



Kobayashi-Maskawa Institute  
for the Origin of Particles and the Universe

# Fast-rolling relaxion

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# Introduction

# Little Hierarchy

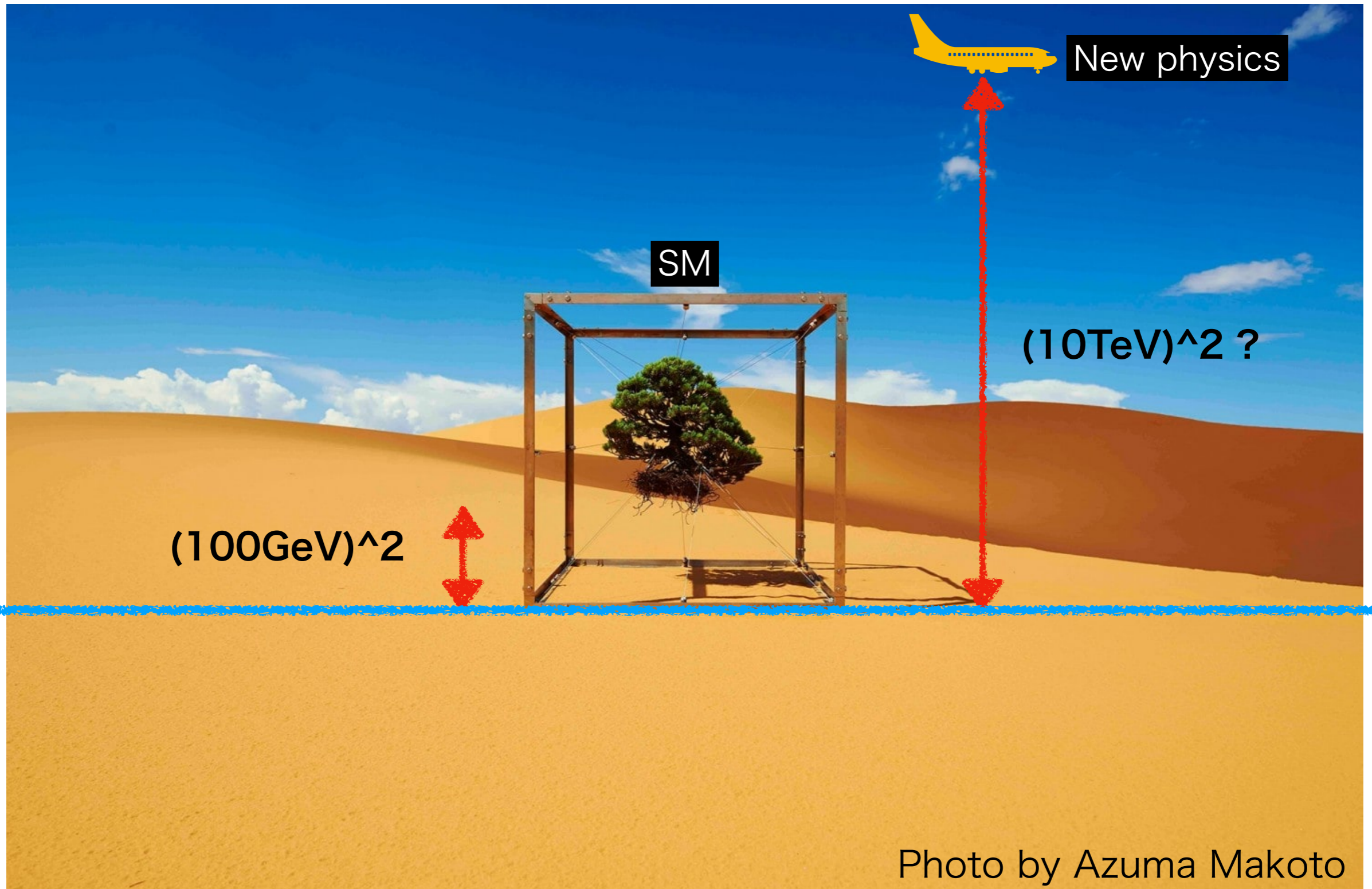
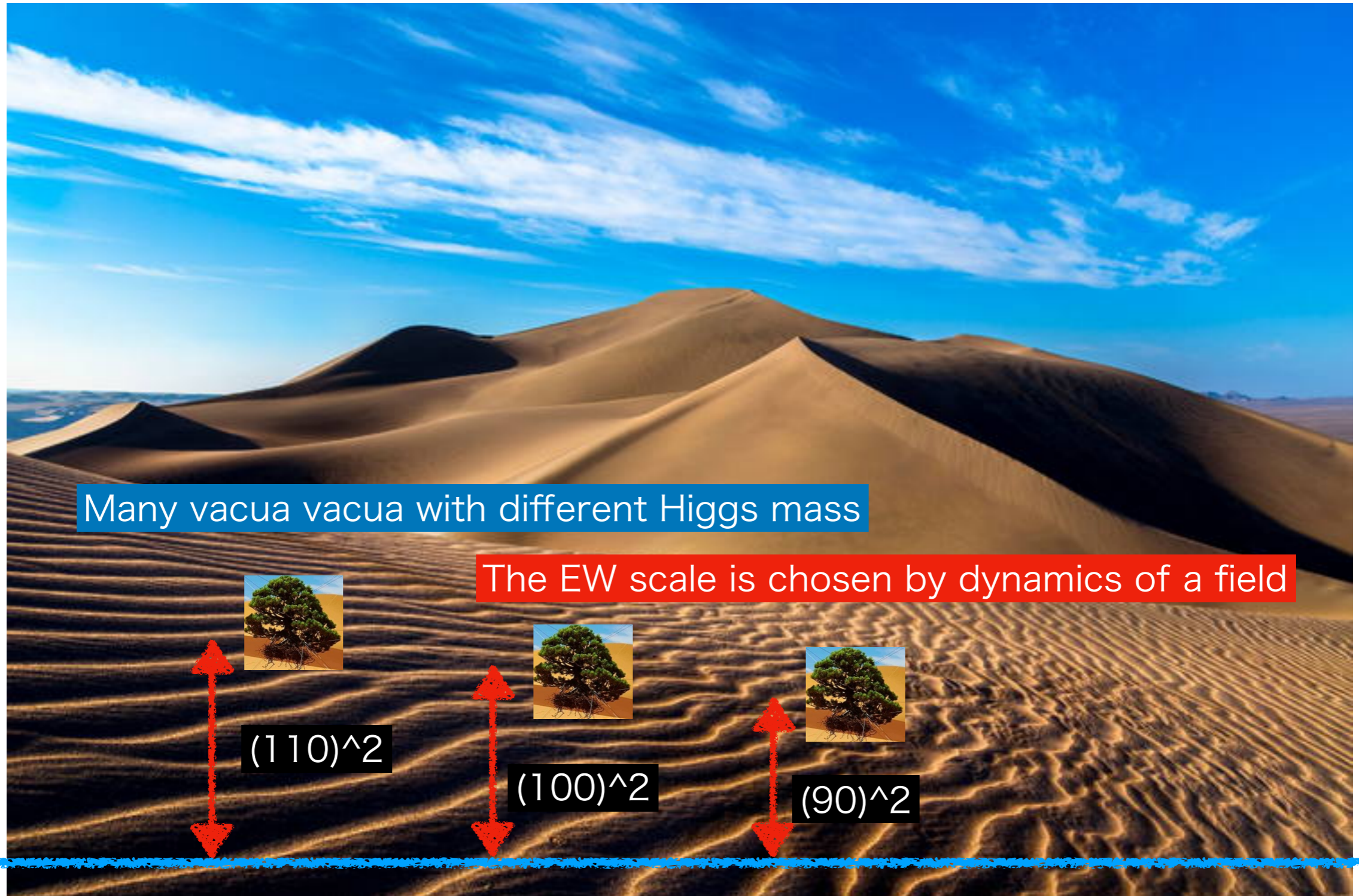


