}^{(1/2,1/2)}O_{16}\right)-\bar C_8\left(V_{16}^{(1/2,1/2)}C_{16}+C_{16}^{(1/2,1/2)}V_{16}\right)\Big\}$$

condition ①　$\tilde\tau_1 = n_1/\sqrt{2} \quad n_1 \in 2\boldsymbol{Z}$

new massless states：• two $\mathbf{8}_V \otimes (\mathbf{1},\mathbf{14})$　• two $\mathbf{8}_S \otimes (\mathbf{16},\mathbf{1})$

$$\Rightarrow \begin{cases} SO(16)\times SO(14)\times U(1) \longrightarrow SO(16)\times SO(16) \\ \mathbf{8}_S\otimes(\mathbf{16},\mathbf{14}) \longrightarrow \mathbf{8}_S\otimes(\mathbf{16},\mathbf{16}) \end{cases}$$

$$n_F - n_B = 64$$

Interpolation between $SO(32)$ and $SO(16)\times SO(16)$ with WL

- Massless spectrum $\quad$ ∃ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_B^{(7)} \Big\{ & \bar V_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16}\right) - \bar S_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16}\right) \\
& + \bar V_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16}\right) - \bar S_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16}\right) \\
& + \bar O_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16}\right) - \bar C_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16}\right) \\
& + \bar O_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16}\right) - \bar C_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16}\right) \Big\}
\end{aligned}
$$

condition ② $\quad \tilde\tau_1 = n_2/\sqrt{2} \quad n_2 \in 2\mathbf{Z}+1$

new massless states： • two $\mathbf{8}_V \otimes (\mathbf{16},\mathbf{1})$ • two $\mathbf{8}_S \otimes (\mathbf{1},\mathbf{14})$

$$
\begin{cases}
SO(16)\times SO(14)\times U(1) & \longrightarrow & SO(18)\times SO(14) \\
\mathbf{8}_S \otimes (\mathbf{16},\mathbf{14}) & \longrightarrow & \mathbf{8}_S \otimes (\mathbf{18},\mathbf{14})
\end{cases}
$$

$$\boxed{n_F - n_B = 0}$$

Interpolation between $SO(32)$ and $SO(16) \times SO(16)$ with WL

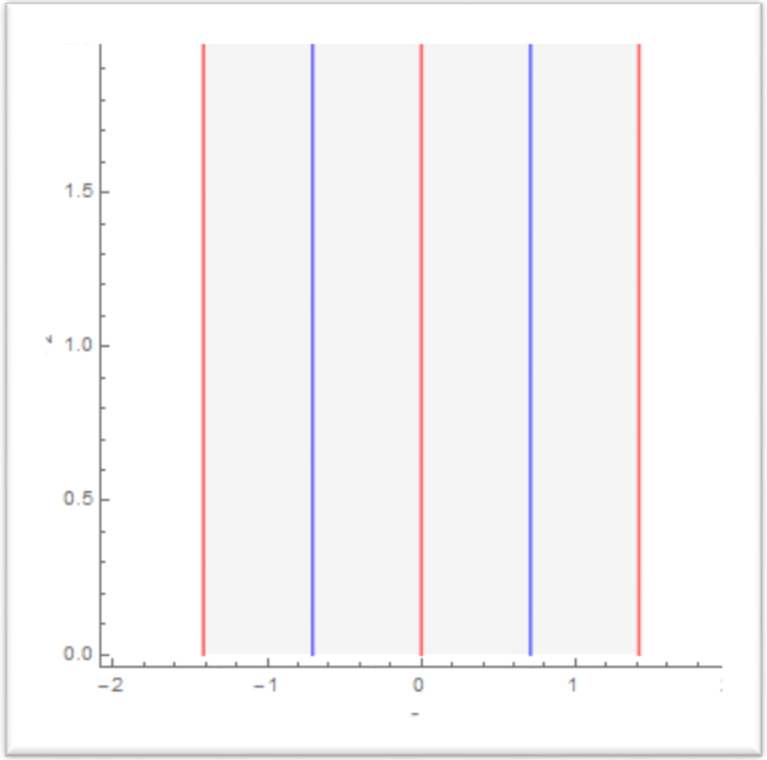
● **Summary of the conditions**

We have found the two conditions under which the additional massless states appear:

condition ① $\tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1 \in 2Z$

condition ② $\tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2Z+1$

$$\left(\tilde{\tau}_1 = \frac{A}{\sqrt{1+A^2}} a^{-1} \right)$$



Actually, there are only four inequivalent orbits in the fundamental region:

Condition	$n_1 = 0$ and 2 (or -2)	$n_2 = -1$ and 1
Gauge gp	$SO(16) \times SO(16)$	$SO(18) \times SO(14)$
	$n_F > n_B$	$n_F = n_B$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **The one-loop partition function**

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

$$
\begin{pmatrix} O_{16}^{(\alpha,\beta)} \\ V_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7} \left(\Lambda_{(0,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(0,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7 \right) \qquad \begin{pmatrix} S_{16}^{(\alpha,\beta)} \\ C_{16}^{(\alpha,\beta)} \end{pmatrix} = \frac{1}{2\eta^7} \left(\Lambda_{(1/2,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 \pm \Lambda_{(1/2,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7 \right)
$$

- $R \to \infty$: $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)} \right) \longrightarrow \frac{\sqrt{\alpha'}}{2 r_\infty} Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16} \right) \delta_{\beta,0}$

- $R \to 0$: $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)} \right) \longrightarrow \sqrt{\alpha'} r_0 Z_B^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16} \right) \delta_{\alpha,0}$

$$
r_\infty = \frac{R}{\sqrt{1+A^2}}
$$

$$
r_0 = \sqrt{1+A^2} R
$$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

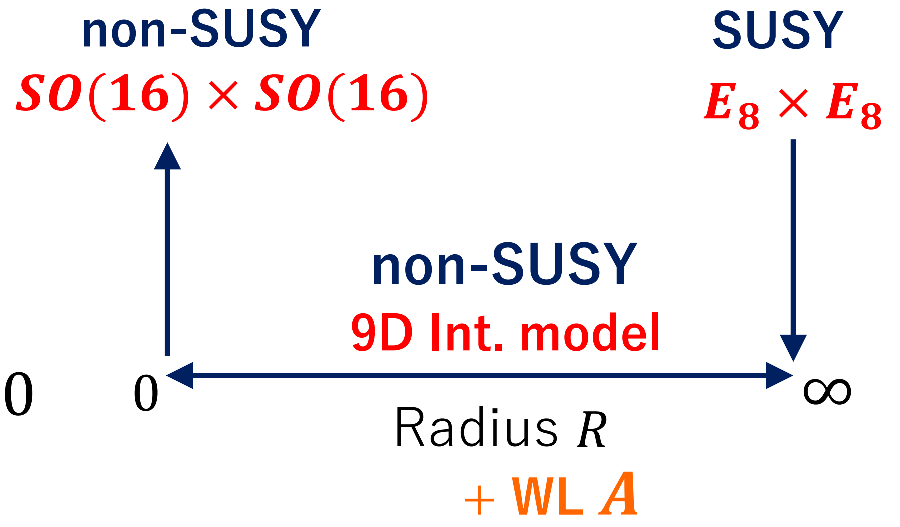
- **The one-loop partition function**

$$
\begin{aligned}
Z_{\mathrm{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

- **The limiting cases**
  - $R \to \infty$: the 1st and 2nd lines survive
  - $R \to 0$: the 1st and 3rd lines survive

For any WL A, $Z_{\mathrm{int}}^{(9)}$ realizes

$$
\begin{cases}
\text{SUSY } E_8 \times E_8 \text{ model in } R \to \infty \\
SO(16) \times SO(16) \text{ model in } R \to 0
\end{cases}
$$



Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Massless spectrum** at generic R, massless states come from $n=w=0$ part

$$Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ \underline{\bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right)}$$
$$+ \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