Photo by Azuma Makoto

# Dynamical solution



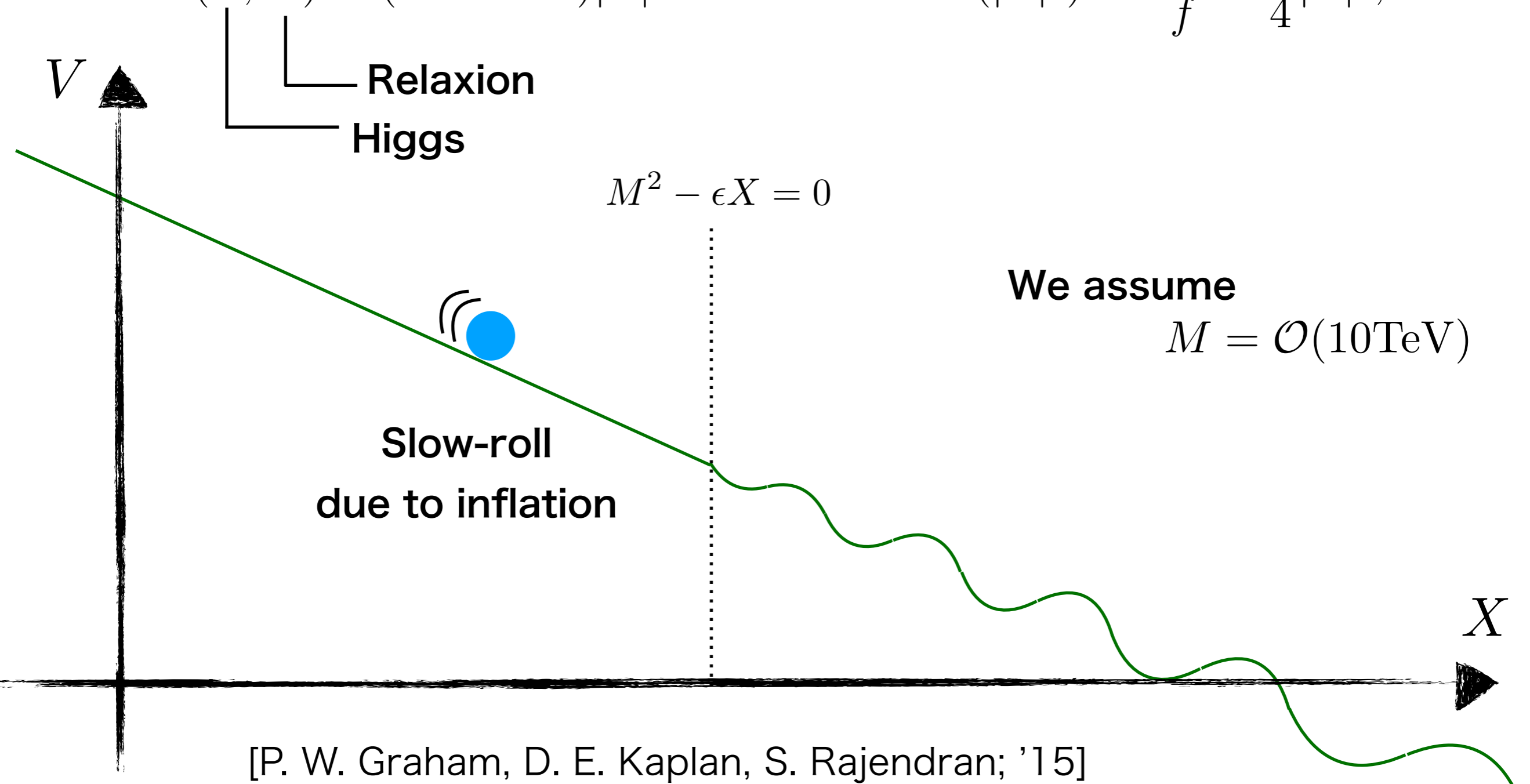
# Relaxion mechanism

Higgs mass

Slope

Back reaction

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2) \cos \frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$



Relaxion  
Higgs

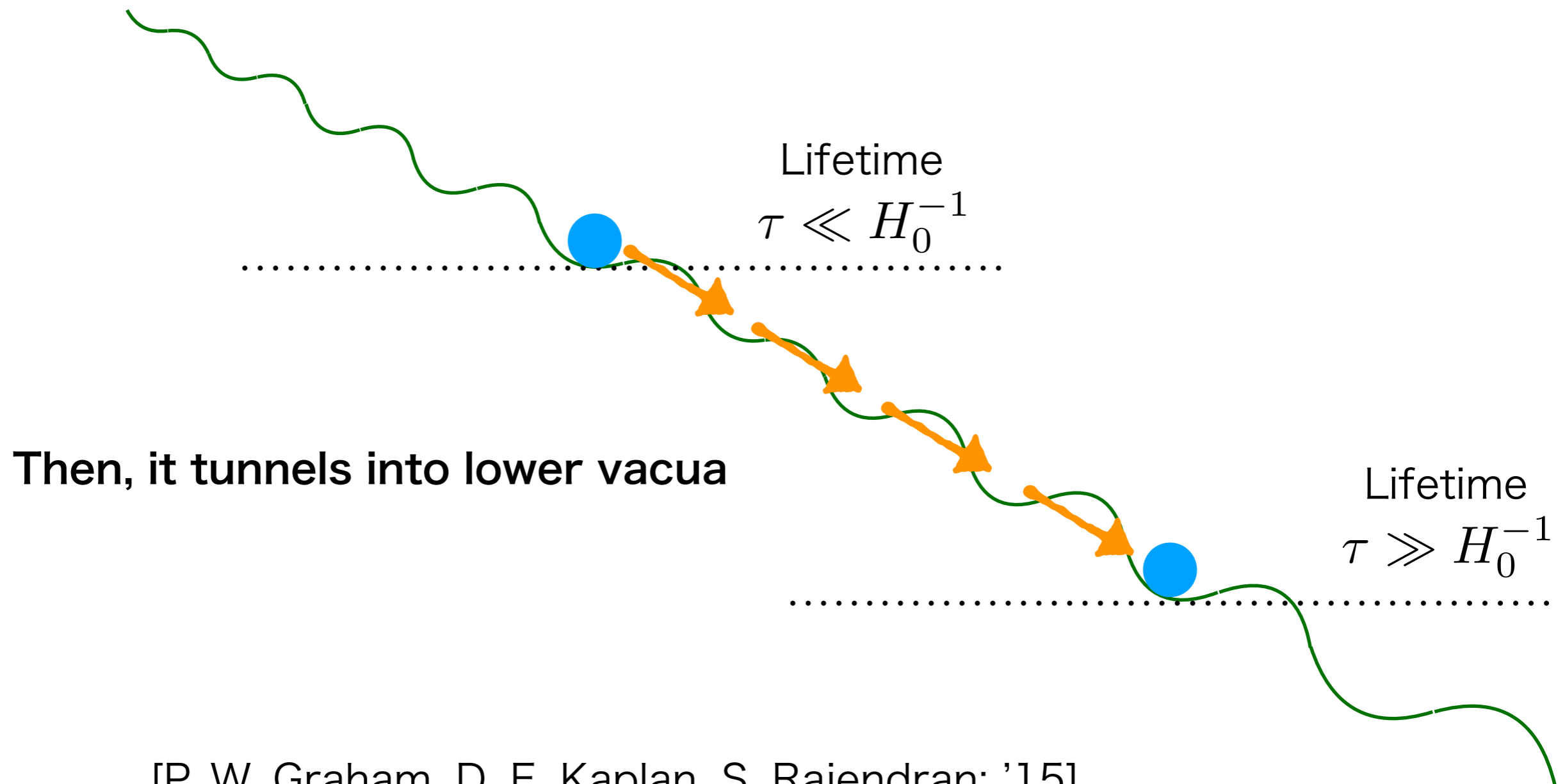
$$M^2 - \epsilon X = 0$$

We assume  
 $M = \mathcal{O}(10\text{TeV})$

Slow-roll  
due to inflation

# Quantum tunneling

The relaxation stops classically at  $\frac{dV}{dX} = 0$



# Timescale of tunneling

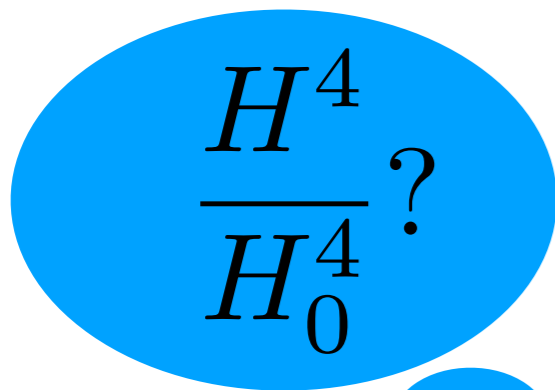
At the last tunneling,  $\gamma \ll H_0^4$

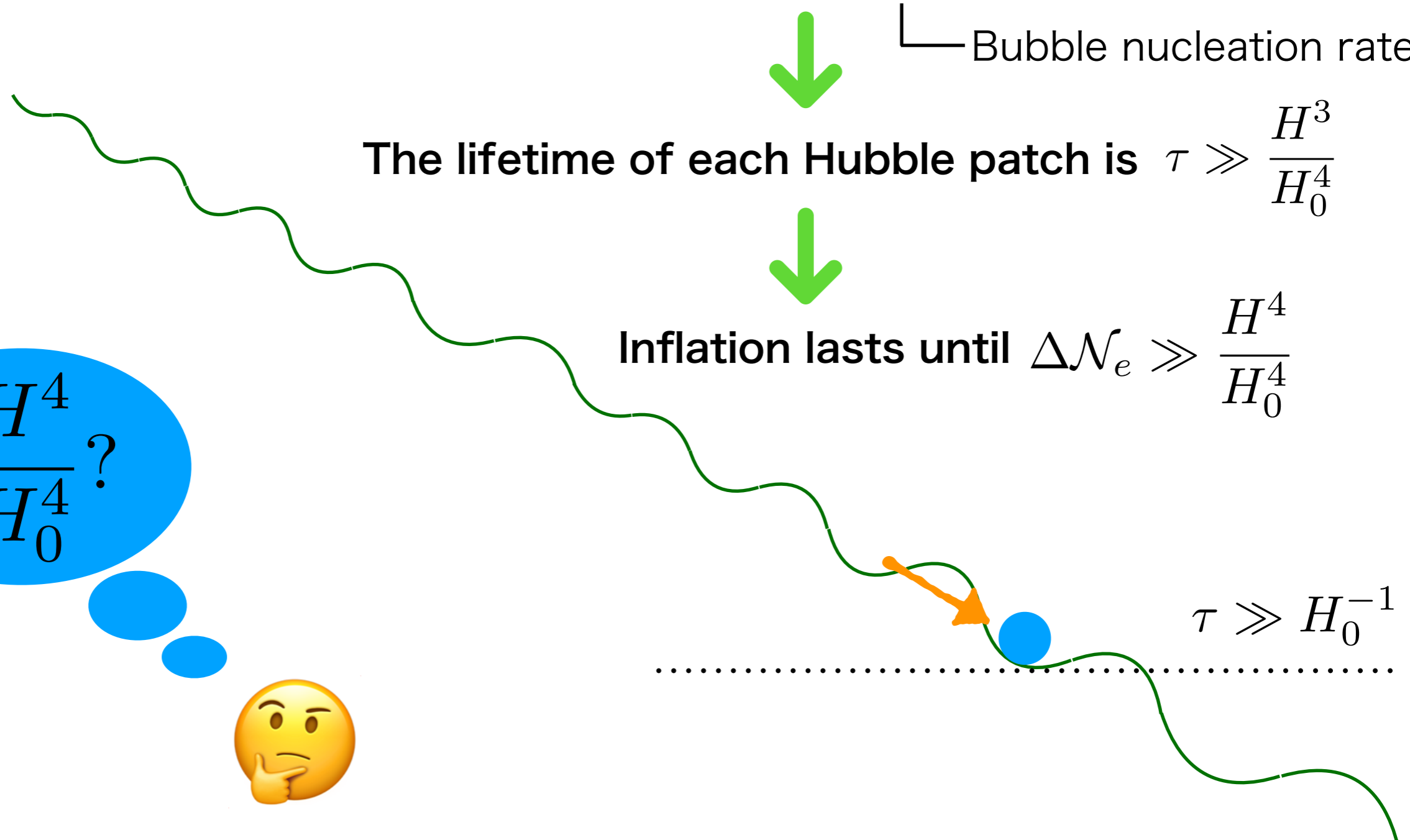
└ Bubble nucleation rate

The lifetime of each Hubble patch is  $\tau \gg \frac{H^3}{H_0^4}$

Inflation lasts until  $\Delta\mathcal{N}_e \gg \frac{H^4}{H_0^4}$

$\tau \gg H_0^{-1}$


$$\frac{H^4}{H_0^4}?$$



# Too large e-folds

$$\mathcal{N}_e \gg \frac{H^4}{H_0^4} \simeq 10^{156} \left( \frac{H}{1 \text{ MeV}} \right)^4$$

**It is argued that a too large number of e-folds will cause problems in inflation sector**

e.g.) [K. Choi, S. H. Im; '16]

To avoid fine-tuning, we need  $\mathcal{N}_e \lesssim 10^{24}$  for slow-roll inflation  
(in the context of scanning time)

[H. Matsui, F. Takahashi; '18, K. Dimopoulos '18, ...]

Eternal inflation is generically incompatible with the (refined) de Sitter swampland conjecture

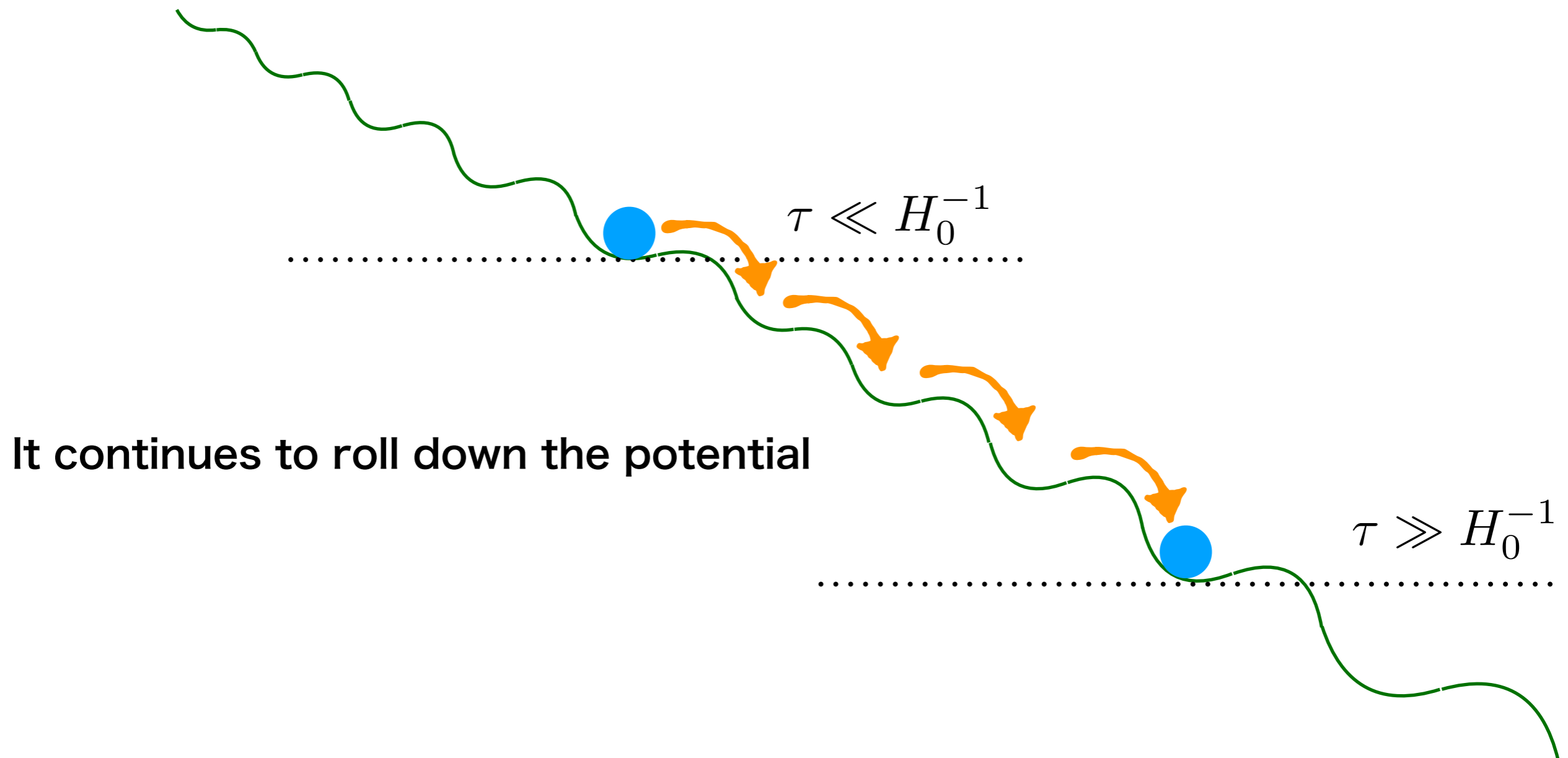
+ Why not multiverse?



Fast-rolling relaxation

# Fast-roll

The relaxion DOES NOT stop classically at  $\frac{dV}{dX} = 0$



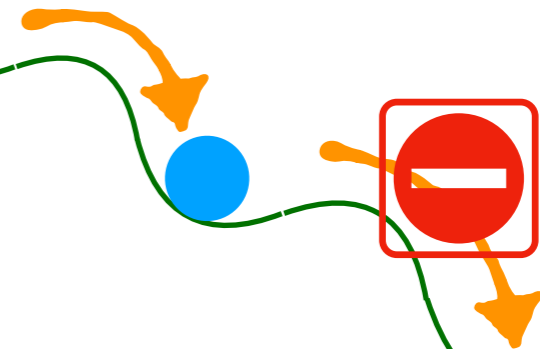
# Stopping mechanism

We need another stopping mechanism

ex) particle production with  $\frac{X}{f} F \tilde{F}$

[A. Hook, G. Marques-Tavares; '16, ...]