Massless bosons

- 9-dim. graviton, anti-symmetric tensor, dilaton: $g_{\mu\nu},\ B_{\mu\nu},\ \phi$
- Gauge bosons in adj rep of $SO(16) \times SO(14) \times U(1) \times U_{G,B}^2(1)$

Massless fermions

- $8_S \otimes (\mathbf{128}, \mathbf{1})$

$$n_F - n_B = -736$$

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- Massless spectrum $^\exists$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\mathrm{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ① $\tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1 \in 2\boldsymbol{Z}$

new massless states： • two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$

$$SO(16) \times SO(14) \times U(1) \longrightarrow SO(16) \times SO(16)$$

Furthermore, the different additional massless states appear depending on whether $n_1/2$ is even or odd.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- Massless spectrum $\quad \exists$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ①-1 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \qquad n_1/2 \in 2Z$

new massless states : • two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ • two $\mathbf{8}_S \otimes (\mathbf{1}, \mathbf{64})$

$$\mathbf{8}_S \otimes (\mathbf{128}, \mathbf{1}) \longrightarrow \mathbf{8}_S \otimes ((\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128}))$$

In the fundamental region, this condition is $\tilde{\tau}_1 = 0$, which corresponds to the no WL case.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- Massless spectrum $\quad {}^{\exists}$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ①-2 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \qquad n_1/2 \in 2\boldsymbol{Z} + 1$

new massless states： • two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ • two $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{64})$

$\Rightarrow \quad \boldsymbol{SO(16) \times SO(14) \times U(1)} \longrightarrow \boldsymbol{SO(16) \times E_8}$

In the fundamental region, this condition is $\tilde{\boldsymbol{\tau}}_1 = \sqrt{2}$ (or $\tilde{\boldsymbol{\tau}}_1 = -\sqrt{2}$).

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- Massless spectrum $\exists$ a few conditions under which the additional massless states appear

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_B^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(O_{16}^{(0,0)} S_{16} + S_{16}^{(0,0)} O_{16} \right) \\
& + \bar{V}_8 \left(O_{16}^{(1/2,0)} S_{16} + S_{16}^{(1/2,0)} O_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(V_{16}^{(0,1/2)} V_{16} + C_{16}^{(0,1/2)} C_{16} \right) \\
& + \bar{O}_8 \left(V_{16}^{(1/2,1/2)} V_{16} + C_{16}^{(1/2,1/2)} C_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}
\end{aligned}
$$

condition ② $\tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2\mathbf{Z}+1$

new massless states： • two $8_S \otimes (1, 14)$ Gauge group is not enhanced

In the fundamental region, this condition is $\tilde{\tau}_1 = \sqrt{2}/2$ and $\tilde{\tau}_1 = -\sqrt{2}/2$.

There is no condition such that $n_F = n_B$ in this example.

Interpolation between $E_8 \times E_8$ and $SO(16) \times SO(16)$ with WL

- **Summary of the conditions**

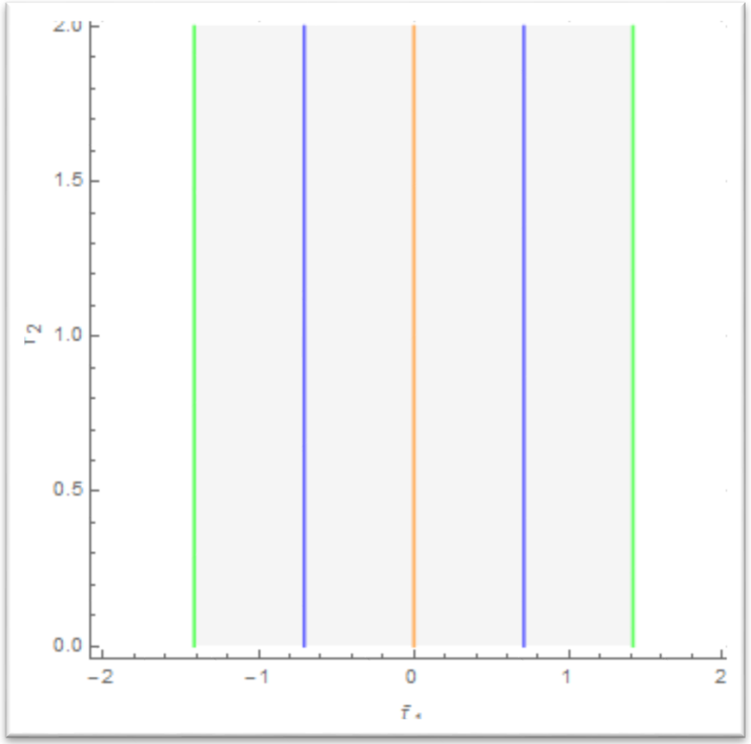
We have found the three conditions under which the additional massless states appear:

condition ①-1 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1/2 \in 2\boldsymbol{Z}$

condition ①-2 $\quad \tilde{\tau}_1 = n_1/\sqrt{2} \quad n_1/2 \in 2\boldsymbol{Z}+1$

condition ② $\quad \tilde{\tau}_1 = n_2/\sqrt{2} \quad n_2 \in 2\boldsymbol{Z}+1$

$$\left(\tilde{\tau}_1 = \frac{A}{\sqrt{1+A^2}} a^{-1}\right)$$



Actually, there are only four inequivalent orbits in the fundamental region:

Condition	$n_1 = 0$	$n_1 = 2$ (or -2)	$n_2 = 1$ and -1
Gauge gp	$SO(16) \times SO(16)$	$SO(16) \times E_8$	$SO(16) \times SO(14) \times U(1)$
	$n_F > n_B$	$n_F < n_B$	$n_F < n_B$

The leading terms of the cosmological constant

The cosmological constant is written as

$$\Lambda_{int}^{(9)}(a,R) = -\frac{1}{2}\left(4\pi^2\alpha'\right)^{-9/2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}Z_{int}^{(9)}(a,R;\tau)$$

Up to exponentially suppressed terms, the results are

- $SO(32)$ - $SO(16)\times SO(16)$ interpolation

$$\Lambda_{int}^{(9)}(a,R) \simeq 48\pi^{-14}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9\times 8\left\{(224-220)+2(16-14)\cos\left(\sqrt{2}\pi\tilde{\tau}_1\right)\right\}$$

- $E_8\times E_8$ - $SO(16)\times SO(16)$ interpolation

$$\Lambda_{int}^{(9)}(a,R) \simeq 48\pi^{-14}\left(\frac{a_0}{\sqrt{\alpha'}}\right)^9\times 8\left\{(2^7-220)-2\cdot 14\cos\left(\sqrt{2}\pi\tilde{\tau}_1\right)+2\cdot 2^6\cos\left(\frac{\sqrt{2}}{2}\pi\tilde{\tau}_1\right)\right\}$$

These results reflect the shift symmetry $\tilde{\tau}\to\tilde{\tau}+2\sqrt{2}$ and the conditions under which the additional massless states appear.

Outline

1. Introduction
2. Heterotic Strings
3. 9D Interpolating models
4. 9D Interpolating models with Wilson line
5. **Summary**

Conclusion

- We have constructed 9D interpolating models with two parameters, radius R and WL A, and studied the massless spectra.

- We have found some conditions for (R, A) under which the additional massless states appear.

- We have found that an example under which the cosmological const. is exponentially suppressed simultaneously with the gauge group enhancement to $SO(18) \times SO(14)$.

Outlook

- How are SM-like or GUT-like 4D models with $n_F = n_B$ constructed ?

- We can generalize by putting more WL and the other backgrounds. In fact, compactifying d-dimensions, the compactifications are classified by $\frac{SO(16+d,d)}{SO(16+d)\times SO(d)}$, whose DOF is $d(16+d)$.

- Are the conditions found in this work preferred ? Where are stable points in moduli space ?

Thank you!