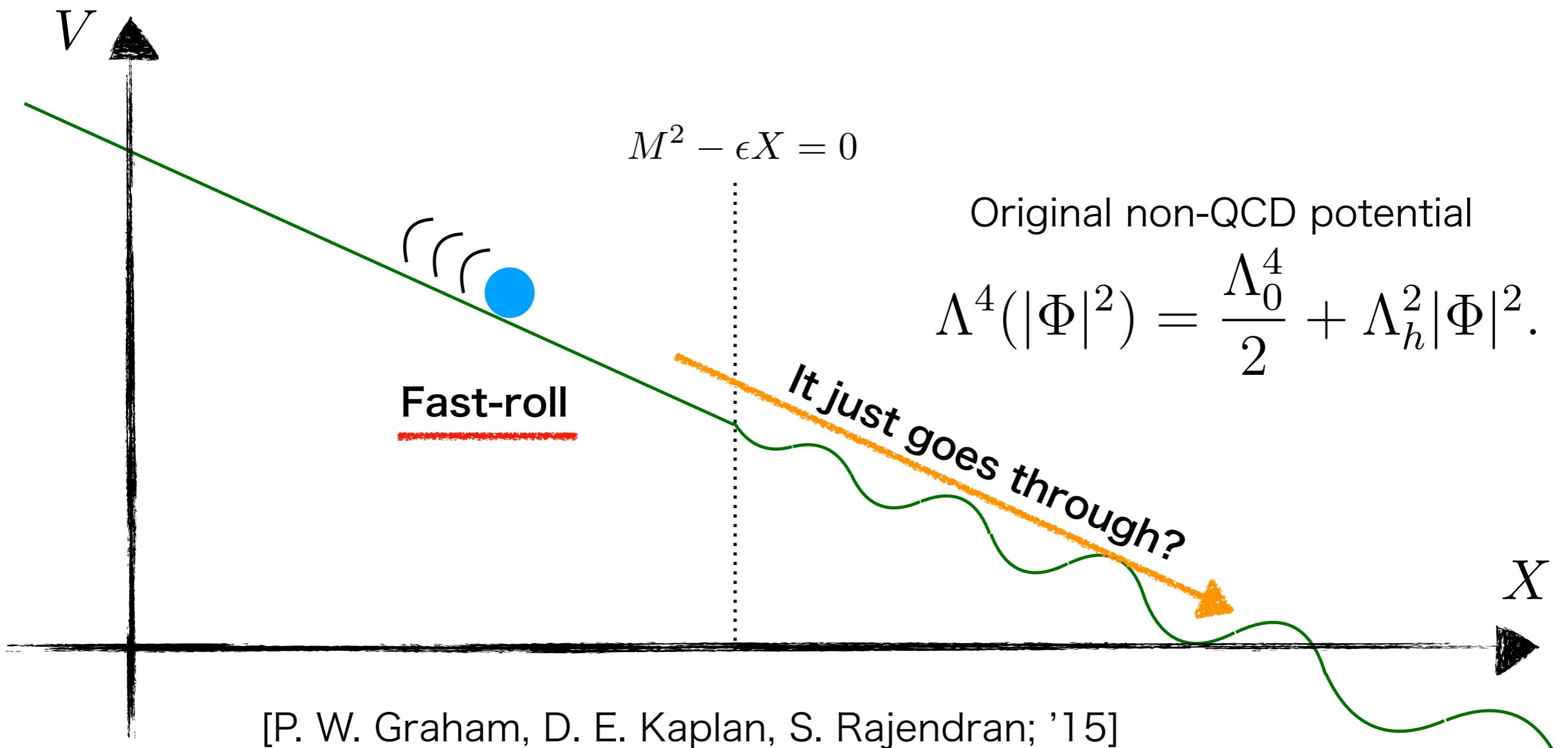
But, we want something



Model

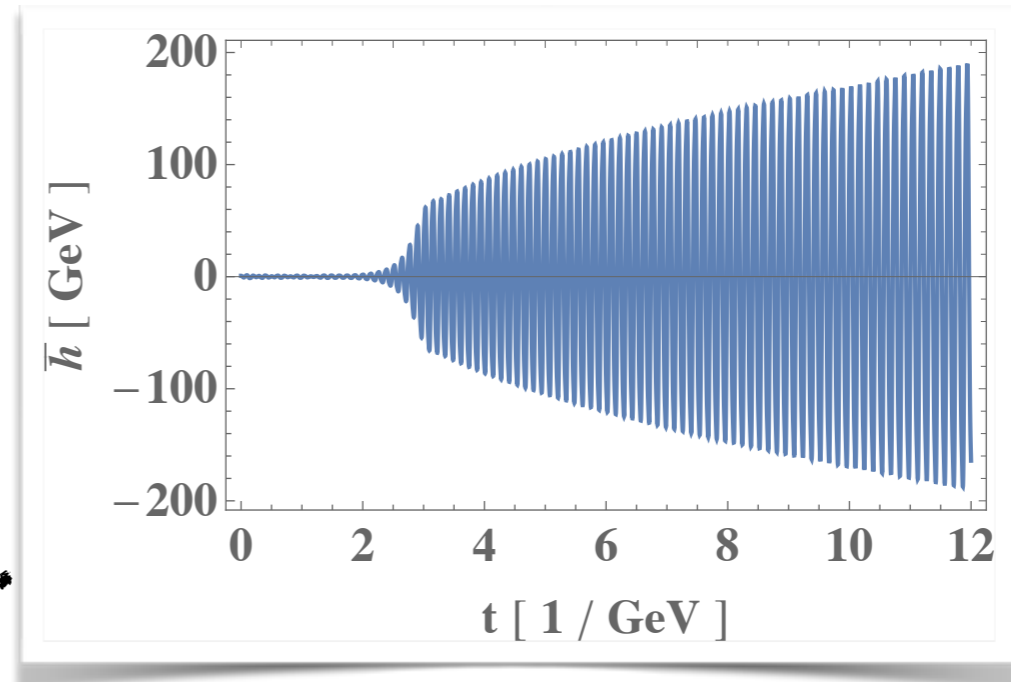
# No extension!

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2) \cos \frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

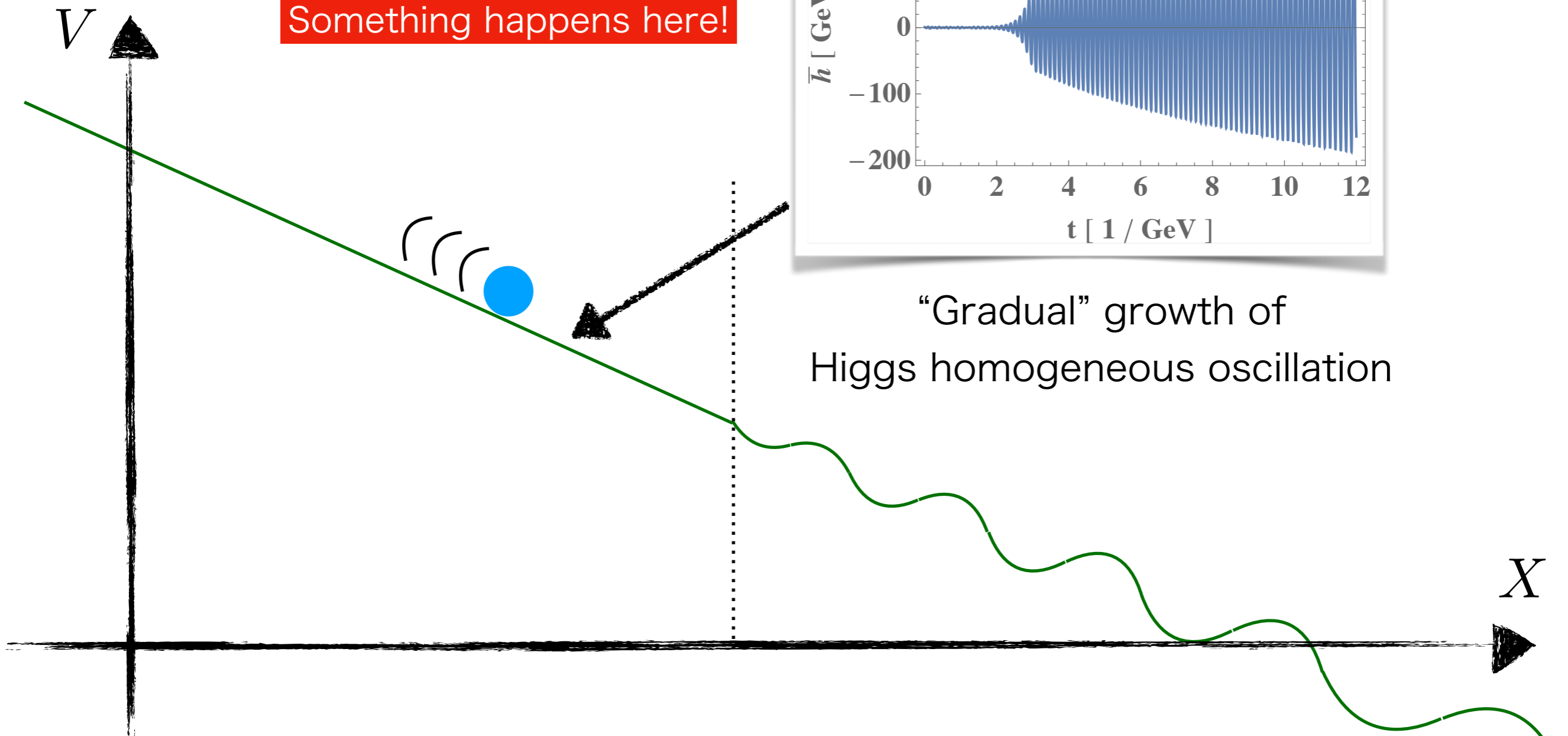


# Higgs homogeneous oscillation

Something happens here!



“Gradual” growth of Higgs homogeneous oscillation



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Edge solution



# Equations of motion

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon \left( rM^2 + \frac{\bar{h}^2}{2} \right) + \frac{\cancel{\Lambda_h^4} + \Lambda_h^2 \bar{h}^2}{2f} \sin \frac{\bar{X}}{f},$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon\bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 \bar{h} \cos \frac{\bar{X}}{f},$$

Terminal velocity

$$\dot{\bar{X}} \simeq \frac{\epsilon r M^2}{3H}$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos \omega t$$

$$\delta m^2 = f\omega = \frac{\epsilon r M^2}{3H}$$

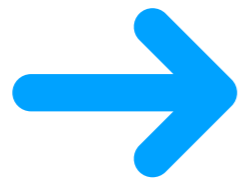
$$m^2 = M^2 - \epsilon\bar{X}(0)$$

$\bar{X}$  : Relaxion homogeneous mode

$\bar{h}$  : Higgs homogeneous mode

# Oscillatory solution

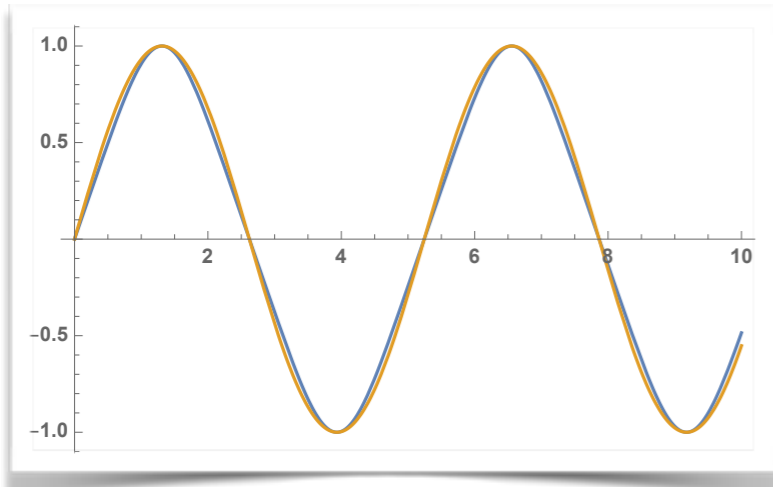
$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \cancel{\Lambda_h^2 \bar{h} \cos \omega t}$$



$$\bar{h}(t) = \mathcal{A} \operatorname{sn} \left( \sqrt{m^2 + \frac{\lambda}{8} \mathcal{A}^2} t, -\frac{\mathcal{A}^2 \lambda}{8m^2 + \mathcal{A}^2 \lambda} \right)$$

$$\simeq \mathcal{A} \sin(\bar{m}(\mathcal{A})t),$$

$\mathcal{A}$  : Amplitude of oscillation



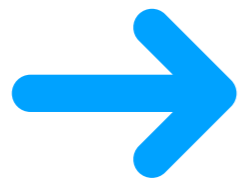
**Effective mass**

$$\bar{m}^2(\mathcal{A}) = \frac{\pi^2(8m^2 + \mathcal{A}^2 \lambda)}{32 \left[ K \left( -\frac{\mathcal{A}^2 \lambda}{8m^2 + \mathcal{A}^2 \lambda} \right) \right]^2} \simeq m^2 + \frac{3}{16} \lambda \mathcal{A}^2$$

# Parametric resonance

$$\ddot{h} + \cancel{3H\dot{h}} \simeq - \underbrace{(m^2 - \cancel{\delta m^2 t})}_{\bar{m}^2(\mathcal{A})} \bar{h} - \frac{\lambda}{4} \bar{h}^3 - \Lambda_h^2 h \cos \omega t$$

Mathieu equation



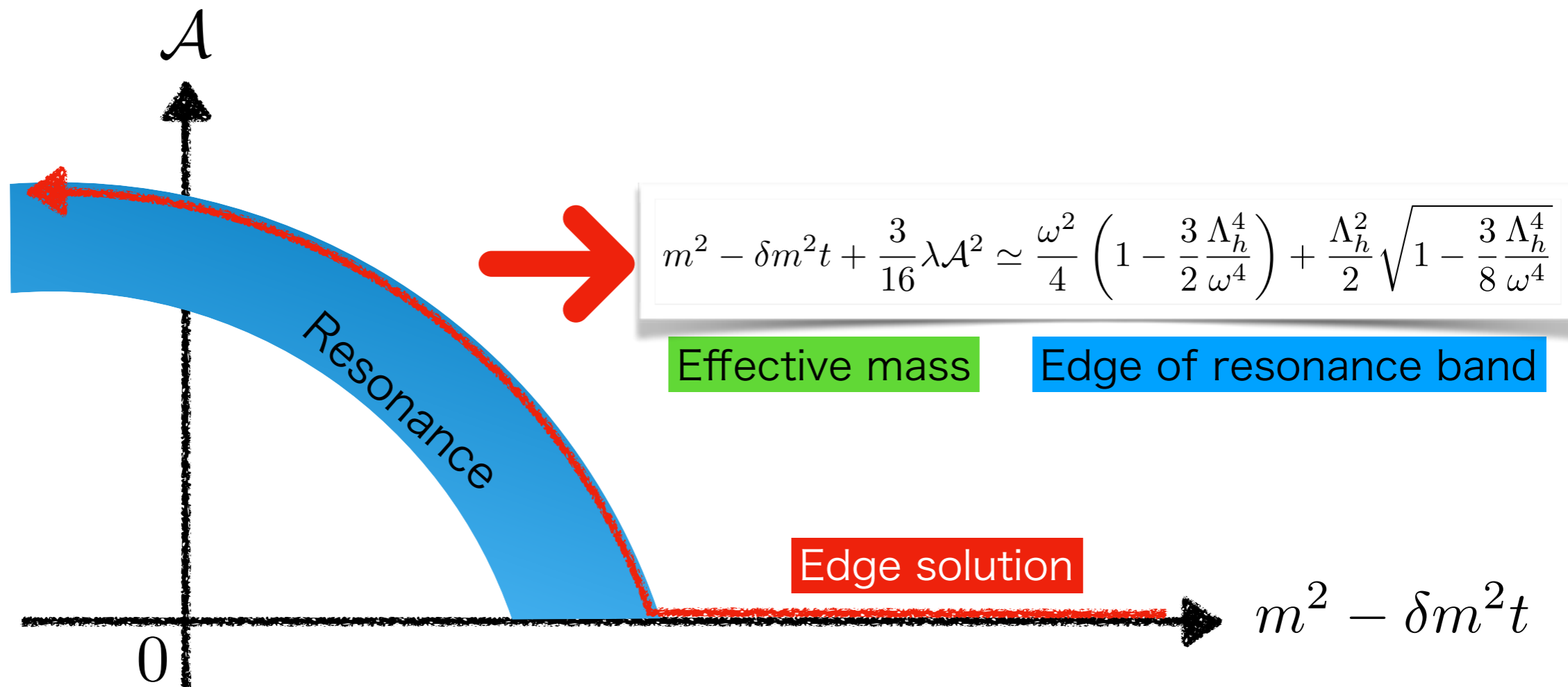
$$\ddot{h} \simeq -\bar{m}^2(\mathcal{A}) \bar{h} - \Lambda_h^2 h \cos \omega t$$

Exponential growth of the amplitude occurs when

$$\frac{\omega^2}{4} \left( 1 - \frac{3}{2} \frac{\Lambda_h^4}{\omega^4} \right) - \frac{\Lambda_h^2}{2} \sqrt{1 - \frac{3}{8} \frac{\Lambda_h^4}{\omega^4}} \lesssim \bar{m}^2(\mathcal{A}) \lesssim \frac{\omega^2}{4} \left( 1 - \frac{3}{2} \frac{\Lambda_h^4}{\omega^4} \right) + \frac{\Lambda_h^2}{2} \sqrt{1 - \frac{3}{8} \frac{\Lambda_h^4}{\omega^4}}$$

# Edge solution

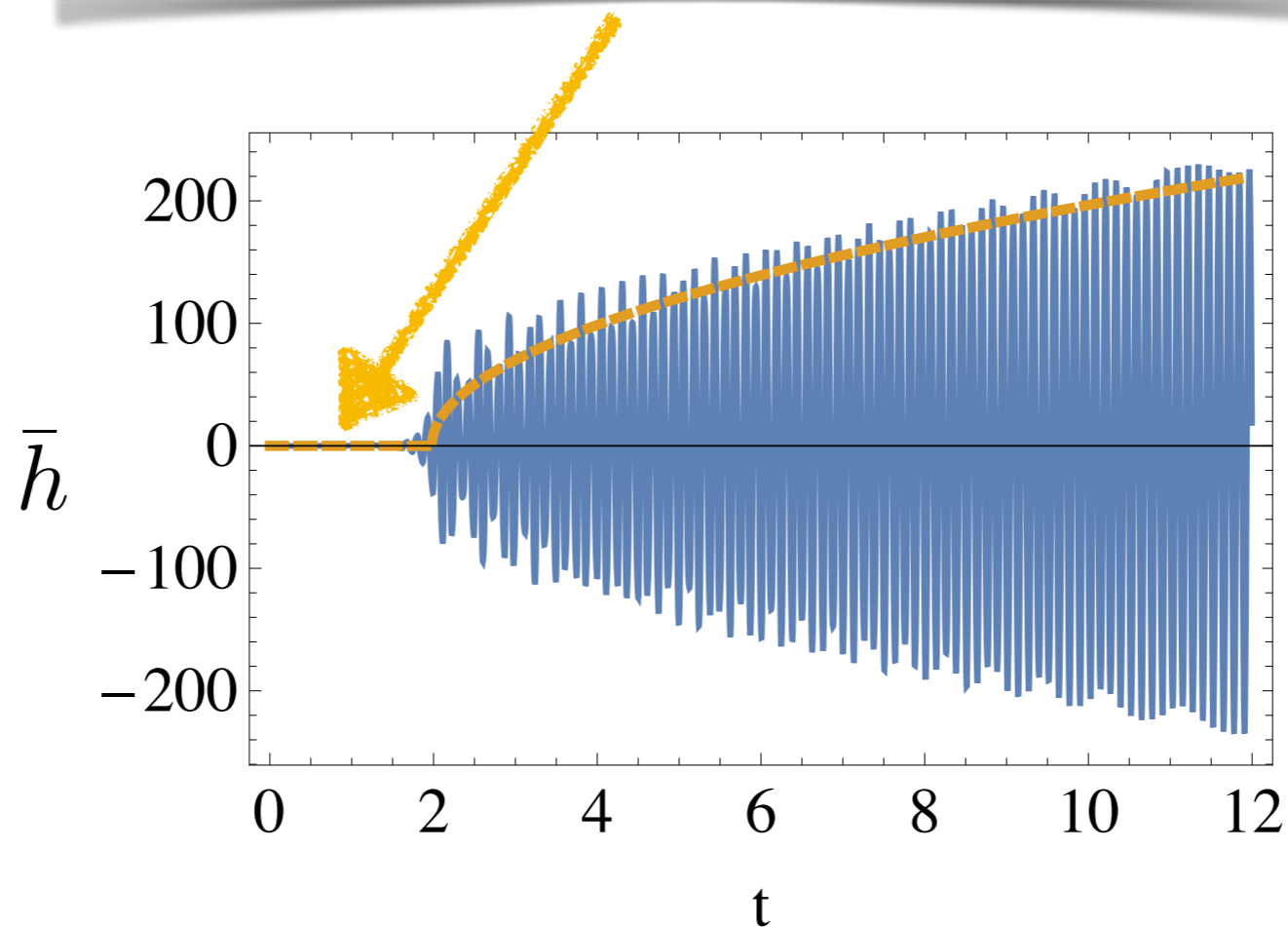
$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \underline{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 \bar{h} \cos \omega t$$



# Numerical check

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 \bar{h} \cos \omega t$$

$$m^2 - \delta m^2 t + \frac{3}{16}\lambda\mathcal{A}^2 \simeq \frac{\omega^2}{4} \left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2} \sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}$$

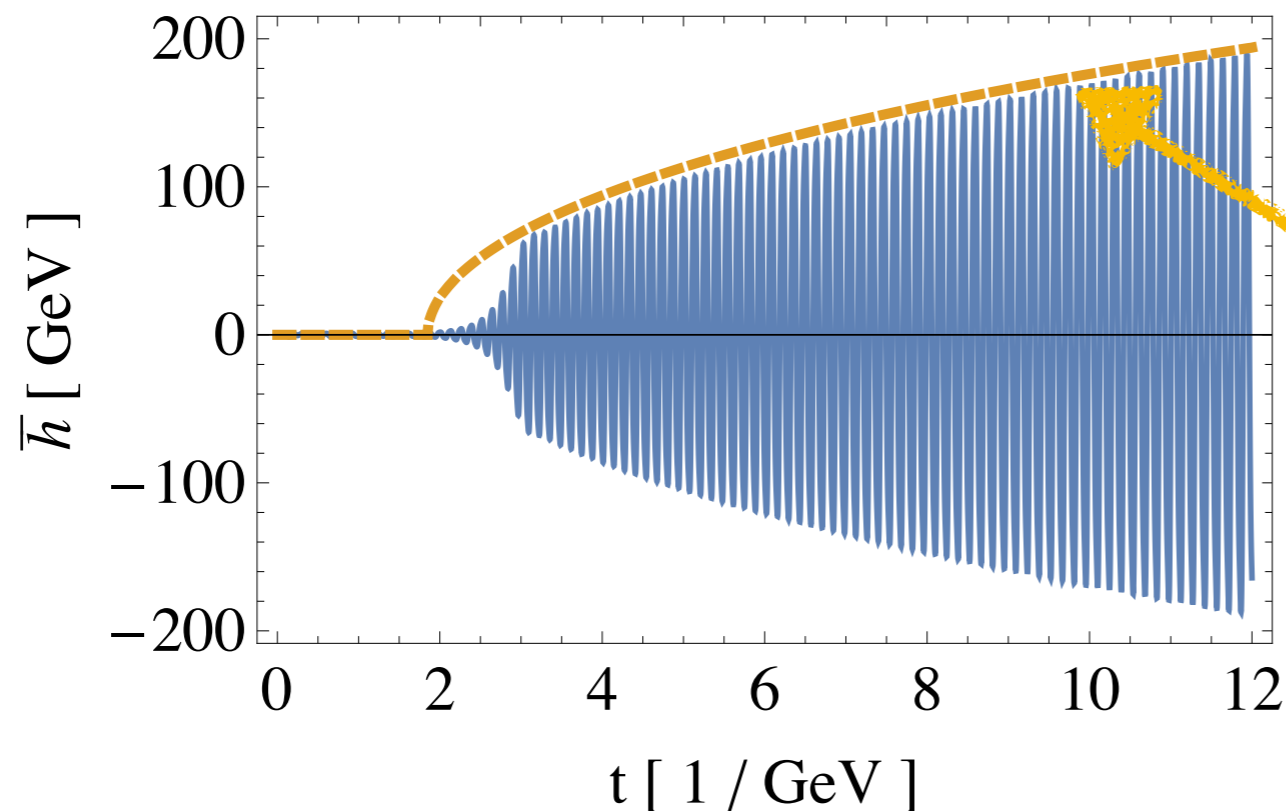


# Edge solution (full)

Hubble friction, Back reactions, ...

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon \left( rM^2 + \frac{\bar{h}^2}{2} \right) + \frac{\Lambda_0^4 + \Lambda_h^2 \bar{h}^2}{2f} \sin \frac{\bar{X}}{f} ,$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon \bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 \bar{h} \cos \frac{\bar{X}}{f} ,$$



**Analytic expression**

$$m_\Phi^2 + \frac{3\lambda}{16}\mathcal{A}_h^2 \simeq \frac{\omega_X^2}{4} + \frac{\Lambda_h^2}{2} - \frac{\Lambda_h^2(32\Lambda_0^4 + 17\Lambda_h^2\mathcal{A}_h^2)}{128f^2\omega_X^2} - \frac{(8\Lambda_h^2 - 3\lambda\mathcal{A}_h^2)(8\Lambda_h^2 - \lambda\mathcal{A}_h^2)}{512\omega_X^2} - \frac{9\omega_X^2 H^2}{4\Lambda_h^2} ,$$

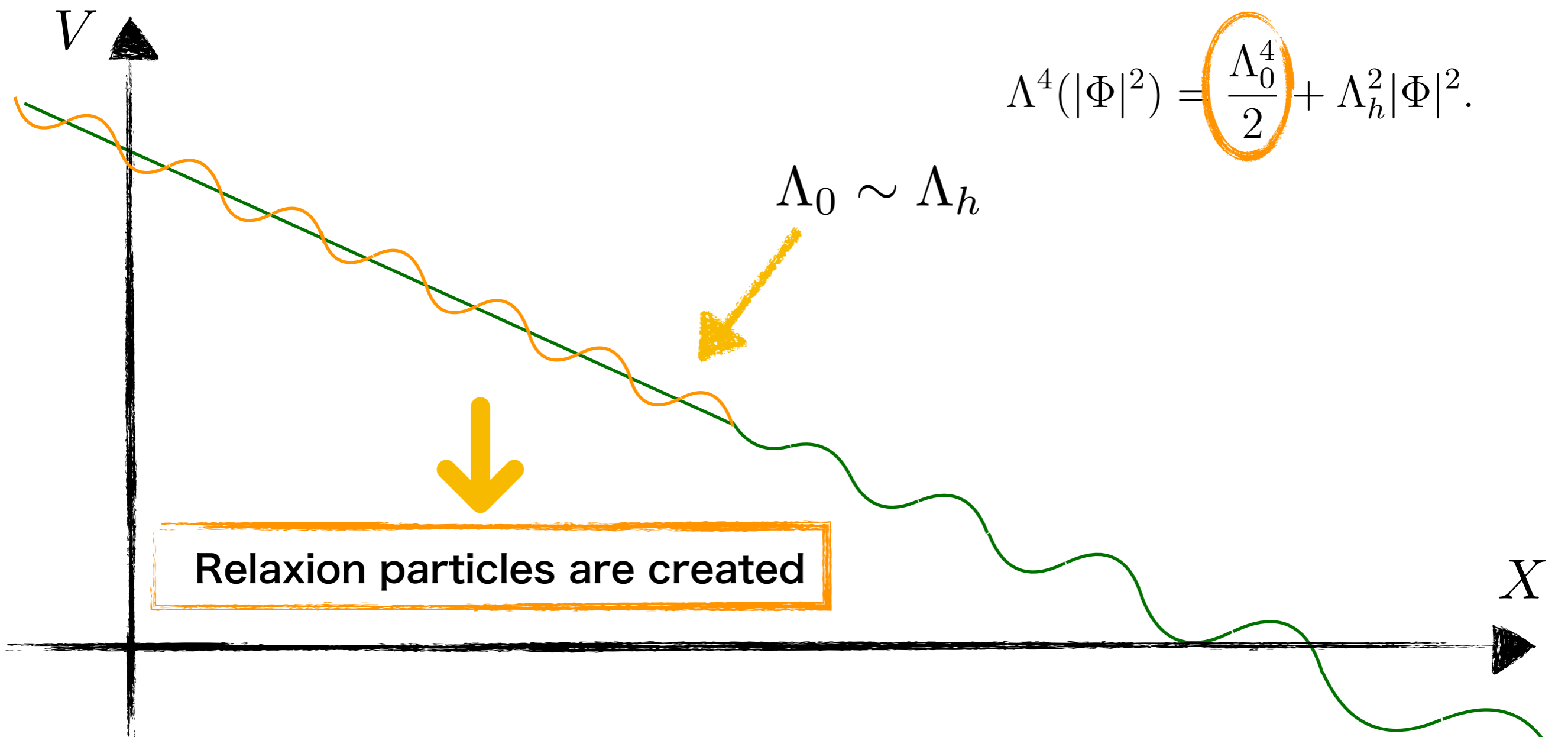
$$\omega_X \simeq \frac{r\epsilon M^2}{3Hf} - \frac{\mathcal{A}_h^2\omega_X}{8f^2} - \frac{\lambda\mathcal{A}_h^4}{128f^2\omega_X} + \frac{3\mathcal{A}_h^2\omega_X^3}{16f^2\Lambda_h^4} H^2 ,$$

# Inhomogeneous modes

# Particle production

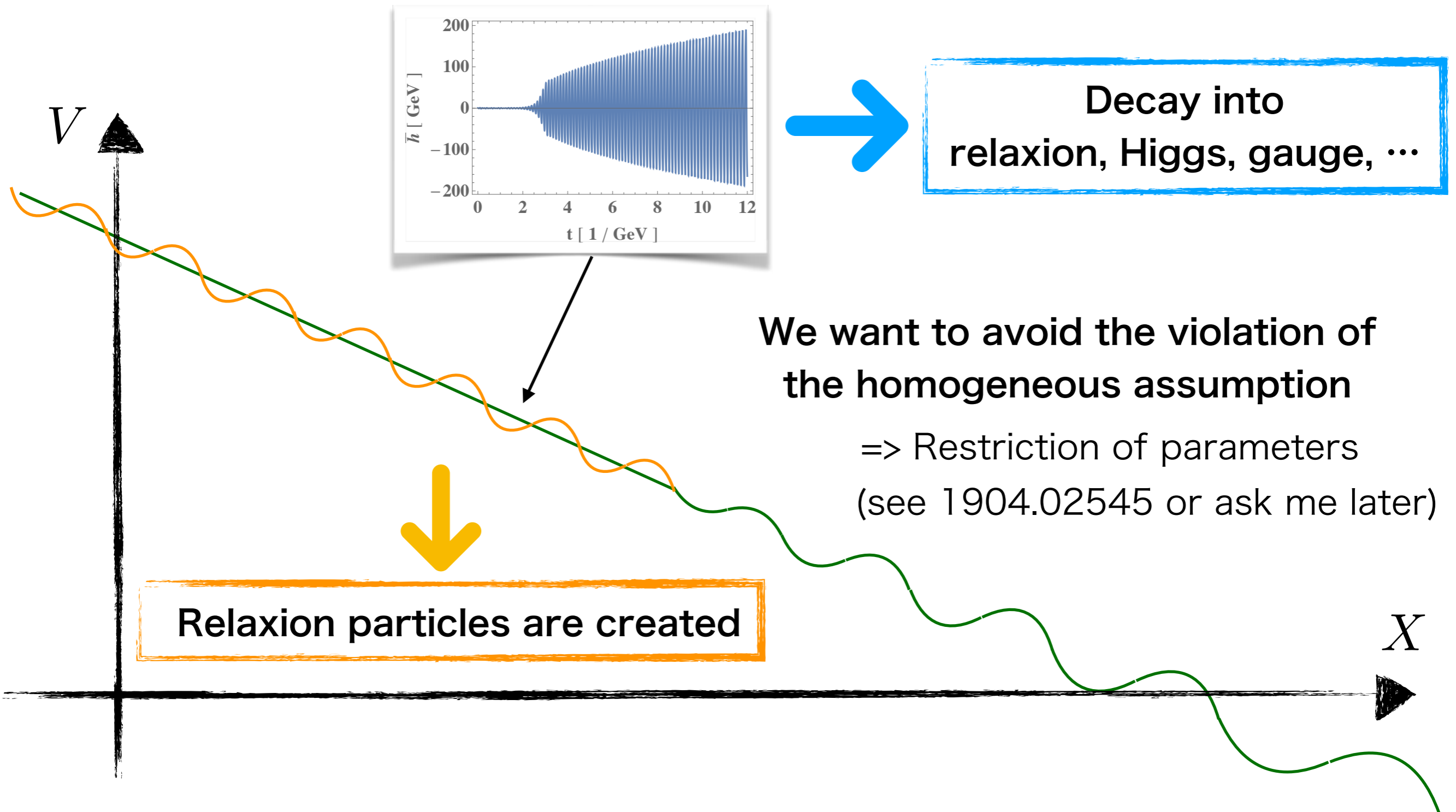
$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2) \cos \frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2 |\Phi|^2.$$



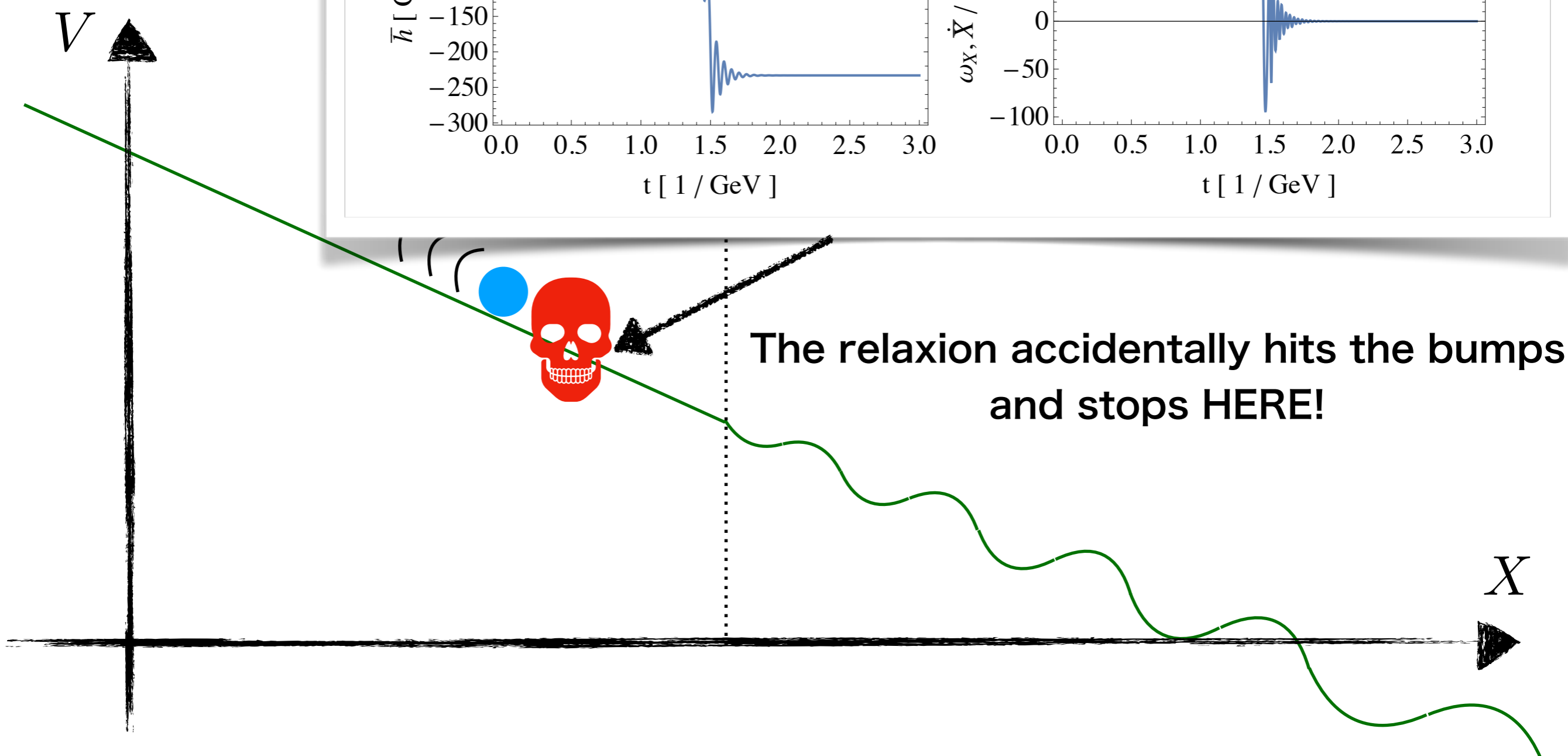
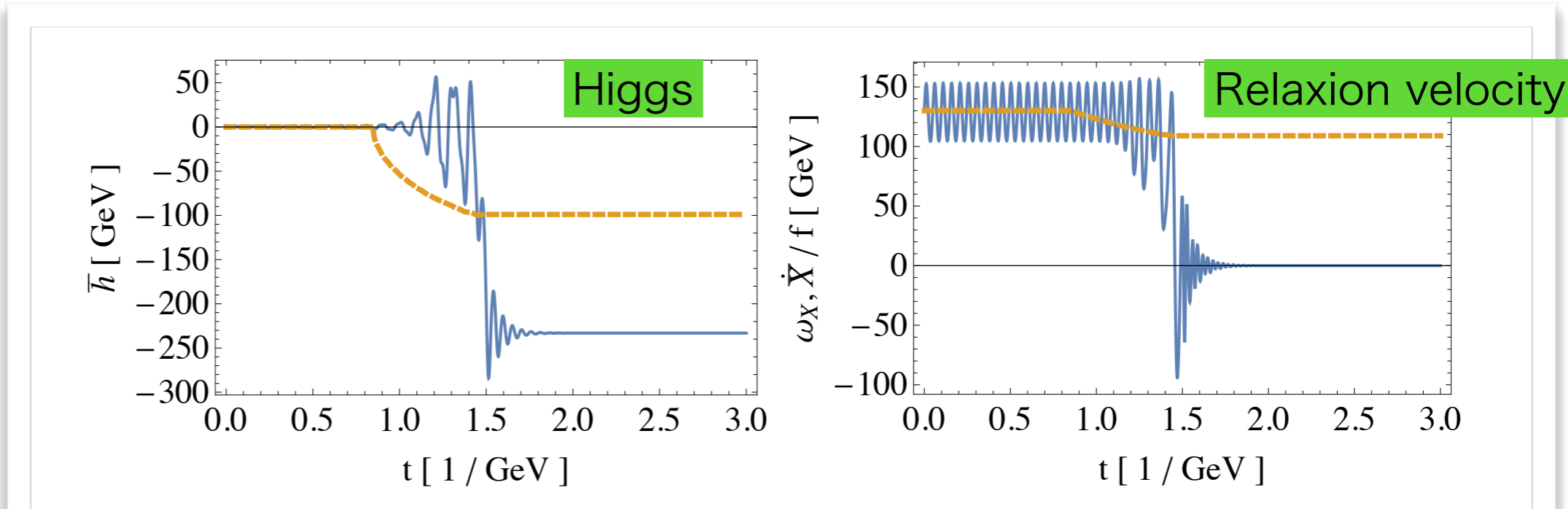


# Particle production



# Stopping mechanism

# Stopping mechanism



The relaxion accidentally hits the bumps and stops HERE!

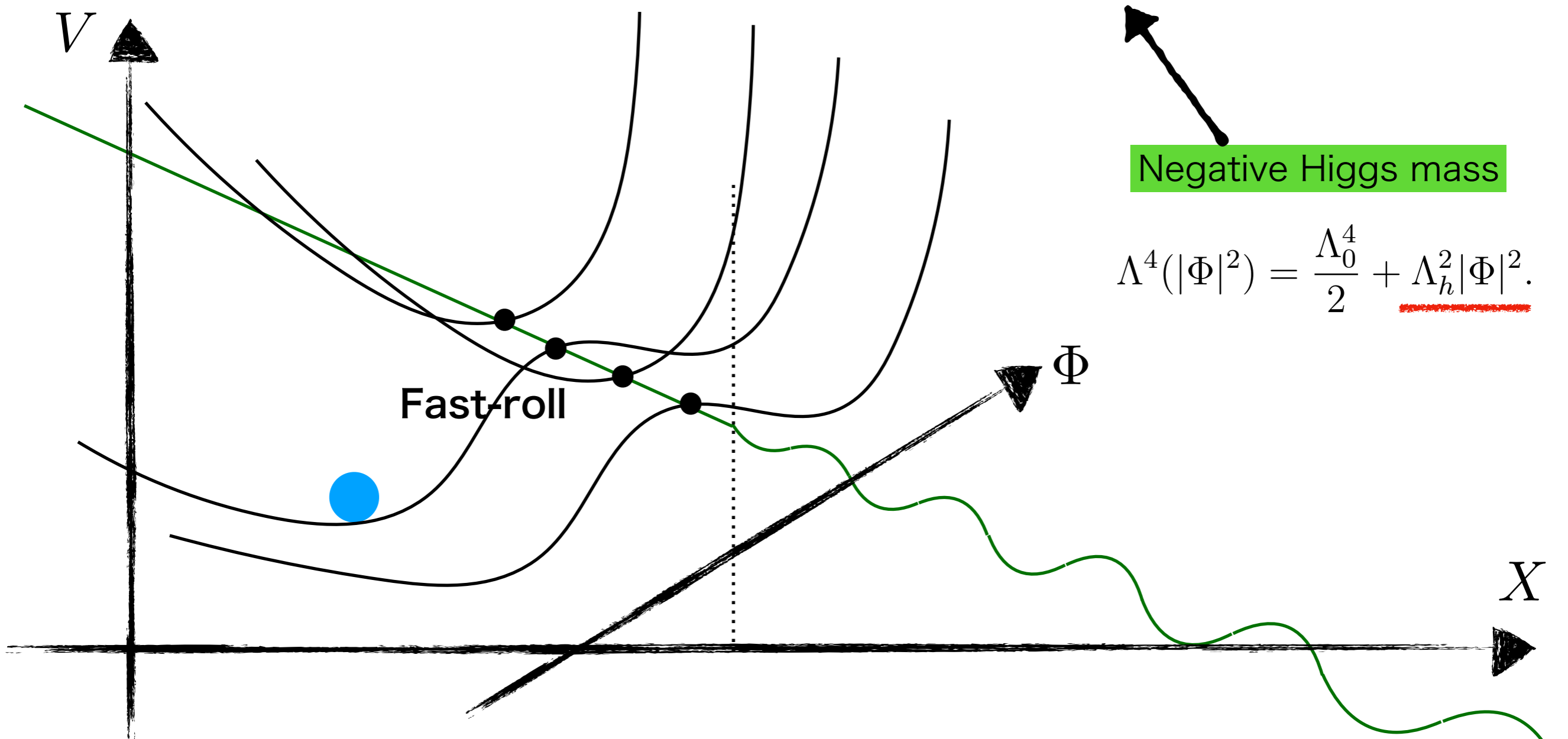
# Negative Higgs mass

Positive but small

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2) \cos \frac{X}{f} + \frac{\lambda}{4} |\Phi|^4,$$

Negative Higgs mass

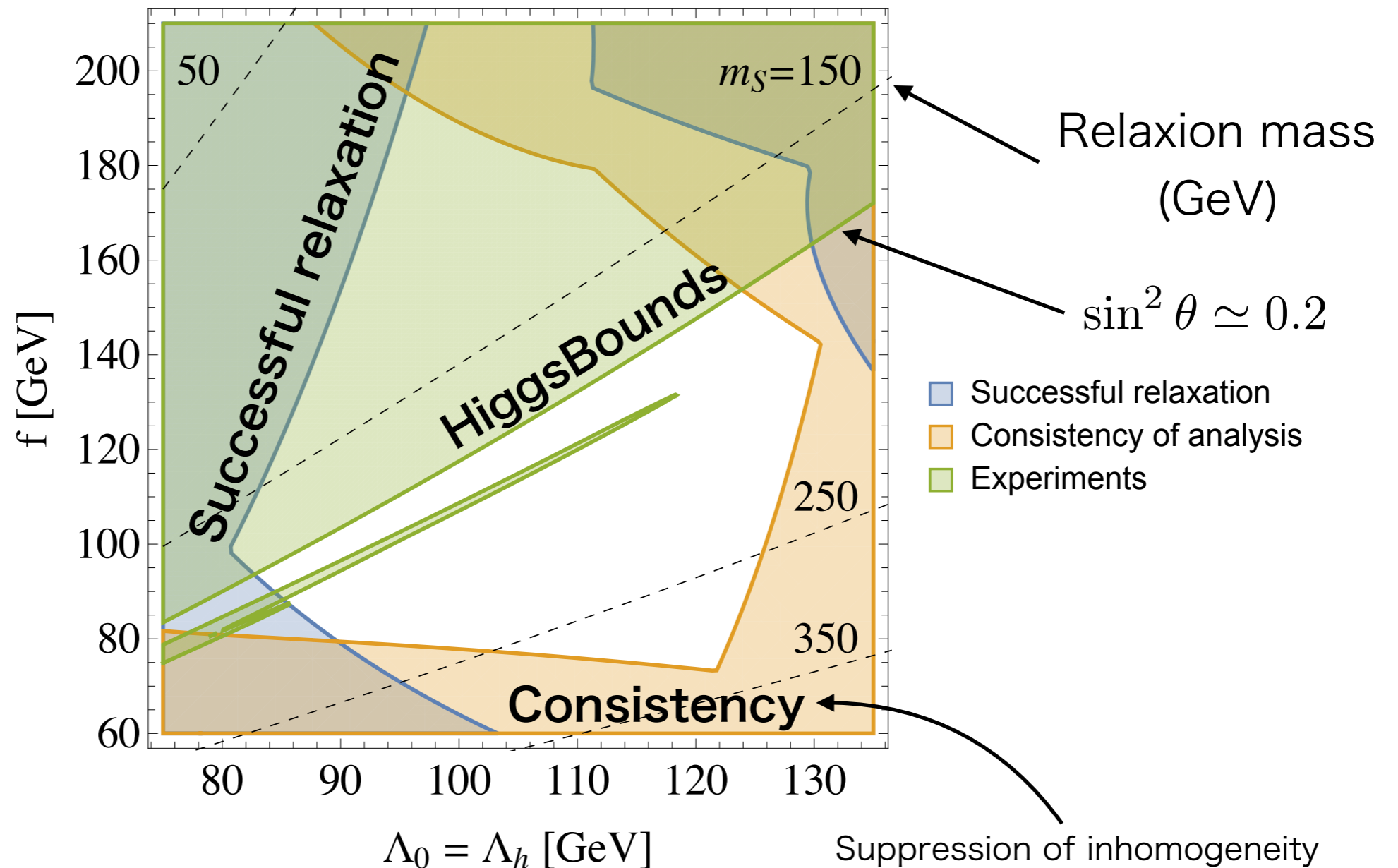
$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2 |\Phi|^2.$$



Parameter space

# Parameter space

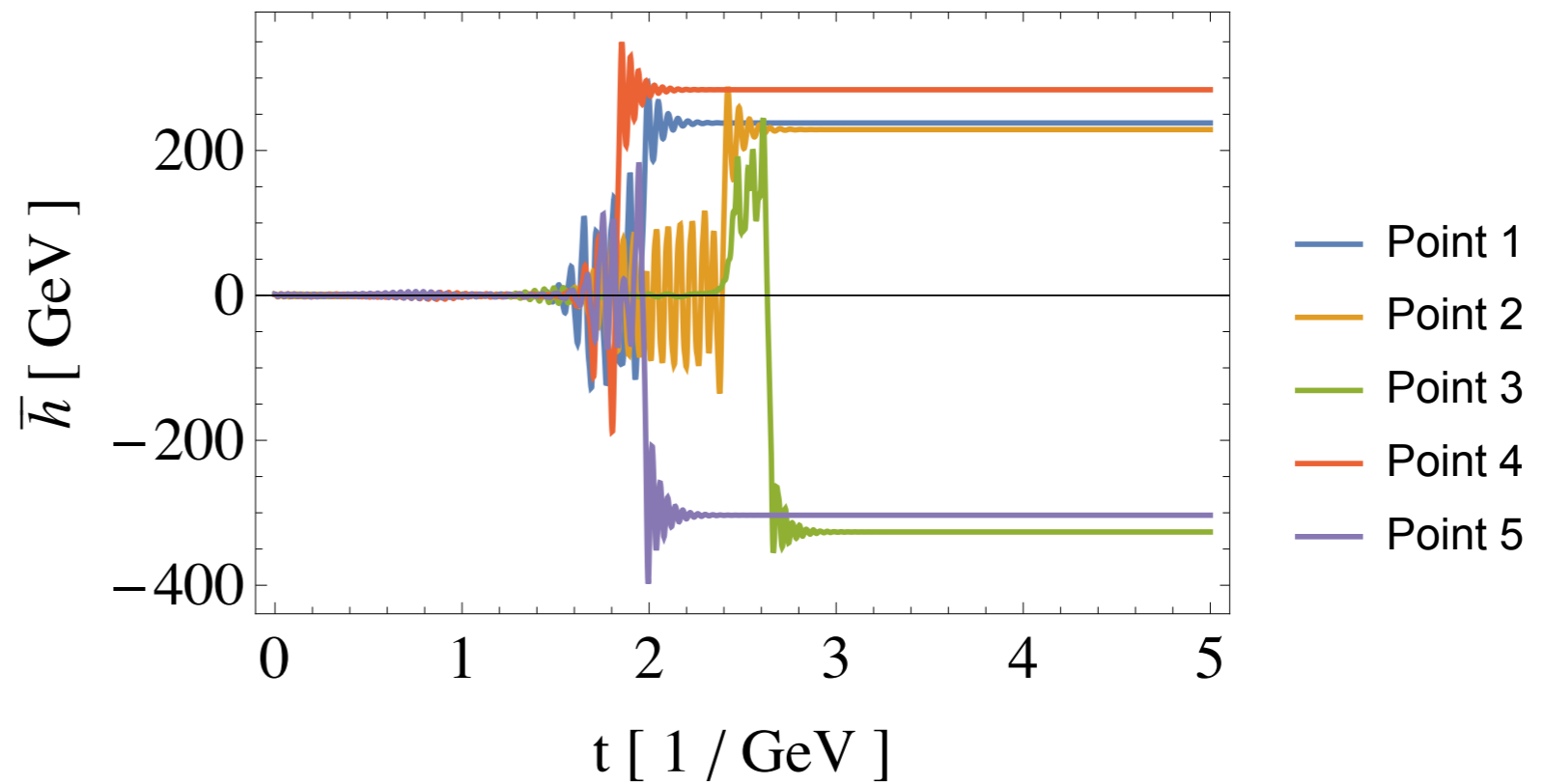
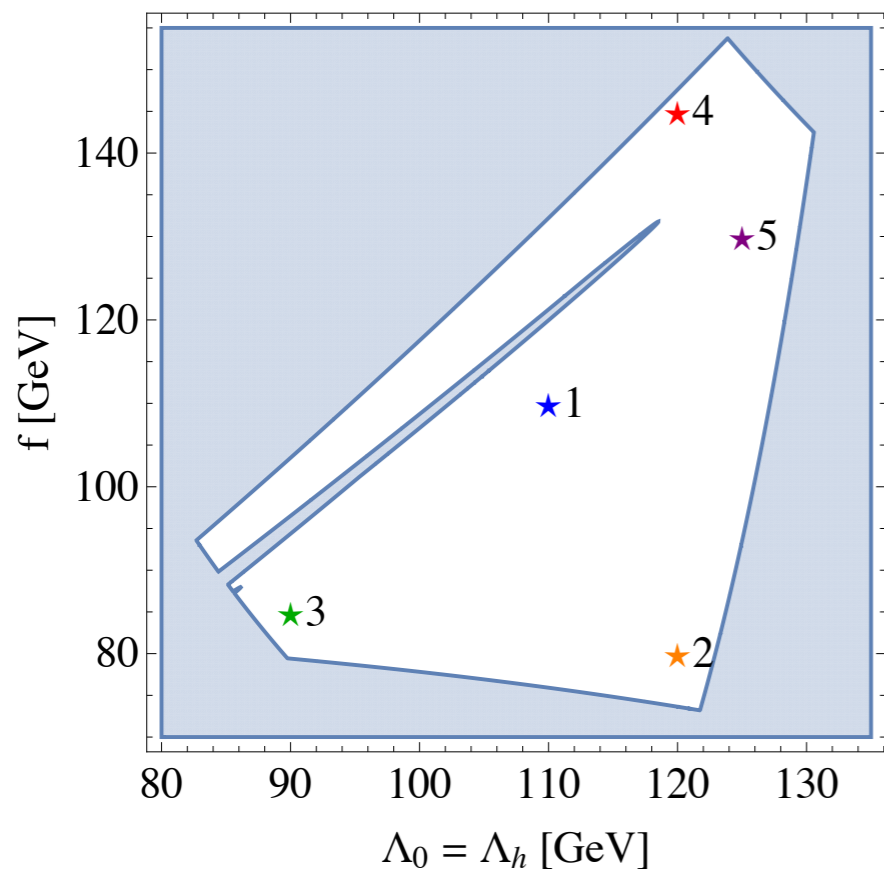
$$H = 10 \text{ GeV}, \quad \epsilon = 0.8 \text{ GeV}, \quad r = 0.002, \quad M = 20 \text{ TeV}.$$



The lifetime of each vacuum is much longer than the age of the universe

# It stops!

$H = 10 \text{ GeV}$ ,  $\epsilon = 0.8 \text{ GeV}$ ,  $r = 0.002$ ,  $M = 20 \text{ TeV}$ .



# Summary

- In the original relaxion model, the tunneling phase requires an unacceptably large number of e-folds.
- The fast-roll relaxion can easily solve this problem, but we can not use the original stopping mechanism.
- We propose a mechanism to stop the relaxion without extending the original model.
- The mechanism predicts a relaxion that has a mass of  $O(100)$  GeV and mixes with the Higgs boson. It improves the testability of our mechanism.